



39 Much of the sensitivity of pHMMs is due to their natural representation of profiles – when a  
40 collection of sequence family members is used to train the model, a pHMM captures the position-  
41 specific letter and gap frequencies inherent to the family. Profile representation of a family of  
42 sequences allows for improved search sensitivity relative to search using a collection of individual  
43 sequences (*Gribskov et al., 1987; Eddy, 2011; Krause et al., 2024*), and these families also enable  
44 faster annotation time when sequences can be compared to a single family profile rather than  
45 the family's constituent members. This pair of benefits has driven the development and use of  
46 databases of sequence families and accompanying pHMMs all across bioinformatics, e.g. (*Mistry*  
47 *et al., 2021; Mi et al., 2019; Gibson et al., 2015; Grazziotin et al., 2016; Storer et al., 2021; Huerta-*  
48 *Cepas et al., 2019*).

49 Perhaps less appreciated is the fact that pHMM-based software is typically more sensitive than  
50 BLAST even when aligning to a database of individual sequences rather than profiles (*Wheeler*  
51 *and Eddy, 2013; Steinegger and Söding, 2017; Frith, 2023; Krause et al., 2024*). Unlike other align-  
52 ment methods that compute just a single highest-scoring alignment (akin to a maximum proba-  
53 bility Viterbi alignment (*Viterbi, 1967*) in pHMM terminology *Durbin et al. (1998)*), pHMMs enable  
54 computation of support for homology based on the sum of the probabilities of *all* alignments via  
55 the Forward/Backward (FB) algorithm (*Rabiner, 1989; Krogh et al., 1994*). Posterior probabilities re-  
56 sulting from FB also enable greater alignment accuracy (*Holmes and Durbin, 1998; Do et al., 2005;*  
57 *Frith, 2023*) as well as improved mechanisms for addressing composition bias and determining  
58 alignment boundaries (*Eddy, 2008*).

59 Computing FB is computationally expensive – to align a pair of sequences, FB requires com-  
60 pletion of a dynamic programming matrix with size determined by the product of the sequence  
61 lengths, with each matrix cell requiring additional calculations to capture the sum of alignment  
62 probabilities (see *Eddy (2011)* for discussion). HMMER3 introduced a pipeline in which most can-  
63 didates are never subjected to expensive FB analysis, thanks to a series of earlier filter stages.  
64 In common use cases, the first filter of HMMER3 (called MSV) consumes ~70% of HMMER's run  
65 time, while FB consumes ~20% of time and is primarily responsible for large memory usage due  
66 to the quadratic-sized dynamic programming matrix required for recovering the alignment. FB  
67 dominates run time in cases of queries with high length or large numbers of true matches, and be-  
68 comes the primary run time bottleneck in the event of improved speed for the earlier filter phases  
69 (*Anderson and Wheeler, 2023*).

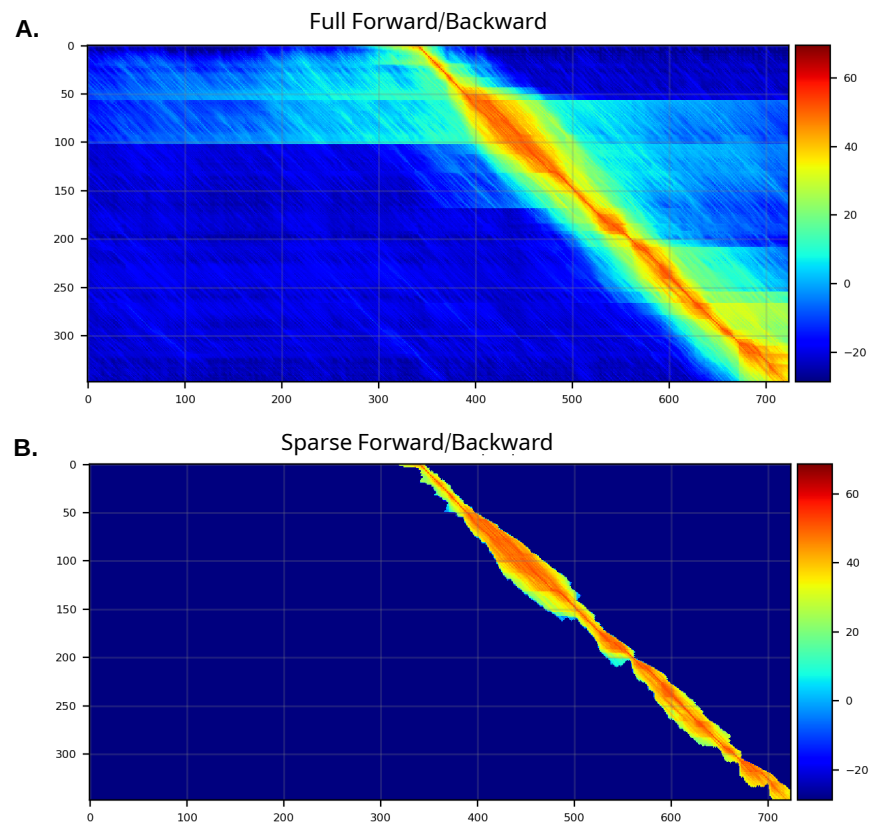
## 70 Algorithms for high speed

71 Recent years have seen remarkable speed gains for sequence alignment methods, including those  
72 targeting alignment of highly-similar sequences (*Langmead and Salzberg, 2012; Li, 2013; Kim et al.,*  
73 *2019; Edgar, 2020; Li, 2021; Sahlin, 2022; Li, 2023*) and those reporting BLAST-like sensitivity in the  
74 context of high sequence divergence (*Steinegger and Söding, 2017; Buchfink et al., 2021*). We  
75 focus here on MMseqs2 (*Steinegger and Söding, 2017*), a profile alignment tool that achieves ex-  
76 ceptional speed gains relative to BLAST. The speed of MMseqs2 is primarily due to two innovations  
77 in its analysis pipeline. First, an optimized lookup table is used to restrict alignment computation  
78 to only involve matches with two very short high scoring length-k matches; these are extended to  
79 compute an ungapped alignment filter like that used in HMMER3. Next, MMseqs2 avoids the FB  
80 alignment step entirely, simply computing a highest-scoring alignment and using that as the basis  
81 of reported results. This approach produces impressive speed gains, and benefits from the advan-  
82 tages of position-specific scores, but misses out on the benefits of the more robust FB algorithm  
83 (*Frith, 2023*), resulting in modest loss in sensitivity relative to pHMM search (*Krause et al., 2024*).  
84 Another search tool, DIAMOND (*Buchfink et al., 2021*), has also demonstrated excellent speed, but  
85 its sensitivity does not appear to rival that of MMseqs2 (*Krause et al., 2024*).

## 86 **A hybrid pipeline for high-speed and sensitive alignment**

87 Here, we describe a sequence search pipeline that utilizes the MMseqs2 software suite to rapidly  
88 identify candidate sequence matches, then employs a fast FB heuristic to improve alignment sen-  
89 sitivity. The fast heuristic limits search space in the FB dynamic programming (DP) matrix to a high-  
90 probability cloud, as demonstrated in Figure 1, and results in calculations that closely approximate  
91 the results of the full FB algorithm, while providing a substantial reduction in space requirements  
92 and run time. The sparse FB implementation, along with downstream analyses making use of the  
93 resulting sparse posterior probability matrix, are based on methods in HMMER3, but are imple-  
94 mented from scratch in the Rust programming language, with the aim of creating a modern and  
95 stable codebase for reduced runtime and memory requirements of highly-sensitive sequence an-  
96 notation. The software, called `nai1` (for `nai1` is an `alignment inference tool`), is released under an  
97 open (BSD 3-clause) licence; source code is available at <https://github.com/TravisWheelerLab/nai1>  
98 and is hosted on the official Rust package registry at <https://crates.io/crates/nai1>.

99 In the following sections, we demonstrate the efficacy of `nai1`'s sparse FB implementation,  
100 demonstrate the impact of the overall pipeline on speed and sensitivity of sequence search, and  
101 provide a thorough description of its implementation.



**Figure 1. Sparsely filled Forward/Backward matrix capturing most of the probability mass.** Top panel (A) shows heatmap of scores per cell in the Match State matrix for the sequence Q01LW8\_ORYSA aligned to the model for its matching family, DAO (FAD dependent oxidoreductase); bottom panel (B) shows the sparse set of (non-blue) cells that make up the cloud used for computing sparse Forward/Backward. The model positions are aligned along the y-axis and the sequence positions are aligned along the x-axis.

