A Powered Simple Walking Model Modifies the Rate of Recovery Definition

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Abstract:

This study proposes a modified definition of the original rate of recovery (ROR) in human walking, derived from a powered simple walking model. The metric captures the proportion of energy transferred from one step to the next by incorporating mechanical work during step-to-step transitions. Our findings show that the modified ROR decreases nearly linearly with increasing walking speed on even terrain due to collision impulses scaling with velocity ($co \sim v_{ave}$) and energy dissipation scaling quadratically ($w_{co} \sim v_{ave}^2$). For nominal even walking at $1.25\ \mathrm{m.s^{-1}}$, the analytical ROR was 85%, reflecting efficient energy transfer. Experimental data, however, is expected to show lower ROR values due to additional energy dissipation factors, such as soft tissue work that are not reflected in the analytical model. For uneven terrains with step height variations of ±0.036 m, the average ROR was 82%, suggesting that a completely random uneven terrain may be analytically approximated by even walking. However, the elevated energetic demands of uneven walking, primarily due to suboptimal pre-emptive push-offs, must further decrease the ROR. Similarly, walking conditions associated with increased metabolic costs, such as aging or restricted lookahead, are also expected to result in reduced RORs. These findings underscore the critical role of optimal pre-emptive push-offs and proper foot placement in minimizing collision energy dissipation and enhancing walking efficiency.

Introduction:

Humans tend to optimize the metabolic energy required for walking [4]. A significant portion of this metabolic energy is used to perform mechanical work on the Center of Mass (COM), propelling it forward and compensating for mechanical energy losses [10]. Consequently, conserving mechanical energy during walking can also reduce the associated metabolic cost [10]. One specific instance of mechanical energy loss occurs during the transition between steps, where the heel-strike dissipates energy by performing negative mechanical work, referred to as "collision" [14, 15]. The rate of recovery (ROR), defined as the fraction of energy transferred from one step to the next [4], may be redefined to account for step-to-step transition energy dissipation. In this framework, the legs' external work compensates for the energy lost during heel-strike [10, 14]. Factors such as age, state of lookahead, walking speed, and terrain may also influence heel-strike energy losses, impacting the rate of recovery.

Human locomotion is hypothesized to mimic pendulum and spring-like mechanics to minimize active muscle work [4]. Specifically, the single support phase of walking aligns with the motion of an inverted pendulum [15]. In an idealized scenario with no energy dissipation, a walker starting with sufficient initial energy would require no external input. Thus, human walking primarily involves converting potential and kinetic energy within a conservative system [4].

According to Cavagna et al. [4], walkers perform works against gravity (W_v) to lift the COM and to accelerate it forward (W_f). When potential and kinetic energies are completely out of phase, mechanical energy transitions seamlessly, with no dissipation. In this state, gravitational potential energy is converted into kinetic energy, propelling the walker forward. However, when these energies become aligned, energy exchange is disrupted, resulting in dissipation. Humans then exert external COM work (W_{ext}) to compensate for this energy loss. This assumption hinges on the notion that human walking always resembles inverted pendulum mechanics and incurs no energy waste (Figure 1A).

Donelan et al. [9], however, proposed that humans simultaneously perform positive and negative work during step-to-step transitions. This suggests a significant deviation from the inverted pendulum model, as the COM mechanics transition to the next stance (Figure 1B). Donelan et al. [10] further argued that these simultaneous COM works are mechanically costly and serve as a rough indicator of walking energetics.

When considering the right and left legs as separate actuators exerting toe-off and heel-strike impulses [14], it becomes reasonable to investigate their individual effects on the COM. Similar to other mechanical actuators, the work of each leg is determined by the force applied and the corresponding displacement. Thus, the mechanical work of each leg cannot mathematically cancel the other's work. While the net effect on the COM may reflect the resultant force, the independent work of each leg contributes its own metabolic cost [11].

Both toe-off and heel-strike impulses generate upward forces on the COM in the frontal plane, but their sagittal plane effects differ. The toe-off impulse propels the pelvis forward, while the heel-strike impulse opposes forward progression, partially wasting momentum and mechanical energy (Figure 1C & D) [9, 10, 14]. Consequently, the toe-off impulse (push-off, po) contributes to forward momentum, whereas the heel-strike impulse (collision, co) results in energy loss [10, 14].

A simple powered walking model demonstrates the independent contributions of the leading and trailing legs during step transitions [14]. For a given walking speed, it predicts the active work required (push-off) for optimal steady-state walking, where active work matches dissipation. However, suboptimal conditions can compromise optimal push-off. Pre-emptive push-off is hypothesized to limit the magnitude of subsequent collision [14, 15]. When heel-strike occurs earlier than expected [6], pre-emptive push-off can only partially mitigate the resulting collision [13]. To maintain steady walking, additional positive mechanical energy is required during the subsequent stance phase [5, 12, 14]. Suboptimal push-off ($po_{sub} < po_{opt}$) coincides with increased dissipation and lower rates of recovery. Factors such as age,

restricted lookahead, and complex terrain may induce greater step dissipations [12], and as such further reduce the rate of recovery.

Walking energetics increase with terrain complexity [12, 20, 22] and age [8, 12]. Restricted lookahead might also impair the ability to select appropriate footholds, limiting the benefits of human walking dynamics [17]. Consequently, restricted lookahead may adversely affect the rate of recovery.

This study redefines the rate of recovery based on heel-strike energy dissipation. We develop a simulation to derive analytical rates of recovery for nominal (optimal) walking across various walking speeds. The simulation is extended to scenarios with suboptimal push-off impulses ($po_{sub} < po_{opt}$) and cases where COM elevation changes, i.e. step-ups and step-downs. Without loss of generality, we examine a scenario where pre-emptive push-off provides the entire step transition and compensates for work against gravity.

