Simultaneous stabilizing feedback control of linear and angular momentum in human walking.

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Abstract
Stabilizing bipedal gait is mechanically challenging. Previous studies have used the assumption that the linear momentum of the body center of mass (CoM) is controlled to analyze how gait is stabilized. In these studies, the position of the center of pressure (CoP) or of the foot relative to the COM is often used as an indicator of corrections of the CoM state. Mechanically, neither of these variables is directly related to changes in linear momentum, whereas they do directly affect whole-body angular momentum (WBAM), which has also been suggested to be a controlled variable. We show that, in human walking, linear and angular momentum follow quasi-periodic functions with similar periodicity and phase. Combining the equations of linear and rotational motion for a system of linked rigid segments shows that, in this case, the horizontal distance between CoP and CoM is a good predictor of horizontal forces in the corresponding direction. This suggests that linear and angular momentum are simultaneously controlled to follow similar quasi-periodic functions and may explain the success of preceding studies that correlated CoM states to CoP or foot locations. We developed feedback models to predict the ground reaction force and its moments along the sagittal and transverse axes from the preceding CoM state and WBAM respectively. These models were fitted to experimental data of participants walking at normal and slow speeds. The consistent, good fit of both models supports that linear and angular momentum are controlled simultaneously in human walking.

Nomenclature
All kinematic variables are expressed with respect to an inertial reference frame affixed to the world. All moments of inertia are with respect to the center of mass of the respective segment/body. Dot notations denote derivatives with respect to time.
$\boldsymbol{a}_{CoM'}$ = linear acceleration of the projection of body center of mass on the support surface.
CoM = whole-body center of mass.
CoM' = projection of the whole-body center of mass on the support surface.
CoP = center of pressure, or point of application of the ground reaction force.
$\boldsymbol{F}_{gr}$ = ground reaction force vector.
g = gravitational constant.
$\boldsymbol{H}_{CoM}$ = angular momentum of the whole body around its center of mass.
$\boldsymbol{I}_i$ = inertia tensor of segment *i.*
m = body mass.
m_i = mass of segment ***i.***
$\boldsymbol{p}_{CoM}$ = linear momentum of the whole body.
$\boldsymbol{r}_{CoM}$ = position vector of the body center of mass.
$\boldsymbol{r}_{CoM'}$ = position vector of the projection of the body center of mass on the support surface.
$\boldsymbol{r}_{CoP}$ = position vector of the point of application of the ground reaction force or center of pressure.
$\boldsymbol{r}_i$ = position vector of the center of mass of segment *i* relative to the body center of mass.
$\boldsymbol{v}_{CoM}$ = velocity of the body center of mass.
$\boldsymbol{v}_i$ = velocity of segment *i* relative to the body center of mass.
$\boldsymbol{\omega}_i$ = angular velocity of segment *i.*

Introduction

Stabilizing bipedal gait is challenging as illustrated by the high incidence of falls in toddlers learning to walk (Ghassabian et al., 2016) and in older and diseased adult populations (Robinovitch et al., 2013). The mechanical challenge of stabilizing bipedal gait arises from the body being upright and segmented and the small base of support. Additionally, the anatomy is typically "top-heavy," with heavier segments supported by lighter segments. Consequently, perturbations, whether internal or external, are amplified by gravity, potentially causing instability. Robust gait, therefore, necessitates the control of the entire body through muscles delivering moments around the joints they span.

Conditions for stabilization of gait can be obtained from reduced order models, focusing on the "external coordination", such as the control of linear momentum of the CoM (De Comite and Seethapathi, 2024; Seethapathi and Srinivasan, 2019; van Leeuwen et al., 2022; Wang and Srinivasan, 2014). Hof (Hof, 2007) proposed an analysis of control of the CoM based on the equation of rotational motion for the whole body modeled as a system of linked rigid segments. He rewrote the equation of rotational motion to describe rotation around the projection of the CoM on the support surface CoM':

$$(\boldsymbol{r}_{CoP} - \boldsymbol{r}_{CoM'}) \times \boldsymbol{F}_{gr} - \dot{\boldsymbol{H}}_{CoM} = (\boldsymbol{r}_{CoM} - \boldsymbol{r}_{CoM'}) \times m\boldsymbol{a}_{CoM'} \qquad 1$$