## 102 Results

103 The primary innovation of `nail` is the development of an approximate method that reduces the  
104 time and memory required for computation of the Forward and Forward/Backward (FB) algorithms  
105 for pHMMs, along with downstream analyses that are based on posterior probabilities resulting  
106 from FB (including creation of an alignment). The approach is a close cousin to the X-drop heuristic  
107 used in BLAST: start with a seed that establishes a region of interest within the DP matrix, and  
108 expand DP calculations out in both directions until pruning conditions are met – details are pro-  
109 vided in the Methods section. Figure 1 presents a single example of the reduced computation  
110 required by `nail`'s sparse Forward/Backward for a relatively short alignment of one Pfam-based  
111 pHMM against a sequence belonging to the family. Seeds for `nail`'s FB heuristic are acquired by  
112 running MMseqs2 as a subroutine for candidate identification.

113 We begin by describing the data used for evaluation, then demonstrate the space-pruning ef-  
114 ficacy of `nail`'s Cloud Search approach. We then show that annotation with `nail` significantly  
115 improves accuracy over maximum probability alignment, while adding only a small amount of  
116 processing time. Scripts and notes to reproduce benchmarking results can be found at <https://github.com/TravisWheelerLab/nail-benchmarks>.  
117

## 118 Benchmarks

### 119 Pfam domain benchmark

120 Assessment was performed primarily using a benchmark created with software (*create-profmark*)  
121 available in the HMMER3 release (*Eddy, 2011*). The benchmark consists of 1,339 families from  
122 Pfam-A v35.0 (*Mistry et al., 2021*) that could be split into a training and test set such that the test  
123 set contained at least 10 sequences and no training-test pair of sequences shares greater than  
124 25% identity. The training set defines a multiple sequence alignment for each family, and we refer  
125 to the collection of training families as the query. For each family, sequences from the group not  
126 included in the training set were down-sampled such that at most 30 sequences were used for the  
127 family and no two sequences were >50% identical; this left 25,688 total sequences, which serve  
128 as the test set. Each true test sequence was embedded in a larger unrelated sequence, to simu-  
129 late the sub-sequence nature of the protein domains in Pfam; specifically, unrelated sequences  
130 were produced by sampling from uniprot\_sprot (2023\_05), shuffling, then splicing the true test se-  
131 quence into the middle of the shuffled sequence. This set of sequences containing true positives  
132 was supplemented with 2 million additional sequences sampled and shuffled as above, but with  
133 no embedded matches. By construction, this benchmark contains cases that are highly difficult  
134 for sequence alignment tools to recognize (train and test sequences are less than 25% identical),  
135 in order to emphasize differences in sensitivity. Note that the benchmark does not include re-  
136 versed sequences, as these are prone to producing an excess of unexpected positives due to the  
137 surprising distribution high scores when aligning sequences to their reversals or even reversals of  
138 their homologs (*Glidden-Handgis and Wheeler, 2023*). For more details on benchmark construction  
139 method and philosophy, see (*Eddy, 2011*).

### 140 Long protein data set

141 Alignment with Pfam models represents a common use case for sequence alignment, but one that  
142 involves relative short sequences – the median Pfam domain length is just over 300. The purpose  
143 of `nail`'s sparse Forward/Backward implementation is to avoid calculation over a full quadratic-  
144 sized dynamic programming matrix, and longer sequences are the ones that suffer most from  
145 this quadratic scaling; we therefore performed some tests using sequences on the longer end  
146 of the protein sequence length distribution. Specifically, we captured 6 pairs of long sequences  
147 from Uniprot (Table 1), and performed experiments to assess time and space efficiency along with

148 approximation accuracy. For each pair, one sequence was designated the *query* and the other the  
149 *target*.

Query		Target	
Name	Length	Name	Length
TITIN_HUMAN	34,350	TITIN_MOUSE	35,213
EBH_STAAC	10,498	EBH_STAEQ	9,439
VLMS_LECSP	8,903	W4932_FUSPC	8,892
R1AB_CVH22	6,758	R1AB_BC512	6,793
HMCN1_HUMAN	5,635	HMCN1_MOUSE	5,634
RYR1_HUMAN	5,038	RYR1_PIG	5,035

**Table 1.** Long sequence pairs

### 150 **Analysis pipeline – a sketch**

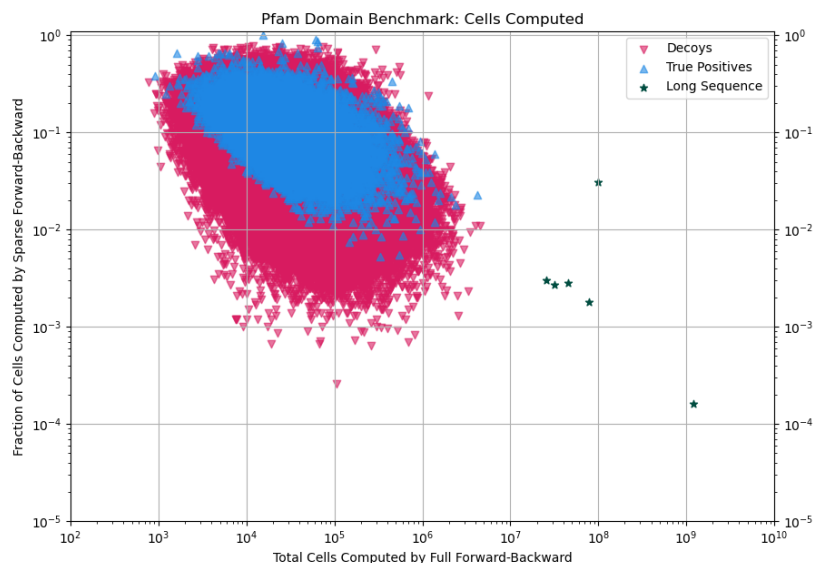
151 As a first step, the `na1` pipeline runs MMseqs2 search, which rapidly produces a set of candidate  
152 query/target pairs by performing k-mer-based seed selection followed by fast local alignment. `na1`  
153 runs the standard MMseqs2 search pipeline (with a few parameters adjusted as in Table 2): (i) a  
154 k-mer match stage identifies candidate matches based on the presence of two co-diagonal length-  
155 k matches with score above a threshold score; (ii) a parameterized number of above-threshold  
156 paired k-mer matches are extended to capture only those candidates with good-scoring ungapped  
157 alignments, then (iii) surviving candidates are aligned to produce the single highest-scoring gapped  
158 alignment for each candidate query/target pair. After running MMseqs2 search, `na1` retains all re-  
159 sults with P-value less than 0.01. The first and last positions of each surviving MMseqs2 alignment  
160 are mapped to corresponding cells (i.e. target and query positions) in a hypothetical FB alignment  
161 matrix. Using the mapped cells as a starting point, a heuristic search algorithm (Cloud Search) iden-  
162 tifies a contiguous subset of FB matrix cells with non-negligible probability. Within this reduced set  
163 of matrix cells, `na1` then completes a sparse variant of Forward/Backward, producing an over-  
164 all alignment score along with position-specific posterior probabilities that positions are aligned;  
165 these posterior probabilities are used to compute a composition bias score adjustment along with  
166 the final sequence alignment. See Methods for more details.

### 167 **Sparse Forward/Backward reduces computation, is a good approximation**

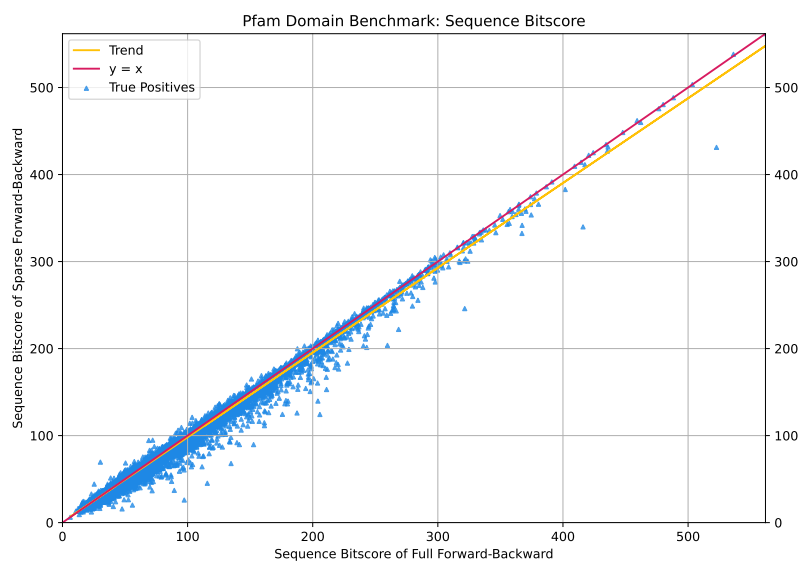
168 To evaluate `na1`'s sparse Forward/Backward method, we tested the extent to which it reduces the  
169 number of computed cells, as this directly impacts time and space utilization. We also measured  
170 how well the sparse analysis approximates alignment scores computed using full Forward/Backward.

