A polymer physics view of peripheral chromatin: de Gennes' self-similar carpet

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(Dated: January 5, 2024)

Using scaling arguments to model peripheral chromatin localized near the inner surface of the nuclear envelope (NE) as a flexible polymer chain, we discuss the structural properties of the peripheral chromatin composed of alternating lamin-associated domains (LADs) and inter-LADs. Modeling the attraction of LADs to NE by de Gennes' self-similar carpet model explains two major experimental observations: (*i*) The high density of chromatin close to the nuclear periphery decays to a constant density as the distance to the periphery increases. (*ii*) Due to the decreasing mesh size towards the nuclear periphery, the chromatin carpet inside NE excludes molecules above a threshold size that depends on the distance from the nuclear periphery. The exclusion size decreases with the second power of the local concentration (chromatin volume fraction) until it reaches its bulk value in the nuclear interior.

I. INTRODUCTION

Inside the highly crowded volume of the eukaryotic cell nucleus, chromatin is not randomly distributed [1–7]. While euchromatin, which allows access to lineage-specific genes, is often observed away from the nuclear surfaces, heterochromatin, which is commonly associated with gene silencing, is localized near the nuclear surfaces [4–10]. This 3d organization scheme is also one of the epigenetic factors regulating the biological characteristics and health of a cell [11, 12]. Consistently, many genetic disorders with a terminal nature, such as various types of laminopathies [13–18] and carcinoma-type cancers [19, 20], manifest themselves as alterations in peripheral chromatin. Understanding the origins of such distorted distribution patterns of the genome requires characterizing the structural and conformational properties of peripheral chromatin localized near the inner surface of the so-called nuclear envelope (NE).

Progress in sequence-based chromatin conformation capture technologies (such as Hi-C [21], ChIP [22], and DamID [23]) in addition to developments in high-resolution-microscopy (such as FISH [24]), have provided the most detailed structural properties of peripheral chromatin so far [7, 25]. These experiments report that peripheral chromatin, also referred to as lamina-associated domains (LADs), interact with nuclear boundary components and have the characteristics of gene-inactive heterochromatin. These domains have sharply defined boundaries, and their lengths range between 0.1 Mb (mega-basepairs) to 10 Mb with a median length of about $\sim$ 0.5 Mb. [7, 26–31] LADs are connected by relatively longer strands of inter-LADs, which form chromatin loops protruding towards the nuclear interior (Fig. 1). Interestingly, unlike LADs, inter-LADs exhibit the characteristics of gene-active euchromatin [31–34]. Consistently, experiments reported that large multiprotein complexes maintaining gene transcription (*e.g.*, RNA polymerase II) and enzymes involved in DNA repair machinery (*e.g.*, DNA glycosylase) interact less with LADs than with inter-LADs. [7, 9, 26] Relatedly, experiments probing chromosome structures with dextran particles of nanometer sizes showed that large probe particles could not penetrate near the nuclear boundary [35], which suggests a denser environment with smaller mesh size at the nuclear periphery, in comparison to central nuclear space.

LADs are anchored to the inner surface of NE (see Fig. 1) via molecular interactions provided by a zoo of lamina-associated proteins such as LBR, LAP1, LAP2β, LEMD2, MAN1, PRR14, *etc.* (see, *e.g.*, Refs. [7, 17, 30] and references therein). As such, given the size of LADs, it is intuitive to think that strands of LADs bind to the inner-nuclear surface via multivalent molecular contacts, which can provide cumulative interaction energies on the order of $\sim 10\ k_{\rm B}T$ [36, 37] easily, where $k_{\rm B}$ is the Boltzmann constants and T is the absolute temperature. This cumulative attraction strength can stabilize LADs strongly enough that the average distribution of peripheral chromatin may not change drastically during the lifetime of a healthy cell, even though individual chromatin segments can lose contact with the NE temporarily, and turnover can occur. Similarly, inter-LADs can interact with the nuclear pore complexes of NE via weak molecular interactions [4, 7]. This conventional picture can allow peripheral chromatin to be considered as an adsorbed polymer layer to obtain insights into peripheral chromatin structures.

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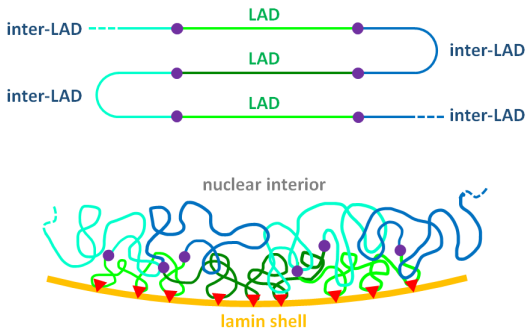

FIG. 1. Illustration of a train of alternating LADs (in purple and blue) and inter-LADs (in yellow and green) along a peripheral chromosome. Circles indicate the sharp LAD/inter-LAD borders. Different types of proteins that anchor LADs to lamina and inter-LADs to nuclear pore complexes are represented with triangles. Section of a peripheral chromosome is sketched in the open state (top) and a natural state at the nuclear periphery (bottom). The real biological environment at the eukaryotic nuclear periphery may have a higher chromatin density than shown here.

