A Powered Simple Walking Model Modifies the Rate of Recovery Definition: Human Walking Deviates Drastically from Pendular Motion During the Double Support Phase

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Conflict of Interest: None

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Abstract:

This study proposes a modified definition of the original rate of recovery (ROR) in human walking, derived from a powered simple walking model and center of mass work. The metric captures the proportion of energy transferred from one step to the next by incorporating total step mechanical dissipation. Our simulation results show that the modified ROR decreases nearly linearly with increasing walking speed on even terrain due to collision impulses scaling with velocity ($co \sim v_{ave}$) and energy dissipation scaling quadratically ($w_{co} \sim v_{ave}^2$). For nominal even walking at $1.25\ \mathrm{m.s^{-1}}$, the analytical ROR was 85%, depicting higher rates than previously proposed. Empirical data, however, reveal lower ROR values due to additional energy dissipation factors, such as elevated collision dissipation or negative net single support work, which are not captured by the analytical model. In contrast to the analytical model, the experimental results also show a maximum rate of recovery occurring at $1.21\ \mathrm{m.s^{-1}}$ (59.4%). This peak appeared when total step dissipation was considered. We also demonstrated that the ROR trajectory was not symmetrical as previously presented but depended on the total step dissipation. For uneven terrains with step height variations of ±0.036 m, the average analytical model ROR was 82%, suggesting that a completely random uneven terrain may be analytically approximated by even walking. However, the elevated energetic demands of uneven walking, primarily due to suboptimal pre-emptive push-offs (timing or magnitude), must further decrease the ROR. Similarly, other walking conditions associated with increased metabolic costs, such as aging or when vision about the coming terrain is denied (restricted lookahead: max rate 58.5%), are also expected to result in reduced RORs.

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Keywords: rate of recovery, walking energetics, center of mass work, step-to-step transition, push-off, collision dissipation

Introduction:

A substantial portion of walking metabolic energy is used to perform mechanical work on the Center of Mass (COM), propelling it forward and compensating for mechanical energy losses (Donelan et al., 2002a). Consequently, conserving mechanical energy during walking can also reduce the associated metabolic cost (Donelan et al., 2002a). One specific instance of mechanical energy loss occurs during the transition between steps, where the heel-strike dissipates energy by performing active negative mechanical work, referred to as "collision" (Kuo, 2002; Kuo et al., 2005). The rate of recovery (ROR) is defined as the fraction of mechanical energy transferred from one step to the next (Cavagna et al., 1977). It is a key metric in walking biomechanics that measures the efficiency of energy transfer between steps, reflecting how well mechanical energy is conserved and reused during gait. A higher ROR indicates a better walking economy, reduced metabolic costs, and efficient coordination of passive (e.g., tendon elasticity) and active (e.g., muscle contractions) walking mechanisms. Since human walking is not always conservative, ROR may be redefined to account for step-to-step transition active energy dissipation. In this framework, the legs' external work compensates for the energy lost during heel-strike (Donelan et al., 2002a; Kuo, 2002). Factors such as age, state of lookahead, walking speed, and terrain conditions may also influence heel-strike energy losses, impacting the rate of recovery.

Human locomotion is hypothesized to mimic pendulum and spring-like mechanics to minimize active muscle work (Cavagna et al., 1977). Specifically, the single support phase of walking aligns with the motion of an inverted pendulum (Kuo et al., 2005). In an idealized scenario with no energy dissipation, a walker starting with sufficient initial energy would require no external mechanical energy input. Thus, human walking primarily involves converting potential and kinetic energy within a conservative system (Cavagna et al., 1977).

According to Cavagna et al. (Cavagna et al., 1976, 1977), walkers perform works against gravity (W_v) to lift the COM and to accelerate it forward (W_f). When potential and kinetic energies are completely out of phase, the mechanical energy transitions with no dissipation. In this state, gravitational potential energy is converted into kinetic energy, propelling the walker forward. However, when these energies become aligned, energy exchange is disrupted, resulting in dissipation. Humans then perform work (W_{ext}) to compensate for this energy loss. This assumption hinges on the notion that human walking always resembles inverted pendulum mechanics and incurs no energy waste departing from a pendular motion (Figure 1A).

Donelan et al. (Donelan et al., 2002b), however, proposed that humans simultaneously perform positive and negative work during step-to-step transitions. This suggests a significant deviation from the inverted pendulum model, as the COM mechanics transition to the next stance (Figure 1B). Donelan et al. (Donelan et al., 2002a) further argued that these simultaneous COM works are energetically costly and serve as a rough indicator of walking energetics. Post step transition, the human walking tends to resemble pendular motion again (Donelan et al., 2002a; Kuo et al., 2005).

When considering the right and left legs as separate actuators exerting toe-off (push-off) and heel-strike (collision) impulses (Kuo et al., 2005), it becomes reasonable to investigate their individual effects on the COM. Similar to other mechanical actuators, the work of each leg is determined by the force applied and the corresponding displacement. Thus, the mechanical work of each leg cannot mathematically cancel the other's work. While the net effect on the COM may reflect the resultant force induced, the independent work of each leg incurs its own metabolic cost (Griffin et al., 2003).

Both toe-off (push-off) and heel-strike (collision) impulses generate upward motions on the COM in the frontal plane, but their sagittal plane effects differ. The toe-off (push-off) impulse propels the pelvis forward, while the heel-strike (collision) impulse opposes the forward progression, partially wasting

momentum and mechanical energy (Figure 1C & D) (Donelan et al., 2002a, 2002b; Kuo, 2002). Consequently, the toe-off (push-off) impulse contributes to forward momentum, whereas the heel-strike (collision) impulse results in energy loss (Donelan et al., 2002a; Kuo, 2002).

A simple powered walking model demonstrates the independent contributions of the leading and trailing legs during step transitions (Kuo, 2002). For a given walking speed, it predicts the active work required (push-off work) for optimal steady-state walking, where active work matches dissipation. Pre-emptive push-off is hypothesized to limit the magnitude of subsequent collision (Kuo, 2002; Kuo et al., 2005). When heel-strike occurs earlier than expected (Darici et al., 2018), the pre-emptive push-off can only partially mitigate the resulting collision (Kuo, 2001). Hence, to maintain steady walking, additional positive mechanical energy is required during the subsequent stance phase (Darici & Kuo, 2023; Hosseini-Yazdi Seyed-Saleh, 2024; Kuo, 2002). Suboptimal push-off ($po_{sub} < po_{opt}$) results in increased dissipation and a reduced amount of energy transferred to the subsequent step.

