Top-down origins of the predator-prey power law?

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Abstract

The ratio of predator-to-prey biomass density is not constant along ecological gradients: denser ecosystems tend to have fewer predators per prey, following a scaling relation known as the "predator-prey power law". The origin of this surprisingly general pattern, particularly its connection with environmental factors and predator-prey dynamics, is unknown. Here, we explore some ways that a sublinear predator-prey scaling could emerge from density-dependent interactions among predators (which we call a top-down origin), rather than among prey (bottom-up origin) as proposed in Hatton *et al.* (2015). We combine two complementary theoretical approaches. First, we use phenomenological differential equations to explore the role of environmental parameters and dynamical properties in controlling the predator-prey ratio. Second, we simulate an agent-based model with tunable predator self-regulation to investigate the emergence of predator-prey scaling from plausible microscopic rules. While we cannot rule out alternative explanations, our results show that specific forms of predation and intraspecific predator interactions are potential mechanisms for the predator-prey power law.

1 Introduction

General biological patterns are few and offer precious opportunities for synthesis and unification [1–8]. Hatton *et al.* [9] put forward evidence of such a general pattern, holding across many terrestrial and aquatic ecosystems, dubbed the "predator-prey power law". The analysis in Ref. [9] reveals a decrease in the ratio of predator to prey when comparing ecosystems at increasingly higher predator and prey densities. The law holds for focal predator species as well as for aggregate communities and has been recently shown to be robust in ecosystems with more complex trophic structures than previously analyzed (e.g., including omnivores) [10].

In this paper, we try to understand this observation, focusing on a central dataset: the total of large mammalian herbivores and carnivores showing a large gradient of biomass density across a range of 23 African reserves (Fig. 1 and Supplementary Material (SM)). Species composition is similar across the gradient and includes herbivores with a body mass ranging from 5 to 500 kg and large carnivores (20 to 140 kg) [9, 11, 12]. A reason for focusing on this dataset is the robustness of the pattern to various factors, including the taxonomy of species included in the

communities, variations in species body mass, the possibility of systematic bias in sampling, and alternative regression approaches [9].

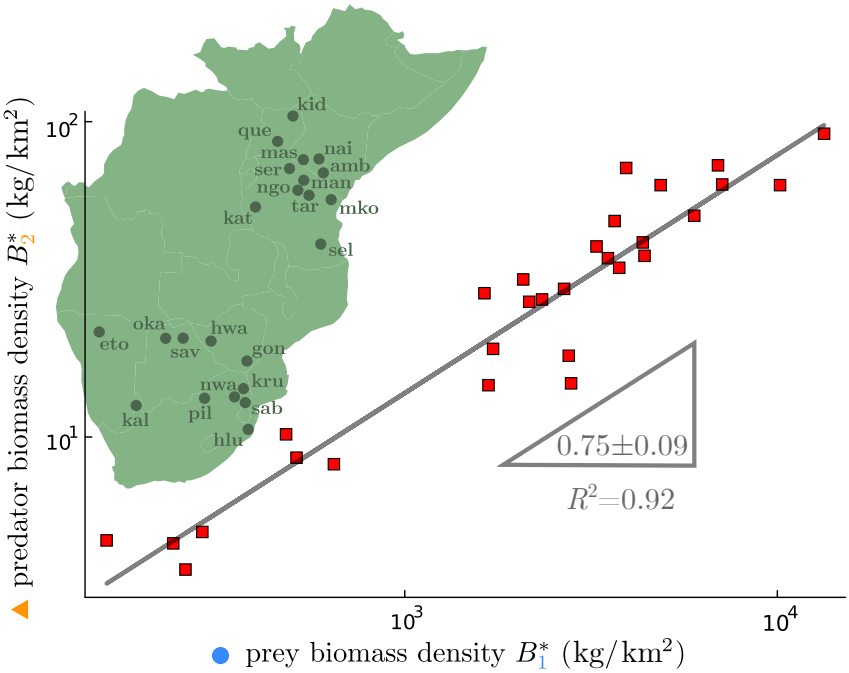

Figure 1: Predator biomass density B_2^* scales sublinearly with prey biomass density B_1^*, as $B_2^* \sim (B_1^*)^k$ with $k \simeq 3/4$. Data refers to predator-prey communities in African reserves obtained from the meta-analysis in Ref. [9] (see SM). The pattern is robust across ecosystems [9].

The first step towards understanding this scaling is explaining what could cause the stark differences in density between different sites. Here, we assume that these densities are close to their equilibrium value given the abiotic and biotic environment (rather than, for instance, low and high densities representing early and late stages in a colonization process), as suggested by the analysis of time series reported in Ref. [9]. Under this assumption, we could start by imagining two very distinct explanations.

One possibility is that the gradient is driven by environmental differences among the parks, therefore, by variation in the conditions in which the species find themselves interacting. The other possibility is that, at an equivalent environmental conditions, there are differences (phenotypic differences) in the species in the different parks. In the latter case, reaching different equilibria would require having different species, e.g. that vary in body mass, size of territory or home range, resource needs, etc. Yet, evidence reported in Ref. [9] stands against the idea that the different parks have significantly different composition in species or individual size, at least none that would systematically create such a gradient. Therefore, we assume the biomass density gradient is caused by environmental factors.

We can proceed now to explore the reasons behind the specific form of the scaling along the biomass gradient. Predator and prey densities could both be controlled entirely by changes in external factors, such as more or less productivity and mortality due to aridity or landscape features, irrespective of any dynamics within and between these trophic levels. In that case, a robust scaling law between predator and prey density, where both increase clearly but sublinearly, would require a "conspiracy" of parameters. Thus we wish to posit that the observed variation of densities and scaling law between them requires a combination of environmental variation and dynamical mechanisms.

A first explanation of this kind was proposed by Hatton *et al.* [9], suggesting that the driving dynamical mechanism is a sublinear relationship between prey density and prey biomass production. They showed, and we recall below, that the environmental variation implied by

this explanation is that different sites must have different attack rates between predator and prey [9].

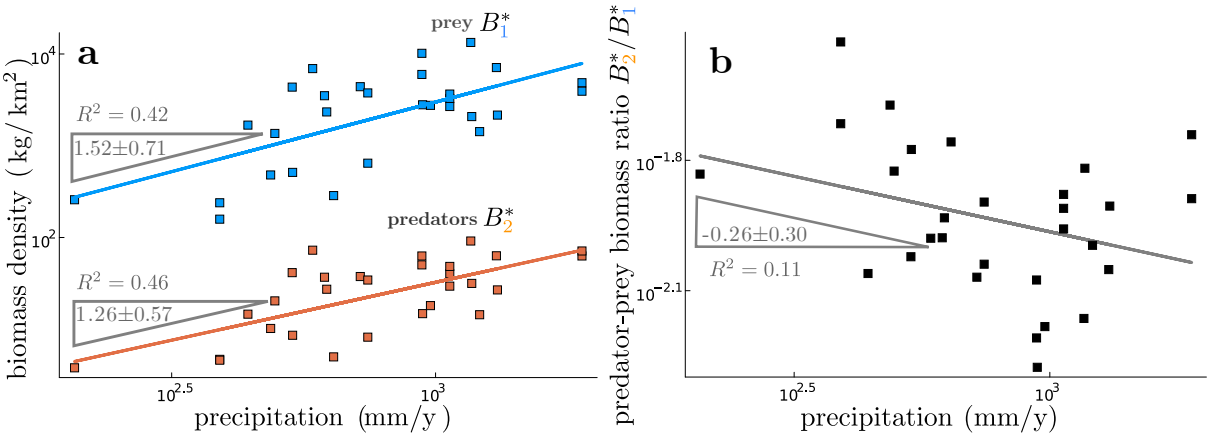

Figure 2: Predator and prey biomass densities scale differently with respect to park precipitation, with their ratio scaling approximatively as $-1/4$, suggesting that precipitation may be a good proxy for the relevant environmental parameter responsible for the biomass density gradient across different parks. In (**a**), we report predator and prey biomass density vs. park precipitation and, in (**b**), their ratio, again vs. precipitation.