New Definition for Rate of Recovery:

According to a powered simple walking model [14], the active push-off (po) is applied pre-emptively to the center of mass (COM) to mechanically energize the step-to-step transition [6]. This action compensates for the dissipation caused by the heel-strike collision, which occurs after the push-off. Consequently, the required external work (W_{ext}) is assumed to offset the energy lost during the heel-strike [4]. Considering 'u' as the transition impulse, its associated work is calculated as $\frac{1}{2}u^2$ [6, 14]. The total mechanical energy of the human body is the sum of its potential and kinetic energies ($E_{tot} = E_P + E_K$), Assuming that the lowest elevation of the COM during the stance phase is the reference for gravitational potential energy ($E_P = 0$), the total energy at the end of the stance phase corresponds to the maximum kinetic energy ($E_{K_max} = E_{tot}$, Figure 1E & F). As originally defined, the rate of recovery is given by [4]:

$$Rate\ of\ Recovery = \frac{System\ total\ energy\ before\ transition\ - W_{ext}}{System\ total\ energy\ before\ transition} \qquad \text{Eq. 1}$$

This equation quantifies the proportion of mechanical energy transferred from one step to the next. Since the system's total energy equals the maximum kinetic energy just before the step transition, the rate of recovery becomes:

$$Rate\ of\ Recovery = \frac{Maximum\ kinetic\ energy - W_{ext}}{Maximum\ kinetic\ energy} \quad \text{Eq. 2}$$

Assuming the COM velocity just before the transition approximates the average walking velocity (v_{ave}) [14], and that w_{ext} represents the normalized external work per unit mass, the rate of recovery can be rewritten as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - w_{ext}}{\frac{1}{2}v_{ave}^2} \quad \text{Eq. 3}$$

To sustain steady walking, the walker's energy must remain constant before and after the step-to-step transition. Denoting the velocity before and after the transition as v^- and v^+, respectively, steady walking implies, $|v^+| = |v^-| = v_{ave}$. With the kinetic energy conserved across the transition, the active work energy addition (W_{ext}) equals the heel-strike energy dissipation or collision work (W_{co}). Therefore, the rate of recovery can be further expressed as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - w_{co}}{\frac{1}{2}v_{ave}^2} \quad \text{Eq. 4}$$

Here, w_{co} represents the normalized energy loss due to the heel-strike collision.

Simulation based on a powered simple walking model:

The nominal push-off impulse for a given walking speed is expressed as $po = v_{ave} \cdot \tan\alpha$ where '2α' represents the angle between the leading and trailing legs at the point of transition [14]. Consequently, the nominal push-off work becomes $w_{po} = \frac{1}{2}v_{ave}^2 \cdot \tan^2\alpha$ [14]. Assuming $w_{co} = w_{po}$ and $w_{ext} = w_{co}$, the rate of recovery is:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - \frac{1}{2}v_{ave}^2.\tan^2\alpha}{\frac{1}{2}v_{ave}^2} = 1 - \tan^2\alpha \qquad \text{Eq. 5}$$

For small angles, the approximation $\tan\alpha \sim \alpha$ holds, leading to $po = v_{ave} \cdot \alpha$ and $w_{po} = \frac{1}{2}v_{ave}^2 \cdot \alpha^2$ [14]. Assuming a unit leg length, the step length (s) becomes proportional to 2α [13, 14]. Since step length follows a power law with walking speed [13], we have: $s \sim v^{0.42}$, yielding $2\alpha = c \cdot v^{0.42}$ and $\alpha = \frac{1}{2}c \cdot v^{0.42}$. Substituting α into Eq. 6 with $\tan\alpha \sim \alpha$:

$$Rate\ of\ Recovery = 1 - \alpha^2 = 1 - \frac{1}{4}c^2 \cdot v_{ave}^{0.84} \qquad \text{Eq. 7}$$

This analytical expression suggests that the rate of recovery decreases nearly linearly with walking speed. Applying the principles of the powered simple walking model to a suboptimal push-off (po_{sub}, Figure 1G) [14] the mid-transition velocity (v^0) forms an angle (γ) with the COM velocity before transition (v^-) which depends on the induced push-off impulse: $\gamma = \arctan(\frac{po_{sub}}{v_{ave}})$. Since $v^- \sim v_{ave}$ [14] , the transition collision impulse (co) becomes:

$$\begin{cases} \frac{co}{v^0} = \sin(2\alpha - \gamma) \\ \frac{v_{ave}}{v^0} = \cos\gamma \end{cases} \rightarrow co = v_{ave}\frac{\sin(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 8}$$

The normalized collision work is $w_{co} = 0.5co^2$ [6, 14], and the modified rate of recovery is:

$$Rate\ of\ Recovery = \frac{\frac{1}{2}v_{ave}^2 - w_{co}}{\frac{1}{2}v_{ave}^2} = 1 - (\frac{\sin(2\alpha - \gamma)}{\cos\gamma})^2 \qquad \text{Eq. 9}$$

Using small angle approximations for 'α' and 'γ':

$$Rate\ of\ Recovery = 1 - \left(2\alpha - \frac{po_{sub}}{v_{ave}}\right)^2 = 1 - \left(c.v_{ave}^{0.42} - \frac{po_{sub}}{v_{ave}}\right)^2 \qquad \text{Eq. 10}$$

This result shows that the rate of recovery becomes a nonlinear function of both the suboptimal push-off magnitude and the average walking speed.

