1

A Computational Framework for Understanding the Impact of Prior Experiences on Pain Perception and Neuropathic Pain

Malin Ramne[1*] & Jon Sensinger[2,3]

1. Department of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden
2. Institute of Biomedical Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada
3. Department of Electrical and Computer Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada

*Corresponding author
E-mail: malin@ramne.com

Abstract

Pain perception is influenced not only by sensory input from afferent neurons but also by cognitive factors such as prior expectations. It has been suggested that overly precise priors may be a key contributing factor to chronic pain states such as neuropathic pain. However, it remains an open question how overly precise priors in favor of pain might arise. Here, we first verify that a Bayesian approach can describe how statistical integration of prior expectations and sensory input results in pain phenomena such as placebo hypoalgesia, nocebo hyperalgesia, chronic pain, and spontaneous neuropathic pain. Our results indicate that the value of the prior, which is determined by generative model parameters, may be a key contributor to these phenomena. Next, we apply a hierarchical Bayesian approach to update the parameters of the generative model based on the difference between the predicted and the perceived pain, to reflect that people integrate prior experiences in their future expectations. In contrast with simpler approaches, this hierarchical model structure is able to show for placebo hypoalgesia and nocebo hyperalgesia how these phenomena can arise from prior experiences in the form of a classical conditioning procedure. We also demonstrate the phenomenon of offset analgesia, in which a disproportionally large pain decrease is obtained following a minor reduction in noxious stimulus intensity. Finally, we turn to simulations of neuropathic pain, where our hierarchical model corroborates that persistent non-neuropathic pain is a risk factor for developing neuropathic pain following denervation, and additionally offers an interesting prediction that complete absence of informative painful experiences could be a similar risk factor. Taken together, these results provide insight to how prior experiences may contribute to pain perception, in both experimental and neuropathic pain, which in turn might be informative for improving strategies of pain prevention and relief.

Author summary

To efficiently navigate the world and avoid harmful situations, it is beneficial to learn from prior pain experiences. This learning process typically results in certain contexts being associated with an expected level of pain, which subsequently influences pain perception. While this process of pain anticipation has evolved as a mechanism for avoiding harm, recent research indicates overly precise expectations of pain may in fact contribute to certain chronic pain conditions, in which pain persists even after tissue damage has healed, or even arises without any initiating injury. However, it remains an open question *how* prior experiences contribute to such overly precise expectations of pain. Here, we mathematically model the pain-learning-process. Our model successfully describes several counterintuitive but well-documented pain phenomena. We also make predictions of how prior experiences may contribute the perception of pain and how the same learning process could be leveraged to improve strategies of pain prevention and relief.

Introduction

Pain is beneficial for survival as it can motivate an organism to withdraw from harmful situations and avoid such harmful situations in the future. Despite its clear importance for survival the underlying mechanisms of pain are an active area of research. This limitation in knowledge, along with a wide variability in the pain

experience, render many pain conditions difficult to treat. Chronic pain is one of the leading global burdens of health [1] as well as an enormous personal and economic burden, affecting more than 30% of people worldwide [2]. A first step toward understanding any complex phenomenon is formulating a precise definition. The International Association for the Study of Pain (IASP) have defined pain as [3]:

> *"An unpleasant sensory and emotional experience associated with, or resembling that associated with, actual or potential tissue damage"*

As a mechanism of protection from harm, pain should typically only arise when there is actual or potential tissue damage present, and the level of pain experienced should be proportional to the level of that tissue damage. However, pain can sometimes persist after the noxious stimulus has been removed or the tissue damage has healed. In fact, in some cases pain may arise even in absence of initiating tissue damage, as is the case with neuropathic pain (pain caused by a lesion or disease of the somatosensory nervous system [3]). Such maladaptive pain does not accurately reflect the level of harm to which the organism is exposed. How and why neuropathic pain may arise and persist in the absence of tissue damage is poorly understood, and consequently is often very difficult to treat.

Mathematical models of neuropathic pain have primarily focused on mechanisms in peripheral afferent neurons [4–6] and spinal cord neural circuits [6–8]. While neurophysiological changes at this level of the nervous system do contribute to neuropathic pain following peripheral nerve lesions, pain perception is also influenced by cognitive factors such as expectations based on previous experiences. It is suggested that this occurs through statistical integration of sensory input and prior information, for example through Bayesian inference [9–12]. Such models have been used to describe placebo hypoalgesia and nocebo hyperalgesia [13–16], and statistical pain learning [17] under experimental pain paradigms. As for chronic pain, Eckert et al., recently applied a Bayesian framework to illustrate how overly precise priors for being in a state of pain, combined with an ambiguous likelihood of perceiving pain from a sensory stimulation, increases the probability of transitioning to a chronic pain state [18]. Bayesian inference seems like a promising framework for modelling neuropathic pain, as damage to the nervous system often results in disrupted sensory input (corresponding to increased ambiguity of the likelihood), yet it remains an open question how overly precise priors in favor of pain might arise.

How the prior distribution is defined plays an important role in Bayesian inference [19]. One common Bayesian approach for tracking a state that may change over time is Kalman filtering [20], in which the prior is produced by a generative model of the world [19,20]. Here, we present a modified Kalman filter model of neuropathic pain, adapted from previously published models on motor adaptation [21,22]. In our model the state (pain) is estimated using the standard Kalman algorithm. A second Kalman filter is used to update the parameters of the generative model in the first Kalman filter, based on the difference between the predicted and the perceived pain. This reflects that people integrate previous experiences of pain in their future expectations. Our model predicts that previous experiences of pain and stochastic fluctuations of the parameters in the generative model, coupled with ambiguity in sensory input following damage to the nervous system, may be key drivers of neuropathic pain.

Results

Single-layer Kalman filter

The level of pain we experience is typically a result of sensory input arising from noxious stimuli but is also modulated by expectations based on previous experiences and the context in which the stimuli is delivered [9]. Such modulation is not unique to pain; integration of sensory input and expectations occur for any kind of perception. This process is often formalized through statistical accounts, such as Bayesian inference, where different sources of information are combined to form an estimate of the world [19]. An optimal estimate is obtained if the contribution of each source is determined by their respective level of uncertainty. Bayesian inference has successfully been applied to describe a range of perceptual and sensorimotor tasks, both qualitatively and quantitatively [21–23]. Just as for other forms of perception, Bayesian models provide a framework for how expectations can modulate pain perception [13,18,24,25].

Pain is dynamic, meaning that the intensity, quality, and characteristics of the experience change over time. One common Bayesian approach for modelling such temporal dynamics is Kalman filtering [20]. In the Kalman filter a state, x, is estimated based on sensory input and a prior estimate, which is produced by a generative model of the world [19,20]. In the context of pain, we suggest that the state x represents the true level of actual or potential tissue damage, and the estimate $\hat{x}$ represents the perceived pain. We assume that the level of actual or potential tissue damage can be influenced by actions u such that $x^{(k)} \propto Bu^{(k)}$. In addition to the influence of such actions, we assume that tissue damage develops over time as $x^{(k)} \propto Ax^{(k-1)}$, where $0 < A < 1$ implies healing. For ease of communication, we use linear concepts in this model, but the ideas we are exploring extend also to nonlinear concepts. The generative model in the Kalman filter thus consists of parameters $\hat{A}$ and $\hat{B}$ as estimates of the real-world parameters A and B. In this first part we will start by showing that, similar to other Bayesian models explored in literature [13,18,24,25], this Kalman filter can successfully model a variety of pain phenomena, including placebo hypoalgesia, nocebo hyperalgesia, and chronic pain and additionally can produce dynamics representative of spontaneous neuropathic pain.

