

A Bayesian decision-making model of implicit motor learning from internal and external errors

Hyosub E. Kim^{1,2,*}, Romeo Chua¹, and Davin Hu¹

¹School of Kinesiology, The University of British Columbia, Vancouver, B.C., Canada

²Graduate Program in Neuroscience, The University of British Columbia, Vancouver, B.C., Canada

*For correspondence: hyosub.kim@ubc.ca

Abstract

A key challenge for the sensorimotor system is deciding which errors to learn from and which to ignore. Recent work has shown that humans are remarkably precise in parsing movement errors into internally- and externally-generated components for this purpose: Participants automatically ignore internally-generated reaching errors caused by motor noise, yet implicitly adapt to size-matched externally-generated errors caused by visuomotor rotations (Ranjan & Smith 2018, 2022). Following replication of these results with 16 neurotypical adults, we formalized our understanding of this behavior with a novel Bayesian decision-making model. The Parsing of Internal and External Causes of Error (PIECE) model frames adaptation as a process of causal inference regarding the source of error, with the magnitude of motor corrections reflecting a combination of state estimation and the observer's degree-of-belief that their movement was externally perturbed. Thus, PIECE presents a challenge to a class of computational models that frames adaptation as a process of re-aligning the perceived hand position with the movement goal. We show that only PIECE can capture the precise parsing of internal versus external errors observed. Combined, this work provides a normative explanation of how the nervous system discounts intrinsic motor noise and adapts to perturbations, keeping movements finely-calibrated.

23 Introduction

24 Not all motor errors are created equal. Imagine reaching for your morning coffee, but instead of your fingers
25 landing gracefully on the mug, they unintentionally touch down with more force than intended and tip it.
26 This error could be the result of intrinsic motor variability (i.e., noise), the random internally-generated
27 fluctuations in movement parameters that occur even during overlearned movements like reaching. In
28 such a case, it would be counterproductive for your motor system to recalibrate its sensorimotor mapping.
29 Adapting to motor noise, or internally-generated error (IGE), which by definition is unpredictable, would
30 result in highly unstable behavior and potentially amplify the magnitude of future errors even during
31 unperturbed movements (Chaisanguanthum et al., 2014; Faisal et al., 2008; van Beers, 2009; van Beers
32 et al., 2013). On the other hand, if your reach is inaccurate because of an external perturbation, such as
33 the extra inertia from a heavy watch that you just put on, then it is critical to recalibrate your sensorimotor
34 mapping in order to account for the added weight and keep future movements accurate and precise. The
35 sensorimotor system is constantly tasked in this manner with deciding which errors to learn from and
36 which to ignore.

37 While from a strictly logical viewpoint, it is clear the motor system should not adapt to motor noise
38 but should learn from externally-generated error, what is the empirical evidence that this is actually
39 the case? Perhaps the strongest evidence comes from a remarkable study by Ranjan & Smith which
40 showed that humans are able to accurately parse total movement error into its constituent parts: the
41 error component due to motor noise, or internally-generated error (IGE), and the error component due
42 to an external perturbation, or externally-generated error (EGE). In their study, participants experienced
43 small, randomized visuomotor rotations in which the cursor feedback was rotated relative to their actual
44 hand trajectory by $\pm 2^\circ$ or $\pm 4^\circ$ on every other trial (Ranjan, 2022; Ranjan and Smith, 2018). Despite the
45 distribution of rotations (EGEs) being matched to baseline motor variability, participants demonstrated
46 robust implicit single-trial adaptation to only the externally-generated component of the total error while
47 effectively ignoring their IGE. That is, immediately following a perturbation trial, the subsequent adaptive
48 response was opposite in direction and proportional to the rotation size (EGE), while also statistically
49 independent of the IGE magnitude. Consistent with this finding, a study of saccadic eye movements also
50 reported more robust adaptation to EGE than IGE (Collins and Wallman, 2012), and we have also shown
51 that visually-clamped errors (Morehead et al., 2017) as small as 1° elicit robust adaptation (Kim et al.,
52 2018). Combined, these studies indicate that, for the motor system, the source of the error determines the
53 implicit adaptive response.

54 According to standard theories of adaptation, the mismatch between predicted and actual sensory
55 consequences of a motor command, called a “sensory prediction error”, is the main driver of implicit
56 adaptation (Kim et al., 2018, 2019; Mazzoni and Krakauer, 2006; Morehead et al., 2017; Shadmehr et al.,
57 2010; Tseng et al., 2007). The neurophysiological underpinning of sensory predictions is the efference copy,
58 also known as “corollary discharge” (Carriot et al., 2013; Sommer and Wurtz, 2008; Sperry, 1950; Von Holst
59 and Mittelstaedt, 1950), which refers to the copy of the motor command that gets sent from motor cortices
60 to subcortical and sensory regions of the brain. Importantly, computational theories of adaptation typically
61 assume that the sensory prediction is centered on the motor goal, or more specifically, the explicit aiming
62 location, i.e., the path the end effector would travel in the absence of any IGE or adaptation (Mazzoni and
63 Krakauer, 2006; Taylor and Ivry, 2011; Taylor et al., 2014). However, Ranjan & Smith’s study provides
64 compelling behavioral evidence that the sensory prediction also contains information regarding the amount
65 of central motor noise associated with each motor command (Churchland et al., 2006). They argue that the
66 brain generates a highly accurate sensory prediction of the hand’s actual trajectory (minus any peripheral
67 contributions to movement error), rather than the intended trajectory to the target, which explains how
68 the motor system is able to effectively cancel out IGE from the total error. This refined definition of SPE
69 provides a plausible explanation of how the motor system parses total movement error, but what remains
70 to be found is an overarching computational-level theory that explains this behavior.

71 The aim of the current study was to understand the computational principles underlying such discrim-
72 inative and finely-tuned adaptive responses. In parallel with replication of the main results of Ranjan and
73 Smith, we formalized our understanding of this behavior by developing a novel Bayesian model, referred
74 to as the Parsing of Internal and External Causes of Error (PIECE) model. Similar to prior Bayesian
75 models of motor adaptation (e.g., Wei and Körding, 2009) and error detection (Gaffin-Cahn et al., 2019),
76 causal inference regarding whether the feedback was perturbed or not is central to the PIECE model.
77 However, PIECE is structurally different from these models in that it assumes that, in addition to vision
78 and proprioception, the observer also has access to an efference copy-based cue of the motor command
79 which includes associated central motor noise (Ranjan and Smith, 2018; Szarka et al., 2024). The PIECE
80 model conceptualizes adaptation as a process of utilizing all three cues to form a posterior belief regarding
81 the presence (or absence) of a perturbation. Critically, the adaptive motor output reflects the estimated
82 perturbation size, which is inferred by the observer by combining their prior on the perturbation with their
83 visual likelihood, weighted by this degree of belief. This framework runs counter to a powerful class of
84 computational models that frames adaptation as a process of aligning the perceived hand position with

85 the movement goal (Tsay et al., 2022; Wei and Körding, 2009; Zhang et al., 2024). The present study
86 challenges such a view, as only the PIECE model could accurately capture the precise parsing of IGE and
87 EGE observed. Our work instead supports a normative view of implicit adaptation in which prediction,
88 perception, and error correction are unified.