In this study, by employing de Gennes' self-similar carpet model and scaling arguments, we consider peripheral chromatin as a flexible polymer chain adsorbed onto a structureless planar, attractive surface, as a simplified view of the inner nuclear surface. Using scaling predictions, we estimate the thickness of the LAD layer and relate the polymer nature of that high-density chromatin layer to the experimentally observed steric hindrance of large molecules from the peripheral region of the nucleus. We relate our results and predictions to the available experimental data of nuclear chromatin distribution profiles, and lastly, discuss the implications and weaknesses of our model, together with future perspectives.

II. SCALING THEORY OF PERIPHERAL CHROMATIN

During interphase, individual chromosomes occupy distinct territories, but not all neighbor the nuclear boundary [2, 38]. This study only considers peripheral chromosomes, forming a straightforward polymer interface near the NE.

A. Assumptions and justifications

In polymer physics, a Kuhn segment of size $b = 2\ell_{\rm p}$ is defined as the largest segment of a polymer chain below which the polymer behaves as a rigid rod. The chemical composition of a polymer affects the persistence length $\ell_{\rm p}$, and thus, the Kuhn size b. However, when considering chains composed of a sufficiently large number of Kuhn segments (*i.e.*, $N \gg 1$), polymer chains with distinctly different chemistry can exhibit universal conformational characteristics.

Considering the persistence length of chromatin on the order of $\ell_{\rm p} \sim 1$ kb to 10 kb [39–45], even the shortest reported LADs can correspond to $N_{\rm L} \sim 10^2$ to $\sim 10^3$ Kuhn segments respectively, which are much shorter than a single chromosome (*i.e.*, with $N \sim 10^6$ Kuhn segments), but long enough to be considered as a statistical flexible polymer chain adsorbed on a surface.

Chromosomes are organized as chromatin fiber, which is a protein-DNA complex. One major protein complex organizing DNA into chromatin fiber is the histone proteins. Since the one-dimensional packing density of histone proteins varies along the fiber, chromatin is a highly heterogeneous polymer. Due to the different packing densities of histone proteins, heterochromatin and euchromatin are structurally distinct and possibly have non-identical persistence lengths [46]. For the same reason, values reported in the literature for chromatin persistence length vary largely, ranging between $\ell_{\rm p} \approx 30$ nm and $\ell_{\rm p} \approx 300$ nm [47–49]. It is not straightforward to assign a clear difference between the persistence lengths of heterochromatin and euchromatin. Thus, we assume an identical Kuhn monomer size $b \sim 10$ nm for heterochromatin-rich LADs and euchromatin-rich inter-LADs. Since scaling theories focus on the order magnitudes of the effects but not on exact numbers, ignoring all prefactors on the order of unity, assuming equal Kuhn lengths for LADs and inter-LADs should not affect our scaling analysis. Note that throughout the article, $\simeq$ signs denote scaling relations with neglected prefactors of order unity (see *e.g.*, Eq. (1) below).] Hereon, by the term "*monomer*", we refer to a "*Kuhn monomer*" of size b.

We assume that the chromatin in its native nuclear environment can be modeled as a polymer solution in the concentrated regime. That is, the volume fraction of chromatin is $\phi > 0.1$. Single-particle tracking experiments reporting Rouse-type dynamics for the subsection of chromosomes support this assumption [50, 51].

We ignore the effects of electrostatic interactions since at physiological salt conditions (*i.e.*, $c_s \approx 100$ mM), chromatin could be treated as a neutral polymer for scaling purposes [47, 52]. In our calculations, there is also no selective attraction between chromatin segments that can lead to a phase separation.

Since the characteristic size of the chromosome is an order of magnitude smaller than the curvature radius of the nucleus (*i.e.*, $\sim 10^0$ μm *vs.* $\sim 10^1$ μm), we treat the inner NE surface as planar.

To distinguish between a surface-adsorbed and a non-adsorbed chromatin monomer, the adsorption of LAD segments to NE is modeled under the "*weak adsorption*" limit. The weak adsorption scheme also considers all weak interactions such as hydrogen bonding ionic interactions, *etc.* Thus, instead of considering a subset of chromatin monomers anchored to NE by associated proteins with adsorption energies on the order of $-U \gg 1k_BT$, we assume that every chromatin monomer (whether LAD or

