Robust variability of grid cell properties within individual grid modules enhances encoding of local space

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Abstract

Although grid cells are one of the most well studied functional classes of neurons in the mammalian brain, the assumption that there is a single grid orientation and spacing per grid module has not been carefully tested. We investigate and analyze a recent large-scale recording of medial entorhinal cortex to characterize the presence and degree of heterogeneity of grid properties within individual modules. We find evidence for small, but robust, variability and hypothesize that this property of the grid code could enhance the ability of encoding local spatial information. Performing analysis on synthetic populations of grid cells, where we have complete control over the amount heterogeneity in grid properties, we demonstrate that variability, of a similar magnitude to the analyzed data, leads to significantly decreased decoding error, even when restricted to activity from a single module. Our results highlight how the heterogeneity of the neural response properties may benefit coding and opens new directions for theoretical and experimental analysis of grid cells.

The discovery of grid cells in medial entorhinal cortex (MEC) [1] has led to considerable experimental and computational work aimed at identifying their origin [2–12] and their function [13–21]. The organization of grid cells into discrete modules [22, 23], with particular grid properties (grid spacing and orientation) conserved within module, but not between modules, has fundamentally shaped this research. For instance, the increasing size and spacing of grid modules along the dorsal-ventral axis of MEC, by discontinuous jumps of a near constant ratio, has been argued to be optimal for encoding local spatial information [24, 25], when grid cell activity across all modules is integrated together [24–28]. This is despite the fact that the hippocampus, a downstream target of the MEC, receives inputs from only a portion of the dorsal-ventral axis [29]. The modularity of the grid system has also been proposed to simplify the wiring necessary for generating continuous attractor dynamics [2, 3, 5, 16], a computational mechanism theorized to underlie grid cells function that enjoys considerable experimental support [23, 30–34].

Much of this prior theoretical work has made the additional assumption that grid cell properties are identical, up to a phase shift, within a single module. However, the distributions of measured orientation and spacing show non-zero variability [22, 30, 34]. This could be due to finite recording time, neurophysiological noise, or the sensitivity of numerical methods used to fit grid properties. Alternatively, this variability could reflect underlying inhomogeneity, at a fine scale, within modules. Despite the fundamental way in which grid cell function has been informed by their modular organization, to the best of our knowledge, no characterization of the degree and robustness of variability in grid properties within individual modules has been performed.

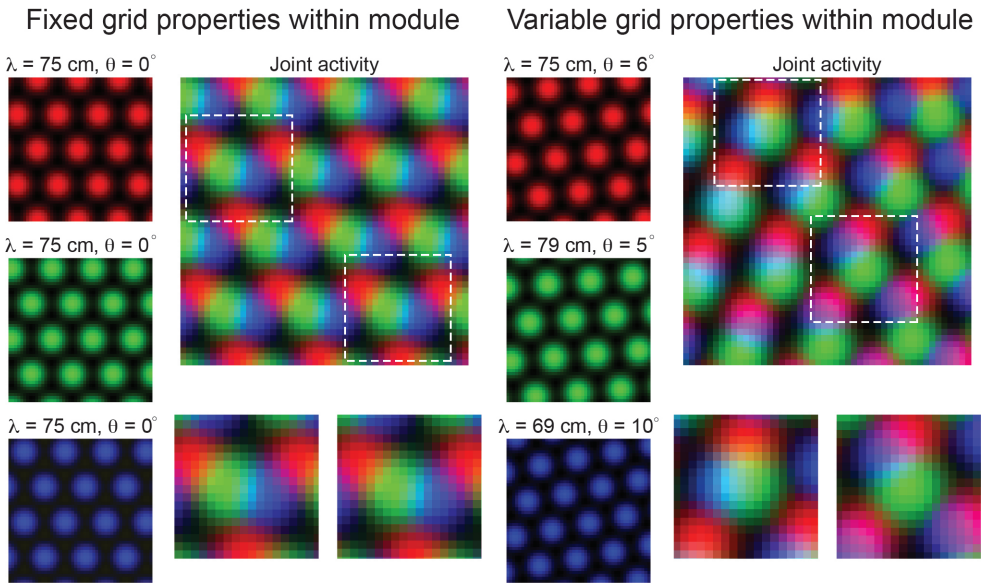

Figure 1: **Variability in grid cell properties within a module leads to enhanced encoding of local space.** When the activity of three idealized grid cells, all with the same grid spacing and orientation, are considered, the periodicity of the responses limits the amount of information conveyed about local space (Left column – "Fixed grid properties within module"). That is, there are multiple locations in physical space with identical population level activity. However, when three grid cells with variable grid spacing and orientation (in the realm of what is measured within individual grid modules – see Results), their joint activity contains considerably more information (Right column – "Variable grid properties within module"). This benefit of spatial inhomogeneity is expected to increase with larger populations of grid cells. Dashed squares in the joint activity map are enlarged below.

If robust variability of grid properties does exist, then it is possible that this heterogeneity could be exploited to encode additional information about local space, reducing the requirement of integration across multiple grid modules (Fig. 1). In particular, when all grid cells have the same grid orientation and spacing, the joint population activity has a translational invariance that persists, even as larger populations of grid cells are considered (Fig. 1, "Fixed grid properties within module"). In contrast, if grid cells within a module have variability in their grid orientation and spacing, then – over a *finite* area – the translational invariance of the population activity is broken, and distinct patterns emerge in distinct locations of space (Fig. 1, "Variable grid properties within module"). As an example, the blue and green grid cells in Fig. 1 show the most overlap in the upper left half of the arena (denoted by cyan pixels), while the red and blue grid cells show the most overlap in the lower right portion of the arena (denoted by purple pixels). This is despite the fact that the variability in spacing and orientation is only on the order of a few centimeters and degrees, respectively.

In this paper, we perform detailed analysis of recent state-of-the-art MEC electrophysiological recordings, which were made publicly available [34]. We characterize the variability of grid orientation and spacing within individual modules, and find evidence for small, but robust, variability. This variability is present whether a single value of grid orientation and spacing is assigned to each grid cell, or whether grid distortion [35, 36] is taken into account by considering three grid orientations and spacings independently. To assess the functional implications of this heterogeneity, we perform simulation experiments with synthetic, noisy grid cell populations, where we have complete control over the distribution of grid orientation and spacing. We find that the variability in grid cell orientation and spacing, at a similar degree as present in the data we analyze, leads to lower decoding error of local space when using the activity of a *single* module.

Taken together, our results challenge a long-held assumption and support a growing understanding of the spatial information encoded by grid cell populations [37–40]. Additionally, they encourage consideration of the broader benefits that multiple modules may provide, beyond the encoding of local space [41–43].

Results

Robust differences in grid cell properties within individual modules

To determine the extent to which variability of grid properties in individual modules exists, and to what extent this variability is a robust property of the grid code, we analyzed previously published MEC recordings [34], which include tens to hundreds of grid cells simultaneously recorded. This allows us to characterize the distribution of grid properties within a single grid module, to an extent not possible with other data sets.

For each grid cell, we compute the grid spacing (λ) and orientation (θ) by measuring properties associated with the six hexagonally distributed peaks of the spatial autocorrelogram (SAC), as traditionally performed [44] (Fig. 2A; see Materials and Methods). For clarity, we begin by focusing on a single module, recorded from an open field environment (recording identifier: Rat R, Day 1, Module 2 – R12). This module was picked for its long recording time (approximately 130 minutes of recorded activity, as compared to the other open field recordings that have 30 minutes or less of recorded activity) and for its large number of simultaneously recorded putative grid cells ($N = 168$). In this module, we find that grid cells with high grid scores (> 0.85; $N = 74$) have a range of grid orientation and spacing (example cells shown in Fig. 2B; distributions across module shown in Fig. 2E, F), with λ ranging from 65 cm to 90 cm and θ ranging from 2° to 9°. Overlaying the SAC of pairs of grid cells with similar grid spacing and different grid orientation (Fig. 2C) or vice versa (Fig. 2D) enables visualization of the extent of this variability.

Because individual grid fields can be cut-off by the boundaries of the environment, it is possible that computing the grid spacing from the SAC (which considers all grid fields) could lead to an under-estimate of λ, for some grid cells. To verify that the width of the distribution of grid spacing, within the same module, that we see is not due to the specifics of the SAC, we re-computed the grid spacing for all grid cells directly from their rate maps, considering only the three grid fields closest to the center of the environment (Fig. S2A; see Materials and Methods). We find that, while the grid spacing estimated from the SAC tends to be larger than the grid spacing estimated from the rate maps, a broad range of λ is again present (Fig. S2B, C).

To assess whether the heterogeneity of grid properties present in a single grid module is a robust feature or attributable to noise (either in the recording or the grid property fitting procedure), we measure the variability in grid orientation and spacing **within** a single grid cell and **between** pairs of grid cells. If the heterogeneity is explainable by noise, then we expect that the within-cell variability will be of the same size as the between-cell variability. In contrast, if the heterogeneity is a robust feature of the grid code, then we expect the within-cell variability will be significantly smaller than the between-cell variability.

To measure the within- and between-cell variability, we split the recording into evenly spaced 30 second bins, randomly assigning each temporal bin to one of two equal length halves and recomputing the grid properties for each half of the recording (Fig. 3A; see Materials and Methods). We set inclusion criteria to filter out cells that did not have consistent hexagonally structured SACs across splits of the recording (see Materials and Methods). While these requirements are strict (see Fig. S1, for percent of cells rejected), they set a conservative estimate on the amount of grid property variability, ensuring that we did not artificially inflate the variability due to weak grid cells. We do not find that the length of the temporal bin used to split the data has a large impact on the percent of cells accepted by this criteria (Fig. S3; see Materials and Methods).