89 Results

90 Differential adaptation to IGE-EGE

91 We began our study by attempting to replicate the main findings of Ranjan and Smith regarding the senso-
92 rimotor system's decomposition of total error into internally- and externally-generated components (Ranjan
93 and Smith, 2018). We tested 16 healthy, neurotypical young adults on a visuomotor task that involved fast
94 point-to-point reaches to a visual target while controlling a small white cursor, which, depending on the
95 trial, could be rotated by 0° , $\pm 2^\circ$, or $\pm 4^\circ$. These rotations were pseudorandomized across the experimental
96 block (Fig. 1a shows the experimental set-up) and were always surrounded by null (no rotation) trials. For
97 our analyses, the overall motor error was decomposed into its internally- and externally-generated parts.
98 The internally-generated error (IGE) is defined as the random error that occurs on every reach (i.e., any
99 angular deviation of the hand trajectory from the target), and on unperturbed reaches, serves as the only
100 contribution to total error. IGE is the result of intrinsic motor variability plus any intrinsic bias in reach
101 direction (Wang et al., 2024). Externally-generated errors (EGE) are caused by external perturbations
102 (e.g., visuomotor rotations) and are under experimental control (Fig. 1b). In this experiment, EGEs were
103 defined as the visuomotor rotations, and thus small and statistically independent of IGE. The intention
104 behind utilizing such small perturbations was to keep IGE and EGE on an equal footing in terms of their
105 magnitudes. With each perturbation trial being surrounded by null trials, adaptation was quantified as
106 the difference in reach angle between the trial immediately following versus immediately preceding the
107 perturbation trial: $\text{adaptation}_t = \text{hand}_{t+1} - \text{hand}_{t-1}$, where t indexes the perturbation trial. Combined,
108 these methods allowed us to easily dissociate the influence of IGE versus EGE on implicit adaptation on
109 a trial-by-trial basis (see Data Analysis).

110 In Fig. 1c-d we present data from an example participant. Separately plotting adaptation to EGE and
111 IGE shows a clear dichotomy in the responses to these two different types of error. Consistent with the
112 findings of Ranjan & Smith, there is robust adaptation to the externally imposed visuomotor rotations but
113 no evidence of adaptation to spontaneously generated errors due to motor noise, which spanned a similar

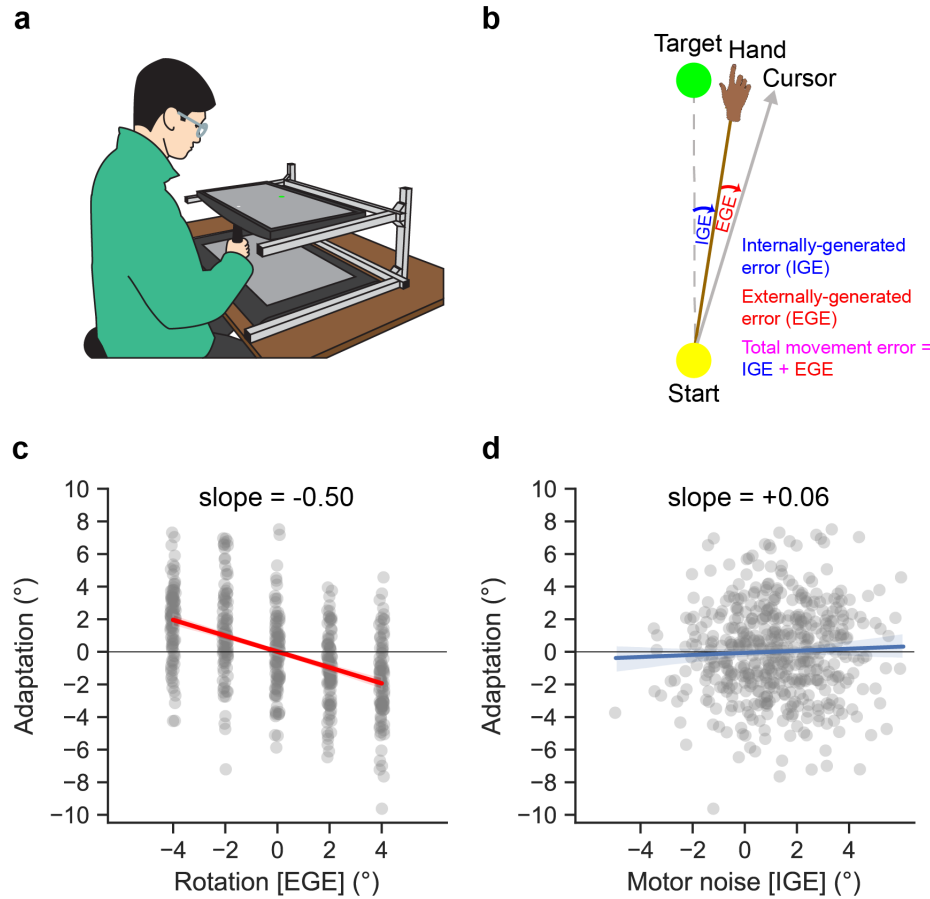


Figure 1: (a) Basic experimental set-up involved participants making quick point-to-point reaches by sliding a stylus along a graphics tablet. Participants could not see their hand, as experiments were conducted in a darkened room and the horizontally-oriented monitor blocked vision of their hand. (b) Schematic showing how we operationally defined the two sub-components of the total error. The internally-generated error (IGE) is equivalent to motor noise, or how far off the intended aim (i.e., the target) the reach was. The externally-generated component (EGE) refers to the external perturbation—in this case, the magnitude of the visuomotor rotation. (c) Individual participant data showing adaptation as a function of EGE (gray dots represent single-trial adaptive responses). There is a clear, distinct response to EGE. (d) When the same adaptive responses are plotted as a function of IGE, there is no discernible relationship between the variables. Note that the data are shifted slightly rightwards due to a small counter-clockwise reaching bias. The shaded regions in (c) and (d) represent bootstrapped 95% CIs (difficult to see due to low variance).

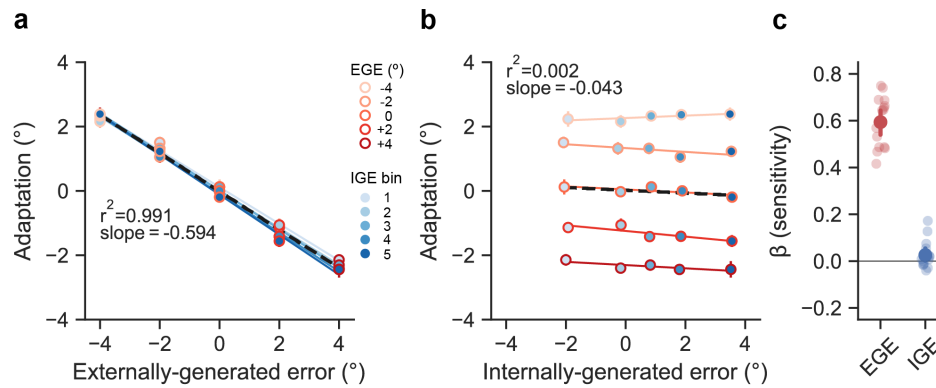


Figure 2: (a) Population-averaged adaptive responses were binned based on the level of EGE (shade of red) and IGE (shade of blue). The filled-in circles represent the mean of each quintile and are plotted as a function of EGE in (a), and as a function of IGE in (b) (data are shifted right due to a small counter-clockwise reaching bias of less than 1° across participants). The vast majority of the variance in adaptive responses (99.1%) was explained by EGE. (c) Linear regression coefficients from an analysis of unbinned data (signs are flipped for ease of comparison). Error bars represent bootstrapped 95% CIs and are quite small in (a) and (b). More translucent dots in (c) represent individual participants.

114 range as the EGEs. This dissociation between adaptive responses to EGE versus IGE was consistent across
 115 our entire sample.

116 We performed a similar set of group-level analyses as Ranjan & Smith to better understand the
 117 population-averaged adaptive responses to errors. Briefly, we first binned the data based on the level
 118 of EGE, and then for each of the five levels of EGE, we further binned the data based on the level of IGE
 119 into quintiles (see Methods). This procedure was applied separately for each participant. The results of
 120 these analyses are presented in Fig. 2a-b, where each point represents the average across every partici-
 121 pant's mean response for the corresponding EGE/IGE level. As seen in Fig. 2a, there was a clear linear
 122 response to EGE (black dashed line, mean slope= -0.594 , bootstrapped 95% CI: $[-0.643, -0.545]$, $r^2=0.991$,
 123 $p < 10^{-24}$), with over 99% of the variance in the EGE/IGE grid being explained by the level of EGE.
 124 In stark contrast, Fig. 2b shows that there was no systematic relationship between adaptation and IGE,
 125 with less than 0.2% of the variance in the EGE/IGE grid being explained by IGE (dashed black line, mean
 126 slope= -0.043 $[-0.071, 0.000]$, $r^2=0.002$, $p=0.82$).