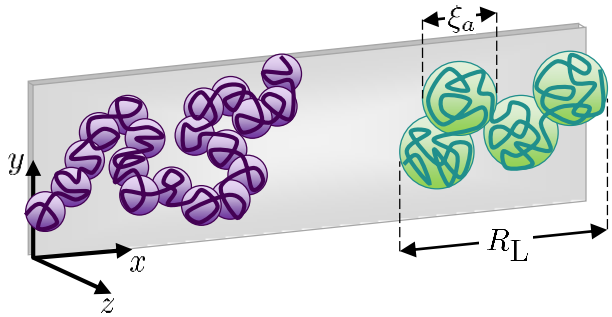

FIG. 2. Illustration of two isolated chains adsorbed on a flat surface. Both polymer chains have the same number $N_{\rm L}$ of Kuhn monomers of size b. The polymer on the left is adsorbed more strongly (with a larger ϵ) compared to the polymer on the right. For the more strongly adsorbed polymer on the left, number of monomers in an adsorption blob g_a and adsorption blob size ξ_a are smaller [see Eqs. (1) and (3)], while number of blobs $N_{\rm L}/g_a$ and end-to-end size $R_{\rm L}$ are larger [see Eq. (4)].

inter-LAD) has an equal probability of interacting with the surface with an adsorption energy of $U \approx -\epsilon kT$, where $0 < \epsilon < 1$. LADs will be distinguished from inter-LADs *via* the formation of an adsorbed layer (see below).

All parameters we consider are the averages of quantities that have Gaussian distributions. As argued above, an average number of Kuhn monomers per LAD and per inter-LAD ($N_{\rm L}$ and $N_{\rm I}$) are both considered to be much larger than unity. This regime of *long* polymers provides an excellent playground for scaling theories.

B. Single-chain adsorption model for an isolated LAD: adsorption blobs at the periphery

We begin our discussion by considering a single LAD composed of $N_{\rm L}$-monomers. While this discussion will provide insight for only an isolated LAD adsorbed on NE, it will help us introduce the concepts of chain adsorption, which will be used in the next section to describe the biologically relevant scenario of multiple independent LADs of a chromosome, overlappingly adsorbed on NE.

Consider an n-mer segment of a LAD, which is long enough to be considered as a flexible chain, but still much shorter than the whole LAD, *i.e.*, $1 \ll n \ll N_{\rm L}$. If the total attraction energy acting on all of its monomers is smaller than the thermal energy $k_{\rm B}T$, thermal fluctuations dominate over the cumulative surface attraction. As a result, the n-mer segment has an unperturbed configuration; its root-mean-square size, $r(n) \simeq bn^{\nu}$, is the same as that of an unperturbed/non-adsorbed n-mer segment. The exponent ν depends on the solvent quality, polymer volume fraction, and the molecular architecture of the polymer. In the biologically relevant concentrated regime of our concern, $\nu = 1/2$ for linear polymers [53] and $\nu < 1/2$ for cyclic polymers [54–59].

As n increases, the number of contacts that the n-mer segment can form with the surface, and hence, the magnitude of surface-attraction energy of the segment, increases. At a length scale

$$\xi_a \simeq b g_a^{\nu}, \tag{1}$$

which will be referred to as the size of an "*adsorption blob*". At this scale, the cumulative adsorption energy of monomers in contact with the surface becomes on the order of thermal energy $k_{\rm B}T$. In Eq. 1, g_a is the number of monomers per adsorption blob. The monomer volume fraction within an adsorption blob can be written as $\phi_a \simeq b^3 g_a/\xi_a^3$. Using Eq. (1), we can write the volume fraction as a function of adsorption blob size as

$$\phi_a \simeq (\xi_a/b)^{(-3\nu+1)/\nu}. \tag{2}$$

Thus, the number of monomers in contact with the surface inside an adsorption blob is $(\phi_a/b^3) \times (\xi_a^2 b)$. Here, ϕ_a/b^3 is the number density of monomers inside an adsorption blob, and $\xi_a^2 b$ is the contact volume of the blob within a distance b to the surface. Thus, the adsorption energy of a blob is $(\epsilon k_{\rm B}T) \times (\phi_a/b^3) \times (\xi_a^2 b) \simeq \epsilon k_{\rm B}T\,(\xi_a/b)^{1/\nu-1}$. Equating this energy to thermal energy $k_{\rm B}T$, we obtain the size of an adsorption blob as

$$\xi_a \simeq b\epsilon^{-\nu/(1-\nu)}, \tag{3}$$

or the number of monomers in an adsorption blob as $g_a \simeq \epsilon^{-1/(1-\nu)}$.

There are $N_{\rm L}/g_a$ adsorption blobs along a single LAD. An isolated single LAD chain adsorbed on the nuclear periphery, therefore, behaves as a two-dimensional self-avoiding random walk of adsorption blobs on the surface (Fig. 2). Such a random walk of $N_{\rm L}/g_a$ steps, each of size ξ_a, has a root-mean-square end-to-end size of $R_{\rm L} \simeq \xi_a\,(N_{\rm L}/g_a)^{3/4}$ [60], from which, we obtain

$$R_{\rm L} \simeq b\epsilon^{(3/4-\nu)/(1-\nu)} N_{\rm L}^{3/4}, \tag{4}$$

via Eq. (3). As the surface attraction becomes stronger with increasing ϵ, the adsorption blob size ξ_a shrinks [see Eq. (3)], but the number $N_{\rm L}/g_a$ of adsorption blobs per chain increases. The combined effect increases the two-dimensional end-to-end LAD size $R_{\rm L}$.