171 To analyze search space reduction, we computed the percentage of the full quadratic search  
172 space that is explored by the sparse approach. In Figure 2A, matrix sparseness (y-axis) is plotted  
173 against matrix size (x-axis – the product of the lengths of the query pHMM and target sequence).  
174 Reduction in search space is modest for alignments of shorter sequences; this is not surprising, as  
175 the total size of the matrix is not particularly large, so that even a modestly-wide band around the  
176 maximum-scoring alignment will consume much of the full analysis space. For longer sequences  
177 (for example with a length-400 model aligned to a length-2500 protein, creating a matrix of size  $10^6$ ),  
178 `na1`'s sparse method often restricts the total number of computed cells to 1% or less of the full  
179 size of the matrix. Note that the sparsification is slightly greater on average for alignments involv-  
180 ing false positive matches. Though `na1`'s implementation is not SIMD vectorized as in HMMER3  
181 (Farrar, 2007; Eddy, 2011), the dramatic reduction in computed cells results in notable speed gains  
182 (see below). Figure 2B shows, for true positives from the domain benchmark, that the Forward

183 score computed on the sparse matrix typically closely matches the score computed by Forward on  
184 the full matrix.

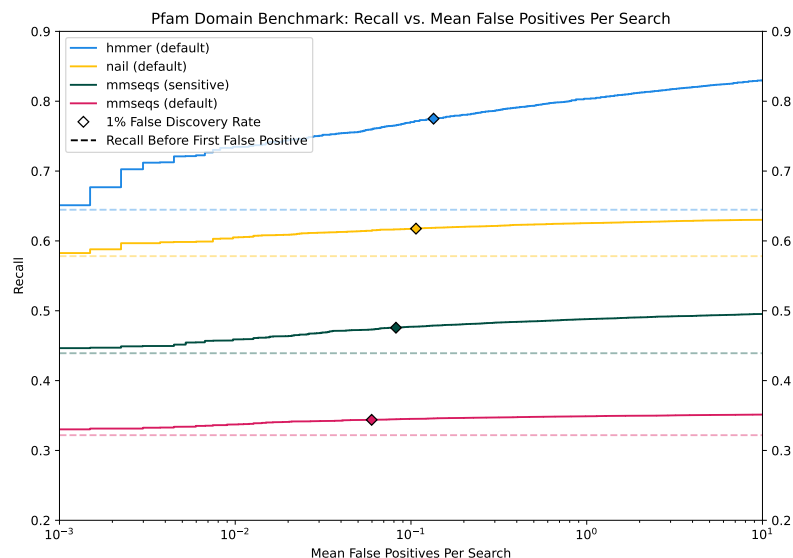


(a) Each point represents a candidate alignment that survived the MMseqs2 Viterbi filter, and plots the fraction of the full dynamic programming matrix included in `naa1`'s sparse cloud computations (y-axis) against the full matrix size (x-axis). Alignments of true domain matches are plotted in blue; red points show sparsification for false positive alignments; green stars (bottom right) show sparsification for long-sequence pairs, and follow the general trendline of space reduction as a function of matrix growth.



(b) Each point represents the relationship between sparse and full Forward scores for a true match in the benchmark. Loss of score shows up as vertical depression below diagonal. In some cases, a sparse alignment is reported with bias-adjusted score that is greater than the full matrix score; this is because `naa1` follows HMMER3's heuristics for bias score adjustment, but matrix sparsity sometimes causes the bias-induced score adjustment (which decreases scores) to be smaller in scale.

**Figure 2. Efficacy and impact of sparsifying Forward/Backward matrix.**



**Figure 3. Recall as a function of false annotation rate.** The protein domain benchmark consists of 25,688 true target sequences from 1,339 Pfam families, mixed with 2 million shuffled sequences from Uniprot. `nail` and HMMER were each tested with default parameters; MMseqs2 was tested with both default and sensitive (`-s 7.5 --max-seqs 1000`) settings.

### 185 Recall as a function of false annotation

186 We used the Pfam-based benchmark described above to assess the accuracy gains achieved with  
187 the Forward/Backward algorithm relative to MMseqs2 alignment, and to measure the efficacy  
188 of `nail`'s sparse implementation in retaining these gains. Each of the 1,339 query alignments  
189 was used to search for matching family members in the test database containing 25,688 true-  
190 embedded sequences mixed with 2 million simulated sequences. An alignment was considered  
191 to be 'true positive' if at least 50% of the length of an embedded target sequence was covered by  
192 an alignment with the query from the same family. A hit that entirely matched shuffled sequence  
193 was defined as a 'false positive'. Alignments between a query and target of differing families were  
194 treated as neutral (ignored) rather than being penalized, since it is not possible to ensure lack of  
195 homology across assigned family labels.

196 Figure 3 presents recall (fraction of all true positives that are recovered at a specific E-value  
197 cutoff) as a function of false annotation rate (number of false positive matches per query with that  
198 E-value or better). For each tested method, all resulting alignments were gathered together and  
199 sorted by increasing E-value, so that a recall curve can be plotted. HMMER3's `hmmsearch` tool  
200 was run with default settings (`-E 10`), and establishes a sensitivity target; since `hmmsearch` can  
201 produce multiple 'domain' alignments for a matched query-target pair, the domain with the best  
202 score (lowest E-value) was retained. Curves are plotted for MMseqs2 with default and sensitive  
203 (`-s 7.5 --maxseqs 1000`) settings, and indicate a large sensitivity loss relative to pHMM annotation  
204 with HMMER. `nail` closes this gap, particularly at low false positive levels, producing near-HMMER  
205 sensitivity with MMseqs2-like speed (see below).

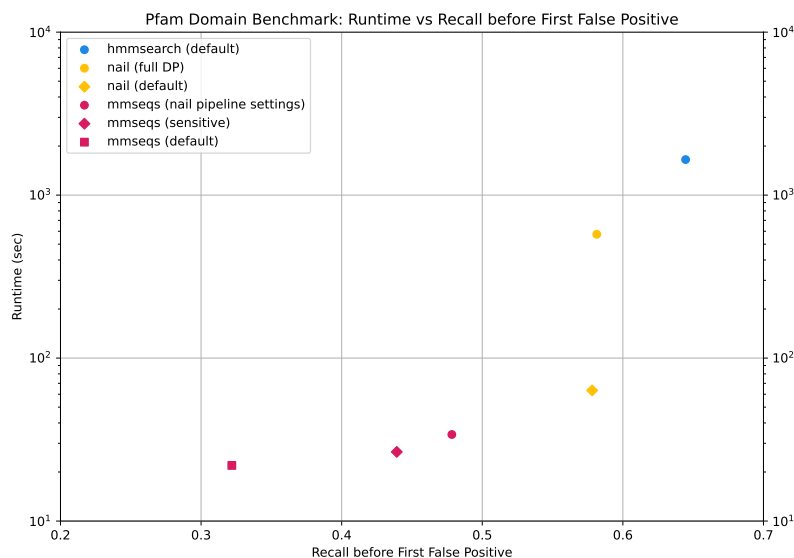
206 The horizontal dashed lines in Figure 3 represent the recall before the first false positive for  
207 each tool, which we refer to as *recall-0* and is equivalent to the measure primarily reported in  
208 (Steinegger and Söding, 2017) and (Buchfink et al., 2021). All tools show at least several percent  
209 gain in recall beyond that first false positive, with HMMER showing the steepest recall gains. As  
210 implemented, `nail` essentially re-scores candidate matches produced by MMseqs2. To establish

211 an upper bound on the recall gains possible with sparse Forward/Backward, `nai1` includes an option to compute the *entire* Forward/Backward matrix for all candidates reported by MMseqs2 stage  
212 ('--full-dp'). The corresponding curve is not shown here because it is essentially identical to that of  
213 the sparse implementation in `nai1`. This demonstrates that loss of recall in `nai1` relative to HMMER  
214 is due to limitations in candidates passing the initial filter, not failure of the sparse method, and  
215 highlights the value of future developments in fast candidate identification. We note that another  
216 high-speed annotation tool, DIAMOND (*Buchfink et al., 2021*), was omitted from analysis due to  
217 much lower benchmark sensitivity, in agreement with *Krause et al. (2024)*.

219 Note: this analysis accentuates the difference in real world performance of the tools because  
220 the benchmark is constructed to consist exclusively of hard-to-find matches. Furthermore, the  
221 performance gap may also be overstated due to the fact that the benchmark is built from Pfam  
222 sequences, which themselves were partly gathered using HMMER. Even so, the analyses agree with  
223 other observations of superior pHMM sensitivity (*Steinegger and Söding, 2017; Krause et al., 2024*).

