eCV: Enhanced coefficient of variation and IDR extensions for reproducibility assessment of high-throughput experiments with multiple replicates.

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ABSTRACT

Motivation

Reproducibility assessment is essential in extracting reliable scientific insights from high-throughput experiments. Inconsistency between technical replicates poses a challenge, particularly clear in next generation sequencing technologies based on immunoprecipitations, where the assessment of reproducibility in peak identification is a critical analytical step. While the Irreproducibility Discovery Rate (IDR) method has been instrumental in assessing reproducibility, its standard implementation is constrained to handling only two replicates. In the current era of steadily growing sample sizes, eased by multiplexing and reduced sequencing costs, highly performing methods that handle any number of replicates are desirable.

Results

We introduce three novel methods for reproducibility assessment in high-throughput data that handle an arbitrary number of replicates. The first, general IDR (gIDR), extends the standard IDR by adapting its Expectation-Maximization (EM) algorithm to handle distributions of any dimensions dictated by the number of replicates. The second, meta-

IDR (mIDR), employs a meta-analysis approach, calculating local IDR scores for all pairs

of replicates and combining them using standard probability rules. The third method

introduces an "enhanced" Coefficient of Variation (eCV), ranking features based on

intensity and variability, using a parametric bootstrap approach to obtain an index

analogous to local IDR. Comparative analysis with traditional IDR in simulated and

experimental data reveals the heightened performance of the proposed multivariate

alternatives under varying scenarios, thereby addressing the critical challenge of

reproducibility assessment in contemporary high-throughput experiments.

Availability and implementation

The described methods are implemented as an R package:

https://github.com/eclipsebio/eCV

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1 INTRODUCTION

The reproducibility of high-throughput experiments is essential in assuring high-quality results and valid scientific insights (Nicholson and Holmes, 2017). Lack of consistency within technical replicates, whether coming from heightened variability or diminished mean intensities, can undermine the reliability of the finding. For example, the lack of reproducibility between peaks in separated ChIP-seq libraries has been recognized as one of the most important pitfalls of the technology (Mundade *et al.*, 2014; Landt *et al.*, 2012). Li et al. (2011) introduced the Irreproducibility Discovery Rate (IDR) to measure the reproducibility of high-throughput data. This method robustly models the distribution of omic feature intensity along each technical replicate using a copula model (CM), and models conveniently the joint distribution of omic features in each sample replicate based only on the marginal distributions (Genest and Nešlehová, 2013).

Within the IDR framework, features are transformed and assumed to come from a mixture of two normal distributions, each representing one of two possible states: reproducible or irreproducible. An expectation-maximization (EM) algorithm (Dempster *et al.*, 1977) is then applied to estimate the parameters of this mixture, including the mean and variance of each normal distribution along with the mixing probability. Upon convergence, the likelihood of an omic feature belonging to the reproducible state is used to compute a "local" IDR score. To account for situations where the distribution of omic-feature values is unknown or too complex to approximate via elemental distributions (e.g., normal, negative binomial, gamma), a "semiparametric" CM approach is used, where marginal empirical cumulative distributions of each replicate are fed into the CM (Tsukahara, 2005). The actual IDR is calculated analogously to an adaptation step-up procedure to control

the false discovery rate (FDR) (Sun and Cai, 2007; Li *et al.*, 2011). After the model is

fitted, features with small IDR scores are taken as "reproducible," while features with

higher IDR scores are excluded as "irreproducible."

IDR has shown robust performance in extensive simulations (Li et al., 2011) and has also

been adopted by the ENCODE project as the gold standard to assess the reproducibility

of ChIP-seq data (Landt *et al.*, 2012). Nevertheless, the classical implementation of IDR

has the limitation of being unable to handle more than two replicates. The reason for this

constraint imposed on IDR is the conclusion from early ChiP-seq experiments that more

than two replicates would increase the assay's cost while not significantly improving

performance values, such as site discovery (Rozowsky *et al.*, 2009). However,

multiplexing and the continuous drop in sequencing costs make it more affordable for

researchers to increase sample size (Chen *et al.*, 2012). Larger sample sizes can improve

estimates of the variability associated with noise and artifacts effects and increase

statistical power (Li *et al.*, 2017).

Several alternatives to measure reproducibility in an arbitrary number of replicates have

been proposed, including pooling of all replicates (Chen *et al.*, 2011; Landt *et al.*, 2012),

selection of the best pairs of replicates (Revilla-i-Domingo *et al.*, 2012), or "majority rule"

(Haecker *et al.*, 2012). However, as Yang et al. (2014) discussed, the quality of these

data-combination strategies may be susceptible to outliers, diminish signal-to-noise

ratios, or compromise subsequent quantitative comparisons among samples.

To overcome these limitations, reproducibility assessment methods exploiting the true

multivariate nature of multi-replicate experiments are needed. Here, we present three

novel approaches to generalize the reproducibility assessment of high throughput data

on an arbitrary number of replicates. The first one, general IDR (gIDR), expands the

classic IDR by extending its EM approach to handle distributions of arbitrary dimensions

dictated by the number of replicates. The second one called meta-IDR (mIDR), resembles

a meta-analysis procedure where local IDR scores are calculated for every pair of

replicates and combined using standard probability rules. The third method is based on

the coefficient of variation rather than IDR. It ranks features in terms of their intensity and

variability and uses a parametric bootstrap approach to estimate an index analogous to

the local IDR. This method is an "enhanced" Coefficient of Variation (eCV).