Assuming a median value of $N_{\rm L} \approx 10^3$ for a single LAD, we obtain $R_{\rm L} \simeq 100$ nm for the size of a single LAD after assuming $\epsilon = 1$. Assuming a smaller ϵ decreases the LAD's size since the segment loses contact with the surface with a weaker attraction. Nevertheless, when the distance between neighboring adsorbed LADs is smaller than $R_{\rm L}$, those LADs interact by sterically repelling each other, and the above picture for isolated LADs loses validity. As will be discussed next, such a dense LAD environment changes chain conformations and leads to a steric environment for specific molecular sizes.

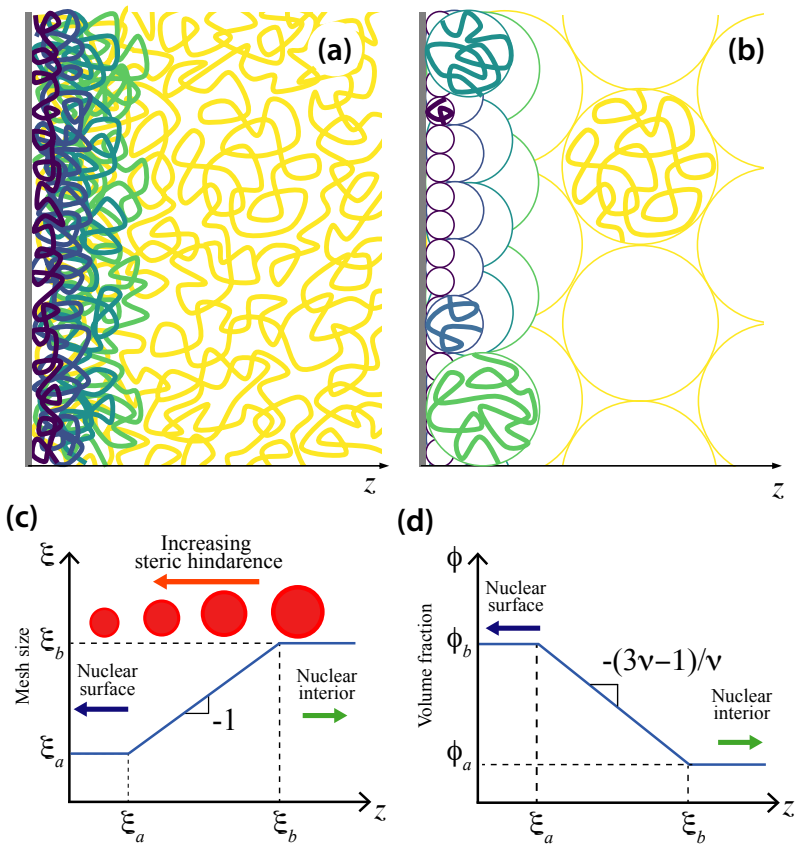

FIG. 3. Illustration of multi-chain adsorption on a flat surface (a-b), and log-log plots of chromatin volume fraction ϕ (c) and blob size ξ (d) as functions of distance z from the adsorbing surface. In panels (a) and (b), osmotic pressure on chain segments is color-coded from high pressure close to the surface (purple) to low pressure away from the surface (yellow). The adsorbed polymer shown in (a) is modeled in terms of adsorption and correlation blobs, as shown in (b). As distance to surface increases, density decreases, pressure relaxes, and the adsorption blobs on the surface (purple, size ξ_a) grow towards correlation blobs in bulk (yellow, size $\xi = \xi_b$). Chromatin has the same unperturbed chain statistics inside each blob, which is depicted only for some selected blobs for the sake of graphical simplicity.

C. Multi-chain adsorption model for a single chromosome: de Gennes' self-similar carpet

Consider an individual chromosome in contact with NE located at $z = 0$, where z defines the distance from the surface toward the nuclear interior. A certain fraction of the chromosome monomers should be in contact with the NE surface to form LADs, while the rest forms loops (*i.e.*, inter-LADs) over the surface (Fig. 1). Due to the interaction of neighboring LADs adsorbed on the surface, the picture we present in the previous section for the adsorption of a single isolated LAD loses validity. Experiments show that only $\sim 30\%$ of LADs identified by sequencing techniques are located at the nuclear periphery [7, 24, 61]. Thus, we can argue that there is insufficient space at the periphery to adsorb all LADs.

If LADs and inter-LADs are sufficiently long, we may treat different LADs and different inter-LADs of the same chromosome as independent chains. Nevertheless, we suggest a molecular picture in which, near the nuclear periphery, these LADs compete for adsorption to the same limited surface space and sterically repel each other. The inter-LADs, although not attracted to the surface energetically, can be pushed entropically from the nuclear bulk towards the periphery by the osmotic pressure of the confined chromosomes.