127 To further quantify the impact of EGE versus IGE on implicit adaptation, we performed a bivariate
 128 regression analysis of the unbinned data from each participant, using EGE and IGE as predictor variables.
 129 The coefficients of each predictor (sign-flipped for more convenient comparisons) were markedly different
 130 (mean difference = 0.577 , 95% CI for difference scores: $[0.508, 0.645]$; $t_{15} = 17.9$, $p < 10^{-10}$), reflecting the
 131 high sensitivity of each participant to EGE ($0.599[0.550, 0.648]$, $t_{15} = 23.1$, $p < 10^{-12}$), with correspondingly

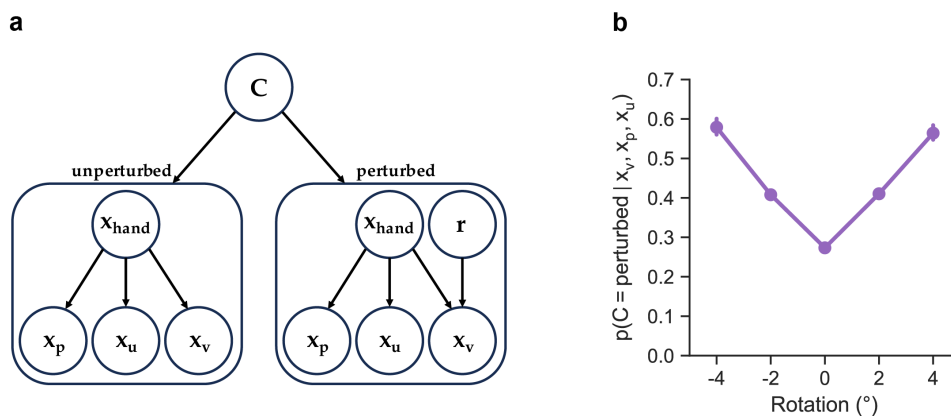


Figure 3: (a) Generative model for PIECE. In this model, the observer uses proprioceptive (x_p), motor predictive (x_u), and visual cues (x_v) to compute posterior probabilities of C , the causal node, which determines whether the feedback was perturbed or not. In the unperturbed case visual feedback is a function of the actual hand position only, whereas in the perturbed case visual feedback is a function of both the actual hand position, x_{hand} , and the rotation, r . On a trial-by-trial basis, the posterior over C weights the observer’s posterior estimate of r , which is computed by combining the prior on the rotation and the likelihood associated with the visual measurement. (b) The posterior, $p(C = \text{pert} | x_v, x_p, x_u)$, as a function of the rotation size (EGE) for the same participant shown in Fig. 1. This function was derived by first finding the maximum-likelihood estimates (MLEs) of this participant’s data with PIECE (i.e., the parameter set that maximizes the probability of their observed data), and then using these MLEs to simulate behavior across the experimental protocol and recording the posterior estimates.

132 low sensitivity to IGE (0.022[−0.004, 0.052]; $t_{15} = 1.51, p = 0.153$). In line with the original results of
 133 Ranjan & Smith, our behavioral results highlight the motor system’s clear parsing of total movement error
 134 into internally- and externally-generated components, effectively discounting the former and learning from
 135 the latter.

136 **The Parsing of Internal and External Causes of Error (PIECE) Model**

137 The primary purpose of the current study was to understand the computational principles underlying the
 138 motor system’s incredibly accurate parsing of movement error into IGE and EGE. Towards this aim, we
 139 have developed a Bayesian model that optimally combines all available sensory cues in order to form a
 140 posterior belief regarding the presence (or absence) of a perturbation, with motor output reflecting the
 141 weighting of the estimated perturbation size by this degree of belief. As seen in the graphical generative
 142 model of Fig. 3, the observer has access to cues regarding hand position from vision (x_v), proprioception
 143 (x_p), and an efference copy-based sensory prediction of hand position (x_u), which we refer to as a “motor
 144 prediction” since it stems from the motor command. The proprioceptive and motor prediction cues are
 145 unbiased estimates of the actual hand position, x_h . That is, $x_p \sim N(x_h, \sigma_p^2)$, and $x_u \sim N(x_h, \sigma_u^2)$. During

146 perturbed trials with a visuomotor rotation, the visual cue is offset from the actual hand location by the
147 size of the visuomotor rotation, $x_v \sim N(x_h + r, \sigma_v^2)$, while, by definition, $r = 0$ during unperturbed trials.
148 Based on a recent adaptation study that showed that participants' visual uncertainty increases as a function
149 of the error size, we also assume the observer's visual uncertainty, σ_v , increases as a linear function of the
150 distance of the feedback cursor from the target (i.e., the target error, e): $\sigma_{v,t} = \alpha + \beta \cdot e_t$, where t indexes
151 the trial number (Zhang et al., 2024).

152 With respect to the other nodes in the generative model, the observer's prior on hand position, x_h , is
153 normally distributed: $x_h \sim N(b, \sigma_h^2)$, where b represents any directional bias in reach angle relative to the
154 target (Wang et al., 2024; see Fig. 2b) and σ_h is equal to the participant's intrinsic motor variability. This
155 prior on x_h reflects the fact that human participants reach in a manner consistent with internal knowledge
156 of the distribution of their intrinsic motor noise (Trommershauser et al., 2003; Trommershäuser et al.,
157 2008). The observer also has a prior for the rotation magnitude, $p(r) \sim N(0, \sigma_r^2)$. We assume the observer
158 believes the distribution of perturbations to be normally distributed, with σ_r representing the range of
159 perturbation sizes they believe is plausible. The prior used by the observer on the causal node, which
160 reflects their degree of belief in whether the reach was perturbed or not, matches the statistics of the task
161 and is flat: $p(C = \text{perturbed}) = p(C = \text{unperturbed}) = 0.5$.

162 For our model-fitting, we calculated each participant's motor variability during the last 50 trials of
163 baseline reaches and used this value for σ_h . To further limit the number of free parameters, rather than
164 fitting α and β of the visual uncertainty function, we used the reported mean values from the study by
165 Zhang and colleagues (see Table S1 from Zhang et al., 2024). In total, PIECE has 3 free parameters: σ_r , b
166 and σ_{combined} , the latter representing the standard deviation that results from combining random variables
167 x_p and x_u . Briefly, while the observer may have access to separate proprioceptive and motor prediction
168 cues, from the experimentalist's perspective, x_p and x_u are not dissociable due to their both being internal
169 measurements centered on the true hand location. See Modeling Analysis for the derivation of σ_{combined}
170 from x_p and x_u .

171 Causal inference in PIECE

172 Beginning with Bayes' Rule, the observer combines the prior, $p(C)$, and the likelihood, $p(x_v, x_p, x_u|C)$, to
173 compute the posterior for the causal node, C :

$$p(C|x_v, x_p, x_u) = \frac{p(C)p(x_v, x_p, x_u|C)}{p(x_v, x_p, x_u)} \quad (1)$$

174 As in most Bayesian cue combination models, we assume the cues are conditionally independent and
 175 their respective Gaussian distributions (likelihoods) are multiplied. Performing causal inference and com-
 176 puting the posterior for the unperturbed case (abbreviated as “ \neg pert” below) requires marginalization over
 177 the hand position, x_h :

$$p(C = \neg\text{pert}|x_v, x_p, x_u) \propto p(C = \neg\text{pert})p(x_v, x_p, x_u|C = \neg\text{pert}) \quad (2)$$

$$= p(C = \neg\text{pert}) \int p(x_v|x_h)p(x_p|x_h)p(x_u|x_h)p(x_h|C = \neg\text{pert})dx_h \quad (3)$$

The posterior for the perturbed case (abbreviated as “pert”) requires marginalization over the hand posi-
 178 tion, x_h , and the rotation magnitude, r :

$$p(C = \text{pert}|x_v, x_p, x_u) \propto p(C = \text{pert})p(x_v, x_p, x_u|C = \text{pert}) \quad (4)$$

$$= \iint p(C = \text{pert})p(x_h|C = \text{pert})p(r|C = \text{pert})p(x_v|x_h, r)p(x_p|x_h)p(x_u|x_h)drdx_h \quad (5)$$

$$= p(C = \text{pert}) \iint p(x_h|C = \text{pert})p(r|C = \text{pert})p(x_v|x_h, r)p(x_p|x_h)p(x_u|x_h)drdx_h \quad (6)$$

179 The denominator in Eqn. 1 (i.e., the “marginal likelihood”) serves as a normalization factor that is
 180 common to both $C = \neg$ pert and $C = \text{pert}$ and thus does not need to be formally computed. Instead, as a
 181 final step, we can divide the unnormalized posteriors for unperturbed and perturbed world states by their
 182 sum to normalize them.

