Interpretable representation learning for 3D multi-piece intracellular structures using point clouds

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Abstract

A key challenge in understanding subcellular organization is quantifying interpretable measurements of intracellular structures with complex multi-piece morphologies in an objective, robust and generalizable manner. Here we introduce a morphology-appropriate representation learning framework that uses 3D rotation invariant autoencoders and point clouds. This framework is used to learn representations of complex multi-piece morphologies that are independent of orientation, compact, and easy to interpret. We apply our framework to intracellular structures with punctate morphologies (e.g. DNA replication foci) and polymorphic morphologies (e.g. nucleoli). We systematically compare our framework to image-based autoencoders across several intracellular structure datasets, including a synthetic dataset with pre-defined rules of organization. We explore the trade-offs in the performance of different models by performing multi-metric benchmarking across efficiency, generative capability, and representation expressivity metrics. We find that our framework, which embraces the underlying morphology of multi-piece structures, facilitates the unsupervised discovery of sub-clusters for each structure. We show how our approach can also be applied to phenotypic profiling using a dataset of nucleolar images following drug perturbations. We implement and provide all representation learning models using CytoDL, a python package for flexible and configurable deep learning experiments.

Introduction

A central goal of cell biology is to understand the spatial and dynamic organization of the components within the cell and how their interactions contribute to cell function. Enabled by advances in imaging methods, we are now at the dawn of the big data era for cellular imaging[1-4], in which unprecedented amounts of rich image datasets can enable quantitative characterization of cellular organization and its connections with cellular phenotype.

The term cellular organization encompasses multiple aspects of a cell's configuration that must be

unpacked before further discussion. Here we focus on two of these aspects: spatial protein distributions

and shape of multi-piece intracellular structures. For example, the spatial pattern of fluorescently labeled

PCNA, representing the punctate morphology of DNA replication foci, changes throughout the cell cycle,

making it difficult to quantify due to its dynamic and complex nature. These types of spatial distributions

are usually analyzed via the texture patterns they represent, for example computing Haralick texture

features[5]. However, the biological meaning of some of these features, such as the *second angular moment*

of texture, is difficult to understand. Therefore, for spatial protein distributions, we face the challenge of

developing a robust and generalizable analysis workflow that facilitates biological interpretation.

On the other hand, major organelles or subcellular structures can often be analyzed by segmentation,

which separates the foreground signal from the background. Intracellular structures composed by a single

segmented piece, such as the cell itself or the nucleus, can then be studied via a range of features

including, among other methods, shape decomposition using spherical harmonic expansion[1,6,7]. This

approach is, however, mainly used for cell and nuclear shapes because it is limited to continuous shapes,

and does not easily apply to complex, multi-piece structures like the Golgi apparatus, which has a

discontinuous shape. In fact, most intracellular structures exhibit a polymorphic morphology consisting of

multiple pieces, which presents an additional challenge for interpretable image analysis pipelines. While

each individual piece could be segmented and measured, the entirety of the multi-piece structure cannot

be easily represented as a whole.

To overcome these two challenges, we demonstrate the use of 3D point clouds to encode biological data

in microscopy images, combined with an unsupervised *representation learning* framework for single cell

feature extraction. Representation learning is a field of machine learning that has become an increasingly

popular way to extract meaningful features directly from raw data without the need for hand-engineered

features[8,9]. These features are in the form of latent variables learned by neural networks during training,

which we refer to as *representations*.

An important aspect of the proposed learning framework is that it is generative, meaning we can switch

between the original point clouds and the single-cell representations learned, resulting in highly

interpretable features and addressing the first challenge described. A key contribution of this work is the

use of point clouds to incorporate intensity information present in large 3D images representing spatial

protein distribution in a segmentation-free manner. Another important contribution is the adaptation of the

point cloud-based approach to handle segmented multi-piece shapes. This is achieved using the concept

of signed-distance field, allowing us to generalize our framework to more complex intracellular

structures, thus overcoming the second challenge described.

The representations learned by neural networks normally depend on the orientation of an object in the

image. Despite the fact that the orientation of the cells is important in many contexts, such as when cells

are subject to shear stress, during development or direct migration, it may not be of any biological

relevance in other contexts. For example, the orientation of a cell in a monolayer colony may merely

reflect the orientation of that colony relative to the microscope stage and not anything biological.

Therefore, it would be desirable to design analysis workflows where the image orientation can be factored

out of the learned representations if appropriate. We achieved this by leveraging the notion of 3D rotation

invariance to extract features that do not depend on an object's orientation. The incorporation of

geometric information in the form of the object orientation into the representation learning process is an

example of *geometric deep learning*[10]. By using point clouds as an unifying way of encoding image data,

we are able to overcome the challenges described above and take advantage of previous implementations

for rotation-invariant feature learning[11–13] while extending their applications to quantitative cell biology.

Here, we first develop a rotation invariant representation learning framework that uses point clouds to

encode relevant information about the underlying biological data. We then use a synthetic dataset of

punctate structures to confirm that rotation invariant representations are not sensitive to data orientation

and are more compact when learned from point clouds compared to images. We show how 3D rotation

invariant features learned from point clouds can be used to recover unique morphological changes of

DNA replication foci across the cell cycle without supervision. We also explore the localization patterns

of multiple punctate structures and discover novel patterns of intracellular organization. Next, by adapting

our framework to handle more complex multi-piece structures, we systematically characterize sources of

shape variation of other major intracellular structures like nucleoli, Golgi, and lysosomes. Finally, we

demonstrate how the learned representations based on this framework can be used for detecting

morphological alterations in a nucleolar drug perturbation dataset, and for visualizing the average

phenotype for each drug to aid interpretability.

Results

A novel morphology-appropriate framework for learning 3D rotation invariant representations of

complex intracellular structures and holistic model evaluation

The 3D rotation invariant representation learning framework has two main components. The first consists

of encoding the raw single-cell image data into a point cloud. This encoding process is done in a

morphology-appropriate manner for punctate structures and polymorphic intracellular structures, such as

DNA replication foci and nucleoli, respectively. The biological meaning of shape differs between these

two types of morphologies; we focus on representing the relative location of individual pieces in punctate

structures (Fig. 1a), while both relative location and shape of individual pieces are considered important

for polymorphic structures (Fig. 1b).

The second part of the framework consists of a neural network model that consumes the generated point

clouds for learning 3D rotation invariant representations of the biological data (Fig. 1c). We define

rotation invariance using the group of all rotations in 3D. All the neural network models are designed as

autoencoders[14]: First, an *encoder network* compresses the generated point clouds into vector latent

representations. Next, these latent representations are used by a *decoder network* to reconstruct the input

data.

To evaluate the utility of the 3D point cloud encoding, we performed benchmarking against traditional

methods using neural network models trained on 3D images directly. We trained classical (rotation

dependent) and rotation invariant versions of both image- and point cloud-based models to evaluate the

impact of adding the geometric constraint of rotation invariance. We expected point cloud-based models

to outperform image-based models for two reasons. First, point clouds are a less redundant way of

representing sparse multi-piece intracellular structures compared to image-based models. Second, image-

based autoencoders often generate blurry reconstructions that can be particularly problematic for small

objects[15,16]. More details about all models used in this paper can be found in *Section 4 of Methods*.

We used a multi-metric approach to evaluate our models and representations. Our goal is to increase

transparency behind the performance of these models, and to explore trade-offs. Importantly, we hope to

identify models that are quantitatively useful across a broad set of tasks to make gaining biological insight

from the learned representations more likely, not necessarily the model that is best for any one metric..

The models were evaluated with respect to their *efficiency*, *generative capabilities* and *representation*

expressivity as detailed in Figure S1 (also in *Section 6 of Methods*).

This multi-metric evaluation approach tested different models beyond their ability to reconstruct the

input, which is the primary task of autoencoders. For example, this included efficiency metrics that

gauged the compute resources and time required to evaluate each model, including the model size,

inference time and carbon emissions. We also included generative metrics like the evolution energy

between two sample shapes (Fig. S1b). In addition, we also evaluated the models' expressivity by

quantifying the biological information content in the learned representations. Specifically, we used

representations for classifying or predicting (via regression) biologically-relevant measurements in an

application-appropriate manner (*Section 6.2 of Methods*). For example, we used the representations to

predict volume or number of pieces of a given intracellular structure when these properties are relevant

for the application. Considering all these metrics together, we quantified the holistic utility of each model

(Fig. S1c) and the pros and cons of using each approach.

To aid reproducibility and empower researchers with the ability to independently test and apply these

models to their own data, we provide all the representation learning models used in this study via

CytoDL, a Python package for configurable 2D and 3D image-to-image deep learning transformations

and representation learning, available at https://github.com/AllenCellModeling/cyto-dl. CytoDL is

designed consistent with FAIR[17] practices, and is built to work for diverse use cases, thus making it

robust, modular, and flexible to the evolving nature of research.

3D rotation invariant point cloud models are efficient, produce low rotation invariance errors and

generate good reconstructions in a synthetic dataset of punctate structures

We started by evaluating the effectiveness of 3D rotation invariance and the choice of using point clouds

to encode punctate structures using synthetic data. We used cellPACK to create a synthetic dataset of

punctate structures with known rules of organization. cellPACK generates 3D models of complex

biological environments using novel packing algorithms[18]. To create the synthetic dataset, we used

multiple spatial rules for packing spheres in real 3D nuclear shapes based on gradient algorithms (*Section*

1.3 of Methods). These algorithms include a rotationally-dependent *planar gradient* where spheres are

packed away from a plane oriented at 0-, 45-, or 90-degrees about the z-axis, and a set of three

rotationally-independent gradients including a *radial gradient* where spheres are packed with a gradient

away from the centroid of the nucleus, a *random gradient* where spheres are packed randomly, and a

surface gradient where spheres are packed close to the nuclear boundary, resulting in six total packing

rules (Fig. 2a). We packed spheres according to each of these six rules into 254 real nuclear shapes

selected from the DNA replication foci dataset (*Section 1.1 of Methods*). Importantly, the stochastic

nature of the packing algorithm generates heterogeneity in the distribution of spheres across the simulated

nuclei, thus making the recovery of rules via unsupervised learning more challenging, and evaluating the

robustness of each representation learning method.

Since 3D rotation is an important variable associated with the planar rules, we expected 3D rotation

invariant models to give us the most compact representations by factoring out this variable. We

additionally hypothesized that point cloud models (Fig. 1a) would provide better representations than

image models since they better describe the punctate nature of the synthetic data represented by the

centroid of the packed spheres (Fig. 2a). To test this hypothesis, we trained two classical and two 3D

rotation invariant models using images and point clouds as input data respectively (*Sections 2.1, 3.1, and*

4 of Methods).

