Phase diversity-based wavefront sensing for fluorescence microscopy

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16 Abstract

Fluorescence microscopy is an invaluable tool in biology, yet its performance is compromised
when the wavefront of light is distorted due to optical imperfections or the refractile nature of
the sample. Such optical aberrations can dramatically lower the information content of images
by degrading image contrast, resolution, and signal. Adaptive optics (AO) methods can sense
and subsequently cancel the aberrated wavefront, but are too complex, inefficient, slow, or
expensive for routine adoption by most labs. Here we introduce a rapid, sensitive, and robust
wavefront sensing scheme based on phase diversity, a method successfully deployed in
astronomy but underused in microscopy. Our method enables accurate wavefront sensing to
less than $\lambda/35$ root mean square (RMS) error with few measurements, and AO with no
additional hardware besides a corrective element. After validating the method with simulations,
we demonstrate calibration of a deformable mirror > 100-fold faster than comparable methods
(corresponding to wavefront sensing on the ~100 ms scale), and sensing and subsequent
correction of severe aberrations (RMS wavefront distortion exceeding $\lambda/2$), restoring diffraction-
limited imaging on extended biological samples.

32 Introduction

Fluorescence microscopy is invaluable in biological research due to its contrast,
resolution, speed, and potential for live imaging. However, the refractile nature of biological
tissues or misaligned optics often cause undesirable bending of illumination and emission light,
introducing wavefront distortion (**Fig. 1a**). Such 'optical aberrations' prevent diffraction-limited
imaging, degrading image contrast, resolution, and signal.

Adaptive optics technologies (AO[1,2]) attempt to restore imaging performance by (i)
sensing the distorted wavefront and (ii) applying a corrective wavefront of equal amplitude but
opposite sign (typically via an adaptive element such as a deformable mirror or spatial light
modulator), thereby suppressing aberrations. The key to implementing effective AO is the
sensing, which would ideally be performed as rapidly and accurately as possible. Despite its
importance, there is no consensus on best practices for wavefront sensing, which remains an
active area of research.
Wavefront sensing methods can be broadly classified as 'direct' or 'indirect', which offer
their own advantages and disadvantages (**Fig. 1b**). Both classes have been used effectively for
AO in confocal[3-5], multiphoton[6-10], light sheet[11-13], and super-resolution imaging[14-16]. Direct
approaches use dedicated hardware (often a Shack-Hartmann wavefront sensor, SHWFS) to
rapidly sense the wavefront, and are most effective in conjunction with point-like sources
('guide stars') that are usually exogenously introduced[6,17,18]. Typical disadvantages of direct
approaches include the need to form an image (which requires sufficient ballistic signal to reach
the detector), limiting these methods to weakly scattering samples; the cost of the sensing
hardware (which can be considerable[5]); low sensitivity of the sensing hardware; and differential
aberrations in the sensing vs. imaging paths.
By contrast, indirect sensing methods reconstruct the wavefront from a sequence of
intensities or images, each with a corresponding change applied to the adaptive element[3,8-10,19].
Advantages of indirect approaches include their relatively low cost and simple hardware (since
no wavefront sensor is needed) and improved performance in opaque tissue (since the
requirement for ballistic signal is usually relaxed). The Achilles heel of indirect methods is their
slow speed, as the need for many sequential measurements and the associated computational
burden results in slow wavefront sensing that often far exceeds the time for image acquisition.
Within the plethora of indirect sensing methods, phase diversity methods (PD[20-23]) are
under-explored in fluorescence microscopy. Originally developed for astronomical applications,
phase diversity approaches can efficiently estimate both an extended object and the associated
aberrated wavefront from as few as two images. Despite its potential, we are aware of only a
handful of studies that have attempted to apply PD-based sensing to biological samples[24-26].
Here we present a rapid and accurate implementation of PD and demonstrate its use in
wavefront sensing and AO. First, we conducted simulations to benchmark our algorithm,
exploring the regime in which the algorithm excels. Second, we built a simple widefield
microscope incorporating a deformable mirror, demonstrating that our PD implementation can
calibrate the mirror orders of magnitude faster than comparable indirect methods, and faster
than a commercially available Shack-Hartmann wavefront sensor (SHWFS) while providing
similar accuracy (producing a flat wavefront within 15 nm root mean square (RMS) error).
Finally, we show that PD can be used in an AO loop, correcting severe optical aberrations
induced within biological specimens, thereby restoring diffraction-limited imaging.
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Results

Phase diversity for fluorescence microscopy

In fluorescence imaging, we can represent the formation of the image d as a convolution
between the object f and the point spread function $s(\phi)$, $d = f * s(\phi)$. Here ϕ represents a phase
aberration, we have neglected noise, and have assumed that s is spatially invariant. For AO
applications it would be desirable to estimate ϕ to compensate for it, thereby improving image
quality. In general, f and ϕ are both unknown and thus we cannot estimate either from the
single image d. Phase diversity methods address this problem with the collection of additional
diversity images $d_1,...,d_k$ in which known diversity aberrations $\theta_1,...,\theta_k$ are purposefully
introduced. The additional information contained within such images allows us to estimate f
and the unknown aberration ϕ (**Fig. 1c**).
Previous applications of phase diversity have mainly focused on astronomical
applications in which aberrations are caused by atmospheric turbulence. We developed an
implementation well suited to fluorescence microscopy (**Supplementary Note 1**). Given the
original aberrated image d, the ideal PSF s, the additional diversity images $d_1,...,d_k$ and the
known diversity aberrations $\phi_1,...,\phi_k$, our algorithm uses the Gauss-Newton algorithm to rapidly
estimate both f and ϕ (**Methods, Supplementary Fig. 1**).
We used simulations to initially benchmark our algorithm, and to characterize its
performance (**Fig. 2, Supplementary Figs. 2, 3**). We added synthetic aberrations and noise to a
variety of noiseless phantom objects to produce aberrated images, finding that using a single
additional diversity image was sufficient to estimate the aberration, the underlying object, and
to enable image correction via AO (**Fig. 2a-c, Supplementary Fig. 2a**). Simulations also allowed
us to investigate the relative efficacy of different types of diversity aberrations (**Fig. 2d,**
Supplementary Fig. 3), the effect of increasing the number of diversity images (**Fig. 2e**), and the
influence of signal-to-noise ratio (SNR) in the aberrated image (**Supplementary Fig. 2c**). In
addition, simulations aided us in selecting model parameters for our algorithm (**Methods,**
Supplementary Fig. 2b).
For example, we found that astigmatism, primary spherical, or defocus Zernike basis
functions were effective as diversity aberrations (**Fig. 2d**), whereas coma and trefoil were not
(**Supplementary Fig. 3**). Using more diversity images provided better aberration estimation than
fewer (**Fig. 2e**), although for the diversity phases we chose, performance saturated at four
diversities (for a total of five images, including the initial aberrated image). Finally, the quality of
wavefront sensing deteriorated at lower input image SNR, although the algorithm provided
usable wavefront estimation for input root-mean-square (RMS) wavefront distortions up to
~0.25 μm even with input image SNR as low as ~3 (**Supplementary Fig. 2c**).

Rapid sensorless calibration of a deformable mirror
To test the performance of our method in an experimental setting, we built a widefield
fluorescence microscope incorporating a widely used, commercially available deformable mirror
(DM) in a conjugate pupil plane (**Supplementary Fig. 4**). The DM uses 52 electromagnetic
actuators to manipulate the shape of a reflective membrane (**Fig. 3a**), thereby introducing or

correcting aberrations, but relies on accurate calibration of each actuator before use.
Calibration in this context means determining the 'influence function' of each actuator (the
induced wavefront as a function of applied voltage). Once known, the influence functions can
be inverted, thereby generating a 'control matrix' that provides the per-actuator voltage needed
for a desired aberration (**Methods**, ref.[27]). Since calibration relies on accurate wavefront sensing,
it provides an excellent initial test of our phase diversity method.

We began by obtaining a benchmark calibration using a gold standard, commercially
available Shack-Hartmann wavefront sensor (SHWFS, **Fig. 3b**). We applied positive and negative
voltages to each DM actuator (**Fig. 3a**), imaging the associated fluorescence from a single 500
nm bead (the 'guide star') at each voltage onto the SHWFS. We then converted the measured
SHWFS images into wavefronts by using the manufacturer's software, then determined the
linear relationship between wavefront and voltage, yielding the influence functions for each
actuator (**Fig. 3b**). We empirically determined the illumination power and exposure time (1 s)
necessary to produce high quality wavefronts, finding that with these settings we could reliably
calibrate the mirror in 118 s using the SHWFS (115 s acquisition and 3 s for processing, **Fig. 3e**).