All three methods presented here were compared against traditional IDR in simulated and

experimental data of varying numbers of replicates. Simulated data followed three

scenarios used in the original IDR publication, demonstrating situations where the

reproducible features are a) many and strongly correlated, b) a few and weakly correlated,

and c) even fewer but strongly correlated. The methods were also compared in miR-

eCLIP (Manakov *et al.*, 2022) and RBP-eCLIP (Van Nostrand *et al.*, 2016) data. We show

that IDR is outperformed by multivariate alternatives, particularly by gIDR when many

highly correlated genuine features are present (such as miR-eCLIP, where RNAs form

chimeras with highly abundant miRNAs). Methods eCV and mIDR, on the other hand,

performed well when a small number of highly correlated reproducible features were

present (e.g., reads clustering at transcription factor or RNA binding protein binding sites).

In addition, the eCV method resulted in the most stable performance across all scenarios

when a liberal threshold on the probability is used (50%). All approaches are implemented

in the R package *eCV*.

2 MATERIALS AND METHODS

2.1. Data sets

2.1.1 Simulated data sets

Simulated data was generated using an extension of the copula mixture model presented in Li et al. (2011), assuming 1,000 omic features with values coming from two, three, and four replicates, respectively. In each simulation, the values of the i-th feature were defined as $y_{ij} = -\log\left(\frac{1}{1+e^{x_{ij}}}\right)$. The vector holding the values of feature i, $\boldsymbol{x}_i = [x_{i1}, \dots, x_{ir}]$, is assumed as coming from a mixture of two multivariate normal distributions (ϕ) of dimension r:

$$\boldsymbol{x}_i \sim \phi_0(\mu_0 \mathbf{1}, \boldsymbol{\Sigma}_0)^{(1-K_i)} \phi_1(\mu_1 \mathbf{1}, \boldsymbol{\Sigma}_1)^{K_i} \tag{1}$$

Such as

$$\phi_k \propto |\boldsymbol{\Sigma}_k^{-1}|^{\frac{1}{2}} \exp\left(-\frac{1}{2}\operatorname{tr}(\boldsymbol{\Sigma}_k^{-1}(\boldsymbol{x}_i - \mu_k \mathbf{1})(\boldsymbol{x}_i - \mu_k \mathbf{1})^T)\right) \tag{2}$$

Where the dummy variable K_i, standing for the reproducible or irreproducible classes, was sampled from a Bernoulli variable K_i with probability π.

$$K_i \sim \operatorname{Ber}(\pi) \tag{3}$$

Parameters μ_k and $\boldsymbol{\Sigma}_k$ represent a mean scalar and an $r\mathrm{x}r$ variance-covariance matrix, respectively. The quantity $\mathbf{1}$ was a vector of ones of length r. When $K_i = 0$, irreproducible features values were sampled from ϕ_0, with $\mu_0 = 0$ and $\boldsymbol{\Sigma}_0 = \mathbf{I}$ being the identity matrix (i.e., a matrix with all zeros, except for ones on the diagonal). Whenever $K_i = 1$, features values were sampled from ϕ_1.

In all cases, features were assumed to have unit variance, the same mean intensity ($\mu_1 = 2.5$) and the same correlation values. This implied following a compound-symmetric

model for the variance-covariance matrix $\boldsymbol{\Sigma}_1 = \sigma_1^2(1-\rho_1)\left[\mathbf{I} + \frac{\rho_1}{1-\rho_1}\mathbf{J}\right]$, where σ_1^2 and ρ_1 are

the reproducible class variance and correlation, respectively, $\mathbf{I}$ is an rxr identity matrix

and $\mathbf{J}$ is an rxr matrix with all ones.

A summary of each parameter's value defining each simulation scenario is presented in

Table 1. Scenario 1 corresponded to a case where most features came from the

reproducible class ($\pi = 0.65$) and were highly correlated ($\rho_1 = 0.84$). Scenario 2

corresponded to a case with few ($\pi = 0.25$) lowly correlated ($\rho_1 = 0.40$) reproducible

features. Scenario 3 represented cases with even fewer ($\pi = 0.05$) but highly correlated

($\rho_1 = 0.84$) reproducible features. The instructions to generate the simulated data can be

found in Supplementary File S1.

Table 1: Parameters values defining each simulation scenario. Each simulated

scenario (**Scenario 1-3**) was defined by assigning values to population parameters

(**Parameter values**); proportion (π), mean intensity (μ_1), variance (σ_1^2), and correlation

(ρ_1) of reproducible features.

Parameters values	Scenario 1	Scenario 2	Scenario 3
π	0.65	0.25	0.05
μ_1	2.50	2.50	2.50
σ_1^2	1.00	1.00	1.00
ρ_1	0.84	0.40	0.84

2.1.2 Experimental data sets

Experimental data came from two miR-eCLIP (Manakov *et al.*, 2022) and one RBFOX2

eCLIP (Van Nostrand *et al.*, 2016) data sets. The first scenario corresponded to miR-

eCLIP on HEK293T cells transfected with two microRNAs (miRNA), *hsa-miR-1-3p* (miR-

1) and *hsa-miR-124-3p* (miR-124). The data was arranged as intensity of messenger

RNA and miRNA interactions (mRNA:miRNA), in the form of chimeric reads along two

technical replicates. In this scenario, chimeric peaks annotated with the transfected

miRNA (i.e., mRNAs forming chimeras with either one of the exogenous miRNAs) were

used to measure performance. In this context, peaks annotated with the transfected

miRNA and classified as reproducible, were taken as "true positives". Meanwhile,

endogenous mRNA:miRNA chimeras, classified as irreproducible, were taken as "true

negatives". We justified this choice by the fact that most endogenous chimeras in this

data set had significantly lower intensities than the ones involving transfected miRNAs

(Supplementary File S2). Since only two replicates were available ($r = 2$), the only

relevant comparisons were between IDR and eCV, since both gIDR and mIDR converge

exactly to IDR when $r = 2$.