183 The posterior on C defines the observer’s relative degrees of belief in both the unperturbed and per-
 184 turbed world states. Fig. 3b shows the posterior on $C = \text{pert}$ as a function of the rotation size (EGE) for
 185 the same participant shown in Fig. 1c-d. As seen in this figure, the observer assigns more credibility to the
 186 hypothesis that their reach was perturbed as EGE increases. However, some degree of belief gets assigned
 187 to the perturbed hypothesis even during unperturbed trials, highlighting the probabilistic nature of motor
 188 adaptation (Berniker and Kording, 2008). In the PIECE model, these degrees of belief are combined with
 189 optimal state estimation of the perturbation magnitude.

190 State estimation in PIECE

191 The observer updates their state estimate of the perturbation with each observation (measurement). This
 192 *estimate* is denoted \hat{r} , whereas r represents the true perturbation, which is unknown to the observer.
 193 For this state estimation step, we assume the observation on a given trial, z_t , which is distributed as
 194 $\sim N(r, \sigma_{v,t}^2)$, is an unbiased measurement of the perturbation on that trial (i.e., the actual rotation size;
 195 Burge et al., 2008; Haith and Krakauer, 2013; Wei and Körding, 2010), as opposed to $x_{v,t}$, which is a
 196 function of the perturbation and the true hand location. Since the perturbations varied randomly in this
 197 experiment, we assumed the trial-by-trial effects were independent of each other and that there was no
 198 memory across trials (Ranjan, 2022; Zhang et al., 2024).

199 To update their estimate of the rotation size, the observer should combine their measurement with
 200 their prior on the rotation. Doing so leads to the proportion of the error that is corrected for on each
 201 trial being specified by K_t (Eqn. 10), where $\frac{1}{\sigma_{v,t}^2}$ and $\frac{1}{\sigma_r^2}$ represent the measurement and prior precisions,
 202 respectively (Burge et al., 2008; Haith and Krakauer, 2013; Korenberg and Ghahramani, 2002). The key
 203 intuition here is that the learning rate (K_t) should decrease with increasing visual uncertainty ($\sigma_{v,t}$), and
 204 conversely, increase the more volatile the observer believes the environment is, represented by a higher
 205 value of σ_r .

206 Taken together, the overall state estimate of the perturbation on trial t , \hat{r}_t , is a linear combination of
 207 the state estimates under the unperturbed and perturbed world state hypotheses:

$$\hat{r}_t = p(C = \text{-pert} | x_{v,t}, x_{p,t}, x_{u,t}) \hat{r}_{\text{-pert},t} + p(C = \text{pert} | x_{v,t}, x_{p,t}, x_{u,t}) \hat{r}_{\text{pert},t} \quad (7)$$

208 In the unperturbed world state, there is no perturbation and so $r = 0$ by definition. Thus, the
 209 contribution of $\hat{r}_{\text{-pert}}$ to the overall state estimate is zero. The full expression for how much the observer
 210 adapts their overall state estimate, \hat{r}_{t+1} , is therefore expressed as follows:

$$\hat{r}_{t+1} = p(C = \text{pert} | x_{v,t}, x_{p,t}, x_{u,t}) \cdot K_t(z_t), \quad (8)$$

$$(9)$$

211 where

$$K_t = \frac{\frac{1}{\sigma_{v,t}^2}}{\frac{1}{\sigma_{v,t}^2} + \frac{1}{\sigma_r^2}} \quad (10)$$

212 Lastly, we assume the observer’s motor output is a reflection of their estimate of the perturbation (thus
213 opposite-signed) plus their intrinsic motor bias and motor noise:

$$x_{h,t+1} = -\hat{r}_{t+1} + b + \epsilon_{t+1} \quad (11)$$

$$\epsilon \propto N(0, \sigma_h^2), \quad (12)$$

214 where σ_h is measured during baseline reaches.

215 As expressed in Eqn.11, the observer’s motor output reflects the combination of the estimated rotation
216 size and the strength of their belief in the presence of an external perturbation. Thus, by combining
217 Bayesian cue combination, causal inference, and state estimation, the PIECE model unifies competing
218 notions of the computational goal of implicit adaptation.

219 **Model-based analyses**

220 We evaluated the performance of four computational models of adaptation on our behavioral data: PIECE,
221 the Proprioceptive Recalibration Model (PReMo) (Tsay et al., 2022), the Perceptual Error Adaptation
222 Model (PEA) (Zhang et al., 2024), and the Relevance Estimation Model (REM) (Wei and Körding, 2009).
223 The models competing with PIECE, collectively referred to here as “hand-to-target alignment” models,
224 were chosen because of their previously demonstrated ability to explain a wide range of adaptation phe-
225 nomena, the fact that they are among the most prominent Bayesian, or Bayesian-inspired, models of
226 adaptation, and, importantly, because they each frame the computational goal of adaptation as one of
227 aligning the perceived hand position with the movement goal. While the development of the PIECE model
228 was informed by these other models, in stark contrast to them, PIECE posits the computational goal of
229 adaptation as being one of correcting for the externally-generated error (EGE). By objectively comparing
230 the performance of these four models, we can adjudicate between the hand-to-target versus error correction
231 views of adaptation.

232 The high-level intuition of PReMo is that mismatches between visual, proprioceptive, and efference
233 copy-based cues result in a recalibrated estimate of the “felt” hand position, and implicit adaptation
234 serves to minimize any misalignment between this perceived hand position and the motor goal, or target.
235 PEA also posits adaptation as being driven by a perceptual error. The derivation of the perceived hand
236 position in PEA follows optimal integration principles (Ernst and Banks, 2002; Landy et al., 1995), and
237 as mentioned earlier, the model assumes visual uncertainty increases with error size. REM is closest in

