Neural geometrodynamics: a psychedelic perspective

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Abstract

We explore the mathematical framework for the interaction of neural and connectivity dynamics of neural models in the light of distinct timescales and in the context of psychedelics. The shortest timescales characterize the system's dynamics as determined by (very fast) ephaptic and (fast) synaptic coupling. Trajectories within the resulting dynamical landscape are shaped by the structure of a differential equation and its connectivity parameters. However, as in terrestrial landscapes, connectivity evolves over slower timescales reflecting state-dependent and state-independent plasticity mechanisms. These two forms of plasticity are described by an equation governing the dynamical adaptation of connectivity. Finally, we introduce an ultra-slow time scale relevant to the dynamics of plasticity processes (metaplasticity) and discuss it in the context of psychedelics. Drawing inspiration from the equations of general relativity, which define the interlocked dynamics of spacetime geometry and matter/energy, we employ the term *neural geometrodynamics* to describe the coupled dynamics of neuron population and connectivity state. Just as the spacetime metric evolves, so does the geometry of the neural phase space, shaped by the complex interplay between state-dependent and state-independent plasticity mechanisms. Inspired by general relativity, this work aims to bring a novel perspective to the understanding of neural system dynamics across various timescales.

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1 Introduction

> Spacetime tells matter how to move;
> matter tells spacetime how to curve.
>
> *John Archibald Wheeler*, in
> *Gravitation* (1973)

In this paper, we explore new perspectives to interpret changes in the brain's landscape and connectivity, focusing on the subtle interplay between structural and dynamical aspects across timescales (fast, slow and ultraslow). Our primary goal is to present a framework that enhances the understanding of the intricate relationships among brain dynamics, structure, and plasticity. This framework, which we call "neural geometrodynamics", draws on principles from non-linear dynamics and is further inspired by conceptual links to general relativity in physics.

In describing neural dynamics, we will employ the mathematical formalism of neural mass models (NMMs), although other computational neuroscience formulations are equally relevant [1, 2]. Neural mass models have been extensively utilized to model various brain activities, from localized brain functions to the coordinated activity observed in different brain regions. By employing mathematical formulations that include essential features like synaptic connectivity and neuronal excitability, NMMs enable the simulation and analysis of complex brain activities in various dynamic regimes [3]. NMMs are particularly useful because they provide a link between the mesoscopic physiological scale and macroscopic brain function, allowing for the connection of effects on neurons at the molecular level, such as those of psychedelics, with those of whole-brain connectivity [4, 5].

Psychedelics serve as an illustrative case of the framework, given the immediate and potentially lasting plastic changes these substances can provoke in the brain [6]. By altering neural dynamics and connectivity, psychedelics are thought to induce both transient and sustained shifts in cognition and perception [7]. Several studies underscore the role of psychedelics in inducing neuroplasticity with antidepressant effects, revealing mechanisms at molecular, synaptic, and dendritic levels [8, 9], and with significant potential for treating neuropsychiatric disorders [10, 11], although the duration and permanence of these effects remain to be fully understood.

Recent conceptual perspectives have enhanced our understanding of the brain's response to psychedelics, combining biological, dynamical systems, complexity science, and artificial intelligence viewpoints. The REBUS (RElaxed Beliefs Under pSychedelics)

framework [12], grounded in the Free Energy Principle and the entropic and anarchic brain models, offers a perspective on the effects of psychedelics on the brain whereby psychedelic action results in the collapse of brain functional hierarchies or, in other words, in the "flattening of the landscape" of brain's dynamics to allow the brain state to escape a deep local minimum. The term *annealing* is also used in this context in relation to physical annealing in metallurgy and simulated annealing in numerical optimization [13].

Another important perspective for this paper is the CANAL framework [11] for describing the pathological plasticity of "being stuck in a rut" in certain mood disorders and the potential therapeutic role of psychedelics through the concept of metaplasticity. The CANAL framework can be further extended by establishing connections with Deep Neural Networks [14] to introduce a distinction between two distinct pathological phenomena — one related to fast brain dynamics and their slow and ultraslow counterparts. These distinctions will be naturally integrated into our framework.

While the discussion is centered on the effects of psychedelics, the proposed framework readily extends to other phenomena related to plasticity, including pathological plasticity in mood disorders [15] and interventions like transcranial brain stimulation (tES) [16].

In what follows, we formalize the notions of brain dynamics, plasticity, and their associated timescales and subsequently use them to study the impact of psychedelics on the brain. In the last section, we draw conceptual connections between our proposed framework and concepts from general relativity in physics. We hope these parallels will illuminate the complex relationship between the structure and function of brain dynamics.

2 Dynamics across timescales

The state of a system can be defined by a set of coordinates in phase space: a multidimensional manifold in which each dimension corresponds to one of the variables. For a single particle moving in one dimension, the phase space is two-dimensional, with one axis representing its position and the other representing its momentum. For example, Figure 1 illustrates the phase space of a pendulum with friction. In phase space, and perhaps after some transient period, the possible trajectories of the states of the system lie in a reduced or invariant manifold which we may refer to as the "geometry" or latent "structure" of the phase space. In convergent phase space dynamics, these are called

Box 1 - GLOSSARY

State of the system: Depending on the context, the state of the system is defined by the coordinates x (Eq. 1, fast time view) or by the full set of dynamical variables (x, w, θ) – see Eqs. 1, 2 and 3.

Effective connectivity (or connectivity for short): In our high-level formulation, this symbolized by w. It represents the connectivity relevant to state dynamics. It is affected by multiple elements, including the structural connectome, the number of synapses per fiber in the connectome, and the synaptic state (which may be affected by neuromodulatory signals or drugs).

Plasticity: By plasticity, we mean here the ability of the system to change its effective connectivity (w). The plasticity of the system may vary over time.

Metaplasticity: The ability of the system to change its plasticity over time. Dynamics of plasticity.

State or Activity-dependent plasticity: Mechanism for changing the connectivity (w) as a function of the state (fast) dynamics and other parameters (α). See Eq. 2.

State or Activity-independent plasticity: Mechanism for changing the connectivity (w) independently of state dynamics, as a function of some parameters (γ). See Eq. 2.