The influence of expectations on pain perception is commonly and perhaps most clearly demonstrated by the phenomenon of placebo hypoalgesia (or the reverse effect, nocebo hyperalgesia), where identical noxious stimuli may be perceived as less (more) painful depending on the expectations associated with the context in which the stimuli is delivered [26]. Using a Kalman filter, we simulate the placebo (nocebo) effect in the context of a brief noxious stimulus by allowing the generative model parameter $\hat{B}$ be lower (higher) than the real-world value of B, see Figure 1. It is also suggested that the effects of expectations can extend beyond acute pain and could contribute to pain persisting even after the noxious stimulus has been removed and tissue damage has healed [9–12]. In our model this disproportionate pain results from a mismatch between the generative model parameter $\hat{A}$ and the real-world value of A. The contribution of the expectation and the sensory input to the final estimate is determined by their respective uncertainty. The uncertainty in sensory

165 input is denoted by R, where larger values of R indicate more uncertainty. The
166 simulation results presented in Figure 2 demonstrate elevated uncertainty in the
167 sensory input (higher value of R), in combination with the generative model
168 parameter $\hat{A} \geq A$ results in the perceived pain being significantly higher than the true
169 level of tissue damage and persisting even when the tissue damage has healed.

170
171 *Figure 1 Placebo hypoalgesia (or the reverse effect, nocebo hyperalgesia) is the*
172 *phenomenon where identical noxious stimuli may be perceived as less (more) painful*
173 *depending on expectations. In the Kalman filter, the placebo (nocebo) effect corresponds to*
174 *the generative model parameters being lower (higher) than the real-world values.*

175
Figure 2 In the Kalman filter model chronic pain corresponds to elevated uncertainty in the
sensory input, in combination with the generative model parameter $\hat{A} \approx 1$. Left: when R is
low the perceived pain will primarily be influenced by the sensory input and closely
correspond to the true level of tissue damage, regardless of the predictions of the generative
model (expectations). Right: for a larger value of R, predictions from the generative model
influence the perceived level of pain, possibly resulting in a chronically elevated level of pain
even after the tissue damage has healed. The shaded areas indicate the interquartile range.

In the context of Bayesian models, elevated uncertainty of sensory input is a
requirement for expectations to have a significant enough influence that they cause
the level of pain to deviate from what is signaled by the sensory input. We assume
that tissue damage itself can cause some disturbance to the sensory receptors that
innervate the affected area, resulting in increased uncertainty in sensory input

relating to that region. Sensory input may also be affected by damage directly to
peripheral afferent nerves or regions of the central nervous system involved in
processing of sensory input. Under these circumstances, pain may arise
spontaneously, even without some initiating action or tissue damage. This type of
pain is commonly referred to as neuropathic pain. The gradual increase of pain
before the onset of tissue damage in the bottom right panel of Figure 2 is one
example of how pain can arise spontaneously when the uncertainty in the sensory
input is elevated. In Figure 3 we provide additional examples of how, in our model,
the expected level of spontaneous neuropathic pain following for example a nerve
injury depends on the level of uncertainty in the sensory input (value of R) and the
value of the generative model parameter $\hat{A}$.
199

200
Figure 3 Damage to the sensory nervous system may result in increased uncertainty of
sensory input relating to the level of tissue damage. Depending on the level of uncertainty
(value of R) and the value of the generative model parameter $\hat{A}$, these changes could result
in spontaneous neuropathic pain.

By modeling pain perception as a Kalman filter our results so far have successfully
demonstrated pain phenomena such as placebo hypoalgesia, nocebo hyperalgesia,
chronic pain, and spontaneous neuropathic pain. We have shown that the level of
uncertainty in the sensory input and the values of the generative model parameters
may be key contributors to these phenomena. What we have not been able to
account for with this model is *how* the values of the parameters in the generative
model are determined. The generative model parameters are typically tuned by hand
or fit to experimental data (e.g., [13–16]). Although these manually tuned models do
give insight to how expectations can influence pain perception, they do not account
for how previous experiences shape those expectations. To incorporate how
previous experiences contribute to pain expectations we must turn to descriptive
models of how the generative model is updated.
217

The Hierarchical Kalman filter

The traditional Kalman filter gives us a framework for understanding how loss of
certainty in sensory input, combined with predictions from a generative model of the
world, might contribute to pain phenomena such as placebo hypoalgesia, nocebo

hyperalgesia and spontaneous neuropathic pain. However, in this framework the
generative model parameters are manually tuned, and therefore the traditional
Kalman filter alone cannot account for how previous experiences influence the
generative models and the resulting pain expectations. To address this shortcoming,
we turn to recent work on modelling motor adaptation, where a second Kalman filter
has been used to update the parameters of the generative model based on the
difference between the prediction and the posterior estimate [21,22]. This process
reflects that people integrate previous experiences in their future expectations. Here,
we adapt the hierarchical Kalman filter to the context of pain and demonstrate with
an example of classical conditioning that the model successfully reflects how
previous pain experiences influence future expectations.
233
To verify that the hierarchical model reproduces similar results as the single-layer
Kalman filter regarding placebo hypoalgesia and nocebo hyperalgesia, we turn to the
concept of classical conditioning. Classical conditioning is a procedure that is often
used to induce placebo or nocebo responses in experimental studies [27]. In the
conditioning procedure a conditioned stimulus (previously neutral stimulus, e.g., an
auditory cue) is repeatedly paired with with an unconditioned stimulus (e.g., a
noxious stimulus). Through this process, the conditioned stimulus can come to elicit
a response originally associated with the unconditioned stimulus, a so-called
conditioned response. In Figure 4 we demonstrate the results of the simulated
placebo and nocebo conditioning. First, we show that there is no expectation of pain
associated with the conditioned stimulus at baseline. We then assess the expected
and perceived pain during the conditioning procedure and note that the expected
pain increases for repeated exposure to the unconditioned stimulus. Finally, the
effect of the conditioning is tested by pairing each conditioned stimulus with an
intermediate-intensity unconditioned stimulus, revealing that the simulated
conditioning procedure results in placebo hypoalgesia and nocebo hyperalgesia that
qualitatively matches the pattern produced by the single-layer Kalman filter in Figure
1.
252

253
Figure 4 We simulate a placebo (nocebo) conditioning procedure by assessing the expected
and perceived pain for a conditioned stimulus paired with repeated low (high)-intensity
unconditioned stimulus. The top panels show how the parameters of the generative model
are updated over time based on the difference between the expected and perceived pain. At
baseline, the conditioned stimulus is not associated with pain. During conditioning the
expected pain increases for repeated exposures to the unconditioned stimulus. After
conditioning, placebo hypoalgesia (nocebo hyperalgesia) is obtained in response to an

In the single-layer Kalman filter we noted that the value of the generative model
parameter $\hat{A}$ and the certainty in the current sensory input, R, influence the level of
spontaneous neuropathic pain following nerve injury (*Figure 3*). In the hierarchical
Kalman filter model, the value of $\hat{A}$ changes over time, which affects the level of
predicted pain. Furthermore, the value of R influences how the generative model
updates. These differences result in pain dynamics that differ from what was
observed in the single-layer Kalman filter. In particular, the model predicts that the
value of $\hat{A}$ at the time of a nerve injury and the level of sensory disruption (i.e., value
of R) may play a role in the level and variance of subsequent neuropathic pain, see
Figure 5.

273
Figure 5 The value of $\hat{A}$ at the time of a nerve injury and the level of sensory disruption (i.e.,
value of R) may play a role in the characteristics of subsequent neuropathic pain. For $R =$
0.6^2 (left), the perceived pain is strongly influenced by sensory input and remains close to 0.
For intermediate valued of R, e.g., $R = 6^2$ (middle), $\hat{A} \approx 1$ regardless of the initial value, and
the perceived pain is tonically at an intermediate intensity with little variance. For $R = 600^2$
(right), the value of $\hat{A}$ is no longer at all influenced by sensory input and reduces to a random
walk centered at the initial value $\hat{A}_0$ and variance equal to the noise in the generative model.
In this scenario, if $\hat{A}_0 > 1$ the pain is likely to stay tonically high, and similarly if $\hat{A}_0 < 1$ the
level of pain is likely to drop to stay at 0. If $\hat{A}_0 \approx 1$, the perceived level of pain may vary
widely as the value of $\hat{A}$ fluctuates above and below 0.