and stated that the first term reflects the 'move the center of pressure (CoP)'-mechanism. In similar vein, two of the authors of this paper (van Dieën and Lemaire, 2023) stated that this shows that the CoP can be shifted relative to CoM' to control linear momentum. This notion is reflected in many studies on stabilization of bipedal gait from our and other groups, whether explicitly using the position of the CoP, or the position of the foot relative to the CoM {Wang, 2014 #8917;van Leeuwen, 2022 #9524;van Leeuwen, 2021 #8622;van den Boogaart, 2022 #8469;Hof, 2010 #3612;Seethapathi, 2019 #9671;Vlutters, 2016 #8817;De Comite, 2024 #9859}. However, shifting the CoP does not directly affect linear momentum. To explain this, we consider a system of rigid linked segments and for the sake of conciseness ignore forces other than gravity and the ground reaction force. The equation of linear motion of the CoM of such a system is:

$$\boldsymbol{F}_{gr} + m\boldsymbol{g} = \dot{\boldsymbol{p}}_{CoM} \qquad 2$$

The equation of rotational motion of such a system about its CoM is:

$$(\boldsymbol{r}_{CoP} - \boldsymbol{r}_{CoM}) \times \boldsymbol{F}_{gr} = \dot{\boldsymbol{H}}_{CoM} \qquad 3$$

Evidently, the CoP does not appear in equation 1 and thus changing its position relative to the CoM does not change linear momentum. Rather, a change in CoP causes an instantaneous change in angular momentum (equation 3). Similarly, a change of other terms in equation 1 not present in equation 2 cannot be considered strategies to control the body's linear momentum.

The above triggers the question why analyses using the vector between CoP and CoM' have been successful in making predictions about the mechanics and control of standing and walking. It appears that these analyses were derived from models representing the human body by a mathematical inverted pendulum, see for example for standing (e.g., Morasso and Schieppati, 1999) and for walking (Townsend, 1985). By definition, a mathematical pendulum has its mass concentrated at the CoM and hence does not have an angular momentum around its CoM. As such, the ground reaction force necessarily points at the CoM, or in other words the external moment ($(\boldsymbol{r}_{CoP} - \boldsymbol{r}_{CoM}) \times \boldsymbol{F}_{gr}$) is always zero. Obviously, the human body is not a mathematical pendulum. It is better approximated by a physical pendulum, i.e., a pendulum with distributed mass, but even this may not be an appropriate model, when trying to understand the control of linear and angular momentum. In this

study, we used existing data on human walking 1) to assess the correlation between the distance between from CoM' to CoP and the horizontal ground reaction force in the corresponding direction, 2) to show that simultaneous control of linear and angular momentum would cause such correlation, and 3) to identify simultaneous feedback control of linear and angular momentum.

Methods

EXPERIMENTAL DATA

Ground reaction forces and whole-body CoM trajectories during treadmill walking were obtained from healthy young adults, participating in a previous experiment (van Leeuwen et al., 2020). Data from 13 participants (10 females, 3 males; mean (SD) age: 30 (9) yrs, mass 73 (17) kg, length 1.71 (0.09) m), for whom complete data for both normal and slow walking speeds were available, were included in the analysis.

Participants walked on a instrumented dual-belt treadmill (Motek-Forcelink, Amsterdam, Netherlands) at normal and slow (1.25 and 0.6125 × √(leg length) m/s) normalized (Hof, 1996) speeds. Stride frequency was enforced by means of a metronome, at the average preferred frequency determined during the final 100 steps of a familiarization trial. Trials lasted five minutes for normal walking and ten minutes for slow walking.

Ground reaction forces were recorded from force plates embedded in the treadmill and sampled at 200 Hz. Full body kinematics were measured using Optotrak cameras (Northern Digital Inc, Waterloo Ontario, Canada) and sampled at 50 samples/s. Cluster markers were attached to the feet, shanks, thighs, pelvis, trunk, upper arms and forearms. Corresponding anatomical landmarks were digitized using a six-marker probe.

For all trials, we analyzed 200 consecutive strides. Heel strikes were detected based on the combined CoP as derived from force plate data (Roerdink et al., 2008). The mass and moment of inertia of each segment were estimated using its length and circumference as predictors and regression coefficients based on gender (Zatsiorsky et al., 1990). The segment's center of mass location was estimated to be at a percentage of the longitudinal axis of the segment (de Leva, 1996; Zatsiorsky et al., 1990). The CoM was estimated as a weighted sum of the centers of mass of the body segments and its numerical derivative was used as an estimate of CoM velocity. Whole body angular momentum was calculated as:

$$\boldsymbol{H_{CoM}} = \sum \boldsymbol{I}_i\, \boldsymbol{\omega}_i + m_i \boldsymbol{r_i} \times \boldsymbol{v}_i \qquad 4$$

with subscripts i referring to body segments and $\boldsymbol{v}$ the velocity of the segment center of mass relative to the whole-body CoM. The moment of the ground reaction force relative to the CoM was calculated as the numerical derivative of $\boldsymbol{H_{CoM}}$.