Monomers in steric contact with the surface gain adsorption energy $\epsilon k_{\mathrm{B}}T$ per contact. Because of this surface attraction, monomer concentration near the surface is at its maximum. Contrarily, monomers positioned away from the surface do not gain net energy, and consequently, the steric repulsion becomes more dominant, favoring a less dense chromatin environment (Fig. 3). Such a multi-chain adsorption scheme can be described by de Gennes' self-similar carpet model [62], in which the competition between steric chain-chain repulsion and chain-surface attraction determines the properties of the adsorbed polymer layer.

As discussed in the previous subsection, conformations of the adsorbed segments of this multi-chain structure are determined by the adsorption blobs of size ξ_a (i.e., Eq. (3)). The steric repulsion between chains defines a

second length scale, the "*correlation blob*" size ξ, which is the average distance between chain segments. At the length scale of the correlation blob, the inter-chain steric repulsion energy is defined to be on the order of thermal energy $k_{\rm B}T$. Hence, at length scales below ξ, inter-chain steric repulsion is compensated by thermal energy, and chain segments have conformation unperturbed by steric repulsion, which is $\xi \simeq bg^{\nu}$, where g is the number of monomers in a correlation blob. That is to say, at length scales shorter than ξ, a chain segment and its subsections are unaware of other segments and behave as if it is isolated. Here,

The above definition of correlation blob size, ξ, gives the osmotic pressure (steric repulsion energy density) as $\pi \simeq k_{\rm B}T/\xi^3$. At the surface, since the steric interactions ($k_{\rm B}T$ per correlation blob) are compensated dominantly by the adsorption energy ($k_{\rm B}T$ per adsorption blob), the correlation blob coincides with the adsorption blob (*i.e.*, $\xi = \xi_a$). The high osmotic pressure, due to the high density of monomers close to the surface, is relaxed gradually as the distance z to the surface increases. Such a relaxation increases correlation length ξ and decreases volume fraction ϕ. Inside the "*bulk*" of the nucleus, far away from NE, the correlation blob size ξ_b and the volume fraction ϕ_b are both independent of the distance z, and they are related by [53]

$$\phi_b \simeq (\xi_b/b)^{(-3\nu+1)/\nu} \qquad \text{and} \qquad \xi_b \simeq b\phi_b^{\nu/(-3\nu+1)}. \quad (5)$$

The above relations also hold at an arbitrary distance z to the surface. The volume fraction of monomers inside a correlation blob is the same as the overall local volume fraction $\phi(z)$ at z, *i.e.*,

$$\phi \simeq b^3 g/\xi^3 \simeq (\xi/b)^{(-3\nu+1)/\nu}\,. \quad (6)$$

Hence, de Gennes' self-similar carpet can be seen as layers of correlation blobs of increasing size $\xi(z)$ with increasing distance z from the surface (Figs. 3). The original model [62] argues a simple z-dependence for the correlation blob size, namely,

$$\xi(z) \simeq z \quad (7)$$

suggesting an increasing correlation length with the distance z from the surface until it reaches the bulk value at $z = \xi_b$. Plugging this relation into Eq. (6) also provides a distance-dependent density profile:

$$\phi(z) \simeq (z/b)^{(-3\nu+1)/\nu}\,. \quad (8)$$

Below, we summarize Eqs. 5- 8 predicting the chromatin volume fraction and the correlation blob size profiles between nuclear boundary and bulk as follows

$$\phi(z) \simeq \begin{cases} \phi_a & , \quad z < \xi_a \\ \phi_a\,(z/\xi_a)^{(-3\nu+1)/\nu} & , \quad \xi_a < z < \xi_b\,, \\ \phi_b & , \quad \xi_b < z \end{cases} \quad (9)$$

$$\xi(z) \simeq \begin{cases} \xi_a & , \quad z < \xi_a \\ z & , \quad \xi_a < z < \xi_b\,, \\ \xi_b & , \quad \xi_b < z \end{cases} \quad (10)$$

Eqs. 9 and 9 are plotted in Figs. 3 in log-log scales. Notably, the correlation blob size ξ is proportional to mesh size (*i.e.*, the size sterically available to nuclear proteins). Thus, the smaller the ξ is, the stronger the steric hindrance near the nuclear boundary becomes. Figs. 3 highlights three distinct compartments of our model, each with a different chromatin density and mesh (correlation) size profiles. These three regions can interpreted as follows:

1. *At close proximity of the nuclear envelope*

At $z < \xi_a$, the surface is covered with a layer of adsorption blobs of size ξ_a. This high-density layer has a thickness of ξ_a (Eq. 3) and can be interpreted as 'true' LAD layer, which can also provide extra strength to NE [63]. Thus, the nuclear volume occupied by such layer can be expressed as

$$V_{\rm LAD} \simeq V(\xi_a/R), \quad (11)$$

where V and R are nuclear volume and radius, respectively. We will further discuss the amount of LADs in the nuclear volume per nucleus in the following sections. Note that some LAD sections (or whole LADs) cannot find space on the surface to get adsorbed to, and hence, would be pushed towards the nuclear interior by the high chromatin-osmotic pressure at this layer.