Walking energetics increase with terrain complexity (Hosseini-Yazdi & Kuo, 2025; Pandolf et al., 1976; Voloshina et al., 2013) and age (Das Gupta et al., 2019; Hosseini-Yazdi Seyed-Saleh, 2024). Restricted lookahead might also impair the ability to select appropriate footholds, limiting the benefits of human walking dynamics (Matthis & Fajen, 2013). It is shown that walking with a restricted lookahead also coincides with elevated walking energetics (Hosseini-Yazdi Seyed-Saleh, 2024). Since step mechanical work provides a rough approximation of walking energetics (Donelan et al., 2002a), it is reasonable to infer that any walking regime requiring increased metabolic rates corresponds to higher active mechanical work performance, and as such lower rates of recovery.

This study redefines the rate of recovery based on step energy dissipation. We develop a simulation to derive analytical rates of recovery for nominal (optimal) walking across a range of walking speeds. The simulation is extended to scenarios with suboptimal push-off impulses ($po_{sub} < po_{opt}$) and cases where

COM elevation changes; step-ups and step-downs (i.e. uneven walking). Similar to even walking (Kuo, 2002), we examine a scenario where pre-emptive push-off provides the entire step transition energy and compensates for work against gravity during uneven walking. Further we compare the analytical results with ROR derived from COM work trajectories (empirical data) for walking over smooth surfaces.

New Definition for Rate of Recovery:

According to a powered simple walking model (Kuo, 2002), the active push-off (po) is applied pre-emptively to the COM to mechanically energize the step-to-step transition (Donelan et al., 2002a). This action compensates for the dissipation caused by the heel-strike collision, which occurs after the push-off (Donelan et al., 2002b). Consequently, the required external work (W_{ext}) is assumed to offset the energy lost during the heel-strike (Cavagna et al., 1977). Considering 'u' as the transition impulse, its associated work is calculated as $\frac{1}{2}u^2$ (Kuo, 2002). We count the total mechanical energy of the human body as the sum of its potential and kinetic energies ($E_{tot} = E_P + E_K$). Assuming that the lowest elevation of the COM during the stance phase as the reference for gravitational potential energy ($E_P = 0$), the total energy at the end of the stance phase corresponds to the maximum kinetic energy ($E_{K_max} = E_{tot}$, Figure 1E & F). As originally defined, the rate of recovery based on the step works is given by (Cavagna et al., 1977):

$$Rate\ of\ Recovery = \frac{E_{tot} - W_{ext}}{E_{tot}} \qquad \text{Eq. 1}$$

This equation quantifies the proportion of mechanical energy transferred from one step to the next. Since the system's total energy equals the maximum kinetic energy just before the step transition, the rate of recovery may be re-written as:

$$Rate\ of\ Recovery = \frac{E_{K_max} - W_{ext}}{E_{K_max}} \qquad \text{Eq. 2}$$

Assuming the COM velocity just before the transition approximates the average walking velocity (v_{ave} in which $\vec{v}_{ave} = \vec{v}_f + \vec{v}_v$) (Kuo, 2002), and w_{ext} represents the normalized external work per unit of mass, the rate of recovery can be derived as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - w_{ext}}{\frac{1}{2}v_{ave}^2} \qquad \text{Eq. 3}$$

To sustain a steady walk, the walker's energy must remain constant before and after the step-to-step transition. Denoting the velocity before and after the transition as $v^-$and v^+, respectively, steady walking implies, $|v^+| = |v^-| = v_{ave}$. With the kinetic energy conserved across the transition, the active work energy addition (w_{ext}) equals the heel-strike energy dissipation or collision work (w_{co}). Therefore, the rate of recovery can be further expressed as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - w_{co}}{\frac{1}{2}v_{ave}^2} \qquad \text{Eq. 4}$$

Here, w_{co} represents the normalized energy loss due to the heel-strike collision.

Simulation based on a powered simple walking model:

The nominal push-off impulse for a given walking speed is expressed as $po = v_{ave} \cdot \tan\alpha$ where '2α' represents the angle between the leading and trailing legs at step transition point (Kuo, 2002). Consequently, the nominal push-off work becomes $w_{po} = \frac{1}{2}v_{ave}^2 \cdot \tan^2\alpha$ (Kuo, 2002). Assuming $w_{co} = w_{po}$ and $w_{ext} = w_{co}$, the rate of recovery is:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - \frac{1}{2}v_{ave}^2 . \tan^2\alpha}{\frac{1}{2}v_{ave}^2} = 1 - \tan^2\alpha \qquad \text{Eq. 5}$$

For small angles, the approximation $\tan\alpha \sim \alpha$ holds, leading to $po = v_{ave} \cdot \alpha$ and $w_{po} = \frac{1}{2}v_{ave}^2 \cdot \alpha^2$. Assuming a unit leg length, the step length (s) becomes proportional to 2α (Kuo, 2001, 2002). Since step

length follows a power law with walking speed (Kuo, 2001); $s \sim v^{0.42}$, yielding $2\alpha = c \cdot v^{0.42}$ and $\alpha = \frac{1}{2} c \cdot v^{0.42}$. Substituting '$\alpha$' into Eq. 5 with $\tan\alpha \sim \alpha$:

$$Rate\ of\ Recovery = 1 - \alpha^2 = 1 - \frac{1}{4} c^2 \cdot v_{ave}^{0.84} \qquad \text{Eq. 6}$$

This analytical expression suggests that the rate of recovery decreases nearly linearly with walking speed. Applying the principles of the powered simple walking model to a suboptimal push-off (po_{sub}, Figure 1G) (Kuo, 2002) the mid-transition velocity (v^0) forms an angle (γ) with the COM velocity before transition (v^-) which depends on the induced push-off impulse magnitude: $\gamma = \arctan(\frac{po_{sub}}{v_{ave}})$. Since $v^- \sim v_{ave}$ (Kuo, 2002) , the transition collision impulse (co) becomes:

$$\begin{cases} \frac{co}{v^0} = \sin(2\alpha - \gamma) \\ \frac{v_{ave}}{v^0} = \cos\gamma \end{cases} \rightarrow co = v_{ave} \frac{\sin(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 7}$$

The normalized collision work is $w_{co} = 0.5 co^2$ (Kuo, 2002), and the modified rate of recovery is:

$$Rate\ of\ Recovery = \frac{\frac{1}{2} v_{ave}^2 - w_{co}}{\frac{1}{2} v_{ave}^2} = 1 - (\frac{\sin(2\alpha - \gamma)}{\cos\gamma})^2 \qquad \text{Eq. 8}$$

Using small angle approximations for 'α' and 'γ':

$$Rate\ of\ Recovery = 1 - \left(2\alpha - \frac{po_{sub}}{v_{ave}}\right)^2 = 1 - \left(c.v_{ave}^{0.42} - \frac{po_{sub}}{v_{ave}}\right)^2 \qquad \text{Eq. 9}$$

This result shows that the rate of recovery becomes a nonlinear function of both suboptimal push-off magnitude and the average walking speed.