We found that point clouds were more efficient across all efficiency metrics (Model size, Inference time

and Emissions in Fig. 2b). In addition, point clouds also produced better reconstructions ("Reconstruction

error") and had low evolution energy scores, meaning that the interpolations between two shapes are

smooth. Next, we tested whether the implementations of the rotation invariant models were indeed

generating representations that were not sensitive to orientation of the input data. We confirmed that this

was the case as shown by similar orientation of reconstructions in the last row of Figure S2. We found

that rotation invariant representations from point clouds were more compact using the Levina-Bickel

intrinsic dimensionality metric ("Compactness")[19] and had much lower rotation invariance errors

compared to its image-based counterpart (Fig. 2b, *Section 6 of Methods*). All four models were able to

reconstruct the unique morphologies associated with each packing rule (Fig. S3). However, we found that

representations from both rotation invariant models were slightly worse than their classical counterparts at

classifying the six rules ("Rule classification" in Fig. 2b). This was an expected outcome since rotation is

an important distinguishing feature of the planar rules and rotation invariant representations are

insensitive to this feature. Overall, the 3D rotation invariant point cloud model was an efficient generative

model that learned compact and orientation-independent representations for synthetic punctate structures.

Having established the holistic utility of the rotation invariant point cloud model on synthetic data, we

next performed principal component analysis (PCA; S*ection 7.1 of Methods*) on the learned

representations using this model to interpret their meaning. We performed this PCA on a version of this

model trained with jitter augmentations to improve reconstruction quality (Fig S4; also see jitter details in

Section 5.2 of Methods). By visualizing the first principal component (PC1) of the reconstructions for

each rule via a latent walk, we found that PC1 recovers how nuclear size affects each rule's packing (Fig.

2c). Notably, the rotation invariant reconstructions for all planar rules are aligned in the same direction,

allowing us to focus on the subtle differences in spatial distribution between 0- and 45-degrees

orientation.

Next, we performed an archetype analysis[20] to find extremal points in the representations of the synthetic

dataset (*Section 7.2 of Methods*). Archetypes are determined so that observations can be approximated by

convex combinations of the archetypes. By setting the number of archetypes to six, we found each

archetype represented one of the six rules used in cellPACK to generate the synthetic dataset. These

results show that the obtained point cloud rotation invariant representations can enable unsupervised rule

discovery for a synthetic dataset of punctate structures.

Rotation invariant point cloud representations recover cell cycle-dependent spatial patterns of DNA

replication foci

After establishing its applicability to synthetic data, we tested the representation learning framework on a

real single-cell image dataset of punctate structures for biological discovery and hypothesis generation.

The dataset contains single-cell images of DNA replication foci in hIPS cells expressing fluorescently

tagged PCNA (N=2,420, *Section 1.1 of Methods*). DNA replication foci are punctate and display a

continuous change in their overall localization pattern and intensity throughout cell cycle[21] (Fig. 3a). Due

to tagged PCNA fluorescence intensity being an important source of variation for DNA replication foci

patterns, we adapted the point cloud sampling strategy so that the raw image intensity is treated as a

fourth coordinate, in addition to the XYZ spatial coordinates (Fig. S5a-b, and *Section 3.1.2 of Methods)*.

This additional coordinate ensures that intensity information is captured in the learned representations

without impacting the rotation invariance of the XYZ spatial coordinates (Section 4.2 of Methods).

To test whether the representations learned with this data capture biologically relevant features about

DNA replication foci localization, we manually classified each single-cell image in this dataset into one

of eight cell-cycle stages based on the spatial pattern of PCNA (*Section 1.1 of Methods*). We also

manually labeled cells as outliers if they were dead, dying, or did not express PCNA. Next, we used the

representations learned by each of the four models to benchmark their performance on various tasks,

including classification of cell cycle stages and detection of outliers from the DNA replication foci

dataset.

We found that point cloud models were more efficient but not as compact as the rotation invariant image

model (Fig. 3b). Point cloud models also provided better overall reconstructions compared to image

models (compare reconstructions in Fig. S6f-h to Fig. S6b-c). Despite the poor reconstruction of both

image models (Fig. S6b-c), we found that the rotation invariant image model was the best at classifying

cell cycle stages ("Cell cycle classification" in Fig. 3b, 81% accuracy vs 80% accuracy for the best point

cloud model). This result in particular demonstrates the limits of evaluating models using a single metric

alone. We confirmed that poor reconstructions of image models were not due to dataset size or image

normalization issues using an alternative approach (Fig. S6d and *Section 4.1.1 of Methods*). We also

found that point cloud models had slightly better performance detecting outliers compared to image-based

models ("Outlier classification" in Fig. 3b, ~100% accuracy vs 98% accuracy for the best image model).

Finally, we found that the rotation invariant point cloud model had lower rotation invariance error scores

compared to its image counterpart. Overall, the results elucidate the challenge of reconstructing sparse

intracellular structures using classical image autoencoders[24], and highlight the good performance of the

rotation invariant point cloud representations across many metrics evaluated for the DNA replication foci

dataset.

To interpret the representations learned by the rotation invariant point cloud model per cell cycle stage,

we performed PCA on this dataset stratified by the manual annotations of cell cycle stages. A latent walk

along PC1 for each cell cycle stage revealed some overlap in the morphology and intensity of DNA

replication foci between neighboring cell cycle stages (earlyS $\sigma=2$ and earlyS-midS $\sigma=-2$, for example).

This highlights the inherent uncertainty that is present in the task of manual annotation of a continuous

process in discrete classes.

We found that an archetype analysis with eight archetypes was able to recover expected cell cycle

patterns of DNA replication foci (order of archetypes in Fig. 3d resembles examples in Fig. 3c for $\sigma=0$).

The archetypes capture three main sources of variation in the dataset as expected. First, overall nuclear

shape which is mainly represented by archetypes displaying different elongations. In addition to nuclear

shape, the intensity and localization of DNA replication foci are significantly different between

archetypes. These two sources of variation seemed correlated as we observe in real PCNA images. Lastly,

the spatial pattern of PCNA changes from a dim signal uniformly distributed in the nucleus at G1 to

compact bright spots well localized in lateS.

Next, we asked if the learned representations could be used for interpreting the evolution of the DNA

replication foci spatial pattern as a function of the cell cycle stage. To test this, we binned nuclear volume

into five bins to create a pseudo cell cycle axis (*Section 7.3 of Methods*), and we averaged the rotation

invariant point cloud representations of all cells within each bin (Fig. 3e). We observed a continuous

orientation-invariant transition that recapitulates the coalescence of uniformly distributed dim DNA

replication foci puncta in G1 into a few bright dots in late S. This pattern is followed by signal

sparsification into uniform dim punctate again in G2. We observed a moderate distinction between spatial

patterns in Figure 3e, which suggests that nuclear size is only a weak proxy for cell cycle stages. These

results demonstrate that the learned point cloud rotation invariant representations can recover the overall

behavior of DNA replication foci in an unsupervised manner.

Representation learning framework reveals interpretable spatial patterns for other punctate

structures from the WTC-11 hiPSC Single-Cell Image Dataset v1

To assess whether our approach would generalize to other intracellular structures with punctate

morphology, we analyzed a larger dataset of punctate structures from the WTC-11 hiPSC Single-Cell

Image Dataset v1 (*Section 1.2 of Methods*). This dataset comprises centrioles (N=7.575), peroxisomes

(N=1,997), endosomes (N=2,601), nuclear pores (N=17,703), nuclear speckles (N=2,980), cohesins

(N=2,380), and histones (N=15,875). Examples of these structures are shown in Figure 4a. Once again,

we trained classical and rotation invariant image- and point cloud-based models on this larger dataset (Fig

S5c, *Sections 2.1 and 3.1 of Methods)*. In addition to the usual set of evaluation metrics, we tested the

applicability of the learned representations for two classification tasks. The first task focused on

identifying the specific intracellular structure from the seven options available in the dataset. The second

task involved classifying cell-cycle stages (interphase or mitosis) based on the annotations provided

within the dataset (*Section 6.2 of Methods*).

Overall, we found that classical image-based models provide better reconstructions when trained with the

combination of these seven different punctate structures relative to what we observed for the same models

trained on DNA replication foci dataset alone (Fig. S7a-b). Despite this improvement in reconstruction,

we noticed that the classical image-based model poorly reconstructs some of these structures including

centrioles, peroxisomes, and endosomes (see blurry reconstructions in Fig. S7b). In addition, we found

that imposing rotation invariance further deteriorates reconstruction of image models across all structures

(Fig. S7c). Once again, an alternative approach confirmed that poor reconstruction was not due to dataset

issues (Fig. S7d and *Section 4.1.1 of Methods*).

Both classical and rotation invariant point cloud models produced more accurate and comparable

reconstructions, but with spatial distribution artifacts for structures with fewer training samples, like

endosomes and peroxisomes, similar to the classical image model (as shown by arrows in Fig. S7f-g). In

addition to providing improved reconstructions compared to image models, we found that the rotation

invariant point cloud representations performed well at both structure classification (~95% accuracy vs

90% accuracy for best image model) and cell stage classification (~58% accuracy vs 57% accuracy for

best image model), while being more compact and orientation independent (Fig. 4b).

We then analyzed rotation invariant point cloud representations by computing the first principal

component (PC1) for each structure for cells in interphase. To do this, we sampled real single-cell images

along PC1 for each structure as shown in Figure 4c. Top and side views in Figure 4c suggest the major

source of variation across all seven intracellular structures in this dataset appears to be aspects of cell and

nuclear shape like height and elongation. Next, we noticed that centrioles are localized near the nucleus at

one extreme of PC1 and gradually migrate towards the cell membrane at the other extreme (column

"Centrioles" in Fig. 4c). Additionally, we observed nuclear speckles to be more uniformly distributed

within the nucleus at one extreme of PC1 and more concentrated near the nuclear shell forming a ring-like

pattern at the other extreme of PC1. Surprisingly, latent walks along PC1 revealed similar patterns as

additional sources of variation for these two structures (Fig. 4d). We found that centrioles polarize by

moving away from the cell center (represented by a dark cross in Fig. 4d), and nuclear speckles

concentrate in a ring-like pattern (last row of column "Nuclear speckles" in Fig. 4d). Overall, these results

highlight the ability of our rotation invariant point cloud representations to capture meaningful and

biologically relevant variations in the spatial pattern of multiple intracellular structures.