Another classic assumption is that the various parks differ by their primary productivity, positively correlated with herbivore growth (secondary productivity) [13]. Annual precipitation is a good proxy for primary productivity in African parks [14]. In Fig. 2 (**a**), we report predator biomass density vs. precipitation, scaling with an exponent around 5/4 and prey biomass density vs. precipitation, scaling with an exponent close to 3/2 (consistent with Ref. [14]). This results in predator-prey biomass ratio vs. precipitation scaling as $-1/4$ (Fig. 2 (**b**)). Within this assumption, we show that a range of explanations are possible, but they are all top-down, in the sense of relying on predation dynamics, and specifically on density-dependent predator interactions. We explore phenomenological models of these interactions and propose an ABM that shows that they can arise microscopically.

2 Predator-prey dynamics and macroecological patterns

We aim to compare ecosystems hosting the same predator and prey species pairs at different stationary biomass densities. We encode the *environmental* parameters responsible for the biomass gradient in the vector $\vec{\rho}$. These parameters can represent biotic and abiotic factors affecting the population dynamics (e.g., amount of primary resources or shelters) but also phenotypic traits, which are sensitive to a change of environment but decoupled from the population densities. We collect parameters of the model which are considered independent of the species' density and environmental conditions in the vector $\vec{\sigma}$. These may be representative of *phenotypic* (or taxonomic) traits. Finally, we define the species density vector $\vec{B}$: all the model ingredients representing the dynamics in a given environment are functions of it. We consider a two-dimensional system with one prey species whose density is denoted by B_1, and one predator species, B_2, that feeds on it. This coarse-grained setting allows us to highlight how density dependence clearly can shape the trophic structure at different biomass densities, even though it is not suited to describing a realistic food web in detail.

The equations describing the dynamics of the density of prey B_1 and of predators B_2 can

be given, with some degree of generality, by

$$\frac{dB_1}{dt} = P_1(\vec{\rho}, \vec{\sigma}, B_1) - L_1(\vec{\rho}, \vec{\sigma}, B_1) - C(\vec{\rho}, \vec{\sigma}, \vec{B}) \,, \tag{1}$$

$$\frac{dB_2}{dt} = P_2(\vec{\rho}, \vec{\sigma}, \vec{B}) - L_2(\vec{\rho}, \vec{\sigma}, B_2) \,, \tag{2}$$

where P stands for production, L for internal losses and C for predation losses (C/B_1 is the functional response). In general, predation losses and predator production depend on both species.

Any parameter in $\vec{\rho}$ could, in principle, differ between sites and induce a variation in observed prey and predator biomass density, but each choice would have a distinct ecological meaning. Changing, e.g., a coefficient in the functional response means that, in different ecosystems, something unrelated to the density of the species, $\vec{B}$, or the taxonomic properties, $\vec{\sigma}$ is affecting the probability of a catch. An example can be a different concentration of shelter for prey [15]. Another common reason behind different biomass densities of predators and prey in different environments is a variation in the energy influx in the system through primary production, which can be encoded in a changing coefficient in the prey production term.

The observed gradient might not be due to variation in one parameter alone, and we must carefully consider how covariance between the environmental parameters may affect our model predictions [16]. Consider a choice of the production and losses that correspond to the Lotka-Volterra model with self-regulation,

$$\frac{dB_1}{dt} = rB_1 - cB_1^2 - qB_1B_2 \,, \tag{3}$$

$$\frac{dB_2}{dt} = gqB_1B_2 - mB_2 - aB_2^2 \,, \tag{4}$$

where r is the growth rate of the prey, q is the catch rate, g the numerical response of the predators, m the death rate of the predators, and c and a the self-regulation strength of prey and predators respectively. It is instructive to write down the stationary solution for the two species' densities to highlight the explicit dependence on the parameters

$$B_1^* = \frac{ar}{ca + gq^2} \,, \tag{5}$$

$$B_2^* = \frac{gqr}{ca + gq^2} \,, \tag{6}$$

where we considered negligible predator mortality, $m = 0$, for clarity. If the prey growth rate r is the single most relevant environmental parameter that changes along the biomass gradient, i.e., $\rho = r$ and $\vec{\sigma} = (a, g, q, m)$, then linear scaling between predator and prey density $B_2^* \sim B_1^*$ emerges, both being linear in r. If, however, the interaction strength q is also relevant and it co-varies with r, for example as $q \sim r^{k-1}$, with $k < 1$, then the observed scaling is sublinear $B_2^* \sim (B_1^*)^k$.

In summary, the choice of the parameters that are supposed to model a biomass gradient has to be informed by observation, and different choices of parameters produce different macroecological patterns starting from the same dynamical model. We want to explore how dynamical, density-dependent effects can be relevant for the scaling. Therefore, we use as a working assumption that only one environmental parameter, possibly different in other datasets, is responsible for the gradient.

Within this working assumption, a linear relation and, therefore, a constant average number of prey per predator in a patch across a biomass gradient would imply that only the stationary number of predators and prey matters, and how densely packed the two species are is irrelevant. If species density matters [17–20], a sublinear scaling is expected when the net effect of crowding penalizes more the standing biomass of predators than that of prey. In Fig. 3 we pictorially