2. *In between the nuclear periphery and the nuclear bulk*

As z increases along the interface region of $\xi_a < z < \xi_b$, the high pressure and density at the nuclear periphery relax towards bulk values. The relaxation of surface effects translates into increasing correlation length ξ with increasing z. This increasing correlation size can allow large molecules to approach the surface. In principle, at $\xi_a < z < \xi_b$, layers of blobs in this interface contain subsections of both LADs and inter-LADs (Fig. 3). Consistently, a recent study [64] discusses that surface-attached LADs have less gene activity than non-attached LADs, which reside at the boundary between the so-called "compartment B" and "compartment A" domains.

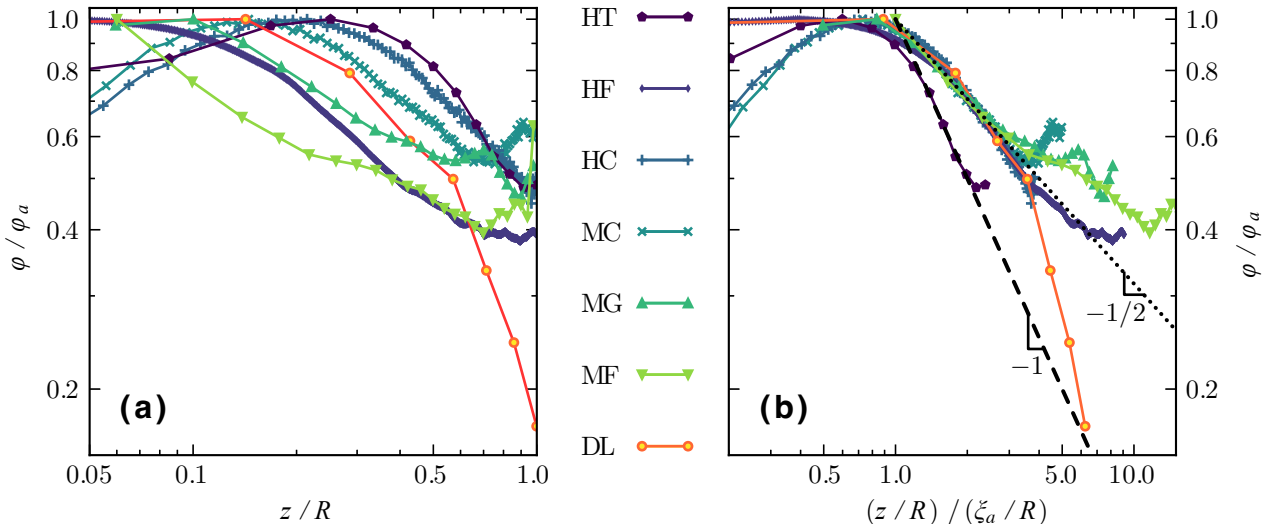

FIG. 4. Log-log plots for the dependence of chromatin volume fraction ϕ on distance to nuclear periphery z. Data are from various eukaryotic cell types; see legend and Table I. In both panels, ϕ data is scaled by the maximum measured volume fraction ϕ_a for that cell type. In the left panel, z is scaled by cell radius R, such that $z/R = 1$ is at the nuclear center. The decrease in chromatin density close to nuclear periphery at small z/R, is due to a depletion region we do not account for in our theory. In the right panel, z/R is scaled with ξ_a/R to obtain a data collapse in accordance with Eq. (9). Numerical values for the fitting-ξ_a/R are provided in Table I. The dashed and dotted lines show our predictions of $\phi/\phi_a \simeq (z/\xi_a)^{-1}$ and $\phi/\phi_a \simeq (z/\xi_a)^{-1/2}$, see Fig. 3(c) and Eq. (9) with $\nu = 1/2$ (for linear polymers) and $\nu = 2/5$ (for cyclic polymers) respectively. The data points are points to guide the eye.

3. *Inside the nuclear bulk*

Far away from the nuclear periphery at $z > \xi_b$, the abovementioned pressure relaxation is complete. The chromatin sections in this region do not *feel* the adsorbing surface from this distance. Here, the only effect of NE on the chromatin is confinement. This region of low chromatin density, referred to as "compartment A" [4, 6, 25, 29], is filled with transcriptionally-active inter-LADs, and has a constant correlation length ξ_b.

Code	Ref.	Cell type	ξ_a/R
HT	[65]	Human T-cell	0.42
HF	[66]	Human fibroblast	0.11
HC	[67]	Human cardiomyocyte	0.26
MC	[67]	Mouse cardiomyocyte	0.20
MG	[68]	Mouse ganglion†	0.12
MF	[68]	Mouse fibroblast†	0.06
DL	[69]	Drosophila larval muscle	0.16

TABLE I. Parameters, ξ_a/R, used to collapse the experimental data on a master curve given in Fig. 4(b). († Includes only gene-poor heterochromatin, but not gene-rich euchromatin.)