Our sensorless PD algorithm requires known diversity aberrations to sense an unknown
wavefront. Given that our microscope also incorporated a piezoelectric stage, we reasoned that
stage defocus could readily provide such known diversities and is an especially convenient
choice as it does not rely on a calibrated mirror. For each actuator, we applied the same voltages
as for the SHWFS calibration, but also applied different stage defocus values (± 2 µm, ± 1 µm, 0
µm) at each voltage and captured the associated images using our electron multiplying CCD
(EM-CCD) detector (**Fig. 3c**). Feeding each set of five images into our PD algorithm produced a
wavefront estimate for each voltage (**Methods**), subsequently enabling us to determine the
induced wavefront as a function of voltage. Unlike the SHWFS, PD cannot directly estimate
tip/tilt components in the wavefront. We thus developed an indirect method for estimating the
contribution of these Zernike modes, relying on PD's ability to estimate the object (and thus, the
tip/tilt-induced displacement of the object at each voltage, **Methods**, **Supplementary Fig. 5**).

The tip/tilt corrected influence functions closely resembled the SHWFS ground truth
(**Fig. 3d**). Even though the PD-derived influence functions required ten measurements (five
defocus values x two voltages) per actuator instead of only two, we found that the greater
sensitivity of our EM-CCD allowed us to acquire each image in only 30 ms, and with ~3-fold less
power than when using the SHWFS. Thus, including both the shorter acquisition time (53 s) and
computation time (11 s), the total time for calibrating all actuators was only 64 s, faster than the
calibration with the SHWFS (**Fig. 3e**). We note that our PD-derived calibration time is also
several hundred times faster than previous indirect methods[28,29].

To compare the PD-derived calibration more quantitatively to that derived from the
SHWFS, we conducted more detailed assays[27] in which we inverted the influence functions to
obtain control matrices, applied known Zernike aberrations (Zernike modes 1-20, each with
amplitude 150 nm) and measured the degree to which the correct aberration was applied with
the SHWFS (**Fig. 3f**). The SHWFS-calibration (**Fig. 3g**) provided near ideal performance (**Fig. 3i**,

Supplementary Fig. 6, error 0 ± 7 nm, measured across on-target as well as off-target entries in
our assay, N = 3543 measurements). Applying the same assay using the PD calibration produced
a similar result (**Fig. 3h**), albeit with more noise (**Fig. 3i, Supplementary Fig. 6,** error -1 ± 12 nm,
N = 3581). Given that the PD calibration used ~20-fold less fluorescence than the SHWFS, it is
perhaps unsurprising that it produced a noisier result. We found that using alternate diversity
phases (e.g., astigmatism, **Supplementary Fig. 7**) improved in the PD response, suggesting that
the choice of diversity may also influence PD performance in this assay. Incorporating tip/tilt
modes into our calibration was essential, as characterization assays without tip/tilt
compensation were considerably worse (**Supplementary Fig. 5**). We also found that PD yielded
calibrations of similar quality on an extended sample consisting of multiple 500 nm beads
(**Supplementary Fig. 8**), which was impossible with the SHWFS calibration scheme as
implemented here, since it required a single bead.
Perhaps most importantly, we investigated whether the PD-derived calibration could
remove system aberrations by 'flattening' the wavefront measured from our beads, a key
prerequisite for successful AO. To this end, we first used PD to estimate the wavefront from
single 500 nm beads (using the same defocus diversities as for the calibration), finding small but
non-negligible aberrations (**Fig. 3j**, typical RMS wavefront error of 46 nm measured over the
first 20 Zernike modes). These aberrations also manifested as slight asymmetries in bead
images, evident in axial views (**Fig. 3k**). Next, we implemented a feedback scheme whereby we
used our PD calibration to induce an equal and opposite wavefront, thereby reducing the
aberration (**Methods**). This method reliably reduced the wavefront error to less than 15 nm (**Fig.
3j**) within a few iterations, resulting in more symmetric bead images (**Fig. 3l**) and submicron
axial extent (**Fig. 3m**).
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Aberration correction on extended samples using phase diversity
Having demonstrated that phase diversity may be used to calibrate a deformable mirror,
we next turned to aberration correction on extended samples consisting of multiple 500 nm
fluorescent beads (**Fig. 4**). We introduced test aberrations ϕ_{Abe} of varying composition and
magnitude, collected additional images with added diversity aberrations, and evaluated the
extent of our algorithm to sense and correct the test aberrations after one or more cycles of
wavefront sensing and correction (**Fig. 4a**, **Methods**).
On extended, densely labeled imaging fields, a single cycle using the aberrated image
and four diversity images (±1 μm oblique astigmatism; ±1 μm vertical astigmatism) proved
sufficient to restore images contaminated with up to ~200 nm RMS wavefront distortion (**Fig.
4b, c**). At this level of degradation, individual beads were severely blurred and could not be
easily discerned in the input images, yet correction resulted in the same clarity as ground truth
images without system or added aberrations. As expected, increasing the magnitude of
aberrations caused progressive deterioration in the quality of our correction, yet we observed
noticeable improvement in signal even when the aberrated images were so blurred that no hint
of bead structure remained (**Fig. 4c**).

We found the Pearson correlation coefficient (PCC) between corrected and ground truth
images useful in characterizing correction performance over a variety of test aberrations (**Fig.
4d**). PCC values above ~0.9 usually were associated with corrections that provided clear
improvement over the aberrated input, and we found that most data collected with test
aberrations with < 250 nm RMS wavefront distortion fell into this category. PCC values below
0.9 usually indicated obvious aberration. Although in some cases we obtained a PCC > 0.9 even
for test aberrations with RMS wavefront distortion exceeding 300 nm, in general PCC values
exhibited a noticeable 'shoulder' at ~250 nm RMS distortion, with an increasingly steep drop
and spread in PCC extent below this shoulder.

To better understand this behavior, we also examined $\phi_{Estimate}$, the wavefront estimate
produced by phase diversity (**Fig. 4c**, **e**). If the test aberration ϕ_{Abe} was being accurately sensed,
we would expect $\phi_{Estimate}$ to closely resemble ϕ_{Abe}. When examining the RMS distortion in both
quantities, we found that they agreed well until ~250 nm, where we again observed a 'shoulder'
beyond which we observed an increasing difference between the RMS values of $\phi_{Estimate}$ and ϕ_{Abe}
(**Fig. 4e**). These results suggest that inaccurate wavefront sensing is a primary reason that
correction fails past ~250 nm RMS test aberration.

Next, we investigated methods to improve performance past this limit. First, we
explored varying the magnitude of the diversity phases. In general, we found that larger
diversity magnitudes offered better sensing, although in all cases sensing still degraded in the
presence of an increasing magnitude of test aberration (**Supplementary Fig. 9**). Second, we
varied the number and type of diversity phases, using different Zernike modes as the different
diversities (**Fig. 4f**). Using a test aberration with 250 nm RMS wavefront distortion, we found
that performance improved up to four diversity phases (the number used thus far, **Fig. 4b-e**)
and then plateaued, at least for the choice of diversities we tried here. Further testing of
different diversities (e.g., random phases) might offer improved performance with fewer
diversities. Interestingly, we also confirmed the prior result from simulations (**Supplementary
Fig. 3**), whereby coma and trefoil Zernike modes proved ineffective for wavefront sensing.
Finally, we explored the effect of multiple cycles of wavefront sensing and correction (**Fig. 4g**).
This strategy proved the most effective, as we were able to fully correct larger aberrations (up to
450 nm RMS distortion, the largest value we tested) simply by increasing the number of
correction cycles.

The time required for a single cycle of wavefront sensing and correction depended on
the magnitude of test aberration, with larger aberrations requiring more time (**Fig. 4h**). For test
aberrations with RMS wavefront distortions < 250 nm, the computational burden associated
with wavefront sensing was on par with acquisition related timing (1.7 s, including file saving
and hardware-related delays, see **Methods**). For larger aberrations, the computational burden
was several-fold larger. Nevertheless, given anticipated improvements in both data acquisition
and algorithmic efficiency, we are confident that the second-level time resolution we report for
our imaging field (512 x 512 pixels, ~53 x 53 μm^2) could be substantially reduced in the future.