For uneven terrain with a step-up, maintaining the average walking speed (v_{ave}) at mid-stance requires the post-transition COM kinetic energy to account for the work against gravity. The post-transition velocity (v^+) must satisfy (Figure 1H):

$$\frac{1}{2}v^{+2} - \frac{1}{2}v_{ave}^2 = g.\Delta h \rightarrow v^+ = \sqrt{v_{ave}^2 + 2g.\Delta h} \qquad \text{Eq. 11}$$

If the entire uneven step energy is provided by a pre-emptive push-off, then:

$$\begin{cases} \frac{v^+}{v^0} = \cos(2\alpha - \gamma) \\ \frac{v_{ave}}{v^0} = \cos\gamma \end{cases} \rightarrow v^+ = v_{ave}\frac{\cos(2\alpha - \gamma)}{\cos\gamma} \rightarrow \sqrt{v_{ave}^2 + 2g.\Delta h} = v_{ave}\frac{\cos(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 12}$$

The mid-transition velocity angle change (γ) must be determined numerically for a given walking condition. The uneven step collision impulse becomes:

$$\frac{co}{v^0} = \sin(2\alpha - \gamma) \rightarrow co = v_{ave}\frac{\sin(2\alpha - \gamma)}{\cos\gamma} \qquad \text{Eq. 13}$$

Thus, the rate of recovery for an uneven step can be derived as in even walking when a suboptimal push-off is applied, with $\gamma > \alpha$. Using the small angle approximation, the rate of recovery for uneven walking becomes: $1 - (2\alpha - \gamma)^2$. For a step-down, the process is similar, with Δh being negative. Here, the potential energy converts to kinetic energy, increasing momentum, and the required push-off magnitude decreases as the step-down amplitude increases.

Results:

We redefined the rate of recovery based on step-to-step mechanical work dissipation [10], focusing on scenarios where the COM trajectory significantly deviates from conservative inverted pendulum motion. Originally, the rate of recovery was defined as the percentage of total mechanical energy carried over from one step to the next [4]. We assumed that the external work equates to heel-strike energy dissipation, enabling us to analytically estimate the rate of recovery for various walking conditions using a simple powered walking model (whether the push-off was nominal or suboptimal) [14].