We find that the distribution of within-cell variability of grid orientation and spacing is more concentrated around 0 than the between-cell variability, across all 100 random shuffles of the data into two halves. Comparing the average within- and between-cell variability of grid spacing and orientation reveals that nearly all of the grid cells that passed the criteria for inclusion ($N = 82$) exhibit more between-cell than within-cell variability (Fig. 3D, E): 95.1% grid cells for orientation ($\langle\overline{\Delta\theta}_{\text{within}}\rangle = 1.2^\circ$, $\langle\overline{\Delta\theta}_{\text{between}}\rangle = 2.0^\circ$; $\langle\cdot\rangle$ denotes mean across cells) and 100% of grid cells for spacing ($\langle\overline{\Delta\lambda}_{\text{within}}\rangle = 1.9$ cm, $\langle\overline{\Delta\lambda}_{\text{between}}\rangle = 6.9$ cm). A Wilcoxon-Signed-Rank Test indicates that between-cell variability is significantly higher than within-cell variability for both orientation and spacing ($\Delta\theta$: $p < 0.001$, $\Delta\lambda : p < 0.001$).

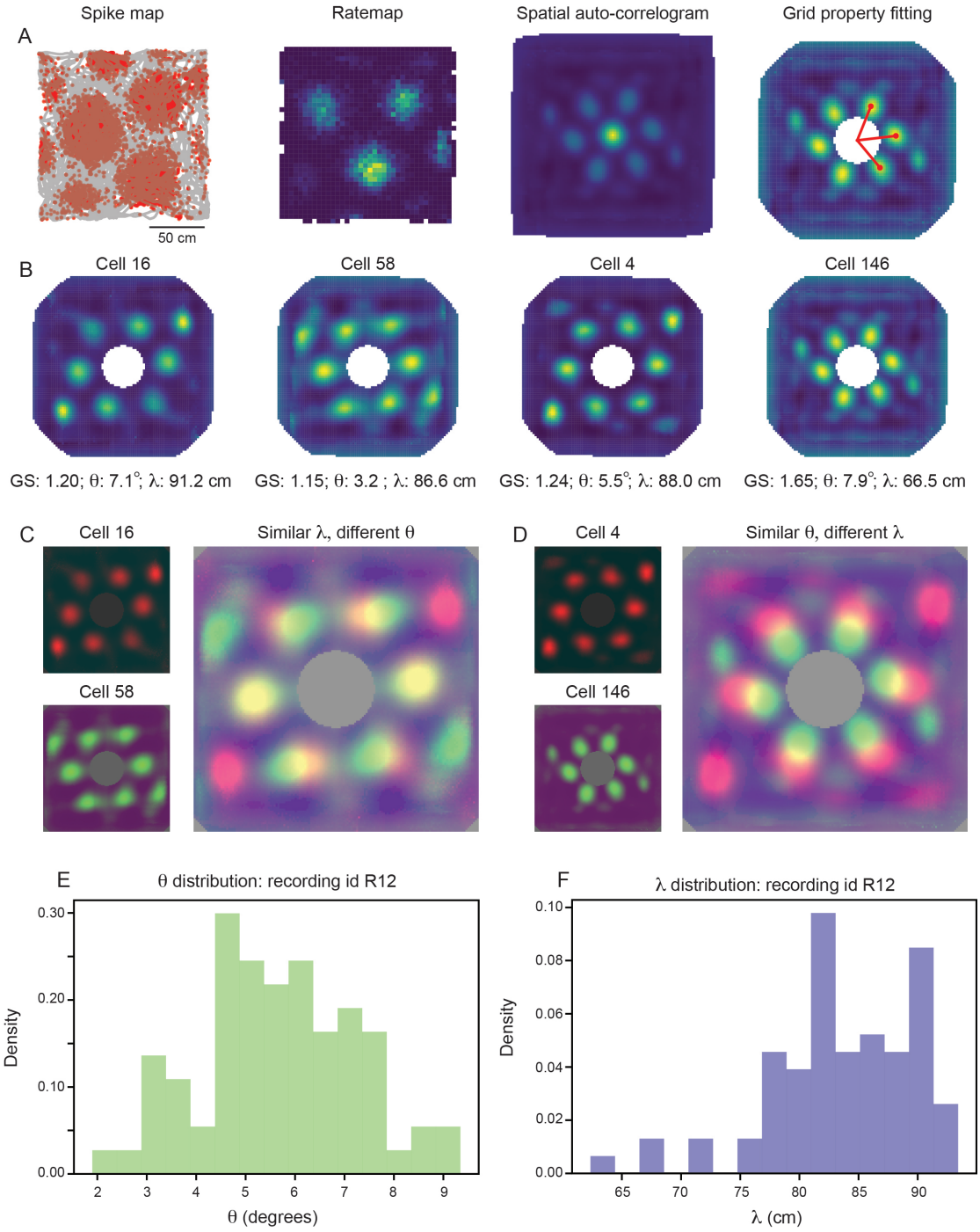

Figure 2: **Grid properties are variable within a single grid module (recording ID R12).** (A) Overview of the standard procedure used to calculate the grid spacing and orientation of a given grid cell. First, spike maps are computed by identifying the location of the animal at the time of each spike. Gray line denotes the trajectory of the rat, red dots denote locations of spikes. A rate map is constructed by binning space and normalizing by the amount of time the rat spent in each spatial bin. A spatial autocorrelogram (SAC) is computed and, after the center peak is masked out (white pixels in the center of the spatial autocorrelogram – leading to change in color scale), the grid properties are fit by measuring the length and angle of the three peaks closest to 0°. (B) Example grid cells from the same module (recording ID R12), with estimated grid score, orientation (θ), and spacing (λ). (C)–(D) SAC overlaid for two pairs of grid cells [from (B)]; one pair with different θ and similar λ (C) and the other with similar θ and different λ (D). (E)–(F) Distribution of θ (E) and λ (F) across all grid cells with grid score > 0.85 (N = 74).

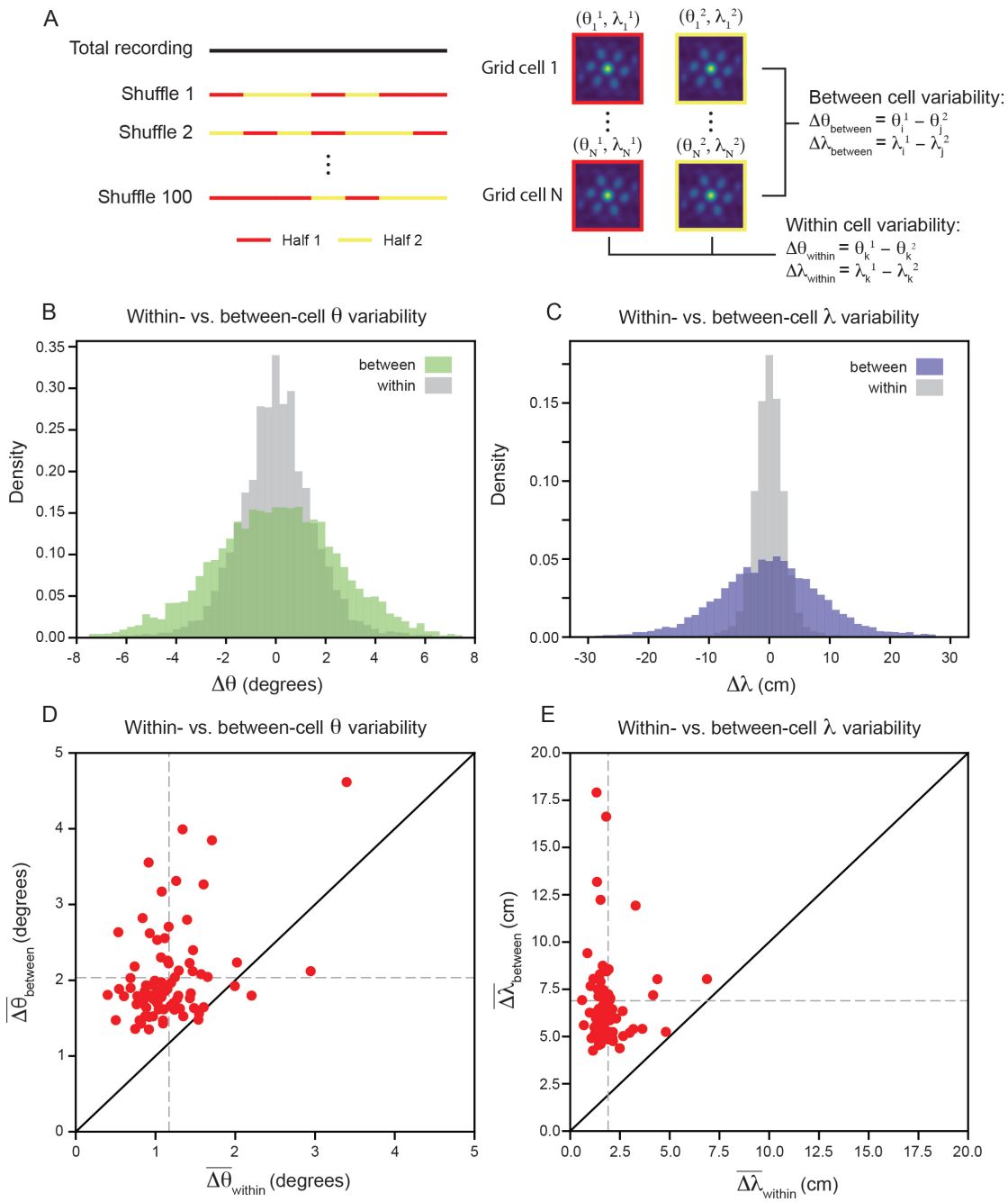

Figure 3: **Variability of grid properties is a robust feature of individual grid module (recording ID R12).** (A) Schematic overview of approach used to compute the between- and within-cell variability of grid orientation and spacing. (B)–(C) Distribution of within- and between-cell variability of θ and λ, respectively. Note that the distribution is across all 100 random shuffles of the data into two halves. (D) Average within-cell variability of grid orientation ($\overline{\Delta\theta}_{\text{within}}$), compared to average between-cell variability of grid spacing ($\overline{\Delta\theta}_{\text{between}}$). (E) Same as (D), but for λ. 1 cell was excluded from (E) for visualization ($\overline{\Delta\lambda}_{\text{between}} > 20$ cm), but was included in non-parametric statistical analysis. For (D)–(E), $N = 82$.