Connectodynamics: Equations governing the dynamics of w in slow or ultraslow time.

Fast time: Timescale associated to state dynamics pertaining to x.

Slow time: Timescale associated to connectivity dynamics pertaining to w.

Ultraslow time: Timescale associated to plasticity dynamics pertaining to θ (where $\theta = (\alpha, \gamma)$). See Eq. 3.

Phase space: Mathematical space, also called **state space**, where each point represents a possible state of a system, characterized by its coordinates or variables. phase space in our equations has dimension $\mathbb{R}^n$, with each dimension corresponding to one of the components of x.

Geometry of reduced phase space: The phase space has dimension $\mathbb{R}^n$, but the state trajectories lie in a submanifold (the reduced or invariant manifold). We call the geometry of this submanifold and its topology the "structure of phase space" or "geometry of dynamical landscape".

Manifold hypothesis: High-dimensional real-world data lies in a low-dimensional invariant manifold within the high-dimensional space.

Invariant manifold: A submanifold within (embedded into) the phase space that remains preserved or invariant under the dynamics of a system. It represents a geometric structure that remains unchanged as the system evolves over time.

Stable manifold or attractor: A type of invariant manifold defined as a subset of the phase space to which trajectories of a dynamical system converge or tend to approach over time.

Latent space: A compressed, reduced-dimensional data representation (see Box 2).

Topological tipping point: A sharp transition in the topology of attractors due to changes in system inputs or parameters.

stable manifolds, including attractors or repellors. Together, the structure of the phase space with its invariant properties can be referred to as the dynamical landscape, where

Box 2 - The manifold hypothesis and latent spaces

The dimension of the phase (or state) space is determined by the number of independent variables required to specify the complete state of the system and the future evolution of the system. The **Manifold hypothesis** posits that high-dimensional data, such as neuroimaging data, can be compressed into a reduced number of parameters due to the presence of a low-dimensional invariant manifold within the high-dimensional phase space [17, 18]. **Invariant manifolds** can take various forms, such as **stable manifolds or attractors** and unstable manifolds. In attractors, small perturbations or deviations from the manifold are typically damped out, and trajectories converge towards it. They can be thought of as lower-dimensional submanifolds within the phase space that capture the system's long-term behavior or steady state.
Such attractors are sometimes loosely referred to as the **"latent space"** of the dynamical system, although the term is also used in other related ways. In the context of using variational autoencoders, latent space is the result of a projection or embedding of the original high-dimensional data or some data derivatives (e.g., functional connectivity [19, 20]) into a lower-dimensional space. This mapping, which exploits the underlying invariant manifold structure, can help reveal patterns, similarities, or relationships that may be obscured or difficult to discern in the original high-dimensional space. If the latent space is designed to capture the full dynamics of the data (i.e., is constructed directly from time series) across different states and topological tipping points, it can be interpreted as representation of the invariant manifolds underlying system.

the depth or shallowness of the "valleys" can in some cases be interpreted as the stability of the state in that location given some stochastic forcing. For example, in mechanics, the landscape can be labeled by potential energy isolines, e.g., in a physical system such as in the pendulum example in Figure 1 (bottom right), or their generalization, Lyapunov functions [21].

Fast time: neural dynamics

Here, we discuss the first equation in neural geometrodynamics in the context of neural mass models, but the ideas are applicable more extensively in computational neuroscience. The standard equation we use in neural mass modeling is a multidimensional ODE of the form

$$\dot{x} = f\big(x; w, \eta(t)\big) \tag{1}$$

with $x \in \mathbb{R}^n$ and where w are connectivity parameters[1] and where, as usual, a dot over a variable denotes its time derivative. This equation governs dynamics at short time scales (seconds or less) when connectivity parameters w are assumed to be constant.

[1] In the REBUS model [22], from the Free Energy perspective, w would correspond to the weights or precision assigned to priors/beliefs; from the Entropic Brain perspective, w would correspond to the weights of the effective connectivity between neuronal populations on the macroscopic scale.

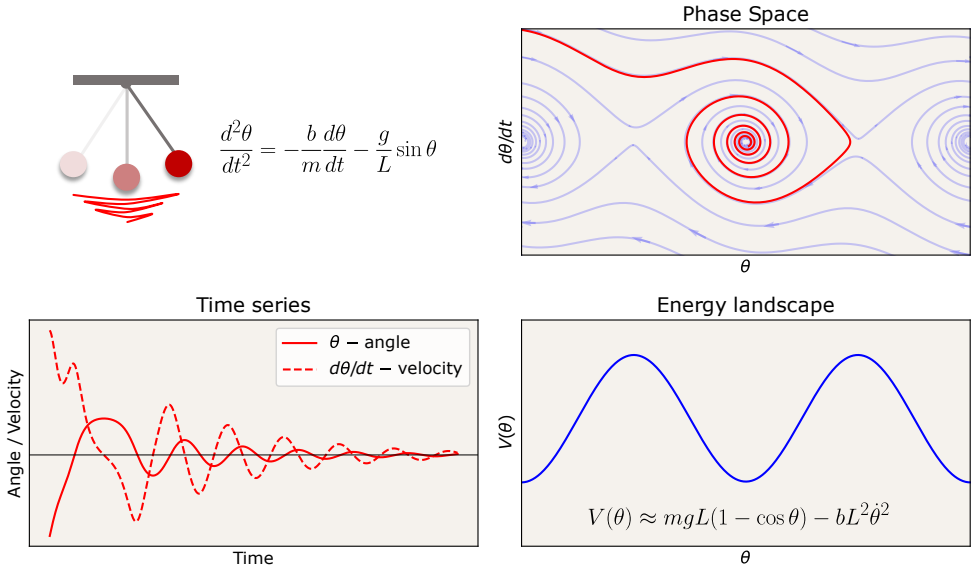

Figure 1: Dynamics of a pendulum with friction: time series, phase space, and energy landscape. Attractors in phase space are valleys in the "energy" landscape (or more generally, defined by the level sets of a Lyapunov function).

We may think of this equation describing phenomena in **fast time** scales as providing the "structure" for the dynamics of neuronal population state. The fast timescale is set by synaptic transmission (milliseconds) and by ephaptic coupling (electromagnetic waves) [23–26] in a nanosecond or subnanosecond scale [25].