Rotation invariant representation learning framework generalizes to polymorphic multi-piece

structures

We next asked if we could adapt our approach to learn 3D rotation invariant representations for non-

punctate intracellular structures, such as nucleoli and Golgi apparatus. These organelles are polymorphic

structures where the shape of individual pieces, in addition to the location of these pieces, may be

important for the underlying biological process and therefore should be captured by the learned

representations[22,23]. We combined the point cloud approach with a signed distance field (SDF[11,24,25])

computed from segmented images, which is a function that represents the signed distance of a position to

the nearest part of a shape, to incorporate the shape information of individual pieces into the

representation learning framework (Fig. S8 and *Section 3.2 of Methods*). To make this possible, a few

modifications had to be made to the autoencoder architecture. First, while the encoder was left

unchanged, its input point clouds are now sampled from segmented images instead of raw data,

specifically from the surfaces of each of the multi-piece substructures. Next, a second point cloud is

sampled from the 3D segmentation volume, and each point is assigned to its local SDF value. We

replaced the point cloud decoder with an implicit decoder, which takes the latent representation and

spatial coordinates of each point's location in the second point cloud as input, to reconstruct SDF values
at each spatial location (Fig. 1b). Thus, both the encoder and decoder represent the 3D shape in different
formats, allowing the model to capture complex shape information using SDFs, while remaining less
sensitive to orientation changes. Additionally, for the reasons described in Section 4.2.2 of Methods, we
had to relax the generative requirement of the model to make the generalization to polymorphic structures
possible.
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We applied the adapted framework to images of the granular component (GC) of nucleoli via
fluorescently-tagged nucleophosmin (NPM1, N=11,814, Fig. 5 and *Section 1.2 of Methods*) which are
part of the WTC-11 hiPSC Single-Cell Images Dataset v1[1]. Nucleoli are multi-compartment condensates
that exhibit a broad distribution in both the number of pieces and size[26] and exhibit rapid rotation in 3D
[27,28]. Given these properties, we expected that 3D rotation invariant representations learned using an
implicit definition of the nucleolar surface via an SDF would be more interpretable than representations
learned by classical models directly from segmented images. To evaluate this, we trained two classical
image models using segmentations and SDFs, two 3D rotation invariant image models using
segmentations and SDFs, and one 3D rotation invariant point cloud model using SDFs (*Sections 2.2, 3.2*
of and 4 of Methods). Examples of inputs and outputs of each of these models can be seen in Figure S9.
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We found that the two classical image models based on segmentations and SDFs, and the 3D rotation
invariant point cloud model generate similar quality reconstructions ("Reconstruction error" in Fig. 5b).
However, the point cloud model was less efficient in terms of emissions and inference time (Fig. 5b). We
also found that 3D rotation invariant image models produce lower quality reconstructions compared to
classical image models, as we had observed for models trained on punctate structures (see *Section 6.1 of*
Methods for details on how reconstruction error was computed for each model). The results also indicate
rotation invariant representations from point clouds are more orientation-independent compared to
representations learned from both segmentations and SDFs ("Rotation invariance error" in Fig. 5b). Next,
we asked which representations would capture more relevant morphological attributes of nucleoli. To
answer this question, we used the learned representation to classify the number of nucleolar pieces in the
segmented images and to predict the size, surface area and relative distance between pieces (*Section 6.2 of*
Methods). We found that rotation invariant point cloud representations performed best on all of these
tasks ("Classification of number of pieces", "Average feature regression" and "Average distance
regression" in Fig. 5b), suggesting that these representations contain relevant biological information.
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Next, we used PCA on data grouped by number of pieces to interpret the rotation invariant point cloud

representations. Since we had to relax the generative capabilities of this model to achieve rotation

invariance, we retrieved the closest real cells while performing a latent walk of PC1 (Fig. 5c) and PC2

(Fig. 5d). We found height and elongation to be the major source of variation for single piece-nucleoli

(~30% of the examples in the dataset; N=3,499, explained variance of PC1was 16% and PC2 was 7%).

This was confirmed by computing Pearson correlation with structure elongation (r=0.56 for PC1, r=0.05

for PC2) and height (r=0.13 for PC1, r=0.01 for PC2). In the remaining 70% of the dataset (N=8,315),

where nucleoli consist of multiple pieces, the predominant source of variation appears to be the distance

between pieces and the relative size of these pieces. For example, when considering nucleoli composed of

two pieces, we observe PC1 (explained variance was 19%) to represent the height of the larger piece and

the size of the small piece (Fig. 5c). In addition, we found both PC1 and PC2 to correlate with the average

distance between pieces (r=0.42 for PC1, r=0.17 for PC2). By performing an archetype analysis with five

archetypes, we found that three archetypes represent nucleoli with a single piece but different elongations

(archetypes 1-3 in Fig 5e). Archetype four represents nucleoli with one large piece and one small piece,

which is a common configuration in the dataset, and archetype five represents nucleoli fragmented in

many small pieces.

Motivated by previous observations of cell cycle-dependent nucleolar morphology[29], we asked whether

rotation invariant representations would capture nucleolar changes as a function of the cell cycle. We

again used nuclear volume bins to create a pseudo cell cycle axis and visualized the closest real example

to the average representation within each bin (*Section 7.3 of Methods*). Consistent with previous

observations[29], we found that cells exiting division (small nuclear volume) have nucleoli that are

fragmented into multiple pieces that coalesce into a single large piece as the cells grow and progress

towards mitosis (Fig. 5f). Altogether, the results show that this representation learning framework can be

successfully adapted using SDFs to polymorphic structures and that it provides representations that

capture relevant aspects of the nucleolar biology.

Learned representations enable shape variation profiling on multiple polymorphic structures

We next expanded this method to profile multi-piece shape variation on the granular component (GC) of

nucleoli (N=11,814) and three other polymorphic structures. To do this, we analyzed a subset of

polymorphic structures from the WTC-11 hiPSC Single-Cell Image Dataset v1 (*Section 1.2 of Methods*).

This subset comprised the dense fibrillar component (DFC) of nucleoli (N=9,923), lysosomes

(N=10,114), and Golgi (N=6,175). Representative images of each structure are shown in Fig. 6a. We

applied the SDF point cloud rotation invariant representation learning framework to this dataset of four
different polymorphic structures, and once again benchmarked it against classical and rotation invariant
segmentation-based and SDF-based image models (Fig. S8). We focused here on learning scale invariant
representations by scaling all intracellular structures on a cell by cell basis (*Section 2.2.2 of Methods*).
While this scaling strategy prevents us from comparing sizes across different intracellular structures, it
helps preserve the resolution of structures occupying only a few voxels. Examples of inputs and outputs
of each of the models used can be seen in Figure S10.

We again observed that the rotation invariant point cloud representations continued to display the lowest
rotation invariance errors of any model tested (Fig. 6b). We found that all rotation invariant models were
more compact than their classical counterparts. We also found that both rotation invariant image models
were worse at reconstruction and evolution energy than their classical counterparts (Fig. 6b).
Interestingly, all models were able to classify structure equally well ("Structure classification" in Fig. 6b).
This suggests that the morphology of these structures is distinct enough to be easily discerned using
unsupervised learned representations, despite all being referred to as "polymorphic".

We performed PCA on rotation invariant representations learned for each of the four intracellular
structures and visualized the first two principal components (Fig. 6d-e). We identified features like overall
elongation and number of pieces as primary sources of variation for both components of nucleoli, which
are known to be nested via liquid-liquid phase separation[30]. In the case of lysosomes, we observed PC1
and PC2 to be related to aspects of cell shape. This was further confirmed by computing Pearson
correlation between PC1 (explained variance ~15%) and cell elongation (r=-0.42), and PC2 (explained
variance ~8%) and cell height (r=0.28). PC1 also displayed strong correlation with the average distance
between lysosome pieces (r=-0.45 for PC1), suggesting that the first principal component also captures
information about lysosome spatial clustering. In the case of Golgi, the first two principal components
(explained variance ~13% for PC1 and 9% for PC2) appeared to capture aspects of elongation and
fragmentation. This was corroborated by computing correlations with structure elongation (r=-0.61 for
PC1 and r=0.06 for PC2), the average distance between Golgi pieces (r=-0.55 for PC1 and r=0.27 for
PC2), and number of pieces (r=0.14 for PC1 and r=0.05 for PC2). An archetype analysis with four
archetypes retrieved two granular component (GC) nucleolar morphologies that represent aspects of
elongation (Archetype 1) and number of pieces (Archetype 2). The other two archetypes were examples
of the dense fibrillar component (DFC; Archetype 3) of nucleoli and the Golgi (Archetype 4).

Overall, representations from all SDF-based models were able to regress information about distances

between pieces better than segmentation-based models, whereas the rotation invariant representations

were able to regress information about volume and area of pieces better than classical models. While the

point cloud rotation invariant model performed well in generation and representation expressivity metrics,

no single model excelled across all metrics. This observation suggests that application-appropriate model

selection is key for achieving optimal results. For example, if the goal of an analysis is to learn

representations that are not sensitive to rotations, then the rotation invariant point cloud approach would

be most appropriate, but if the goal is to learn a model that reconstructs the data well, then a classical

segmentation model would be most appropriate in this case.

Evaluating drug effects on nucleolar morphology using unsupervised representation learning

We then proceeded to test the applicability of the representation learning approach to a perturbation

detection task using a drug screening dataset. We imaged WTC-11 hiPS cells expressing an

endogenously, fluorescently tagged nucleophosmin, representing the granular component of nucleoli.

Cells were treated with 16 different drugs at relatively low concentration to induce subtle phenotypic

alterations (*Section 1.4 of Methods*). Analysis was conducted on cells imaged two hours after treatment.

We used the representation learning framework to extract unsupervised representations for cells in the

dataset (N=1,025). To do so, we fine-tuned the models trained on the dataset of nucleolar (GC) single-cell

images described in the section "Rotation invariant representation learning framework generalizes to

polymorphic multi-piece structures". We followed the methods described in Chandrasekaran et al.[2] to

evaluate the performance of these fine-tuned models. To do so, we computed the *mean average precision*

to measure how distinguishable different single cells of a drug-treated set are from untreated cells

(DMSO; N=140), and a q-value statistic based on permutation testing.

The results are summarized in Figure 7a, where we plot the q-value per drug for each model. Drugs with

q-value under the significance threshold of 0.05 are considered by that model as causing significant

alterations in nucleolar morphology. Aside from the first two drugs, we found a significant difference in

the behavior of segmentation- and SDF-based models. Therefore, we sorted the x-axis from low to high

average q-value based on all SDF models. Consequently, drugs on the left side of the plot induce a

stronger phenotypic change compared to drugs on the right side. Figure 7b shows a table with more

details about each drug, such as name, concentration, molecular target or mechanism of action, effect

based on literature review and effect observed on nucleoli based on visual inspection of this drug dataset.

Representative examples of the range of phenotype of each drug are shown in Figure 7c.

The first drug to appear on the x-axis of Figure 7a is Actinomycin D, indicating that this drug is the one

with the strongest effect on nucleolar morphology. This drug works as a control in this analysis since it is

the only well characterized drug in this study that is known to target the DNA and cause significant

alteration in nucleolar size, as described in Figure 7b and shown in Figure 7c. Next, we found

Staurosporine to have the second strongest effect. A visual inspection of images of cells treated with this

drug reveal the presence of many dead cells where nucleoli display very abnormal morphology (Fig 7c).

Both Actinomycin D and Staurasporineno were identified as being distinguishable from the control

(below q=0.05 threshold) by all models.