For uneven terrain with a step-up, maintaining the average walking speed (v_{ave}) at mid-stance requires the post-transition COM kinetic energy to also account for the work against gravity. The post-transition velocity (v^+) must therefore satisfy (Figure 1H):

$$\frac{1}{2} v^{+2} - \frac{1}{2} v_{ave}^2 = g.\Delta h \rightarrow v^+ = \sqrt{v_{ave}^2 + 2g.\Delta h} \qquad \text{Eq. 10}$$

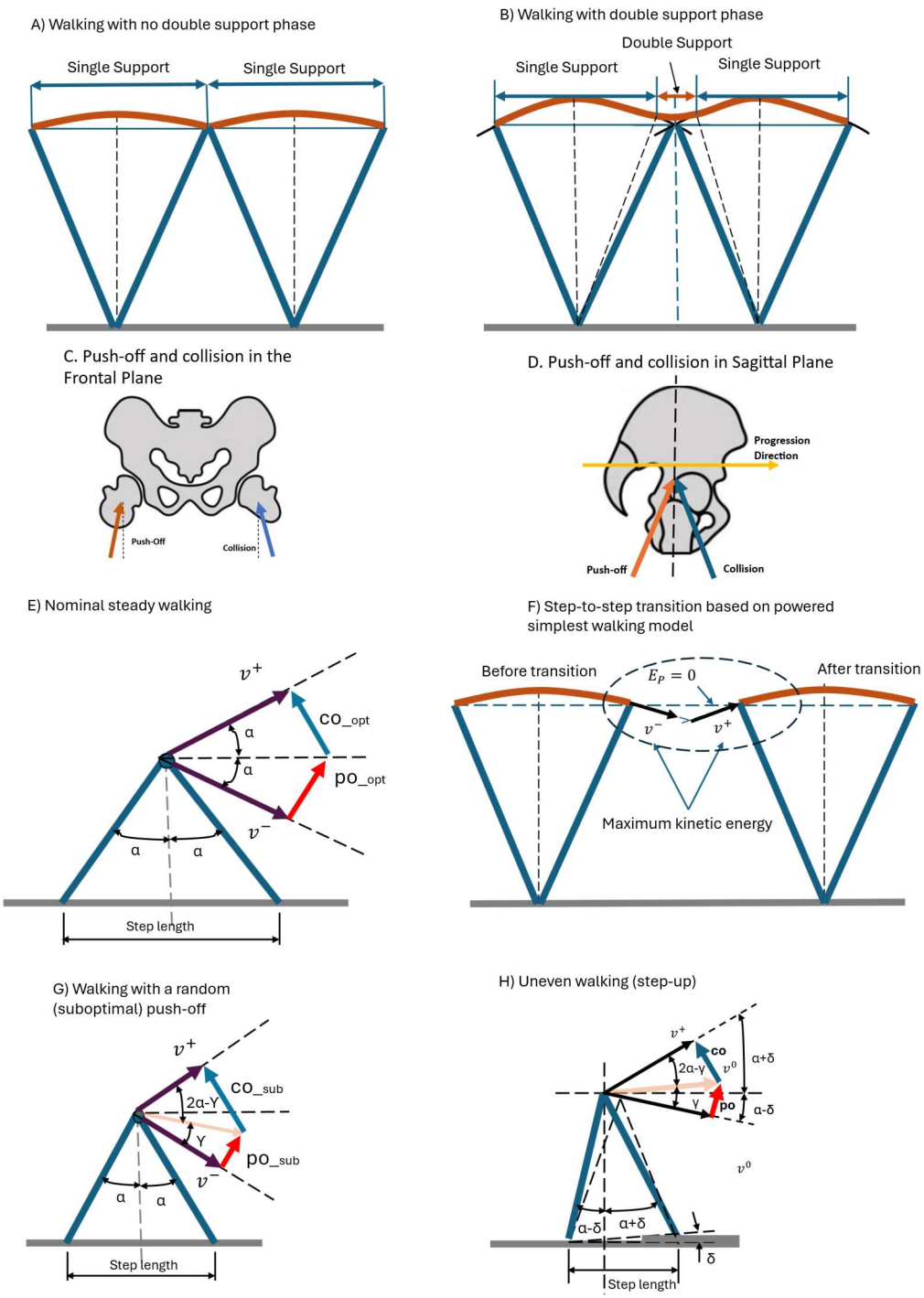

Figure 1: Schematics showing the step-to-step transition during even walking: (A) pendular walking model with rigid legs in which the step-to-step transition is instantaneous (no double support phase), (B) human walking demonstrates a double support phase in which humans switch from one inverted pendulum to another, (C) push-off and collision impulses have similar effects in the vertical direction, (D) push-off impulse adds to the COM mechanical energy in the progression direction while collision impulse reduces the momentum, (E and F) In nominal steady walking, the push-off and collision work, and pre (v^-) and post transition (v^+) speeds are equal, the pre transition velocity may be approximated by v_{ave}, (G) if the pre-emptive push-off is less than optimal (suboptimal), the magnitude of the subsequent collision impulse grows and the post transition speed declines, (H) when the total step transition and gravity works are provided by the pre-emptive push-off impulse, optimal uneven walking takes place.

If the entire uneven step energy is provided by a pre-emptive push-off, then:

$$\begin{cases} \frac{v^+}{v^0} = \cos(2\alpha - \gamma) \\ \frac{v_{ave}}{v^0} = \cos\gamma \end{cases} \rightarrow v^+ = v_{ave}\frac{\cos(2\alpha - \gamma)}{\cos\gamma} \rightarrow \sqrt{v_{ave}^2 + 2g.\Delta h} = v_{ave}\frac{\cos(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 11}$$

The mid-transition velocity angle change (γ) must be determined numerically for a given walking condition. The uneven step collision impulse becomes:

$$\frac{co}{v^0} = \sin(2\alpha - \gamma) \rightarrow co = v_{ave}\frac{\sin(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 12}$$

Thus, the rate of recovery for an uneven step can be derived similar to even walking when a suboptimal push-off is applied, yet with $\gamma > \alpha$. Using the small angle approximation, the rate of recovery for uneven walking becomes: $1 - (2\alpha - \gamma)^2$. For a step-down, the process is similar, with Δh being negative. Here, the potential energy converts to kinetic energy, increasing momentum, and the required push-off magnitude decreases as the step-down amplitude increases.