Equation 1 characterizes the **dynamical landscape**, which is established through the geometric structure of the phase space, where trajectories are shaped by the given set of ordinary differential equations. Such landscape is analogous to the Neural Activation Landscape proposed in [14]. More specifically, we talk about the landscape as defined by the manifold generated by the motion of trajectories with coordinates $x \in \mathbb{R}^n$. Typically, trajectories lie in a reduced manifold of dimensionality lower than $\mathbb{R}^n$. The fact that such a reduced space exists means that it can be generated by coordinates in a reduced latent space.

As we can see, the landscape is determined by the functional form of $f(x; w, \eta(t))$ and by the parameters w. The external input term $\eta(t)$ makes the equations non-autonomous (an autonomous ODE does not explicitly depend on time). This term can refer to external forces providing random kicks to the trajectory or to a more steady and purposeful forcing from unspecified internal systems or external inputs.

Slow time: connectodynamics

The landscape, like that on planet Earth, may appear to be static, but in reality, it is not fixed. It also flows in **slow time**. We thus consider changes in connectivity in the system, that is now $w = w(t)$. We call the potential for such changes **plasticity of the system**. The general form of this equation is $\dot{w} = g(x, w; \theta)$, with θ standing for a set of parameters controlling plasticity.

To be more concrete, we can think of two types of process: one that modifies the connectivity parameters independently of the system's state (ψ) and another which is a function of the state (e.g., Hebbian plasticity [27], h). We express this by writing

$$\dot{w} = \psi(w; \gamma) + h(x, w; \alpha) \tag{2}$$

This decomposition separates out *state-dependent* (via the term $h(x, w; \alpha)$) and *state-independent plasticity* (with $\psi(w; \gamma)$) processes. The set of parameters θ is similarly decomposed as $\theta = (\alpha, \gamma)$: we separate out the plasticity-controlling parameters in order to differentiate the *state-dependent (α)* and *state-independent (γ)* plasticity control parameters (e.g., Hebbian vs. drug-enhanced structural plasticity [11]).

In summary, the functions h and ψ with parameters α and γ regulate *connectodynamics*, defining to where and how fast the effective connectivity will change in a state-dependent or state-independent way.

These connectodynamics differential equations define a new dynamical landscape, which we can call the *plasticity landscape* (analogous to the Synaptic Weight Landscape in [14]). The state w in this plasticity landscape will determine the shape of the neural dynamics landscape.

Ultraslow time: metaplasticity

We now allow the plasticity parameters also to be dynamic, i.e.,

$$\dot{\theta} = \xi(x, w, \theta) \tag{3}$$

These dynamics reflect changes in plasticity, such as the changes regulating state-dependent (Hebbian) plasticity (changes in α) during neurodevelopment, and state-independent plasticity, such as the ones induced by psychedelics in the acute or post-acute phases (changes in ψ). In the latter case of plasticity, there are numerous candidates for these plastic processes, such as heterosynaptic plasticity [28], synaptic scaling and homeostatic plasticity [29], critical-period plasticity [30] and metaplasticity [31]. They all reflect a

common physiological principle well described by Abraham et al. in the definition of metaplasticity:

> **Metaplasticity** *[...] is manifested as* **a change in the ability to induce subsequent synaptic plasticity**, *such as long-term potentiation or depression. Thus, metaplasticity is a higher-order form of synaptic plasticity [31].*

Thus, metaplasticity and its counterparts are terms used in neuroscience to refer to the plasticity of synaptic plasticity. That is, the idea that the ability of synapses to strengthen or weaken in response to increases or decreases in their activity (which is called synaptic plasticity) can be modulated based on the history of the synaptic activity.

There are various mechanisms through which metaplasticity can occur, such as changes in the amount of neurotransmitters that are released, alterations in the types or numbers of receptors on the post-synaptic neuron, changes in the internal signaling pathways within neurons, and more. Metaplasticity has important implications for the learning and memory of an organism, as it can regulate the ability of synaptic connections to change and adapt over time. In extreme scenarios, such regulation can lead to two opposite outcomes in which metaplasticity can either become overly exuberant, reflecting the notion of catastrophic forgetting in artificial neural networks, or impoverished, reflecting general plasticity loss [14].

Metaplasticity may also play a role in certain neurological and psychiatric conditions. For example, reduced metaplasticity could underlie conditions such as major depressive disorder, obsessive-compulsive disorder, anxiety, or substance abuse on one hand, and overly malleable metaplasticity could define bipolar and personality disorders on the other [14, 32].

We call the set of equations 1,2 and 3 — somewhat whimsically — the equations for *neural geometrodynamics* in reference to the equations of general relativity in physics. We recall that general relativity provides equations defining the dynamics of spacetime geometry (via the "metric") coupled with matter (see Section 4) [33].

3 Dynamics under psychedelics

Psychedelics like psilocybin and LSD act as agonists or partial agonists for serotonin 5-hydroxytryptamine 2A (5-HT_{2A}) receptors, specifically targeting Layer V cortical pyramidal neurons [11, 22, 34–36]. Recognized for their potent and immediate impact on