No other drug was identified by either the classical or rotation invariant image-based segmentation

models. On the other hand, the SDF based models identified several other drugs that could be associated

with off-target effects. Starting from the left side of the plot in Figure 7a, these models next identified

Paclitaxel and Nocodazole, which were associated with cell death 24 hours after treatment, while also

locking cells in mitosis (Fig. 7b-c). SDF-based models also detected Jasplakinolide which is known to

promote actin polymerization and is associated with higher nuclear volumes[31]. Visual inspection of the

image data did not reveal any apparent nucleolar alteration (Fig. 7c) or change in cell health within 24

hours after treatment, suggesting that the effect of this drug is subtle. Next, Torin was detected by all SDF

models, which was observed by visual inspection to induce cell death, like Staurosporine. Lastly,

Rascovitine at 10μM was detected only by the SDF rotation invariant models. While there was no visible

alteration of nucleoli morphology in this dataset at this concentration, Roscovitine at 10μM caused cell

death at 4 hours in images of a different dataset of H2B-tagged cells (Histones, data not shown), and is

known to cause nucleolar segregation[32,33] at higher concentrations. These results suggest that our

representation learning framework captures subtle concentration-dependent phenotypes that are not

visible by eye. Overall, we observed three different categories of drugs that were retrieved using different

models; "sledgehammer" phenotypes that were detected by all models, subtle off-target effects that were

detected by SDF models, and subtle concentration-dependent phenotypes that were detected only by the

rotation invariant SDF models. The remaining nine drugs in the dataset did not induce visible alterations

to nucleolar morphology, although four of them induced cell death at later time points, including H89,

Chloroquine, Rotenone and Brefeldin.

Visualization of the closest real example to the average representation using the point cloud model

emphasized that Actinomycin D, Staurosporine and Torin-treated nucleoli were visually distinct from the

control, confirming that nucleolar morphology is strongly affected by these drugs (Fig. 7d). These results

illustrate that our 3D representation learning approach can enable perturbation detection and phenotype

profiling, and indicate the need for follow up experiments to confirm the impact of some of these drugs,

like Jasplakinolide, on nucleolar morphology.

Discussion

In this paper, we developed a morphology-appropriate 3D rotation invariant representation learning

framework for multi-piece intracellular structures using point clouds. We benchmarked this framework

against classical and rotation invariant image-based models using a novel multi-metric evaluation

criterion that focuses not only on traditional reconstruction quality but also on measurements that can be

important for downstream analysis and biological discovery. We found that our morphology-appropriate

approach can be more efficient, can produce meaningful reconstructions, and can learn more compact and

expressive representations across a range of tasks than classical image and point cloud models. We

applied this framework to synthetic and real single-cell image datasets for punctate structures, like DNA

replication foci, and polymorphic structures, such as nucleoli. Our results reveal that geometry-aware

choices of encodings and neural network architectures can enable unsupervised discovery and

interpretation of variability in the morphology of several multi-piece intracellular structures.

We first demonstrated this principle by recovering the rules used to create a synthetic dataset of punctate

structures using cellPACK in a fully unsupervised way. We then recovered expected morphological

patterns for different intracellular structures using unsupervised methods, such as archetype analysis and

PCA, and by identifying biologically interpretable localization patterns for structures, like DNA

replication foci and centrioles. Interestingly, the good performance achieved by different models in

classifying cell cycle stages using PCNA suggests that manual annotation of this information can be

replaced or assisted by unsupervised representation learning. In addition, the learned representations for

centriole capture its repositioning from the cell center towards the periphery, a behavior which is known

to be mediated by the microtubule network[34,35]. The learned representations also recapitulate a known axis

of morphological change of nuclear speckles, which goes from many, small, irregularly shaped speckles

to larger, rounder shaped speckles. This is known to occur when transcription is inhibited in cells and is

also the primary axis of variability between cell types[36].

We further showed how the framework can be extended to polymorphic structures and characterized the

full complexity of nucleolar shape variation as an example. When applied to a larger dataset of

polymorphic structures, the unsupervised representation learning revealed interesting spatial organization

aspects of these structures. For example, Golgi fragmentation captured by the first two principal

components of rotation invariant representations is often described as an important morphological feature

under both normal and physiological conditions, and has connections with different signaling pathways[23].

However, we observed a trade-off when using the rotation invariant point cloud model where the model

produced good reconstructions and learned expressive representations, but was inefficient compared to

image models, potentially limiting its ability to scale to much larger datasets and its utility to users with

limited computational resources.

Finally, we evaluated the utility of our approach on phenotypic profiling of a nucleoli perturbed image

dataset and demonstrated the interpretability of the learned representations. In general, we noticed that

segmentation-based models were not able to detect drugs like Torin-2 that caused clear alterations on

nucleolar morphology as a result of cell death. This result suggests that SDF encodes information relevant

for perturbation detection. It is important to note that our assessment of cell health based on the number of

dead cells in the FOV does not distinguish between different processes by which cells can die. This could

be a possible explanation for why some drugs that induce death were detected as being distinguishable

from the control, while others were not. A larger drug screening single-cell image dataset is necessary to

comprehensively validate our findings and evaluate the robustness of our approach.

In this work, we focused the experiments on 3D confocal images of intracellular structures in the hiPS

cell, but our framework is general and can be applied to other cell types and imaging modalities. For

example, single-molecule localization microscopy (SMLM) naturally generates large 3D point cloud data

for single molecule interaction at the 10-20 nm resolution range[37]. Similarly, imaging-based spatial

transcriptomics methods that measure a small subset of genes using single-molecule fluorescence in-situ

hybridization (smFISH) technologies typically generate point locations of RNA localization[38], and

antibody-based immunofluorescence imaging methods generate protein localization maps in 3D systems

such as organoids[39,40]. These datasets are often quantified using point cloud clustering analysis and could

benefit from unsupervised representation learning frameworks such as the one proposed here.

Many current techniques for analyzing single-molecule localization microscopy operate in two

dimensions, forcing researchers to project their 3D data into 2D for analysis purposes[41,42]. This trend is

prevalent in cell biology and medicine, where numerous publicly accessible datasets are predominantly in

2D[2,3,43]. The development of 3D analysis frameworks combined with availability of 3D data, can mitigate

barriers to conducting comprehensive 3D analyses across diverse imaging modalities. This, in turn,

reduces the risk of artifacts and allows analysis to be centered on the native data formats.

Previous studies have introduced unsupervised representation learning approaches for cell images using

autoencoders with geometric deep learning[44,45]. Our work complements these approaches in three ways:

first, by incorporating the notion of orientation invariance into our intracellular structure morphology-

dependent framework for representation learning; second, by providing a systematic multi-task

benchmark to evaluate the utility of each model that goes well beyond traditionally assessed

reconstruction quality; third, by focusing our analysis on 3D multi-piece intracellular structures with

complex morphology and spatial distribution. Our approach was tested across datasets of different sizes

commonly obtained in typical single cell imaging studies, ranging from a few hundred to tens of

thousands samples, thus providing a guideline for the quality of unsupervised representations in different

cases. For instance, we observed poor image-based reconstructions on small datasets like the DNA

replication foci dataset. This observation might be closely related to our deliberate choice of using vanilla

autoencoders throughout our analysis, and many modifications of autoencoders have been shown to

improve different aspects of reconstruction[46,47], disentanglement[44,48,49], causal inference[50,51], and

dynamics[52,53].

Our framework can be further improved in multiple ways. For example, our results indicate cell and

nuclear shape are major sources of variation because that information was not factored out of our learning

framework and therefore, become confounding variables. While this reflects a true coupling between cell

and nuclear shape and structure localization, alternative approaches may offer a way to decouple these

confounding variables from learned representations. For instance, one could incorporate reference

information about other intracellular structures for answering questions about intracellular structure co-

localization[54]. Another possibility for improving our framework could be adding mutual exclusivity rules

to further constrain the models and move towards a better understanding of compartmentalization[15].

Finally, given the limited ability to simultaneously tag and visualize multiple structures[55,56], extending our

framework to predict spatial patterns of a set of structures given the representation from another set could

help build a holistic description of intracellular organization. The morphology-appropriate representation

learning using point clouds and SDFs that we have described here is flexible and can be modified to

incorporate several such improvements.

In summary, we have begun to develop a computational analysis pipeline for interpretable representation learning of complex multi-piece intracellular structures. An important goal of this work is to make the data, models and analysis tools freely available to the community, so that it can serve as a benchmark for further methods development for 3D analysis. We hope that this work can spur the interest of the cell biology community into new ways of analyzing and interpreting complex intracellular organization.

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Author contributions

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Declaration of interests

The authors declare no competing interests.

Methods

1. single-cell Image Datasets

1.1 DNA replication foci dataset

Spinning-disk confocal 3D images taken of a fluorescently tagged cell line that targets proliferating cell nuclear antigen (PCNA) labeling DNA replication foci with mEGFP were processed to create the DNA replication foci dataset[57]. Fluorescent cell-membrane and DNA dyes tagged the cell boundary and nucleus, respectively. Nuclear segmentations were obtained using the protocol described in Viana et al. [1], with the only difference being that nucBlue dye was replaced with nucViolet dye. Segmentations of DNA replication foci were generated for each FOV, using three different segmentation workflows created using the *Allen Cell & Structure Segmenter*[58] to segment specific DNA replication foci morphologies. Next, we visually identified which segmentation workflow was best for each cell and saved the result in an empty FOV at that cell's correct location. More details about the dataset and images are available here https://open.quiltdata.com/b/allencell/packages/aics/nuclear_project_dataset_4.

Cells in interphase were labeled by an expert as belonging to one of nine classes - G1, earlyS, early-midS, midS, midS-lateS, lateS, lateS-G2, G2, and unclear. Unclear labels were dropped during analysis. About 3% of cells were labeled as outliers based on bad segmentations of DNA replication foci, cells appearing dead or dying, no EGFP fluorescence, and bad segmentations of cells and nuclei. Dead cells and no fluorescence were used for the outlier detection task, accounting for 16 cells out of a total of 2,420 cells.

1.2 WTC-11 hiPSC Single-Cell Image Dataset v1

Spinning-disk confocal 3D images taken from 25 endogenously tagged hIPS cell lines were processed to create the WTC-11 hiPSC Single-Cell Image Dataset v1[1]. Fluorescent cell-membrane and DNA dyes tagged the cell boundary and nucleus, respectively. Cell, nuclear and structure segmentations were used as provided in the dataset release available here https://open.quiltdata.com/b/allencell/packages/aics/hipsc_single_cell_image_dataset.

We performed analysis on histones via H2B (N=15,875) , nuclear pores via Nup153 (N=17,703), peroxisomes via PMP34 (N=1,997), endosomes via Rab-5A (N=2,601), centrioles via centrin-2 (N=7,575), cohesins via SMC1A (N=2,380) and nuclear speckles via SON (N=2,980) as selected punctate structures from this dataset. We selected nucleoli (DFC) via fibrillarin (N=9,923), nucleoli (GC) via nucleophosmin (N=11,814) , lysosomes via LAMP-1 (N=10,114), and Golgi via sialyltransferase (N=6,175) as polymorphic structures. While we used all single-cell images for training our models, we limited our analysis to interphase cells.