In keeping with convention, the reported grid cell properties are the average of those computed for each of the three independent axes in the SAC (Axis 1: aligned to $\approx 0°$; Axis 2: aligned to $\approx 60°$; Axis 3: aligned to $\approx -60°$ [45]). To ensure that this averaging is not contributing to the greater between-cell variability, we repeated the analysis above, restricting ourselves to each axis separately. The results again demonstrate that grid properties are significantly more robust within-cell than between-cell (Fig. 4A, B). For each axis, the average between-cell variability for every cell was significantly higher than the average within-cell variability (Fig. 4C, D), as reported by the Wilcoxon-Signed-Rank Test for orientation ($p < 0.001$, for all three axes) and for spacing ($p < 0.001$, for all three axes)

Having demonstrated that grid cell properties are robustly heterogeneous in a single module, we proceed to analyze the remaining recordings in the data set ($N = 420$ cells across 8 modules; one module has no cells that pass our criteria) [34]. Although there are differences across recordings, we find larger between- than within-cell variability for grid orientation and spacing is present across all recordings (Fig. 5A, B; 80.0% of grid cells have greater between- than within-variability for orientation, $\langle \overline{\Delta\theta}_{\text{within}} \rangle = 1.4°$ and $\langle \overline{\Delta\theta}_{\text{between}} \rangle = 1.9°$; 87.9% of grid cells have greater between- than within-variability for spacing, $\langle \overline{\Delta\lambda}_{\text{within}} \rangle = 1.7$ cm and $\langle \overline{\Delta\lambda}_{\text{between}} \rangle = 3.6$ cm). A Wilcoxon-Signed-Rank Test finds that these differences are significant ($\Delta\theta$: $p < 0.001$; $\Delta\lambda : p < 0.001$). We do not find evidence suggesting that the within-cell variability is influenced by grid score (Fig. 5C, D; linear regression for θ: $R^2 = 0.03$, $p = 0.55$ Wald Test; linear regression for λ: $R^2 = 0.07$, $p = 0.14$ Wald Test).

Taken together, our analysis of large-scale MEC recordings demonstrates that grid cells in the same grid module do not have a single grid spacing and orientation, but instead have a restricted range of values around the module mean. This property is a robust feature that cannot be explained by noise from the recording or fitting procedures.

Variability in grid properties within single modules improves the encoding of local space

The variability of grid cell properties within individual grid modules, while statistically significant, is small in magnitude. Can a computational benefit in the encoding of local space be gained from this level of inhomogeneity? How sensitive might such a computational benefit be to the exact amount of variability present?

To address these questions, we generate populations of synthetic grid cells (see Materials and Methods), where we have complete control over the number of grid cells and their firing properties. For simplicity, we assume that all grid cells in our population have grid orientation and spacing sampled from Gaussian distributions, with means $\hat{\lambda}$ and $\hat{\theta}$ and standard deviations σ_λ and σ_θ, respectively. Assigning each grid cell in our population a grid orientation and spacing, we are able to generate "ideal" rate maps [14]. Sampling from a Poisson process on these ideal rate maps, we generate noisy grid cell rate maps (Fig. 6A). Leveraging a simple linear decoder (see Materials and Methods), we can examine how decoding error of local space is affected by the number of grid cells in the population and the amount of variability (σ_λ and σ_θ).

We begin by investigating the decoding capabilities of a synthetic module with properties similar to that of the experimentally recorded module that was analyzed in detail (recording identifier R12; Figs. 2, 3). We therefore set $\hat{\lambda} = 85$ cm and $\hat{\theta} = 6°$. To determine an appropriate value for σ_λ and σ_θ, we subtract the mean between-cell variability by the mean within-cell variability ($\langle \overline{\Delta\theta}_{\text{between}} \rangle - \langle \overline{\Delta\theta}_{\text{within}} \rangle = 2.0° - 1.2° = 0.8°$ and $\langle \overline{\Delta\lambda}_{\text{between}} \rangle - \langle \overline{\Delta\lambda}_{\text{within}} \rangle = 6.9$ cm $- 1.9$ cm $= 5.0$ cm). We interpret these values as the amount of variability in grid spacing and orientation that is not due to noise. Therefore, we set $\sigma_\theta = 1°$ and $\sigma_\lambda = 5$ cm. This amount of variability leads to similar sampled distributions of λ and θ as was found in the real data (compare Fig. 2E, F with Fig. 6B, C).

Applying the linear decoder to populations of synthetic grid cells, we find that decoding error decreases as the number of grid cells is increased (Fig. 6D, blue line). When $N = 1024$, the decoding error is ≈ 20 cm, substantially better than random decoding (Fig. 6D, dashed black line). As expected, this result is not seen in a synthetic population of grid cells with no variability in grid properties ($\sigma_\theta = 0°$ and $\sigma_\lambda = 0$ cm) (Fig. 6D, black line). In particular, while the restricted number of grid firing fields enables a decoding error

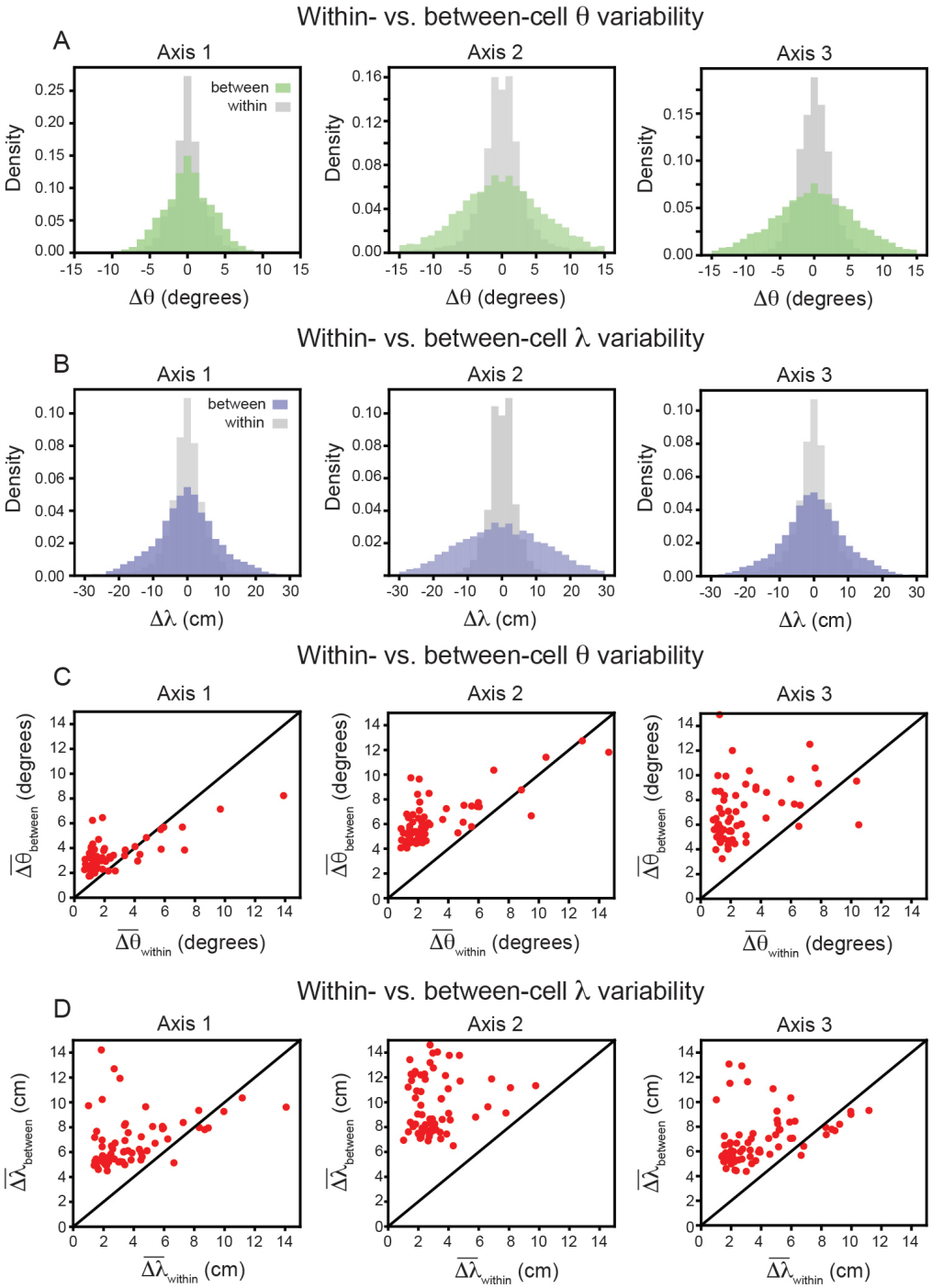

Figure 4: **Variability of grid properties, restricted to the same axis, is a robust feature of individual grid module (recording ID R12).** Same analysis as in Fig. 3 (B) – (E), but for variability measured on each axis independently. For visualization, we exclude a small number of cells that were outside the axes limits, including 2, 5, and 10 cells for Axes 1–3, respectively (C); and 3, 4, and 3 cells for Axes 1–3, respectively (D); these cells were included in non-parametric statistical analyses.