DATA ANALYSIS

To answer our first question, we calculated Pearson's coefficients of correlation between the time series of the distance from CoM' to CoP and of the horizontal ground reaction forces for the mediolateral and anteroposterior directions separately.

For our second aim, we calculated the time series of linear and angular momenta from the experimental data described above. Subsequently, we simulated equations 2 and 3, prescribing $\dot{\boldsymbol{p}}_{\boldsymbol{CoM}}$ and $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ as functions of the gait cycle derived from our experimental data. Horizontal force was calculated from equation 2. Vertical force was considered constant and horizontal force in the

orthogonal direction was set to zero. The resulting ground reaction force was inserted into equation 3 to obtain the distance between CoP and CoM'. Finally, Pearson's coefficient of correlation between this distance and the horizontal ground reaction force was assessed.

To assess whether feedback control of linear and angular momentum can be identified from gait data, we used an approach similar to that previously described for the control of linear momentum (van Dieën et al., 2024). We considered the target values of $\boldsymbol{v}_{com}$ and $\boldsymbol{H}_{com}$ to have quasi-periodic patterns ($\boldsymbol{v}_{target}$ and $\boldsymbol{H}_{target}$) that can be estimated from the average patterns over gait cycles. Next, we predicted deviations in linear and angular momentum from the averages as follows:

$$\dot{\boldsymbol{p}}_{\boldsymbol{CoM}}(t) = \beta_1(i)(\boldsymbol{r}_{\boldsymbol{CoM}}(i-\partial_1) - \boldsymbol{r}_{\boldsymbol{target}}(i-\partial_1)) + \beta_2(i)(\boldsymbol{v}_{\boldsymbol{CoM}}(i-\partial_1) - \boldsymbol{v}_{\boldsymbol{target}}(i-\partial_1)) \quad 5$$

$$\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}(t) = \beta_3(i)(\boldsymbol{H}(i-\partial_2) - \boldsymbol{H}_{\boldsymbol{target}}(i-\partial_2)) \quad 6$$

in which i refers to the phase of the gait cycle (in %), $\partial_{1,2}$ are delays (in % of the gait cycle) of the feedback response, and $\beta_{1,2,3}(i)$ are regression coefficients, which represent phase-dependent feedback gains. For linear momentum, we redefined $\boldsymbol{r}_{\boldsymbol{CoM}}$ to the position of the initial stance foot in each stride and we used position and velocity as predictors based on previous literature (Wang and Srinivasan, 2014) For angular momentum the integral term is undefined, hence we used a proportional term only. Both models were fit in a least-squares sense to the n observations (strides) at each phase i. Assuming phase-dependent feedback (e.g., Berger et al., 1984; Bruijn et al., 2015; Golyski et al., 2021; Magnani et al., 2021), we assessed the models separately for every phase $i \geq 50\%$. We started at 50% of the stride to allow for using data during the preceding half of the gait cycle. We normalized strides based on left heel strikes. Normalizing strides from right heel strike to right heel strike provides complementary, but similar information and is hence not reported here. The value for the delay is unknown and may vary between individuals, so we optimized this across all phases in the gait cycles of a trial. This was done based on maximizing the goodness of fit of equations 5 and 6. To find the best fit, we fitted the models for all possible delays from 1 to 49 % of the gait cycle and retrieved the model that yielded the minimum, i.e., most negative, (sum of) regression coefficients. Models were fit separately for the frontal and sagittal plane. For every resulting model we checked for which phases of the gait cycle the model fit was significant.

Results

The distance between CoM' and CoP was strongly correlated to the horizontal ground reaction force for all participants, at both slow and normal speeds and in both anteroposterior and mediolateral directions (Figure 1).