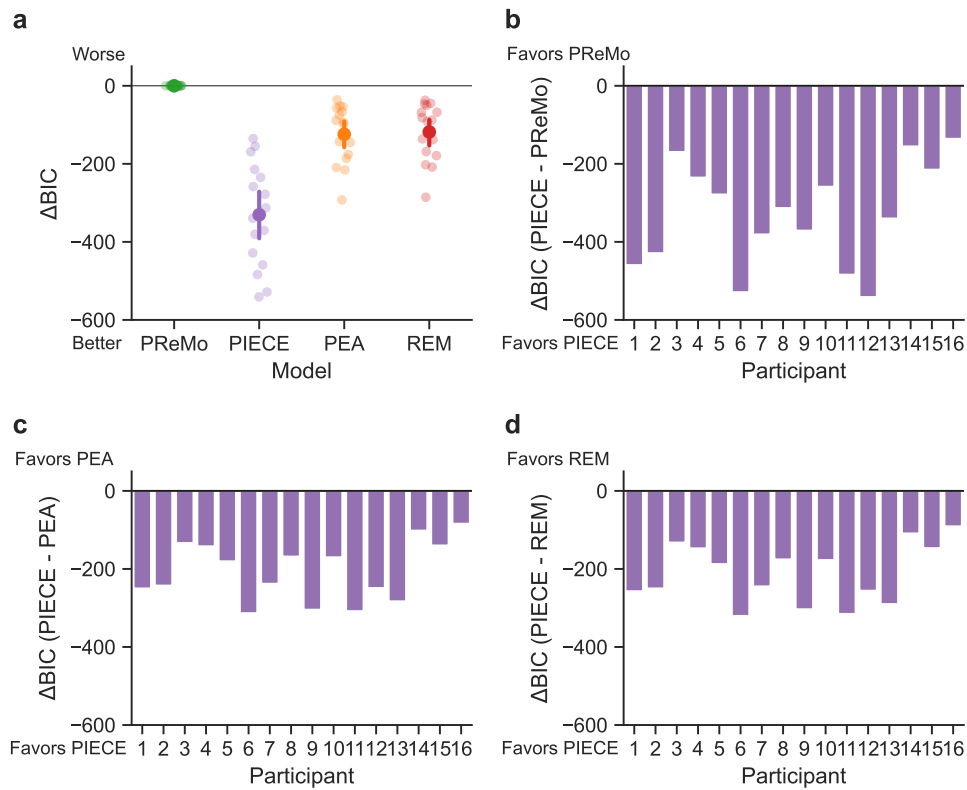


Figure 4: (a) Direct comparison of BIC scores of each model to PReMo's BICs. Lower scores indicate a better model fit. (b-d) For all 16 participants, PIECE outperformed each of the other three models.

238 structure to PIECE, requiring a causal inference step. However, similar to PReMo and PEA, REM also
239 assumes realignment of the estimated hand position with the target as the primary goal of adaptation.

240 We started our analyses by using maximum likelihood estimation to separately fit each individual's
241 data with all four models. After finding the set of parameters that maximized the probability of each
242 participant's data, we computed and compared Bayesian Information Criterion (BIC) scores (Schwarz,
243 1978). Fig. 4 shows the results of this analysis, with panel A showing ΔBIC scores for each model when
244 benchmarked to PReMo's BIC scores for each individual. (We chose PReMo as the primary comparison
245 model based on our involvement in its development and also because it is most explicit in promoting the
246 hand-to-target alignment idea.) Clearly, PIECE had the lowest (best) BIC scores, indicating that out of
247 all of the models it most successfully captured the parsing of total movement error into IGE and EGE
248 demonstrated by all participants. When directly comparing BIC scores between PIECE and each of the
249 other three models, one can see that PIECE was not just the best overall model, but that it unanimously
250 did the best job of fitting each individual's data.

251 In addition to objective model selection criteria, we also performed a posterior predictive check of our

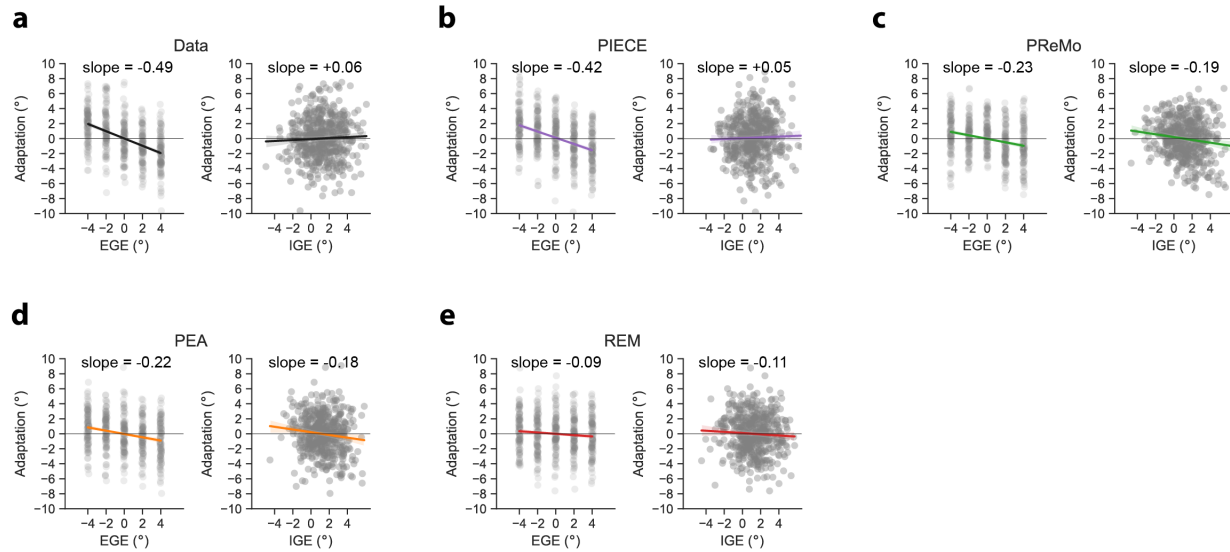


Figure 5: Posterior predictive check via simulation of entire experiment using MLEs of each model's parameter set. (a) Representative data from a single participant (P13). Adaptive responses are a linear function of EGE and remain statistically independent of IGE. (b) PIECE model mimics the empirical data: Slope values indicate high sensitivity to EGE and insensitivity to IGE. (c-e) PReMo, PEA, and REM all fail to capture the accurate parsing of errors into EGE and IGE. Gray dots represent adaptation measures of individual trials. Thick colored lines and shading represent line of best fit and associated bootstrapped 95% CI, respectively. Posterior predictive checks of other 15 participants followed qualitatively similar pattern.

252 models by generating simulated data with the best-fit parameters (see for parameter values). The logic of
253 this procedure is that any successful model should be able to approximate the observed data (Wilson and
254 Collins, 2019). Fig. 5 shows data from a representative participant, along with simulated data using each
255 model's maximum likelihood estimates of parameter values for this participant. Only the PIECE model
256 successfully captured the sensorimotor system's ability to robustly adapt to small external perturbations
257 while simultaneously discounting motor noise. Combined, we see that PIECE is the most appropriate
258 model for these data based on objective model selection criteria, and that, qualitatively, it was the only
259 model that could capture participants' highly accurate decomposition of total error into constituent parts.

260 The behavioral data from the current study and from Ranjan & Smith make clear that responses to
261 IGE and EGE are statistically independent. For this reason, any computational framework that posits
262 the goal of adaptation as one of aligning the hand with the target—as PReMo, PEA, and REM do—is
263 bound to fail. While those models capture adaptation to EGE, they also incorrectly predict adaptation
264 to be driven by IGE, as clearly illustrated in Fig. 5. Even on unperturbed trials, the misalignment of the
265 hand with the target due to intrinsic motor variability will trigger an adaptive response in all models other

266 than PIECE. However, a plethora of data now show that human participants do not treat IGE and EGE
267 equally, which only the PIECE model was able to explain.

268 Discussion

269 As first reported by Ranjan & Smith, the adaptation system shows a remarkable capability to filter out
270 IGE from total error in order to implicitly recalibrate its sensorimotor mapping in response to external
271 perturbations (Ranjan, 2022; Ranjan and Smith, 2018). Our successful replication of these results supports
272 this finding and points to its robustness. To provide a computational-level explanation of the observed
273 error parsing behavior (Blohm et al., 2020; Krakauer et al., 2017; Marr, 2010), we developed the PIECE
274 model. PIECE is a Bayesian decision-making model that incorporates ideas of cue combination and causal
275 inference, and takes inspiration from the three models it was compared to in this study, PReMo, PEA,
276 and REM. In contrast to these other models, though, PIECE returns to classical ideas of adaptation as a
277 process of state estimation with regard to the external perturbation (Krakauer et al., 2019; Shadmehr et al.,
278 2010). Indeed, if one were to simply cast adaptation as a process of proportionally updating motor output
279 by a fraction of the experienced rotation size, as in standard state-space models of adaptation (Donchin
280 et al., 2003; Smith et al., 2006; Thoroughman and Shadmehr, 2000), we could recapitulate the differential
281 adaptation to IGE-EGE observed in our study. However, such a model does not tell us *why* or *how* the
282 sensorimotor system decides which errors to adapt to, and which errors to ignore, not to mention other
283 limitations of state-space models (Kim et al., 2018; Krakauer et al., 2019). The unique advance offered by
284 the PIECE model is that it provides a coherent computational-level explanation of how the sensorimotor
285 system parses motor errors into their respective internally- and externally-generated components in order
286 to ignore the former and adapt to the latter.