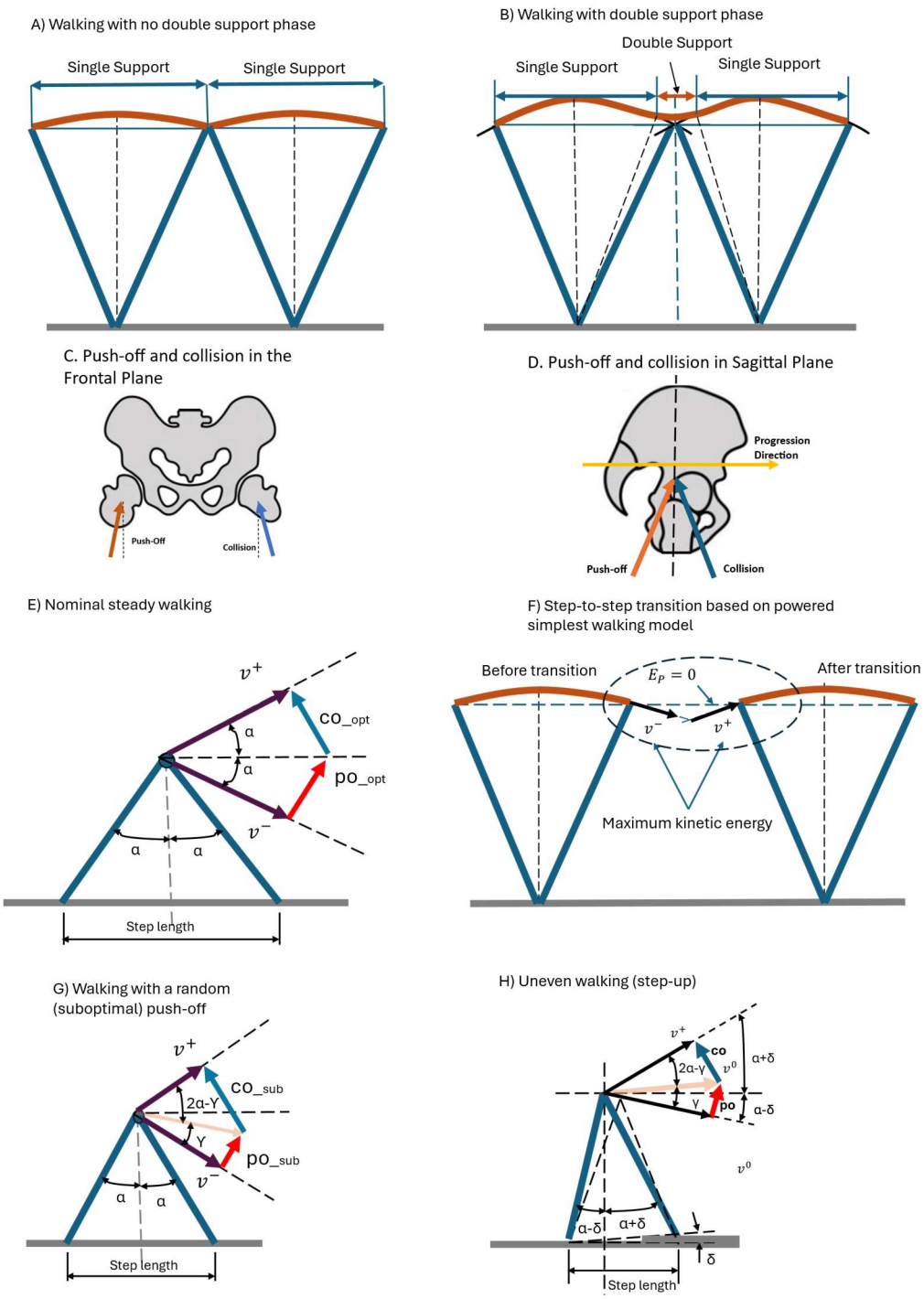

Figure 1: Schematics showing the step-to-step transition during even walking: (A) pendular walking model with rigid legs in which the step-to-step transition is instantaneous (no double support phase), (B) human walking demonstrates a double support phase in which humans switch from one inverted pendulum to another, (C) push-off and collision impulses have similar effects on the COM in the vertical direction, (D) push-off impulse adds to the COM mechanical energy in the progression direction while collision impulse reduces the momentum, (E and F) In nominal steady walking, the push-off and collision works, and pre (v^-) and post transition velocities (v^+) are equal, the pre transition velocity may be approximated by v_{ave}, (G) if the pre-emptive push-off is less than optimal (random magnitude or suboptimal), the magnitude of the subsequent collision impulse grows and the post transition velocity declines, (H) during uneven walking (step-up) the optimal walking takes place when the total step transition and gravity works are provided by the pre-emptive push-off impulse.

The step length was estimated using an empirical power law for a given velocity [13]. As walking velocity increased from 0.8 m·s^{-1} to 1.6 m·s^{-1}, the estimated step length increased from 0.61 m to 0.82 m (a 34.4% increase for a walker with a leg length of 1 m). Correspondingly, the analytical total energy of walking (COM kinetic energy just before transition) and the push-off work during the step-to-step transition increased significantly—by 400% (from 0.32 J·kg^{-1} to 1.28 J·kg^{-1}) and 733.3% (from 0.03 J·kg^{-1} to 0.22 J·kg^{-1}), respectively. However, the rate of recovery declined from 90.6% to 83.1% (an 8.3% decrease). The small angle approximation yielded a rate of recovery that was, on average, only 1.5% higher than the analytical rate across the examined range. Figure 2 illustrates the variation in walking biomechanical features with velocity.

For suboptimal push-offs, where push-off magnitudes varied from zero to nominal (velocity = 1.25 m·s^{-1}, $\alpha =$ 0.37 rad), the mid-transition velocity angle relative to the pre-transition velocity (γ) increased from zero to 0.37 rad. Simultaneously, the analytical collision impulse decreased from 0.84 m·s^{-1} to 0.49 m·s^{-1} (a 71.4% reduction), while analytical push-off work increased from zero to 0.12 J·kg^{-1} and collision work declined from 0.36 J·kg^{-1} to 0.12 J·kg^{-1} (a 66.7% reduction). The analytical rate of recovery increased from 54.5% to 84.9% (a 55.8% increase). The small angle approximation initially suggested a lower recovery rate but exceeded the analytical trajectory when the suboptimal push-off impulse reached approximately 70% of the optimal push-off magnitude (Figure 3).

For uneven walking (step-up), we assumed that optimal walking occurs when the entire step transition energy and work against gravity are provided by the pre-emptive push-off. As the step-up amplitude increased, the optimal angle of the mid-transition COM velocity change (γ) also increased. Simulations indicated that push-off magnitudes for step-up walking exceeded those for nominal even walking at the same speed but were constrained by the maximum exertable magnitude ($v_{ave}.\tan 2\alpha$), where analytical collision became zero [14].