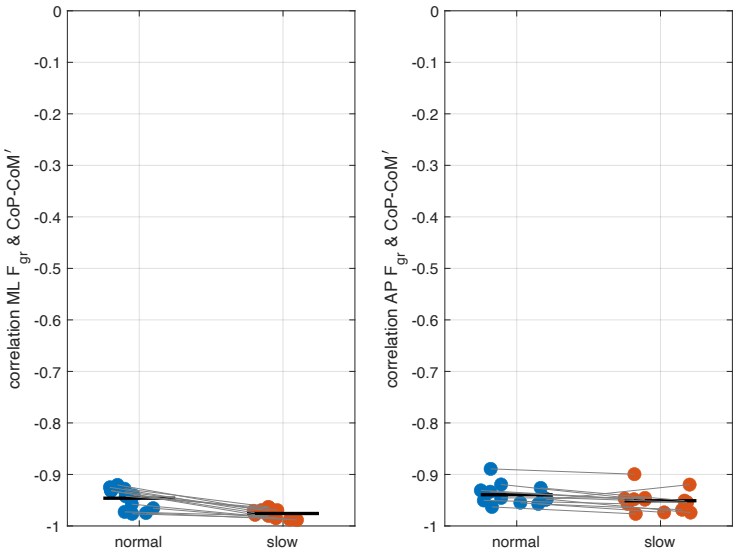

Figure 1. Coefficients of correlation between the mediolateral (left) and anteroposterior (right) ground reaction forces and the corresponding distances between CoM' and CoP in normal and slow walking are all close to -1. Each dot represents the coefficient of correlation for a single subject, the bar represents the mean correlation.

If $\boldsymbol{F_{gr}}$ is closely correlated to distance from CoM' to CoP, as in Figure 1, this may suggest that $\boldsymbol{H_{CoM}}$ is controlled to be (close to) zero, in line with suggestions in the literature (Herr and Popovic, 2008). Under the assumption that $\boldsymbol{H_{CoM}}$ is close to zero throughout the gait, which implies that $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ should be close to zero, we can readily predict the horizontal ground reaction force. Using a right handed coordinate system, where positive x is forward, positive y is upward, and positive z is right, we define $(\boldsymbol{r_{CoP}} - \boldsymbol{r_{CoM}}) = [x\ y\ z]'$. We assume that a correction of the linear momentum of the CoM is required in the fore-aft direction only and we assume y and thus F_y to be constant (mg). Thus $\boldsymbol{F_{gr}} = [F_x\ mg\ F_z]' = [F_x\ mg\ 0]'$, with F_x required to correct linear momentum according to equation 2. If the change in angular momentum is constrained to be zero as suggested (Herr and Popovic, 2008), equation 3 yields:

$$\begin{bmatrix} yF_z - zF_y \\ zF_x - xF_z \\ xF_y - yF_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ xF_y - yF_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad 7$$

This shows that, the horizontal ground reaction force can be linearly predicted by x, the distance between CoM' and CoP. Obviously, any deviation from $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}} = \boldsymbol{0}$ will cause an error in the prediction of the horizontal ground reaction force, with a maximum error equal to the instantaneous $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ divided by the vertical ground reaction force. In experimental data of walking, $\boldsymbol{H_{CoM}}$ is not equal to zero. Instead, it shows quasi-periodic patterns in the sagittal and frontal planes (Figure 2).

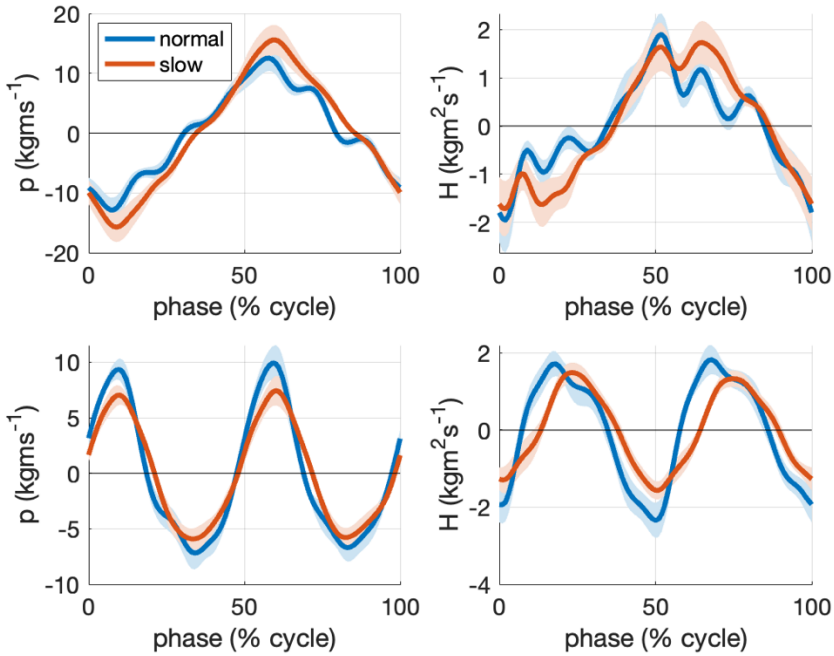

Figure 2. The linear (p; left) and angular (H; right) momenta in the frontal (top) and sagittal (bottom) planes averaged over strides and participants, show that in each plane these signals have similar frequency and phase at both speeds. The shaded areas indicate the confidence intervals.