The second experimental scenario also consisted of miR-eCLIP data, but this time on

wild-type K562 cells, using two, three, and four replicates. Since all chimeras involved

endogenous RNAs, we used a different definition of "truth". In this scenario, true positives

were defined by chimeras with positive seed match on the 3' untranslated region (UTR).

A positive seed match (i.e., high-quality alignment between the seed region of a miRNA

and the complementary sequence of its target mRNAs) on the 3'UTR of a gene (the region

of the targeted gene most likely to generate stable and effective RNA duplexes) show a

potential regulatory interaction (Gu *et al.*, 2009). Seed matching analysis was performed

with *scanMiR* (v4.2.1) and *scanMiRData* (v4.2.1) (Soutschek *et al.*, 2022).

The third experimental scenario comprised RBFOX2-eCLIP data, organized as peaks

generated around clusters created by crosslinking and immunoprecipitating RBFOX2

across two, three, and four technical replicates from wild-type HEK293T cells. Peaks

enriched with the RBFOX2 motif were detected using *HOMER* (Heinz *et al.*, 2010). Peaks

enriched with the RBFOX2 motif and mapping onto introns or the 3'UTR of targeted

transcripts were taken as proxies of "truly reproducible" features. Every other type of peak

was considered "truly irreproducible." A summary of the characteristics of each

experimental scenario is provided in Table 2.

Table 2: Characteristics of each experimental data scenario.

Characteristics	Scenario 1	Scenario 2	Scenario 3
Technology	miR-eCLIP	miR-eCLIP	RBP-eCLIP
Target of immune precipitation	AGO2	AGO2	RBFOX2
Experimental condition	Transfections with miR-1 and miR-124	Wild type	Wild type
Cell type	HEK293T	K562	HEK293T
Feature type	Peaks formed by chimeric reads	Peaks formed by chimeric reads	Peaks formed by IP reads normalized by input reads

Number of replicates (r)	Two	Four	Four
Proxy for truly reproducible features ($K_i = 1$)	Chimeric peaks annotated with the transfected miRNA	Chimeric peaks on 3' UTR with positive seed matching	Peaks on introns or 3' UTR with RBFOX2 motif

The protocol details for both technologies can be found in Manakov et al. (2022) and Van

Nostrand et al. (2016), respectively. Briefly, miR-eCLIP is an extension of the standard

eCLIP protocol using an anti-AGO2 antibody and modified to enable chimeric ligation of

miRNA and target mRNA. RBFOX2 RBP-eCLIP (RBFOX2-eCLIP), on the other hand,

was obtained by applying RBP-eCLIP against RBFOX2, an RNA-binding protein with a

widespread role in several cellular mechanisms and tissue-specific effects (Arya *et al.*,

2014). RBP-eCLIP extends the single-end eCLIP protocol (Van Nostrand *et al.*, 2016) by

optimizing several aspects of the original technique, like cell input requirement, UV

crosslinking settings, and the lysis protocol.

The resulting sequencing data went through multiple steps of preprocessing and then

clusters of aligned reads were found using CLIPper (v2.0.1). For the miR-eCLIP data sets,

each cluster was annotated with the names of miRNAs responsible for that target. For the

RBFOX2 eCLIP, the IP levels were normalized with input levels to calculate fold

enrichments, and p-values were calculated using a Yates' Chi-Square test (or Fisher

Exact Test if the observed or expected read number was below 5). In all datasets, peaks

were annotated using transcript information from GENCODE release 41 (GRCh38.p13)

(Frankish *et al.*, 2022) with the following priority hierarchy to define the final annotation of

overlapping features: protein-coding transcript (CDS, UTRs, intron), followed by non-

coding transcripts (e.g., exon, intron).

2.2 Reproducibility assessment methods: models and algorithms

2.2.1 Estimation of mixture copula model parameters

Suppose $\mathbf{X} = (x_{ij})_{i=1,\dots,n}^{j=1,\dots,r}$ is a matrix with positive values standing for significance or

intensity of n omic features in r technical replicates. As in standard IDR, gIDR and mIDR,

used a copula mixture model on $\mathbf{X}$. In the case of mIDR, the model fits in every pairwise

combination of replicates. The derivation of the copula model and parameters estimation

is as follows. First, feature values are transformed according to the probability integral

transform (Savits, 1994) and re-scaled to avoid infinities:

$$u_{ij} \equiv \frac{n}{n+1}\hat{F}_j(x_{ij}). \tag{4}$$

Where $\hat{F}_j$ is the empirical cumulative probability distribution (ECDF) of omic feature values

across replicate j.

Second, the vector of model parameter values is initialized $\boldsymbol{\theta} = \boldsymbol{\theta}^{(0)} = \left(\pi_1^{(0)}, \mu_1^{(0)}, \Sigma_1^{(0)}\right)$.

Where $\Sigma_1 = \sigma_1^2(1 - \rho_1)\left[\mathbf{I} + \frac{\rho_1}{1-\rho_1}\mathbf{J}\right]$, σ_1^2 is the reproducible class variance, ρ_1 is the

reproducible class correlation, $\mathbf{I}$ is a $r\mathrm{x}r$ matrix with ones in the diagonal and zero

otherwise and $\mathbf{J}$ is $r\mathrm{x}r$ matrix with all ones. In the case of the irreproducible class, $\Sigma_0 = \mathbf{I}$.