## 224 Exploring the tension between speed and accuracy

225 Assessment of sequence annotation methods must consider the tradeoff between speed and sen-  
226 sitivity. In doing so, it is helpful to summarize the full recall curves from Figure 3 with a simple statisti-  
227 c. Here, we use the value *recall-0*, which is computed as the fraction of planted positives assigned  
228 an E-value better than the best-scoring false positive. This summary statistic is easy to interpret  
229 and generally agrees with relative ordering of methods in analyses such as Figure 3. Figure 4, plots  
230 run time (y-axis) and *recall-0* (x-axis) for annotation of the Pfam-based benchmark described above  
231 – an idea tool will produce a point that is low (fast) and to the right (sensitive). We view these results  
232 as a conservative estimate of the speed benefits of the sparse Forward/Backward approach, be-  
233 cause the Pfam-based domain sequences are often quite short – the relative speed/recall tradeoff  
234 is expected to be increasingly in favor of sparse Forward/Backward for longer sequence elements  
235 (see Figure 2).



**Figure 4. Run time vs. recall.** Pfam-based benchmark was searched with tool variants to demonstrate performance-runtime tradeoffs. These include MMseqs2 variants (default; sensitive: `-s 7.5 --max-seqs 1000`; nail pipeline settings: `--k-score 80 --min-ungapped-score 15 --max-seqs 1000`), HMMER3's `hmsearch` (default), and `nai1` variants (default; `--full-dp`). All tools were run with 8 threads.



236 Figure 4 includes results of searching with HMMER3, which produces the highest *recall-0* values  
237 at the cost of ~62-fold increase in run time relative to sensitive MMseqs2. Recall and times for  
238 MMseqs2 default and sensitive are shown, along with values for MMseqs2 as parameterized when  
239 used within the `nail` pipeline (see Table 2). Meanwhile, `nail` recovers more than half of MMseqs2's  
240 lost sensitivity, while increasing run time only ~2.4-fold. The full matrix variant of `nail` is also  
241 plotted, to demonstrate the speed boost achieved with sparse alignment, with essentially no loss  
242 in recall. A large majority of the sensitivity difference between `nail` and HMMER3 is the result of  
243 aggressive candidates filtering by the k-mer match stage in MMseqs2, suggesting that an alternate  
244 ultra-fast alignment seed detection method is warranted.

## 245 Methods

### 246 MMseqs2 as a prefilter for `nail`

247 The first step in the `nail` pipeline is to identify a collection of promising query-target candidates,  
248 along with alignment matrix positions that will serve as seeds for sparse matrix calculations. `nail`  
249 identifies candidates by running the MMseqs2 'search' command with two non-default parameters  
250 (see Table 2). This produces a maximal-scoring alignment and E-value for each reported query-  
251 target pair. The E-value is a measure of significance of an alignment computed by internally adjust-  
252 ing the alignment's P-value by the size of the search space (the P-value indicates, for an alignment  
253 with score  $s$ , the probability of a non-homologous pair of sequences producing score  $\geq s$ ). Echoing  
254 the filtering strategy used in HMMER3, the `nail` pipeline converts MMseqs2 E-values into P-values  
255 (by inverting the database size adjustment), then filters out candidates with P-Value > 0.01 (i.e. 1%  
256 of non-homologous query-target pairs are expected to pass the filter).

Parameter	MMseqs2 sensitive	<code>nail</code> default	Description
<code>--k-score</code>	auto (88)	80	k-mer threshold for generating similar k-mer lists
<code>--min-ungapped-score</code>	15	15	Accept only matches with ungapped alignment score above threshold
<code>--max-seqs</code>	300	1000	Maximum results per query sequence allowed to pass the prefilter

**Table 2.** MMseqs2 parameters that can be altered through `nail`'s command line interface, along with brief descriptions of their effects (copied from `mmseqs prefilter -h` command). Standalone MMseqs2 internally determines a value for `--k-score` based on a combination of sensitivity settings and system information; this table presents the value selected by MMseqs2 for sensitive search on our benchmark tests, with kmer size of 6. `nail` overrides this setting with a more permissive default. Note: further reduction to `--k-score` will increase `nail` sensitivity and runtime.

### 257 Mapping the MMseqs2 profile to a pHMM

258 Ideally, the previous step would provide landmarks (begin/end cells) in the pHMM alignment ma-  
259 trix for each identified candidate query-target pair. Because the alignment results correspond to  
260 an MMseqs2-style profile, and those profile positions do not necessarily map to the HMMER3-style  
261 pHMM positions used in `nail`'s Forward/Backward alignment, `nail` must map MMseqs2 profile po-  
262 sition to the corresponding HMMER3 pHMM position. This is accomplished by performing an align-  
263 ment of each MMseqs profile against the consensus sequence generated from the corresponding  
264 HMMER3 pHMM, using the MMseqs 'search' tool. The resulting alignment is used to map between  
265 the two profile representations through a linear scan.

266 **Default implementation of the Forward/Backward algorithm**

267 To prepare for discussion of a sparse alignment implementation, we first describe the standard  
 268 implementation of the Forward/Backward algorithm for aligning a query profile HMM (or sequence)  
 269 to a target sequence. Input to the algorithm consists of:

- 270 • An alphabet  $\Sigma$  of size  $k$  ( $k = 20$  for the amino acid alphabet).
- 271 • A length- $n$  target sequence  $T = t_1, t_2, \dots, t_n$ , with all  $t_j \in \Sigma$ .
- 272 • A query pHMM  $Q$  defined by a collection of values organized around three core states for  
 273 each of  $m$  positions:
  - 274 – Match states ( $M$ ) emit residues (letters) from  $\Sigma$  with a position-specific distribution, and  
 275 during alignment are used to associate (match) a residue  $t_j$  from  $T$  to a position  $q_i$  in  $Q$ ;
  - 276 – Insert states ( $I$ ) emit residues in between match-state residues, and during alignment  
 277 allow some residues in  $T$  to not correspond to positions in  $Q$  (to lie between matched  
 278 residues). In principle, position-specific insertion emission probabilities are legal, but  
 279 `naal` follows the common convention of employing a single emission distribution for all  
 280 insert states (which matches the background distribution);
  - 281 – Delete states ( $D$ ) are silent states (no emission) that, in alignment, allow some positions  
 282 in  $Q$  to be deleted (not represented) in  $T$ .
  - 283 – Note: though this description introduces the query as a pHMM, `naal` is capable of search-  
 284 ing with a single sequence. A single sequence will correspond to a pHMM in which emis-  
 285 sion probabilities are not position-specific, but instead depend simply on the observed  
 286 residue at each position. Transition probabilities are uniform.

287 In support of these states,  $Q$  is described by two matrices (see *Durbin et al., 1998* for more  
 288 detail):

- 289 1. For each position  $i$ , emissions of match state  $M_i$  are defined by a vector  $q_{i1}, q_{i2}, \dots, q_{ik}$ ,  
 290 where a value  $q_{ic}$  corresponds to the model's probability of observing residue  $c$  at posi-  
 291 tion  $i$ .
- 292 2. A transition matrix captures the probability of transitioning from one state to another  
 293 in sequential positions (transitions between D and I states are not included):  
 294  $t(M_i, M_{i+1}), t(M_i, D_{i+1}), t(M_i, I_i), t(I_i, I_i), t(I_i, M_{i+1}), t(D_i, D_{i+1}), t(D_i, M_{i+1})$

295 With this input, the Forward algorithm fills in three  $(m + 1)(n + 1)$  matrices,  $F^M$ ,  $F^I$ , and  $F^D$ , one for  
 296 each state. The value stored at a cell  $(i, j)$  in a state's matrix corresponds to all ways of aligning the  
 297 first  $j$  letters of  $T$  with the first  $i$  model positions, ending in that state. After initializing  $F_{0,0}^M = F_{0,0}^D =$   
 298  $F_{0,0}^I = 0$ , the remaining matrix cells are computed via the recurrence equations:

$$c = t_j$$

$$F_{i,j}^M = q_{ic} \cdot \text{sum} \begin{cases} F_{i-1,j-1}^M \cdot t(M_{i-1}, M_i), \\ F_{i-1,j-1}^I \cdot t(I_{i-1}, M_i), \\ F_{i-1,j-1}^D \cdot t(D_{i-1}, M_i) \end{cases}$$

$$F_{i,j}^I = \text{sum} \begin{cases} F_{i,j-1}^M \cdot t(M_i, I_i), \\ F_{i,j-1}^I \cdot t(I_i, I_i) \end{cases}$$

$$F_{i,j}^D = \text{sum} \begin{cases} F_{i-1,j}^M \cdot t(M_{i-1}, D_i), \\ F_{i-1,j}^D \cdot t(D_{i-1}, D_i) \end{cases}$$