Phase diversity-based adaptive optics restores cellular images contaminated with aberrations

Finally, we investigated the use of phase diversity-based wavefront sensing for AO

correction on cellular samples (**Fig. 5**). On fixed U2OS cells immunostained for microtubules

(**Fig. 5a, e**)**,** aberrations with 219 nm RMS wavefront distortion blurred images to the extent that

individual fibers were indiscernible (**Fig. 5b, f**). We used PD to sense the aberrated wavefront,

finding that we could dramatically improve image quality with only two correction cycles, largely

restoring the appearance of microtubule fibers (**Fig. 5c, g**). However, close inspection of the

corrected result also revealed that correction was incomplete (compare **Fig. 5e, g**). We suspect

the reason for this is mainly due to the out of focus fluorescence resulting from widefield

illumination and detection on these 3D samples. As our PD algorithm does not currently

account for the 3D nature of the object, it has no way to model such background, and likely

misinterprets it as out of focus background from the focused image plane as well as other

aberrations. In support of this hypothesis, we were able to partially compensate for such effects

by intentionally neglecting to correct the defocus Zernike mode (**Fig. 5d**, **Methods**). The

resulting AO correction without defocus produced better restoration than the full AO correction

(**Fig. 5h**), albeit still worse than the ground truth result (**Fig. 5e**).

We were nevertheless encouraged by the performance of our current 2D PD algorithm

on these cellular volumes, as it suggests that PD will likely perform even better on microscope

systems with optical sectioning. As another test case more suited to the widefield microscope

employed here, we examined immunostained myosin in fixed Ptk2 cells (**Fig. 5i**). As these

samples were much thinner (generally of submicron thickness), they provided a closer

approximation to a 2D biological sample than the U2OS cells. Myosin is also known to assemble

into bipolar filaments of length ~300 nm in vitro[30] and in cells[31], providing an additional test of

the resolution of our system.

In ground truth images, we resolved periodic striations of myosin along stress fibers (**Fig.**

5i, l, m), with individual puncta sized ~300 nm or above. While we could not universally resolve

the separation between bipolar myosin heads, Fourier transforms revealed spectral density out

to ~300 nm (**Fig. 5i**), and we occasionally found puncta separated by ~300 nm (**Fig. 5p**). By

contrast, introducing aberrations with 311 nm RMS wavefront distortion (**Fig. 5j**) reduced spatial

resolution to the point that myosin striations (**Fig. 5l, n**) or individual puncta (**Fig. 5p**) were

completely absent. A single cycle of PD-based AO correction largely restored imaging

performance (**Supplementary Fig. 10**), and after two cycles we could no longer discern

differences between the ground truth and AO-corrected images (**Fig. 5k, l, o, p**).

Discussion

In samples thicker than a single cell, the presence of optical aberrations hinders

diffraction-limited imaging. AO provides a means of counteracting such aberrations, yet despite

ongoing refinement, these methods remain the province of relatively few labs. We believe

broad dissemination of AO has been hampered for several key reasons. First, additional

hardware must be integrated into the optical path, which is not trivial to set up, and often adds

considerable complexity and cost relative to the base microscope. In this regard, phase diversity
(along with many other indirect methods) requires the addition of an adaptive element and
accompanying relay optics only, simplifying microscope design, and improving emission-side
light efficiency compared to direct sensing approaches. Second, AO methods require dedicated
algorithms and software for wavefront sensing and corrective feedback, which until recently
have not been freely available[27]. Finally, many AO methods (especially indirect) require
substantially more measurements (or time) to sense the aberrations than to acquire the
aberrated image. We think phase diversity is highly promising in this regard, as only a few
additional images are required to sense the wavefront, and sensing can occur on sub-second
timescales.

As we demonstrate when calibrating a deformable mirror (**Fig. 3**), our existing
implementation of phase diversity is orders of magnitude faster than related indirect
approaches[28,29], and also provides faster calibration than direct sensing using a commercially
available SHWFS. We also show that our method can correct severe aberrations induced in
biological samples (**Fig. 5**), recovering diffraction-limited performance. We believe these results
pave the way for a future generation of adaptive microscopes that do not rely on SHWFS,
minimize instrument complexity, and provide AO capability rapidly and with minimal extraneous
dose or loss in temporal resolution.

Our method can be improved, and its scope extended beyond the proof-of-concept
studies demonstrated here. First, the 3D object is not modeled in our current 2D phase diversity
method. Three-dimensional phase diversity[24] provides a more realistic imaging model and might
improve the accuracy of wavefront sensing and subsequent correction on 3D biological samples,
particularly if there is substantial out-of-focus light (**Fig. 5a-h**). Second, more accurately
modeling the PSF[32] or noise (e.g., Poisson[23] instead of Gaussian, as assumed here) might
similarly improve both wavefront sensing and object estimates. Third, here we considered only
a relatively small field of view, over which aberrations were isoplanatic. Imaging larger, more
heterogeneous samples will likely require tiling, with sensing and correction (or at least differing
object and phase estimates) applied for each tile[33,34]. Fourth, the question of which diversity
aberrations are best applied to optimally sense an aberrated wavefront is an open one[35], and
deserves further study. Fifth, although we have focused here on deformable mirror calibration
and aberration correction, our phase diversity method is likely to facilitate other methods that
rely on wavefront sensing, such as remote refocusing[36,37]. Sixth, approaches that use neural
networks[38] in conjunction with additional diversity images[25,39] for wavefront sensing may offer
improved performance in highly aberrating tissue[40], but are still quite slow compared to
classical methods like ours. An intriguing direction might be to combine these approaches,
offering improved speed and performance relative to that offered by either class alone.
Seventh, the speed of our algorithm could likely be further improved if the current MATLAB
implementation was rewritten in a compiled programming language, and perhaps if a different
optimization method were used. Finally, a clear next direction is the application of our method
to systems that provide optical sectioning, including light sheet and confocal microscopy.

Methods

Sample preparation

To adhere 500 nm diameter yellow-green fluorescent beads (ThermoFisher Scientific, F8813) to #1.5 coverslips (Thorlabs, CG15XH), we (i) cleaned coverslips with methanol; (ii) pipetted ~5 µL Poly-L-Lysine solution (Millipore Sigma, P4832-50ML) onto the center of the cleaned coverslips; (iii) waited ~5 minutes and rinsed the coverslips with methanol; (iv) allowed coverslips to dry; (v) diluted 500 nm beads ~10,000-50,000-fold in methanol; (vi) pipetted diluted bead solution onto the center of the coverslips. Beaded coverslips were then stored dry until use.

Ptk2 cells were grown in Eagle's Minimum Essential medium (ATCC, 30-2003) supplemented with 10% Fetal bovine serum (Gibco, 26140079), in a 5% CO_2 incubator. They were dissociated with 0.05% Trypsin-EDTA (Gibco, 25300054) and replated on #1.5 coverslips (Warner Instruments, 64-0734). Cells were kept in the incubator for 24 hr, before being fixed in pre-warmed (5 min in 37°C water bath) 4% formaldehyde (Electron Microscopy Sciences, 15710) in PBS solution for 5 min and permeabilized in pre-warmed (to 37°C) 0.2% Triton X-100 (Sigma, T8787) in PBS for 5 minutes. After a quick rinse with PBS, cells were incubated in 1mg/mL L-Lysine (Sigma, L5501) + 1% IgG free BSA (Biotium, 22013) in PBS solution for 15 min at room temperature. For staining, cells were incubated with mouse anti-myosin heavy chain Clone H11, IgM (Biolegend, 922704) and Rabbit anti Phospho-Myosin Light Chain 2 (Thr18/Ser19) (Cell Signaling Technology, 3674), both at 1:200 dilution in PBS containing 1% BSA. Samples were incubated in primary antibodies at 37°C for 1 hour, washed once (with 5 min incubation) in PBS containing 0.05% Tween-20 (Fisher BioReagents, BP337-500), and washed twice in PBS (5 min incubation each time). Secondary antibody staining was performed at 37°C for 1 hour, using a solution containing 1% IgG free BSA in PBS, goat anti-rabbit IgG secondary antibody labeled with Alexa Fluor 568 (ThermoFisher Scientific A-11011) at 1:50 dilution, and goat anti-mouse IgM secondary antibody labeled with Alexa Fluor 488 (ThermoFisher Scientific A-21042) at 1:400 dilution. Cells were incubated in PBS containing 0.05% Tween-20 for five minutes, then incubated twice for five minutes in PBS. Finally, immunostained cells were mounted in ProLong Gold, Antifade Mountant (Thermo Fisher, P10144), cured in the dark at room temperature for 24 hours, and stored at -20°C until imaging.