Third, $\mathbf{X}$ is turned into "pseudo data" $\mathbf{Z} = (z_{ij})_{i=1,\dots,n}^{j=1,\dots,r}$, where $z_{ij} = G^{-1}(u_{ij}|\boldsymbol{\theta})$, with

$$u_{ij} \equiv G(z_{ij}) = \frac{\pi_1}{\sigma_1} \Phi\left(\frac{z_{ij} - \mu_1}{\sigma_1}\right) + \Phi(z_{ij}) \tag{5}$$

and Φ is the univariate standard normal cumulative density function. The value of G^{-1} is

obtained by calculating G on regular grid of 1,000 u_{ij} values followed by linear

interpolation as in Li et al. (2011).

Then, an EM algorithm on the augmented pseudo data $y_i = (K_i, z_i)$ is used to obtain $\boldsymbol{\theta}^{(t)}$

(i.e., the estimates of parameter values at iteration t) where $z_i = [z_{i1}, \dots, z_{ir}]$. After setting

$\boldsymbol{\theta} = \boldsymbol{\theta}^{(t)}$, the last two steps are repeated until convergence or a maximum value to t (t_{MAX})

are reached.

2.2.2 gIDR

The principal difference between IDR and gIDR is that the latter incorporates information

from all replicates by using the following complete log-pseudolikelihood formula:

$$l(\boldsymbol{\theta}) = \sum_{i=1}^{n} \begin{bmatrix} (1 - K_i)(\log(1 - \pi) + \log(\phi_0(z_i|\mu_0, \Sigma_0))) + \\ K_i(\log(\pi) + \log(\phi_1(z_i|\mu_1, \Sigma_1))) \end{bmatrix}. \tag{6}$$

219   where ϕ_k is the probability density function of a "General" Multivariate Normal distribution,

220   with dimension equal to the number of replicates r (see equation (2))

221   Since the model is in essence a mixture of two normal distributions on the pseudo data,

222   standard EM formulas apply (Xu *et al.*, 2016). Hence, for the Expectation step (*E-step*)

223   we have

$$K_i^{(t+1)} = E(K_i|z_{i1}, \dots, z_{ir}, \boldsymbol{\theta}^{(t)}) = \frac{\pi_1^{(t)} \phi_1\left(z_{i1}, \dots, z_{ir}|\mu_1^{(t)}, \Sigma_1^{(t)}\right)}{\sum_{k=0}^{1} \pi_k^{(t)} \phi_k\left(z_{i1}, \dots, z_{ir}|\mu_k^{(t)}, \Sigma_k^{(t)}\right)}, \tag{7}$$

224   For the Maximization step (*M-step*), we have:

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \pi_1} = \sum_{i=1}^{n} \frac{(1 - K_i)}{1 - \pi_1} - \frac{K_i}{\pi_1} = 0, \tag{8}$$

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \mu_1} = \sum_{i=1}^{n} K_i \, \boldsymbol{z}_i^T \boldsymbol{\Sigma}_1^{-1} - \mu_1 \mathbf{1}^T \boldsymbol{\Sigma}_1^{-1} \sum_{i=1}^{n} K_i = 0, \tag{9}$$

226   and

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\Sigma}_1^{-1}} = \boldsymbol{\Sigma}_1 \sum_{i=1}^{n} K_i - \sum_{i=1}^{n} K_i \, (\boldsymbol{z}_i - \mu_k \mathbf{1})(\boldsymbol{z}_i - \mu_k \mathbf{1})^T = 0 \tag{7}$$

227   Where in equation (10) we used a) the derivatives of the logarithm of a matrix's

228   determinant with respect to $\boldsymbol{\Sigma}_1^{-1}$ and b) the derivative of the trace of the product between

229   a matrix and $\boldsymbol{\Sigma}_1^{-1}$ with respect to $\boldsymbol{\Sigma}_1^{-1}$ (Dwyer, 1967).

230   The maximum likelihood estimates of each parameter in the $(t + 1)$-th iteration are given

231   by the following expressions:

$$\pi_1^{(t+1)} = \sum_{i=1}^{n} \frac{K_i^{(t+1)}}{n}, \tag{11}$$

$$\mu_1^{(t+1)} = \frac{\sum_{i=1}^{n} K_i^{(t+1)} \sum_{j=1}^{r} z_{ij}}{r \sum_{i=1}^{n} K_i^{(t+1)}} \tag{12}$$

233   and

$$\Sigma_1^{(t+1)} = \frac{\sum_{i=1}^{n} K_i^{(t+1)} \left(\mathbf{z}_i - \mu_1^{(t+1)} \mathbf{1}\right)\left(\mathbf{z}_i - \mu_1^{(t+1)} \mathbf{1}\right)^T}{\sum_{i=1}^{n} K_i^{(t+1)}}. \tag{13}$$

234   Finally, since $\Sigma_1 = \sigma_1^2 (1 - \rho_1) \left[\mathbf{I} + \frac{\rho_1}{1-\rho_1}\mathbf{J}\right]$, we have:

$$(\sigma_1^2)^{(t+1)} = \frac{\mathrm{tr}\left(\Sigma_1^{(t+1)}\right)}{r} = \frac{\sum_{i=1}^{n} K_i^{(t+1)} \sum_{j=1}^{r} \left(z_{ij} - \mu_1^{(t+1)}\right)^2}{r \sum_{i=1}^{n} K_i^{(t+1)}} \tag{8}$$

and

$$\rho_1^{(t+1)} = \frac{\mathrm{trU}\left(\Sigma_1^{(t+1)}\right)}{(\sigma_1^2)^{(t+1)} \binom{r}{2}} = \frac{2 \sum_{i=1}^{n} K_i^{(t+1)} \sum_{j\neq l} \left(z_{ij} - \mu_1^{(t+1)}\right)\left(z_{il} - \mu_1^{(t+1)}\right)}{(\sigma_1^2)^{(t+1)} r(r-1) \sum_{i=1}^{n} K_i^{(t+1)}}, \tag{9}$$

where tr and trU are the trace and upper-triangular trace of a matrix, respectively, and

$\binom{r}{2}$ is the number r replicates combinations in two. Finally, the local IDR is approximated

as $\Pr(K_i = 0 | \boldsymbol{\theta}, \mathbf{X}) \approx 1 - K_i^{(t_{MAX})}$, where t_{MAX} is the number of iterations at convergence.