299 Notes:

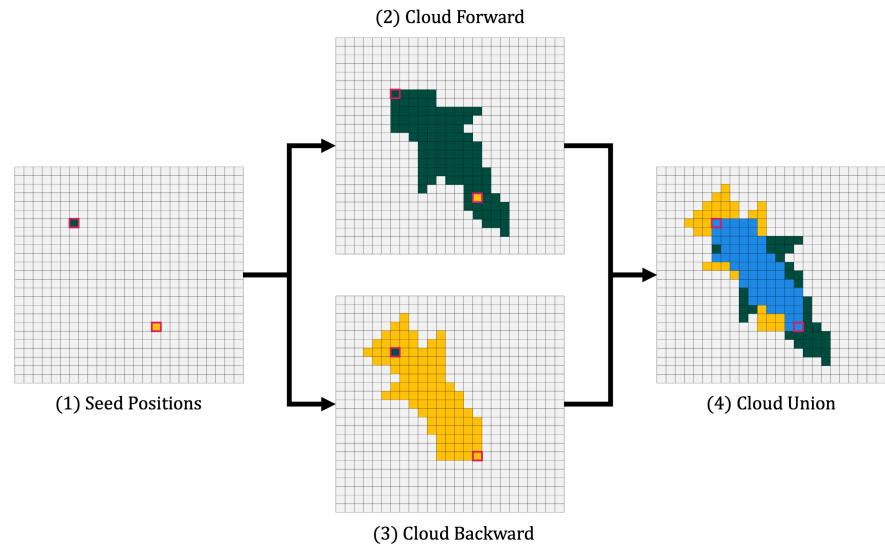
- 300 • The result of the Forward algorithm is a ratio of the sum, over all possible alignments, of  
301 the probability of observing  $T$  under the assumption of relationship to  $Q$ , divided by the  
302 probability of observing  $T$  under a random model. The log of this ratio is a score, and the  
303 E-value of an alignment can be computed based on how this score relates to the distribution  
304 of scores for alignments involving random sequences (see *Eddy, 2008*).
- 305 • This recurrence is similar to the Viterbi recurrence (*Viterbi, 1967*) for finding the highest-  
306 probability alignment; it differs in that it sums the values of alternate paths, rather than se-  
307 lecting the maximum probability path. Viterbi is essentially equivalent to the scheme used in  
308 Smith-Waterman, BLAST, MMseqs2, DIAMOND, and others (*Durbin et al., 1998; Frith, 2020*).
- 309 • This description addresses only the core model and assumes global alignment; local align-  
310 ment, and additional states, require straightforward modifications to the recurrence, see  
311 *Eddy (2008)*.
- 312 • The recurrence involves calculation of the products of probabilities, and can suffer from nu-  
313 merical underflow. The Viterbi (max) method avoids underflow by performing all computa-  
314 tions in log space. This is not possible for the Forward algorithm, due to the fact that it adds  
315 probabilities. This is often addressed by moving values in and out of log space (supported by  
316 fast approximation of  $\log(p_1 + p_2)$ ); this is the method used in `naïl`'s implementation. Some  
317 implementations achieve further acceleration by scaling values directly in order to avoid con-  
318 version to log space entirely (*Eddy, 2011*).
- 319 • Though the recurrence suggests recursive function calls, the matrix can be computed by fill-  
320 ing a table in an ordered fashion, due to the ordered local dependencies of computations.  
321 This is usually performed in row-major order (filling from upper left to lower right, one row  
322 at a time), though dependencies allow for other orders, such as filling in sequential anti-  
323 diagonals (*Ropelewski et al., 1997*), as is done in `naïl`.

324 The Forward algorithm computes a measure of support for the relationship between  $T$  and  $Q$ , but  
325 does not directly produce a specific alignment between the two. One important byproduct of the  
326 calculation is that each  $(i, j)$  cell in the *Forward* matrices represents the probability of all alignments  
327 *ending* in the corresponding state, having accounted for the first  $j$  letters of  $T$  and the first  $i$  posi-  
328 tions of  $Q$ . A common followup to Forward is to perform the same sort of computation in reverse,  
329 filling in tables from lower-right to upper-left based on an inversion of the recurrence for Forward.  
330 This *Backward* algorithm computes, for each cell, the probability of all alignments *starting* at  $t_j$  and  
331 model position  $i$ . The Forward and Backward matrices can be combined (*Durbin et al., 1998*) to  
332 produce a posterior probability that each cell is part of the correct alignment. This posterior prob-  
333 ability matrix can serve as the basis of an alignment with maximum expected accuracy (*Holmes  
334 and Durbin, 1998; Durbin et al., 1998*). We omit details, as they are not required to understand the  
335 work here, but note that typical calculation of each of these matrices is performed across the full  
336 quadratic alignment space.

### 337 **Efficient search for high-probability cloud in Forward/Backward matrices**

338 The Forward/Backward computation described above captures the total probability of all possi-  
339 ble alignments, and in doing so, fills in multiple matrices with quadratic size (the product of the  
340 lengths of  $T$  and  $Q$ ). `naïl` improves computational efficiency with a heuristic that exploits the fact  
341 that this is usually overkill – most possible alignments have such low probability that excluding  
342 them from computation has no relevant impact on the overall sum of probabilities (see Figure 1).  
343 `naïl`'s sparse matrix approach aims to identify which matrix cells contain non-negligible probabil-  
344 ity, and limit calculations to touch only those cells. Doing so minimally impacts computed scores  
345 and resulting sequence alignments, while substantially reducing the total computation. In this sec-  
346 tion, we describe a heuristic approach for achieving this goal. The method, which we call *Cloud  
347 Search*, resembles the well-known X-drop algorithm used in maximum-score alignment methods

348 such as BLAST (Altschul et al., 1990). *na1* begins with a seed that provides guidance on where  
 349 high-probability cells are likely to be found, then expands a search forward and backward across  
 350 the matrices for a cloud of cells around this seed that appear to contain essentially all relevant  
 351 probability mass. This constrained space is then used as the basis for all downstream analysis.



**Figure 5. Cloud Search.** In this schematic representation of Cloud Search: (1) An alignment from MMseqs2 is used as the source of begin- and end-points (green and yellow; these could come from any source). (2) Calculation is performed in the forward direction (moving down and to the right) from the begin point by filling in one anti-diagonal at a time, pruning each diagonal in from the ends based on score-dropoff conditions; this typically extends beyond the provided end point. (3) A similar flood fill pass is performed in the reverse direction starting from the provided end point, moving up and to the left. (4) The union of the two resulting spaces is identified as the sparse cloud.

### 352 Cloud Search by pruned anti-diagonal completion

353 The method proceeds as follows:

- 354 • Cloud Search is initiated with a pair of alignment matrix cells, *begin* and *end*. As currently  
 355 implemented, this pair is taken from an MMseqs2 alignment between *Q* and *T* (Figure 5: (1))  
 356 – the first and last positions of the alignment specify the begin cell  $(i_b, j_b)$  and end cell  $(i_e, j_e)$ .  
 357 In principle a cell pair could be produced by some other seed finding approach, and could be  
 358 initialized by more than one such pair of begin/end cells.
- 359 • Cloud Search flood-fills the matrices forward (down and right) from the *begin* cell, extending  
 360 out until pruning conditions are reached – Figure 5: (2). After initializing  $F_{i_b, j_b}^M = F_{i_b, j_b}^D = F_{i_b, j_b}^I = 0$   
 361 (green cell in upper left), neighboring cells down and right of  $(i_b, j_b)$  are computed in anti-  
 362 diagonal fashion, first filling the two cells  $(i_{b+1}, j_b)$  and  $(i_b, j_{b+1})$ , then the three cells below these,  
 363 and so on. Based on the recurrence, each cell on one anti-diagonal pushes values to recipient  
 364 cells in subsequent anti-diagonals; based on this push-based transfer of information, the  
 365 only cells touched on one anti-diagonal are those that are reachable from some active cell  
 366 on the previous two anti-diagonals. Beginning from  $(i_b, j_b)$ , all reachable anti-diagonal cells  
 367 are computed and retained, until the anti-diagonal achieves length  $\gamma$  (default: 5). After this,  
 368 when an anti-diagonal has been computed, two pruning conditions are applied to constrain  
 369 expansion of search space.
  - 370 – Once all values in an anti-diagonal  $d$  have been computed, the maximum value for that  
 371 anti-diagonal is captured as  $\max_d$ . All cells with  $F_{i,j}^M \geq \max_d - \alpha$  are retained, and others

372 are pruned. Scores at this point are captured in *nats* (natural logarithms), with default  
373  $\alpha = 12$ , so that this effectively prunes cells on an anti-diagonal that have probability that  
374 is  $\sim 1$  million-fold lower than the most-probable cell on that anti-diagonal.  
375 - As flood fill continues, the overall best-seen score across all computed anti-diagonals is  
376 captured as  $\max_o$ . Any cell with score  $F_{i,j}^M < \max_o - \beta$  is pruned. With a default  $\beta = 20$ ,  
377 this prunes cells with  $\sim 1$  billion-fold reduction from the best seen overall value (this is  
378 analogous to X in the X-drop heuristic). When all cells in an anti-diagonal are pruned, the  
379 flood fill stops.