287 Implicit adaptation is not driven by perceptual error

288 PReMo and PEA are well-developed models that explicitly formulate the computational goal of adaptation
289 as one of realigning the perceived hand position with the target. Both models posit that in a perturbed
290 environment, such as a visuomotor rotation, the mismatch between sensory and motor prediction-based
291 cues results in a proprioceptive shift towards the visual cursor, generating the driving signal for adaptation
292 (Tsay et al., 2022; Zhang et al., 2024). Although these models differ in their explanation of how this
293 shift occurs and what constraints are placed on it, both are incapable of accommodating the error parsing
294 observed here and first reported by Ranjan & Smith (Ranjan and Smith, 2018). As our data and analyses

295 make demonstrably clear, the perceived hand position, and its relationship to the target, cannot serve as
296 the primary driver of single-trial adaptation. Fig. 5 shows that, while these models can accommodate
297 participants' responses to EGE, they incorrectly predict adaptation in response to IGE, since the hand is
298 by definition misaligned with the target when there is motor noise, even in the absence of a perturbation.
299 These models fall short, in part, because they do not incorporate causal inference.

300 **The role of causal inference in adaptation**

301 Our results indicate that the implicit adaptation system performs causal inference with respect to the
302 sources of the observed total error. Bayesian causal inference models were originally developed to explain
303 why auditory and visual cues are sometimes fused, forming a coherent percept, and why they are sometimes
304 perceived as having two separate sources (Körding et al., 2007; Sato et al., 2007). In the realm of motor
305 adaptation, causal inference was formalized by Wei & Kording with their Relevance Estimation Model
306 (REM)(Wei and Körding, 2009). Although REM and PIECE are structurally similar in terms of causal
307 inference, the posterior on whether the reach was perturbed or not is used in nearly opposite ways in the
308 two models—to correct for errors that are believed to be self-generated (i.e., IGE), in the case of REM,
309 and to correct for errors that are believed to be externally-generated (i.e., EGE), in the case of the PIECE
310 model.

311 According to REM, visual errors that fall within the range one should expect to observe, based on
312 normal motor variability, are deemed more relevant and thus require an adaptive response. In their paper,
313 the authors tested a much larger range of error sizes ($\approx 4^\circ - 58^\circ$) and were not at all concerned with
314 explaining parsing of total movement error into IGE and EGE. And similar to nearly all adaptation
315 studies, the methods they used did not allow them to specifically examine responses to IGE (see Methods),
316 so perhaps it should not be surprising that their proposed explanation of adaptation differs from ours.
317 Regardless of these methodological differences, the current data and the PIECE model make clear that,
318 depending on their inferred source, size-matched errors are treated drastically differently by the motor
319 system, and only the external component of error should be corrected for.

320 **The role of prediction in adaptation**

321 REM includes only proprioceptive and visual cues regarding actual hand position, while both PReMo and
322 PEA also include an efference copy-based predictive cue of hand position. However, in PReMo and PEA
323 the motor prediction is assumed to be centered on the motor goal (i.e., the target), rather than where

324 the central motor system actually directs the hand, which would equal the participant's explicit aim plus
325 any associated IGE (and, possibly, any implicit adaptation). In this manner, the predictive cues in PEA
326 and PReMo are akin to the prior on hand position in the PIECE model, $p(x_h)$, which is equal to the
327 actual distribution of unperturbed baseline reaches. In other words, the observer in PIECE possesses
328 knowledge of their own intrinsic motor variability (and bias), an assumption made based on findings from
329 a series of reaching studies examining rapid value-based decision making (Trommershauser et al., 2003;
330 Trommershäuser et al., 2008). In the PIECE model, the combination of the prior on hand position and
331 the predictive cue centered on the actual hand position contribute to effective error parsing.

332 Based on our model fitting results, the motor prediction cue provides highly accurate and precise
333 information regarding the actual motor command, including noise. The distribution of σ_{combined} values
334 across participants was quite tight, ranging from 0.2 to 1.4. This suggests that for many participants, there
335 was very little uncertainty regarding where the hand was actually sent. These values must be interpreted
336 with caution, however, as they actually represent the uncertainty associated with the combination of
337 proprioceptive and motor prediction-based cues, as the two could not be dissociated in this paradigm (see
338 Methods). Regardless, these parameter estimates still suggest a highly accurate and precise prediction, as
339 proprioception is known to be quite noisy. This finding further supports the idea first presented by Ranjan
340 & Smith that, in order to effectively parse errors as small as those used in our paradigms, the precision of
341 the motor prediction must be much greater than that of the movement itself (Ranjan, 2022).

342 **PIECE and other sensorimotor learning phenomena**

343 PIECE incorporates a key insight from the PEA model related to goal-directed reaching—that visual
344 uncertainty increases with error size. Zhang and colleagues empirically showed that, during reaching, the
345 distance of the cursor from the target will increase visual uncertainty (decrease precision). An important
346 caveat is that the authors were applying this principle to data involving implicit adaptation to visual
347 error clamps, in which participants are explicitly instructed to ignore the cursor feedback and to fixate on
348 the target, and the cursor trajectory is clamped to a specified angle relative to the target. Despite the
349 difference between our tasks, our use of the same linear function for visual uncertainty is justified based on
350 prior work which showed that the eyes automatically direct themselves towards a reaching target (Neggers
351 and Bekkering, 2002). The success of our models further extends the findings of Zhang and colleagues,
352 advancing the notion that whether the cursor feedback is contingent on hand position (as in our study)
353 or not (as in a clamp study), the visual estimate of the cursor position will increase in uncertainty as a

354 function of cursor-to-target distance.

355 In the Zhang et al study, this increasing visual uncertainty is used to explain non-linear adaptive
356 responses to increasing perturbation sizes. Several studies have now shown that sensitivity to external
357 perturbations is only constant over a narrow range of error size, and that adaptive responses quickly
358 saturate as perturbation size increases (Hayashi et al., 2020; Kim et al., 2018; Marko et al., 2012; Morehead
359 et al., 2017; Wei and Körding, 2009). Importantly, the PIECE model can also account for the saturation
360 phenomenon. As the learning rate (K_t) in PIECE is derived from optimally combining prior and likelihood,
361 it is a function of the observer’s prior uncertainty regarding the rotation magnitude, σ_r , and their visual
362 uncertainty, $\sigma_{v,t}$. As visual uncertainty increases with rotation size, the learning rate decreases, helping to
363 capture the saturation of adaptive responses in a principled manner, avoiding the use of an arbitrary free
364 parameter for the learning rate as used in most state-space models. Indeed, when visual uncertainty is
365 combined with the observer’s prior over possible perturbation sizes, PIECE can flexibly accommodate any
366 non-monotonocities in adaptation as a function of error size, as shown in Fig. 6. It should also be noted that
367 if one were to simply add a retention parameter to PIECE (or equivalently, assume temporal dynamics
368 in the state estimation step—see 7), this model could also accommodate the saturation of asymptotic
369 adaptation in response to consistent perturbations (Bond and Taylor, 2015; Morehead et al., 2017; Zhang
370 et al., 2024).