The classical conditioning example demonstrated how repeated painful experiences
allow the generative model to update over time to give more accurate predictions of
the real world. Later, we saw that the value of $\hat{A}$ at the time of a nerve injury may
determine the existence and level of neuropathic pain. Taken together, the same
processes as in classical conditioning could occur also in other contexts, and result
in values of the generative model parameters that predispose certain individuals to
neuropathic pain. In the hierarchical Kalman filter the value of $\hat{A}$ is determined by
previous painful experiences. Intuitively, exposure to persistent pain ($A \approx 1$) typically
results in values of $\hat{A}$ close to 1, while exposure to quickly transient pain ($A < 1$)
typically gives lower values of $\hat{A}$.
294

But what happens if there is no exposure to painful experiences? Since there are no
periods of decaying pain to inform the generative model, a tonically pain-free state
also results in $\hat{A} \approx 1$, regardless of the true value of A. Figure 6 shows how persistent
pain (left), quickly transient pain (middle) and complete absence of pain (right) each
influence the generative model, and the resulting level of neuropathic pain following
a nerve injury (vertical dashed line). Our model corroborates that persistent non-
neuropathic pain is a risk factor for developing neuropathic pain following
denervation [28–30], and additionally offers an interesting prediction that complete
absence of informative painful experiences could be a similar risk factor.
304

305
Figure 6 The value of the generative model parameters are determined by previous painful
experiences. In the left panels, $A = 0.99$, yielding persistent pain following noxious stimuli
(indicated by stars). This value of A results in $\hat{A} \approx 1$, and a high risk of spontaneous
neuropathic pain following a nerve injury (vertical dashed line). In the middle panels $A = 0.9$,
resulting in quickly transient pain, a lower value of $\hat{A}$ and a lower risk of spontaneous
neuropathic pain. In the right panels $A = 0.9$ again, but in this simulation, there are no
noxious stimuli. These circumstances result in $\hat{A} \approx 1$ and a higher risk of spontaneous
neuropathic pain than in the example in the middle. The shaded areas indicate the
interquartile range.

Another phenomenon that relates to the temporal dynamics of pain perception is
offset analgesia, which is defined as a disproportionally large pain decrease after a
minor noxious stimulus intensity reduction [31]. The phenomenon can be elicited by
a simple experimental paradigm: noxious thermal stimuli are applied to an area of
skin in three consecutive time intervals T1, T2 and T3, where the same temperature
is applied during T1 and T3, and the temperature during T2 is slightly higher. This
stimulation paradigm typically results in a disproportionate pain reduction in
response to the temperature reduction from T2 to T3 as compared to a control
condition where a constant temperature is applied throughout all three time intervals.
We were curious to see if the adaptive nature of the hierarchical Kalman filter would
enable the model to reproduce this phenomenon. As can be seen in Figure 7, the
model produces an underdamped response following a reduction in noxious stimuli.
This effect is qualitatively similar yet smaller than what is typically observed in
experimental studies of offset analgesia. We will elaborate on possible reasons for
the magnitude discrepancy in the discussion section.

Figure 7 Offset analgesia is defined as a disproportionally large pain decrease after a minor noxious stimulus intensity reduction. This phenomenon is often elicited by applying noxious thermal stimulation of the same temperature in time intervals T1 and T3, separated by time interval T2 with slightly higher temperature. In the control condition a constant temperature is applied throughout all three time intervals. Our simulation results show a qualitatively similar underdamped response to the minor temperature reduction as is observed in experimental studies of offset analgesia.

Discussion

Bayesian inference has been particularly useful in modelling how expectations influence sensory perception [32]. Since the contributions of sensory input and expectations are weighted by their respective certainty, this framework allows for interesting predictions for perception under different levels of certainty in sensory input. In the context of pain, Bayesian models have successfully described several phenomena, including placebo hypoalgesia, nocebo hyperalgesia and chronic pain [13,18,24,25]. However, these models rely on parameters to be tuned by hand or fit to experimental data. While such manually tuned models do give insight to how expectations can influence pain perception, they do not account for how previous experiences shape those expectations, and accordingly cannot explain phenomenon such as classical conditioning, offset analgesia or how certain events might predispose individuals to chronic or neuropathic pain.

Here, we first verified that the Kalman filter, a common Bayesian approach for modelling temporal dynamics, produced results in line with previous, similar Bayesian models, and additionally offered insight to how disruption of sensory input may contribute to spontaneous neuropathic pain. Next, to address how previous experiences contribute to pain expectations we applied a second Kalman filter to update the parameters of the generative model based on the difference between the prediction and the posterior estimate. With this hierarchical model structure, we could again model placebo hypoalgesia and nocebo hyperalgesia that qualitatively matches results from previous Bayesian models, but also show how these phenomena can arise from previous experiences in the form of a classical conditioning procedure. We also showed that, due to the adaptive nature of the hierarchical Kalman filter, the model produced a disproportionally large pain decrease after a minor noxious stimulus intensity reduction, a phenomenon that is commonly referred to as offset analgesia. To our awareness, this phenomenon has not previously been reproduced in mathematical models of pain.

The results on classical conditioning and offset analgesia indicated that the
hierarchical Kalman filter model can account for pain phenomena occurring at
different time scales. This led us to believe that similar processes might affect pain
perception also in the long term. Like simulations with the traditional Kalman filter,
the value of the internal model parameter $\hat{A}$ at the time of a nerve injury and the level
of sensory disruption (i.e., value of R) played a role in the level and variance of
subsequent simulated neuropathic pain. However, with the hierarchical model we
could explore how the value of $\hat{A}$ is determined by the pain dynamics before the
nerve injury. Our simulations corroborated that persistent non-neuropathic pain is a
risk factor for developing neuropathic pain following denervation, which is well
established in previous literature [28–30]. The model additionally offered an
interesting prediction that complete absence of informative painful experiences could
be a similar risk factor. Taken together, these results provide insight to how previous
experiences may contribute to pain perception, in both experimental and neuropathic
pain, which in turn might be informative for improving strategies of pain prevention
and relief.
383
Neuropathic pain is often resistant to common pharmacological pain treatments
[33,34]. Therefore, finding alternative ways of relieving or preventing neuropathic
pain is of utmost interest. There exist several mechanisms by which neuropathic pain
may arise, such as spontaneous activation of nociceptive afferent sensory fibers,
which in our model corresponds to sensory input signaling tissue damage with high
certainty. Our model also predicts that spontaneous neuropathic pain will arise if the
following two criteria are fulfilled: 1) the generative model predicts pain, and 2) the
uncertainty in the sensory input, denoted R, is large. The first of these criteria
provides a hint at possible preventative measures, such as avoiding persistent pain
before denervation. This is in line with existing literature on risk factors for
developing neuropathic pain [28–30]. Another prediction from the model, which might
seem controversial, is that exposure to transient pain before denervation might be
another strategy for mitigating the risk of developing neuropathic pain. This
prediction remains to be established empirically.
398
When it comes to relieving existing neuropathic pain, we must turn to the second
criteria; the uncertainty in the sensory input, R. If R is large, there will be no
meaningful input for the generative model to update from, therefore the current
model does not offer an avenue for pain relief by addressing the generative model
directly. However, if R can be reduced and the sensory input can be controlled such
that it signals lower levels of tissue injury, this should lead to lower levels of
perceived pain. We intentionally left the definition of *sensory input* somewhat vague
in defining the model. Input relating to tissue injury could come in several different
sensory modalities, such as nociception, somatosensation and vision. After nerve
injury some (or all, e.g., following amputation) of these sensory modalities could be
affected, resulting in increased uncertainty of the sensory input. If some of the
sensory input could be restored, this could possibly offer an avenue for pain relief.
Thus, our model offers a possible explanation for the working mechanism of existing
non-pharmacological treatments for neuropathic pain involve some form of sensory
restoration [35–38].
414
In addition to giving clues of possible avenues for pain prevention and relief, our
model also offers a prediction for how different levels of sensory disruption may