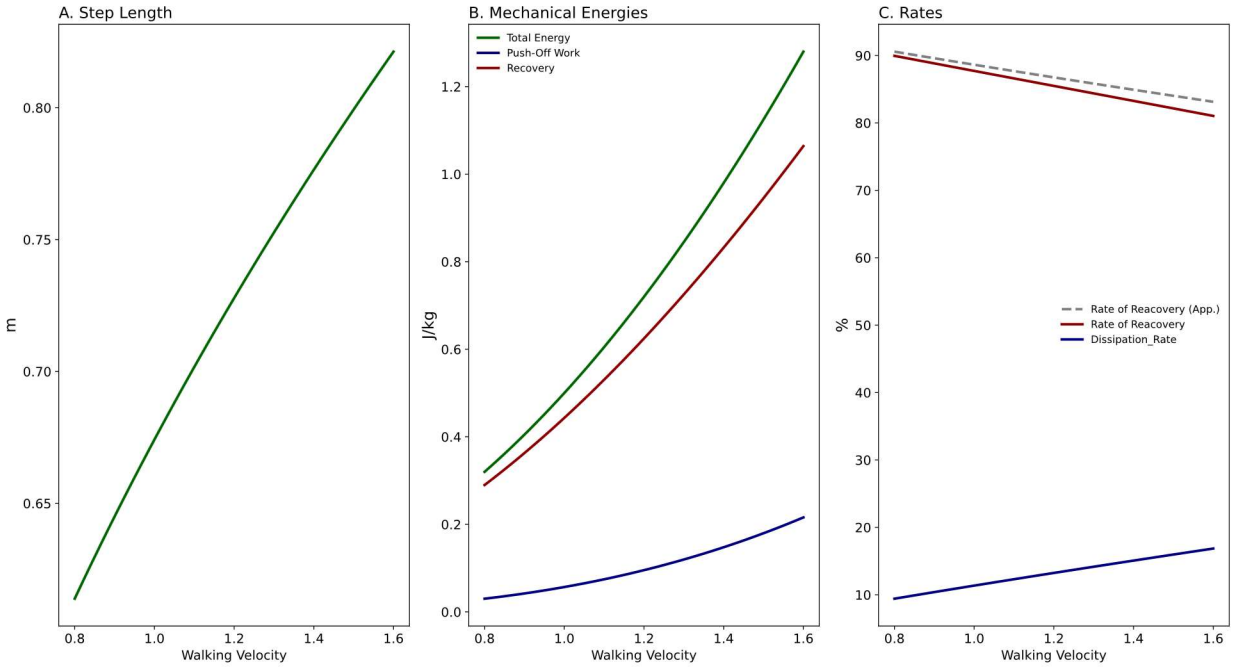

Figure 2: Summary of analytic simulation for nominal walking for any given walking speed: with the walking velocity increase (A) the step length also increases (leg length = 1m), (B) the total walker's mechanical energy, push-off work and the magnitude of the mechanical energy transferred from one step to the next (recovery) rises, (C) while the dissipation rate increase, the rate of recovery declines. The dashed line shows the rate of recovery when small angle approximation is employed.

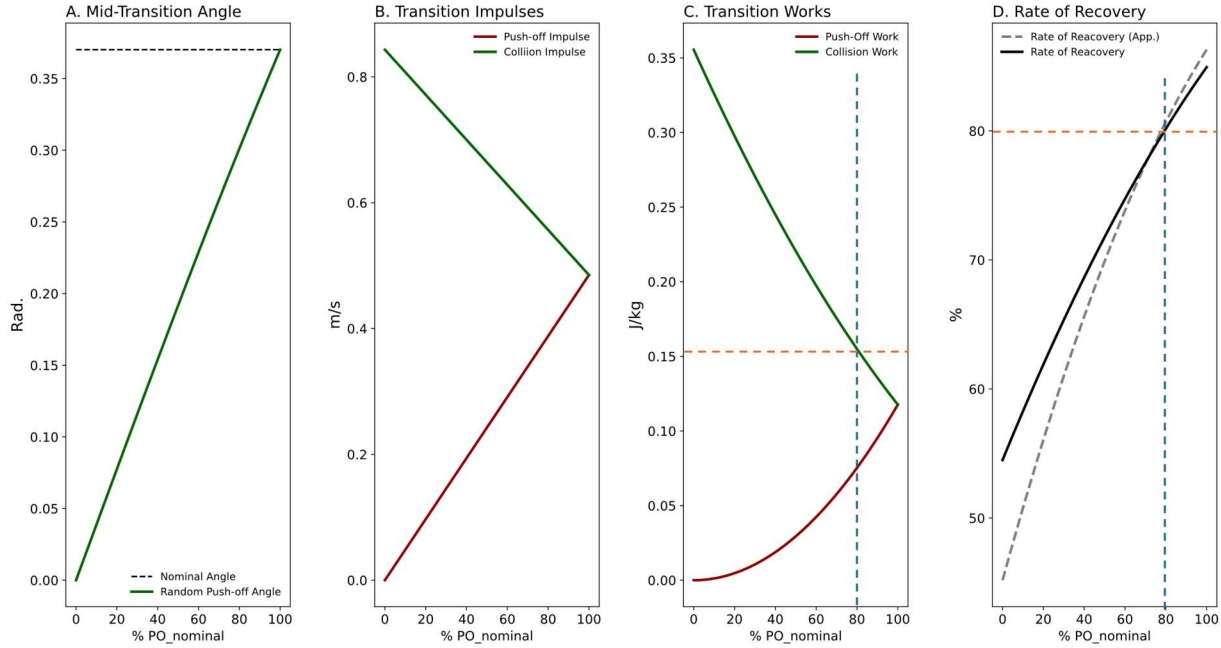

Figure 3: With a random push-off exertion (suboptimal) for $v_{ave} = 1.25\ m.s^{-1}$, as the magnitude of the induced push-off increases (A) the mid-transition velocity angle (γ) with pre-transition velocity rises, (B) resulted collision impulse declines, (C) the associated collision work dissipation decreases, and (D) the modified rate of recovery for the suboptimal push-off elevates. The dashed line depicts exerting 80% of the optimal push-off. The associated transition dissipation is 0.16 $J \cdot kg^{-1}$ and the estimated rate of recovery is 80%.

On average, the small angle approximation overestimated the analytical rate of recovery by only 1.3%. The analytical rate of recovery increased from 84.9% to 99.3% as step elevation changes varied from 0 to 0.05 m, indicating that maximum push-off eliminates dissipation, allowing full transfer of the previous step's mechanical energy (Figure 4).

For step-down walking, gravitational potential energy contributed to step transition work, reducing the required push-off as step-down amplitude increased. Simulations showed that for Δh = -0.036 m, no push-off was needed since gravitational potential energy fully supported the step, maximizing collision energy dissipation. We terminated simulations when pre-emptive push-off reached zero, defining this as the maximum step-down amplitude. Beyond this point, analytical collision could not offset excess gravitational energy, and walkers could not sustain the average prescribed speed. When pre-emptive push-off was zero, the mid-transition COM velocity equaled the pre-transition speed ($\gamma = 0$). The rate of recovery decreased with greater step-down amplitudes, declining from 85% to 54% as collision dissipation increased from 0.12 $J \cdot kg^{-1}$ to 0.36 $J \cdot kg^{-1}$. For even walking, the small angle approximation overestimated the analytical rate, but the trajectories intersected, with the approximation underestimating recovery at the maximum step-down amplitude (Figure 5).

Discussion:

In this study, we propose a modified definition for the rate of recovery based on a powered simple walking model [14]. This metric quantifies the proportion of a step's total energy that is transferred to the subsequent step [4]. Energy dissipated during this transition is compensated by the active work exerted on the center of mass (COM) at the start of the next step [4, 9]. Unlike earlier definitions, which suggested that energy dissipation stems from an incomplete conversion between potential and kinetic energy [4], our approach attributes the dissipation to negative work performed during the heel strike [9, 10, 14]. This reflects the deviation from conservative inverted pendulum motion during the step-to-step transition [14, 15].