III. COMPARISON TO EXPERIMENTAL DATA FROM LITERATURE

To obtain a more quantitative validation of our model, we facilitate the experimentally available chromatin density profiles published in the literature, Fig. 4 and Table I. First, we compare our scaling predictions with the chromatin density profiles.

The nuclear chromatin density profiles were often reported in the fluorescence-intensity units or arbitrary units for each cell type as a function of nuclear radial distance [35, 65–73]. Therefore, to compare our results summarized in Eqs. 9 and 10) with such data, we normalize the density axis by the maximum value of the signal for the corresponding cell type. The maximum value is matched to the density near the surface in our model (i.e., ϕ_a). Then, we compare the data with our normalized density distribution (i.e., ϕ/ϕ_a) (Fig. 4).

In general, as shown in Fig. 4a, decreasing chromatin concentration with increasing distance from the nuclear periphery is a common hallmark of the density profiles in all simulations and experiments [35, 65–73] and agrees well with our model, see Eq. (9) and Fig. 3c. This can be seen more clearly in the master curve combining all the available data (Fig. 4b).

To obtain the master curve, we use ξ_a/R as a free fit parameter for all cell types (Table I). We observe that the thickness of the LAD layer relative to the nuclear radius is in the range of about $\xi_a/R \simeq 0.1 - 0.2$, which agrees well with high-resolution FISH microscopy images [24,

61]. The only exception is the human T-cell, for which we predict a ξ_a/R of about 40% (Table I). Also, note that data for this cell type is singled out from the other collapsed data sets in Fig. 4b;

While we observe decay of $\phi/\phi_a = (z/\xi_a)^{-1}$ for the human T-cell, which our model predicts for linear chains of $\nu = 1/2$ under melt conditions, all other data can be collapsed onto a universal line of $\phi/\phi_a = (z/\xi_a)^{-1/2}$, particularly near the periphery. This expression is obtained if an exponent of around $\nu = 2/5$ is chosen for the statistical size of the LAD and inter-LAD segments; see Eq. (9). While $\nu = 2/5$ describes the data relatively well, and it is larger than some predictions for cyclic chains (i.e., $\nu = 1/3$) [54, 59] but consistent with others [55–58].

In fact, an inter-LAD in between two consecutive LADS, can be attached to NE from both ends and effectively could have a cyclic polymer topology. In addition, on smaller length scales, chromatin loops are formed by the interactions between topologically associated domains along the chromosome [5, 6, 10, 25, 74, 75]. Hence, the peripheral chromatin could be modeled as a polymer that can be described by the exponent $\nu = 2/5$. In that sense, the outlier data point in Fig. 4b, human T-cells, can be rationalized by the argument that the inter-LADs in human T-cells are tethered to NE at distant positions, and as such, the peripheral chromatin cannot be treated as a concentrated solution of cyclic chains but as linear chains in this case. Moreover, the T-cell chromatin distribution being different than that of other cell types (Fig. 4b) may also be due to the constrained release of lamina-associated enhancers and genes from the nuclear envelope, as observed during Jurkat T-cell activation [76], which changes the 3D organization of the genome.

Next, we compare the correlation lengths that we predict with the available experimental data reporting the effective porosity of nuclear chromatin. By microinjecting fluorescent-labeled dextran probe molecules of varying molecular mass M into interphase HeLa nuclei, Görisch *et al.* correlated the nuclear distribution of such particles with the nuclear porosity near and away from the periphery [35]. While small dextran molecules (i.e., $d \lessapprox 20$) nm with $M \leq 77$ kDa can diffuse close to the nuclear periphery, larger molecules cannot. For those larger molecules, the dextran density increases away from the NE and levels off inside the bulk. This finding is in qualitative agreement with our prediction for the mesh size, again by using $\nu = 2/5$,

$$\xi(z) \simeq (b\phi(z))^{-2}\,. \tag{12}$$

that the mesh size ξ increases with the distance; see Eq. (10) and Fig. 3. Note that this prediction is much stronger than the square root dependence obtained by considering chromatin as hexagonal packed crystals [77].

The smallest length scale in our model is the Kuhn length (or the persistence diameter), $b = 2\ell_p$. Such a scaling model can only make limited predictions about the phenomena related to length scales smaller than b. One such prediction is that, since the Kuhn length defines the size of the smallest loops that can be formed by the polymer chain inside the dense chromatin environment, any molecule of size smaller than b will be able to find pores to diffuse along. This simple picture is adequate to describe the smallest mesh size of $b \approx 20$ nm observed by Görisch *et al.* close to the nuclear periphery. In fact, this is on the order of the Kuhn size of chromatin, which $b \simeq 10$ nm.