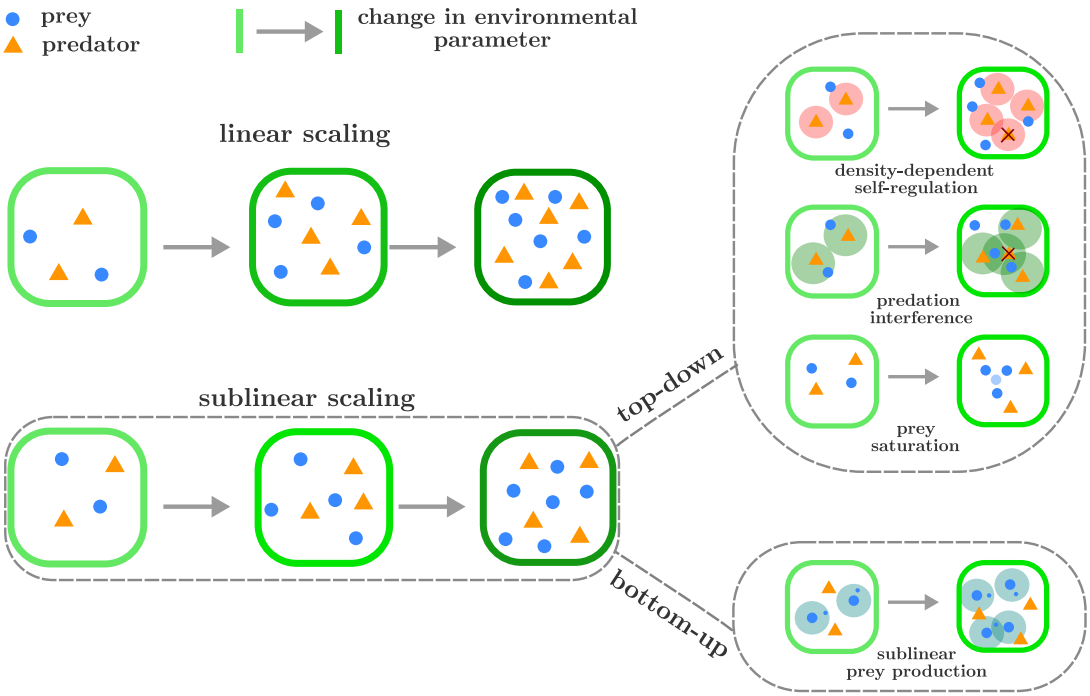

Figure 3: Sublinear scaling reflects the relevance of density-dependent effects for the predator-prey ratio. The figure portrays a *Gedankenexperiment* in which we start with a patch of an ecosystem hosting at stationarity a given number of predators and prey. Then we change the environmental conditions so that the stationary number of prey is the same in the two cases but the number of predator scale linearly in one case and sublinearly with the one of prey in the other. Examples of mechanisms that can lead to sublinear scaling are also portrayed, covering bottom-up and top-down explanations.

summarize this observation and show examples of mechanisms that could dynamically lead to sublinear scaling. The bottom-up explanation, in the sense of relying on prey dynamics and specifically on density dependence in prey production, emerges from sublinear growth of herbivores [9]. The main classes of mechanisms behind the top-down explanation are predator density-dependent self-regulation, predation interference, and prey saturation.

3 Bottom-up, sublinear prey production

We summarize here the bottom-up explanation proposed in Ref. [9], which invokes sublinear prey production to describe the observed predator-prey scaling. The model is defined as

$$\frac{dB_1}{dt} = rB_1^k - qB_1B_2, \tag{7}$$

$$\frac{dB_2}{dt} = gqB_1B_2 - mB_2, \tag{8}$$

where r is the growth rate of the prey, q is the catch rate, g the growth rate of the predators, m the death rate of the predators and $k < 1$ is an exponent characterizing the density dependent effects on the per-capita growth rate of the prey. To be dimensionally consistent, we should multiply the prey production term by a factor B_0^{1-k}, where B_0 has the dimensions of a density. Here, and *mutatis mutandis* in the other sections, we consider $B_0 = 1$ without loss of generality for the purpose of this work.

The equilibrium prey density B_1^* and predator density B_2^*, are respectively

$$B_1^* = \frac{m}{gq}\,, \tag{9}$$

$$B_2^* = \frac{rm^{k-1}}{g^{k-1}q^k} = \frac{rg}{m}(B_1^*)^k\,. \tag{10}$$

Therefore rg/m has to be constant and q has to decrease (increase) to move up (down) across the biomass gradient in order to obtain the scaling $B_2^* \sim (B_1^*)^k$. If, instead, r is primarily responsible for the biomass increase, this model predicts a constant prey density while the predators grow. In terms of the general definition above, the two cases correspond, respectively, to $\rho = q$, $\vec{\sigma} = (r, g, m)$ and $\rho = r$, $\vec{\sigma} = (q, g, m)$.

This bottom-up explanation is informed by another macroecological observation, namely the sublinear scaling of biomass production with respect to biomass density [9, 21]. Sublinear production in the dynamical equation for prey does not represent a unique possible explanation for the macroecological pattern observed across ecosystems. However, it can be realistic and parsimonious, as discussed in depth in Ref. [21].

Support for q as the relevant parameter to move along the biomass density comes from cross-system studies, which have shown that density increases across prey causes a decrease of nutritional quality and edibility, which may tend, in turn, to weaken interaction strength with consumers [9, 22–26].

4 Top-down, density-dependent predation and predator self-regulation

Here we explore how the predator-prey scaling can have a top-down origin, emerging from density dependence in predation and predator self-regulation. We consider a phenomenological model in which density-dependent effects are important at all scales, following the conceptualization of Ref. [16]. The functional form of the terms involved could, in principle, emerge from underlying spatiotemporal details [27] and should not be necessarily rooted in mechanistic explanations [28]. The model reads

$$\frac{dB_1}{dt} = rB_1 - cB_1^2 - qB_1^\alpha B_2^\beta\,, \tag{11}$$

$$\frac{dB_2}{dt} = gqB_1^\alpha B_2^\beta - mB_2^{1+\gamma}\,, \tag{12}$$

where r is the growth rate of the prey, c the strength of prey's self-regulation emerging, e.g., from competition for resources [29, 30], q, g, and m are, respectively, related to the catch rate, predator growth rate, and predator death rate (by properly accounting for dimensionality, as in the section above). The exponent α encode saturation, β is connected to predator interference, and γ models the strength of predator self-regulation. The functional response is scale-free, in the sense that complete saturation with respect to prey density is never achieved. However, this is not unrealistic as evidence suggests that predator feeding rates may often be unsaturated under typical conditions [31]. It is also possible to use other models such as the Hassell-Varley-Holling model [32–35], especially if the systems are far from complete saturation.

In this work, we are only quantitatively interested in the scaling exponents that emerge from the dynamics. Therefore, Eqs.(12) and (11) can be reduced, through non-dimensionalization (see SM), to the form