371 Conclusions

372 There are three important insights provided by the PIECE model. The first insight is that causal inference
373 is critical for adaptation, both to infer the source of error and also to regulate how much weight to
374 apply to the observer’s estimate of the perturbation, which in turn dictates the magnitude of single-trial
375 adaptation. Without causal inference, other models such as PReMo and PEA are unable to capture the
376 statistical independence between responses to IGE and EGE, since the perceptual error driving adaptation
377 in these models is a function of hand position (IGE). Relatedly, the second insight is that accurate error
378 parsing can only be understood through the lens of state estimation with respect to the EGE, i.e., the
379 perturbation. Despite incorporating causal inference, the REM model also failed to account for the data
380 because it too frames adaptation as a process of inferring the hand position for purposes of aligning it with
381 the motor goal. However, our data and analyses make clear that adaptation responds almost exclusively to
382 EGE. Any minimal response to IGE is attributable to the probabilistic nature of inferring and estimating
383 the source of error—in other words, the motor system is not perfect in its error parsing. Despite this latter

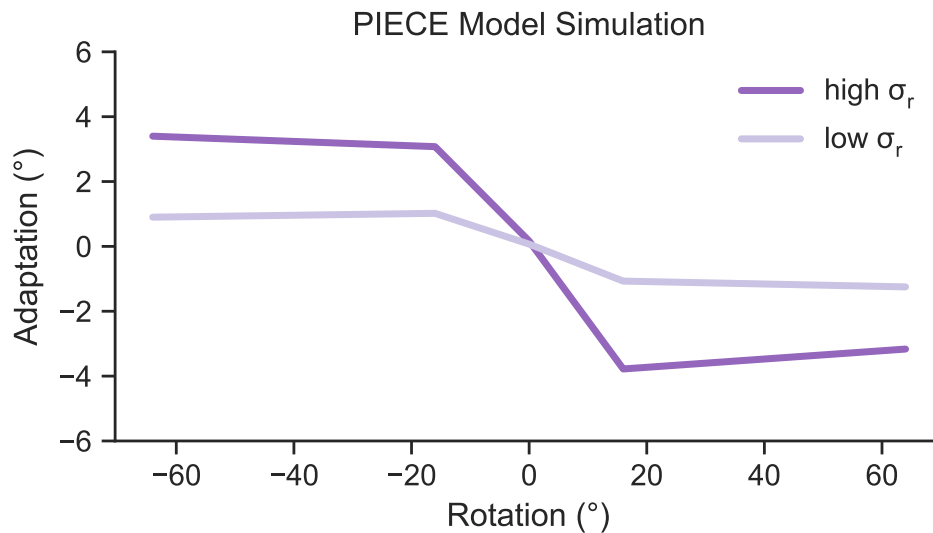


Figure 6: PIECE can accurately capture the non-linear adaptive response to a wide range of error sizes. Here, we simulated responses to $\pm 4^\circ$, $\pm 16^\circ$, and $\pm 64^\circ$ errors, as in Experiment 4 from Zhang et al., 2024. Note that both the inflection point and magnitude of adaptation can be flexibly accommodated by PIECE. They are both functions of the observer's degree of belief that they are perturbed, and therefore depend on σ_r and σ_v , the width of the observer's prior on rotation size and visual uncertainty, which increases with error size (Zhang et al 2024).

384 point, the third insight is that the internal prediction of where the hand is sent during a reach is highly
385 accurate and precise. This was made clear in the original work of Ranjan & Smith, while PIECE embeds
386 this phenomenon within a Bayesian framework. When considered within the context of high-level motor
387 skills where the margin of error is miniscule, like playing the violin or grabbing our morning coffee in a
388 rush, perhaps it should not be surprising that our sensorimotor system has evolved to keep accurate tabs on
389 where it sends our limbs so that it can finely recalibrate its mapping in response to external perturbations.

390 Methods

391 Participants

392 Healthy, young adults were recruited from the University of British Columbia community (N = 16, 8
393 females; average age = 22.3 years old, range: 19-29). All participants were naive to the purpose of
394 the experiment and provided their written informed consent (consistent with the Helsinki Declaration).
395 Participants were remunerated \$15 for their participation. Experimental procedures were approved by the
396 behavioral research ethics board at the University of British Columbia under study ID H23-02324.

397 **Experimental Set-Up**

398 Participants sat in front of a testing set-up comprised of a horizontally-oriented, 144 hertz refresh rate
399 monitor (53.2 cm by 30 cm, ASUS), mounted directly above a graphics tablet (49.3 cm by 32.7 cm, Intuos
400 4XL; Wacom, Vancouver, WA), as shown in Fig. 1a. Participants made reaching movements while holding
401 a stylus (power grip) embedded within a 3D-printed handle and sliding it along the graphics tablet. The
402 stylus position was recorded at 200 Hz. On each trial, the monitor displayed a yellow start location (6
403 mm diameter circle), a green circular target (6 mm diameter), and the real-time position of the stylus as
404 indicated by a white circular cursor (3 mm diameter). Both the yellow start location and green target
405 were positioned at the participant's midline. When seated for testing, participants could not see their arm
406 or hand. The experimental protocol was conducted in the dark to further minimize peripheral vision of
407 the arm. The experimental software was custom written using the Psychtoolbox extension in MATLAB
408 (Brainard, 1997; Pelli, 1997).

409 **Reaching Task**

410 Participants made quick point-to-point reaches to the straight ahead target located 9 cm from the start
411 position. To initiate each trial, participants moved the stylus so that the white cursor entered the yellow
412 start target. The participant had to maintain their hand in the start position for at least 300 ms (hold
413 time drawn from a uniform distribution, $U[300, 500]$) before the green target appeared 9 centimeters away.
414 Participants were instructed to make quick and accurate point-to-point reaches to the target. If the
415 movement time, defined as the elapsed time from movement onset to movement endpoint, was greater
416 than 500 ms, the target turned red to indicate to the participant that their movement was not quick
417 enough. Movement onset was defined as the first time point that movement velocity was ≥ 1 cm/s and the
418 hand had traveled 0.5 cm from the center of the start target. Movement end was defined as the first time
419 point following movement onset that the velocity fell below 1 cm/s. The endpoint hand location was taken
420 as the hand position at movement end, with frozen endpoint feedback of the cursor position provided for
421 500 ms. Following the completion of endpoint feedback, the yellow start target reappeared to prompt the
422 participant to bring the cursor back to "home" to continue the next trial.

423 **Experimental Schedule**

424 The experiment commenced with a baseline block containing 70 null (unperturbed) trials with veridical
425 feedback. The baseline block was used to familiarize participants with the experimental protocol, ensuring

426 that participants attempted to make quick and accurate point-to-point reaches. Participants were informed
427 that once they started moving, they were to “follow through with their reach to the end” without correcting
428 their movement. We were successful in ensuring participants’ reaction times ($282 \text{ ms} \pm 40 \text{ ms}$; mean of
429 medians \pm SD) and movement times (365 ± 39) were brisk.

430 At the end of the baseline block, the experimenter provided the participant with the following instruc-
431 tions: ‘You may notice some changes to your cursor. Regardless of whether you notice these changes or
432 not, continue to aim directly for the target and reach as quickly and accurately as possible in a straight
433 line.’

434 Trials alternated between null and perturbation trials within each experimental block. On perturbation
435 trials involving a visuomotor rotation, the cursor was rotated away from the hand by 0° , $\pm 2^\circ$, or $\pm 4^\circ$ relative
436 to the start position (100 trials / perturbation level).

437 The participants also experienced trials where there was no visuomotor rotation, but the target jumped
438 by $\pm 2^\circ$, or $\pm 4^\circ$ mid-reach (hand distance of 4.5 cm). As the focus of the current study is on the results
439 from the visuomotor rotation trials, results from the target jump trials are provided in the Supplement.

440 Full visual feedback was provided during the reach as well as the return home during all visuomotor
441 rotation trials, with the perturbation left on during both the outbound and inbound portions of the trial.
442 On half of the null trials, randomly selected, visual feedback of the cursor was completely absent during
443 the reach. On these no visual feedback trials, following reach completion, the cursor remained hidden on
444 the return to the start position until the hand was <1 cm from the center of the start target. For the
445 other 50% of null trials with visual feedback, participants received full veridical feedback following reach
446 completion and during the return to the start position to begin the next trial. The schedule of perturbation
447 trials was randomized.