result in different pain characteristics. Matching sensory symptoms to pain
mechanisms is a challenge that researchers and clinicians are grappling with, to
better predict treatment responses of individual patients [39,40]. Regarding
neuropathic pain, certain sensory characteristics have been found to be particularly
common, such as burning pain, numbness, paresthesia (tingling or prickling
sensation, "pins and needles"), and pain attacks, sudden bursts of increased pain, or
electric shocks [41–44]. The simulation results presented in Figure X indicate that
moderately high values or R result in an intermediate level of pain with relatively little
variance, which might correspond to a monotonous pain sensation or paresthesia
reported by some individuals with neuropathic pain. For higher values of R, more
diverse pain dynamics are possible. For example, for $\hat{A} \approx 1$ at the time of the nerve
injury, large variance of pain intensity is obtained. Under these conditions pain may
vary between high and low intensities, possibly reflecting the pain attacks-descriptor
that is often used when screening for neuropathic pain. Thus, our model predicts that
monotonous or paresthetic sensations of intermediate intensity might be more
common in patients with partial denervation (R intermediate) while pain attacks,
monotonously high intensity or complete absence of pain would be expected in
patients with complete denervation (R high).
435
We have focused on modeling pain at the information processing level and have
assumed a linear relationship between tissue damage and sensory input arriving to
the central nervous system. However, there are a myriad of physical, chemical, and
neurobiological processes that modulate sensory afferent signals before they reach
the brain. Due to these mechanisms, the mapping from actual or potential tissue
damage to sensory input can be quite complex. As an example, thermal
discriminative abilities are modified by the temperature to which the skin has been
adapted prior to delivering the thermal stimuli, indicating that the relationship
between thermal stimulation and sensory input may be non-linear [45]. Omission of
this relationship, along with other modulatory mechanisms such as sensitization and
habituation, could be an explanation for why our offset analgesia simulations
produced an effect that is smaller than typically observed in experiments [46,47].
This limitation also means that our model cannot account for pain phenomenon that
may arises because of impairment of pain modulation, such as certain forms of
chronic pain.
451
Predictive coding is a modeling framework that has been suggested to be applicable
to pain [48,49]. This framework has a lot in common with the model proposed here,
including a hierarchical structure and precision modulated integration of sensory
input and predictions. One key difference is the level of abstraction; while our model
deals with information processing at the level of cognitive computation, predictive
coding is based on processes at the level of neuronal hardware. While a lot remains
to be determined regarding the specific function and contributions of various brain
regions in pain processing, structures and processes have been identified in which
predictive coding may be applicable as a modeling framework [49]. For example,
Song et al., successfully modeled local field potentials in ACC and S1 during pain
perception in rats using predictive coding [50]. Models targeting different levels of
abstraction can generate different predictions related to pain processing. While our
model provides predictions regarding pain prevention and treatment through
behavioral interventions, models at the implementation level of abstraction, such as

predictive coding, may be able to generate insights regarding neural processes
involved in pain perception.

Defining specific sensory modalities is an interesting avenue for future development
of the model. Doing so could provide insight to how central processing might
contribute to pain conditions such as allodynia, a neuropathic pain phenomenon
where pain is elicited by otherwise non-painful stimulation such as cold, warmth or
touch. This development of the model might also allow for exploration of the possible
mechanism of treatments for neuropathic pain that involve some form of sensory
restoration. One example of such a treatment is mirror therapy, which is commonly
used for phantom limb pain [36,51]. Phantom limb pain is a particular form of
neuropathic pain, where pain is experienced to arise in a missing limb, for example
following amputation. In mirror therapy a mirror is used to create a visual illusion that
the missing limb is intact, which could be seen as a form of partial sensory
restoration. The working mechanism and efficacy of mirror therapy is disputed
[52,53], although it seemingly does to provide pain relief to some individuals
[51,54,55]. Incorporating several sensory modalities in the model and probing the
effect of restoring some of these modalities could shed light on why mirror therapy
provides pain relief for some individuals and not in others.

Another limitation of the current model is the lack of actions and decision making. As
a mechanism of protection from harm, pain should elicit actions that allow the
organism to withdraw from harmful situations that is cause pain and inform decisions
on how to avoid such situations in the future. A few different frameworks have been
suggested to model this process, commonly involving some variation of
reinforcement learning [48,56]. Active inference is another framework that has been
suggested to be relevant for the context of pain [57]. This framework has a lot in
common with the model presented here, such as updating a generative model of the
world, and also includes the possibility of holding the model fixed and altering actions
within the world to sample information that better reflects the predictions [58].
Incorporating the feedback between actions and pain perception in our model could
provide insights to how behavior might contribute to persistence of pain and what
actions might be helpful for pain relief.

Finally, throughout this work we have entirely focused on the *sensory* aspect of pain.
According to the definition, pain is a sensory *and emotional* experience, yet pain is
often treated as a purely sensory experience both in clinical practice and in research.
Meanwhile, chronic pain is commonly comorbid with psychological disorders such as
depression and PTSD [59–61], although the linking mechanism between these
conditions remains unclear. Ignoring such as significant portion of the pain
experience creates a risk of overlooking mechanisms that might contribute to
persistence of pain in chronic pain conditions, as well as possible avenues for new
and improved pain treatments. For example, neuroimaging studies have found that
initially greater functional connectivity within certain motivation–valuation circuitry
predicts pain persistence [62] and that spontaneous fluctuations of chronic pain
intensity is associated with activity in brain regions that are typically ascribed to
emotional processing [63]. Furthermore, there is tentative evidence that
incorporating psychological components in pain treatments yields significant
improvements in pain relief [64]. We believe that modeling the emotional aspects of

pain is a highly interesting avenue for future research, as it could provide insight to previously underexplored mechanisms contributing to pain persistence and relief.

A framework from psychological and behavioral science that seems to have some relevance to both decision-making and emotion related to pain is *prospect theory*, which has been highly impactful in modeling decision-making under risk [65]. While this framework most commonly is applied to evaluation of financial outcomes, the underlying principles are common to many processes of perception and emotion. Prospect theory relies on a few key cognitive features, including that evaluation is relative to reference point, which is something we have already encountered in the examples of offset analgesia, and loss aversion. *Loss aversion* is the concept where psychological value of loss looms larger than the value of equivalent gain. In the context of pain, loss would typically correspond to an increase in pain, whereas gain corresponds to pain relief. It is not hard to imagine that loss aversion may influence decision-making in pain (in fact there are at least tentative results corroborating this [66]). Furthermore, the psychological value of recent changes in pain intensity may influence the valence of emotional aspects of pain. Taken together, it seems that concepts form prospect theory may offer an avenue for incorporating aspects of the pain experience that have not been captured by this model.