U2OS cells were fixed and immunostained for microtubules as previously described[41].

Widefield adaptive optics system

To demonstrate our phase diversity wavefront sensing method and compare its performance to Shack-Hartmann based wavefront sensing, we constructed a widefield system with deformable mirror and Shack-Hartmann sensor in the emission path.

All optics and optomechanics were bolted to a 3'x5' table (TMC,784-636-02DR and 12-416-84) floated with room air. To excite fluorescence, the beam from a 488 nm laser (Coherent, Sapphire 200 mW, 1137967) was expanded 15x (Thorlabs, GBE15-A), reflected using a

dichromatic mirror (DC, Semrock, Di03-R488-t3-25x36), and focused with lens L1 (Thorlabs, 250

mm focal length achromat, AC254-250-A-ML) at the back focal plane of a 60x, 1.2 NA water

immersion objective (OBJ, Evident Scientific, 1-U2B893) held in a rapid automated modular

microscope (RAMM) system equipped with motorized (for coarse axial objective positioning and

lateral sample positioning) and piezoelectric (150 micron travel, for fine axial sample

displacement) stages (Applied Scientific Instrumentation, RAMM-BASIC and PZ-2150-FT). We

used an automated beam shutter (Thorlabs, SH05R) and neutral density filters (Thorlabs,

NE20A-A) for beam shuttering and attenuation as needed.

Fluorescence was collected in epi-mode through the same objective and was

transmitted through L1 and DC. A 500 mm focal length achromat (L2, Thorlabs, ACT508-500-A-

ML) placed in 4f relation to L1 served to magnify and image the objective's pupil 2x (from 7.2

mm to 14.4 mm), thereby serving to nearly fill the ~15 mm diameter of our deformable mirror

(DM, Imagine Optic, MIRAO 52ES), placed at ~5 degree incidence to minimize off-axis

aberrations. Post DM, a 750 mm focal length achromat (L3, Thorlabs, ACT508-750-A-ML) placed

one focal length from the DM served to image the sample plane onto an electron multiplying

charge coupled device (EM-CCD, Oxford Instruments, iXon Ultra 888, DU-888U3-CS0-#BV). The

pixel size in image space, S_{px}, was 104 nm (= 13 μm/[(250/3)*(750/500)]) and the illuminated

area spanned the majority of the ~106.5 μm field of view. We placed a bandpass filter (EF,

Semrock, FF03-525/50) upstream of the camera to reject pump light and water-cooled the EM-

CCD to minimize fan induced vibrations (Solid State Cooling Systems, ThermoCube, 10-400-1C-1-

AR).

For some experiments, we diverted the fluorescence to a Shack-Hartmann wavefront

sensor (SHWFS, Imagine Optic, HASO4 First Shack-Hartmann Wavefront Sensor, IO-6WFS201)

with a flip mirror (FM, Newport, 8893-K) placed post-L3, prior to the EM-CCD. A 180 mm focal

length achromat (L4, Thorlabs, AC508-180-A-ML) placed in 4f relation to L3 served to demagnify

and relay the image of the objective pupil from DM to the Shack-Hartmann sensor. When using

the Shack-Hartmann wavefront sensor, we also placed an iris at the intermediate image plane

(the focal point midway between the L2/L1 lens pair) to isolate the signal from a single 500 nm

diameter fluorescent bead, and placed a bandpass emission filter (EF, Semrock, FF03-525/50) in

front of the sensor. These optics are shown in **Supplementary Fig. 4**.

Bead studies employing phase diversity used a laser power of 180 μW, measured by

placing a power sensor (PM100D Console, S170C Sensor, Thorlabs) after the objective. Bead

studies employing the SHWFS used 524 μW, and cell imaging studies 1.07 mW.

Microscope control software and hardware

The microscope was controlled using a custom-designed program in LabVIEW Version

2023 Q1 (National Instruments) which calls the Andor SDK V2.104.30084.0 (for control of the

EM-CCD camera) and the Imagine-Optic WaveKit V4.3.2 software (for control of the Shack-

Hartmann sensor and deformable mirror). The LabVIEW program also controls a Piezo stage (PZ-

2150-FT, ASI), laser shutter (SH05R, Thorlabs), and motorized flipper mount (8893-K, Newport).

The control software was run on a Colfax ProEdge™ TRX4400 Windows 10-based workstation PC
featuring an AMD Ryzen Threadripper PRO 3995WX 64-core CPU, 256 GB system RAM and an
Nvidia RTX A5000 with 24 GB video RAM. This computer was also used for all computational
processing.
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Deformable mirror voltage offset
Significant aberration is introduced when the deformable mirror has no voltage applied.
A voltage offset can be applied to cancel these and other sources of system aberrations, a
process we refer to as "flattening" the mirror. This offset then serves as the base set of voltages
to which all other voltages are added to. Such offsets were used for all experiments. An initial
set of voltages is typically provided by the DM manufacturer, but this offset does not account
for any other sources of system aberration or longer-term drift. An updated flat was generated
using the AO correction loop procedure described below.
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Phase Diversity algorithm
The goal of the phase diversity algorithm is to take the input diversity images, their
corresponding known diversity wavefronts, and parameters of the system (including numerical
aperture, 1.2; image pixel size, 104 nm; and central emission wavelength 532 nm) and return an
estimate of the unknown wavefront. We implement this algorithm in MATLAB.
When performing on-line wavefront estimation, LabVIEW initiates the algorithm using a
MATLAB script node which calls the MATLAB function `processPhaseDiversityImages`,
a wrapper function which passes the inputs from LabVIEW to MATLAB and calls the function
chain to implement the phase diversity algorithm. This set of functions performs a series of pre-
processing steps, passes the resulting inputs to the algorithm, and returns the wavefront and
object estimates as outputs which are passed back to LabVIEW.
First in this chain is the function `reconstructZernikeAberrations`, which
initiates data pre-processing and passes this result as input to the algorithm function. Image
pre-processing is handled by `fileIO_lvtiff2mat`, which transfers the saved EM-CCD
camera images from disk into MATLAB. The full-frame images are loaded from disk, cropped to
the intended size for processing, a constant background value is subtracted, and an edge-
tapering applied using a Gaussian smoothing kernel with size 10x10 pixels and standard
deviation of 3 pixels.
Additional pre-processing steps are performed by the function `zernretrieve_pre`.
First, the images passed from `fileIO_lvtiff2mat` are Fourier transformed. Second, the
Zernike polynomial functions for the modes to be estimated are computed. Finally, wavefronts
corresponding to the diversity phases are constructed from their known coefficients. The pre-
processed data and parameters are then passed as inputs to the algorithm function,
`zernretrieve_loop`.
The algorithm uses Gauss-Newton optimization to find the wavefront that minimizes an
objective function *J* (**Supplementary Note 1**). In frequency space, it is given in terms of the

optical transfer functions (OTFs, S_k), modulation transfer functions (MTFs, $|S_k|$), and frequency-
space diversity images (D_k):
442

$$J = \frac{\sum_1^K |D_k|^2 - \sum_1^K |D_k^* S_k[\phi]|^2}{\sum_1^K |S_k[\phi]|^2 + \gamma},$$ (1)

with γ being a regularization term set to 1×10^{-6} (determined from simulations, **Supplementary**
Fig. 2b) and * denotes the complex conjugate.
We express the wavefront in a basis of M Zernike polynomials and update the Zernike
coefficients **c** by computing the gradient, **g**, (an M x 1 vector) and the pseudo-Hessian, **H** , (an
M x M matrix) of the Gaussian likelihood function. The update equation from iteration q to
iteration $q + 1$ is given by:

$$\mathbf{c}^{(q+1)} = \mathbf{c}^{(q)} + \Delta\mathbf{c},$$

$$\Delta\mathbf{c} = -\mathbf{H}^{-1}\mathbf{g}.$$ (2)

In practice, the wavefront is initialized by setting all coefficients to zero $\mathbf{c}^{(0)} = \mathbf{0}.$
We evaluate J for convergence by comparing the marginal change in J to the total
change since iteration began. Further iteration is terminated if this quantity is less than 0.001, if
J begins increasing, or if the maximum number of iterations (100 in all tests) is reached. The
change in J is given by
459

$$\Delta J = \frac{|J^{(q+1)} - J^{(q)}|}{|J^{(0)} - J^{(q)}|}.$$ (3)