Since linear and angular momentum show cyclic patterns with a similar frequency and phase (Figure 2), so do their derivatives, the horizontal ground reaction forces and the external moments in the corresponding planes. If linear and angular momentum have the same frequency and phase, equations 2 and 3, yield a negative linear relationship between $\boldsymbol{r_{CoP}}$-$\boldsymbol{r_{CoM'}}$ and the horizontal component of $\boldsymbol{F_{gr}}$ (Figure 3). With limited differences in phase (as visible in the bottom row of Figure 3), the correlation remains close to -1.

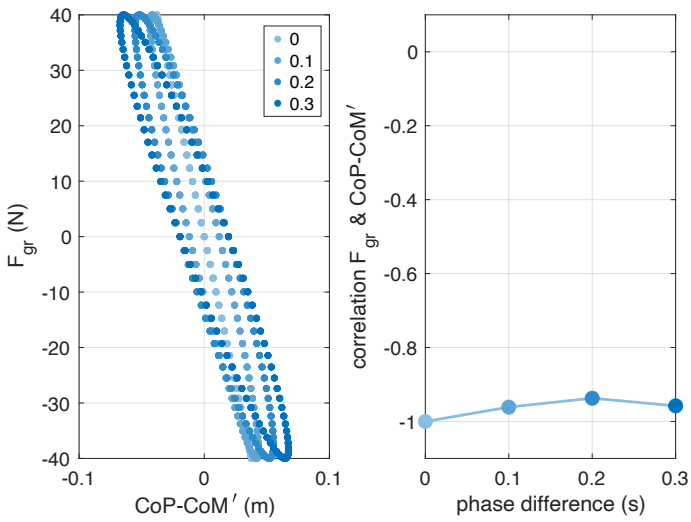

Figure 3. Results of simulations of equations 2 and 3. $\dot{\boldsymbol{p}}_{\boldsymbol{CoM}}$ and $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ were defined as 1Hz sinusoids with amplitudes of 40 N and 14Nm respectively (resembling data presented above). F_x was calculated from equation 2, F_y was considered constant and mass was set to 70 kg, F_z was set to zero. $\boldsymbol{F_{gr}}$ was inserted into equation3 to obtain CoP-CoM'. The results, show the strong negative correlation between $\boldsymbol{r_{CoP}}$-$\boldsymbol{r_{CoM'}}$ and $\boldsymbol{F_{gr}}$ and its limited sensitivity to differences in phase (ranging from 0 to 0.3 s) between $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ and $\boldsymbol{F_{gr}}$.

As is clear from equations 2 and 3, $\dot{\boldsymbol{p}}_{\boldsymbol{CoM}}$ and $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ are not independent, control of one of them constrains the control of the other. Previous successful identification of control models for the linear position and velocity of the CoM, reviewed in the introduction, and the preceding results combined indicate that linear and angular momentum are controlled simultaneously in human walking. Therefore, we fitted control models for linear and angular to data of human walking.

In the frontal plane, the control model for linear momentum suggests phase-dependent control with significant negative gains between 60 and 70% of the gait cycle (early single stance; Figure 4). The time delay for this model was estimated to be around 40% of the gait cycle (Table 1), corresponding to 450 and 610 ms, for normal and slow walking respectively. This indicates that these early stance phase corrections of linear momentum occurred based on information on CoM state around mid-stance of the preceding step. Walking speed did not have clear effects on estimated gains or delays expressed as percentage of the gait cycle.

The control model for angular momentum in the frontal plane (around a sagittal axis) indicates more or less continuous control with a relatively constant gain. The time delay was estimated at around 8% of the gait cycle, corresponding to about 90 and 120 ms for normal and slow walking respectively. Walking speed did not have clear effects on estimated gains (Figure 4) or delays expressed as percentage of the gait cycle (Table 1).