Results:

We redefined the rate of recovery based on step mechanical work dissipation (Donelan et al., 2002a; Ruina et al., 2005), focusing on scenarios where the COM trajectory significantly deviates from conservative inverted pendulum motion. Originally, the rate of recovery was defined as the percentage of total mechanical energy carried over from one step to the next through conversion of potential and kinetic energies (Cavagna et al., 1977). Nevertheless, we assumed that the walker's mechanical energy declined by the step collision dissipation, when the walker transitioned from one step to the next. It enabled us to analytically estimate the rate of recovery for various walking conditions using a simple powered walking model, whether the push-off was nominal or suboptimal (Kuo, 2002).

The step length was estimated using an empirical power law for a given velocity (Kuo, 2001). As walking speed increased from 0.8 $\mathrm{m.s^{-1}}$ to 1.6 $\mathrm{m.s^{-1}}$, the estimated step length increased from 0.61 m to 0.82 m

(a 34.4% increase when the leg length is 1 m). Correspondingly, the analytical total energy of walking (COM kinetic energy just before transition) and the push-off work during the step-to-step transition increased significantly—by 400% (from 0.32 $\mathrm{J.kg^{-1}}$ to 1.28 $\mathrm{J.kg^{-1}}$) and 733.3% (from 0.03 $\mathrm{J.kg^{-1}}$ to 0.22 $\mathrm{J.kg^{-1}}$), respectively. However, the rate of recovery declined from 90.6% to 83.1% (an 8.3% decrease). The small angle approximation yielded a rate of recovery that was, on average, only 1.5% higher than the analytical rate (Figure 2).

For suboptimal push-offs, where push-off magnitudes varied from zero to nominal (velocity = 1.25 $\mathrm{m.s^{-1}}$, $\alpha =$ 0.37 rad.), the mid-transition velocity angle relative to the pre-transition velocity (γ) increased from zero to 0.37 rad. Simultaneously, the analytical collision impulse decreased from 0.84 $\mathrm{m.s^{-1}}$ to 0.49 $\mathrm{m.s^{-1}}$ (a 71.4% reduction), while analytical push-off work increased from zero to 0.12 $\mathrm{J.kg^{-1}}$ and collision work declined from 0.36 $\mathrm{J.kg^{-1}}$ to 0.12 $\mathrm{J.kg^{-1}}$ (a 66.7% reduction). The analytical rate of recovery increased from 54.5% to 84.9% (a 55.8% increase). The small angle approximation initially suggested a lower rate of recovery but exceeded the analytical trajectory when the suboptimal push-off impulse reached approximately 70% of the optimal push-off magnitude (Figure 3).

For step-up, we assumed that optimal walking occurred when the entire step transition energy and the work required to overcome gravity were supplied by the pre-emptive push-off. As the step-up amplitude increased, the optimal angle of the mid-transition COM velocity change (γ) also increased. Simulations indicated that push-off magnitudes for step-up walking exceeded those for nominal even walking at the same speed but were constrained by the maximum inducible analytical magnitude ($v_{ave}.\tan 2\alpha$), where analytical collision became zero (Kuo, 2002).

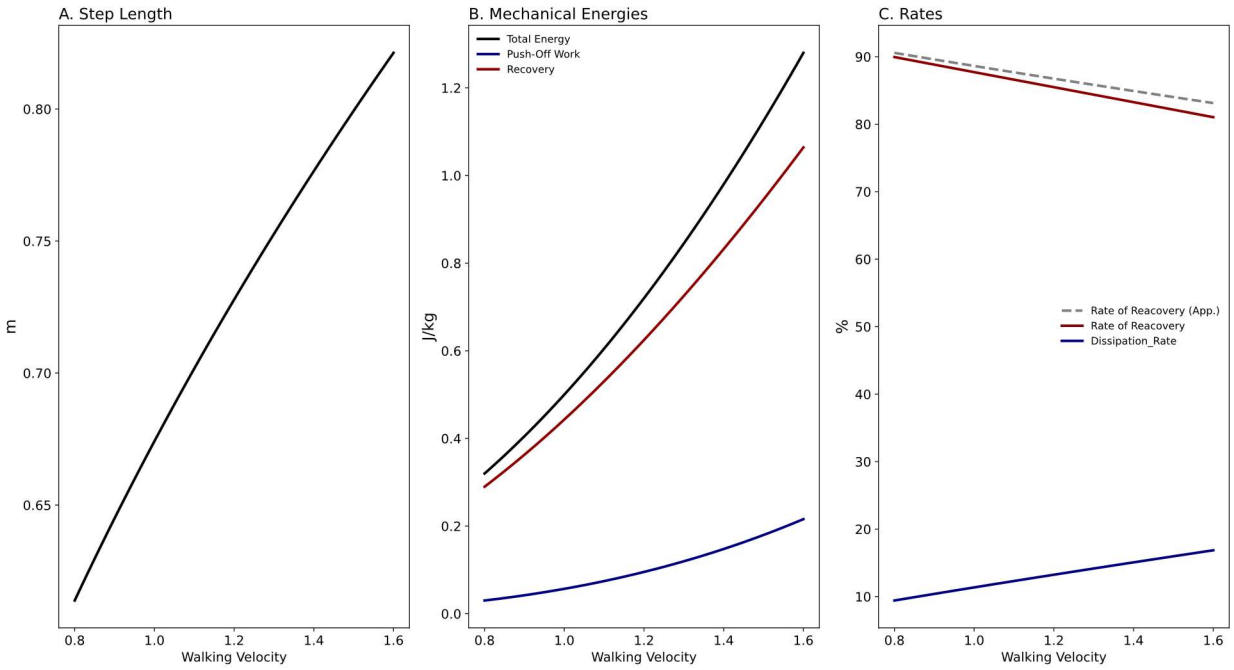

Figure 2: Summary of the analytic simulation for nominal walking across the given walking speed range: (A) As walking speed increases, the step length also increases (leg length = 1m). (B) The total mechanical energy of the walker, push-off work, and the magnitude of mechanical energy transferred from one step to the next (recovered energy or recovery) all rise. (C) While the dissipation rate increases, the rate of recovery declines. The dashed line represents the rate of recovery when a small angle approximation is employed.