380 Pruning is performed based entirely on values stored in the Match state matrix  $F^M$ , and all  
381 scores are maintained in log space. The result of this phase is a set of cells expanding down  
382 and right from  $(i_b, j_b)$ , schematically represented as dark green cells in Figure 5: (2). This cloud  
383 of cells typically remains in a fairly tight band around the maximum probability (Viterbi) path.  
384 Importantly, this cloud search approach typically extends well beyond the initial *end* cell  $(i_e, j_e)$ ,  
385 meaning that a conservative selection of initial points does not constrain the Forward cloud  
386 search.

- 387 • After the Forward Cloud Search phase, a similar Backward pass is performed, beginning at  
388  $(i_e, j_e)$ , and flood filling as in the previous stage, but up and to the left (Figure 5: (3); yellow  
389 cells).
- 390 • Cloud Search concludes by selecting the union of the Forward and Backward clouds (Figure 5:  
391 'Cloud Union'). This establishes a set of cells that hold a non-negligible expansion around the  
392 range bounded by the initiating cells  $(i_b, j_b)$  and  $(i_e, j_e)$ .

### 393 Linear space requirement for computing Cloud Search

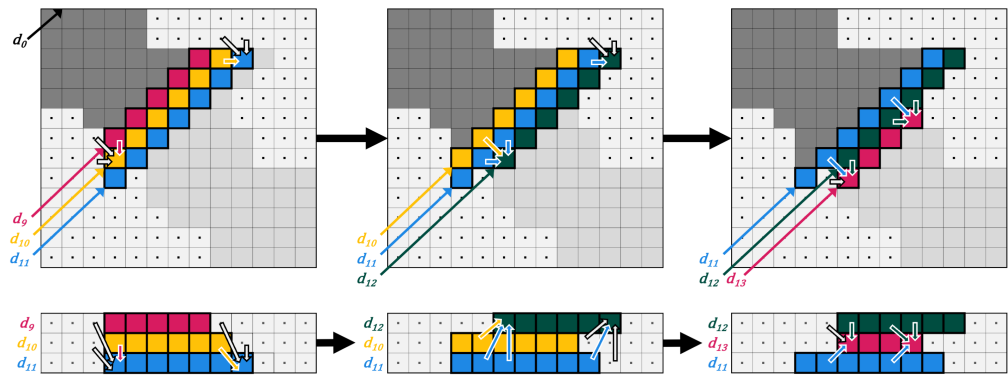
394 The forward and reverse passes of Cloud Search can be computed in linear space, using a 3 by  
395  $m$  matrix, in which each row holds the dynamic programming values computed along one anti-  
396 diagonal. In general, the  $n^{\text{th}}$  anti-diagonal,  $d_n$ , is assigned to row  $n \bmod 3$ , and each column in the  
397 cloud matrix  $C$  corresponds to a column in the implicit DP matrix. For a given matrix cell  $F_{i,j}$ , its  
398 anti-diagonal is given by  $d_n = i + j$ . Then, the value is stored in the cloud matrix at row  $(i + j) \bmod 3$ ,  
399 column  $j$ . Modifications to the recurrence equations follow naturally.

400 During the forward pass of Cloud Search, the values computed along anti-diagonal  $d_n$  depend  
401 on the values computed along the previous anti-diagonals  $d_{n-1}$  and  $d_{n-2}$ . The cloud matrix access  
402 pattern satisfies those dependencies: the values along  $d_n$  are stored in the row that previously  
403 contained the (now retired) values of  $d_{n-3}$ , while the previously computed values of  $d_{n-1}$  and  $d_{n-2}$   
404 remain available. Similarly, during the reverse pass, the values along  $d_n$  are stored in the row  
405 previously containing the values of  $d_{n+3}$  with the values on  $d_{n+1}$  and  $d_{n+2}$  retained. Figure 6 gives an  
406 example of the cloud matrix access pattern during a forward pass of Cloud Search.

407 Once an anti-diagonal has been computed and pruned, the positions (in the implicit complete  
408 matrix) of its lower left and upper right cells are stored; these two cells describe an anti-diagonal  
409 cloud bound: each cell along the anti-diagonal between the two bounding cells is included in the  
410 sparse cloud. In this fashion, the cloud bounds are stored in linear space, with at most  $4m$  values  
411 (two  $(i, j)$  pairs per anti-diagonal) describing a cloud that spans the full width and height of the  
412 complete matrix.

### 413 Cloud union, trimming, and reorientation

414 After completing the forward and reverse passes of Cloud Search, the union of the two clouds is  
415 taken, as shown in Figure 5: (4). This is done by iterating across the forward and reverse bounds  
416 from the left-most anti-diagonal present,  $d_{\text{start}}$ , to the right-most anti-diagonal present,  $d_{\text{end}}$ , and  
417 producing a new bound at each  $d_n$  by combining the two bounds such that the resulting anti-  
418 diagonal covers the ranges of both the forward and reverse anti-diagonals. On anti-diagonals for



**Figure 6. Example anti-diagonal access pattern.** This example shows the implicit DP matrix (top) and the state of the cloud matrix (bottom) when anti-diagonals  $d_{11}$  (left example),  $d_{12}$  (middle example), and  $d_{13}$  (right example) are being filled during the forward pass of Cloud Search. In both representations, the data dependency patterns are shown with arrows. Note that in the implicit DP matrix, the dependencies follow the same patterns at each step, but, in the cloud matrix, the relative positioning (in memory) of the dependencies changes.

419 which there is exclusively a forward or a reverse bound, the union step simply uses that bound.

420 It is possible for the clouds identified by the forward and reverse pass to not intersect; this is  
 421 typically caused by a region of very low homology that Cloud Search does not pursue. In such cases,  
 422 `naïl` discards the search clouds and defaults to filling in a rectangular DP matrix bounded by the  
 423 start and end seed positions. In our experience, these situations are exceedingly rare, occurring in  
 424 less than 0.0001% of alignments of true positive sequences in our benchmarks.

425

426 Cloud trimming: The union of the forward and reverse clouds typically results in a cloud shape  
 427 with protrusions along the edges of the cloud, as shown in Figure 7: (1). The protrusions contain  
 428 cells that either (a) can't include probability from paths originating in the first anti-diagonal in the  
 429 cloud, or (b) can't propagate probability along a path toward the last anti-diagonal in the cloud. In  
 430 other words, in sparse FB, the cells in such protrusions either contain a likely negligible amount  
 431 of probability, or do not contribute to the total probability captured at the end of FB. Additionally,  
 432 certain protrusions cause the cloud to have gaps between contiguous runs of included cells in  
 433 one row of the matrix. Both classes of protrusions slightly complicate the process of reorienting  
 434 the anti-diagonal-based cloud bounds into row-based cloud bounds (discussed next in this section),  
 435 and row-gap-inducing protrusions dramatically complicate the bookkeeping involved with a sparse  
 436 matrix data structure (discussed in the section 'Sparse matrix organization'). Because they can not  
 437 be involved in both Forward and Backward paths without passing through pruned cells, they can  
 438 be safely removed from the cloud.

439 To remove these protrusions, we run a simple linear time algorithm that makes both a forward  
 440 and reverse pass iterating across each bound in the cloud union. Pseudocode for the algorithm is  
 441 given in Algorithm 1, and a visual representation can be found in Figure 7: (2).

442

443 Cloud reorientation: Although the Cloud Search computations may be performed anti-diagonal  
 444 by anti-diagonal, we reorient the anti-diagonal-based cloud bounds into row-based cloud bounds  
 445 (primarily in preparation for a future `naïl` implementation that will implement the J state used  
 446 in HMMER3 to support multi-domain matches (Eddy, 2008). Reorientation is performed using a  
 447 simple linear time algorithm that iterates across the trimmed cloud union bounds. Pseudocode  
 448 for the algorithm is given in Algorithm 2, and a visual representation can be found in Figure 7: (3).