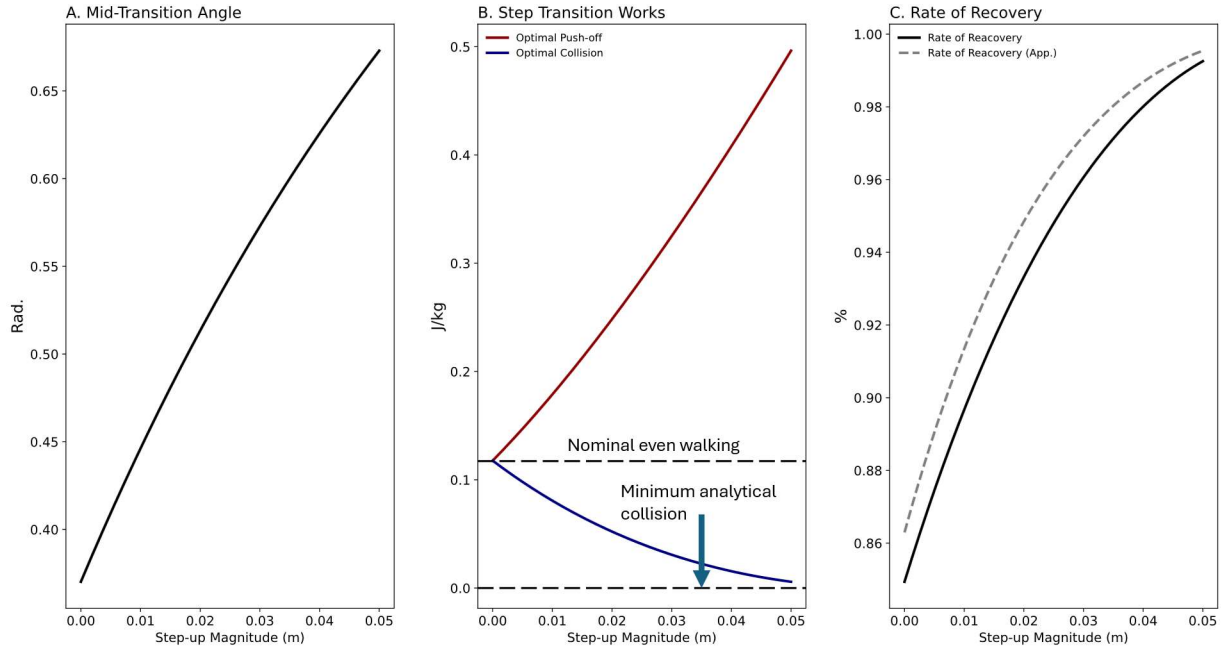

Figure 4: Simulation result for step up when the entire step energy and gravity work is provided by a pre-emptive push-off: with step-up amplitude increase (A) the mid-transition velocity angle change (γ) increases, (B) as the required push-off increases from nominal magnitude to maximum, the collision declines to zero, (C) the analytical rate of recovery increases. The increase of rate of recovery is because of the decrease in the heel-strike collision dissipation. At maximum push-off impulse, the model predicts no dissipation and as such the entire step's energy is transferred to the next step with the makeup needed for the gravity work.