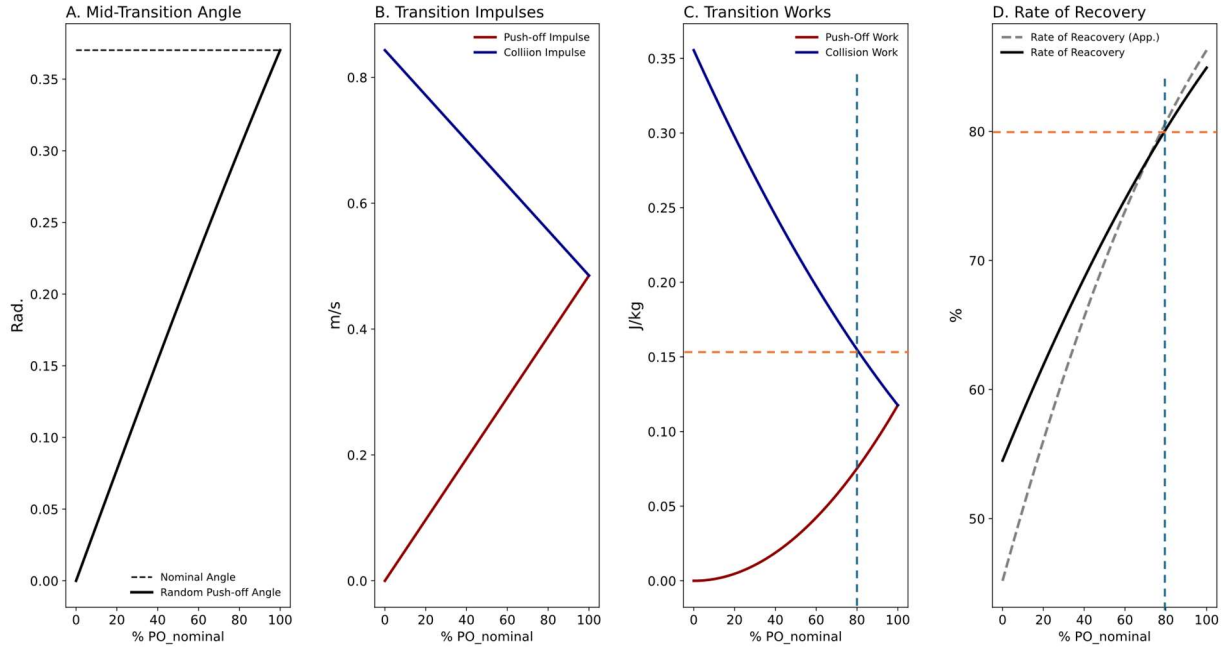

Figure 3: With a suboptimal push-off exertion ($v_{ave} = 1.25\ \mathrm{m.s^{-1}}$), as the magnitude of the induced push-off increases (A) the mid-transition velocity angle change (γ) rises, (B) resulted collision impulse declines, (C) the associated collision work dissipation decreases, and (D) the modified rate of recovery for the suboptimal push-off elevates. The vertical dashed line depicts exerting 80% of the optimal push-off. The associated transition dissipation is 0.16 $\mathrm{J \cdot kg^{-1}}$ and the estimated rate of recovery is 80%.

On average, the small angle approximation overestimated the analytical rate of recovery by only 1.3%. The analytical rate of recovery increased from 84.9% to 99.3% as step elevation changes varied from 0 m to 0.05 m, indicating that maximum push-off eliminated dissipation, allowing full transfer of the previous step's mechanical energy (Figure 4).

For step-down, the gravitational potential energy contributed to step transition work, reducing the required push-off as its amplitude increased. Simulations showed that for Δh = -0.036 m, no push-off was required, as gravitational potential energy fully compensated for the step transition energy, resulting in maximum collision energy dissipation. We terminated simulations when pre-emptive push-off reached zero, defining this as the maximum step-down amplitude. Beyond this threshold, analytical collision could not offset excess gravitational energy conversion, and walkers could not sustain the average prescribed walking speed. The rate of recovery decreased with greater step-down amplitudes, declining from 85% to 54% as collision dissipation increased from 0.12 $\mathrm{J.kg^{-1}}$ to 0.36 $\mathrm{J.kg^{-1}}$. For step-down, the small angle approximation overestimated the analytical rate, but the trajectories intersected. As such, the small angle approximation underestimated the rate of recovery at the maximum step-down amplitude (Figure 5).

Rate of recovery based on empirical data:

For experimental data analysis, the collision work at heel-strike can be estimated by integrating COM power trajectories during the collision phase (Figure 6) (Kuo et al., 2005). Using this approach, the rate of recovery can be expressed as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - \frac{W_{co_exp}}{M}}{\frac{1}{2}v_{ave}^2} = 1 - \frac{2\,W_{co_exp}}{M.\,v_{ave}^2} \qquad \text{Eq. 13}$$

Here, 'M' is the subject's mass. We expect lower rates of recovery for experimental data due to additional dissipation, such as soft tissue (Zelik & Kuo, 2010) or peripheral work (Van Der Zee & Kuo, 2021) that are not reflected in the COM analytical work.