---

### Algorithm 1: Cloud trimming

---

**Data:**  $d_{start} \leftarrow$  first anti-diagonal in cloud  
 $d_{end} \leftarrow$  last anti-diagonal in cloud  
Vector  $D_{left}$  of left anti-diagonal bounds; ( $row, col$ ); result of Cloud Search  
Vector  $D_{right}$  of right anti-diagonal bounds; ( $row, col$ ); result of Cloud Search

**Result:** Cloud bounds trimmed to remove all protrusions

```

for  $d \leftarrow d_{start} + 1$  to  $d_{end}$  do
  ( $prev_{row}, prev_{col}$ )  $\leftarrow D_{left}[d - 1]$ 
  ( $curr_{row}, curr_{col}$ )  $\leftarrow D_{left}[d]$ 
   $\Delta_{left} \leftarrow \max(prev_{col} - curr_{col}, 0)$ 
   $D_{left}[d] \leftarrow (curr_{row} - \Delta_{left}, curr_{col} + \Delta_{left})$ 

  ( $prev_{row}, prev_{col}$ )  $\leftarrow D_{right}[d - 1]$ 
  ( $curr_{row}, curr_{col}$ )  $\leftarrow D_{right}[d]$ 
   $\Delta_{right} \leftarrow \max(prev_{row} - curr_{row}, 0)$ 
   $D_{left}[d] \leftarrow (curr_{row} + \Delta_{right}, curr_{col} - \Delta_{right})$ 

for  $d \leftarrow d_{end}$  to  $d_{start} - 1$  do
  ( $next_{row}, next_{col}$ )  $\leftarrow D_{left}[d + 1]$ 
  ( $curr_{row}, curr_{col}$ )  $\leftarrow D_{left}[d]$ 
   $\Delta_{left} \leftarrow \max(curr_{col} - next_{col}, 0)$ 
   $D_{left}[d] \leftarrow (curr_{row} - \Delta_{left}, curr_{col} + \Delta_{left})$ 

  ( $next_{row}, next_{col}$ )  $\leftarrow D_{right}[d + 1]$ 
  ( $curr_{row}, curr_{col}$ )  $\leftarrow D_{right}[d]$ 
   $\Delta_{right} \leftarrow \max(curr_{row} - next_{row}, 0)$ 
   $D_{left}[d] \leftarrow (curr_{row} + \Delta_{right}, curr_{col} - \Delta_{right})$ 

```

---



---

### Algorithm 2: Cloud reorientation

---

**Data:**  $row_{start} \leftarrow$  first row in cloud  
 $row_{end} \leftarrow$  last row in cloud  
Vector  $D_{left}$  of left anti-diagonal bounds; ( $row, col$ ); result of Cloud Search  
Vector  $D_{right}$  of right anti-diagonal bounds; ( $row, col$ ); result of Cloud Search  
Vector  $R_{left}$  of left row bounds; ( $col$ ); initialize all to MAX\_INT  
Vector  $R_{right}$  of right row bounds; ( $col$ ); initialize all to 0

**Result:** Vectors  $R_{left}$  and  $R_{right}$  contain the column indices of the left-most and right-most cells in each row included the cloud across  $[row_{start}, row_{end}]$   
Indices of rows that are not included in the cloud retain initialization values

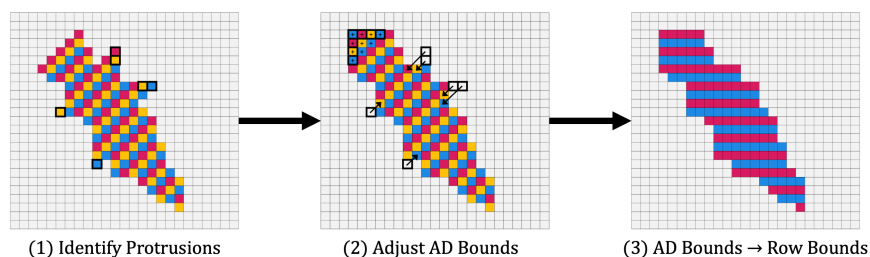
```

for ( $row, col$ )  $\in D_{left}$  do
   $R_{left}[row] \leftarrow \min(R_{left}[row], col)$ 

for ( $row, col$ )  $\in D_{right}$  do
   $R_{right}[row] \leftarrow \max(R_{right}[row], col)$ 

```

---



**Figure 7. Cloud Search.**

---

449 **Sparse Forward/Backward to recover score and alignment**

450 With the cloud of non-negligible alignment matrix cells in hand, it is possible to compute an ap-  
451 proximation of the full Forward/Backward alignment algorithm by filling in only cells in the cloud.  
452 This implicitly treats all other cells as if they carry a probability of zero.

453 **Sparse matrix organization**

454 To compute a Forward/Backward approximation, the ranges defined in the row-based cloud bounds  
455 are used as the basis for creating a sparse version of each of the matrices  $M$ ,  $I$ , and  $D$ . Since the  
456 row bounds describe exclusive contiguous runs of columns present in a row, we can store the  $M$ ,  
457  $I$ ,  $D$  values of the entire cloud in a single flat array, with padding cells between each run of con-  
458 tiguous values to accommodate the data dependencies described in the FB recursion. This flat  
459 array layout is supported by a table of complementary offsets that enable rapid identification of  
460 locations in the flat array corresponding to positions in the implicit matrix (Figure 8B), with two  
461 offsets for each block of active cells. In practice, the space required to hold active and padding  
462 cells is generally only slightly larger than the number of active cells. This layout is used to allocate  
463 a sparse  $M$ ,  $I$ , and  $D$  matrix in the form of an array for computing sparse Forward, another three  
464 arrays for computing Backward, and a single array for computing per-cell posterior probabilities  
465 in support of optimal accuracy alignment.

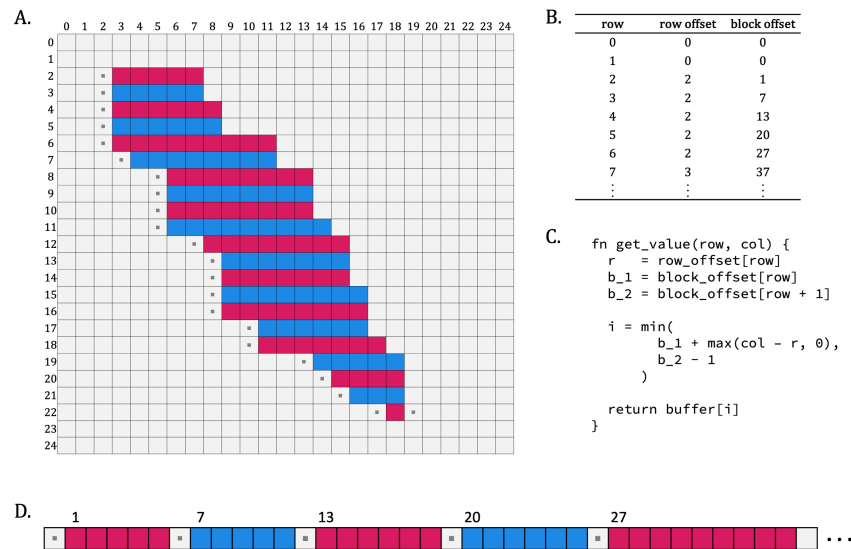
466 **Sparse Forward-Backward**

467 Computing the sparse approximation of Forward/Backward is a matter of traversing the com-  
468 pressed arrays in increasing order for Forward, and decreasing order for Backward, in runs defined  
469 by blocks of active cells. When filling in the sparse matrices, pad cell values are set to zero, and  
470 other cells are computed based on the standard recurrence equations, with retrieval of data via  
471 logical row and column indices supported by the function given in Figure 8C. To compute cell-wise  
472 posterior probabilities, the product of the Forward and Backward matrices are computed in the  
473 usual fashion. A Maximum Expected Accuracy alignment is identified based on these posterior  
474 probabilities (*Durbin et al., 1998*).

475 **Cloud filter, and Forward filter:**

476 Though reduced space Forward/Backward is fast, many of the input alignment candidates will pro-  
477 duce such a low-quality alignment that they will not end up being reported. To avoid time spent  
478 analyzing such candidates, `naal` performs two consecutive filters. The more robust of these is a  
479 filter applied after computing the sparse Forward score within the sparse cloud: using the sparse  
480 Forward score, a P-value is computed and alignments with  $P > 1e-4$  are removed (so that 0.01% of  
481 unrelated sequences are expected to pass the filter; this is similar in function to the Forward filter  
482 used in HMMER3).





**Figure 8. Sparse Matrix.** Example organization of a sparse cloud into a flat array with supporting offset data, and demonstration of its use. (A) sparse cloud cells in pink/blue are supplemented with the set of padding cells (white with ■) that ensure that any Forward/Backward calculation dependencies will refer to either a cloud or padding cell (to avoid conditionals in the DP inner loop). (B) Table of values required to compute offsets into flat array during DP recurrence computation: the row offset is the column index of the first cell in the row; the block offset is the index in the flat array of the first cell in the row. (C) Pseudocode for retrieving a value from the flat array given logical (i.e. implicit full matrix) row and column indices. The retrieval function is fast in practice, and circumvents the use of conditional logic. Note: This is a slight simplification of the actual implementation, which must support access to each of the  $M$ ,  $I$ , and  $D$  values that correspond to the same logical row and column. (D) Representation of the flat array in memory. Note: the visualization has been simplified for clarity; in practice, each element in a block shown actually corresponds to a tuple of three values, one for each of the  $M$ ,  $I$ , and  $D$  matrices. Similarly, each padding cell shown in the flat representation corresponds to a group of three identical padding values.