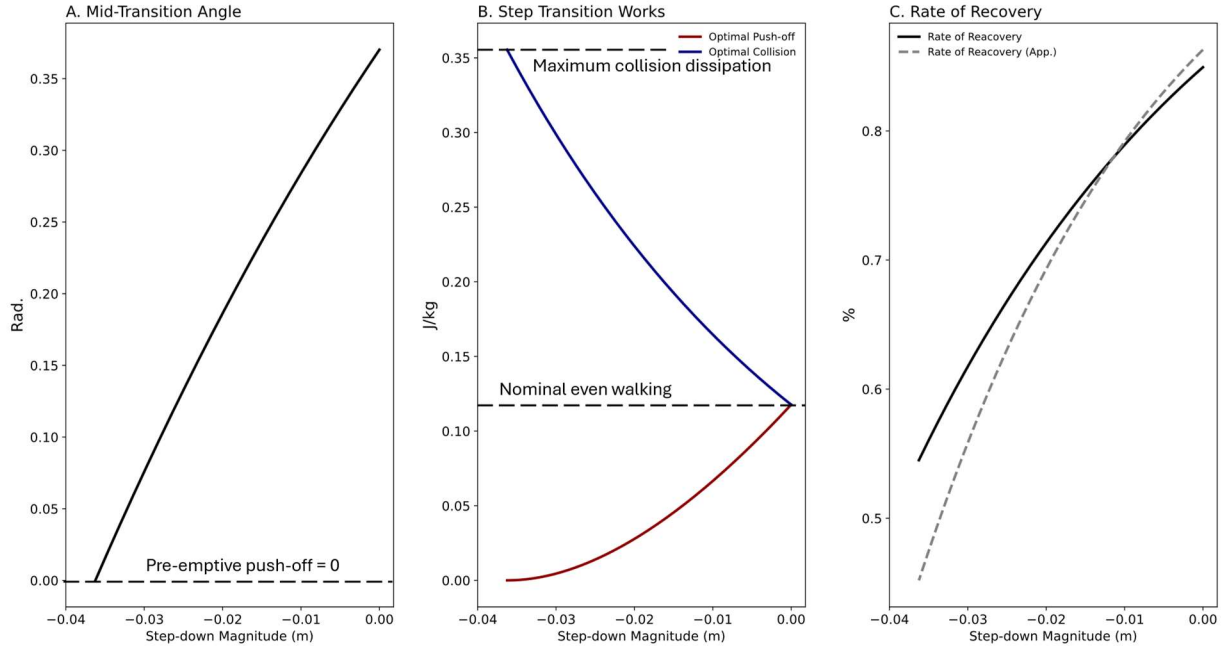

Figure 5: Simulation result for the step down when the entire step energy and gravity work is provided by a pre-emptive push-off and conversion of the gravity potential energy: with step-down amplitude increase (A) the mid-transition velocity angle change (γ) declines, (B) as the required push-off decrease from nominal magnitude to zero, the collision reaches its maximum, (C) the analytical rate of recovery also decreases. Beyond the maximum collision impulse, the model predicts the walker's speed increases beyond the prescribed average speed.

Contrary to earlier models, which posited a maximum rate of recovery at a specific walking velocity [4], our modified definition reveals a nearly linear decline in recovery rates with increasing walking velocity for nominal walking. This decrease occurs because collision impulse is directly proportional to walking velocity ($co \sim v_{ave}$) [14], leading to energy dissipation proportional to the square of velocity ($w_{co} \sim v_{ave}^2$). As a result, this method cannot predict preferred walking speeds.

The modified model also shows significantly higher recovery rates (ROR > 80%) while walking with optimal pre-emptive push-offs, indicating that most energy from the previous step is transferred to the next. This is likely due to the pre-emptive push-off redirecting COM velocity and reducing collision energy dissipation [1]. However, when the push-off is suboptimal, additional active mechanical work must be exerted post-transition, often involving hip torque (active rebound work) [10, 14], or delayed push-offs [6]. This increases heel-strike dissipation, halving the recovery rate, making it mechanically costly.

For experimental analysis, the collision work at heel-strike can be estimated by integrating COM power trajectories during collisions [15]. Using this approach, the rate of recovery can be expressed as:

$$Rate\ of\ Recovery = \frac{\frac{1}{2} v_{ave}^2 - \frac{W_{co_exp}}{M}}{\frac{1}{2} v_{ave}^2} = 1 - \frac{2\, W_{co_exp}}{M.\, v_{ave}^2} \qquad \text{Eq. 14}$$

Here, 'M' is the subject's mass. Experimental observations are expected to yield lower rates of recovery due to additional dissipative factors, such as soft tissue mechanical work [24] that are not reflected in the COM analytical works.

Visual information about terrain is suggested to be crucial for selecting appropriate footholds [18], enabling more efficient potential-to-kinetic energy conversions and reducing active work requirements [16, 17]. Restricted lookahead is proposed to reduce the ability to optimize foothold placement, leading to higher collision energy dissipation and lower recovery rates [12, 17]. Restricted lookahead might also increase mechanical work in the lateral direction as a function of step width [12, 22], though this

component remains small [2, 19], it further elevates the COM active work. Hence, we also expect smaller rates of recovery while walking with restricted lookahead.

The elevated metabolic costs observed in older adults' walking [8] can be linked to age-related muscular changes [3, 23] and altered walking patterns [21, 21]. Given that COM work is a proxy for walking energetics [10], older adults are also expected to exhibit lower recovery rates, consistent with their higher energy costs.

For uneven walking, the modified rate of recovery exhibits high variability. On randomly uneven terrains, the analytical average recovery rate (~82%) approximates that of even nominal walking (~85%) at similar speed ($v_{ave} = 1.25\ \mathrm{m.s^{-1}}$). However, uneven walking is energetically more demanding [20], as reflected in greater mechanical work [12, 22]. Suboptimal timing of pre-emptive push-offs during uneven walking increases the reliance on stance-phase rebound work, leading to expected lower recovery rates [6, 7].

In summary, during single support phases, human walking may approximate inverted pendulum motion [10, 15], but significant deviations occur during double support phases [9, 10, 15]. These deviations incur considerable mechanical costs during step-to-step transitions [10]. The modified rate of recovery underscores the mechanical and metabolic efficiency of pre-emptive push-offs, highlighting their role in energy conservation. However, experimental applications of this model will likely yield lower rates of recovery due to additional dissipative factors such as soft tissue work [24]. Rates of recovery also expected to decline under metabolically demanding conditions, such as restricted lookahead, aging, or uneven terrains.

References:

[1] Peter G. Adamczyk and Arthur D. Kuo. 2009. Redirection of center-of-mass velocity during the step-to-step transition of human walking. *J Exp Biol* 212, Pt 16 (August 2009), 2668–2678. https://doi.org/10.1242/jeb.027581

[2] C. E. Bauby and A. D. Kuo. 2000. Active control of lateral balance in human walking. *J Biomech* 33, 11 (November 2000), 1433–1440. https://doi.org/10.1016/s0021-9290(00)00101-9

[3] Nisim Benjuya, Itshak Melzer, and Jacob Kaplanski. 2004. Aging-induced shifts from a reliance on sensory input to muscle cocontraction during balanced standing. *J Gerontol A Biol Sci Med Sci* 59, 2 (February 2004), 166–171. https://doi.org/10.1093/gerona/59.2.m166

[4] G. A. Cavagna, N. C. Heglund, and C. R. Taylor. 1977. Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure. *Am J Physiol* 233, 5 (November 1977), R243-261. https://doi.org/10.1152/ajpregu.1977.233.5.R243

[5] Osman Darici and Arthur D. Kuo. 2023. Humans plan for the near future to walk economically on uneven terrain. *Proc. Natl. Acad. Sci. U.S.A.* 120, 19 (May 2023), e2211405120. https://doi.org/10.1073/pnas.2211405120