1.3 cellPACK synthetic single-cell dataset

We used cellPACK to create synthetic point clouds within real nuclear shapes[18]. cellPACK provides an algorithm to create high-resolution 3D representations of the biological mesoscale based on specified rules. Segmentation of 254 randomly chosen nuclei from the DNA replication foci dataset were converted into a triangulated mesh and used as input to cellPACK. cellPACK then packed 256 spheres with a radius of 1 voxel within these meshes based on four distinct rules. **Random**: points were generated uniformly at random inside the nucleus; **Planar gradient rule**: points were generated inside the nucleus with a bias away from a plane. The plane contains the centroid of the nucleus, and its orientation is specified by a normal vector. We used normal vectors with three different orientations: i) $\theta=0$, the normal vector points along the z–axis $(0x + 0y + 1z)$ where the longest axis of the nucleus is the y–axis. ii) $\theta=45°$, the normal vector is $(0x + 1/\sqrt{2}y + 1/\sqrt{2}z)$. iii) $\theta=90°$, the normal vector points along the y–axis; **Surface gradient rule**: points were generated with a strong bias towards the nuclear surface **Radial Gradient**: points were generated with a bias towards the centroid of the nucleus. For each rule, cellPACK generated a point cloud with 256 points for each nucleus shape. This dataset is available for download at https://open.quiltdata.com/b/allencell/tree/aics/morphology_appropriate_representation_learning/cellPACK_single_cell_punctate_structure/

 1.4 Drug dataset

A collection of well-characterized drugs was used to perturb the Allen Institute for Cell Science cell line

AICS-50 (WTC-11 hiPSC endogenously tagged for mEGFP-NPM1, tagging nucleoli (GC)). Drugs and

concentrations were selected because cell treatment with each of them induced a well-characterized effect

on one major cellular structure morphology that could be visually observed within 24 hours of treatment

(Table 1) and was not associated with massive cell death within the first two hours of treatment, except

for Jasplakinolide. Cells were seeded on a 96-well glass bottom plate using the protocol described in

Gregor et al.[59]. Four days post seeding 2D brightfield low magnification well overviews were acquired

and used for position selection following the same criteria as described in Viana et al.[1]. Following

position selection cells were washed once with pre-warm phenol red-free mTeSR then media was

replaced with drug-containing phenol red-free mTeSR media at the indicated concentration (Supp table

X). The cells were then placed back on the spinning-disk confocal microscope stage where they were

maintained at 37°C with 5% CO2 for two hours before the start of imaging at high magnification (120X).

Images were acquired with three identical ZEISS spinning-disk confocal microscopes with 10X/0.45 NA

Plan-Apochromat (for well overview and position selection) and 100X/0.8 NA Plan-Apochromat (Zeiss)

(for high-resolution imaging) and ZEN 2.3 software (blue edition; ZEISS). The spinning-disk confocal

microscopes were equipped with a 1.2X tube lens adapter for a final magnification of 12X or 120X,

respectively, a CSU-X1 spinning-disk scan head (Yokogawa) and two Orca Flash 4.0 cameras

(Hamamatsu). 3D FOV image stack acquisition was performed with two cameras allowing for

simultaneous acquisitions of a brightfield and an mEGFP (excited with 4.6mW of a 488nm laser) channel.

Exposure time was 100ms. The resulting images were of 16 bits and 924x624 pixels^2 in xy dimension

after 2x2 binning. FOVs had a final xy pixel size of 0.108 μm and z-stacks composed of 100 z-slices (to

encompass the full height of the cells within an FOV) acquired at a z interval of 0.29μm. Transmitted

light (bright-field) images were acquired using a red LED light source with a narrow range peak emission

of 740 nm and a BP filter of 706/95 nm for bright-field light collection. A Prior NanoScan Z 100 mm

piezo z stage (ZEISS) was used for fast acquisition. Optical control images of field of ring (Argolight)

and dark current were acquired daily at the start of each data acquisition to monitor microscope

performance. Laser power was measured monthly and the corresponding percentage was adjusted to

consistently expose the sample to the same laser power. This dataset is available for download at

https://open.quiltdata.com/b/allencell/tree/aics/NPM1_single_cell_drug_perturbations/

Drug	Manufacturer	ID	Concentration(s)
Actinomycin D	Selleckchem	S8964	0.5μg/mL

BIX 01294	Selleckchem	S8006	1μM
Bafilomycin A	Millipore Sigma	SML1661	0.1μM
Brefeldin	Selleckchem	S7046	5μM
Chloroquine	Medchemexpress	HY-17589A	40μM
H89	Selleckchem	S1582	10μM
Jasplakinolide	Millipore Sigma	420127	50nM
Latrunculin A1	Millipore Sigma	428026	0.1μM
Monensin A	Medchemexpress	HY-N0150	1.1μM
Nocodazole	Medchemexpress	HY-13520	0.1μM
Paclitaxel	Selleckchem	S1150	5μM
Rapamycin	Selleckchem	S1039	1μM
Roscovitine	Selleckchem	S1153	5μM and 10μM
Rotenone	Medchemexpress	HY-B1756	0.5μM
Staurosporine	Selleckchem	S1421	1μM
Torin-2	Selleckchem	S2817	1μM

Table 1 - List of drugs and concentrations used in the perturbed nucleolar GC dataset

1.4.1 Cell health assessment: We assessed cell health at 4 and 24 hours after drug treatment using for

each drug both the AICS-57 (WTC-11 hiPSC endogenously tagged for mEGFP-NMP1) and AICS-61

(WTC-11 hiPSC endogenously tagged for mEGFP-HIST1H2BJ) cell lines. FOVs of this cell line were

visually inspected to determine the extent of cell death induced by each drug. If cell death at either 4 or 24

hours was approximately 50% more prevalent than compared to the control, then cells were classified as

unhealthy after 2 hours. Otherwise cells were classified as healthy. Results from this assessment are

summarized in the last column of the table shown in Figure 7b.

2. Input Data Preprocessing for Image Models

2.1 Punctate structures

2.1.1 cellPACK synthetic dataset: Packing results were voxelized into images of size 238x472x472 voxels. The z-coordinate of these images was padded with zeros to be the same size as X and Y, and the resulting images were downsampled to 118x118x118 voxels via block reduce operation with block size of 4x4x4 voxels and then used as input for image-based models.

2.1.2 DNA replication foci dataset: 3D raw fluorescence intensity single-cell images of DNA replication foci were masked, centered, and aligned by the corresponding nuclear segmentation dilated by 8x8x8 voxels. Images were cropped and then padded to the largest nuclear bounding box in the dataset. Images were then padded and resized to 118x118x118 voxels. Images were globally contrast adjusted to be within the intensity range 0 to 6000, which was empirically determined to remove dead pixels present in a few images and scaled per image using *monai.transforms.ScaleIntensity*[60] to be in the range (0,1).

2.1.3 Expanded dataset of punctate structures: Similar preprocessing was applied to a subset of punctate structures from the WTC-11 hiPSC Single-Cell Image Dataset v1[1], including DNA replication foci, histones, nuclear pores, nuclear speckles, cohesins, peroxisomes, endosomes and centrioles. However, the images of cytoplasmic structures (peroxisomes, endosomes, and centrioles), were masked by the cell membrane segmentation, instead of nuclear segmentation. Images were contrast adjusted using structure-specific intensity ranges reported in[1]. Images were finally scaled per image using *monai.transforms.ScaleIntensity*[60] to be in the range (0,1). The preprocessing code used to generate this dataset is available at https://github.com/AllenCell/benchmarking_representations/tree/main/br/data/preprocessing/image_preprocessing.

2.2 Polymorphic structures

2.2.1 Nucleoli (GC) dataset: Segmentations of nucleoli (GC) available in the WTC-11 hiPSC Single-Cell Image Dataset v1[1] were masked by corresponding nuclear segmentations. We used a hole filling algorithm to fill in holes in the segmented images that were then converted into 3D meshes for subsequent preprocessing. Meshes were downscaled to fit within a cube of size 32x32x32 voxels using a global scaling factor to preserve the relative scale of nucleoli in learned representations. For segmentation models the downscaled meshes were voxelized to create binary images. For SDF models the downscaled meshes were used to compute signed distance field images that were clipped to be in the range (-2,2).

2.2.2. Expanded dataset of polymorphic structures: Segmentation of the nucleolar GC, nucleolar DFC,

Golgi and lysosomes (available in the WTC-11 hiPSC Single-Cell Image Dataset v1) were masked by

either nucleus or cell mask if the structure localizes to nucleus (nucleoli) or cytoplasm (Golgi and

lysosomes). Subsequent preprocessing followed Section 2.2.2, except 3D meshes were downscaled on a

per cell basis based on the cell's intracellular structure bounding box. This downscaling avoids losing

small nuclear structures given the large bounding box of cytoplasmic structures.

2.2.3 Perturbed nucleoli (GC) dataset: We used the *Allen Cell & Structure Segmenter*[58] to segment raw

fluorescence intensity fields of view (FOVs) of perturbed nucleoli (GC) (section 1.4). Nuclear

segmentations for each FOV were produced by applying a UNet model trained on the WTC-11 hiPSC

Single-Cell Image Dataset v1[1] to predict 3D nuclear segmentations from brightfield images. We manually

selected nuclear segmentations in each FOV that covered the corresponding the entirety of the nucleoli

signal. The selected masks were used to generate single-cell images and they were processed as described

in Section 2.2.1.

3. Input Data Preprocessing for Point Cloud Models

3.1 Punctate structures

3.1.1 cellPACK synthetic dataset: The list of N=256 centroids of spheres packed by cellPACK was

extended to 2,048 points by adding a small jitter to each input point cloud 8 times. This jitter was clipped

at a value of 0.2, and the typical range of XYZ coordinates was -10 to 10. This was then used as the 3D

point cloud input. To improve reconstruction quality, this augmentation process was repeated 10 times for

each input. Details regarding the jitter augmentation are described in section 5.2.

3.1.2 DNA replication foci dataset: We started by applying the same preprocessing used in the DNA

replication foci dataset described above for image-based models, except for the last linear scaling step.

Instead, we used an exponential function $e^{\lambda(skewness * intensity)}$, where the skewness is a statistic that

indicates the deviation of a distribution from a normal distribution. The scaled images were then

normalized to obtain a probability density. The coefficient λ is an intracellular-specific scale factor that

was empirically determined based on the visualization of sampled points from random images for each

intracellular structure. We used $\lambda = 100$ for DNA replication foci images. The resulting probability

density was used to sample a point cloud with N=20,480 4D (XYZ+intensity) points. Sparse point cloud

versions with N=2,048 points were sampled from the original point cloud during training. Point clouds

were centered according to the nuclear mask and the intensity coordinate was scaled using by 0.1 to

match the magnitude of the spatial coordinates.

3.1.3 Expanded dataset of punctate structures: We started by applying the same preprocessing used in

the expanded dataset of punctate structures described above for image-based models, except the last linear

scaling step. We again used an exponential function $e^{\lambda(skewness*intensity)}$, with $\lambda = 100$ for nuclear

structures, and $\lambda = 500$ for cytoplasmic punctate structures. The scaled images were then normalized to

obtain a probability density. We followed the same procedure described above for DNA replication foci

to sample point clouds for each of these punctate structures. The intensity co-ordinate was then

normalized using structure specific contrast ranges.

3.2 Polymorphic structures datasets:

3.2.1 Nucleoli (GC) dataset: For each single-cell nucleoli (GC) image, we started by sampling a point

cloud of N=8,192 points from its corresponding 3D mesh generated as described in section 2.2.1. A

second point cloud with N=20K points is sampled during training from the corresponding SDF volume,

yielding a 4D point cloud consisting of SDF value + XYZ coordinates.