To construct the gradient, we evaluate

$$\mathbf{g}[\phi] = -2 \sum_{k=1}^{K} Imag\left\{H_k^*[\phi]\mathcal{F}\left\{h_k[\phi]Real\{\mathcal{F}^{-1}\{V_k[\phi]\}\}\right\}\right\},$$ (4)

where

$$V_k[\phi] = F^*[\phi]D_k - |F[\phi]|^2 S_k[\phi];$$

$\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively; $H_k(\phi)$ and $h_k[\phi]$ denote
the pupil function and its inverse Fourier transform, respectively; and $F[\phi]$ is the Fourier
transform of the object.
To reduce the dimensionality and obtain the gradient in terms of a reduced set of
Zernike coefficients, we take the inner product between this gradient and the Zernike
polynomials to produce an Mx1 gradient vector.
The Hessian **H** is constructed by calculating the elements of the matrix as

473
$$H_{mn} = \langle H_{GN}[\phi]\phi_n, \phi_m \rangle. \tag{5}$$

Following ref.[22], this expression can be evaluated as

$$H_{GN}[\phi]\phi_n = 4 \sum_{k=1}^{K} \sum_{j=1}^{k-1} Imag \left\{ H_j^*[\phi]\mathcal{F}\left\{h_j[\phi]\mathcal{F}^{-1}\{\widetilde{D}_k^*\widetilde{U}_{jk}\}\right\} - H_k^*[\phi]\mathcal{F}\left\{h_k[\phi]\mathcal{F}^{-1}\{\widetilde{D}_j^*\widetilde{U}_{jk}\}\right\}\right\}, \tag{6}$$

where

$$\widetilde{U}_{jk} = \widetilde{D}_j \mathcal{F}\left\{Imag\{h_k^*[\phi]F^{-1}\{H_k[\phi]\phi_n\}\}\right\} - \widetilde{D}_k \mathcal{F}\left\{Imag\left\{h_j^*[\phi]F^{-1}\{H_j[\phi]\phi_n\}\right\}\right\}, \tag{7}$$

$$\widetilde{D}_j = \frac{D_j}{Q^{1/2}}, \tag{8}$$

and

$$Q[\phi] = \gamma + \sum_{k=1}^{K} |S_k[\phi]|^2. \tag{9}$$

To speed up this computation, we performed the calculations on our GPU using
MATLAB's `gpuArray` functionality. Further, we optimized the calculations by avoiding looping
over the sums, but instead calculating the terms in Eq. 6 for all indices in parallel. As the outer
sum is taken from $k = 1$ to K, and the inner sum from $j = 1$ to $k - 1$, we first determined all
required index pairs and then calculated the term for the required pairs only.
To further increase the computation speed, we implemented the calculation of the
Zernike polynomials as a handle class, i.e., all required Zernike polynomials are calculated only
once and then cached, avoiding duplicated computation. The aberrations in each iteration are
then computed by multiplying the new coefficient estimates with the precomputed Zernike
polynomials.
Finally, the coefficients are updated using the previous coefficients, the Hessian, and the
gradient using Eq. 2. These steps are shown in schematic form in **Supplementary Fig. 1**.
Our implementation can estimate a wavefront in ~100 ms (e.g., for the 128 x 128 crops
around a single bead used for DM calibration, **Fig. 3**), with time cost generally increasing in
proportion to the image area (e.g., on the seconds level for the first correction cycle in the
multi-bead tests shown in **Fig. 4h**). We also found that computational cost increased as the
magnitude of wavefront distortion increased, which was due to the increased number of
required iterations.

Phase diversity-based calibration of the deformable mirror
The deformable mirror calibration procedure was implemented using the custom
LabVIEW program described above. To acquire the control matrix, negative and positive
voltages (±0.03 V) were applied to a single actuator and image stacks were acquired with axial
stage positions ±2 μm, ± 1 μm, 0 μm, with each axial defocus used as a diversity phase. Images

were exposed for 30 ms for each voltage value at each stage position, and overall acquisition
time was minimized by moving the stage during the camera readout period, resulting in a total
time of 1.03 s per actuator, including moving the stage, exposing, readout, and switching mirror
voltages. This process was repeated for each of the 52 mirror actuators. After acquisition, the
influence functions for all 52 actuators were computed off-line using MATLAB.
To reduce readout time, images were acquired with a cropped sensor mode of 512x512
pixels. For DM calibrations featuring a single bead (**Fig. 3**), images were further cropped to a size
of 128x128 pixels to minimize processing time. For calibrations featuring multiple beads the full
512x512 acquisition size was used.
Off-line phase diversity processing was performed using the script
`script_Calibration`. When using stage defocus, we computed the high NA defocus
phase[32,42] corresponding to each stage position Z:
516

517
$$\phi(u, v, Z) = Z\left[\frac{2\pi n}{\lambda}\sqrt{1 - \left(\frac{\lambda}{n}\right)^2 (u^2 + v^2)}\right] \tag{10}$$

519   where u,v ranges from 0 to $\frac{NA}{\lambda}$, n is the refractive index and λ is the emission wavelength; and
520   used these phases as the diversity phases in `reconstructZernikeAberrations`,
521   subsequently estimating the wavefront as described above. After estimating the wavefront for
522   each voltage for a given actuator, the influence function for each of the 52 actuators were
523   computed from the slopes of each coefficient value given by
524

525
$$IM_{Z,A} = \frac{dc_Z}{V_2 - V_1}, \tag{11}$$

527   where $IM_{Z,A}$ is the interaction matrix slope value representing the change in Zernike coefficient
528   dc_Z for mode Z, as DM actuator A changes voltage from V_1 to V_2. In all cases here we used V_1 = -
529   0.03V and V_2 = +0.03V. After all values were computed the control matrix was obtained by
530   pseudo-inverting the interaction matrix using the MATLAB function `pinv` with a tolerance
531   value of 0.005.
532
DM calibration with alternative diversity phases
In some experiments, we calibrated the DM using astigmatism as a diversity phase
(**Supplementary Fig. 7**), finding that this choice gave lower off-target noise than our stage
defocus-based calibration. In this case the applied astigmatism commands were derived from a
previous calibration, and we used four permutations of horizontal or oblique astigmatism with
values of ± 1 μm as our diversities. In addition to the offset voltage and diversity phase voltages,
we added additional voltages (-0.03V/+0.03V) to the actuator being measured. Wavefronts were
measured using the on-line wavefront estimation procedure described above. Off-line
processing was performed to obtain the influence functions from the slope of the individual

wavefronts for each actuator and to calculate tip/tilt from the object estimates as was done in
the standard PD DM calibration procedure.
544
Estimating tip and tilt
Tip and tilt are first-order Zernike modes which represent vertical and horizontal
translations in the image plane. These wavefront components are not accounted for by the
phase diversity algorithm but are critical to include in DM calibration to ensure that image
translations are not inadvertently included when applying commands. To estimate residual
tip/tilt that arise when different voltages are applied to the DM, we rely on object estimates
produced by PD, whereby relative differences in tip/tilt manifest as displacements in object
position resulting from the different voltages.
To estimate displacement between object estimates, we compute their cross-correlation
with sub-pixel accuracy using a single-step discrete Fourier transform algorithm (ref.[43],
https://www.mathworks.com/matlabcentral/fileexchange/18401-efficient-subpixel-image-
registration-by-cross-correlation, **Supplementary Fig. 5a**). The computed shift in each axis p_j is
given in units of pixels, which must be converted to units in pupil space to be incorporated into
our control matrix. To determine the appropriate conversion factor, we simulated object
estimates with different magnitudes of tip/tilt. Computing the translation as a function of tip/tilt
Zernike coefficient amplitude yields a conversion factor δ of 0.8352 μm image space/μm
coefficient amplitude. δ can then be used to convert p_j to coefficient amplitudes via
562

$$dc_{1,2} = p_j \frac{S_{px}}{\delta}, \qquad (12)$$

564
where $dc_{1,2}$ are the relative Zernike coefficient amplitudes for tip/tilt (units of μm) and S_{px} is the
size of one pixel (units μm).
This computation is performed when calibrating the DM, and when using phase diversity
to validate a calibration using the characterization assay (**Supplementary Fig. 5d, e**). For the DM
calibration, coefficient amplitudes as a function of actuator voltage are obtained for each of the
52 actuators using the object estimates obtained for each DM voltage pair. The slope value
(units μm coefficient amplitude/V) that is used in the interaction matrix, $IM_{(1,2),A}$ is given by:
572

$$IM_{(1,2),A} = \frac{dc_{1,2}}{V_2 - V_1}. \qquad (13)$$

574
575   When validating the calibration using phase diversity (**Supplementary Fig. 5e**), we find
576 the relative displacements between an object estimate arising from the issued command and an
577 object estimate associated with a reference wavefront without the command. Eq. 13 is then
578 applied to obtain the relative tip/tilt.
579
Shack-Hartmann operation