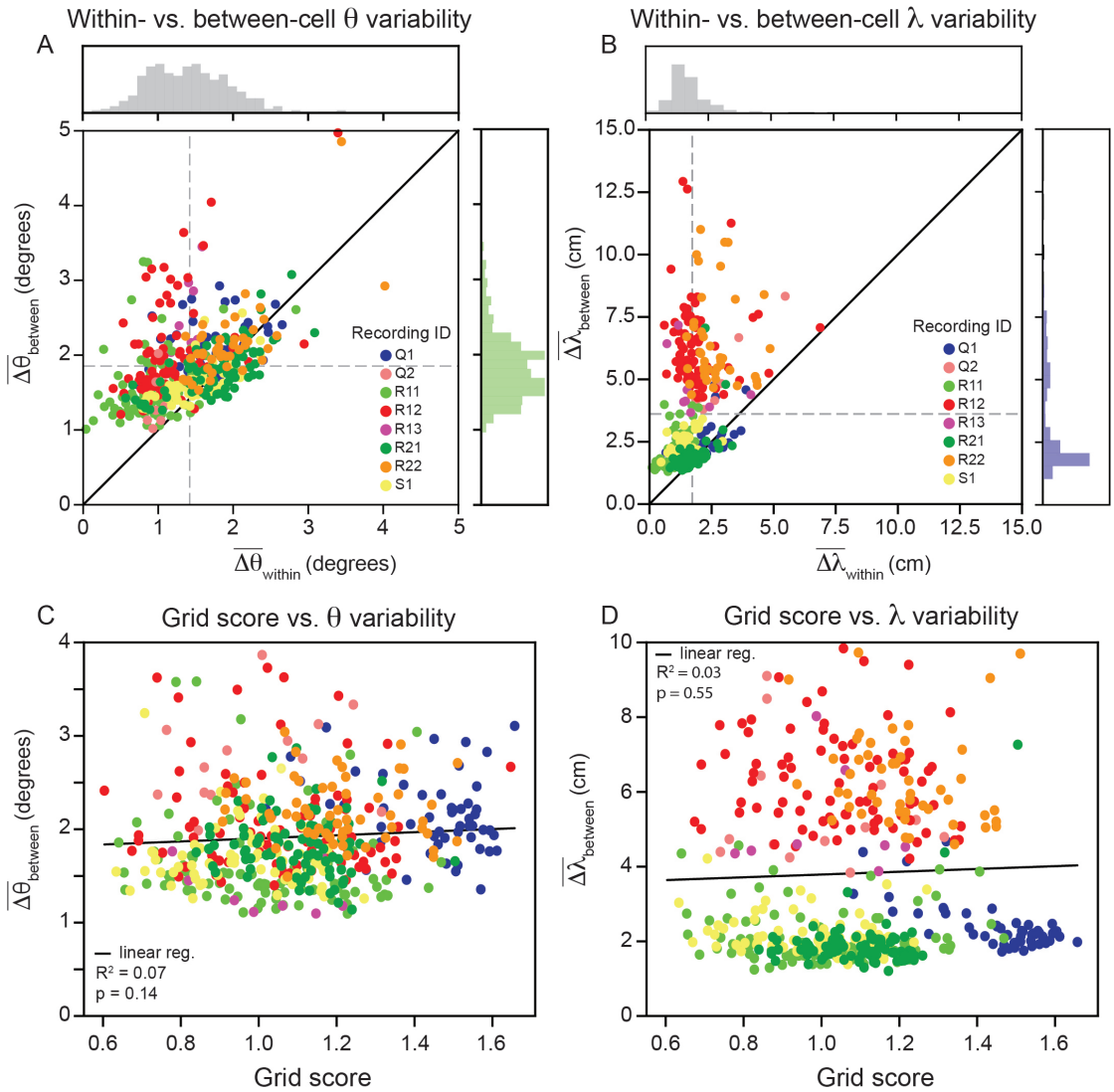

Figure 5: **Within module grid property variability is a robust feature across modules.** (A) Average within-cell variability of grid orientation ($\overline{\Delta\theta}_{\text{within}}$), compared to average between-cell variability of grid orientation ($\overline{\Delta\theta}_{\text{between}}$) for each cell ($N = 420$) across 8 modules (cells colored by their corresponding recording ID). The histogram above the plot shows the distribution of $\overline{\Delta\theta}_{\text{within}}$ and the histogram to the right shows the distribution of $\overline{\Delta\theta}_{\text{between}}$. (B) Same as (A), but for grid spacing. For visualization, 5 cells are excluded ($\overline{\Delta\lambda}_{\text{between}} > 15$ cm), but are included in non-parametric statistical analyses. Dashed gray lines show the population mean. (C)–(D) Average within cell variability of θ and λ (respectively), as a function of grid score. For visualization, 3 and 22 cells are excluded from (C)–(D), respectively, but are included in statistical analyses. Black solid line is linear regression, with R^2 and p-value reported above.

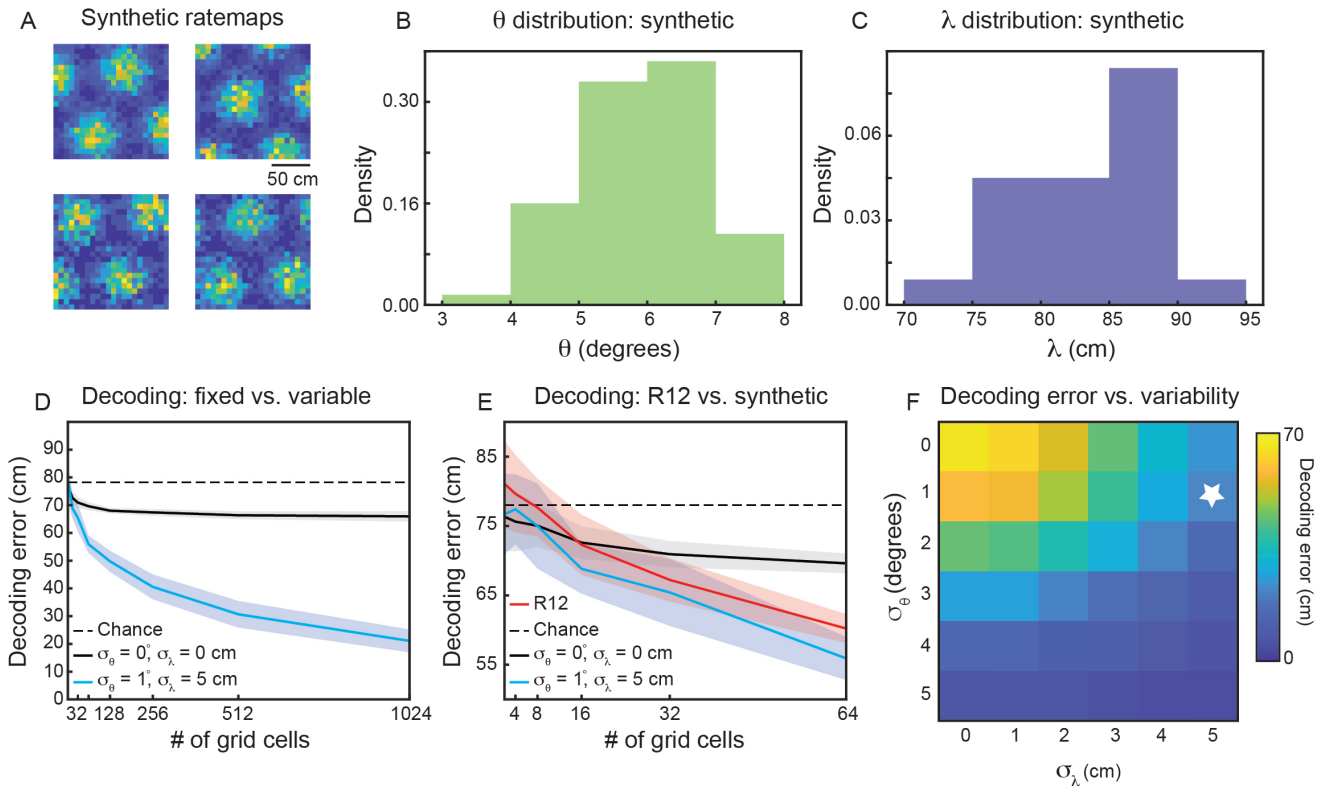

Figure 6: **Variability in grid properties enables improved decoding of local space from the activity of grid cells within a single module.** (A) Example noisy grid cell rate maps generated from a Poisson process. The size of the square arena is set to 1.5 m × 1.5 m to be consistent with what was used in the experimental set-up analyzed [34]. (B)–(C) Distribution of sampled grid spacing and orientation from synthetic population, when using $\hat{\lambda} = 85$ cm, $\hat{\theta} = 6°$, $\sigma_\lambda = 5$ cm, and $\sigma_\theta = 1°$; compare to the distribution measured from real data (Fig. 2E, F). (D) Decoding error, as a function of grid cell population size, with populations having either no variability in grid properties (black line) or variability similar to what was present in the data analyzed (blue line). The solid line is the mean across 25 independent grid cell populations and the shaded area is ± standard deviation of the 25 independent populations. The dashed black line shows chance level decoding error. (E) Decoding error for synthetic populations and real data for up to $N = 64$ cells (red line). (F) Decoding error, over a grid of σ_θ and σ_λ values, for populations of $N = 1024$ grid cells. White star denotes values used in (D).

smaller than chance, the population with fixed grid properties exhibits little change in decoding error with increasing numbers of grid cells. This demonstrates that the improved encoding is specific to populations with inhomogeneity in their grid spacing and orientation.