3.2.2. Expanded dataset of polymorphic structures: Each single polymorphic structure image

underwent a similar process. First, a point cloud of 8,192 points was sampled from the corresponding 3D

mesh (see section 2.2.2). During training, an additional 20K points were sampled from the SDF volume,

producing a 4D point cloud (SDF value + XYZ coordinates).

3.2.3 Perturbed nucleoli (GC) dataset: The perturbed nucleoli (GC) dataset followed the same sampling

strategy. For each single-cell nucleoli (GC) image, an initial 8,192 point cloud was sampled from the 3D

mesh generated as described in section 2.2.3. During training, another point cloud containing 20K points

was drawn from the SDF volume, yielding a 4D point cloud (SDF value + XYZ coordinates).

4. Model Background

Deep learning for feature extraction has been shown to be powerful in the context of cell biology, in

particular, for analyzing images in 2D[61–63]. Despite its success, feature interpretability and generalizability

to unseen image data continues to be a major challenge[64,65]. To alleviate some of these problems, it has

been shown that imposing additional constraints corresponding to prior biological knowledge to models

helps to reduce the space of admissible solutions and improve the likelihood that the learned features can

be useful for scientific discovery[66].

One approach for imposing extra constraints informed by prior knowledge is the integration of known

symmetries into the learning process. This has been the guiding principle for the field of geometric deep

learning, a subset of machine learning that aims to exploit geometric principles and avoid the curse of

dimensionality associated with learning generic functions in high-dimensional spaces[10]. While identifying

symmetries in real-life datasets is not always straightforward, enforcing that learned representations are

invariant or equivariant under simple Euclidean geometric transformations of the input data has been

shown to improve data efficiency and generalization in fields like protein structure prediction [67], medical

image analysis [68], and cell biology [44]. The distinction between invariant and equivariant representations is

of note. While the first type of representation stays the same when the input data is subject to some

transformation, the second changes predictably and equivalently. Rotation is a natural choice of

geometrical transformation to extract features that do not depend on the object's orientation.

4.1 Image Models

To implement 3D rotation invariant image autoencoders, we used image encoders equivariant to the

group of 3D rotations (*SO3 group*) using R^3 steerable kernels as described in Weiler et al.[69] and

implemented in the *escnn* library [70]. Compared to conventional convolutions, R^3 steerable kernels are

equivariant under rotations in R^3. We used scalar fields to learn invariant scalar features in R^3, and vector

fields to learn equivariant vector features in R^3. We used vector features to reconstruct the 3D rotation

matrix as described in Deng et al.[14] and Winter et al.[77].

We used seven layers of steerable kernels with an equal number of hidden scalar fields using trivial

representations and vector fields using irreducible representations. Using a (filter, stride, kernel size)

convention, the convolutions were (8, 1, 3), (16, 1, 3), (32, 2, 3), (64, 2, 3), (128, 2, 3), (512, 2, 3), (N, 1,

1), where N was the size of the latent dimension. In the final layer we used N scalar fields and 2 vector

fields. Each convolutional block also included a batchnorm and ReLU activation[71]. We used average

pooling in the last 5 layers and checked that this did not break equivariance (Fig. S2). We spatially pooled

the scalar embedding in the final layer to get the final N dimensional rotation invariant latent embedding.

We used a bottleneck size of 512 for polymorphic structures and 256 for punctate structures.

The decoding function was a conventional CNN decoder with 6 layers of convolutions. We used

upsampling blocks with a scale factor of 2 in between convolutions. Using a (filter, stride, kernel size)

convention, the convolutions were (512, 1, 3), (256, 1, 3), (128, 1, 3), (64, 1, 3), (32, 1, 3), (16, 1, 3). We

rotated the canonical reconstruction with the rotation matrix computed from the vector representation. We

used a cylinder mask using *escnn.nn.modules.masking_module.build_mask* to mask reconstructions and

reduce interpolation artifacts. We set the background value to 0 for segmentations, and 2 for clipped SDF

images where the maximum value was 2 and positive values were located outside the object. We used the

same settings with classical autoencoders by swapping out equivariant convolutions with regular

convolutions and keeping other details the same.

4.1.1: Masked autoencoders using vision transformers: We also trained masked autoencoders (MAE)

using vision transformers[72] in two stages as an alternative to the *vanilla* autoencoders described above.

We performed this training in two stages. First, we pretrained a MAE[73] using a ZYX patch size of (2,2,2),

a mask ratio of 0.75, and learnable positional embeddings. The encoder was made up of 8 identical

transformer blocks, each with 4 heads and an embedding dimension of 256. The decoder had 2 layers

with 8 heads and an embedding dimension of 192. We then employed a second phase of training with a

mask ratio of 0 (i.e. all image patches are visible to the encoder) where we froze the MAE-trained encoder

and trained a freshly initialized decoder to reconstruct the input image. We trained all models with a mean

squared error (MSE) loss.

4.2 Point cloud models

To implement 3D rotation invariant point cloud autoencoders, we used a 3D rotation equivariant point

cloud encoder using vector neurons (VN[11]), which lifts classical neurons to 3D vectors resulting in 3D

vector representations. VN layers are equivariant to rotations by construction, and have been shown to

outperform other equivariant architectures for tasks like classification, segmentation, and reconstruction.

We incorporate VN layers into a Dynamic Graph CNN (DGCNN)[74] backbone for point cloud encoding.

DGCNN uses network modules called EdgeConvs to perform CNN-like local neighborhood feature

extraction. These EdgeConvs can be stacked to extract global features[74]. Dynamic graphs are computed

by constructing k-nearest neighbor graphs on points. We used k=20 based on previous works as a balance

between computational complexity and local structure information[45]. We concatenated the cross-product

of the neighbor features and input points as well as the input points themselves to the hidden

representation. As described in section 3, we included raw image intensity in addition to XYZ coordinates

in some cases to generate 4D point clouds. This coordinate was included with the same vector orientation

as the XYZ coordinates, and thus remains equivariant under rotations in R^3. For the cellPACK dataset, we

used a 3D point cloud as input. We used 6 convolutional blocks where each block comprises a VN Linear

layer and a VN Leaky ReLU layer. We collated intermediate outputs before a final 1D convolution. We

took the norm of the final vector embedding to get a rotation invariant representation. We also trained

classical point cloud autoencoders with DGCNN encoders as described in Vries et al.[45], where VNLinear

and VN LeakyReLU layers are replaced with edge convolutions and ReLU layers.

4.2.1 Decoder for punctate structures: We reconstructed the rotation invariant representation for

punctate structures using a folding net decoder[75]. This decoder concatenates the latent embedding with

source points sampled from a template shape, and then applies two folding operations with ReLU

activations interleaved in between to reconstruct a point cloud. We used a 2D plane as a template in all

cases except for the cellPACK synthetic dataset, where a sphere was used as a template. Next, we used

the learned rotation matrix from the vector embedding to re-orient the canonical reconstruction. We

optimized the model using an earth mover's distance[76].

4.2.2 Decoder for polymorphic structures: We reconstructed the rotation equivariant representation

computed using vector neurons via an inner product decoder[11] to get signed distance function values at

query points. These query points correspond to the points of the second point cloud sampled from the

SDF volume as described in section 3.2. The model is optimized using an L1 loss on the SDF values. We

took the norm of the embeddings after training to compute a rotation invariant representation. This relaxes

the generative nature of the model as we are no longer able to obtain a rotation invariant reconstruction.

This trades off the ability to generate rotation-invariant predictions with reconstruction quality, deemed

necessary for the more complex shapes of the polymorphic structures. Instead, we visualize the closest

real examples to different map points in representation space (Fig. 5, 6).

5. Model Training

5.1. Train, validation and test splits

894 We used a train/valid/test split ratio of 70%, 15%, 15% respectively across all datasets. For the expanded

895 polymorphic dataset, we stratified the split using the structure class. For the perturbed nucleoli (GC)

896 dataset, we stratified the split by drug.

5.2 Jitter augmentation

We trained the rotation invariant point cloud models with an augmented version of the DNA replication

foci dataset (N=2,420) and the synthetic dataset from cellPACK (N=254) because of the small size of

these datasets. This augmentation was done by adding jitter to each input point cloud (with 2,048 points)

10 times during training. The jitter was added via Gaussian noise with a standard deviation of 0.1. The

noise was clipped at a value of 0.2. The typical range of XYZ coordinates was -10 to 10. Adding

augmentations helped improve the quality of reconstructions (Fig. S4).

6. Multi-metric Model Benchmarking

We computed three sets of metrics to quantitatively benchmark our trained models (Fig. S1a-b).

Efficiency metrics including model size, inference time, and emissions to assess ease of inference and

training. Generative metrics included reconstruction error and evolution energy[7] to assess the quality of

generated shapes. Finally, representation expressivity metrics evaluated classification/regression scores

for biologically-relevant features, rotation invariance error, and interpolation distances in embedding

space.

Efficiency: *Model size* is the number of parameters in the model, *inference time* is the time to run GPU

inference on a single input, and *carbon emissions* is an estimate of hardware electricity power

consumption in kWh. All measurements use a single A100 GPU.

Generative capacity: *Reconstruction error* is the average test set reconstruction error using the Jaccard

similarity score for images and Chamfer distance for point clouds, respectively. More details on

classification and regression calculations can be found in section 6.1. *Evolution energy* is the normalized

energy of deformation from one shape to another averaged across many random test set pairs in a dataset

(Fig. S1b). Here, the normalized energy of deformation is computed as the sum of the reconstruction error

between an interpolated shape reconstruction and initial and final reconstructions, divided by the

reconstruction error between initial and final reconstructions. This energy is computed across 10

interpolations between two given shapes.

Expressivity: *Rotation invariance error* is computed as the norm of the Euclidean distance between the

embedding of a test set shape and its rotated version, divided by the sum of the norms of the two

embeddings. The metric aims to quantify how much the embeddings change in representation space with

rotation, while accounting for the size of the embedding space itself via

$$error = \frac{|z_\theta - z_0|}{||z_\theta| + |z_0||}, \text{ where } \theta \text{ is the rotation angle}$$

By dividing by the sum of the norms of the two embeddings, we establish their size relative to the origin.

We computed this metric for four 90 degree rotations of the input in the XY plane. This is then averaged

across many test set examples. *Average interpolation distance* is computed as the distance to the closest

real example in representation space for interpolations between random test set examples (Fig. S). We did

this for 10 interpolations between randomly sampled pairs of test set examples and report the average

across pairs and interpolations. *Compactness* is an intrinsic dimensionality measure calculated using the

Levina-Bickel score[19]. All metrics were z-scored across models for polar plot visualization. The sign of

all metrics except for classification and regression scores were flipped to ensure that a higher value

indicates better performance. *Classification* and *regression scores* are the respective cross-validated test

set accuracies and R^2 values for selected features. More details on classification and regression

calculations can be found in section 6.2.

 6.1 Reconstruction error

Punctate structures: We used the Chamfer loss to compute the reconstruction error of image- and point

cloud-based models. Because this loss is normally applied to point clouds, we convert inputs and

respective reconstructions from image models into point clouds by sampling points using the exponential

sampling function described in section 3.1.