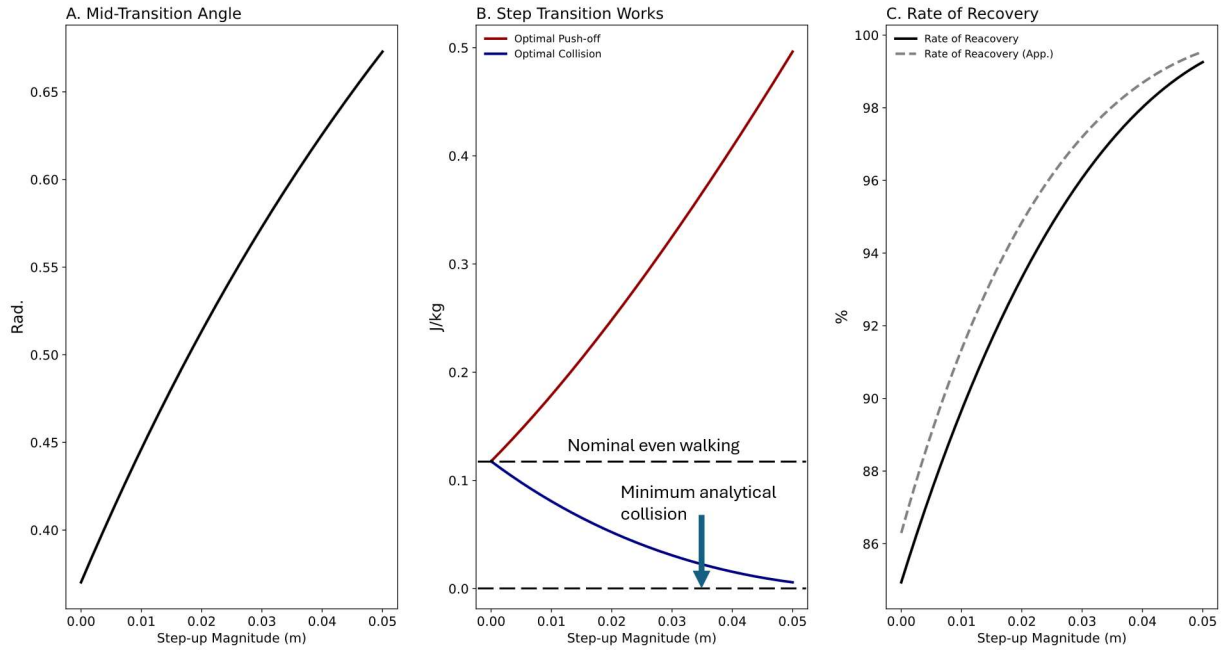

Figure 4: Simulation result for step-up when the entire step energy and gravity work is provided by a pre-emptive push-off: with step-up amplitude increase (A) the mid-transition velocity angle change (γ) increases, (B) as the required push-off increases from nominal magnitude to maximum, the collision declines to zero, (C) the analytical rate of recovery increases. The increase of rate of recovery is because of the decrease in the heel-strike collision dissipation. At maximum push-off impulse, the model predicts no dissipation and as such the entire step's energy is transferred to the next step with the makeup needed for the gravity work.

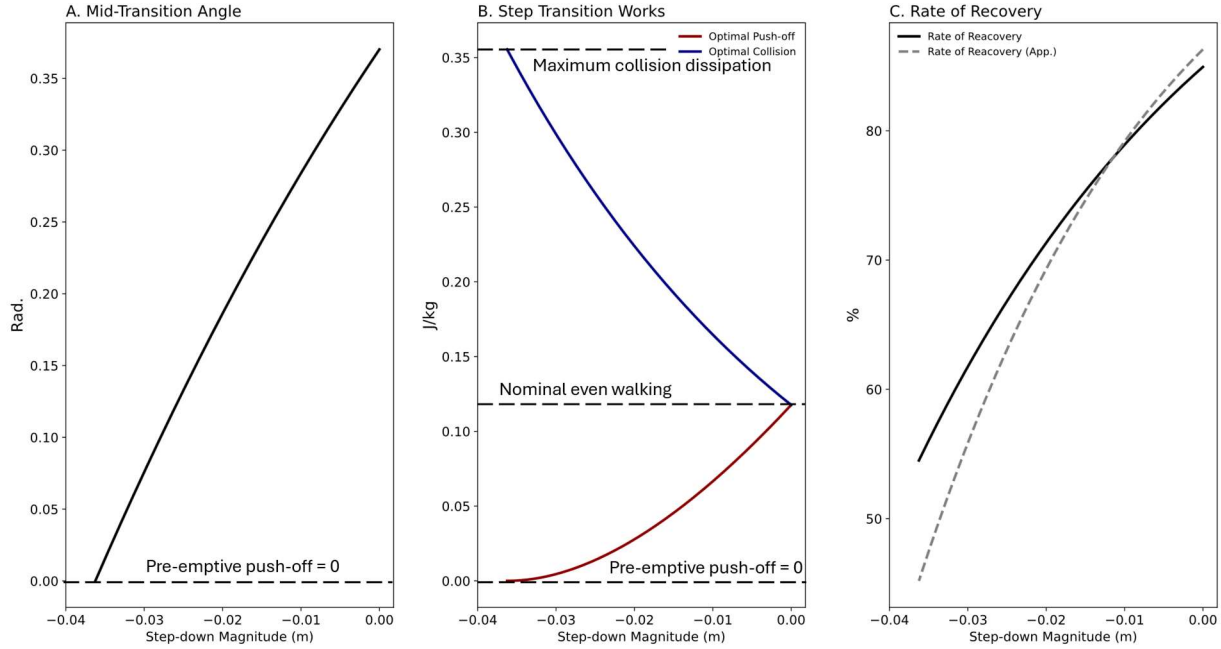

Figure 5: Simulation result for the step down when the entire step energy and gravity work is provided by a pre-emptive push-off and conversion of the gravity potential energy: with step-down amplitude increase (A) the mid-transition velocity angle change (γ) declines, (B) as the required push-off decrease from nominal magnitude to zero, the collision reaches its maximum, (C) the analytical rate of recovery also decreases. Beyond the maximum collision impulse, the model predicts the walker's speed increases beyond the prescribed average speed.

We analyzed the COM power component trajectories derived from empirical data (Hosseini-Yazdi Seyed-Saleh, 2024) to calculate the step positive, negative, push-off, and collision work values for walking speeds ranging from 0.8 $\mathrm{m.s^{-1}}$ to 1.6 $\mathrm{m.s^{-1}}$ on a smooth surface (even terrain). The trajectories were adjusted to correct for random calibration biases, and single support phase positive (rebound) and negative (preload) works were subsequently calculated (Hosseini-Yazdi & Bertram, 2024).

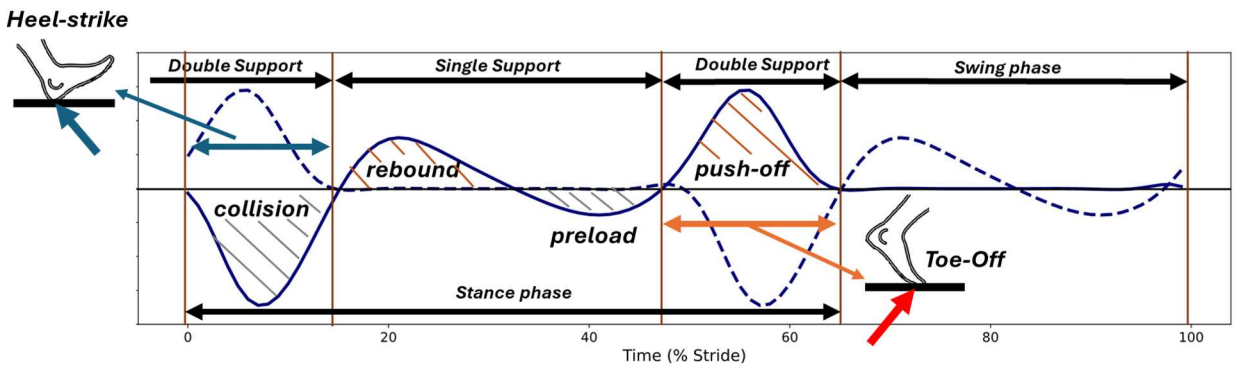

Figure 6: The COM power trajectory during a stride is illustrated, with the power contribution of the opposite leg represented by a dashed line. Each stance begins with a heel-strike, which exerts a collision impulse on the COM. Following the initial double support phase, gait transitions into a single support phase (rebound and preload). In the late stance phase, the trailing leg exerts a push-off impulse during toe-off. The mechanical work for each phase can be determined by integrating the corresponding power trajectory over time.