A second steric barrier in the experiments observed is the mesh size of $\xi \approx 60$ nm, which can be inferred as the size ξ_a of the adsorption blobs of our model [35, 77]. For the HeLa nuclei of radius $R \approx 1$ μm, this adsorption blob size value leads to $\xi_a/R \approx 0.06$, in accord with the range of values presented in Table I. We note that by plugging the values $\xi_a = 60$ nm and $b = 20$ nm into Eq. (1), we obtain the number of Kuhn monomers in an adsorption blob as $g_a \simeq 10$ for both linear ($\nu = 1/2$) and cyclic ($\nu = 2/5$) chains. Recalling that our scaling theory assumes $g_a \gg 1$, a value of $g_a \simeq 10$ is at the limiting boundary of the validity of our model.

Our predictions also provide an estimate of the volume of the LAD layer, $V_{\rm LAD}$ (Eq. 11). For a nuclear radius of around $R \simeq 5$ μm, by using the values in Table I, we obtain nucleus to LAD volume ratios of $V/V_{\rm LAD} \simeq 10 - 20\%$ with an average LAD layer thickness of $\xi_a < 1$ μm. This prediction also seems to be consistent with the electron micrograph of part of a mouse cell nucleus [31].

IV. DISSCUSSION

By revisiting de Gennes' self-similar carpet model, we argue that lamina-associated chromatin domains can be considered as multiple independent chain segments (of a chromosome) attracted to a surface with an energy of $U = -\epsilon k_{\rm B}T$ per adsorbed Kuhn monomer. In our calculations, we assume chromatin layers adjacent to the surface with a thickness ξ_a are all occupied by LADs. In reality, for short LAD and long inter-LAD segments, only a fraction of the surface is covered by isolated LADs that form adsorption blobs of size ξ_a, while the rest of the nuclear surface is covered by inter-LADs of correlation length ξ_b. However, DamID and ChIP-seq experiments associate $\sim 40\%$ of the genome as LADs [7, 9, 78]. For such long LADs, the layers close to the nuclear periphery must be filled almost entirely with adsorption blobs corresponding to LAD chains, per our assumptions.

Our calculations do not distinguish between chromatin sections permanently tethered to NE and those interacting weakly with the periphery. Nevertheless, the statistical nature of the scaling analyses favors surface localization limited by steric repulsion between LAD segments, and thus, the nature of the surface localization does not change the outcome and provides relatively good chromatin distributions as compared to experiments. However, distinguishing tethering from weak attraction or varying the chain stiffness affected the contact probabili-

ties at ~ 100 kb scale [45], but averaging over many such short segments can provide a picture similar to our predictions. It is also intuitive to think that more permanent contacts with NE can give LADs structural properties such as those observed in cyclic polymer brushes [79]. In fact, assuming a cyclic architecture for the inter-LADs that are attached from both ends to NE and protrude towards the nuclear interior, forming loops, explains the chromatin density profile data for the majority of different eukaryotic cell types.

Our model also explains the steric hindrance of large molecules from the nuclear periphery. Adsorption of LADs to NE generates a structurally distinct layer of chromatin with a mesh size smaller than in the bulk of the nucleus (*i.e.*, $\xi < \xi_b$). This picture leads to an effective hindrance mechanism. The smallest mesh size is ξ_a at the proximity of NE. The same ξ_a defines the thickness of the densest layer on the surface, *i.e.*, the thickness of the LAD layer. As the distance to NE increases, the mesh size grows towards that inside the nuclear bulk (ξ_b). This hindrance can exclude large proteins of transcription machinery from the LADs and reinforce the suppressive environment of LADs via this non-specific effect that does not depend on the nucleic acid sequence. The exact mechanism can allow larger mesh sizes, for instance, near the nuclear pore complexes around which euchromatin is relatively sparse, allowing more transcriptional activity near LADs. The hindrance of very small molecules (smaller than the persistence diameter of chromatin, ~ 20 nm) cannot explained by the mechanism of growing mesh size ξ, but could be due to convergence of nucleosome into an effective melt state.

The polymer model does not separate LADs and inter-LADs. Yet, one can conclude that chromatin sections away from the nuclear periphery are of inter-LAD nature implicitly. A more detailed model can separate these two chromatin types, possibly introducing a non-zero Flory-Hugging χ parameter and allowing two different persistence lengths. Nevertheless, we anticipate that within the limits of valid biological parameters, decreasing steric hindrance and decreasing chromatin density towards the nuclear interior will be preserved even in a more complex model. This is also evident from the experimental data in which more compact chromocenter regions of the genome have a mild effect on chromatin distribution, albeit these faces cause higher heterochromatin concentration near their surface. (See, *e.g.*, MG and MF data in Fig. 4(b) and the corresponding Ref. [68].)

Overall, our theoretical approach demonstrates that organizing a meters-long genome inside a micron-size nucleus can require the facilitation of various polymer structures, many of which have been studied extensively in the context of synthetic polymers. Revealing the structural complexity of genome organization can help us extend our knowledge of the relationship between three-dimensional chromosome organization and genetic regulation.

ACKNOWLEDGMENTS

This work has been supported by the National Science Center, Poland (Grant Polonez Bis No. 2021/43/P/ST3/01833) and The Council of Science and Technology of Turkey (TUBITAK) Grant no 122F309.

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