483 Prior to computing the Forward score on the sparse cloud, `na1` is able to *approximate* that  
 484 score using a method that we call ‘cloud filter’, which adds the sparse Forward score (starting at  
 485 the begin cell) and sparse Backward score (starting at the end cell) computed during Cloud Search,  
 486 approximately adjusting for score accumulated in cells shared by the two waves. This adjustment is  
 487 achieved by estimating how much of the forward pass score must have been missed in the reverse  
 488 pass, and vice versa. To do this, `na1` keeps track of the best score observed during forward Cloud  
 489 Search expansion (`best_fwd`), and the best score observed before extending past the anti-diagonal  
 490 containing end cell (`best_infwd`). The difference ( $Z = \text{best\_fwd} - \text{best\_infwd}$ ) is an estimate of the part  
 491 of the Forward pass’s score that is not shared by the two passes of Cloud Search. A similar value  
 492 is captured during the backward pass of Cloud Search ( $A = \text{best\_bkwd} - \text{best\_inbkwd}$ ). The total  
 493 Forward score is then estimated as  $A + \max(\text{best\_infwd}, \text{best\_inbkwd}) + Z$ ; a P-value is computed  
 494 for this, and only candidates with corresponding  $P \leq 1e-3$  are passed on to the Forward stage.

495 **Bias correction, alignment boundaries, alignment:**

496 For all downstream analyses, `na1` follows the methods of HMMER3, but with a sparse matrix im-  
 497 plementation. This includes (i) estimation of the effect of composition bias on the alignment score,  
 498 and corresponding score adjustment, (ii) identification of the start and end of an aligned region  
 499 based on posterior probabilities captured in states that precede and follow the core HMMER3  
 500 model (HMMER’s ‘domain definition’ step), and (iii) maximum expected accuracy alignment. Result-  
 501 ing (bias-corrected) scores are converted to E-values as in HMMER (see *Eddy, 2008*). Note that bias  
 502 correction depends on posterior probabilities, so bias based on sparse computation may be higher  
 503 or lower than in HMMER3 – this may cause the overall (bias-adjusted) score in `na1` to exceed that

504 of HMMER3.

## 505 **Test Environment**

506 All tests were performed using 8 threads on a Linux workstation with an Intel i9-14900KF (6.0GHz  
507 boost) 24 core processor and 128GB RAM. Standard wall clock times were captured.

## 508 **Discussion**

509 As implemented, `nai1` demonstrates that it is possible to employ powerful Forward/Backward in-  
510 ference with significantly reduced time and memory requirements. Here, we highlight ways in  
511 which we expect future advances may lead to superior annotation performance.

### 512 Better candidate seeds

513 `nai1`'s dependency on MMseqs2 creates two common ways that a good alignment can be missed.  
514 In the most straightforward one, the MMseqs2 portion of the pipeline fails to find a good alignment  
515 candidate, so `nai1`'s sparse Forward/Backward stage is never given a chance to identify the match.  
516 The fast k-mer match stage of MMseqs2 is the common cause of such misses, and is responsible  
517 for most of the sensitivity difference between `nai1` and HMMER3. `nai1`'s implementation makes  
518 it possible to explore development of new candidate detection options with no exposure to other  
519 parts of the algorithm. Fast and highly sensitive candidate detection may be improved through  
520 an alternative k-mer matching scheme (perhaps leveraging fast FM-index implementation as with  
521 *Anderson and Wheeler, 2021*), neural networks (*Schütze et al., 2022*), minimizer analogs (*Sahlin,*  
522 *2022; Joudaki et al., 2020*), hardware accelerators (*Anderson and Wheeler, 2023*), or other methods.

### 523 Reporting fragments or multiple domains

524 A more subtle issue is that the current `nai1` pipeline only analyzes the MMseqs2-sourced region  
525 with the highest score; it does not explore lower-scoring MMseqs2 matches to identify a superior  
526 Forward/Backward score/alignment. The most common impact of this will be that only a single  
527 match will be reported when there are in fact multiple hits to be found, as will be true when there  
528 are multiple copies of a query domain, or a highly fragmented sequence match. In some cases, an  
529 unfortunate MMseqs2 seed can mean that the best matching alignment is missed by `nai1` (as in  
530 Figure 9). Mechanisms for identifying multiple good begin/end seeds, and for efficiently managing  
531 the associated sparse cloud(s), will improve `nai1`'s completeness and sensitivity.

### 532 Support for more complex models

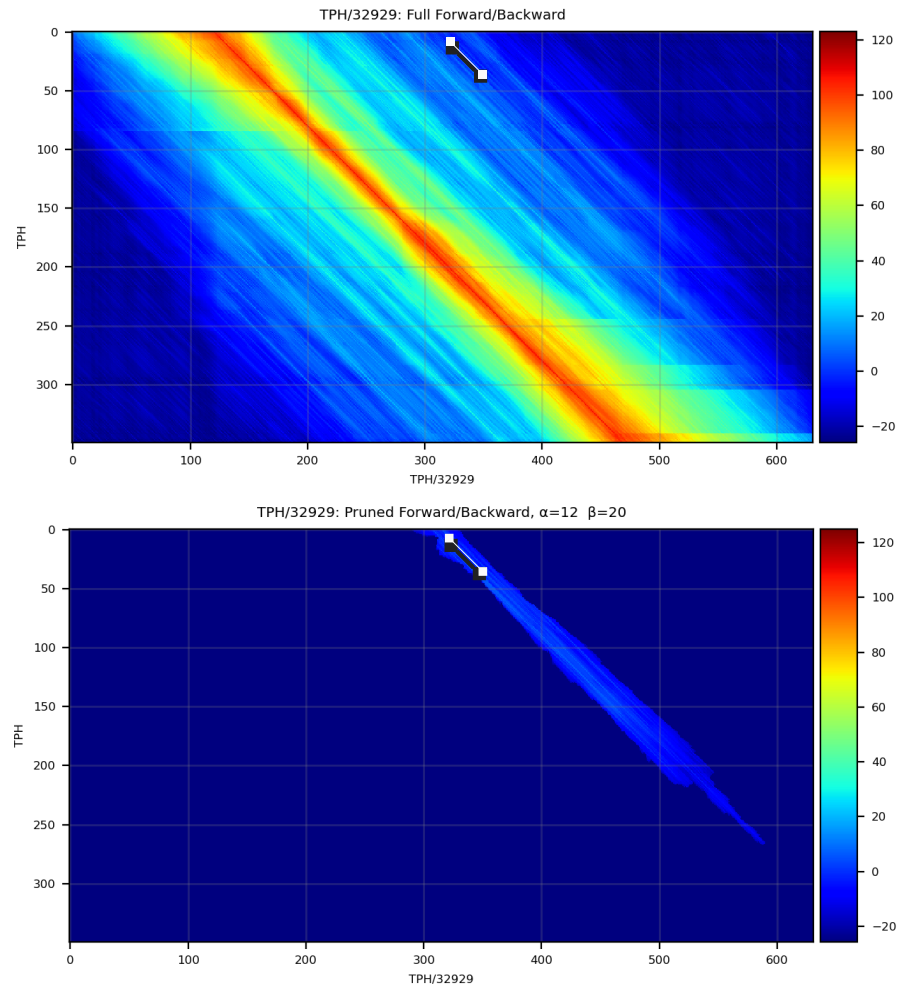
533 `nai1` reduces the computation workspace while retaining the core models of pHMM search. With  
534 this architecture in place, it will be possible to expand model complexity while retaining desirable  
535 run time properties. For example, it will be possible to directly incorporate models of sequence  
536 repetition (*Frith, 2011; Olson and Wheeler, 2018*) and sequencing error (*Krause et al., 2024*) for  
537 improved annotation. Furthermore, `nai1` will also be extended to support nucleotide annotation  
538 as well as annotation of protein-coding DNA (*Krause et al., 2024*).

### 539 Faster computations

540 The Forward/Backward recurrence calculations are modeled after the generic implementation in  
541 HMMER, with significant overhead required to support movement back and forth to log-scaled  
542 representations of odds ratios. Dynamic scaling in probability space is faster (*Eddy, 2011*) and  
543 should be feasible in the sparse representation described here.

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**Figure 9. Example of MMseqs2 producing a seed outside of the dense probability cloud**

Top panel shows heatmap of scores per cell in the Match State matrix for a full-DP alignment of the sequence TPHS\_32929 aligned to the model for its matching family, TPH (Pfam domain PF13868). The location of the poorly placed seed produced by MMseqs2 (white line) is shown at the top center of the matrix. Bottom panel shows the sparse set of low-probability cells identified by Cloud Search based on the MMseqs-derived seed, missing the dense probability mass of the true optimal alignment. The model positions are aligned along the y-axis and the sequence positions are aligned along the x-axis.

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553 Research Technologies department.

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