$$\frac{dB_1}{dt} = \rho B_1 - B_1^2 - \sigma B_1^\alpha B_2^\beta\,, \tag{13}$$

$$\frac{dB_2}{dt} = B_1^\alpha B_2^\beta - B_2^{1+\gamma}\,, \tag{14}$$

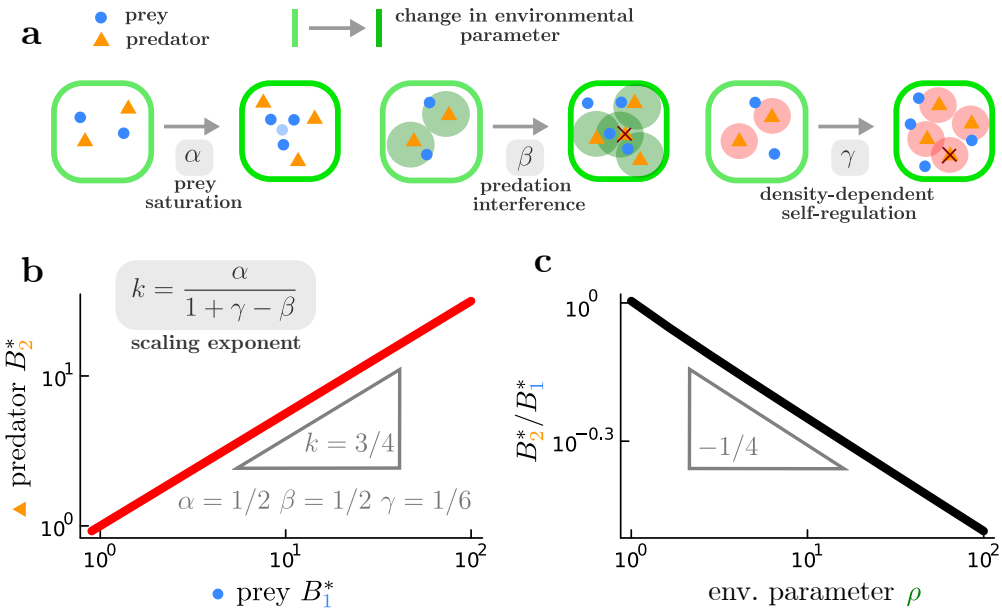

Figure 4: Predator biomass density B_2^* scales sublinearly with prey biomass density B_1^* for given exponents for saturation α, interference β and self-regulation γ (**a**) when herbivore growth is varied to simulate a biomass gradient, shown as straight line on log-log axes. By solving numerically Eqs. (13) and (14), here we show, for $\alpha = \beta = 1/2$, $\gamma = 1/6$, $\sigma = 1/10$ and $\rho \in [1, 100]$, (**b**) the predator stationary biomass density against prey stationary biomass density ($k = 3/4$) and (**c**) their ratio with respect to the environmental parameter ρ (see Eq. (13)), which is proportional to the herbivore growth rate r.

where we call, consistently with our definition of environmental and phenotypic variables, the remaining parameters ρ (which is responsible for the biomass gradients) and σ. These are functions of the original parameters, with $\rho \propto r$ and σ independent from r. The general stationary solution for predator biomass density as a function of prey reads

$$B_2^* = (B_1^*)^k \,, \qquad \text{with} \quad k = \alpha/(1+\gamma-\beta) \,, \tag{15}$$

while the solution for the prey has a closed form only for specific values of the exponents (α, β and γ) and must otherwise be solved numerically. Assuming a varying ρ, corresponding to a variation of primary productivity, every combination of the exponents resulting in $k = 3/4$ predator-prey scaling across ecosystems is viable as long as it produces feasible and stable stationary solutions.

As an example, a meta-analysis of predator–prey pairs of mammals, birds, and reptiles suggests that total predation rates increase as the square root of the product of predator and prey density [16], i.e., $\alpha = \beta = 1/2$. In this case, in order to observe the $k = 3/4$ scaling, the exponent associated to predator self-regulation has to be $\gamma = 1/6$. This means that per-capita losses for predators must scale[1] like $B_2^{1/6}$. In Fig. 4 we report the predator-prey scaling for this choice of parameters.

Differences in primary productivity and herbivore growth are considered among the most common reasons underlying differences in biomass density in otherwise similar ecosystems. Support for considering r the relevant environmental variable comes from data on the relation between biomass density and precipitation in the parks (Fig. 2) and from previous studies connecting precipitation with primary productivity and herbivore growth [13, 14]. If precipitation is associated with the relevant environmental variable ρ in our model (see Fig. 4 (**b**)), we expect

[1]This exponent, incidentally similar to scaling laws found in complex systems outside of ecology [36], suggests that even a seemingly weak density-dependence may play a significant role in biomass patterns.

the scaling in Fig. 2. To rule out an obvious potential confounder, we show in the SM that there is no correlation between park area and precipitation level. This evidence is insufficient to establish primary productivity as the most relevant driver, but it indicates it as plausible.

5 Agent-based model as a proof of concept for the top-down case

Predator-prey agent-based (ABM) models [37–41] can provide a bridge between mathematical models and more realistic behavioral rules, providing more intuition on the dynamics underlying our patterns. We present a simple ABM that produces sublinear predator-prey scaling across a biomass gradient through top-down effects. We focus, as an example, on density dependence in predator self-regulation.

We consider a $L \times L$ grid on which the agents move with Moore connectivity (each site has 8 neighbors) and periodic boundary conditions. No occupancy restriction is enforced: there is no upper bound to the number of agents a grid point can host. There are two species: prey and predators. Agents of both species possess an internal energy that changes during the dynamics; metabolism decreases it, feeding increases it, and during a reproduction event, the parent donates half of its energy to the offspring. The functioning of the model is schematized in Fig. 5. A biomass gradient in the model is obtained by tuning the grass growth rate ρ: the higher ρ, the higher the biomass density at stationarity. The key element of this model is the probability that a predator has of attacking and killing another predator when encountered. We call it *aggressiveness* and denote it by $f(N_2, \xi)$, where N_2 is the number of predators in the grid point and ξ is a parameter that establishes a threshold over which the attack is certain. It controls the sensitivity of this mortality with respect to the predator density (i.e., number at a given grid point). In order to explore more regimes, we study the form

$$f(N_2, \xi) = \min\{(N_2/\xi)^\nu, 1\} \ , \quad \text{with } \nu \geq 0 \ , \tag{16}$$

where the exponent ν modulates the dependence of the aggressiveness on predator density. We choose for our simulations a threshold $\xi = 100$ that allows for a large enough range of prey densities.

Figure 6 shows the equilibrium biomass density of predators B_2^* as a function of the equilibrium biomass density of prey B_1^* (equivalent to the numerical density upon choosing unitary mass for both species without loss of generality) for different values of the grass growth rate ρ. If the aggressiveness does not depend on the number of predators in the grid point ($\nu = 0$) a linear scaling is observed. Any positive value of ν implies sublinear scaling. The general asymptotic expression of the density of predators as a function of the density of prey reads (see SM)

$$B_2^* = (\xi^\nu \omega_2 \delta)^k (B_1^*)^k \ , \qquad \text{with} \quad k = 1/(\nu + 1) \ , \tag{17}$$

where δ is the probability of a catch when a predator engages with a prey.

The model presented here serves as a proof of concept that density-dependent deleterious effect for predators (in this case, self-regulation) can induce sublinear scaling across a biomass gradient. A comparison with the phenomenological model in Eqs. (13) and (14) is possible through the two expressions for the scaling exponent k (Eq. (15) and Eq. (17)) and thanks to the fact that in both models predator density depend on the environmental parameter only through prey density. A parameterization of the phenomenological model that captures the ABM dynamics is $\alpha = 1$, $\beta = 1$ and $\gamma = \nu + 1$.