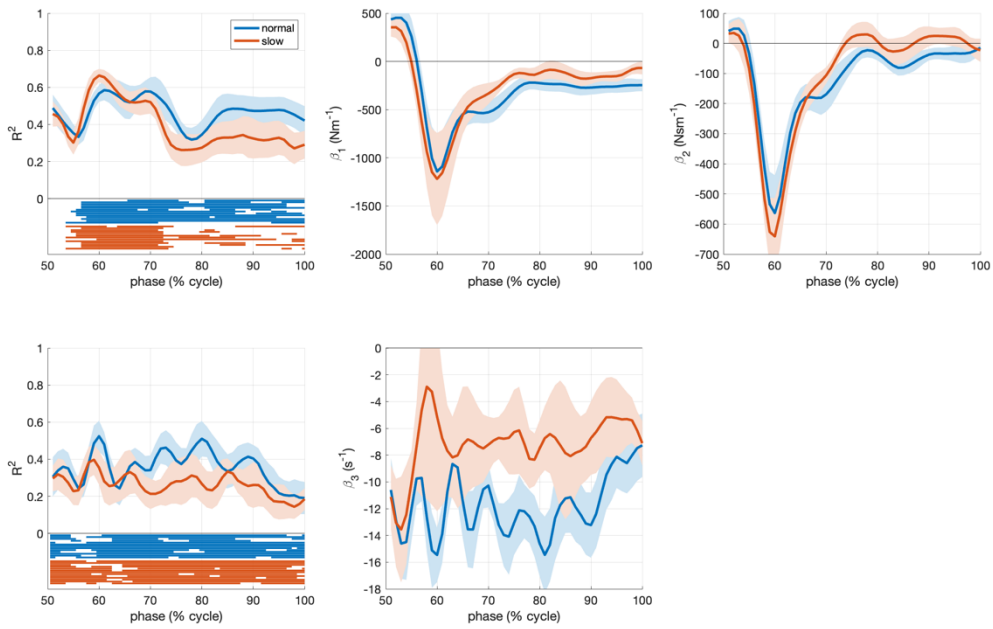

Figure 4. Top row: feedback model described in equation 5, relating the mediolateral ground reaction force at phase *i* in the second half of a stride to deviations in the mediolateral CoM position ($\boldsymbol{r}_{\boldsymbol{CoM}}$) and velocity ($\boldsymbol{v}_{\boldsymbol{CoM}}$) at a phase lead ∂_1 in normal and slow walking. The lines represent the averaged (over participants) phase-dependent goodness of fit (R^2) and feedback gains (β_1 and β_2), the shaded areas indicate the 95% confidence intervals. The horizontal lines in the left panels indicate for each individual participant and each speed when the model fit was significant with negative gains. Bottom row: feedback model described in equation 6, relating $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ at phase i in the second half of a stride to deviations in the angular momentum ($\boldsymbol{H}_{\boldsymbol{CoM}}$) in the frontal plane at a phase lead ∂_2 in normal and slow walking. The lines represent the averaged (over participants) phase-dependent goodness of fit (R^2) and feedback gain (β_3), the shaded areas indicate the 95% confidence intervals.

Table 1. Average (SD) of the delays in percentage of the gait cycle of the control models for linear and angular momentum (equations 5 and 6).

gait speed	plane	linear momentum delay ∂_1 (% gait cycle)	angular momentum delay ∂_2 (% gait cycle)

normal	frontal	45 (8)	8 (4)
slow	frontal	37 (11)	8 (3)
normal	sagittal	35 (12)	5 (1)
slow	sagittal	23 (11)	6 (4)

In the sagittal plane, the control model for linear momentum indicates strongly phase-dependent control with significant negative gains from 60 to 70% of the gait cycle (early single stance; Figure 5). Across speeds, the time delay for this model was estimated to range from 23 to 35% of the gait cycle (Table 1), corresponding to 260 and 540 ms for normal and slow walking respectively. This indicates that the early stance phase corrections of linear momentum occurred based on information on CoM state in mid to late stance of the preceding step. Estimated gains for CoM position were substantially smaller in slow than normal walking.

As in the frontal plane, the control model for angular momentum in the sagittal plane (around a frontal axis) suggests continuous control with a relatively constant gain. The time delay was estimated at averages of 5 and 6% of the gait cycle, corresponding to about 55 and 90 ms for normal and slow walking respectively. Walking speed did not have substantial effects on estimated gains (Figure 5) or delays (Table 1).