Polymorphic structures: We used the Jaccard similarity score on binary masks to compute the

reconstruction error of image- (segmentation and SDF) and point cloud-based models. For segmentation

models, we used Otsu thresholding to binarize each reconstruction and extracted a mesh using marching

cubes. Next, we upscaled the mesh and voxelized it into a 3D binary mask. To generate a binary mask for

SDF images used as input for SDF models and their reconstructions, we first used the marching cubes

algorithm to extract a zero-level set mesh. Then we upscaled the mesh and voxelized it to create a 3D

binary mask. For SDF point cloud models, we used vertices of a 32x32x32 grid as XYZ query points to

perform inference along with ground truth surface point clouds to obtain SDF reconstruction images. We

then used the marching cubes algorithm to extract a zero-level set mesh from each SDF image, upscaled

the mesh, and voxelized it into a 3D binary mask.

6.2 Classification and Regression Scores

We normalized embeddings using *StandardScaler* from scikit-learn[77]. For classification accuracies, we

used a logistic regression classifier with class weights and reported test set classification accuracies with

stratified k-fold cross-validation with five splits. For regression scores, we used a linear regression model

and reported test set R^2 with repeated k-fold cross validation with five splits and 20 repeats.

Classification and regression targets were designed appropriately. For the cellPACK synthetic dataset, we

reported cross-validated accuracy for classifying the six packing rules. For the DNA replication foci

dataset, we reported cross-validated top 2 classification accuracy for 8 interphase cell cycle stages and

cross-validated accuracy for manually annotated outlier labels including dead cells and cells with no

fluorescence. For punctate structures from the WTC-11 hiPSC Single-Cell Image Dataset v1, we reported

cross-validated structure classification accuracy for 7 structures and cross-validated classification

accuracy for cell cycle stages including interphase, prophase, early prometaphase,

prometaphase/metaphase and anaphase/telophase. For the nucleoli (GC) dataset, we reported cross-

validated classification accuracy of thresholded number of pieces of nucleoli present in the segmentation,

including 1, 2, 3, 4 and >=5 piece nucleoli. For this dataset, we also reported an average test set R^2 for

regression of mean and standard deviation of piece volume and area, and an average test set R^2 for

regression of mean and standard deviation of centroid-centroid distances between pieces. For the

polymorphic structure dataset including nucleoli (GC and DFC), lysosomes and Golgi, we reported cross

validated classification accuracy for structures, and regression scores for volume and area of pieces, and

distance between pieces similar to the nucleoli (GC) dataset.

975   7. Data Analysis

976   7.1 Principal component analysis (PCA)

977   We fit PCA to the learned rotation invariant representations using *sklearn.decomposition.PCA* with

978   "auto" SVD solver and the number of components equal to the size of the embedding space. We then

979   performed inverse PCA on different map points (-2σ, 0, 2σ) for different principal components (PCs). We

980   used the inverse PCA reconstruction as input to the decoder of the representation learning model to

981   visualize the reconstructed image/point cloud.

982   7.2 Archetype analysis

983   Archetype analysis approximates the convex hull of data points by identifying extremal points[20]. We used

984   an implementation of the Frank-Wolfe algorithm for archetype analysis[78]. The number of archetypes for

985   each dataset was chosen based on an expected number of clusters in each application.

986   7.3 Nuclear volume binning for nucleoli (GC) and DNA replication foci dataset

987   We clipped the nuclear volume distribution to be within the 2.5% and 97.5% range for both DNA

988   replication foci and nucleolar GC datasets. Next, we binned the data into equal sized bins of 121 μm^3. The

989   bins were <390 μm^3, 390-533 μm^3, 533-676 μm^3, 676-818 μm^3, and >818 μm^3.

991   8. Data Availability

The WTC-11 hiPSC single cell image dataset v1 analyzed in this study is available online at

https://open.quiltdata.com/b/allencell/packages/aics/hipsc_single_cell_image_dataset. The DNA

replication foci dataset analyzed in this study is available online at

https://open.quiltdata.com/b/allencell/packages/aics/nuclear_project_dataset_4. The WTC-11 hiPSC

nucleoli (NPM1) perturbation single cell image dataset analyzed in this study is available online at

https://open.quiltdata.com/b/allencell/tree/aics/NPM1_single_cell_drug_perturbations/. The synthetic

dataset of punctate structures generated using cellPACK and analyzed in this study is available online at

https://open.quiltdata.com/b/allencell/tree/aics/morphology_appropriate_representation_learning/cellPAC

K_single_cell_punctate_structure/. The landing page of the GitHub repository associated with this

manuscript (https://github.com/AllenCell/benchmarking_representations) has additional information for

accessing and processing these datasets.

1003   8. Code Availability

Code for all our representation learning models is available at

https://github.com/AllenCellModeling/cyto-dl/blob/br_release/.

Config files associated with our models, training scripts, and code for multi-metric benchmarking are

available at https://github.com/AllenCell/benchmarking_representations. Our code was all developed in

Python. A full list of dependencies is available at https://github.com/AllenCellModeling/cyto-

dl/tree/br_release/requirements and

https://github.com/AllenCell/benchmarking_representations/blob/main/pyproject.toml. We have released

the code with the Allen Institute Software License.

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Figures

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Figure 1 - Application-appropriate representation learning framework for complex intracellular

structure morphologies. a) Example of a punctate structure using DNA replication foci. Shown are

single-cell segmentations for the nucleus and cell membrane, and raw intensities for DNA replication foci

(via PCNA). 4D (XYZ + intensity) point clouds are sampled from the intensity images by converting

intensities to probabilities. The intensity co-ordinate is scaled to ensure that the range of intensity values

is similar to the range of XYZ coordinate values. This 4D point cloud is then used as input to the

representation learning algorithm. **b)** Example of a polymorphic structure using granular component (GC)

nucleoli (via nucleophosmin). Shown are single-cell segmentations for nucleus, cell membrane, and

nucleoli (GC). Nucleoli segmentation from single-cell data is used to generate a 3D mesh. A surface point

cloud is sampled from the nucleolar mesh. Another point cloud is sampled from the 3D bounding box

volume and its points are assigned local signed distance field (SDF) values relative to the surface of the

nucleolar mesh. These two point clouds are used by the representation learning algorithm. **c)** 3D rotation

invariant representation learning framework using autoencoders. Rotations of the same shape are

projected into vector representations using a 3D rotation equivariant encoder. This vector representation is

then used to reconstruct the original shape. We take the norm of the vector representation to compute a

rotation invariant representation. We do this during training for punctate structures, and after training for

polymorphic structures (*Section 4.2 of Methods*).

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Figure 2 - 3D rotation invariant point cloud models are efficient, produce low rotation invariance errors and generate good reconstructions in a synthetic dataset of punctate structures a) Dataset of synthetic punctate structures generated using cellPACK. A 3D nuclear shape is packed with 6 different rules. This includes Planar 0, Planar 45, Planar 90, Radial, Random, and Surface. Planar rules pack spheres with a gradient away from a plane indicated in red. Radial rule packs spheres close to the centroid. Random rule packs spheres randomly across the 3D nuclear volume. Surface rule packs spheres close to the nuclear boundary. Each rule is used to pack 254 different nuclear shapes. **b)** Benchmarking unsupervised representations across different models and metrics. (Left) Polar plot showing performance of classical and rotation invariant image and point cloud models across efficiency metrics (model size, inference time, emissions), generative metrics (reconstruction, evolution energy), and representation expressivity metrics (compactness, classification of rules, rotation invariance error, average interpolate distance). Metrics are z-scored and scaled such that larger is better (Right) Bar plots showing raw metric values across models for each metric. Error bars are standard deviations. Best model for each metric is indicated. **c)** 1st principal component for each rule using the rotation invariant point cloud model trained with jitter augmentations. PCA is fit to representations of each rule separately. Shown are normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in steps of σ). **d)** 6 archetypes computed from the rotation invariant point cloud representations. Each archetype corresponds to one of

1234   the 6 rules. All reconstructions shown are cut at mid-plane. Color associated with each point is the

1235   distance from the mid-plane in Z.

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Figure 3 - Rotation invariant point cloud representations recover cell cycle-dependent spatial

pattern of DNA replication foci. a) Dataset of DNA replication foci in hIPS cells expressing mEGFP-

tagged PCNA. DNA replication foci have a stereotypical cell cycle dependent localization pattern. Shown

are examples of image and sampled point cloud center slices with adjusted contrast for eight expert-

annotated cell cycle stages. The scale bar is 5 μm. **b)** Benchmarking unsupervised representations across

different models and metrics. (Left) Polar plot showing performance of classical and rotation invariant

image and point cloud models across efficiency metrics (model size, inference time, emissions),

generative metrics (reconstruction, evolution energy), representation expressivity metrics (compactness,

classification of cell cycle via top-2 classification accuracy, rotation invariance error, average interpolate

distance). Metrics are z-scored and scaled such that larger is better (Right) Bar plots showing raw metric

values across models for each metric. Error bars are standard deviations. Best model for each metric is

indicated. **c)** 8 archetypes identified using rotation invariant point cloud representations. Each archetype

corresponds to one of the 8 expert-annotated cell cycle stages. **d)** 1st principal component for each cell

cycle stage using rotation invariant point cloud model. PCA is fit to representations of each cell cycle

stage separately. Shown are normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points

(-2σ to 2σ in steps of σ). **e)** Average canonical reconstructions across five bins of nuclear volume (*Section*

7.3 of Methods). All reconstructions shown are center slices.

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Figure 4 -Representation learning framework reveals interpretable spatial patterns for other
punctate structures from the WTC-11 hiPSC Single-Cell Image Dataset v1 a) Dataset of punctate
structures in hiPS cells from the WTC-11 hiPSC Single-Cell Image Dataset v1 including nuclear pores,
nuclear speckles, cohesins, histones, centrioles, peroxisomes, and endosomes (Viana 2023). Show are
examples of images and sampled point cloud center-slices of the FP-tagged protein. The scale bar is 5
μm. **b)** Benchmarking unsupervised representations across classical and rotation invariant image and
point cloud models across efficiency metrics (model size, inference time, emissions), generative metrics
(reconstruction, evolution energy), and representation expressivity metrics (compactness, classification,
rotation invariance error, average interpolate distance). Classification tasks included classifying 7
different structures, and 6 different interphase/mitotic stages (*Section 6.2 of Methods*). (Left) Polar plot
showing performance across models where metrics are z-scored and scaled such that larger is better
(Right) Bar plots showing raw metric values across models for each metric. Error bars are standard
deviations. Best model for each metric is indicated. **c)** Real examples per map point of PC1 computed
using PCA fit to representations of each structure separately using the rotation invariant point cloud
model. Only cells in interphase were included. Shown are XY and XZ views. The structure channel is
shown as center slices across the nuclear centroid for nuclear pores, cohesins and histones, or as max
projections for nuclear speckles, centrioles, endosomes, and peroxisomes. **d)** Latent walk for PC1. Shown
are normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in steps of σ).
Reconstructions shown are cut at mid-plane. Membrane centroids are marked for centrioles. Only cells in
interphase were considered for this analysis. Centriole reconstructions were rotated to be aligned to the X
axis.
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Figure 5 - Rotation invariant representation learning framework generalizes to polymorphic multi-piece structures. a) Dataset of nucleoli (GC) from the WTC-11 hiPSC Single-Cell Image Dataset v1 (Viana 2023), stratified by number of pieces. Shown are example max intensity projections and corresponding 3D meshes. **b)** Benchmarking unsupervised representations across different models and metrics. (Left) Polar plot showing performance of all models across efficiency metrics (model size, inference time, emissions), generative metrics (reconstruction, evolution energy), representation expressivity metrics (compactness, classification of number of pieces, shape features regression, distance features regression, rotation invariance error, average interpolation distance). Metrics are z-scored and

scaled such that larger is better (Right) Bar plots showing raw metric values across models for each

metric. Error bars are standard deviations. Best model for each metric is indicated. **c)** 1st principal

component for 1 piece, 2 pieces, 3 pieces, 4 pieces, and 5+ pieces examples using rotation invariant point

cloud model. PCA is fit to representations of different numbers of pieces separately. Shown are closest

real examples to normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in

steps of σ). **d)** 2nd principal component for 1 piece, 2 pieces, 3 pieces, 4 pieces, and 5+ pieces examples

using rotation invariant point cloud model. Shown are closest real examples to normalized PCs (standard

deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in steps of σ). **e)** 5 archetypes identified

using rotation invariant point cloud model. **f)** Closest real example to average representations of five

equal sized bins of nuclear volume (*Section 7.3 of Methods*).