6 Discussion

In this work, we showed that top-down mechanisms represent a viable explanation for the origin of the predator-prey power law, alongside the bottom-up explanation in Ref. [9]. Under the

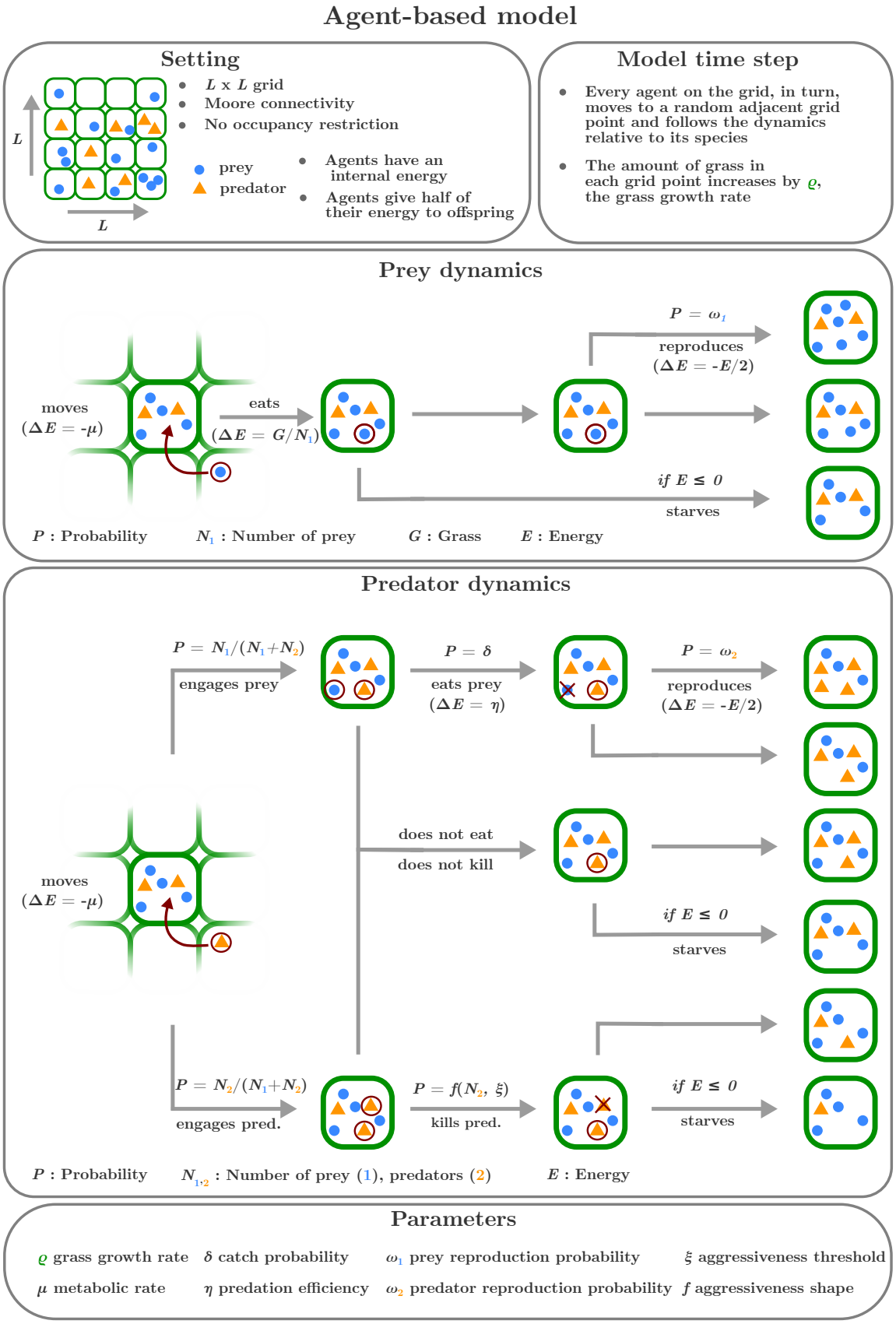

Figure 5: Visual description of the agent based model. The general setting and the parameters are defined as well as predators and prey dynamics.

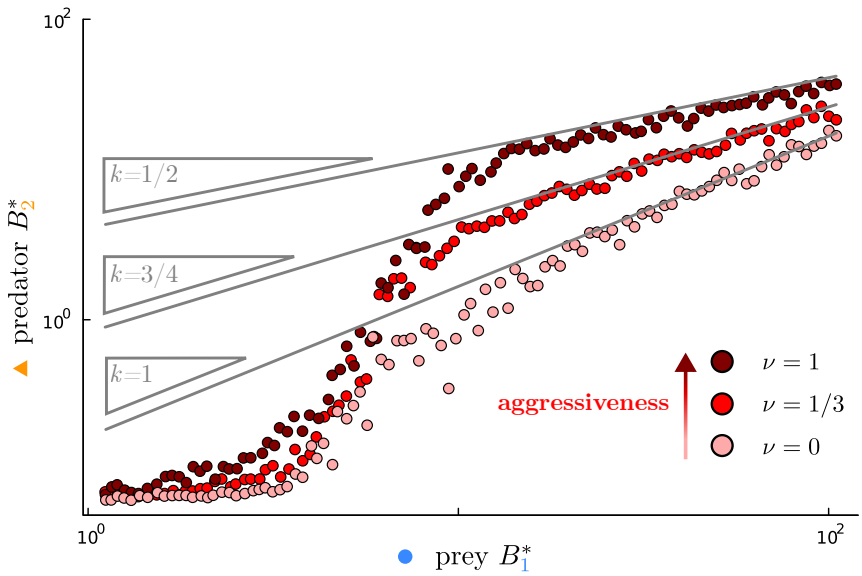

Figure 6: In the ABM, predator biomass density B_2^* scales more and more sublinearly with respect to prey biomass density B_1^* for increasing levels of aggressiveness (encoded in the exponent ν and described by Eq. (16)). Each point is a time average of the densities over simulations that reached stationarity (in particular from step 50 to step 500), for each value of the aggressiveness exponent ν the grass growth rate ρ varies between 1 and 100. The grid area is 4^2, the initial density of prey and predators is 1, the aggressiveness threshold is $\xi = 100$, the reproduction probabilities are $\omega_1 = \omega_2 = 1/2$, the catch probability is $\delta = 1/3$, the predation efficiency is $\eta = 10$, and the metabolic rate $\mu = 1$. The resulting scaling exponents depend only on ν and are robust to changes in the other parameters. The gray lines are theoretical predictions from Eq. (17).

assumption of prey productivity as the primary driver for the biomass density gradient across ecosystems, we demonstrated that combinations of nonlinearities in the functional response and predator self-regulation can produce the predator-prey power law. While there is no single model that can characterize predator functional responses in general [28, 42–44], we considered a scale-free model [16] which capture the relevant dynamical trends that could be behind the macroecological pattern. We also devised and analyzed an agent-based model to provide an example of how predator-prey scaling can emerge as a top-down effect starting from plausible microscopic rules.

We are not able to rule out alternative explanations. Extensive field studies are needed to identify the most relevant environmental factors (and their covariance) behind differences in equilibrium biomass density in ecosystems with similar species composition. This identification is crucial to understand if dynamical ingredients and which ones determine the emergence of predator-prey scaling. We hope to stimulate field work and analysis of existent data sets to test the plausibility of the top-down explanation presented here and, possibly, identify the specific density-dependent mechanisms behind it. Cross-systems studies on predator self-regulation analogous to the one performed for functional response in Ref. [16] would be particularly valuable.

It is also possible to put to the test different models by requiring their consistency with other macroecological patterns, such as the sublinear scaling of production with respect to biomass density [9, 21]. The scaling exponent is approximately the same as in the predator-prey scaling and is recovered for prey production in the bottom-up framework. The top-down model predicts, by the equilibrium requirement on the prey (11), that prey production scales as $B_1^{\alpha+\beta k}$ for a range of densities and then as B_1^2 for higher densities, depending on the value

of σ. For $\alpha = \beta = 1/2$ and $\gamma = 1/6$ the exponent is 0.875, a bit higher than 3/4. If $\gamma = 0$, we have the same exponent $k = \alpha/(1-\beta)$ for both patterns until production scales as B_1^2 for high enough densities. From this perspective, the bottom-up model is more appropriate to capture the general dynamics. Production scaling with biomass density, however, is not recovered for predators; and although available data are still fragmentary, they are indicative that the pattern should hold at every trophic level [9, 21].