Methods

Computational modeling – Single-layer Kalman filter

In modeling the process of pain perception as a Kalman filter, we suggest that the state x represents the true level of actual or potential tissue damage. For discrete point in time $k = 1,...,n$, we assume that the level of actual or potential tissue damage can be influenced by actions u such that the level of tissue damage follows $x^{(k)} \propto B u^{(k)}$, where B is the control gain, determining the change in tissue damage resulting from action u . In addition to the influence of such actions, we assume that tissue damage develops over time as $x^{(k)} \propto A x^{(k-1)}$, where A is the state transition coefficient. If $0 < A < 1$, the tissue is healing. Thus, we obtain the following expression for tissue damage x

$$x^{(k)} = A x^{(k-1)} + B u^{(k)} + \epsilon(0, Q_A) + \epsilon(0, |u^{(k)}| Q_B),$$

where $\epsilon(0, Q_A)$ is the noise related to the state transition $A x^{(k-1)}$ and $\epsilon(0, |u^{(k)}| Q_B)$ is the noise related to the control input $B u^{(k)}$. Note that the variance of the control input noise is proportional to the magnitude of the control signal $u^{(k)}$, to reflect that the variance of an action (e.g., a movement) is proportional to the magnitude of the action. However, in the examples presented here we only deal with binary set of values for u, namely $u = 0$ or $u = 4$, and thus do not fully explore the implications of this heteroscedasticity.

In Kalman filtering a prior estimate, $\hat{\bar{x}}$, of the state is obtained from a generative model of the world. In the context of pain, we suggest that the prior estimate corresponds to the predicted or expected level of pain. In this model, the generative model consists of parameters $\hat{A}$ and $\hat{B}$, reflecting estimates of the real-world

561 parameters A and B. Thus, the prior estimate, or expected level of pain, is obtained
562 as

$$\hat{\bar{x}}^{(k)} = \hat{A}^{(k)}\hat{x}^{(k-1)} + \hat{B}^{(k)}u^{(k)}.$$

566 According to optimal Bayesian integration, the posterior estimate, $\hat{x}$, of the state is a
567 combination of the prior estimate and the sensory input. The sensory input from
568 sensory receptors relating to tissue damage is defined as

$$z^{(k)} = Hx^{(k)} + \epsilon(0,R),$$

572 where H is the observation matrix mapping from state space (tissue injury) to
573 measurement space (sensory input) and $\epsilon(0,R)$ is the measurement noise, where R
574 denotes the variance in how the sensory input relates to the level tissue damage. In
575 the context of pain, we suggest that the posterior estimate corresponds to the
576 perceived level of pain. In Kalman filtering the posterior estimate is defined as

$$\hat{x}^{(k)} = \hat{\bar{x}}^{(k)} + K^{(k)}\left(z^{(k)} - \hat{\bar{x}}^{(k)}\right),$$

580 where

$$K^{(k)} = \frac{\overline{P}^{(k)}}{\overline{P}^{(k)} + R},$$

583 and $\overline{P}^{(k)} = \hat{A}^{(k)^2}P^{(k-1)} + Q_A + |u^{(k)}|Q_B$ is the uncertainty of the prior estimate $\hat{\bar{x}}$. The
584 posterior estimate has covariance $P^{(k)} = \left(1 - K^{(k)}\right)\overline{P}^{(k)}$.

586 In simulations of the single-layer Kalman filter parameter values are set according to
587 Table 1, unless otherwise specified.

589 *Table 1 Default parameter values for simulations with the single-layer Kalman filter.*

Parameter	Default value
Variance of sensory noise, R	0.6^2
Variance of state transfer process noise, Q_A	0.05^2
Variance of control input process noise, Q_B	0.1^2
Initial uncertainty in state estimate, $P^{(0)}$	10^6
Real-world state transfer coefficient, A	0.9
Real-world control gain, B	0.8
Estimated state transfer coefficient, $\hat{A}$	0.8
Estimated control gain, $\hat{B}$	0.6

Computational modeling – Hierarchical Kalman filter

This model builds on previous work on motor adaptation [21,22]. Here, pain is
estimated using a Kalman filter, and second Kalman filter is used to update the
parameters of the generative model, $\hat{\boldsymbol{p}} = [\hat{A},\hat{B}]^T$, based on the difference between the
predicted and the perceived pain. This reflects that people integrate previous
experiences of pain in their future expectations. The prior estimate of the generative
model parameters is given by

$$\hat{\overline{\boldsymbol{p}}}^{(k+1)} = \hat{\boldsymbol{p}}^{(k)} + \epsilon(0,Q_p),$$

where $\epsilon(0,Q_p)$ is noise in the generative model with covariance matrix Q_p. For simplicity we assume that $\hat{A}$ and $\hat{B}$ are independent random variables, and thus the correlation matrix collapses to a diagonal matrix with the variance of the noise of each of the two parameters as the diagonal elements, as follows

$$Q_p = \begin{bmatrix} Q_{pA} & 0 \\ 0 & Q_{pB} \end{bmatrix}.$$

The prior estimate of the generative model parameters has covariance $\overline{P}_p^{(k+1)} = P_p^{(k)} + Q_P$, where $P_p^{(k)}$ is the covariance of the generative model estimate at iteration k.

To form the posterior estimate of the generative model, the prior estimate is combined with the prediction error $\hat{x}^{(k)} - \hat{\overline{x}}^{(k)}$, which has covariance $R_p^{(k)} = P^{(k)} + \overline{P}^{(k)}$. This definition of the prediction error covariance differs somewhat from that used in previous published version of the model. Berniker & Körding used $R_p = R$ and Johnson et al., used $R_p = Q + R$ [21,22]. Our reasoning for updating this definition is simply that we assume that the covariance of the error should be a sum of the covariance of the terms used to compute the error, namely P for $\hat{x}$ and $\overline{P}$ for $\hat{\overline{x}}$. When checking the implications of this update, the model behavior aligns with our expectations. The generative model update takes the form

$$\hat{\boldsymbol{p}}^{(k+1)} = \hat{\boldsymbol{p}}^{(k)} + K_p^{(k+1)}(\hat{x}^{(k)} - \hat{\overline{x}}^{(k)}) + \epsilon(0,Q_p),$$

where K_p is the Kalman gain. Like the traditional Kalman filter, the Kalman gain is defined by the covariance of the terms that are combined to form the posterior estimate,

$$K_p^{(k+1)} = \overline{P}_p^{(k+1)} H_p^{(k)^T} \left[H_p^{(k)} \overline{P}_p^{(k+1)} H_p^{(k)^T} + R_p^{(k)} \right]^{-1},$$

where H_p is the mapping from parameter space to state space, defined as

$$H_p^{(k)} = \frac{\partial \hat{x}}{\partial \hat{\boldsymbol{p}}} = \left[\frac{\partial \hat{x}^{(k)}}{\partial \hat{A}^{(k)}} \quad \frac{\partial \hat{x}^{(k)}}{\partial \hat{B}^{(k)}} \right] = [H_A \quad H_B] = (1 - K^{(k)})[\hat{x}^{(k-1)} \quad u^{(k)}].$$

Finally, the covariance of the estimated parameters is given by

$$P_p^{(k+1)} = \overline{P}_p^{(k+1)} (I - H_p^{(k)} K^{(k)} K_p^{(k+1)}).$$

This definition of $P_p^{(k+1)}$ also differs somewhat from what has been used in previous published versions of the model. The difference here is the introduction of $K^{(k)}$ as a scaling factor to $H_p^{(k)} K_p^{(k+1)}$. Without the scaling factor K the covariance of the estimated parameters, P_p, will tend to 0 when $R \rightarrow \infty$. When there is infinite sensory noise there is no reliable input inform the accuracy of the estimated generative model parameters, and thus P_p should not tend to 0 (since $P_p \approx 0$ indicates high

confidence in the estimated parameters). To address this contradictory behavior, we introduce the scaling factor $K^{(k)}$.

Estimation of the level of pain, $\hat{x}$, is similar to the process outlined for the single-layer Kalman filter. The only difference is in the calculation of the prior state estimate covariance, $\overline{P}$, to which a term $H_p^{(k)2}P_p^{(k)}$ has been added to incorporate the certainty in the generative model estimate in the state estimate. Thus, the prior state estimate covariance is defined as

$$\overline{P}^{(k)} = \hat{A}^{(k)2}P^{(k-1)} + Q_A + |u^{(k)}|Q_B + H_p^{(k)2}P_p^{(k)} \ .$$

In simulations of the hierarchical Kalman filter parameter values are set according to Table 2, unless otherwise specified.