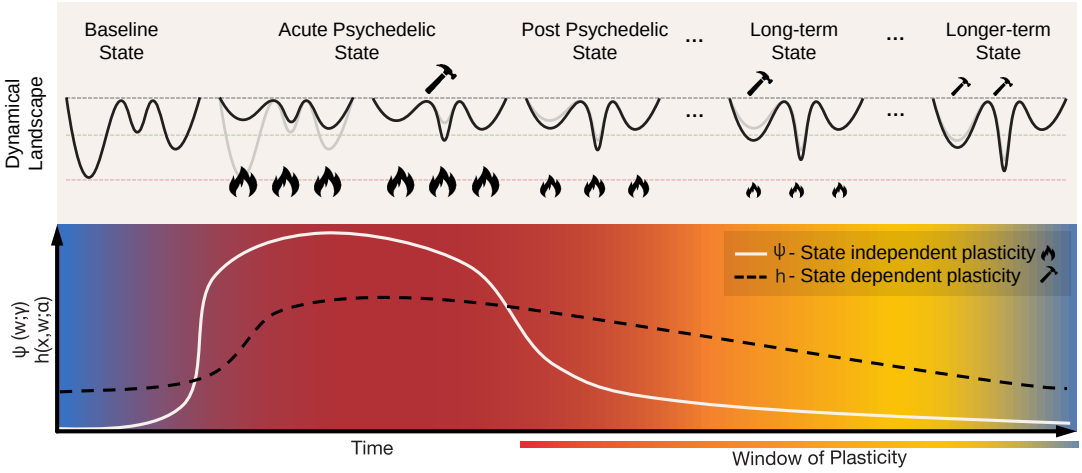

Figure 2: Geometrodynamics of the acute and post-acute plastic effects of psychedelics. The **acute** plastic effects can be represented by rapid state-independent changes in connectivity parameters, i.e., the term $\psi(w;\gamma)$ in Eq. 2. This will result in the flattening or de-weighting of the dynamical landscape. Such flattening allows for the exploration of a wider range of states, eventually creating new minima through state-dependent plasticity, represented by the term $h(x, w; \alpha)$ in Eq. 2. As the psychedelic action fades out, the landscape gradually transitions towards its initial state but with lasting changes due to the creation of new attractors during the acute state. The **post-acute** plastic effects can be described as a "window of plasticity" characterized by a period of enhanced plasticity. These transitions are brought about by time-dependent changes of the parameters γ and α, each controlling the behavior of state-independent and state-dependent plasticity, respectively. In this post-acute phase, the landscape is more malleable to internal and external influences.

the brain, these drugs cause a swift reconfiguration of neural dynamics. As we explain, this immediate effect is represented in our model by alterations in the state-independent connectivity parameters (w).

5-HT_{2A} receptors are strongly expressed in the pyramidal neurons of the cortex along a steep anteroposterior gradient [37]. When psychedelics bind to them, they can lead to a gradual increase of the excitability of these pyramidal neurons — depolarizing them and making them more susceptible to excitatory inputs such as those associated with glutamate receptors [37] — much as the gain knob in an amplifier. This increased excitability and susceptibility to inputs can lead to changes in the firing patterns of these neurons and alterations in the overall neural circuit activity.

How are these effects represented in Equations 1 and 2? If we include the neuromodulatory nodes in our model — the dorsal raphe and median raphe nuclei in the brainstem

are the source of most serotonergic neurons projecting throughout the brain [37] —, the modulation of serotonin receptors can be represented by changes in neuromodulatory connectivity (the subset of w parameters in the model connecting the raphe nuclei to other nodes). Alternatively, if neuromodulatory nodes are not explicitly included in the model, for the purposes at hand, we can think of the changes in the excitability of the nodes affected by neuromodulatory inputs as leading to changes in their effective connectivity to other nodes.

This abrupt shift induced by psychedelics can be thought of as a transformation of the phase space's geometry, allowing the neural state to explore new trajectories. Such changes in connectivity correspond to a flattening of the dynamical landscape [12] or a destabilization of it [14]. Effective connectivity, as inferred using Ising modeling, is seen to decrease in the acute phase of LSD (especially in interhemispheric homotopic links), with a subsequent increase in algorithmic complexity of BOLD signals measured using fMRI imaging [38]. The alteration of effective connective results in an immediate and state-independent remodeling of the dynamical landscape during the acute phase of psychedelics, which is represented by the term $\psi(w;\gamma)$ in the connectodynamics equation (Eq. 2)[2].

The instantaneous modification of the landscape is, however, ephemeral, gradually fading as the acute effects of the psychedelics wear off. The system returns to near its original geometrical configuration but with lasting influences brought about by the plastic changes resulting from the exploration of new trajectories in the acute phase. These residual changes are captured by the state-dependent plasticity term, $h(x, w; \alpha)$, which reflects changes in connectivity due to Hebbian plasticity that arise from the co-activation of neurons during the psychedelic acute stage.

In the literature, there is an increasing body of evidence suggesting a post-acute phase following psychedelic exposure characterized by a period of enhanced plasticity [11, 12, 39–41]. This phase can be interpreted as an extended window of malleability of the landscape, which could have profound implications for learning and therapy. Such window of plasticity has been related to increased neurogenesis and upregulation of Brain-Derived Neurotropic Factor (BDNF) in humans and mice [8]. The activity-dependent release of BDNF plays a crucial role in selectively strengthening active synapses while weakening inactive ones, a critical process for Hebbian-type plasticity. Intriguingly, recent studies

[2] In the REBUS model and the Entropic Brain perspective [22], the weights of the effective connectivity during the psychedelic-induced state are "flattened" or "de-weighted", representing a more symmetrical and non-hierarchical connectivity profile.

with mice have found psychedelic-induced changes in plasticity and antidepressant-like behavior dependent on the increase of endogenous BDNF and TrkB binding (the receptor of BDNF), but independent from the activation of 5-HT_{2A} [9, 10].

In terms of our model, these two pathways correspond to changes of connectivity through Equation 2 due to a temporary modulation of the parameters γ and α (i.e., metaplasticity, see Equation 3) upregulating state-independent and state-dependent plasticity processes, respectively. The strong acute-phase increase of state-independent plasticity (ψ) would be directly associated with the activation of serotonergic receptors, as discussed above, with a possible gradual decrease during the post-acute phase (solid white line in Figure 2). The sustained increase of state-dependent plasticity (h) in the post-acute phase (dashed black line in Figure 2) would be linked to dendritic growth, neurogenesis, upregulation of BDNF, and other related changes. This means that in the post-acute period, the landscape would be more responsive to state changes (itself influenced by external factors), offering a potential mechanism for the long-lasting changes reported after psychedelic experiences. Such external influences are modeled by the external input term $\eta(t)$ in the state equation (Eq. 1) and can represent environmental/sensory inputs, psychotherapy, or neuromodulatory brain stimulation techniques such as transcranial electrical current stimulation (tES).