To validate that the synthetic population was a reasonable surrogate to the experimentally recorded data, we perform the same decoding analysis on grid cells from recording ID R12 (see Materials and Methods). As the real data is limited in the number of cells, we are only able to compare up to populations of $N = 64$ (Fig. 6E). However, in this restricted range, we find agreement between the synthetic population with grid property variability and the recorded data (Fig. 6E, compare red and blue lines). This strengthens our hypothesis that the observed amounts of inhomogeneity in grid spacing and orientation can lead to improved decoding of local space.

To determine the extent to which the decrease in decoding error depends the exact variability of grid spacing and orientation, we perform a grid search over 36 pairs of $(\sigma_\lambda, \sigma_\theta)$ values in $[0°, 1°, \ldots, 5°] \times [0$ cm, 1 cm, ... , 5 cm]. As expected, when the variability of both grid properties is large, we find nearly 0 decoding error of local space (Fig. 6E, bottom right). However, there is additionally a range of σ_λ and σ_θ values that lead to improved decoding, including values smaller than used (Fig. 6E, above and to the left of the white star). We perform the same grid search on synthetic populations of grid cells with different spacings, $\hat{\lambda} = 50$

cm and $\hat{\lambda} = 100$ cm (Fig. S4A, B), finding it again possible for the decoding error to drop well below that of the fixed population, if a sufficient amount of variability exists. For the synthetic module with smaller spacing ($\hat{\lambda} = 50$ cm), the existence of more grid fields in the 1.5 m $\times$ 1.5 m arena leads to greater complexity of interference patterns (Fig. 1), enabling the same amount of variability in grid spacing and orientation to lead to a sharper decrease in decoding error. Additionally, for the synthetic module with smaller spacing, a given value of σ_λ is a larger percentage of $\hat{\lambda}$, as compared to the synthetic module with larger spacing.

Finally, we consider how decoding within a single module compares to decoding across two modules. When the two modules are consecutive (e.g., modules 2 and 3), their grid spacing, λ_2 and λ_3, has been experimentally found to be related via $\lambda_3 \approx \sqrt{2}\lambda_2$ [22]. In such a case, decoding activity from populations with fixed grid properties leads to nearly 0 error (Fig. S5A, black line). The addition of variability in grid properties does not significantly change the behavior of the decoding (Fig. S5A, blue line). This suggests that small amounts of inhomogeneity may not disrupt the previously achieved theoretical bounds on decoding from multiple grid modules [24, 25, 28, 46]. However, if the two modules being decoded from are non-consecutive (e.g., modules 1 and 3) then the grid spacing can be related by an integer multiple, $\lambda_3 = 2\lambda_1$. In such a setting, the grid fields of the larger module are a subset of the grid fields of the smaller module, up to a rotation (due to the difference in orientation between the two modules), and we again find that variability in grid properties can improve decoding accuracy (Fig. S5B, compare black and blue lines).

Taken together, these results suggest that *individual* grid modules can exhibit significant encoding of local space via heterogeneity in their grid properties, even when the extent of the variability in θ and λ is similar to that found in the analysis of the experimental recordings. This benefit can also improve encoding in cases when multiple, non-consecutive modules are considered.

Discussion

The multiple firing fields of grid cells, organized along a triangular lattice, has been historically interpreted as a limiting feature for encoding of local space. Particularly influential in shaping this view has been the discovery of the distribution of grid cells into distinct modules, with grid cell spacing (λ) and orientation (θ) preserved within, but not across, modules [22, 23], making integration across multiple modules necessary for spatial information to be decoded [24–28]. While evidence for discontinuity in the grid cell properties across modules is strong, the corollary assumption, that within-module values of λ and θ are identical (within the bounds of noise), has not been systematically studied.

Analyzing recently collected MEC recordings, we found the range of λ and θ values was large, with examples of grid cell pairs in the same module having over 7° difference in grid orientation and 20 cm difference in grid spacing (Fig. 2). Statistical analysis shows that the variability is more robust than expected from noise, for the majority of grid cells (Fig. 5). This was despite the fact that we used a very conservative criteria for assessing whether a grid cell was robust enough to include in the analysis.

Our comparison of within- and between-cell grid property variability was key to our argument, as it was for previous work using it to study the robustness of differences in peak grid field firing rates [39]. The absence of its use in the characterization of distribution of λ and θ [30] may be why the heterogeneity was not identified until now. We find that our analysis is robust to choices of parameters (Figs. S1, S3) and whether we treat each grid field independently or take the average across grid fields (Figs. 3, 4). This challenges the commonly held belief that the variability observed in the grid orientation and spacing is attributable solely to measurement noise.

We hypothesize that this variability may be used to increase the fidelity at which *individual* grid modules can encode local space. This idea is consistent with a large body of literature showing that heterogeneity in the responses of populations of neurons increases the robustness of encoding [47–50]. We indeed find, in noisy synthetic models of grid cells, that a level of variability in grid properties similar to what is quantified in the real data can be sufficient to accurately decode information of local space (Fig. 6). This benefit is increased with larger numbers of grid cells in the population (Fig. 6D), and is observed over a range of underlying variability values (Fig. 6F). We find that the improvement was most pronounced in modules with

small grid spacing (Fig. S4A), although larger modules can see a decrease in decoding error for amounts of variability consistent with was was found in the analyzed data (Fig. S4B).

We note that our results are additionally aligned with recent findings of heterogeneity in maximum firing rates across individual grid fields [37–39] and grid sheering [35, 36, 40]. Our work further demonstrates that, even in the absence of these perturbations, individual grid modules may encode considerably more local spatial information than previously believed.

Finally, models of the formation of orientation maps in visual cortex have demonstrated that slight angular offsets of retinal mosaics, along which retinal receptive fields are organized, can generate complex patterns [51], similar to those found in visual cortex orientation maps [52]. Our results indicate that grid cells in MEC may be capable of leveraging a similar computational principle, suggesting that mosaic patterns might be a broadly utilized feature in neural coding [53].

Limitations

While the data set we analyzed [34] represents an advance in the ability to simultaneously record from tens to hundreds of putative grid cells, across grid modules, the MEC remains a challenging brain region to access for large-scale neurophysiological experiments. Indeed, with our conservative inclusion criteria, we were ultimately limited by having only 420 grid cells included in our analysis. Future work can perform more detailed and complete characterizations of grid property heterogeneity, as new neurotechnologies that enable larger yield of grid cells are developed [54, 55].

Our decoding analysis, while demonstrating the possibility that variability in grid properties can be leveraged by single grid modules to enhance the encoding of local spatial information, made several simplifying assumptions: Poisson sampling for spike rates; linear decoding; normal distribution of grid properties. While comparison to real data showed that these assumptions are reasonable (Fig. 6E), future work can assess the extent to which these restrictions can be lifted, while still enabling individual grid modules to have low decoding error.

Open questions

The heterogeneity in grid properties we characterize motivates the investigation of several new lines of research. Because these are directions that we believe to be fruitful for the field as a whole, we outline them below, with our hypotheses for possible answers.

Q1: What causes grid property variability? A natural first direction to address is identifying the source of the heterogeneity in grid cell properties we observe. One hypothesis is that this could be driven by "defects" in the specific connectivity pattern that is needed for a continuous attractor. However, whether stable path integration could occur, in the face of such defects, is unclear. As an alternative, the coupling between hippocampus and MEC, which has been shown to lead to variability in grid field firing rates (as experimentally observed) [39, 56], may lead to differences in grid orientation and spacing. In particular, a previous computational model that learned grid cells from place cell input, using non-negative principal component analysis, has shown that place field width affects grid cell orientation and spacing [6]. Heterogeneity in the spatial coding properties of place cells that has been found along the transverse axis of CA3 [57–59], suggesting there may be a systematic differences in the place field widths of hippocampal inputs to MEC grid cells. Further, it was shown that this place-to-grid cell computational model has a linear relationship between place field width and grid spacing, and a non-monotonic relationship between place field width and grid orientation [6]. These may explain why we find stronger average variability in grid spacing than grid orientation (Fig. 5A, B).