Based on heel-strike collision dissipation, the modified rate of recovery consistently declined from 82.5% to 51.5%, closely mirroring model predictions but at a substantially faster rate. On average, with restricted lookahead, the rate of recovery was 3.4% lower than normal lookahead. The young and older adults' trajectories did not differ significantly from each other (Hosseini-Yazdi Seyed-Saleh, 2024). Analyzing the COM work components revealed a walking speed threshold below which push-off work exceeded subsequent heel-strike collision work. This surplus of positive energy suggested a COM speed increase beyond the prescribed walking speed. Since total positive and negative step works were balanced and the average prescribed speed was maintained, the extra mechanical energy post-step transition must have been dissipated during the subsequent single support phase.

Examining the associated single support mechanical work indicated net negative work for walking speeds below the identified threshold. Beyond the threshold speed, a net mechanical energy deficit at step transition was observed, which was compensated by the associated net positive single support work post-step transition. Consequently, we adjusted step dissipation by incorporating net single support negative work. This adjustment showed that the rate of recovery increased for walking speeds below the threshold, rising from 48.6% to 59.4%, with the maximum observed at v = 1.21 $\mathrm{m.s^{-1}}$. Beyond this speed, the rate of recovery declined to 51.5%. With restricted lookahead, the maximum rate of recovery was 58.5% at v = 1.13 $\mathrm{m.s^{-1}}$ (Figure 7).

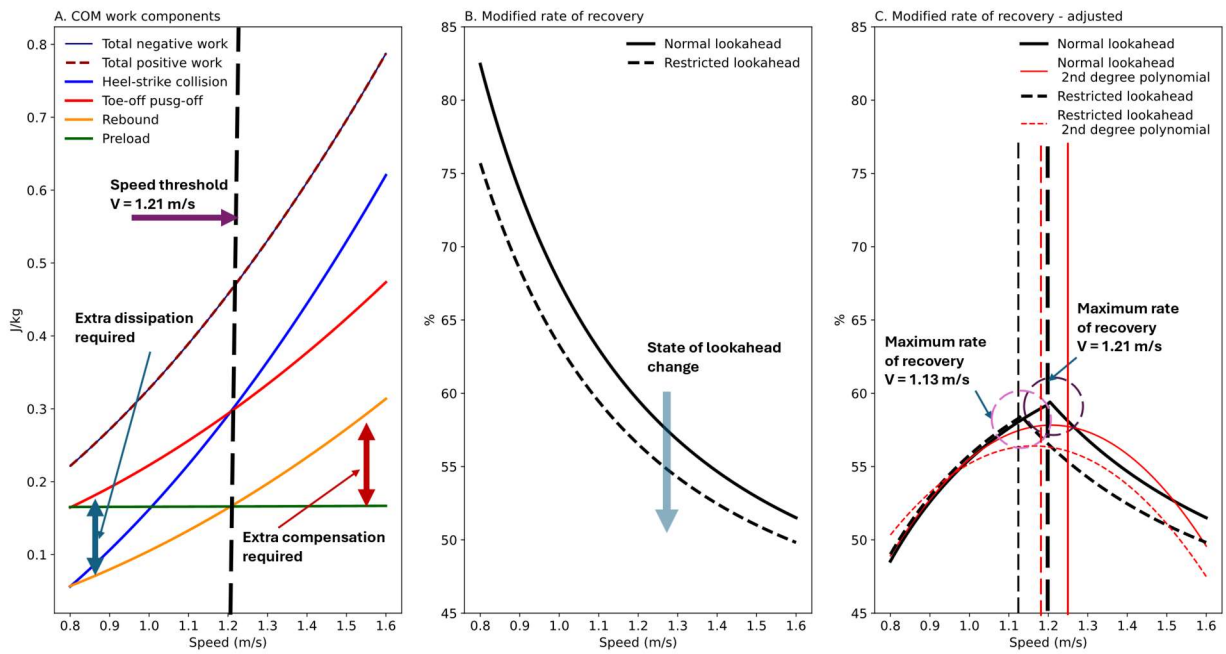

Figure 7: (A) The COM work components derived from the experimental data (Hosseini-Yazdi Seyed-Saleh, 2024) reveal walking speeds where step transition push-off exceeds the associated step collision. This surplus mechanical energy is dissipated during the single support phase, as indicated by the net negative single support work (Rebound – Preload). (B) When only step transition collision dissipation work is considered for the rate of recovery calculation, the trend resembles that of the powered simple walking model, showing a consistent decline. With restricted lookahead, the trajectory shows more step transition dissipation than normal lookahead (C) Considering the total step dissipation as the sum of collision and net single support work when it is negative into the rate of recovery calculation reveals an adjusted trajectory with a maximum point at v = 1.21 $m.s^{-1}$. This speed corresponds to the point where net single support work is zero. With restricted lookahead, the maximum rate of recovery occurs at v = 1.13 $m.s^{-1}$. The modified definition shows that based on the total step dissipation, the trajectory of ROR differs before and after the point of maximum. We also fitted a second order polynomial to the ROR trajectories that are indicated by color red. Their associated maximum RORs are slightly higher than our modified definition, showing a rather symmetrical profile.

Discussion:

In this study, we propose a modified definition for the rate of recovery based on a powered simple walking model (Kuo, 2002). This metric quantifies the proportion of a step's total energy that is transferred to the subsequent step (Cavagna et al., 1977). Energy dissipated during this transition is compensated by the active work exerted on the COM at the start of the next step (Cavagna et al., 1977; Donelan et al., 2002b). Unlike earlier definitions, which suggests that energy dissipation stems from an incomplete conversion between potential and kinetic energy (Cavagna et al., 1977), our approach attributes the dissipation to the active negative work performed during the step (Donelan et al., 2002a, 2002b; Kuo, 2002). This reflects the deviation from conservative inverted pendulum motion during the step-to-step transition or COM energy modulation during the single support phase (Kuo, 2002; Kuo et al., 2005).

Contrary to the original definition, which posited a maximum rate of recovery at a specific walking velocity (v ~ 1.2 $\mathrm{m.s^{-1}}$ or 4.3 $\mathrm{km.hr^{-1}}$)(Cavagna et al., 1977), our analytical model reveals a nearly linear decline in rate of recovery with increasing speed for nominal walking simulation. This decrease occurs because collision impulse is directly proportional to walking velocity ($co \sim v_{ave}$) (Kuo, 2002), leading to energy dissipation proportional to the square of velocity ($w_{co} \sim v_{ave}^2$). As a result, this method cannot predict preferred walking speeds, since the powered simple walking model (Kuo, 2002) assumes that the negative and positive works during step transitions are optimal and equal for a nominal gait.