Dynamics of psychedelics and psychopathology

Recently, psychedelic medicine has emerged as a promising direction for treating mental disorders such as depression or addiction [42]. The nuanced interaction between the brain's neurophysiology and the emergent brain activity underlies the pathophysiology of mood disorders, often resulting in a persistent and maladaptive rigidity in cognitive and emotional processes [43]. Such changes to the brain's neurophysiology can be explained through the CANAL framework whereby pathological plasticity, often caused by a traumatic event, asserts itself and dominates brain activity, driving the brain state to be "stuck in a rut" [11], i.e., a deepening minimum in the dynamical landscape (see Figure 3).

This CANAL framework offers insight into the neural mechanisms underlying the persistence of various brain disorders. In particular, psychedelics may mediate their effects by altering the balance between stability and plasticity in neural networks through metaplasticity and thus act as potential therapeutic treatments. By acting on the serotonergic receptors, they trigger a cascade of neurochemical events, subsequently facilitating the reorganization of entrenched neural patterns. As discussed above, this alteration of the

neural network during the acute phase (connectodynamics) can be interpreted as a rapid deformation or flattening of the landscape that allows the trapped state to escape and access more adaptive cognitive and emotional patterns. The acute phase is believed to be followed by an extended window of malleability of the landscape, otherwise known as a "window of plasticity", where treatments such as psychotherapy and transcranial electrical stimulation can further alter the pathological rigidity characteristic of various brain disorders (see Figure 3).

4 Neural geometrodynamics and general relativity

A parallel can be drawn between neural geometrodynamics and Einstein's equations of general relativity — the original geometrodynamics. Both frameworks involve the dynamical interaction between structure and resulting activity, each influencing and being influenced by the other. The Einstein field equations, including the cosmological constant Λ, are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{4}$$

Here, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu} = R_{\mu\nu}[g_{\mu\nu}]$ is the Ricci curvature tensor and a function of $g_{\mu\nu}$, $R[g_{\mu\nu}]$ is the Ricci scalar (or curvature scalar) and a function of $g_{\mu\nu}$, $T_{\mu\nu}$ is the stress-energy tensor[3], a function of the mass and energy distribution (all the indices refer to spacetime dimensions), G is the gravitational constant, c is the speed of light, and Λ is the cosmological constant. These equations, which can be seen as an equation of the spacetime metric $g_{\mu\nu}$ if the stress-energy tensor $T_{\mu\nu}$ is given, describe the fundamental interaction of gravitation as a result of spacetime being curved by matter and energy. Specifically, they equate local spacetime curvature (on the left-hand side) with the local energy and momentum within that spacetime (on the right-hand side).

To complete these equations, the *geodesic equation* portrays how particles (matter) move in this curved spacetime, encapsulated by the notion that particles follow the straightest possible paths (geodesics) in curved spacetime,

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0 \tag{5}$$

where x^μ are the coordinates of the particle, τ is the proper time along the particle's path, and $\Gamma^\mu_{\rho\sigma}[g_{\mu\nu}]$ are the Christoffel symbols, which are a function of $g_{\mu\nu}$ and encode

[3]The stress-energy tensor (also called the energy-momentum tensor) is a central concept in general relativity. It encapsulates the distribution and flow of energy and momentum in spacetime, and its components include energy density, momentum density, and stress (pressure and shear stress) within a given region.

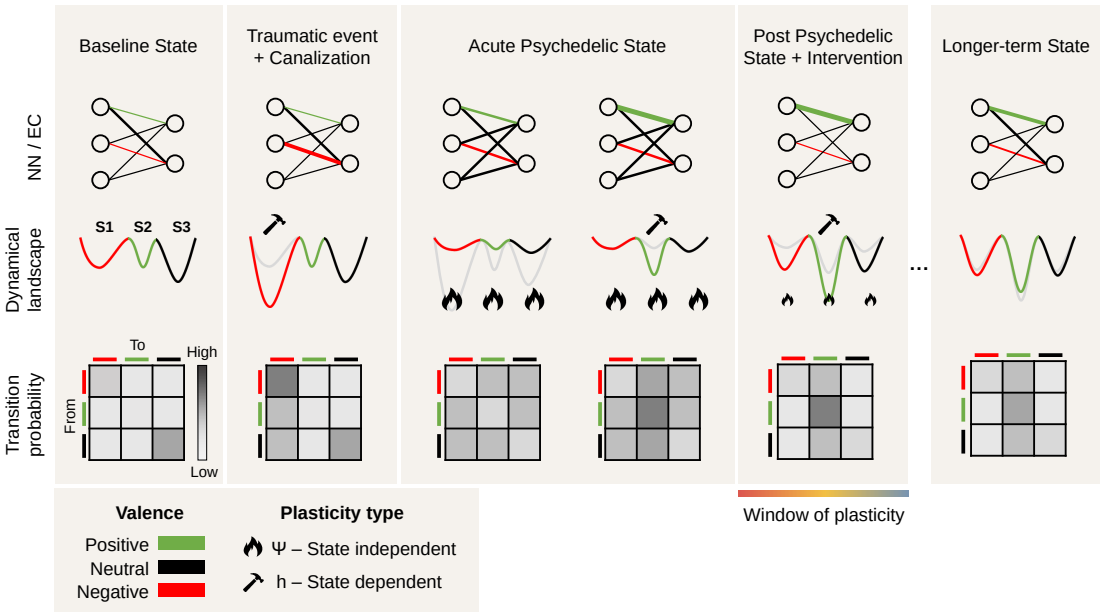

Figure 3: Psychedelics and psychopathology: a dynamical systems perspective. From left to right, we provide three views of the transition from health to canalization following a traumatic event and back to a healthy state following the acute effects and post-acute effects of psychedelics and psychotherapy. The top row provides the *neural network (NN) and effective connectivity (EC) view.* Circles represent nodes in the network and edge connectivity between them, with edge thickness representing the connectivity strength between nodes. The middle row provides *the landscape view*, with three schematic minima and colors depicting the valence of each corresponding state (positive, neutral, or negative). The bottom row represents the *transition probabilities across states* view and how they change across the different phases. Due to traumatic events, excessive canalization may result in a pathological landscape reflected as a deepening of a negative valence minimum where the state may be trapped. During the acute psychedelic state, the landscape is deformed, enabling the state to escape. Moreover, plasticity is enhanced during the acute and post-acute phases, benefiting interventions such as psychotherapy or brain stimulation (i.e., changes in effective connectivity). Not shown is the possibility that a deeper transformation of the landscape may take place during the acute phase (see the discussion on the wormhole analogy in Section 4).