Table 2 *Default parameter values for simulations with the hierarchical Kalman filter*

Parameter	Default value
Variance of sensory noise, R	0.6^2
Variance of state transfer process noise, Q_A	0.05^2
Variance of control input process noise, Q_B	0.1^2
Initial uncertainty in state estimate, $P^{(0)}$	10^6
Real-world state transfer coefficient, A	0.9
Real-world control gain, B	0.8
Variance of generative model parameter $\hat{A}$, Q_{pA}	0.002^2
Variance of generative model parameter $\hat{B}$, Q_{pB}	0.002^2
Initial uncertainty in estimate of $\hat{A}$, $P_{pA}^{(1)}$	10^6
Initial uncertainty in estimate of $\hat{B}$, $P_{pB}^{(1)}$	10^6

Simulations

Since pain commonly is measured on an 11-point scale, where 0=no pain and 10=worst pain imaginable, we let tissue damage x exist on similar 11-point scale of arbitrary units. For simplicity, we let $H = 1$ and dimensionless such that sensory measurements z also exist on an 11-point scale of arbitrary units. In the simulations, $x^{(k)}$ and $z^{(k)}$ are drawn from normal distributions which are truncated at 0 and 10. Truncation is achieved by resampling if the drawn sample falls outside of the permitted range. To account for asymmetry in the resulting distributions due to the truncation, we have chosen to report medians and interquartile ranges, rather than means and standard deviations. These values are computed across 100 simulations, unless otherwise specified.

Matlab code for all simulation results presented in this article are available online at: https://zenodo.org/doi/10.5281/zenodo.10960405.

Figure 1 - Placebo hyperalgesia and nocebo hypoalgesia
In the simulations of placebo hypoalgesia and nocebo hyperalgesia, we let u be the action that precedes or coincides with the noxious stimuli. In experimental paradigms this action typically is in the form of viewing a visual cue or listening to an auditory

677 cue. It is worth noting that in these experimental settings, it is typically not the action
678 (observing a cue) that causes the noxious stimuli, yet the association is being formed
679 between the action and the perceived level of pain.
680
The magnitude of u is identical in the placebo and nocebo conditions, what differs
between the two is the *expected gain in pain* $\hat{B}$, where we let $\hat{B}_{placebo} \sim N(0.4, 0.2^2)$
and $\hat{B}_{nocebo} \sim (0.8, 0.2^2)$. The values are drawn from normal distributions to reflect
inter-individual variations. The expected pain is assessed as the value of $\bar{\hat{x}}$
immediately following the presentation of the cue, while the perceived pain is
assessed as the maximum value of $\hat{x}$ in a time window following the noxious stimuli.
687

688 *Figure 2 – Chronic pain*
689 Default parameter values specified in Table 1 are used in simulations for Figure 2,
690 except for $\hat{A} = 1$, and the value of R which is 0.6^2 in the left panel and 2.0^2 in the right
691 panel. The action resulting in a single brief noxious stimulus is set to $u = 4$.
692

Figure 3 – Neuropathic pain
In Figure 3, different levels of sensory disruption are simulated by letting the value of
R vary, and for each value of R the resulting pain is assessed for different values of $\hat{A}$
. To reflect the transition to a state of nerve injury more accurately, all the simulation
results shown in the figure are preceded by a simulated period of no sensory
disruption ($R = 0.6^2$, default) to allow the estimated state $\hat{x}$ and associated
uncertainty P to settle into baseline values.
700

Figure 4 - Classical conditioning
The classical conditioning simulations follow similar principles as placebo and
nocebo simulations. The action u corresponds to the observation of the conditioned
stimulus. Initially, $\hat{B} = 0$ to reflect that the conditioned stimulus is not associated with
pain before the conditioning procedure. In the placebo conditioning paradigm, the
conditioned stimulus is repeatedly paired with a low intensity noxious stimulus.
Similarly, in the nocebo conditioning paradigm the conditioned stimulus is paired with
a high intensity noxious stimulus. Finally, the effect of conditioning is tested in both
cases by pairing the conditioned stimulus with an intermediate level noxious
stimulus. The expected pain is assessed as the value of $\bar{\hat{x}}$ immediately following the
presentation of the conditioned stimulus, while the perceived pain is assessed as the
maximum value of $\hat{x}$ in a time window following the noxious stimuli.
713

Figure 5 – Spontaneous neuropathic pain
Similar to Figure 3, we simulate different levels of sensory disruption by varying the
value of R. A key difference here is that, here, we assess the resulting pain for
different *initial values* of $\hat{A}$ at the time of nerve injury. However, due to the adaptive
nature of the hierarchical Kalman filter, the value of $\hat{A}$ does not stay constant over
time. This also means that we cannot run a baseline period before nerve injury, as in
Figure 3, since this would alter the value of $\hat{A}$. Instead, we manually tune the
uncertainty of the state and parameter estimations to approximately correspond to
the values obtained from other baseline simulations, such that $P = P_{pA} = P_{pB} = 1$ at
the time of nerve injury.
724

The first period is simulated using baseline values as specified in Table 2, except for the values of A which are specified in the title of each subplot. In the leftmost and middle subplot actions $u = 6$ (indicated by green stars on the x-axis) result in noxious stimuli, in the rightmost subplot there are no such actions. This period is followed by a simulated nerve injury by increasing the value of R to 600^2.

In experimental paradigms of offset analgesia, the subject typically receives noxious stimuli of varying intensity (temperature) without any associated action. Therefore, throughout all offset analgesia simulations $u = 0$. The level of tissue damage is assumed to be linearly proportionate to the applied temperature. At simulation onset the generative model parameters are set to match the real-world parameters, $\hat{A} = A$ and $\hat{B} = B$. The level of perceived pain is assessed as the median value of $\hat{x}$ in intervals $\{T1, T2, T3_1, T3_2, T3_3, T3_4\}$ as specified in Figure 8**Error! Reference source not found.**. The period immediately following ramp-up or ramp-down of temperature (indicated by Δ in Figure 8) is omitted to account for any delay in change of perceived pain relative to the change in temperature.

Figure 8 Pain is assessed in the intervals $\{T1, T2, T3_1, T3_2, T3_3, T3_4\}$ in the offset analgesia simulations. The first period following ramp-up or ramp-down of temperature, indicated by Δ in the figure, is omitted to account for any delay in change of perceived pain relative to the change in temperature.

References

[1] Vos T, Lim SS, Abbafati C, Abbas KM, Abbasi M, Abbasifard M, et al. Global burden of 369 diseases and injuries in 204 countries and territories, 1990–2019: a systematic analysis for the Global Burden of Disease Study 2019. The Lancet 2020;396:1204–22. https://doi.org/10.1016/S0140-6736(20)30925-9.

[2] Cohen SP, Vase L, Hooten WM. Chronic pain: an update on burden, best practices, and new advances. The Lancet 2021;397:2082–97. https://doi.org/10.1016/S0140-6736(21)00393-7.

[3] Raja SN, Carr DB, Cohen M, Finnerup NB, Flor H, Gibson S, et al. The revised International Association for the Study of Pain definition of pain: concepts, challenges, and compromises. Pain 2020;161.