Q2: How does grid property variability affect continuous attractor network structure? Continuous attractor network models of grid cells [2, 3, 5, 16] enjoy considerable experimental support [23, 30–34], making them one of the "canonical" models in neuroscience. However, these models make use of the assumption that all the grid cells in a given module have the same grid orientation and spacing to simplify

the network connectivity. In particular, by assuming equal grid orientation and spacing, it becomes possible to arrange grid cells in a two-dimensional space spanned by their phases (i.e., a "neural sheet" [16]). Neurons close in this space are wired with excitatory connections and neurons far in this space are wired with inhibitory connections. As the data set we analyzed was found to provide strong support for the basic predictions of continuous attractor networks (i.e., toroidal topology of the activity manifold) [34], we do not view our results as directly challenging these models. However, considering how these models can accommodate the observed variability is an important future direction. It is possible that these modifications are learnable, via some plasticity mechanism. These solutions may lead to geometric effects on the underlying manifold, which could be explored by using computational tools [60], as well as affect the dynamical properties of the neural population activity, which could be quantified by data-driven dynamical systems methods [61–63]. We hypothesize that the degree in variability of grid spacing and orientation may strike a balance between being small enough to keep the continuous attractor network structure stable, but large enough to enable encoding of local information of space.

Q3: How does grid property variability shape hippocampal representations? The projections from MEC to hippocampus suggest that the variability in grid properties may influence hippocampal representations (even if grid cells do not comprise the majority of its inputs). We consider two possible ways in which this may happen. First, given that grid cells have been reported to maintain their grid spacing and orientation across exposures to new environments, while undergoing a change in their grid phase [30, 64], the integration across multiple modules has been necessary to explain place field remapping. However, grid phase plays an important role in generating the specific complex interference patterns that emerge when considering the joint activity of grid cells with variable grid properties (Fig. 1). Thus the reported changes in phase may be sufficient to generate large (and seemingly random) changes in local spatial information conveyed by grid cells to hippocampal cells. This could drive additional changes in the local spatial information projected to hippocampus, as well as explain the significant differences in correlation structure between CA1 neurons across different environments [65].

And second, recent work on hippocampal place field drift [66–70] has demonstrated that there is a significant change in the place field location across time, especially in CA1. One possible source of this phenomenon is the reported instability of dendritic spines on the apical dendrites of CA1 place cells [59, 71–73], ostensibly leading to changes in the MEC inputs to these neurons. However, if grid cells across multiple modules are necessary for local spatial information, turnover in synaptic input is unlikely to cause large changes in the spatial preferences of CA1 neurons, as integration over several scales should provide stable encoding properties. In contrast, different subpopulations of grid cells with variable grid properties can lead to differences in the local spatial information encoded by their joint activity, even if they come from a single module, possibly influencing the spatial preferences of CA1 place cells.

Q4: Why are there multiple modules? Given that our results demonstrate the ability of single grid modules to encode information about local space – a feat previously believed to be possible only if activity from multiple grid modules was integrated together – why is does MEC have multiple modules? While the variability removes the necessity for encoding local space with multiple modules, higher fidelity representations is achievable by integrating across multiple modules (Fig. S5). In addition, the use of grid cells beyond spatial navigation [74–79], where hierarchical representations are important [42, 43], may be a sufficient implicit bias for the formation of multiple modules [80]. In particular, encoding information at multiple distinct scales is critical for multi-scale reasoning, a cognitive function grid cells may support.

Materials and Methods

Electrophysiology recordings

The neural activity analyzed in this paper comes from a publicly available data set, which has previously been described in detail [34]. We provide brief summary of the methodology and the experimental paradigms used during the recordings.

Three male rats (Long Evans – Rats Q, R, and S) were implanted with Neuropixels silicon probes [81, 82]. These probes were targeted at the MEC-parasubiculum region and the surgery was performed as described previously [32, 82]. After three hours of recovery, recordings were performed.

In the recordings analyzed, the rats foraged for randomly dispersed corn puffs in a 1.5×1.5 m^2 square open field arena, with walls of height 50 cm. The rats were familiar with the environment and task, having trained 10–20 times prior to the implantation. The rats were food restricted to motivate their foraging, being kept at a minimum of 90% of their original body weight (300–500 grams).

All procedures in the original study were approved by the Norwegian Food and Safety Authority and done in accordance with the Norwegian Animal Welfare Act and the European Convention for the Protection of Vertebrate Animals used for Experimental and Other Scientific Purposes.

Electrophysiology post-processing

The neural activity analyzed in this paper was post-processed, before made publicly available. We describe, in brief, the post-processing performed [34], as well as the post-processing we performed on the downloaded data.

Spike sorting, via KiloSort 2.5 [82], was applied to the data recorded from the Neuropixel probes. Individual units were deemed putative cells if their average spike rate was in the range of 0.5–10Hz, and 99% of their interspike intervals were greater than 2 ms.

For each putative cell, rate maps were constructed by averaging the activity at binned spatial positions in the open field arena. This raw rate map was smoothed, using a Gaussian kernel. The autocorrelation of these rate maps were computed, and a grid score calculated, as described previously [44]. To determine which putative cells were grid cells, and to which module they belonged, autocorrelograms of raw rate maps were constructed, with some of the bins corresponding to the closest and furthest distances removed. These "cropped" rate maps were then clustered by using UMAP [83] to project the data onto a two-dimensional subspace. All the cells in the non-largest cluster were found to have similar grid spacing and orientation, as well as high grid scores. They were therefore deemed putative grid cells. All cells in these clusters had their spike trains placed in the public repository from which we downloaded the data.

From the downloaded spike trains, we constructed rate maps and autocorrelograms in a similar manner, using code made publicly available [8] (Fig. 2).

Computing grid score, orientation and spacing from the spatial autocorrelogram

Grid spacing and grid orientation were computed according to standard methods described in detail previously [22, 84]. Briefly, the goal of the procedure is to identify the location of the six nearest fields in the spatial autocorrelogram (SAC) (Fig. 2A). This was achieved by performing the following steps. First, the SAC was smoothed using a 2D Gaussian filter with $\sigma = 1$. Second, the center of the SAC was excluded by applying a mask. Because the size of the SAC's central peak changes for every module, the radius of such mask was re-computed for each module. Third, we thresholded the SAC by applying an extended-maxima transform `ndimage.maximum_filter`. Fourth, we identified the center of every field by using the function `scipy.stats.find_peaks`.

Once the peaks of every field had been found, we computed the location of every peak in polar coordinates. We then selected the 6 peaks that were closest to the center of the SAC, based on the computed radial components. Because every SAC is symmetric, we considered for further analysis the 3 peaks closest to the X axis in angular distance (Axis 1, 2, and 3 [45]). Grid spacing was computed as the arithmetic mean of the

radial component of the 3 peaks (except when each peak was analyzed separately – Fig. 4). Given that the SAC dimensions are twice of that of the real arena, we multiplied the SAC radial mean by a factor of 2. Grid orientation was computed as the angular mean of orientations (relative to the x-axis) of the three peaks (except when each peak was analyzed separately – Fig. 4).

In order to ensure that subsequent analysis was performed only on cells whose SAC's could be well described by hexagonal structure, we imposed the following constraints: 1) the relative angle between two peaks could not be $< 30°$; 2) the relative angle between two peaks could not be $> 90°$; 3) the values for grid spacing between the peaks could not be substantially different (the ratio of spacings between any two peaks must be > 0.5 and < 2). Cells that did not meet these criteria were determined to have "Poor grid fit" and were rejected from all subsequent analysis. The percentage of all cells that were removed by this inclusion criteria is shown in Fig. S1. We note that cells from one of the nine modules in the publicly available data set that we analyzed had 98.4% of cells rejected by this criteria (recording identifier – R23). We therefore did not include it in any subsequent analysis.

To compute the grid score of recorded MEC cells, we made use of previously published code [8], that is based on metrics that have become standards in quantifying grid cell properties[44].

For analysis of the distribution of grid properties (Fig. 2E, F), we included only grid cells with grid scores greater than 0.85. This was done to demonstrate that variability was present even in cells that exhibit robust grid cell properties. In the subsequent analyses, an alternative criteria is used, which considers the reliability of the grid responses (see below).

Computing spacing from the rate maps

In order to characterize the spacing with a method that is less susceptible to the effects of having grid fields at the boundaries of the environment (as the SAC method could be), we compute the spacing using the rate maps of individual grid cells (Fig S2A). Once the rate map was computed, we smoothed it with a Gaussian filter, setting $\sigma = 1.5$. Then, using the function `scikit-image.feature.peak_local_max` we extract the position of the center of each firing field. Because our goal with this analysis is to show that the fields in the border do not impact our analysis with the SAC, we restrict ourselves to the 3 peaks that are closest to the center of the arena (Fig S2A). We compute the distances between those three peaks to each other and report the spacing as the average of the distances measured.