Establishing if either of these two classes of dynamical explanations for the predator-prey power law is close to a correct description of reality, at least at large scales, may also impact how we can act upon these ecosystems. For example, an increase in prey productivity r would have a completely different effect on species densities in the two cases. Policies of wildlife management based on the two different dynamical models would result in different ecological responses.

Finally, future works might benefit from the inclusion of evolution. Macroecological scaling patterns in plankton communities can emerge from body size scaling through eco-evolutionary dynamics [45]. This, however, requires producer (phytoplankton) mean body size variation with density, which is not observed in other systems, such as the mammalian herbivores and carnivores analysed in this work.

Data are available in the SM and at [9]. The Julia code used for analysis and simulations is available at `https://github.com/onofriomazzarisi/top-down-vs-bottom-up`.

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Supplementary Material

6.1 Data on African communities

In this section, we report the details and references for the data reported in the paper relative to African communities obtained from the meta-analysis provided in Ref. [9]. Moreover, we present further analysis of the same data.

The data chosen as representative of predator-prey systems used to produce the Figures in the main text are reported in Tab. 1. Table 2 associates the full park names to the park codes reported in Tab. 1. Figure 7 reports the behavior of predator vs. prey biomass density, biomass, number density, and number, Fig. 8 reports predator and prey biomass and number density vs. park area, Fig. 9 reports predator and prey biomass and number density vs. precipitation and Fig. 10 reports precipitation vs. park area.

Table 1: Data for African communities obtained from Ref. [9], In particular, these are the aggregated data for predator species and prey species for African national parks, for details and further references see Ref. [9]. For the plots in the main text, the data referring to the same park (when sampled with the same size, e.g., this does not apply for eto26 and eto98) are averaged. The number in the code refers to the year of the measurements, and the D and W refer, respectively, to measurements in the dry and wet seasons. Correspondence between the park code and the park's full name is reported in Table 2.

Park code	**Area (km^2)**	**Precip. (mm/y)**	**Prey (kg/km^2)**	**Pred. (kg/km^2)**	**Prey body (kg)**	**Pred. body (kg)**
amb	390	1304	7114.6	63.2	168.2	55.7
eto26	74000	390	158.4	4.7	111.4	39.8
eto98	16280	390	239.1	4.6	147.4	59.6
gon	5053	487	480.3	10.2	173.9	61.9
hlu82	960	720	3506.1	38.8	94.3	59.0
hlu00	960	720	5295.5	36.3	122.4	58.5
hwa73	14540	536	608.3	7.5	241.5	64.5
hwa96	14651	536	416.4	9.7	130.6	63.2
kal	9590	207	257.9	3.8	158.7	64.2
kat	4471	945	2795.7	14.8	146.9	63.0
kid	1442	1212	1425.3	14.4	445.5	95.9
kru64	18989	496	1197.6	17.5	84.5	62.1
kru75	18989	496	1429.0	21.3	117.2	65.3
kru84	18989	496	1618.8	20.1	132.5	70.3
kru97	18989	496	1205.7	21.8	143.3	70.9
kru09	18989	496	1356.2	21.2	158.2	70.2
man70	84	942	10170.6	62.9	332.5	96.0
man92	110	942	5983.4	50.3	180.6	97.0
mas92	1670	1894	4863.0	62.9	170.0	80.8
mas03	1500	1894	3928.6	71.3	91.0	81.2
mkoD	3245	641	265.4	4.6	215.3	75.4
mkoW	3245	641	307.0	5.4	197.3	80.0
nai66	122	1064	3271.1	40.2	101.9	71.1
nai76	120	1064	3658.2	48.4	139.5	78.5
nai02	117	1064	2677.4	29.5	181.4	93.4

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Table 1 – continued from previous page

Park code	Area (km^2)	Precip. (mm/y)	Prey (kg/km^2)	Pred. (kg/km^2)	Prey body (kg)	Pred. body (kg)
ngo65	260	1166	12945.8	78.8	159.8	55.4
ngo78	260	1166	15247.0	111.4	161.6	65.3
ngo88	260	1166	13038.4	93.6	164.4	68.6
ngo97	260	1166	12307.8	64.3	205.5	64.7
ngo04	260	1166	13225.0	109.7	208.3	58.8
nwa	81	616	3509.9	36.9	48.9	56.5
oka	1	441	1678.0	14.6	86.0	56.6
pil	500	622	2336.3	27.3	157.8	62.0
que	1978	978	2755.8	18.1	262.7	87.0
sab	17	585	6930.8	72.7	43.6	56.0
savD	300	535	3552.2	41.4	1252.2	72.7
savW	300	535	5155.5	41.4	461.3	72.7
sel	43626	1171	2079.3	31.6	328.6	70.3
ser71	25000	1310	2020.1	20.2	159.3	72.8
ser77	25000	1310	2885.2	22.3	153.0	69.6
ser86	25000	1310	2200.3	24.4	127.8	67.8
ser93	25000	1310	2000.5	33.8	122.5	65.3
ser03	25000	1310	1684.6	33.5	117.1	67.0
tar62D	1683	744	1116.6	12.2	192.3	59.2
tar62W	1683	744	174.3	4.2	67.9	55.3
tar00D	2850	744	3763.3	34.4	333.2	66.6

Table 2: Park corresponding to park codes.

Park code	**Park**
amb	Amboseli National Park, Kenya
eto	Etosha National Park, Namibia
gon	Gonarezhou National Park, Zimbabwe
hlu	Hluhluwe iMfolozi National Park, South Africa
hwa	Hwange National Park, Zimbabwe
kal	Kalahari National Park, South Africa
kat	Katavi National Park, Tanzania
kid	Kidepo Valley National Park, Uganda
kru	Kruger National Park, South Africa
man	Lake Manyara National Park, Tanzania
mas	Masai Mara National Reserve, Kenya
mko	Mkomazi Game Reserve, Tanzania
nai	Nairobi National Park, Kenya
ngo	Ngorongoro Crater, Tanzania
nwa	Nwaswitshaka River, Kruger NP, South Africa
oka	Okavango Delta, Botswana
pil	Pilanesburg National Park, South Africa
que	Queen Elizabeth National Park, Uganda
sab	Sabie River, Kruger NP, South Africa
sav	Savuti area of Chobe National Park, Botswana
sel	Selous Game Reserve, Tanzania
ser	Serengeti ecosystem, Tanzania
tar	Tarangire National Park, Tanzania