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Figure 6 - Learned representations enable shape variation profiling on multiple polymorphic structures. a) Dataset of nucleoli GC, nucleoli DFC, lysosomes, and Golgi from the WTC-11 hiPSC Single-Cell Image Dataset v1 (Viana 2023). Shown are example max intensity projections and corresponding 3D meshes. **b)** Benchmarking unsupervised representations across different models and metrics. (Left) Polar plot showing performance for all models across efficiency metrics (model size, inference time, emissions), generative metrics (reconstruction, evolution energy), representation expressivity metrics (compactness, classification of number of pieces, shape features regression, distance features regression, rotation invariance error, average interpolation distance). Metrics are z-scored and scaled such that larger is better (Right) Bar plots showing raw metric values across models for each metric. Error bars are standard deviations. Best model for each metric is indicated. **c)** 1st principal component for each structure using rotation invariant point cloud model. PCA is fit to representations of each structure separately. Shown are closest real examples to normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in steps of σ). **d)** 2nd principal component for each structure

using rotation invariant point cloud model. Shown are closest real examples to normalized PCs (standard deviation (s.d.), σ, units) sampled at 3 map points (-2σ to 2σ in steps of σ). **e)** 4 archetypes identified using rotation invariant point cloud representations.

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Figure 7 - Learned representations allow for morphological profiling of nucleoli under different perturbations. a) Q-value statistics[2] per drug (*Section 1.4 of Methods*) and per model indicating the confidence of each model distinguishing a given drug from control. **b)** Table listing molecular target or

1374 mechanism of action, effects reported in the literature, effects observed on the mEGFP-NPM1 cell line at

1375 2 hours post treatment, and cell health assessment between 4 and 24 hours after treatment for each drug.

1376 **c)** Three representative examples of nucleoli (GC) for the control (DMSO) and each of the 16 drugs used

1377 in this study. **d)** Closest real sample to the average representation for the seven drugs that fall below the q-

1378 threshold using the rotation invariant point cloud model.

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 # Supplemental figures

Figure S1 - Evaluation metrics for representation learning models a) Overview of different evaluation metrics for quantifying the utility of each representation learning framework. Efficiency metrics include model size, inference time, and carbon emissions. Generative ability metrics include reconstruction error and evolution energy. Representation expressivity metrics include rotation invariance error, interpolation distance, feature regression, classification accuracy, and compactness. **b)** Workflow for interpolation distance and evolution energy calculation. Two samples are drawn from the population randomly, and a linear interpolation is performed on the representations of the two samples. The euclidean distance between an interpolation and the nearest real representation is the interpolation distance. The interpolation distance is averaged across many interpolations to compute the average interpolation distance. Each interpolation is reconstructed using the decoder to obtain a reconstruction. The sum of the reconstruction error between the interpolated reconstruction and the reconstructions of the initial and final shapes normalized by the reconstruction error between the initial and final shape is the energy of deformation[7]. The energy of deformation is averaged across many interpolations to compute the evolution energy. Both evolution energy and average interpolation distance are averaged across many random pairs of samples from the test set. **c)** Holistic evaluation of metrics. Metrics are z-scored across models per metric. Z-scored metrics are visualized using a polar plot by flipping the sign for metrics where lower is better (indicated by a negative sign).

Figure S2 - Testing orientation invariance for image and point cloud models for the cellPACK

synthetic dataset a) (Top row) Example image input for the planar 45 rule is rotated by four 90 degree

rotations. (Middle row) Reconstructions using the classical image model (upper) and rotation invariant

image model (lower) for each rotated input. The reconstructions using the rotation invariant model are

pose-corrected using the learned rotation angles. (Bottom row) Rotation invariant reconstructions using

the rotation invariant image model for each rotated input. **b)** (Top row) Example point cloud input for the

planar 45 rule is rotated by four 90 degree rotations. (Middle row) Reconstructions using the classical

point cloud model (upper) and rotation invariant point cloud model (lower) for each rotated input. The

reconstructions using the rotation invariant model are pose-corrected using the learned rotation angles.

(Bottom row) Rotation invariant reconstructions using the rotation invariant point cloud model for each

rotated input. All reconstructions shown are max projections in Z.

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1427 Supp Fig. 3 - **Evaluation of test set model reconstructions for synthetic punctate structures**

1428 **generated using cellPACK.** Test set center slice inputs (**a, d**) and reconstructions using **b**) classical

1429 image model, **c**) rotation invariant image model, **e**) classical point cloud model, and **f**) rotation invariant

1430 point cloud model for each of the 6 packing rules. Both pose-corrected and rotation invariant

1431 reconstructions are shown for the rotation invariant models.

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Figure S4 - Jitter augmentations slightly improve point cloud model reconstructions for synthetic punctate structures generated using cellPACK. Test set center slice reconstructions using rotation invariant point cloud model with jitter augmentations for each of the 6 packing rules.

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Figure S5 - 3D image preprocessing into application appropriate inputs for punctate structures.

Workflow for generating 4D point clouds from 3D intensity images. **a)** single-cell intensity images are

obtained by masking via a dilated nuclear mask (for nuclear structures), followed by alignment to the

longest axis of the nuclear mask. Intensities were then scaled using an exponential function and then

converted to probabilities. These probabilities were then used to sample a dense 4D point cloud with

20480 points and XYZ + intensity coordinates. During training, a sparse point cloud with 2048 points was

sampled from this dense point cloud using the intensities as probabilities. The intensity coordinate was

scaled using a scale factor of 0.1 to ensure that intensity values were in the same range as XYZ coordinate

values. **b)** Examples of dense sample and sparse sample for each cell cycle stage for PCNA dataset.

Shown are center-slice of raw intensity image, center-slice of raw intensity image overlaid with dense

sample, and center-slice of raw intensity image overlaid with sparse training sample. **c)** Examples of

dense sample and sparse sample for each punctate structure from the WTC-11 hiPSC Single-Cell Image

1450   Dataset v1. Structures include histones, nuclear envelope, cohesins, nuclear speckles, endosomes,

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Figure S6 - Evaluation of test set model reconstructions for the DNA replication foci dataset.

Test set center slice inputs (**a, e, g**) and reconstructions using **b)** classical image model, **c)** rotation

invariant image model, **d)** an alternative classical image model via a masked autoencoder with a vision

transformer as an encoder (MAE-ViT), **f)** classical point cloud model, and **h)** rotation invariant point

cloud model for samples from each of the 8 cell cycle stages. Both pose-corrected and rotation invariant

reconstructions are shown for the rotation invariant models.

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1472 **Figure S7 - Evaluation of test set model reconstructions for punctate structures from the WTC-11**

1473 **hiPSC Single-Cell Image Dataset v1.** Visualization of test set reconstructions for sampled histones

1474 (CellId 721646), nuclear pores (CellId 873680), cohesins (CellId 994027), nuclear speckles (CellId

490385), centrioles (CellId 451974), endosomes (CellId 811336), peroxisomes (CellId 835431). Shown

are test set inputs (**a, e, g**) and reconstructions using **b)** classical image model, **c)** rotation invariant image

model, **d)** an alternative classical image model via a masked autoencoder with a vision transformer as an

encoder (MAE-ViT), **f)** classical point cloud model, and **h)** rotation invariant point cloud model for each

structure. Both pose-corrected and rotation invariant reconstructions are shown for the rotation invariant

models. Reconstructions for nuclear pores, cohesins, and histones are center slices, whereas

reconstructions for nuclear speckles, centrioles, endosomes, and peroxisomes are max projections. Spatial

distribution artifacts in reconstructions for endosomes and peroxisomes are highlighted with blue arrows.

Figure S8- 3D image preprocessing into application appropriate inputs for polymorphic structures

a) Workflow for computing signed distance function (SDF) images from segmentations. single-cell

structure segmentations are masked by nuclear segmentation (for nuclear structures), followed by

meshing, centering, and hole filling. The mesh is then rescaled to $32*^3$ cube resolution and then processed

to get a signed distance function. Alternatively, the rescaled mesh is voxelized to get a segmentation. SDF

is clipped to (-2, 2) range for training image models to focus models on the zero level set. Example shown

is for nucleoli (GC). **b)**. Visualization of rescaled segmentation and SDF for examples with different

numbers of pieces of granular component (GC) of nucleoli. Shown are center-slices of raw intensity

images, max projection of the structure segmentation, max projection of the voxelized rescaled

segmentation, and center slice of the rescaled mesh SDF. **c)** Visualization of rescaled segmentation and

SDF for other polymorphic structures from the WTC-11 hiPSC Single-Cell Image Dataset v1 including

lysosomes, Golgi, GC nucleoli, and dense fibrillar component (DFC) nucleoli.

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1507 **Figure S9 - Examples of model inputs and outputs for nucleolar GC dataset.** Test set reconstructions

1508 across all models for sampled 1 piece (CellId 964798), 2 pieces (CellId 661110), 3 pieces (CellId

1509   644401), 4 pieces (CellId 967887) and 5+ pieces (CellId 703621) examples. Max projections of original

1510   structure segmentations overlaid with nuclear and membrane segmentations are shown. Max projections

1511   are shown for segmentations, whereas middle slices are shown for SDFs.

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Figure S10 - Evaluation of model reconstructions for polymorphic structures from the WTC-11

hiPSC Single-Cell Image Dataset v1. A) Test set reconstructions across all models for sampled nucleoli

(GC) (CellId 691110), nucleoli (DFC) (CellId 723687), lysosome (CellId 816468), and Golgi (CellId

800894) examples. Max projections of original structure segmentations overlaid with nuclear and

membrane segmentations are shown. Max projections are shown for segmentations, whereas middle slices

are shown for SDFs.