the *connection* (a mathematical object that describes how vectors change as they are parallel transported along curves in spacetime). The stress-energy tensor $T_{\mu\nu}$ can be computed from the state of the particles, closing the system of equations. For example, for N particles it is given by $T^{\mu\nu} = \sum_i m_i u_i^\mu u_i^\nu \delta(\mathbf{x} - \mathbf{x}_i)$, where m_i and u_i are the mass and velocity of the ith particle. More generally, the stress-energy tensor represents the

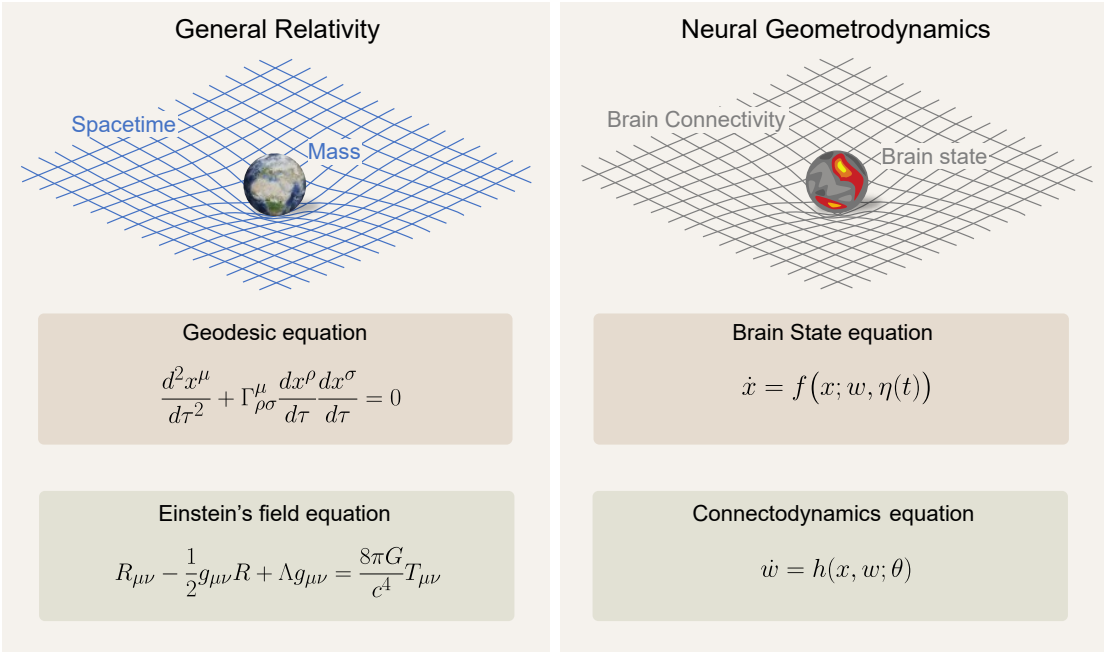

Figure 4: General Relativity and Neural Geometrodynamics. Left: Equations for general relativity (the original geometrodynamics), coupling dynamics of matter with the dynamics of spacetime. Right: Equations for neural geometrodynamics, coupling neural state, and connectivity. Only fast and slow time equations are shown (ultraslow time endows with dynamics the "constants" appearing in these equations).

state of matter and energy, which corresponds to x in our neural model. And the metric $g_{\mu\nu}$, which specifies the geometry of spacetime, is akin to the connectivity w — which shapes the structure of the space where fast dynamics occur.

In the context of neural mass models, the state equation, $\dot{x} = f(x; w)$, is analogous to the geodesic equation – "the state of the system evolves according to the landscape geometry specified by the parameters w". On the other hand, the connectodynamics equation, $\dot{w} = h(x, w; \theta)$ (with θ standing for plasticity parameters), is analogous to Einstein's field equations — the parameters w, which describe the "structure" of the space where dynamics take place, evolve according to the current state of the system x and its 'readiness' for plasticity (parametrized by θ).

This comparison is largely metaphorical: the mutual influence between particles and spacetime in general relative is akin to the state of the neural system and its underlying connectivity parameters. In both cases, dynamics and structure are intertwined (see Figure 4).

Metaplasticity and variable constants in cosmology

In our neural mass model framework, the concept of metaplasticity is introduced as dynamic variations in the plasticity control constants, namely θ in the connectodynamics equation. This set of constants can be represented as evolving over time as a function of the state of the system or other relevant variables,

$$\dot{\theta} = \xi(x, w, \theta; \lambda) \tag{6}$$

In this equation, ξ defines the evolution of the plasticity control constants with parameters λ.

Analogously, in the realm of general relativity and cosmology, it has been speculated that the fundamental constants, such as the speed of light c, the gravitational constant G, or the cosmological constant Λ, may in fact be dynamic. Although not part of the mainstream cosmological model, theories proposing variable constants, such as "Variable Speed of Light" (VSL) or "Variable Cosmological Constant" provide an intriguing parallel. For instance, within VSL theories, the speed of light c is postulated to vary over cosmological time scales. Certain hypothetical dynamical equations could dictate the dynamical evolution of these constants. Although these theories are quite speculative and do not form a part of mainstream physics, they offer an interesting perspective on the concept of metaplasticity and its potential implications for the dynamical evolution of neural mass models and the structure of their landscapes.

Psychedelics as wormholes in the neural landscape

In the parallel of general relativity and neural geometrodynamics, we see the effects of psychedelics as a deformation of the neural landscape (spacetime) that allows the brain state (of a particle or set of particles) to escape from a local minimum and transition to another location in the landscape (spacetime). Although transitions may be smooth and respect the topology of the landscape, deformations of the landscape may also be more extreme — sharp transitions through a *topological tipping point* of the dynamical landscape. This may be due to external inputs ($\eta(t)$), when our system is non-autonomous [44], e.g., from sensory or brain stimulation effect. And, as we have discussed, it may be due to connectivity dynamics.