Notice that when $r = 2$, the *E-step* and *M-step* formulas above reduce to ones derived for

standard IDR (see Li *et al*, 2011 and supplementary materials there).

2.2.3 mIDR

Method mIDR is derived using a simple heuristics method that resembles a meta-analysis

score (Shelby and Vaske, 2008). First, we perform standard IDR in each one of every

pairwise combination of replicates. Then, a meta score is calculated to get the probability

of a feature being irreproducible in at least one of the $\binom{r}{2}$ comparisons:

$$\Pr(K_i = 0|\boldsymbol{\theta}, \mathbf{X}) \approx 1 - \prod_{j \neq l} \Pr(K_i = 1|\boldsymbol{\theta}, x_{ij}, x_{il}) \tag{10}$$

2.2.4 eCV

The eCV method assumes omic feature values across replicates $\boldsymbol{y}_i = [y_{i1}, y_{i2}, \ldots, y_{ir}]$ are

positive and approximately Normal. If x_{ij} stands for the number of aligned sequencings

reads or another measurement of the feature's intensity, with distribution approximately

log-Normal, taking $y_{ij} = \log{(x_{ij} + \in)}$, where $\in$ is an added positive value that prevents

infinite values, yields the same distribution as in (1).

The estimated eCV is calculated as:

$$\widehat{eCV}_i = \frac{|\hat{\sigma}_i^2 - \hat{\mu}_i^2|}{\hat{\mu}_i}, \tag{11}$$

252   where $\hat{\sigma}_i^2 = \frac{1}{(r-1)} \sum_{j=1}^{r} (y_{ij} - \hat{\mu}_i)^2$ and $\hat{\mu}_i = \frac{1}{r} \sum_{j=1}^{r} y_{ij}$. Smaller values of $\widehat{eCV}_i$ would show

253   reproducibility and vice versa.

254   To make inferences on the reproducibility of feature i, we first compute the following

255   expression

$$\Pr(eCV_i \leq \widehat{eCV}_i|\boldsymbol{y}_i, \boldsymbol{\theta}, K_i = 1) \cong \hat{F}(\widehat{eCV}_i|\boldsymbol{y}_i, \boldsymbol{\theta}, K_i = 1) \tag{12}$$

256   With $\hat{F}$ being the ECDF of eCV_i. Equation (18) is analogous to a p-value. The larger the

257   value of $\widehat{eCV}_i$, the larger the value of (18).

258   Second, we apply the Bayes theorem and the chain rule of probability to connect (18)

259   with $\Pr(K_i|\boldsymbol{\theta},\mathbf{X})$:

$$\Pr(K_i = 1|\boldsymbol{\theta},\mathbf{X}) \propto$$

$$F\left(\widehat{eCV}_i\big|\boldsymbol{y}_i,\boldsymbol{\theta},K_i = 1\right)\Pr(\boldsymbol{y}_i|\boldsymbol{\theta},K_i = 1)\Pr(\boldsymbol{\theta}|K_i = 1)\Pr(K_i = 1) \qquad (19)$$

In a fully Bayesian treatment of (19), prior distributions for $\boldsymbol{\theta}$,K_i and their hyperparameters

(e.g., π) might be used. For eCV, however, we assume a somewhat radical "Empirical

Bayes" (EB) (Morris, 1983) method, where we set $\left(\widehat{\boldsymbol{\theta}}\big|K_i = 1\right) \approx \left(\hat{\pi} = 0.5, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)$. The

quantities $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\Sigma}}$ are, respectively, the vector of means and the variance-covariance

matrix across features. The values of $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\Sigma}}$ are assumed as good proxies for the

parameters of prior distribution of reproducible features.

Third, EB estimates are plugged into equation (1) to randomly sample 10,000 values of

$\boldsymbol{y}_i$ ($\widetilde{\boldsymbol{y}}_i$). eCV is calculated in every $\widetilde{\boldsymbol{y}}_i$ to obtain $\widetilde{eCV}_i$ values. The ECDF is used to obtain

$\widehat{F}$. Lastly, $\widehat{eCV}_i$ is plugged in $\widehat{F}$ to obtain an estimate of (19).

2.3 Performance assessment

2.3.1 Defining a common reproducibility index across methods

Methods were compared using estimates of $\Pr(K_i = 0|\boldsymbol{\theta},\mathbf{X})$ as reproducibility index (

$\widehat{\Pr}(K_i = 0|\boldsymbol{\theta},\mathbf{X})$). This index can be interpreted as the probability of an omic feature

belonging to the irreproducible class. In the case of IDR and its extensions, gIDR and

mIDR, the value $\Pr(K_i = 0|\boldsymbol{\theta},\mathbf{X})$ was approximated by the local IDR score (Li et. al 2011).

For eCV, $\Pr(K_i|\boldsymbol{\theta},\mathbf{X})$ was approximated using parametric bootstrap.