The SHWFS was controlled via the custom LabVIEW program described above using
WaveKit v4.3.2 SDK. For all experiments, the device was operated with an exposure time of 1 s.
The processing pipeline to go from image data to Zernike coefficients was performed on-line
using the Imagine Optic Wavekit software. First, the image data of the imaged bead was
converted to wavefront slopes. Second, using the calibration data supplied with the device the
slopes are converted to Zernike coefficients using the modal Zernike reconstruction parameter
which outputs Zernike coefficients under the Wyant convention. To facilitate conversion to ANSI
notation, we output 32 coefficients. Notation conversion was performed using custom LabVIEW
code and the resulting ANSI coefficients were cropped to the desired set of 20, including tip and
tilt.
The procedures for performing a DM calibration and its validation were identical to the
phase diversity version, except that instead of acquiring defocus steps and computing
wavefronts using the PD algorithm, wavefronts were acquired directly using the SHWFS as
described above.
595
Control Matrix Validation
We used a characterization assay to quantitatively assess calibration performance for
Zernike modes 1-20. Voltages corresponding to a 0.15 μm amplitude of each tested mode were
added to the offset voltage and applied to the DM. Defocus diversity image stacks were
acquired using the same axial defocus positions (±2 μm, ±1 μm, 0 μm), exposure time (30 ms)
and image crop size (128 x 128) as for the DM calibration. Wavefront estimates from the
resulting diversity defocus image stacks were obtained as for the calibration data. The phase
diversity algorithm estimates for modes 3-20 and tip/tilt were computed from the object
estimate as described above. As the wavefront measurement may contain nonzero values
corresponding to residual aberration, we acquired an additional reference wavefront by
applying only the offset voltage to the DM and measuring this residual aberration. This
reference wavefront was subtracted from each of the tested mode wavefronts to obtain the
wavefront corresponding to the applied command for each mode.
When assessing calibration performance using the SHWFS, we applied modes 1-20 as
above, then measured the resulting wavefront using the SHWFS. A reference wavefront with
only the offset voltage applied to DM was subtracted from the result using the Imagine Optic
SDK.
For each applied mode, the error was determined based on the target value for the
Zernike coefficients. For example, to validate mode 3 (oblique astigmatism), the expected
Zernike coefficient amplitude was 0.15 μm for oblique astigmatism and 0 for all other
coefficients. We computed the error for every element in the matrices in **Fig. 3g, h** by
subtracting the target coefficients from the measured coefficients.
The control matrix generated by the calibration is sensitive to the orientation of the
measurement device. As our EM-CCD camera and SHWFS were mounted orthogonally with
respect to each other, this change in orientation must be accounted for when validating
calibrations which were originally acquired with a different modality. To perform a SHWFS
validation of a control matrix generated using PD we first rotate the measured PD wavefronts by

-90° and then flip them up-down to obtain the orientation corresponding to the SHWFS.
Likewise, to use PD to validate a calibration acquired with the SHWFS we rotate the wavefront in
the opposite direction (**+90°**) before flipping up-down to obtain the orientation corresponding
to the EM-CCD.
To assess errors across multiple calibrations (**Fig. 3i**) we acquired three DM calibrations,
each using a single bead located in a different field of view. For each acquired calibration (for
both SHWFS- and PD-derived calibrations) we performed validation experiments using both
sensing methods (**Supplementary Fig. 6**) across three different fields of view, resulting in a total
of 9 validations for each sensing method. Each validation consists of a 20x20 matrix of
coefficients, yielding 400 points per validation for a total of N = 3600 points each. Outliers were
removed from validations using the generalized extreme Studentized deviate test for outliers
with the MATLAB function `rmoutliers`, leaving a total of N = 3543 for the SHWFS and N =
3581 for the PD-derived calibration.
636
637   *Root mean square wavefront distortion*
638   Wavefront aberrations are stated in terms of root mean square wavefront distortion of
639   phase computed across N pupil elements relative to a flat wavefront according to
640

$$RMS = \sqrt{\frac{1}{N}\sum_{n=1}^{N}|\phi_n|^2}, \qquad (14)$$

642
643   where ϕ_n is the phase for a given pupil position and n is the pupil pixel index.
644
Random Aberration Generation and Scaling
When choosing aberrations for testing AO correction, Zernike coefficients were
randomly generated mode-by-mode, with a 30% chance for the selected mode to have a non-
zero value. Tip, tilt, defocus, and modes > 20 were excluded from selection. If selected, the
random coefficient value R_C (in μm), was determined based on a variable amplitude, R_A via a
random number R_N between 0 and 1, according to
651

$$R_C = R_A\,(2\,R_N - 1). \qquad (15)$$

653
Values of R_C thus ranged from $\pm R_A$. We varied R_A between 0.1 to 0.8 when testing the effects of
increasing aberration magnitude (**Fig. 4**); R_A was fixed to 0.4 when applying random aberrations
for cellular imaging tests (**Fig. 5**).
In certain test cases, it was desirable to scale the magnitude of an aberration. The scaled
coefficients, C_{sc} were obtained by multiplying each coefficient value C_Z by the ratio of the
desired scaled wavefront RMS value, RMS_{scaled} and the original wavefront RMS value,
RMS_{base}:
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$$662 \qquad\qquad\qquad\qquad C_{sc} = C_Z \frac{RMS_{scaled}}{RMS_{base}}. \qquad\qquad\qquad\qquad (16)$$

663
664
AO Control Loop Software
AO correction was performed using a control loop implemented in the LabVIEW
software program described above. A ground truth image (i.e., no aberration and system
aberrations corrected) was acquired prior to starting the loop procedure. The AO correction
loop was initialized by first generating and applying a random test aberration. The random
aberration Zernike coefficients are then added to the diversity coefficients of each of the 5 total
diversity phases. The four non-zero diversity phases consisted of pure horizontal or oblique
astigmatism at ±1 µm amplitude (**Supplementary Fig. 7b**). Summed coefficients were converted
to DM voltages using the DM control matrix and added to the offset voltages. The offset
voltages served as the initial position for the deformable mirror, and were determined by
removing system aberrations, described below.
After applying the relevant voltages to the DM, we acquired an image corresponding to
each diversity phase. The laser was shuttered after acquisition of all diversity phases completed
to prevent excess photobleaching, and the image stack and diversity phase information was
passed to MATLAB for estimating the wavefront. After estimation, MATLAB passed the object
estimate and estimated coefficients back to LabVIEW. These coefficients were converted to
deformable mirror voltages using the control matrix and subtracted from the voltage previously
applied for each phase, thereby applying AO correction. This process of acquisition,
computation, and correction comprises one correction cycle. The process was repeated for a
desired number of correction cycles.
685
System Aberration Correction
The control loop described above can also be used to correct system aberrations (**Fig. 3j-**
m). The primary difference in procedure is that for system aberration correction, no test
aberrations are applied. Rather, an image is acquired, and the resulting wavefront sensed. Since
this wavefront contains only system aberrations, we applied multiple correction cycles until the
sensed wavefront error did not improve upon successive iterations. The voltages corresponding
to the minimized wavefront error were saved and applied as the base voltage offset.
Axial views of 500 nm beads were acquired by applying the offset voltage to be tested
(before or after correcting system aberrations) and acquiring a defocus image stack consisting
of 101 image planes spaced 100 nm apart (**Fig. 3k-m**). Axial profiles were obtained using a
binary mask to segment beads in the image area and computing the axial FWHM at the centroid
of each segmented bead using the raw maximum intensity as the center position, using code
available at https://github.com/eexuesong/RL_decon_spatial_domain/tree/main/FWHM.
699
Testing Aberration Correction on Extended Samples
To test the capability of PD to correct aberrations induced in extended samples
consisting of multiple 500 nm beads (**Fig. 4b-h**), the AO correction loop described above was
applied n = 105 times with increasing amplitudes of randomly generated test aberrations for
three, 512x512 fields of view selected to include at least 30 beads for a total of n = 315 tests.