[4]   Rho YA, Prescott SA. Identification of molecular pathologies sufficient to cause
neuropathic excitability in primary somatosensory afferents using dynamical
systems theory. PLoS Comput Biol 2012;8.
https://doi.org/10.1371/journal.pcbi.1002524.
[5]   Tigerholm J, Petersson ME, Obreja O, Lampert A, Carr R, Schmelz M, et al.
Modeling activity-dependent changes of axonal spike conduction in primary
afferent C-nociceptors. J Neurophysiol 2013;111:1721–35.
https://doi.org/10.1152/jn.00777.2012.
[6]   Crodelle J, Piltz SH, Hagenauer MH, Booth V. Modeling the daily rhythm of
human pain processing in the dorsal horn. PLoS Comput Biol
2019;15:e1007106.
[7]   Ropero Peláez FJ, Taniguchi S. The Gate Theory of Pain Revisited: Modeling
Different Pain Conditions with a Parsimonious Neurocomputational Model.
Neural Plast 2016;2016:4131395. https://doi.org/10.1155/2016/4131395.
[8]   Crodelle J, Maia PD. A Computational Model for Pain Processing in the Dorsal
Horn Following Axonal Damage to Receptor Fibers. Brain Sci 2021;11.
https://doi.org/10.3390/brainsci11040505.
[9]   Tabor A, Thacker MA, Moseley GL, Körding KP. Pain: A Statistical Account.
PLoS Comput Biol 2017;13:e1005142.
[10]   Tabor A, Burr C. Bayesian Learning Models of Pain: A Call to Action. Curr
Opin Behav Sci 2019;26:54–61.
https://doi.org/https://doi.org/10.1016/j.cobeha.2018.10.006.
[11]   Seymour B, Mancini F. Hierarchical models of pain: Inference, information-
seeking, and adaptive control. Neuroimage 2020;222:117212.
https://doi.org/https://doi.org/10.1016/j.neuroimage.2020.117212.
[12]   Büchel C. The role of expectations, control and reward in the development of
pain persistence based on a unified model. Elife 2023;12:e81795.
https://doi.org/10.7554/eLife.81795.
[13]   Jepma M, Koban L, van Doorn J, Jones M, Wager TD. Behavioural and neural
evidence for self-reinforcing expectancy effects on pain. Nat Hum Behav
2018;2:838–55. https://doi.org/10.1038/s41562-018-0455-8.
[14]   Hoskin R, Berzuini C, Acosta-Kane D, El-Deredy W, Guo H, Talmi D.
Sensitivity to pain expectations: A Bayesian model of individual differences.
Cognition 2019;182:127–39.
https://doi.org/https://doi.org/10.1016/j.cognition.2018.08.022.
[15]   Drusko A, Baumeister D, McPhee Christensen M, Kold S, Fisher VL, Treede
R-D, et al. A novel computational approach to pain perception modelling within
a Bayesian framework using quantitative sensory testing. Sci Rep
2023;13:3196. https://doi.org/10.1038/s41598-023-29758-8.
[16]   Grahl A, Onat S, Büchel C. The periaqueductal gray and Bayesian integration
in placebo analgesia. Elife 2018;7:e32930. https://doi.org/10.7554/eLife.32930.
[17]   Mancini F, Zhang S, Seymour B. Computational and neural mechanisms of
statistical pain learning. Nat Commun 2022;13:6613.
https://doi.org/10.1038/s41467-022-34283-9.
[18]   Eckert A-L, Pabst K, Endres DM. A Bayesian model for chronic pain .
Frontiers in Pain Research 2022;3.
[19]   Ma WJ, Kording KP, Goldreich D. Bayesian Models of Perception and Action
an Introduction. MIT Press 2021;13.

[20]   Kalman RE. A New Approach to Linear Filtering and Prediction Problems.
808   Journal of Basic Engineering 1960;82:35–45.
809   https://doi.org/10.1115/1.3662552.
810   [21]   Berniker M, Kording K. Estimating the sources of motor errors for adaptation
811   and generalization. Nat Neurosci 2008;11:1454–61.
812   https://doi.org/10.1038/nn.2229.
813   [22]   Johnson RE, Kording KP, Hargrove LJ, Sensinger JW. Adaptation to random
814   and systematic errors: Comparison of amputee and non-amputee control
815   interfaces with varying levels of process noise. PLoS One 2017;12:e0170473.
816   [23]   Acerbi L, Vijayakumar S, Wolpert DM. On the Origins of Suboptimality in
817   Human Probabilistic Inference. PLoS Comput Biol 2014;10:e1003661.
818   [24]   Anchisi D, Zanon M. A Bayesian Perspective on Sensory and Cognitive
819   Integration in Pain Perception and Placebo Analgesia. PLoS One
820   2015;10:e0117270.
821   [25]   Grahl A, Onat S, Büchel C. The periaqueductal gray and Bayesian integration
822   in placebo analgesia. Elife 2018;7:e32930. https://doi.org/10.7554/eLife.32930.
823   [26]   Büchel C, Geuter S, Sprenger C, Eippert F. Placebo analgesia: A predictive
824   coding perspective. Neuron 2014;81.
825   https://doi.org/10.1016/j.neuron.2014.02.042.
826   [27]   Colloca L, Petrovic P, Wager TD, Ingvar M, Benedetti F. How the number of
827   learning trials affects placebo and nocebo responses. Pain 2010;151:430–9.
828   https://doi.org/https://doi.org/10.1016/j.pain.2010.08.007.
829   [28]   Limakatso K, Bedwell GJ, Madden VJ, Parker R. The prevalence and risk
830   factors for phantom limb pain in people with amputations: A systematic review
831   and meta-analysis. PLoS One 2020;15:e0240431.
832   [29]   Dijkstra PU, Geertzen JHB, Stewart R, van der Schans CP. Phantom Pain and
833   Risk Factors: A Multivariate Analysis. J Pain Symptom Manage 2002;24:578–
834   85. https://doi.org/https://doi.org/10.1016/S0885-3924(02)00538-9.
835   [30]   Yin Y, Zhang L, Xiao H, Wen C-B, Dai Y-E, Yang G, et al. The pre-amputation
836   pain and the postoperative deafferentation are the risk factors of phantom limb
837   pain: a clinical survey in a sample of Chinese population. BMC Anesthesiol
838   2017;17:69. https://doi.org/10.1186/s12871-017-0359-6.
839   [31]   Grill JD, Coghill RC. Transient Analgesia Evoked by Noxious Stimulus Offset.
840   J Neurophysiol 2002;87:2205–8. https://doi.org/10.1152/jn.00730.2001.
841   [32]   Ma WJ, Kording KP, Goldreich D. Bayesian Models of Perception and Action
842   an Introduction. MIT Press 2021;13.
843   [33]   Finnerup NB, Attal N, Haroutounian S, McNicol E, Baron R, Dworkin RH, et al.
844   Pharmacotherapy for neuropathic pain in adults: a systematic review and
845   meta-analysis. Lancet Neurol 2015;14:162–73. https://doi.org/10.1016/S1474-
846   4422(14)70251-0.
847   [34]   Finnerup NB, Kuner R, Jensen TS. Neuropathic Pain: From Mechanisms to
848   Treatment. Physiol Rev 2020;101:259–301.
849   https://doi.org/10.1152/physrev.00045.2019.
850   [35]   Albright-Trainer B, Phan T, Trainer RJ, Crosby ND, Murphy DP, Disalvo P, et
851   al. Peripheral nerve stimulation for the management of acute and subacute
852   post-amputation pain: a randomized, controlled feasibility trial. Pain Manag
853   2021;12:357–69. https://doi.org/10.2217/pmt-2021-0087.
854   [36]   Limakatso K, Cashin AG, Williams S, Devonshire J, Parker R, McAuley JH.
855   The efficacy of graded motor imagery and its components on phantom limb