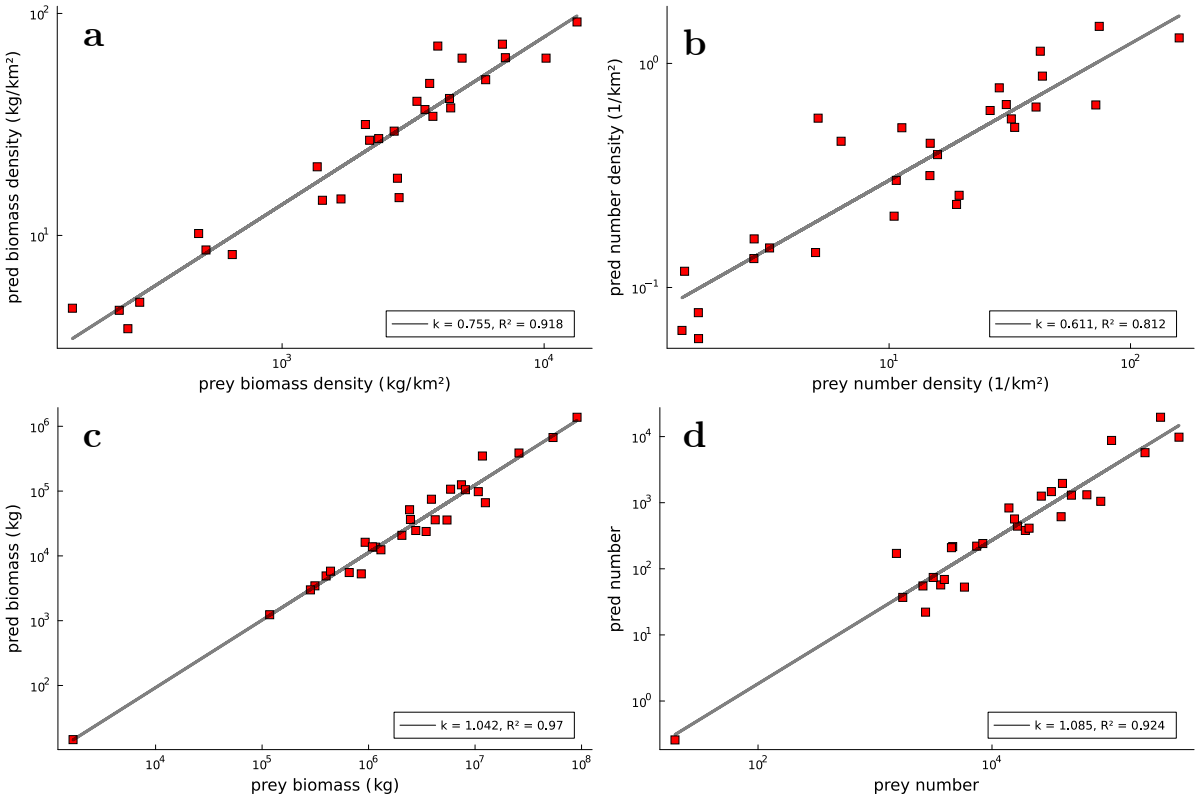

Figure 7: Predator vs prey biomass density (**a**), number density (**b**), biomass (**c**) and number (**d**).

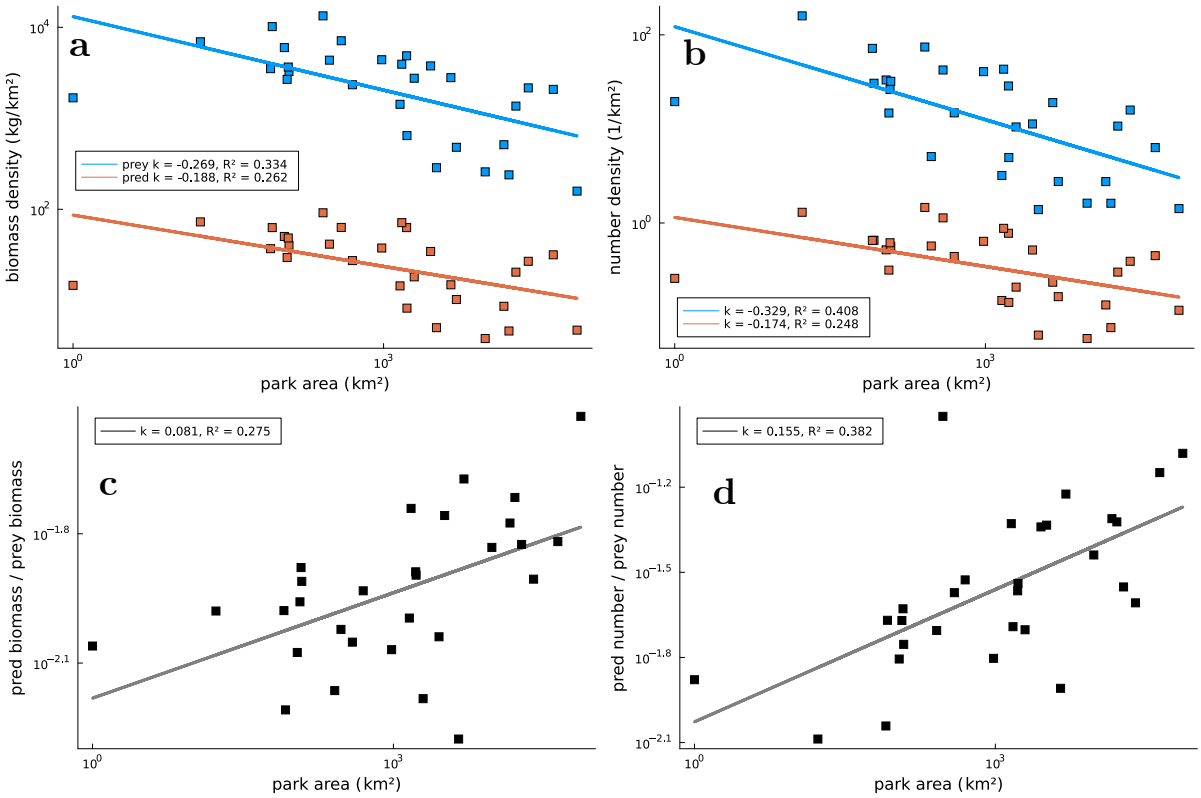

Figure 8: Predator and prey biomass density (**a**) and number density (**b**) plotted against park area. Their ratio is plotted again vs. park area in (**c**) and (**d**).