2.3.1 Performance quantification

In all scenarios, the range of $\widehat{\Pr}\,(K_i = 0|\boldsymbol{\theta}, \mathbf{X})$ was split into 20 thresholds accommodated in the vector $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_{20}]$. The thresholds covered the range between 0 and 100%, with each value τ_m splitting features into reproducible or not. Each threshold was calculated as $\tau_m = \frac{100}{\left(1+e^{(10-m)}\right)}$, where m is an integer from 1 to 20. The individual values of the true state of each feature i (K_i) were imposed in each type of scenario. For simulations, the value of K_i was a priori determined in each simulated data set, depending on the value of π. For the experimental data sets, the value of K_i was imposed based on biological knowledge from each system (Table 2). In all scenarios, the "true" feature state was used to estimate true positives rate (TPR), showing true reproducible signal, and true negative rates (TNR), showing true irreproducible noise.

2.3.2 Inference and summary of performance values

TPR and TNR across $\boldsymbol{\tau}$ values were calculated with the R package *pROC* (Robin *et al.*, 2011). For simulation scenarios, performance values were summarized in terms of median and 90% confidence intervals across simulations. Performance values in experimental data were summarized by the median and 90% confidence intervals across 2,000 bootstrap samples.

2.4 Software implementation and data availability

All methods and simulation scenarios are implemented as functions of the R package *eCV*. All the software required to generate the results presented here are preinstalled in a publicly available Docker image *ecv_results*. The image can be retrieved via *docker pull ecvpaper2024/ecv_results:latest*. The miR-eCLIP data from transfections are available

from the GEO accession <u>GSE198250</u> (miR-1 transfected samples, GSM5942220-1, miR-

124 transfected samples GSM5942222-3). The RBFOX2 eCLIP data are available from

the GEO accession <u>GSE248884</u> (samples GSM7921694-701). The miR-eCLIP data from

wild-type K562 cells are available upon request.

3 RESULTS

The four reproducibility assessment methods (**IDR**, **gIDR**, **mIDR**, and **eCV**) were used to

discriminate between reproducible and irreproducible features. Performance of each

method was evaluated in simulated and experimental data sets. In the case standard IDR

was used in settings with more than two replicates, only the first two were used. An

increasing number of features can significantly increase the computational costs of methods

in the IDR family. Although eCV is less affected by the total number of features, we narrowed

the analysis to only chromosome one to maintain consistency across comparisons.

Therefore, when comparing scenarios, we prioritized performance metrics based on

proportions to minimize the effect of a raw number of features. The instructions for

generating results using simulated data can be located in Supplementary File S1.

Additionally, instructions for generating results from the experimental data for miR-eCLIP

transfection, miR-eCLIP wild-type data, and RBFOX2-eCLIP are provided in

Supplementary Files S2, S3, and S4, respectively.

3.1 Methods performance in simulated scenarios

3.1.1: Simulated Scenario 1: Many highly correlated, truly reproducible features exist.

This scenario represented simulations where many highly correlated features were

reproducible (Figure 1). Performance of the IDR family in this scenario was more robust

to varying reproducibility thresholds τ than eCV, in both true positive (TPR) and true

negative rates (TNR). The method gIDR was consistently high in both TPR and TNR,

particularly for increasing number of replicates r. The performance of mIDR, on the other

hand, was more variable, with TNR increasing with r while TPR reaching lower values

than the ones achieved with standard IDR. Lastly, the performance of method eCV

marginally increased with r but did not reach the TPR performance achieved by IDR and

its extensions. Both eCV and mIDR, however, achieved competitive performance at τ's

values between 50-75%

Figure 1: Methods performance in simulated scenario 1. *Many strongly correlated*

features of genuine signal. Row panels are median and 90% credible intervals (**CI**) of true

negative rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates across 100 simulated

data sets. Column panels are varying number of replicates (r = 2, 3, or 4). The Y-axis is

TPR or TNR values and the X-axis is the threshold on the reproducibility indices.

3.1.2: Simulated Scenario 2: Few weakly correlated truly reproducible features

This scenario represented situations where most features belonged to the irreproducible

class and were lowly correlated (Figure 2). The overall performance values in this

scenario were like the ones in scenario 1, with some differences. For example, gIDR

gained even more accuracy over IDR (through increases in TNR values), but not power

(no significant difference between IDR and gIDR's TPR values). The performance of eCV

was once more overly sensitive to variations in thresholds τ. However, eCV's performance

steadily increased with r, reaching competitive values at $\tau \approx 50\%$. The method mIDR had

marginally higher TNR values compared to IDR, but its TPR values significantly

decreased with the number of replicates r.

Figure 2: Methods performance in simulated scenario 2. *Few lowly correlated features of genuine signal*. Row panels are median and 90% credible intervals (**CI**) of true negative rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates across 100 simulated data sets. Column panels stand for varying number of replicates (r = 2, 3, or 4). The Y-axis is TPR or TNR values and the X-axis is the threshold on the reproducibility indices.

3.1.3: Simulated Scenario 3: Few and highly correlated truly reproducible features.

The final simulated scenario represented situations with an even smaller number of strongly correlated reproducible features (Figure 3). In this scenario, eCV had the best overall performance across number of replicates r, achieving high TPR and TNR even for smaller thresholds τ.. Contrary to the earlier simulated scenarios, the performance of gIDR became severely affected by the increasing number of replicates r. Method mIDR,

on the other hand, systematically improved with r, achieving the best TNR values at $r = 4$.

Figure 3: Methods performance in simulated scenario 3. *Few strongly correlated features of genuine signal*. Row panels stand for median and 90% credible intervals (**CI**) of true negative rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates across 100 simulated data sets. Column panels are varying number of replicates (r = 2, 3, or 4). The Y-axis is TPR or TNR values and the X-axis is the threshold on the reproducibility indices.