Within and between cells splits

To characterize the within- and between-cell variability of grid spacing and orientation, we employed the following approach. First, we split the data into bins of fixed length (30 seconds). From this, we randomly assigned each interval to one of two blocks (denoted as blocks A and B), with exactly half the total number of intervals in each block. For each grid cell, we computed the grid spacing [$\lambda_A^{(i)}$ and $\lambda_B^{(i)}$] and orientation [$\theta_A^{(i)}$ and $\theta_B^{(i)}$], from the data in each block. The within-cell differences of grid spacing and orientation was determined as $\Delta\theta_{\text{within}}^{(i)} = \theta_A^{(i)} - \theta_B^{(i)}$ and $\Delta\lambda_{\text{within}}^{(i)} = \lambda_A^{(i)} - \lambda_B^{(i)}$. Each grid cell's properties were also compared those of another grid cell, with the match being made using random sampling without replacement. These comparisons were determined as $\Delta\theta_{\text{between}}^{(i,j)} = \theta_A^{(i)} - \theta_B^{(j)}$ and $\Delta\lambda_{\text{between}}^{(i,j)} = \lambda_A^{(i)} - \lambda_B^{(j)}$. This process was repeated 100 times (referring to each iteration as a "shuffle"), per recording. Examples of splits of the data, for different shuffles, is schematically illustrated in Fig. 3A). The resulting distributions of $\Delta\theta_{\text{within}}$, $\Delta\lambda_{\text{within}}$, $\Delta\theta_{\text{between}}$, and $\Delta\lambda_{\text{between}}$, across grid cells and splits of the data were then compared. If the within-cell variability in grid spacing and orientation was smaller than between-cell variability, we concluded that the variability of grid cell properties was a robust feature of the data and not due to noise.

When performing this shuffle analysis, we found that some cells, despite having a good grid fit when all the data was considered, did not have SACs that were well described by hexagonal structure when the data was split in half. We viewed this a manifestation of unreliable grid coding and a possible confound in our quantification of variability. As such, we introduced a new inclusion criteria (replacing that of requiring a

grid score of > 0.85), only considering cells that had poor grid fits on $< 5\%$ of all shuffles. The percentage of all cells that were removed by this inclusion criteria is shown in Fig. S1. In general, we found that cells with high grid score were reliable, although there were exceptions. Additionally, we found that size of the bin used for splitting the data did not significantly affect the percent of cells with good grid fits (passed the prior inclusion criteria) that were considered reliable (Fig. S3).

Synthetic grid cells

To study how variability in grid cell properties might endow the grid code with computational advantages, we generated synthetic grid cell rate maps, so that we could have complete control over the distribution of their properties. These synthetic grid cell rate maps were constructed as follows.

First, the lengths of each dimension the "arena" within which the simulated grid cells exist (L_x and L_y) were set. Then, for $N \in \mathbb{N}$ grid cells, the grid spacing and orientation were sampled via $\lambda^{(i)} \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda)$ and $\theta^{(i)} \sim \mathcal{N}(\mu_\theta, \sigma_\theta)$ where $\mathcal{N}(\mu, \sigma)$ is a normal distribution with mean μ and variance σ^2. Grid phase was sampled as $\phi^{(i)} \sim \mathcal{U}([0, L_x] \times [0, L_y])$, where ϕ is a two dimensional vector, with first component uniformly sampled from $[0, L_x]$ and second component uniformly sampled from $[0, L_y]$. To construct a population with no variability in grid properties (to use as a control), we set $\sigma_\lambda = 0$ m. and $\sigma_\theta = 0°$.

For each grid cell, we generated idealized grid responses by summing three two-dimensional sinusoids [14], such that the activity at $\mathbf{x} = (x, y) \in [-L_x/2, L_x/2] \times [-L_y/2, L_y/2]$ is given by

$$X^{(i)}(\mathbf{x}) = X_i^{\max} \frac{2}{3} \left(\frac{1}{3} \sum_{j=1}^{3} \cos \left[R(\theta_i) \mathbf{k}_j (\mathbf{x} + \phi_i) \pi / \lambda_i \right] \right), \tag{1}$$

where $X_i^{\max}$ is the maximal firing rate, $R(\theta_i)$ is the two-dimensional rotation matrix, with rotation θ_i, and $\mathbf{k}_j$ are the wave vectors with 0°, 60°, and 120° angular differences [i.e., $\mathbf{k}_1 = (1, 0)^T, \mathbf{k}_2 = (1/2, \sqrt{3}/2)^T, \mathbf{k}_3 = (1/2, -\sqrt{3}/2)^T$]. To match the recorded neural data, where individual grid cells have distinct maximal firing rates, $X_i^{\max} \sim \mathcal{N}(13, 8)$. We enforced $X_i^{\max}$ to be within $[2, 30]$, by setting any sampled values outside of this range to the boundary values (i.e., 2 or 30).

To determine the extent to which local spatial information can be decoded from the activity of populations of grid cells with different degrees of variability in their grid properties, we performed the following analysis.

We generated noisy synthetic spike rates of N grid cells by assuming a Poisson process and sampling using the idealized rate maps (Eq. 1). More concretely, the activity of grid cell i at position (x, y) was assumed to be a random variable with a Poisson distribution, whose mean was $X^{(i)}(x, y)$. Thus, the probability of observing $\tilde{X}^{(i)}(x, y)$ spikes, at position (x, y), is given by

$$\tilde{X}^{(i)}(x, y) \sim \mathcal{P}\left(X^{(i)}(x, y) \right). \tag{2}$$

Linear decoding synthetic data

For a given resolution of the arena, we generated $\tilde{X}_j^{(i)}(x, y)$, where $j = 1, ..., 10$. That is, we constructed 10 noisy rate maps. We performed cross-validated decoding by averaging across 9 of the 10 rate maps, to get an average rate map $\hat{X}^{(i)}(x, y)$. For sake of simplicity, consider $\hat{X}^{(i)}(x, y) = 1/9 \sum_{j=1}^{9} \tilde{X}_j^{(i)}(x, y)$. To decode local position from the held out noisy rate map [e.g. $\tilde{X}_{10}^{(i)}(x, y)$], we multiplied the activity at each position by all the positions in the average rate map, taking the sum and assigning the decoded position as that with the largest value. The Euclidean distance between the decoded position and the true position is considered the error. This was performed 10 times (holding out each rate map once) and the average error across all positions in the environment was then averaged across all 10 of the validation splits.

Linear decoding experimental data

To decode the electrophysiological data [34], we sampled subpopulations of grid cells from the $N = 82$ units (recording ID R12) that passed the criteria described previously. For each subpopulation, we split the recorded activity into 10 evenly spaced temporal bins, and constructed average ratemaps for each bin. This is consistent with what was done in decoding the synthetic data. All ratemaps, for each cell, were normalized by the maximal activity. We then randomly chose 5 of the 10 time bins to serve as the "train" data, the remaining 5 served as the "test" data. The rate maps were averaged and then the linear decoder was applied. We sampled 10 different choices of splitting the data and 25 choices of subpopulation.

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Supplemental material

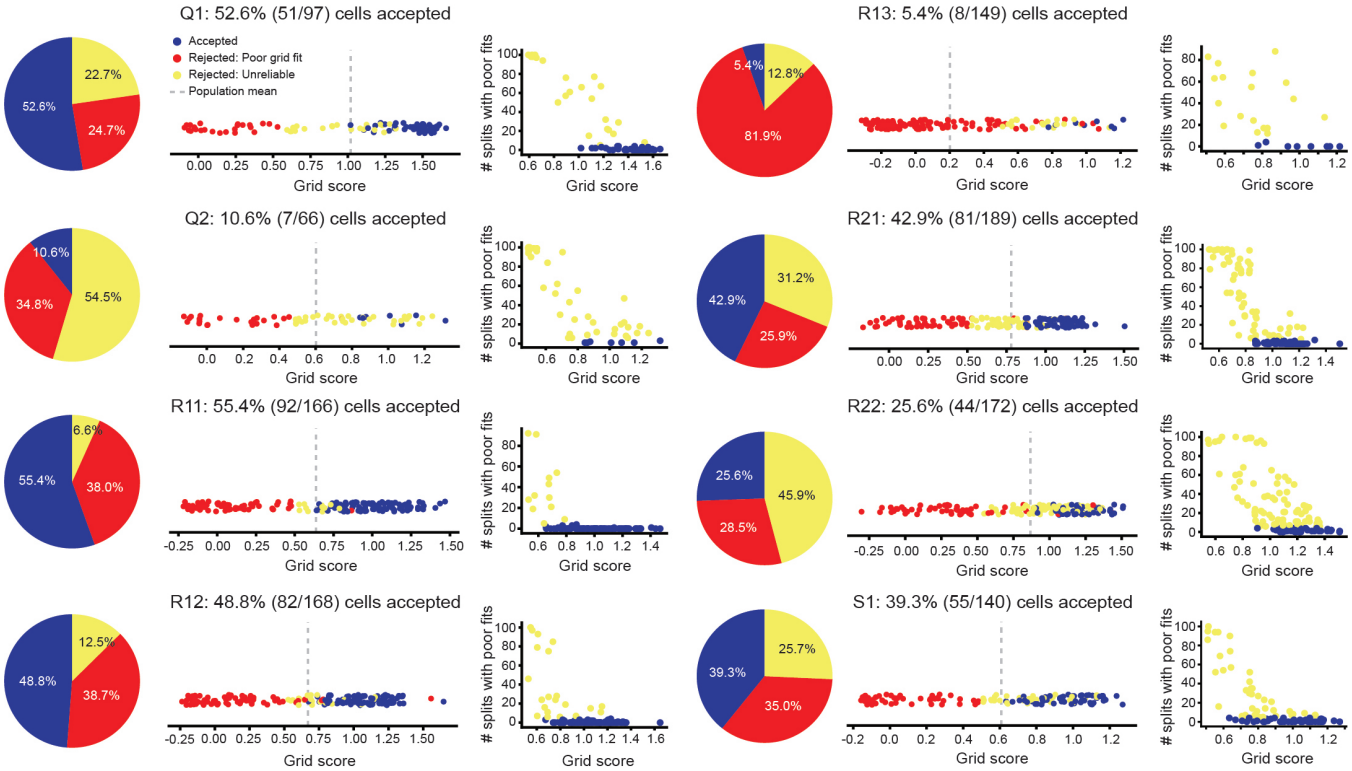

Figure S1: **Accepted and rejected cells, across all grid modules.** (Left) The percent of all cells, across all modules, that were rejected by our inclusion criteria. Cells that were rejected as not having SACs, computed with all data, that were well described by hexagonal structure ("Rejected: Poor grid fits") are shown in red. Cells that were rejected as not reliably having SACs, computed from splits of the data, that were well described by hexagonal structure ("Rejected: Unreliable") are shown in yellow. Cells that met these criteria ("Accepted") are shown in blue. (Middle) Grid score of all cells, with coloring denoting whether they were accepted or rejected. Dashed gray line denotes population mean. (Right) The number of splits of the data (out of 100) that cells had SAC's with poor grid fits, as a function of each cell's grid score.