The creation of a wormhole in general relativity[4] can be viewed as a profound deformation of spacetime, bending and connecting distant parts of the universe in such a way

[4]Wormholes, a term due to John A. Wheeler [45], also known as Einstein-Rosen bridges, are solutions to the Einstein field equations of general relativity which some models suggest could exist under certain

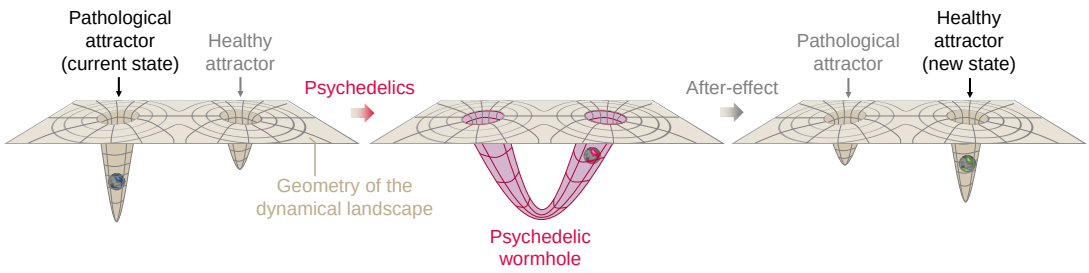

Figure 5: Psychedelic wormhole. On the left, the landscape is characterized by a deep pathological attractor, where the neural state is trapped. After ingestion of psychedelics (middle), a radical transformation of the neural landscape takes place, with the formation of a wormhole connecting the pathological attractor to another, healthier attractor location and allowing the neural state to tunnel out. After the acute effects wear off (right panel), the landscape returns near its original topology and geometry, but activity-dependent plasticity reshapes it into a less pathological geometry.

that an matter/energy, like an astronaut, can travel through vast distances in an instant. This change in the geometry and topology of spacetime can be likened to the effect of psychedelics on the human mind. Just as the wormhole alters the structure of spacetime, psychedelics may radically deform the dynamical landscape of neural dynamics, creating connections across distant landscape locations. In the same way that the astronaut uses the wormhole to bypass vast stretches of space, the deformation caused by psychedelics may allow the state of the brain to tunnel out and escape from a local minimum or stuck pattern of thought, providing access to new areas of the landscape — new perspectives and potentially unexplored territories of consciousness. Both phenomena are characterized by a fundamental transformation that enables traversal into otherwise inaccessible regions, whether in physical space or the brain's dynamical landscape (see Figure 5 for a sketch of this concept).

Characterizing the landscape

An important challenge in the program of neural geometrodynamics is to explore practical methods to characterize the landscape. Here again, we can draw inspiration from physics and mathematics.

The roots of this approach can be traced back to the 19th century when Carl Friedrich

conditions. However, creating or stabilizing a traversable wormhole would likely require forms of exotic matter with properties not yet observed in the known universe.

Gauss pioneered the field of differential geometry. Gauss's Theorema Egregium demonstrated that the curvature of a surface could be determined entirely by measurements within the surface, without any reference to the surrounding space [46]. This seminal insight has laid the groundwork for understanding manifolds in various contexts, including the theory of relativity. In the era of general relativity, the interplay between geometry and physics was further enriched. Differential geometry and algebraic topology — which comes into play when one is interested in the global properties of the manifold, such as its shape, connectedness, and the presence of holes [47, 48] — became essential in describing the fabric of spacetime itself. It enabled physicists to conceptualize how mass and energy warp the geometry of spacetime, thus influencing the motion of objects.

In our current endeavor, these ideas find application in characterizing the complex dynamical landscapes of neural data. Modern tools from deep learning, such as variational autoencoders, can be used to unravel the reduced spaces underlying neuroimaging or neurophysiological data [19, 20], while dynamical systems theory in concert with differential geometry, group theory, and algebraic topology data analysis [49] offer robust frameworks to understand and characterize them [44, 50–54]. The convergence of these mathematical techniques extends to neuroscience the tradition of linking geometry and topology to physical phenomena.

5 Conclusions

In this paper, we have provided the framework of neural geometrodynamics and used it to define the coupling of state dynamics and geometry of reduced space or structure, and discussed it in the context of the concurrent and longer-lasting effects of psychedelics. In passing, we have also established a parallel with other mathematical theories of nature, namely in general relativity, where dynamics and the "kinematic theatre" are intertwined (see the Appendix for a similar parallel with electrodynamics) — which have indeed been a source of inspiration.

Although we can think of "geometry" in neural geometrodynamics as referring to the structure imposed by connectivity on state dynamics (and paralleling the role of the metric in general relativity), it is more appropriate to think of it as the geometry of the reduced phase space (or invariant manifold) where state trajectories lie: this is where the term reaches its fuller meaning. Since the fluid geometry and topology of the invariant manifolds underlying apparently complex neural dynamics may be strongly related to brain function and first-person (structured) experience [55], further research should focus

on creating and characterizing these fascinating mathematical structures.

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A Appendix

A.1 Classical dynamics of particles and fields

Here we provide the equations for other systems where one can think of part of the equation describing the geometry of a space-providing subsystem ("kinematic theatre") and another the subsystem moving in this space, influenced by the structure and then affecting its geometry in return. Several such examples can be found in physics.

Non-relativistic electrodynamics

The non-relativistic dynamics of N-charged particles and the associated electromagnetic field are described by (coupled) Newton's second law and Maxwell's equations. The charged particles are influenced by the electromagnetic field, and at the same time, generate it. This can lead to problematic scenarios, such as the self-interaction problem: charged particles generate an electromagnetic field, and if one considers a particle's interaction with its own field, paradoxical or unphysical results can arise. This self-interaction leads to divergences in calculations and has been a longstanding challenge in classical electrodynamics until recently [56].

The motion of the ith electron is given by $m_i \frac{d^2\mathbf{r}_i}{dt^2} = \mathbf{F}_i$, where $\mathbf{F}_i = q_i(\mathbf{E} + \mathbf{v}_i \times \mathbf{B})$ is the Lorentz force. The electromagnetic field obeys Maxwell's equations:

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},
\end{aligned}
$$

where $\rho(\mathbf{r}, t) = \sum_i q_i \delta(\mathbf{r} - \mathbf{r}_i(t))$ and $\mathbf{J}(\mathbf{r}, t) = \sum_i q_i \mathbf{v}_i(t) \delta(\mathbf{r} - \mathbf{r}_i(t))$ are the charge and current densities.