[6] Osman Darici, Hakan Temeltas, and Arthur D. Kuo. 2018. Optimal regulation of bipedal walking speed despite an unexpected bump in the road. *PLoS ONE* 13, 9 (September 2018), e0204205. https://doi.org/10.1371/journal.pone.0204205

[7] Osman Darici, Hakan Temeltas, and Arthur D. Kuo. 2019. *Anticipatory Control of Momentum for Bipedal Walking on Uneven Terrain*. Bioengineering. https://doi.org/10.1101/769828

[8] Sauvik Das Gupta, Maarten F. Bobbert, and Dinant A. Kistemaker. 2019. The Metabolic Cost of Walking in healthy young and older adults – A Systematic Review and Meta Analysis. *Sci Rep* 9, 1 (July 2019), 9956. https://doi.org/10.1038/s41598-019-45602-4

[9] J. Maxwell Donelan, Rodger Kram, and Arthur D. Kuo. 2002. Simultaneous positive and negative external mechanical work in human walking. *J Biomech* 35, 1 (January 2002), 117–124. https://doi.org/10.1016/s0021-9290(01)00169-5

[10] J. Maxwell Donelan, Rodger Kram, and Arthur D. Kuo. 2002. Mechanical work for step-to-step transitions is a major determinant of the metabolic cost of human walking. *Journal of Experimental Biology* 205, 23 (December 2002), 3717–3727. https://doi.org/10.1242/jeb.205.23.3717

[11] Timothy M. Griffin, Thomas J. Roberts, and Rodger Kram. 2003. Metabolic cost of generating muscular force in human walking: insights from load-carrying and speed experiments. *Journal of Applied Physiology* 95, 1 (July 2003), 172–183. https://doi.org/10.1152/japplphysiol.00944.2002

[12] Hosseini-Yazdi Seyed-Saleh. 2024. Energetics and Biomechanics of Uneven Walking for Young and Older Adults. Retrieved from https://prism.ucalgary.ca/server/api/core/bitstreams/be2fa66b-26a2-4421-9d30-dfcc55dcfb35/content

[13] Arthur D. Kuo. 2001. A Simple Model of Bipedal Walking Predicts the Preferred Speed–Step Length Relationship. *Journal of Biomechanical Engineering* 123, 3 (June 2001), 264–269. https://doi.org/10.1115/1.1372322

[14] Arthur D. Kuo. 2002. Energetics of Actively Powered Locomotion Using the Simplest Walking Model. *Journal of Biomechanical Engineering* 124, 1 (February 2002), 113–120. https://doi.org/10.1115/1.1427703

[15] Arthur D. Kuo, J. Maxwell Donelan, and Andy Ruina. 2005. Energetic consequences of walking like an inverted pendulum: step-to-step transitions. *Exerc Sport Sci Rev* 33, 2 (April 2005), 88–97. https://doi.org/10.1097/00003677-200504000-00006

[16] Jonathan Samir Matthis, Sean L. Barton, and Brett R. Fajen. 2017. The critical phase for visual control of human walking over complex terrain. *Proc. Natl. Acad. Sci. U.S.A.* 114, 32 (August 2017). https://doi.org/10.1073/pnas.1611699114

[17] Jonathan Samir Matthis and Brett R. Fajen. 2013. Humans exploit the biomechanics of bipedal gait during visually guided walking over complex terrain. *Proc. R. Soc. B.* 280, 1762 (July 2013), 20130700. https://doi.org/10.1098/rspb.2013.0700

[18] Jonathan Samir Matthis, Jacob L. Yates, and Mary M. Hayhoe. 2018. Gaze and the Control of Foot Placement When Walking in Natural Terrain. *Current Biology* 28, 8 (April 2018), 1224-1233.e5. https://doi.org/10.1016/j.cub.2018.03.008

[19] J. Maxwell Donelan, Rodger Kram, and Kuo Arthur D. 2001. Mechanical and metabolic determinants of the preferred step width in human walking. *Proc. R. Soc. Lond. B* 268, 1480 (October 2001), 1985–1992. https://doi.org/10.1098/rspb.2001.1761

[20] K. B. Pandolf, M. F. Haisman, and R. F. Goldman. 1976. Metabolic energy expenditure and terrain coefficients for walking on snow. *Ergonomics* 19, 6 (November 1976), 683–690. https://doi.org/10.1080/00140137608931583

[21] Andreas Skiadopoulos, Emily E. Moore, Harlan R. Sayles, Kendra K. Schmid, and Nicholas Stergiou. 2020. Step width variability as a discriminator of age-related gait changes. *J NeuroEngineering Rehabil* 17, 1 (December 2020), 41. https://doi.org/10.1186/s12984-020-00671-9

[22] Alexandra S. Voloshina, Arthur D. Kuo, Monica A Daley, and Daniel P. Ferris. 2013. Biomechanics and energetics of walking on uneven terrain. *Journal of Experimental Biology* (January 2013), jeb.081711. https://doi.org/10.1242/jeb.081711

[23] Elena Volpi, Reza Nazemi, and Satoshi Fujita. 2004. Muscle tissue changes with aging. *Curr Opin Clin Nutr Metab Care* 7, 4 (July 2004), 405–410. https://doi.org/10.1097/01.mco.0000134362.76653.b2

[24] Karl E. Zelik and Arthur D. Kuo. 2010. Human walking isn't all hard work: evidence of soft tissue contributions to energy dissipation and return. *Journal of Experimental Biology* 213, 24 (December 2010), 4257–4264. https://doi.org/10.1242/jeb.044297