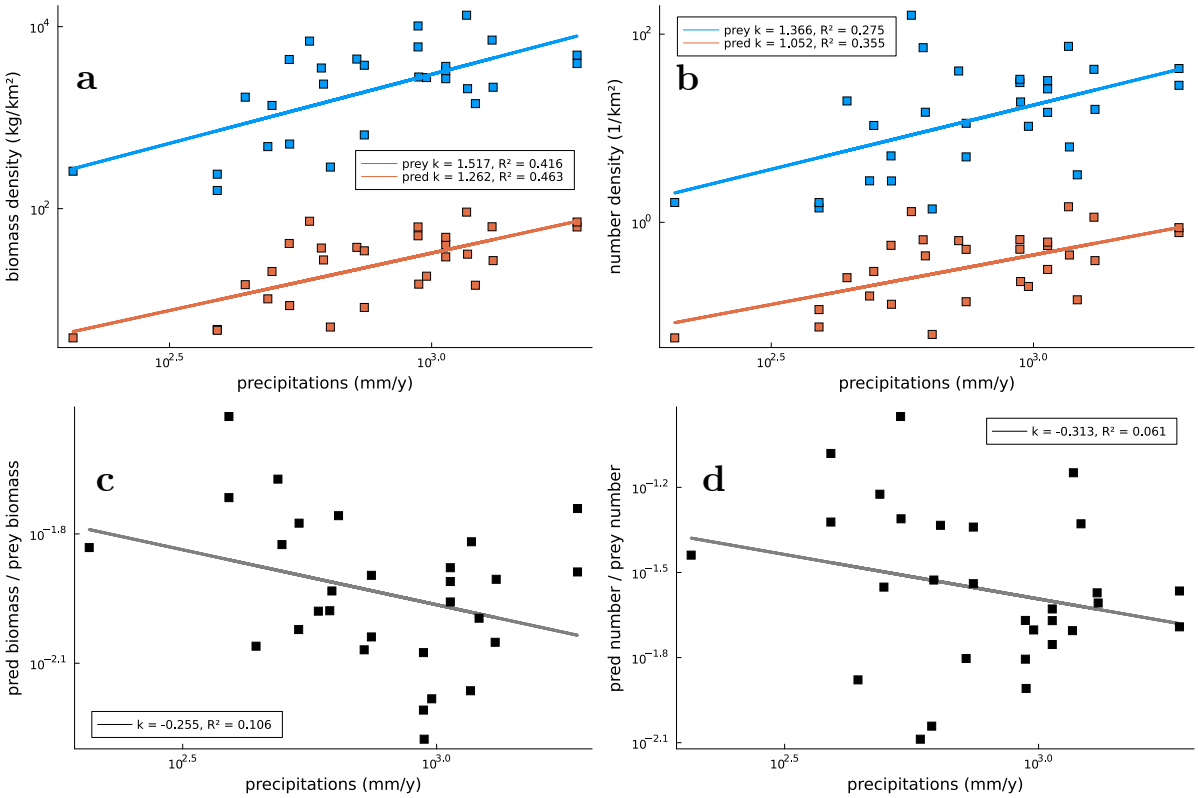

Figure 9: Predator and prey biomass density (**a**) and number density (**b**) vs. precipitation. Their ratio is reported in (**c**) and (**d**).

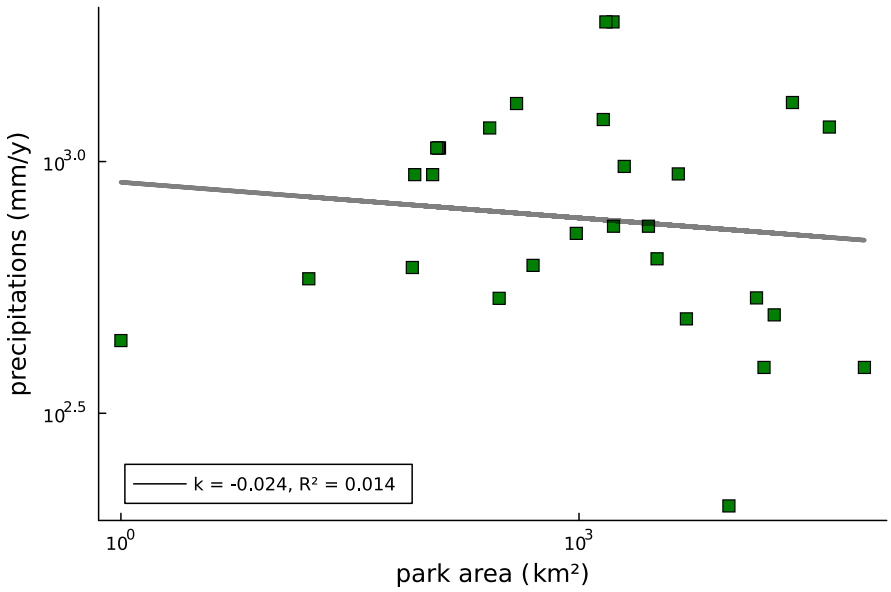

Figure 10: Park precipitation is not correlated with park area.

6.2 Phenomenological model analysis

In this section, we provide details of the analysis of the scale-free phenomenological model discussed in the main text to explore density-dependent effects in predation and self-regulation.

We start by rewriting the model in non-dimensional form. The model describes the dynamics of prey biomass density (B_1) and predator biomass density (B_2) through the following equations

$$\frac{dB_1}{dt} = rB_1 - cB_1^2 - qB_1^\alpha B_2^\beta \,, \tag{18}$$

$$\frac{dB_2}{dt} = gqB_1^\alpha B_2^\beta - mB_2^{1+\gamma} \,, \tag{19}$$

where we refer to the main text for the definition of the parameters. Defining the nondimensional quantities

$$x_1 := \frac{B_1}{d_1} \,, \qquad x_2 := \frac{B_2}{d_2} \,, \qquad \tau := \frac{t}{d_3} \,, \tag{20}$$

The equation above can be written as

$$\frac{dx_1}{d\tau} = rd_3x_1 - cd_3d_1x_1^2 - qd_3d_1^{\alpha-1}d_2^\beta x_1^\alpha x_2^\beta \,, \tag{21}$$

$$\frac{dx_2}{d\tau} = qgd_3d_1^\alpha d_2^{\beta-1}x_1^\alpha x_2^\beta - md_3d_2^\gamma x_2^{1+\gamma} \,. \tag{22}$$

If we make the choices

$$d_3 := \left(\frac{c^\alpha m^{(\beta-1)/\gamma}}{gq}\right)^{\gamma/(\gamma(1-\alpha)+1-\beta)} \,, \tag{23}$$

$$d_2 := (md_3)^{-1/\gamma} \,, \tag{24}$$

$$d_1 := (cd_3)^{-1} \,, \tag{25}$$

the non-dimensional equations become

$$\frac{dx_1}{d\tau} = \rho x_1 - x_1^2 - \sigma x_1^\alpha x_2^\beta \,, \tag{26}$$

$$\frac{dx_2}{d\tau} = x_1^\alpha x_2^\beta - x_2^{\gamma+1} \,, \tag{27}$$

with

$$\rho := rd_3 \,, \qquad \sigma := qd_3d_1^{\alpha-1}d_2^\beta \,. \tag{28}$$

Notice that r does not appear in the definition of d_1, d_2 and d_3. Therefore, when varying r, and therefore ρ as in the main text, we can be sure of the absence of spurious correlations hidden in the redefinition of the densities and time and in the definition of σ.

6.3 Agent-based model analysis

In this section, we derive the asymptotic expression Eq. (17) for the stationary predator density as a function of the prey density across a biomass gradient.

In our ABM, the probability for a predator to reproduce is given by the probability of engaging and eating a prey times the probability of giving birth. Moreover, at high densities, predators mostly die because of attacks from other predators and not from starvation. We can, therefore, write an equation for the evolution of the density of predator x_2 valid at high densities

$$\frac{dx_2}{dt} = \omega_2\delta\frac{x_1}{x_1+x_2}x_2 - \left(\frac{x_2}{\xi}\right)^\nu \frac{x_2}{x_1+x_2}x_2 \,, \tag{29}$$

where x_1 is the prey density, ω_2 is the predator reproduction probability, δ is the catch probability, ξ is the territoriality constant and ν is the aggressiveness exponent. At stationarity, we have

$$0 = \omega_2 \delta x_1^* - \left(\frac{x_2^*}{\xi}\right)^{\nu} x_2^* \ , \tag{30}$$

and therefore Eq. (17)

$$x_2^* = (\xi^{\nu} \omega_2 \delta x_1^*)^{1/(\nu+1)} \ . \tag{31}$$