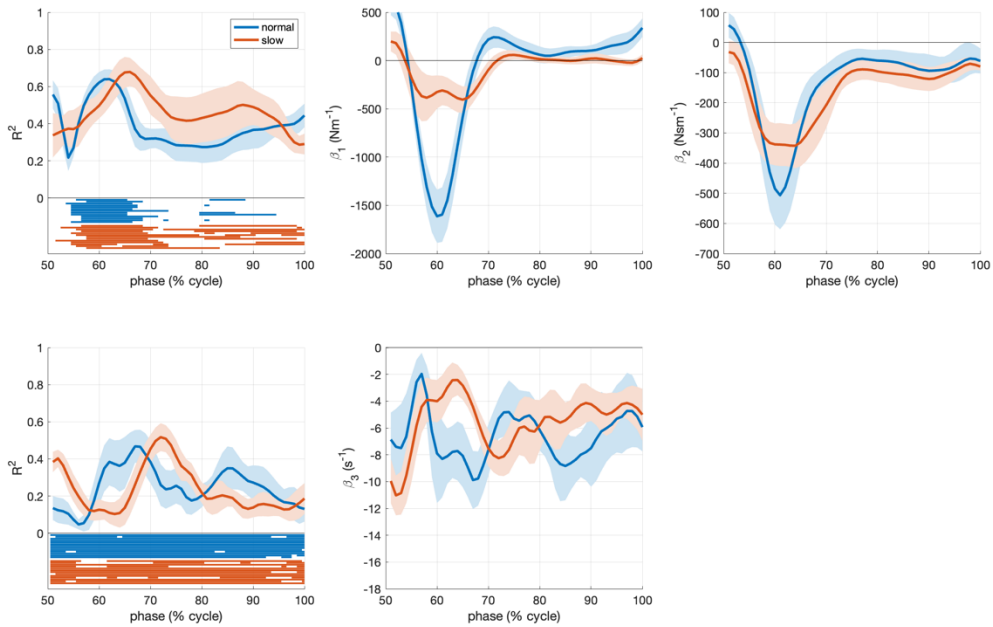

Figure 5. Top row: feedback model described in equation 5, relating the anteroposterior ground reaction force at phase *i* in the second half of a stride to deviations in the anteroposterior CoM position (r_{CoM}) and velocity ($\boldsymbol{v_{CoM}}$) at a phase lead ∂_1 in normal and slow walking. The lines represent the averaged (over participants) phase-dependent goodness of fit (R^2) and feedback gains (β_1 and β_2), the shaded areas indicate the 95% confidence intervals. Bottom row: feedback model described in equation 6, relating $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ at phase *i* in the second half of a stride to deviations in the sagittal plane angular momentum ($\boldsymbol{H_{CoM}}$) at a phase lead ∂_2 in normal and slow walking. The lines represent the averaged (over participants) phase-dependent goodness of fit (R^2) and feedback gain (β_3), the shaded areas indicate the 95% confidence intervals.

Discussion

As mentioned in the introduction, the horizontal location of the CoP relative to the CoM ($\boldsymbol{r_{CoP}}$-$\boldsymbol{r_{CoM'}}$) and related concepts like foot placement have been interpreted as reflecting actions to control whole-body linear momentum. We here argued that mechanically speaking this is incorrect. The use of the

vector between CoM' and CoP and foot placement appears to have been derived from models representing the human body by a mathematical inverted pendulum, see for example for standing (e.g., Morasso and Schieppati, 1999) and for walking (e.g., Townsend, 1985). A mathematical pendulum does not have an angular momentum around its CoM, hence the ground reaction force points at the CoM, or in other words the external moment is always zero. The human body has a distributed mass, and hence it also has a time-varying angular momentum in standing and walking. As such, for adequate control of gait it is necessary that both linear and angular momentum are controlled. Interestingly, the distance between CoM' and CoP has successfully been used to predict linear CoM acceleration from experimental data of human standing (Morasso et al., 1999). Here we showed that also in data from human walking the distance between CoM' and CoP is strongly predictive of the horizontal ground reaction force (Figure 1).

The success of using the vector $\boldsymbol{r_{CoP}}$-$\boldsymbol{r_{CoM'}}$ and related concepts is likely a consequence of constraints arising from the leg's anatomy, force transmission between the foot and the surface, and the presence of a large vertical force component (on average equalling body weight). This means that the variance in direction of the forces applied to the ground by a foot in a particular position of the limb is limited. For example, when standing upright, it is impossible to exert a substantial amount of medio-lateral force on the ground. When a substantial amount of medial ground reaction force is required, one must move the foot laterally, resulting in a shift in CoP. Obviously, this generally results not only in a change of $\dot{\boldsymbol{p}}_{\boldsymbol{CoM}}$, but also in a change of $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$. This is the essence of our idea that simultaneous control of linear and angular momentum is necessary (but not sufficient) and underlies the successful experimental identification of both control processes (Figures 4 and 5).