856		pain and disability: a systematic review and meta-analysis. Canadian Journal
857		of Pain 2023:2188899. https://doi.org/10.1080/24740527.2023.2188899.
858	[37]	Ortiz-Catalan M, Guðmundsdóttir R, Kristoffersen M, Zepeda-Echavarria A,
859		Caine-Winterberger K, Kulbacka-Ortiz K, et al. Phantom motor execution
860		facilitated by machine learning and augmented reality as treatment for
861		phantom limb pain: a single group, clinical trial in patients with chronic
862		intractable phantom limb pain. The Lancet 2016;6736.
863	[38]	Zhang TC, Janik JJ, Grill WM. Mechanisms and models of spinal cord
864		stimulation for the treatment of neuropathic pain. Brain Res 2014;1569:19–31.
865		https://doi.org/https://doi.org/10.1016/j.brainres.2014.04.039.
866	[39]	Forstenpointner J, Otto J, Baron R. Individualized neuropathic pain therapy
867		based on phenotyping: are we there yet? Pain 2018;159.
868	[40]	Ramaswamy S, Wodehouse T. Conditioned pain modulation—A
869		comprehensive review. Neurophysiologie Clinique 2021;51:197–208.
870		https://doi.org/https://doi.org/10.1016/j.neucli.2020.11.002.
871	[41]	Freynhagen R, Baron R, Gockel U, Tölle TR. painDETECT: a new screening
872		questionnaire to identify neuropathic components in patients with back pain.
873		Curr Med Res Opin 2006;22:1911–20.
874		https://doi.org/10.1185/030079906X132488.
875	[42]	Bennett M. The LANSS Pain Scale: the Leeds assessment of neuropathic
876		symptoms and signs. Pain 2001;92:147–57.
877		https://doi.org/https://doi.org/10.1016/S0304-3959(00)00482-6.
878	[43]	Galer BS, Jensen MP. Development and preliminary validation of a pain
879		measure specific to neuropathic pain. Neurology 1997;48:332 LP – 338.
880		https://doi.org/10.1212/WNL.48.2.332.
881	[44]	Bouhassira D, Attal N, Fermanian J, Alchaar H, Gautron M, Masquelier E, et
882		al. Development and validation of the Neuropathic Pain Symptom Inventory.
883		Pain 2004;108.
884	[45]	Johnson KO, Darian-Smith I, LaMotte C, Johnson B, Oldfield S. Coding of
885		incremental changes in skin temperature by a population of warm fibers in the
886		monkey: correlation with intensity discrimination in man. J Neurophysiol
887		1979;42:1332–53. https://doi.org/10.1152/jn.1979.42.5.1332.
888	[46]	Szikszay TM, Adamczyk WM, Panskus J, Heimes L, David C, Gouverneur P,
889		et al. Psychological mechanisms of offset analgesia: The effect of expectancy
890		manipulation. PLoS One 2023;18.
891		https://doi.org/10.1371/JOURNAL.PONE.0280579.
892	[47]	Ligato D, Petersen KK, Mørch CD, Arendt-Nielsen L. Offset analgesia: The
893		role of peripheral and central mechanisms. European Journal of Pain
894		2018;22:142–9. https://doi.org/https://doi.org/10.1002/ejp.1110.
895	[48]	Chen ZS, Wang J. Pain, from perception to action: A computational
896		perspective. IScience 2023;26. https://doi.org/10.1016/j.isci.2022.105707.
897	[49]	Büchel C, Geuter S, Sprenger C, Eippert F. Placebo analgesia: A predictive
898		coding perspective. Neuron 2014;81.
899		https://doi.org/10.1016/j.neuron.2014.02.042.
900	[50]	Song Y, Yao M, Kemprecos H, Byrne Á, Xiao Z, Zhang Q, et al. Predictive
901		coding models for pain perception. J Comput Neurosci 2021:843284.
902		https://doi.org/10.1101/843284.
903	[51]	Limakatso K, Parker R. Treatment Recommendations for Phantom Limb Pain
904		in People with Amputations: An Expert Consensus Delphi Study. PM and R
905		2021;13:1216–26. https://doi.org/10.1002/pmrj.12556.

906   [52]   Schone HR, Baker CI, Katz J, Nikolajsen L, Limakatso K, Flor H, et al. Making
907   sense of phantom limb pain. J Neurol Neurosurg Psychiatry 2022;93:833 LP –
908   843. https://doi.org/10.1136/jnnp-2021-328428.
909   [53]   Aternali A, Katz J. Recent advances in understanding and managing phantom
910   limb pain [version 1; peer review: 2 approved]. F1000Res 2019;8.
911   https://doi.org/10.12688/f1000research.19355.1.
912   [54]   Ramachandran VS, Rogers-Ramachandran D. Synaesthesia in phantom limbs
913   induced with mirrors. Proc R Soc Lond B Biol Sci 1997;263:377–86.
914   https://doi.org/10.1098/rspb.1996.0058.
915   [55]   Herrador Colmenero L, Perez Marmol JM, Martí-García C, Querol Zaldivar M
916   de los Á, Tapia Haro RM, Castro Sánchez AM, et al. Effectiveness of mirror
917   therapy, motor imagery, and virtual feedback on phantom limb pain following
918   amputation: A systematic review. Prosthet Orthot Int 2018;42.
919   [56]   Seymour B. Pain: A Precision Signal for Reinforcement Learning and Control.
920   Neuron 2019;101:1029–41. https://doi.org/10.1016/j.neuron.2019.01.055.
921   [57]   Tabor A, Burr C. Bayesian Learning Models of Pain: A Call to Action. Curr
922   Opin Behav Sci 2019;26:54–61.
923   https://doi.org/https://doi.org/10.1016/j.cobeha.2018.10.006.
924   [58]   Friston K, FitzGerald T, Rigoli F, Schwartenbeck P, O☐Doherty J, Pezzulo G.
925   Active inference and learning. Neurosci Biobehav Rev 2016;68:862–79.
926   https://doi.org/https://doi.org/10.1016/j.neubiorev.2016.06.022.
927   [59]   Scioli-Salter ER, Forman DE, Otis JD, Gregor K, Valovski I, Rasmusson AM.
928   The Shared Neuroanatomy and Neurobiology of Comorbid Chronic Pain and
929   PTSD: Therapeutic Implications. Clin J Pain 2015;31:363–74.
930   https://doi.org/10.1097/AJP.0000000000000115.
931   [60]   Gatchel RJ. Comorbidity of Chronic Pain and Mental Health Disorders: The
932   Biopsychosocial Perspective. American Psychologist 2004;59:795–805.
933   https://doi.org/10.1037/0003-066X.59.8.795.
934   [61]   Yalcin I, Barrot M. The anxiodepressive comorbidity in chronic pain. Current
935   Opinion in Anesthesiology 2014;27.
936   [62]   Baliki MN, Petre B, Torbey S, Herrmann KM, Huang L, Schnitzer TJ, et al.
937   Corticostriatal functional connectivity predicts transition to chronic back pain.
938   Nat Neurosci 2012;15:1117–9. https://doi.org/10.1038/nn.3153.
939   [63]   Baliki MN, Chialvo DR, Geha PY, Levy RM, Harden RN, Parrish TB, et al.
940   Chronic pain and the emotional brain: specific brain activity associated with
941   spontaneous fluctuations of intensity of chronic back pain. J Neurosci
942   2006;26:12165–73. https://doi.org/10.1523/JNEUROSCI.3576-06.2006.
943   [64]   Ashar YK, Gordon A, Schubiner H, Uipi C, Knight K, Anderson Z, et al. Effect
944   of Pain Reprocessing Therapy vs Placebo and Usual Care for Patients With
945   Chronic Back Pain: A Randomized Clinical Trial. JAMA Psychiatry
946   2022;79:13–23. https://doi.org/10.1001/jamapsychiatry.2021.2669.
947   [65]   Kahneman D, Tversky A. Prospect Theory: An Analysis of Decision Under
948   Risk. Handbook of the Fundamentals of Financial Decision Making, vol.
949   Volume 4, WORLD SCIENTIFIC; 2012, p. 99–127.
950   https://doi.org/doi:10.1142/9789814417358_0006.
951   [66]   Brojerdian N, Dagnall N, Fatoye F, Goodwin P. Can loss aversion be applied
952   to health conditions? Physiotherapy 2019;105:e68.
953   https://doi.org/https://doi.org/10.1016/j.physio.2018.11.031.
954