3.2 Methods performance in experimental data

3.2.1: Experimental Scenario 1: miR-eCLIP data on transfected cells

The first experimental data set came from chimeric peaks data (peaks formed by chimeric reads coming from mRNA:miRNA interactions) obtained with miR-eCLIP on HEK293T cells. Peaks annotated with any of the two transfected miRNAs: *hsa-miR-124-3p* (miR-124) and *hsa-miR-1-3p* (miR-1), were used as proxies of truly reproducible features. Endogenous mRNA:miRNA chimeras, categorized as irreproducible, were considered as "true negatives." This selection was justified by the observation that the majority of endogenous chimeras in this dataset exhibited significantly lower intensities compared to those involving transfected miRNAs. Only n 🈁= 3,145). Of these, 1,560 were annotated with miR-124, while only 257 were annotated with miR-1. Confidence intervals at the 95% level suggested a high correlation of the peak intensities for both transfected miRNAs: [0.79, 0.83] for miR-124 and [0.55, 0.70] for miR-1. Since only two technical replicates were available in each transfection, only IDR and eCV were compared.

The results of the method comparisons in the miR-124 transfection resembled those obtained in simulated scenario 1 (i.e., many strongly correlated truly signal features). The results of the methods comparisons in the miR-1 transfection, on the other hand, resembled the ones in scenario 3 (i.e., small number of strongly correlated truly signal features). Although the global behavior of both IDR and eCV was like that in the simulations, eCV had significantly higher global performance than IDR, particularly for $\tau \approx 50\%$.

Figure 4: Methods performance in experimental scenario 1: *miR-eCLIP data from*

transfected cells. Row panels are median and 90% credible intervals (**CI**) of true negative

rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates. Column panels are the

transfections: *hsa-miR-124-3p* (**miR124**, left) and *hsa-miR-1-3p* (**miR1**, right). The Y-axis

is TPR or TNR values and the X-axis is the threshold on the reproducibility indices.

3.2.2: Experimental Scenario 2: miR-eCLIP data from wild-type cells.

The second experimental data set also came from chimeric peaks data obtained with

miR-eCLIP, but this time from two, three, and four replicates from wild-type K562 cells.

Only chromosome 1 peaks were used (n = 1,858). Contrary to the earlier scenario, this

one consisted of only endogenous chimeras. Therefore, only significantly enriched peaks

with positive seed matches on the 3' UTR were used as proxies of "truly reproducible"

features. Although the peaks being "truth" were strongly correlated within groups of genes

targeted by 13 miRNA species involved in those peaks (~0.9), the correlations between

groups miRNA significantly dropped.

Results resembled the ones in simulated scenario 2 (i.e., few lowly correlated truly

reproducible features). Like in scenario 2, gIDR's TNR values improved with increasing

number of replicates r, while TPR values were not significantly different than IDR's. Also,

in agreement with simulated scenario 2, eCV varied more quickly with thresholds τ than

the other methods, while also steadily improving with the number of replicates. As before,

eCV matched the performance of every other method when $\tau \approx 50\%$, except for mIDR's

TNR. The performance of mIDR also mirrored simulated scenario 2, except that here, its

TNR values were significantly higher than any other method.

Figure 5: Methods performance in experimental scenario 2: *miR-eCLIP data from wild-type cells.* Row panels are median and 90% credible intervals (**CI**) of true negative rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates. Column panels are varying number of replicates (*r* = 2, 3, or 4). The Y-axis is TPR or TNR values and the X-axis is the threshold on the reproducibility indices.

3.2.3: Experimental Scenario 3: RBFOX2 eCLIP data on wildtype cells

The third and last experimental data set came from peak data derived from an RBFOX2 eCLIP on HEK293T cells for 2, 3, and 4 replicates. Peaks enriched with the RBFOX2 binding motif and mapping onto the 3' UTR were proxies for "truly reproducible" features. Only chromosome 1 peaks were used (n = 24,703). Of these, only 5,173 fulfilled both conditions, with approximately 20% of the peaks having the RBFOX2 motif and

approximately 13% mapping onto the 3' UTR. Correlations among these peaks were also

highly significant: confidence intervals at 95% = [0.60, 0.63].

Results in this scenario resembled the ones for simulated scenario 3, distinguished by

only a handful of highly correlated truly reproducible features. As for simulated scenario

3, eCV reached higher TPR values with increasing r while systematically keeping higher

TNR, particularly at τ around 50%. Similarly, mIDR improved its TNR values for increasing

r. However, contrary to simulation results, its TPR seemed to decrease. Also, consistently

with simulated scenario 3, gIDR performance was severely affected, particularly for

increasing r.

Figure 6: Methods performance in experimental scenario 3. *RBFOX2 eCLIP data*

from wildtype cells. Row panels are median and 90% credible intervals (**CI**) of true

negative rate (**TNR**, top) and true positive rate (**TPR**, bottom) rates. Column panels are

varying number of replicates (r = 2, 3, or 4). The Y-axis is TPR or TNR values and the X-

axis is the threshold on the reproducibility indices.

4 DISCUSSION

The escalating capacity for multiplexing and diminishing sequencing costs make it

possible to work with increasing sample sizes, thereby augmenting the quality of high-

throughput data by raising power and precision (Chen et al., 2012; Li et al., 2017).

Nevertheless, reproducibility analysis is still needed to ensure valid results. This becomes

a non-trivial problem when using standard techniques, such as irreproducible discovery

rate (IDR), on more than two replicates (Yang et al., 2014). We propose three novel

methods to overcome this limitation as alternatives to the traditional IDR. Results from

both simulations and experimental data suggest that the methods presented here can

outperform IDR in several relevant biological scenarios.