As can be seen from equations 2 and 3, both linear and angular momentum depend on the (sum of) external forces. In terms of underlying degrees of freedom, linear momenta of body segments affect whole-body linear momentum and linear and angular momenta of all segments affect whole-body angular momentum. So, also in this respect linear and angular momentum are not independent. To stabilize upright posture, linear and angular momentum must be controlled simultaneously through moments exerted on body segments. The first term of equation 1 has been related to the effects of ankle moments and foot placement as both affect $\boldsymbol{r_{CoP}}$ and $\boldsymbol{F_{gr}}$ (Hof, 2007; van Leeuwen et al., 2022). However, controlling both $\boldsymbol{r_{CoP}}$ and $\boldsymbol{F_{gr}}$ such that $\dot{\boldsymbol{H}}_{\boldsymbol{CoM}}$ is also controlled cannot be achieved by a single joint moment, as this would leave one moment to control two outputs in each plane. Similarly, foot placement can be considered as a single control variable for each plane and is thus insufficient to control two outputs. As illustrated in the results section, when a change in linear momentum is needed and hence a change in $\boldsymbol{F_{gr}}$, a simultaneous change of the CoP is needed to avoid an undesired change in $\boldsymbol{H_{com}}$. From this perspective, control gain β_3 will depend on gains β_1 and β_2. Of course, this argument can be reversed. Shifting the CoP relative to the CoM allows decoupling of the control of linear and angular momentum. In fact, this may explain why deviations in linear momentum appear to be corrected mainly in early stance. By choosing an appropriate foot placement horizontal forces can be generated to correct linear momentum without causing deviations in angular momentum.

The control models of equations 5 and 6 do not necessarily reflect neural feedback. Passive mechanics may contribute to coupling of the ground reaction force and the external moment to the preceding state of the body (Patil et al., 2019) and will have a delay that does not reflect the time needed for neural transmission. In fact, this can even occur without delay. The delays identified in the control of angular momentum were short and may reflect a combination of fast neural feedback and intrinsic stiffness and damping. On the other hand, for linear momentum delays were long, indicating corrections of linear momentum in early stance. This may reflect that linear momentum can be corrected in this phase of the gait cycle without undue consequence for angular momentum by choosing an appropriate foot placement location. But, also passive mechanics, specifically effects of collision forces at foot placement, can show up as delayed effects. This may be especially relevant for

foot placement in the sagittal plane. Collision forces can provide stability to passive dynamic walkers in the sagittal plane (Mcgeer, 1990) but not in the frontal plane (Bauby and Kuo, 2000). Effects of collision forces, mechanically caused by CoM states right before the collision, would be manifested in early stance. Maximum gains for linear momentum control were found in this phase of the gait cycle. Moreover, position gains in the sagittal plane appeared to be speed dependent, which could be accounted for by differences in collision forces. Experiments involving sensory manipulations may reveal to what extent the control models reflect neural feedback based on sensory information.

Given the effect of the CoM' to CoP vector, equations 5 and 6 do not necessarily describe the data equally well (c.f. Figures 4 and 5). Less good fit of either model might indicate that one variable is controlled at the cost of the other. However, one needs to consider that not only the model degrees of freedom are different, also the signal-to-noise ratios of estimates of CoM states and $\boldsymbol{H_{CoM}}$ may be different. Given the many kinematic degrees of freedom, it is not clear how simultaneous control of angular and linear momentum is realized. Multiple solutions allow control of linear and angular, but not all of these will be desirable, from for example an energetic perspective, or even be in keeping with the aim of locomotion.

We showed that corrections of linear momentum are constrained by angular momentum. Obviously, the reverse also holds, corrections of angular momentum may affect linear momentum. We conclude that studying the control of linear momentum and angular momentum separately has limitations, because 1) linear and angular momentum are mechanically coupled as both depend on external forces (like $\boldsymbol{F_{gr}}$) and 2) due to anatomical constraints and foot-ground interactions, controlling linear and angular momentum generally requires adjusting leg position. Experimental data show that control of linear momentum may be sacrificed to control angular momentum. For example, after a trip, which induces a substantial change in angular momentum, recovery responses caused a further perturbation of forward linear momentum, but restored angular momentum (Pijnappels et al., 2004). Conversely in standing on a narrow beam, where shifts of the CoP are constrained, linear momentum is controlled through hip moments, allowing changes in angular momentum (Otten, 1999).

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