The selected aberrations were randomly generated in each test with the random amplitude R_A
stepped from 0.1 to 0.8. We performed 10 tests each for R_A values from 0.1 to 0.3, and 15 tests
for values from 0.4 to 0.8 to account for increased variability in the outcome of random
coefficient values R_C at higher values of R_A. For display purposes in **Fig. 4d, e** and to account for
the random nature of the test aberrations, we binned the test aberration wavefront error in
fixed 50 nm intervals. Note that x values displayed in **Fig. 4d, e** correspond to the mean values
of the test aberration in each bin.
712
713   *Flat-Field Correction*
714   To correct for non-uniform illumination, a flat-field correction was applied to cellular
715   images (**Fig. 5**) after image acquisition. Flat-fielding was accomplished by dividing the original
716   images by the average of 101 images of an autofluorescent plastic slide (92001, Chroma) and
717   rescaling the quotient to a 14-bit value.
718
Simulations based on synthetic images
To evaluate the performance of our PD method, we also conducted simulations with
synthetic images (**Fig. 2a-c, Supplementary Figs. 2, 3**). We used three synthetic objects: 1) a
resolution target (used for all simulations except those shown in **Fig 2b, c**); 2) images of
microtubules acquired in a real microscope[44] (**Fig 2b**); and 3) the Fishing Boat image (**Fig 2c**,
from https://sipi.usc.edu/database/). Based on the forward imaging model (**Supplementary**
Note 1), we added synthetic aberrations to these objects, creating aberrated images; generated
additional diversity images with known diversity phases, and then used the PD algorithm to
estimate the original aberrations. We generated random aberrations according to Eq. 15 for the
first 20 Zernike modes (excluding piston, tip and tilt). In some cases, we also rescaled the
aberrations to the desired magnitudes based on Eq. 16. Diffraction-limited images were
generated by setting input aberrations to zero. The image size was set to 256 x 256 pixels in all
cases, and simulations were performed within MATLAB.
When mimicking the imaging process, we also added mixed Poisson and Gaussian noise,
generating images with SNR of
734

735 $$SNR = \frac{S}{\sqrt{S + {\sigma_G}^2}}, \qquad\qquad (17)$$

736
where S is the signal defined by the average of all pixels with intensity above a threshold (here
set as 10% of the maximum intensity of the blurred objects in the noise-free image) and σ_G is
the standard deviation of the Gaussian noise (here we used Gaussian noise with a mean value
of zero and σ_G of 10). To determine the accuracy of wavefront estimation, we computed the
root mean square (RMS) wavefront error of the difference between the PD-estimated
wavefront and test aberrated wavefront ($\phi_{Residual}$).
In exploring the effect of parameter γ (in Eq. 1) on the performance of the PD algorithm
(**Supplementary Fig. 2b**), we generated one raw aberrated image and one diversity image and
used them to reconstruct the aberrated wavefront. The test aberrations were randomly
generated and rescaled to have an RMS wavefront distortion of 150 nm. For a diversity phase,

we used 0.5 μm amplitude astigmatism. The γ parameter was varied from 10^{-1} to 10^{-30}, and for
each γ setting, 50 independent simulations were carried out to obtain the mean and standard
deviation of the RMS wavefront error. Simulations were repeated at 3 SNR levels (SNR = 5, 10
and 20).
751   To test different diversity phase choices (**Fig. 2d, Supplementary Fig. 3**), we generated
one aberrated image and one diversity image, and used them to estimate the aberrated
wavefront. Test aberrations were randomly generated and rescaled to an RMS wavefront
distortion of 150 nm. The diversity phase was chosen from one of the following types of Zernike
modes: defocus (Z=4), astigmatism (Z=5), coma (Z=8), trefoil (Z=9), spherical (Z=12), and a
combination of defocus and astigmatism (Z=4 & Z=5). For each choice, we varied the RMS
wavefront distortion of the diversity phase from 0.05 μm to 2 μm. For each diversity phase at
each RMS value, 50 independent simulations (resolution target object, varying test aberrations
and noise) were carried out to obtain the mean and standard deviation of the wavefront RMS
errors ($\phi_{Residual}$). The SNR was set at 10 in these tests.
761   To investigate the effect of varying the number of diversity phases (**Fig. 2e**), we
generated one aberrated image and different numbers of diversity images, using them to
estimate the aberrated wavefront. The diversity phases were sequentially increased to a
maximum of five: 1) astigmatism (Z=5) with amplitude of 0.5 μm; 2) astigmatism (Z=3) with
amplitude of 0.5 μm; 3) astigmatism (Z=5) with amplitude of -0.5 μm; 4) astigmatism (Z=3) with
amplitude of -0.5 μm; and 5) defocus (Z=4) with amplitude of 0.5 μm. In each simulation, the
original test aberrations were randomly generated with RMS wavefront distortion varying from
0.02 μm to 0.24 μm. For each test aberration, 50 independent simulations were carried out to
obtain the mean and standard deviation of the wavefront RMS errors ($\phi_{Residual}$). The SNR was set
to 10 in these tests.
771   To explore the performance of the PD algorithm on images at different input noise
levels (**Supplementary Fig. 2c**), we generated one aberrated image and four diversity images,
and used them to estimate the aberrated wavefront. The four diversity phases were: 1)
astigmatism (Z=5) with amplitude of 0.5 μm; 2) astigmatism (Z=3) with amplitude of 0.5 μm; 3)
astigmatism (Z=5) with amplitude of -0.5 μm; and 4) astigmatism (Z=3) with amplitude of -0.5
μm. Test aberrations were randomly generated with RMS wavefront distortion varying from
0.02 μm to 0.24 μm. For each test aberration, 50 independent simulations were carried out to
obtain the mean and standard deviation of the wavefront RMS errors ($\phi_{Residual}$). Simulations
were conducted at SNR levels of 3, 5, 10, and 20 respectively.
780

781 **Competing Interests**
782   C.J., M.G., P.J.L., and H.S. have filed a provisional patent for the technique described here.
783

Author Contributions
Conceived project and directed research: H.S. Developed and implemented core phase diversity
algorithm: M.G., N.R., Y.W., and P.J.L. Accelerated algorithm: C.J. and M.C.S. Wrote software:
C.J., M.G., and M.C.S. Designed and built optical system: C.J., M.G., and H.S. Prepared samples:
C.J., M.G., Y.S., S.K., and H.S. Performed experiments: . C.J. and M.G. Wrote paper: C.J., M.G.,
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prototype microscope constructed at NIH.
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Fig. 1, Sensing aberrated wavefronts in fluorescence microscopy. a) Left: In an ideal fluorescent
sample, spherical wavefronts are captured from the sample and converted to parallel
wavefronts by the objective lens (OBJ), yielding a flat wavefront at the pupil plane (top left).
Right: in most real refractile samples, bending of light yields a distorted wavefront with
noticeable phase variation at the pupil (top right). **b)** Adaptive optics techniques, which sense
the aberrated wavefront and use this knowledge to apply an equal and opposite corrective
wavefront using a deformable element, can be broadly divided into two classes based on the
method of wavefront sensing. In direct sensing (bottom), a dedicated wavefront sensor is used
in a separate optical path to sense aberrations. By contrast, indirect sensing (top) uses the same
optical path as for fluorescence imaging, employing algorithms to sense the wavefront based on
fluorescent images of the sample. **c)** In this work, we develop a phase diversity method for
wavefront sensing in fluorescence microscopy. Given the unknown object f and unknown
aberrated point spread function $s(\phi)$, the image d does not contain sufficient information to
infer the unknown aberration. In phase diversity, additional aberrated images $d_1,...,d_k$ are
collected, each with known diversity aberrations $\theta_1,...,\theta_k$ purposefully added. The set of diversity
images $d_1,...,d_k$ now contribute additional information, enabling us to estimate the unknown
aberration ϕ.