Both simulated and experimental data scenarios were chosen to show some semblance

to each other. For example, the chimeric peaks in experimental scenario 1 (cells

transfected with miR-124 or miR-1) annotated with the respective transfected miRNA

resulted in significantly high correlations. These high correlations and the fact that miR-

124 had more than 83% of annotated peaks, while miR-1 had about 14%, could explain

the strong resemblance between this data set and simulation scenarios 1 and 3, standing

for many and few strongly correlated truly reproducible features, respectively. A similar

explanation can be applied to the behaviors between simulated scenarios two and three

and the second miR-eCLIP and RBFOX2 eCLIP data sets.

Of all the methods compared, the general IDR (gIDR) was particularly powerful at

detecting truly reproducible features while discarding truly irreproducible ones in many of

the scenarios tested. This behavior manifested in two opposite situations: many highly

correlated reproducible features (e.g., simulation scenario 1) and few lowly-correlated

reproducible features (e.g., simulation scenario 2). Unfortunately, the performance of

gIDR significantly dropped when even fewer but highly correlated reproducible features

were present (e.g., simulation scenario 3). In those cases, gIDR underperformed with an

increasing number of replicates. One compelling explanation for these behaviors is the

high sensitivity of multivariate normal components to the misspecification of initial

parameter values in combination with ill-conditioned variance-covariance matrices (Gray,

1994; Lo, 2011). Consider for example the determinant of the variance-covariance matrix

of the reproducibility component $\mathbf{\Sigma}_1$, $\left|\sigma_1^2(1-\rho_1)\left[\mathbf{I}+\frac{\rho_1}{1-\rho_1}\mathbf{J}\right]\right| = (\sigma_1^2)^r(1+(r-1)\rho_1)(1-$

$\rho_1)^{r-1}$, where as before, σ_1^2 and ρ_1 the reproducible class variance and correlation. If,

say $\sigma_1^2 = 1$ and $\rho_1 = 0.9$, then the determinant $|\mathbf{\Sigma}_1| = \frac{9(1+r)}{10^r}$ quickly approaches zero as

r grows, making $\mathbf{\Sigma}_1$ near singular. This, combined with near-boundary values of the mixing

parameter π (e.g., 0.05, as in simulated scenario 3), might artificially push features into

one mixture components, regardless of the actual reproducibility state. Potential solutions

to these issues are application of regularization during estimations of $\mathbf{\Sigma}_1$ (Liu and Palomar,

2019) and preoptimized initial values of parameters parameter values (Panić *et al.*, 2020).

Meta IDR (mIDR) performance varied more than gIDR, being particularly high in

simulated scenario 3. Although the experimental RBFOX2 eCLIP scenario resembled

simulated scenario 3, the method did not perform as well, particularly in terms of TPR.

Nevertheless, it seems that mIDR would increase its performance with an increasing

number of replicates and be particularly good in situations where only a few features are

truly reproducible. Although the performance of mIDR would increase with r, since

traditional IDR is applied in every pairwise combination, the number of total comparisons

would increase quadratically. Although the calculation of IDR in every pair of replicates

can be achieved in parallel, it is also worth mentioning potentially high computational

costs if several replicates are jointly analyzed.

The global performance of eCV was variable across scenarios. Although with the highest

sensitivity to the choice of the threshold on the reproducibility values, eCV was

undoubtedly the more consistent method across varying numbers of replicates. eCV was

also less potent than IDR when most features were reproducible and highly correlated.

These facts point to eCV being a more stringent method than IDR, potentially due to the

heavy prior information integrated into the model via Empirical Bayes. Nevertheless, eCV

still detected the most reproducible features (e.g., those with higher feature intensities,

Supplementary Figure S1) while calling fewer false positives. In addition, eCV also had

significantly more global accuracy, most clear in the miR-124 miR-eCLIP experiment. As

opposed to gIDR, eCV would also have higher performance in scenarios with fewer

reproducible features.

Further research on the applicability of a fully Bayesian treatment of eCV could alleviate

the potential stringency of the method. Nevertheless, due to the low sample sizes typically

used in high-throughput experiments, a strong prior via EB, like the one used in the *limma*

suit of models (Ritchie et al., 2015) could augment power. The ability to detect highly

reproducible features, combined with its high TNR across different scenarios, makes eCV

a competitive method, particularly when a threshold of 50% was used. This threshold is

also compelling since it matches the intuition of calling features as reproducible; their

values surpass what is expected from entirely random behavior.

All the methods were compared on the same scale. In the case of IDR and its extensions,

this reduced to working with the so-called local IDR. In the case of eCV, this involved

using the probability of the estimated eCV being significantly larger than values expected

for reproducible features. Although a method for FDR control via eCV was not tested, a

similar approach to the one used by IDR could be implemented. This would involve

plugging the eCV's reproducibility indices into a multiple comparison approach, like the

one adopted by IDR following Sun and Cai (2007).

We have shown that multivariate alternatives to standard IDR might perform highly, but

choosing the best model still is a non-trivial issue. Although the results shown here supply

some guidelines, due to most biological problems not having a clear sign of "truth,"

deciding when to use one or another method is not trivial. Therefore, deciding which

method would perform the best would depend on various aspects, such as what type of

statistical error one wishes to control the most. Due to its robustness to varying sample

sizes, stable behavior, and high accuracy, we favor eCV as a suitable high-performance

method across many contrasting scenarios.

Conflict of interest

Eclipsebio funded and conducted the research presented in this paper. The authors are

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