The original method for calculating the rate of recovery reported values ranging from approximately 50% to 70% for walking speeds of 0.8 $\mathrm{m.s^{-1}}$ (2.9 $\mathrm{km.hr^{-1}}$) to 1.6 $\mathrm{m.s^{-1}}$ (5.8 $\mathrm{km.hr^{-1}}$) (Cavagna et al., 1976). Thus, the analytical model demonstrates considerably higher rates of recovery (ROR > 80%) for optimal pre-emptive push-offs simulation, indicating that most of the energy from the previous step is effectively transferred to the next. This is attributed to the influence of the pre-emptive push-off, which limits heel-strike collisions (Adamczyk & Kuo, 2009; Hosseini-Yazdi & Bertram, 2024). However, when the push-off is

suboptimal, post step transition, additional active mechanical work must be exerted (Hosseini-Yazdi & Bertram, 2024; Hosseini-Yazdi & Kuo, 2025). This increases heel-strike dissipation (Hosseini-Yazdi & Bertram, 2024), almost halving the recovery rate when heel-strike happens first.

For uneven walking, the analytical rate of recovery exhibits a high variability. On randomly uneven terrains, the analytical average rate of recovery (~82%) approximates that of even nominal walking (~85%) at a similar speed ($v_{ave} = 1.25\ \mathrm{m.s^{-1}}$). However, uneven walking is energetically more demanding (Pandolf et al., 1976), as reflected in greater mechanical work (Hosseini-Yazdi & Kuo, 2025; Voloshina et al., 2013). During the uneven walking, the proposed suboptimal timing of pre-emptive push-offs must increase the reliance on the subsequent stance-phase rebound work (Hosseini-Yazdi & Kuo, 2025), which leads to amplified total active mechanical work performance and thus projects a lower rate of recovery.

The empirical COM work trajectories (Hosseini-Yazdi Seyed-Saleh, 2024) reveal an imbalance in step transition work, which is regulated during the subsequent single support phase (net single support work, Figure 7A). Consequently, we consider dissipation from both step transition and the subsequent single support phase, when its net work is negative, to calculate the rate of recovery. The walking speed associated with the maximum rate of recovery closely aligns with the preferred or self-selected walking speed on even terrain (Gast et al., 2019; Hosseini-Yazdi & Bertram, 2024; Ralston, 1958), when the net single support work is zero.

The lower rate of recovery in the modified method may be associated with the external work calculation approach. In the modified method, external work is calculated using a limb-by-limb approach (Donelan et al., 2002b; Kuo et al., 2005), which has been shown to yield higher work values than the combined method on which the original ROR calculation was based (Cavagna et al., 1977; Donelan et al., 2002b).

Although the adjusted modified model and the original definition of the rate of recovery both yield a maximum value, they differ significantly in their trajectories. The trajectory of the original rate of recovery

may align with the best-fit curve for the data, which is symmetrical around the point of maximum (Cavagna et al., 1976, 1977). We could attain similar trajectory when we fitted second order polynomial to modified empirical ROR data (Figure 7C). In contrast, based on the total step dissipation, we suggest that the rate of recovery varies depending on two distinct factors: the dissipation of excess mechanical energy and the compensation for mechanical energy deficits after the step-to-step transitions. Consequently, the adjusted modified method portrays a piecewise profile influenced by the magnitude of pre-emptive push-off relative to the total energy required for step transition.

Visual information about terrain is suggested to be crucial for selecting appropriate footholds (Matthis et al., 2018). It is proposed that vision about the coming terrain (lookahead) enables more efficient potential-to-kinetic energy conversions and reduces the active work requirements (Hosseini-Yazdi Seyed-Saleh, 2024; Matthis et al., 2017; Matthis & Fajen, 2013). On the other hand, restricted lookahead is proposed to lead to higher collision energy dissipations (Hosseini-Yazdi Seyed-Saleh, 2024; Matthis & Fajen, 2013). Restricted lookahead might also increase mechanical work in the lateral direction as a function of step width (Hosseini-Yazdi Seyed-Saleh, 2024; Voloshina et al., 2013), although this component may remain small (Bauby & Kuo, 2000; Donelan et al., 2004), it further elevates the COM active work. Hence, the restricted lookahead lower maximum rate of recovery can be associated with the greater step mechanical work performance.

The empirical data (Hosseini-Yazdi Seyed-Saleh, 2024) does not indicate significant mechanical work performance difference between young and older adults for walking over smooth surfaces. Nevertheless, the elevated metabolic costs observed in older adults' walking (Das Gupta et al., 2019) can be linked to age-related muscular changes (Benjuya et al., 2004; Volpi et al., 2004) and altered walking patterns (Skiadopoulos et al., 2020). Given that COM work is a proxy for walking energetics (Donelan et al., 2002a), older adults are also expected to exhibit lower rates of recovery, consistent with their higher energy costs.

In summary, human walking may approximate inverted pendulum motion during single support phases (Donelan et al., 2002a; Kuo et al., 2005), but significant deviations occur during double support phases (Donelan et al., 2002a, 2002b; Kuo et al., 2005). These deviations result in considerable mechanical costs during step-to-step transitions (Donelan et al., 2002a). The modified rate of recovery emphasizes the mechanical and metabolic efficiency of pre-emptive push-offs, highlighting their critical role in energy conservation. Unlike the original definition of the rate of recovery (Cavagna et al., 1977, 1976), our analytical model predicts a declining trend. The analytical result trajectory can be attributed to the assumption that the entire step work is performed at the step transition instance. However, the empirical COM work trajectories show that this assumption holds true for only one walking speed. In other cases, a non-zero net single support work indicates additional work performed after the step transition. Based on empirical data and our adjusted method, we observe a walking speed at which the rate of recovery reaches its maximum. This speed aligns closely with reported human self-selected or optimal walking speeds (Cavagna et al., 1976; Gast et al., 2019; Hosseini-Yazdi & Bertram, 2024; Ralston, 1958). As COM work explains the majority of walking energetics (Donelan et al., 2002a), we anticipate that any walking conditions associated with the elevated energetics will coincide with reduced rates of recovery. Examples include walking with restricted lookahead (maximum rate of recovery = 58.5%), on uneven terrain, or old age walking. Furthermore, the lower rates of recovery based on experimental data may be associated with other costs or dissipation, such as soft tissue or peripheral work, that are not considered in the analytical simulation. Developing a sound metric for the rate of recovery is particularly significant for understanding gait mechanics, assessing mobility in populations such as the elderly or those with impairments, guiding rehabilitation, and optimizing prosthetic designs. Additionally, ROR is influenced by factors like walking speed, terrain, and load, making it a valuable tool for evaluating walking performance and efficiency in both clinical and everyday settings.

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