Relativistic equations

Maxwell's equations are relativistic (they transform properly under Lorentz transformations), but Newton's is not. For the **relativistic version**, the field strength tensor $F^{\mu\nu}$ is defined in terms of the four-potential A^μ [57],

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

The dual tensor $\tilde{F}^{\mu\nu}$ is defined in terms of $F_{\alpha\beta}$ and the Levi-Civita symbol $\varepsilon^{\mu\nu\alpha\beta}$,

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

The homogeneous Maxwell's equations are given by:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

and the inhomogeneous Maxwell's equations are given by:

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$$

where J^ν is the four-current vector, which we now define for a particular case. Given a distribution of N particles each with charge q_i and four-velocity u_i^μ, the four-current J^μ at position $\mathbf{x}$ and time t is given by

$$J^\mu(\mathbf{x}, t) = \sum_{i=1}^{N} q_i u_i^\mu \delta^3(\mathbf{x} - \mathbf{x}_i(t))$$

Here, $\mathbf{x}_i(t)$ is the position of the i-th particle at time t, and δ^3 is the three-dimensional Dirac delta function.

The equation of motion for a charged particle in an electromagnetic field, commonly known as the Lorentz force equation, in its relativistic form is:

$$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}u_\nu$$

Here, p^μ is the four-momentum of the particle, u^ν is the four-velocity of the particle, $F^{\mu\nu}$ is the electromagnetic field tensor, and q is the charge of the particle. The equation describes how the four-momentum of the particle changes with proper time τ under the influence of the electromagnetic field.

A.2 Modeling plasticity in neural mass models

In this section, we provide a brief overview of plasticity mechanisms and how they relate to the terms in our formalism, namely the functions h and ψ. See Table 1 for a summary. Including plasticity in Neuronal Mass Models (NMMs) allows for the modeling of time-varying connectivity strengths that reflect the learning and adaptation processes observed in biological neuronal networks.

Functional Plasticity

The simplest and most common way to include synaptic plasticity is through Hebbian learning rules. Hebbian plasticity, a type of functional plasticity, follows the principle that "neurons that fire together wire together" [27]. It can be included in NMM using the equation

$$\dot{w}_{ij} = \eta x_i x_j - \delta w_{ij} \tag{7}$$

where w_{ij} is the synaptic strength from neuron j to neuron i, x_i and x_j are the neuronal activities, and η and δ are parameters controlling the learning and decay rates.

Homeostatic Plasticity

Homeostatic plasticity, a form of plasticity that adjusts synaptic strengths to keep the overall activity of a neuron or network within a certain range, can be included into an NMM using the equation [29],

$$\dot{w}_{ij} = \eta(x_i - x_{\text{target}})x_j - \delta w_{ij} \tag{8}$$

where x_{target} is the target activity level.

Structural Plasticity

Structural plasticity, where the actual number and dendrites and arrangement of synapses change over time, can be represented in NMMs by modifying (or even adding) rows and columns from the w adjacency matrix to represent the formation or elimination of synapses or even fibers.

Empirically-derived Structural Plasticity

NMMs can be used to infer the structural changes to plasticity without explicitly describing the plastic mechanism per se. For example, in the post-acute psychedelic state, NMMs can be used to infer the plastic changes to w_{ij} by optimizing the model functional

connectivity FS_{ij}^{model} to approximate the empirical functional connectivity FS_{ij}^{model} with a certain learning rate ϵ as in the following equation.

$$w_{ij} = w_{ij} + \epsilon(FS_{ij}^{empirical} - FS_{ij}^{model}) \tag{9}$$

Such optimization is, for example, computed through gradient descent methods with priors on the topology of structural connectivity between brain regions [58]. Recent methods have further extended this framework by adding time-shifted correlation [59] to the optimization as a better description of the overall brain state, as in this case, the post-acute psychedelic state.

Including these forms of plasticity in NMMs allows for more realistic modeling of neural systems in better capturing their adaptive nature and the impact of learning and experience on synaptic connections.

Type	Time Scale	Mechanism	Effects	State-dependence
Functional Plasticity	Milliseconds to minutes	Changes in strength/efficiency of synapses (e.g., Hebbian plasticity, LTP, LTD)	Short and long-term memory, fine-tuning of connections	State-dependent (h)
Acute Psychedelic-induced Plasticity	Minutes to Hours	Targeting of serotonergic neuroreceptors especially the 5-HT_{2A} with a result in overall excitability [11, 12, 39, 40]	Flattens or de-weights the dynamical landscape	State-independent (ψ)
Homeostatic Plasticity	Hours to days	Regulation of overall excitability to maintain stability (e.g., adjusting synapse strength for E/I balance)	Balances and stabilizes network	State-independent (ψ)
Structural Plasticity (e.g., post-acute psychedelic-induced plasticity)	Hours to years	Larger physical changes in neurons (e.g., dendrite growth, synapse formation) through an increase of endogenous BDNF and via TrkB binding (the receptor of BDNF) [41]	Long-term memory, development	Depends on context
Metaplasticity	Various, often longer-term	Changes in mechanisms governing synaptic plasticity (e.g., modulation of thresholds/rules)	Regulates other forms of plasticity, "plasticity of plasticity" [31]	Depends on context

Table 1: Summary of Different Types of Neural Plasticity Phenomena. **State-dependent Plasticity (h)** refers to changes in neural connections that depend on the current state or activity of the neurons involved. For example, functional plasticity often relies on specific patterns of neural activity to induce changes in synaptic strength. **State-independent Plasticity (ψ)** refers to changes that are not directly dependent on the specific activity state of the neurons. For example, acute psychedelic-induced plasticity acts on the serotonergic neuroreceptors and thus acts on the brain networks regardless of specific activity patterns. Some forms of plasticity, like structural plasticity and metaplasticity, may exhibit characteristics of both state-dependent and state-independent plasticity, depending on the context and specific mechanisms involved. Finally, **metaplasticity** refers to the adaptability or dynamics of plasticity mechanisms.