Fig. 2, Simulations confirm that phase diversity enables accurate wavefront sensing. Different objects (1951 USAF resolution test chart, immunolabeled microtubules in fixed cell, Fishing Boat, **a-c)** respectively, left column) are contaminated with aberrations (wavefront ϕ and corresponding point spread function (PSF) $s(\phi)$) and noise, yielding aberrated, noisy images (*Image(ϕ)*). Acquiring additional diversity images (*Image($\phi+\theta$)*) with additional known diversity phases θ enables accurate phase diversity-based estimation of the original aberration ($\phi_{Estimate}$) with small residual wavefront error ($\phi_{Residual}$). **d)** Residual root mean square (RMS) wavefront error for different choices of diversity phase, as a function of the magnitude and type of the

diversity phase. Here the input RMS wavefront distortion was 0.15 μm (dashed red line), and
the signal-to-noise ratio (SNR) of the aberrated image was 10. See also **Methods,**
Supplementary Fig. 3. Means and standard deviations from 50 simulations are shown. **e)**
Residual RMS wavefront error for different numbers of diversity images, as a function of the
magnitude of input aberration. Dashed red line indicates the y = x line, i.e., images
corresponding to points underneath this line have less wavefront distortion than the original
image. SNR of the aberrated image is 10, see also **Supplementary Fig. 2**. Means and standard
deviations from 50 simulations are shown. Scale bars in **a)** 5 μm for objects and images, 2 μm
for PSFs. Results in the last column of **a-c)** and in **d)** are obtained by using two images in our
phase diversity algorithm, the original aberrated image and one diversity image. Simulations in
d, e) are based on the USAF test target shown in **a).** See also **Methods**.

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Fig. 3, Phase diversity enables rapid calibration of a deformable mirror. a) Our 52-element
deformable mirror (left) is calibrated by applying positive and negative voltages (±V) to each
actuator (right, shown for actuator 23) and measuring the induced wavefront as a function of
applied voltage ('influence function'). **b)** Influence functions measured using direct wavefront
sensing with a commercially available Shack-Hartmann wavefront sensor (SH). **c)** For calibration
using phase diversity (PD), we acquire a series of widefield images of 500 nm beads at different
axial positions (defocus diversities) of our stage (±2, ±1, 0 μm) for each voltage, and use the
resulting wavefronts to compute influence functions **d)** that closely resemble those acquired
with SH. **e)** Despite a larger computational burden, PD results in a faster calibration (64 s) than
SH (118 s) due to the shorter acquisition time. **f)** To further investigate the accuracy of our
calibrations, we command the deformable mirror to issue Zernike modes from 1-20 (left, shown
for Zernike mode 3, oblique astigmatism), and then measure the resulting wavefront with the
SH. Ideally, the measurement would show only the target Zernike mode (yellow), but in practice
(middle) we also measure some degree of off-target Zernikes (green). Top row: wavefront

representation; bottom row: amplitudes for each Zernike mode. The measurement of the
response to each Zernike mode (1-20, ANSI convention) represents a single column in our
characterization assay matrix (right). Characterization assays for SH- **g)** and PD- **h)** derived
calibrations closely resemble each other, although the latter produces a noisier assay. Note that
the SH was used to measure responses in both cases, see also **Supplementary Fig. 6e, f** for an
example where PD is used. This is confirmed in quantitative assessment **i)** of the errors between
target and measured coefficient values for each element in SH and PD characterization matrices.
Violin plots are shown, with mean (horizontal black bars) and standard deviations (horizontal
red bars) after outlier removal, SH: 0 ± 7 nm; PD: -1 ± 12 nm (see also **Supplementary Fig. 6**).
The PD-derived calibration is of sufficient quality that it can reduce system aberrations **j)** from >
45 nm (top) to < 15 nm (bottom) root mean square error (RMSE), by iteratively measuring
wavefronts derived from images of 500 nm beads and applying equal and opposite wavefronts
via the deformable mirror. Left: wavefront representation; right: amplitudes for each Zernike
mode (3-20). Representative axial views of 500 nm beads before **k)** and after **l)** correcting
system aberrations are also shown. Images prior to correction show obvious asymmetries (red
arrow, **k)**), which are largely removed post correction, resulting in more symmetric images with
smaller axial (Z) extent **m)**, as quantified by full width at half maximum (FWHM). Scale bar in **k)**:
1 μm. Means and standard deviations in **m)** are derived from N = 25 beads.
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Fig. 4, Phase diversity-based AO on extended, multi-bead samples. a) We introduce known
aberrations (ϕ_{Abe}) to a multi-bead sample and attempt to sense and correct these aberrations
using phase diversity. We (i) collect a series of diversity images (Diversity 1, Diversity 2,...
Diversity *n*) which are then used to (ii) estimate the unknown wavefront ($\phi_{Estimate}$) and object

using phase diversity. The deformable mirror is then programmed to (iii) apply an equal but
oppositely signed wavefront, providing a corrected image. We then re-measure the wavefront
($\phi_{Residual}$) and iterate if necessary. **b)** Example images of 500 nm beads with system aberrations
corrected (ground truth, top), after inducing aberration with RMS wavefront distortion 150 nm
(middle), and after one cycle of phase diversity-based AO correction using four diversity images
(bottom). **c)** Effect of increasing aberrations. Columns from left: same test aberration as used in
b) with indicated RMS wavefront distortion; higher magnification view of associated aberrated
subimage, corresponding to red dashed rectangle in **b)**; estimated wavefront produced by phase
diversity; corresponding image after correction is applied. Rows show effect of increasing the
RMS amplitude of ϕ_{Abe}. These results were obtained with a single cycle of wavefront estimation
and correction. **d)** Assessing correction using Pearson correlation coefficient (PCC, y axis) when
applying random assortment of ϕ_{Abe} with indicated wavefront distortion (x-axis). Results are
shown for a single cycle of wavefront sensing and correction; PCC is measured relative to
ground truth images. Individual trials (N = 315) are shown as scatterplot, mean and standard
deviations are also shown, binning points in 50 nm increments. Data are pooled from three
different fields of view. **e)** As in **d)** but assessing wavefront estimation by examining RMS
wavefront distortion in estimated aberration vs. test aberration. Dashed green line indicates
values in which the input and estimated RMS values are identical. **f)** Quality of images resulting
from single correction (as assessed by PCC, y-axis) as a function of the number of diversity
phases (x-axis) used in wavefront estimation. Here the test aberration was the same shape as in
b, c), with 236 nm RMS distortion. Individual data points as well as means and standard
deviations are shown from three independent imaging fields. Diversity phases (±1 μm amplitude
oblique astigmatism, horizontal astigmatism, defocus, and spherical aberration) are shown
below the x-axis; data shown in **b-e, g, h)** all use the first four phases. **g)** Quality of correction
(PCC, y-axis) as a function of number of correction cycles (x-axis) for ϕ_{Abe} with 50 nm, 150 nm,
250 nm, 350 nm, 450 nm wavefront distortions using same data as in **b, c)**. Insets with dashed
arrows show first and sixth correction with 450 nm RMS wavefront distortion in ϕ_{Abe}. **h)** Total
time (red) and computational time (blue) for first correction, as a function of RMS magnitude of
ϕ_{Abe}. Computational time scales nonlinearly as a function of RMS magnitude due to the
increased number of iterations, whereas acquisition, file saving, and instrument overhead
contribute a fixed time cost (here 1.7 s). Bead crops in **c, g** are scaled to the same intensity
range. Scale bars: 2 μm **a, c,** insets in **g)**, 5 μm in **b)**.
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Fig. 5, Phase diversity-based AO on biological samples. a-d) U2OS cells were fixed and
immunostained for microtubules. Images of ground truth **a)**, after aberration with 219 nm RMS
wavefront distortion **b),** after resulting AO correction (second correction cycle) **c)**, and AO-
corrected but without defocus correction (also second correction cycle) **d)** are shown. **e-h)**

Higher magnification views corresponding to the red dashed rectangular region in **a)**. **i-k)** Ptk2 cells were fixed and immunolabeled against myosin heavy chain. Images of ground truth **i)**, aberrated with 311 nm RMS wavefront distortion **j)** and corrected after measuring wavefront distortion with phase diversity **k)** (result corresponds to second correction cycle, see also **Supplementary Fig. 10**). Insets show Fourier transform magnitudes (displayed after logarithmic transformation), with dashed circle indicating $1/300$ nm^{-1} spatial frequency. **l)** Line profile comparison along green (i), red (ii), and magenta (iii) lines in **i-k)**, showing that AO correction recovers peak profiles which are absent in the aberrated image. **m-o)** Higher magnification views of dashed rectangular regions in **i-k)**. **p)** Higher magnification views of green (iv), red (v), and magenta (vi) rectangles in **m-o)**. Cyan arrows highlight puncta separated by ~300 nm that are resolved in ground truth images, absent in aberrated image, and recovered in AO corrected image. Scale bars: 10 μm **a-d, i-k)**, **m-o);** 2 μm **e-h)**; 1 μm **p).** Images have been flat-fielded (**Methods**).

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