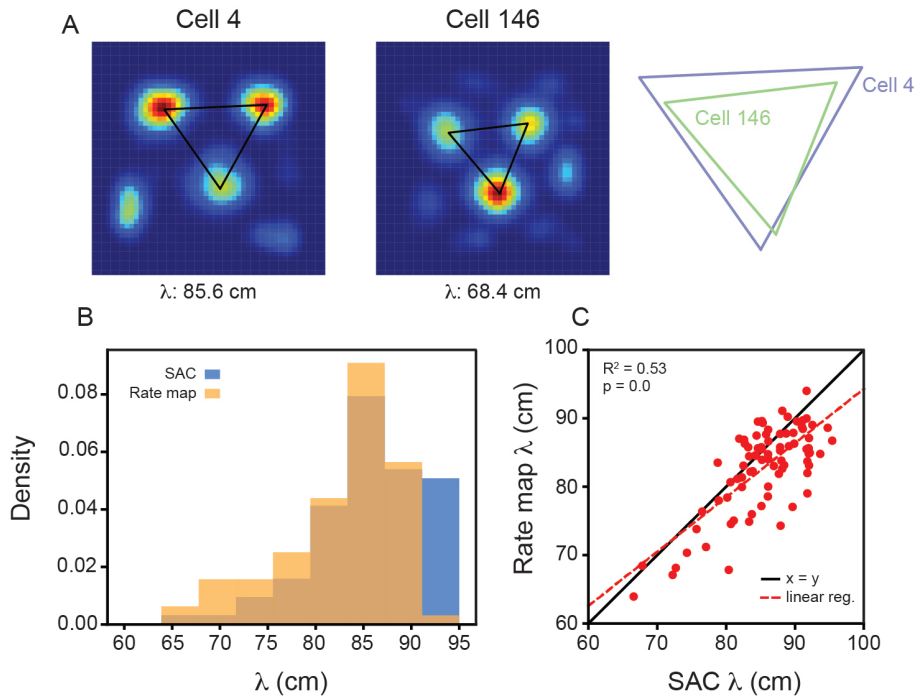

Figure S2: **Variability in grid spacing, within a single module, exists when computing λ directly from the rate maps.** (A) (Left) Example grid cell rate maps from the same module (recording ID R12) with overlaid triangles, corresponding to the spacing between each of the three most central peaks. Grid spacing is computed as the average of the three lengths. (Right) To aid comparison, the triangles are enlarged (with their relative size fixed) and overlaid. Note that these cells are the same ones plotted in Fig. 2B. (B) Distribution of grid spacing computed using the SAC and the rate maps. (C) The grid spacing of all grid cells, from recording R12, computed using the SAC and the rate map. Red dashed line is the linear regression fit with R^2 and p-value reported above.

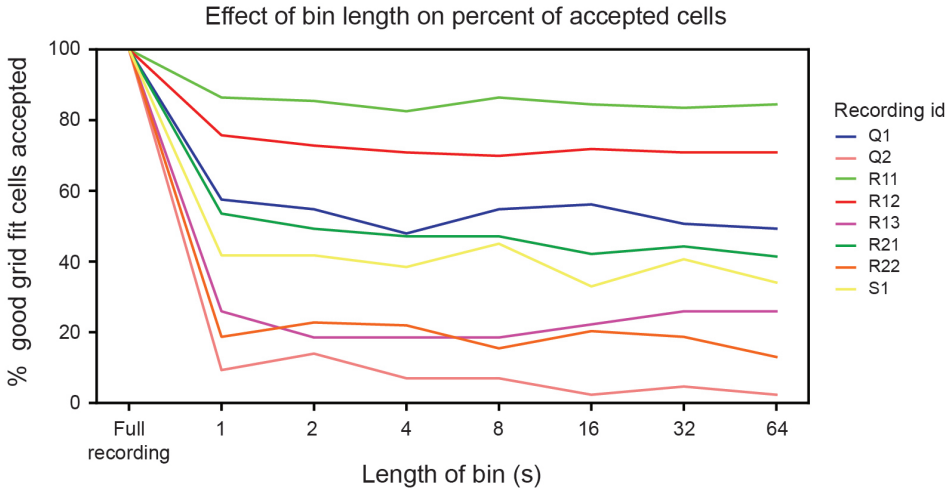

Figure S3: **Bin length does not affect the percent of cells, with good grid fits, that are accepted for analysis.** The percent of cells with good grid fits (i.e., those cells that do not get rejected for having "poor grid fits") that are accepted by not being deemed unreliable, as a function of the size of the bins used in the shuffle analysis. All modules are plotted (each line is colored based on its recording ID).

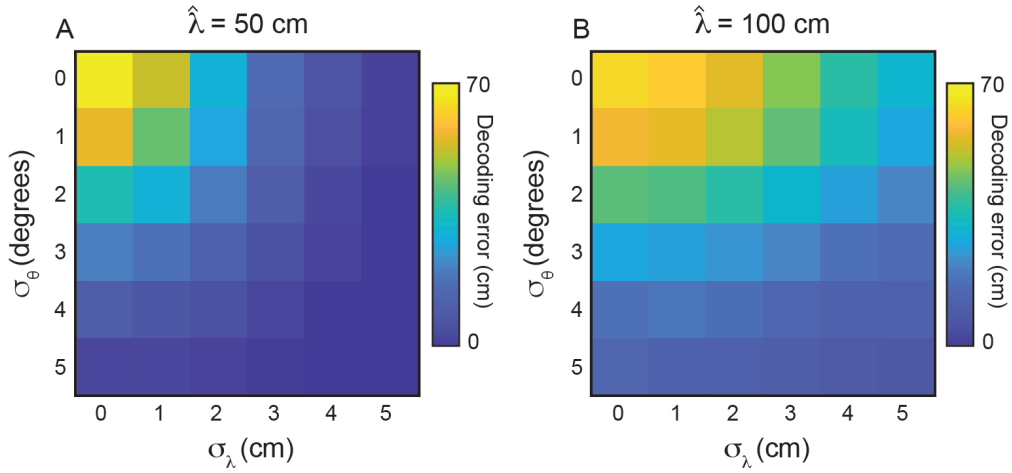

Figure S4: **Variability in grid properties improves decoding of local space for grid modules with different grid spacings.** (A)–(B) Same as Fig. 6F, for $\hat{\lambda} = 50$ cm and $\hat{\lambda} = 100$ cm, respectively. $\hat{\theta}$ is set to 0°.

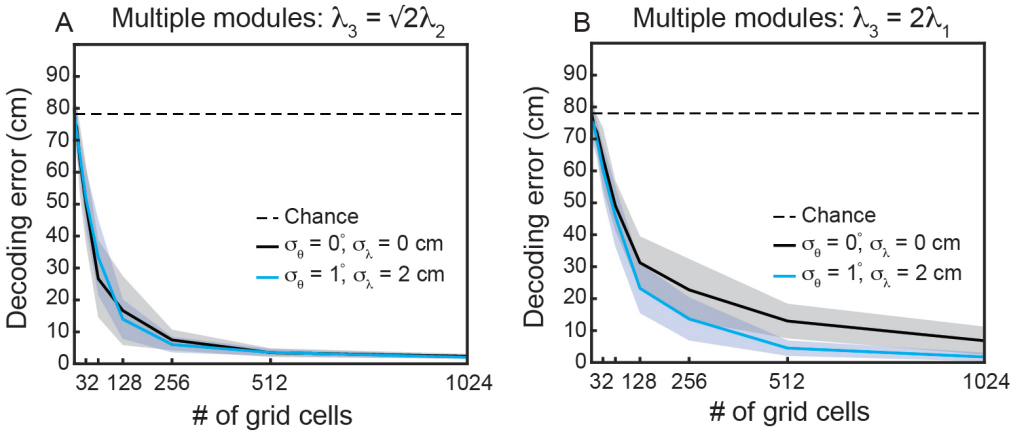

Figure S5: **Variability in grid properties improves decoding of local space for multiple modules, when the modules have integer multiple spacing.** (A)–(B) Same as Fig. 6D, when decoding from multiple modules. (A) $\hat{\lambda} = 70, 100$ cm and $\hat{\theta} = 0°, 10°$. (B) $\hat{\lambda} = 50, 100$ cm and $\hat{\theta} = 0°, 10°$.