448 Participants completed a total of 18 experimental blocks with 100 trials each, totaling 1800 trials. Each
449 experimental block was separated by a minimum 1-minute break.

450 **Data Analysis**

451 All data were analyzed using custom-written Python scripts and utilized standard libraries, including
452 NumPy (Harris et al., 2020), pandas (pandas development team, 2020), Matplotlib (Hunter, 2007), and
453 seaborn (Waskom, 2021). Reach angle was defined as the angle between the straight lines connecting
454 the start position to the target and the hand at peak velocity. This measure also quantified IGE, as
455 we only used the straight ahead target at 0° . Structuring the experiment such that each perturba-

456 tion trial was surrounded by a null trial on either side was critical to our analyses. Single-trial adap-
457 tation was quantified as the difference between reach angles on the post- and pre-perturbation trials
458 ($\text{adaptation}_t = \text{reach angle}_{t+1} - \text{reach angle}_{t-1}$). Operationalizing adaptation this way rather than the
459 more common method of comparing reach angles on post-perturbation to perturbation trials provides an
460 uncontaminated measure of adaptation to internally-generated motor noise. As previously explained (Ran-
461 jan, 2022), defining adaptation as $\text{reach angle}_{t+1} - \text{reach angle}_t$ means the outcome measure (adaptation)
462 contains its putative predictor, IGE (reach angle_n), resulting in spurious correlations. We circumvent this
463 potential confound by using the triplet analysis.

464 For each participant, individual trials were excluded from analysis if the reach angle had an absolute
465 z-score magnitude greater than 3.5. This resulted in <1.1% of trials being removed from any individual
466 participant’s data, with 99.5% of all trials in the experiment being included.

467 We used the following binned analysis method of Ranjan & Smith (see Ranjan, 2022). To estimate
468 sensitivity to EGE, trials were first split based on the level of EGE and further binned into one of five
469 quintiles based on the magnitude of IGE (i.e., 5 levels of EGE x 5 IGE quintiles = 25 bins), separately for
470 each participant. The average magnitude of adaptation within each of these 25 bins was then regressed
471 onto the EGE. For estimating sensitivity to IGE, we regressed the same binned adaptation measures onto
472 IGE. To calculate the r^2 values in Fig. 2, we averaged the 25 bins of adaptation versus EGE (IGE) across
473 participants and regressed the average response onto its respective error signal.

474 Statistical analyses

475 For the analysis of unbinned data, we performed bivariate regression analysis using ordinary least-squares
476 fitting with SciPy’s *linregress* function (Virtanen et al., 2020). After confirming with QQ plots that the
477 data could be modeled as Gaussian distributions, we compared regression coefficients with a paired t-test.
478 We also assessed whether the coefficients were reliably different than zero using a one-sample t-test.

479 Modeling Analysis

480 We separately fit each individual’s dataset with each of the four models using maximum likelihood esti-
481 mation with SciPy’s *minimize* function (Virtanen et al., 2020). In this procedure, our goal was to find a
482 given model’s parameter set, Θ_{model} , that maximized the probability (equivalently, minimized the negative
483 log-likelihood) of the observed trial-by-trial data. We assumed the trials were all independent of each
484 other, and therefore summed the negative log-likelihoods across all trials. For formal model comparisons,

485 we computed and compared Bayesian Information Criterion (BIC) scores (Schwarz, 1978).

486 To visualize how well each model could capture differential adaptation to EGE-IGE, we generated
487 simulated data, using the actual perturbation schedule, for each participant using each model’s maximum
488 likelihood estimates of parameter values. Fig. 5 shows an example “posterior predictive check” for a
489 representative participant. We note that the patterns observed for this participant were qualitatively
490 similar across our entire sample.

491 Model recovery analysis with all four models and parameter recovery analysis for PIECE were performed
492 to further validate our model fits (see Supplementary Information; Wilson and Collins, 2019).

493 Below, we provide details of the models that were implemented.

494 **PIECE**

495 In addition to the description of PIECE provided in the main text, we note that the motor prediction,
496 x_u , and the proprioceptive cue, x_p , are both centered on the true hand location, x_h . As we cannot
497 dissociate them, since they are internal to the observer, we combined them, resulting in random variable
498 $x_{\text{combined}} \sim N(x_h, \sigma_{\text{combined}})$, where $\sigma_{\text{combined}} = \frac{\sigma_u^2 \sigma_p^2}{\sigma_u^2 + \sigma_p^2}$. The variance takes this form because x_u and
499 x_p are derived from Gaussian distributions and thus the distribution of x_{combined} is the product of these
500 underlying distributions (likelihoods).

501 In total, there are 3 free parameters in this model: σ_{combined} , σ_r and b , the participant’s intrinsic motor
502 bias.

503 **Proprioceptive Recalibration Model (PReMo) (Tsay et al., 2022)**

504 PReMo assumes the computational goal of adaptation is to realign the perceived hand position, x_{per} , with
505 the target, T , which is always equal to zero since we only used the straight ahead target. Computing x_{per}
506 on trial t involves the following:

$$x_{\text{per},t} = \beta_p + \frac{\sigma_u^2}{\sigma_u^2 + \sigma_p^2} x_{p,t}, \quad (13)$$

where β_p represents the proprioceptive shift (i.e., how much proprioception gets cross-modally shifted by vision). σ_u represents the uncertainty around the motor prediction, which in PReMo is always centered on the target (in contrast to PIECE, where the prediction is centered on the hand position). σ_p represents proprioceptive uncertainty, and x_p represents the raw proprioceptive input, i.e., the true hand position. The physiological limit of how much proprioception can be shifted is represented by β_p^{sat} , which was set to 5 in order to reduce the number of free parameters:

$$\beta_p = -\min \left(|\beta_p^{\text{sat}}|, \left| \eta_p \left(\frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2} x_{v,t} - \frac{\sigma_u^2}{\sigma_u^2 + \sigma_p^2} x_{p,t} \right) \right| \right) \quad (14)$$

Similar to previous studies that have used randomized perturbations with an average error of zero, we assumed the effects of different perturbations are independent of each other. Therefore, single-trial adaptation (STA) was modeled as resulting only from the current perturbation, with no retention across trials, and is a function of the distance between the perceived hand position and the target. The free parameter B represents the learning rate (i.e., error sensitivity):

$$x_{\text{STA},t+1} = B(T - x_{\text{per}}), \quad (15)$$

For PReMo and all other models, the actual hand angle on the next trial, $x_{h,t+1}$, is modeled as the sum of single-trial adaptation (learning), the participant's intrinsic motor bias, b , a free parameter, and motor noise, which was taken as a draw, ϵ , from a Gaussian distribution, $N(0, \sigma_h^2)$:

$$x_{h,t+1} = x_{\text{STA},t+1} + b + \epsilon_{t+1} \quad (16)$$

σ_h represents motor variability and is taken directly from baseline reaches. Therefore, there are 6 free parameters in this model: $\sigma_u, \sigma_p, \sigma_v, \eta, B$, and b .

512 Perceptual Error Adaptation Model (Zhang et al., 2024)

513 For fitting the PEA model to data, we make the same assumptions as the authors of the original work did.
 514 Namely, we assume x_u and x_p are, on average, centered on the target position, since we used a randomized
 515 perturbation schedule with a mean error of zero. Therefore, instead of serving as separate cues, x_u and
 516 x_p were cue combined, resulting in random variable $x_{\text{int}} \sim N(T, \sigma_{\text{int}}^2)$, where T is the target direction (i.e.,
 517 zero), and $\sigma_{\text{int}}^2 = \frac{\sigma_u^2 \sigma_p^2}{\sigma_u^2 + \sigma_p^2}$ represents the variance of the combined sensory signal from x_u and x_p .

518 Single trial adaptation is a function of the distance between the estimated hand position, $\hat{x}_{h,t}$ and the
519 target:

$$x_{\text{STA}} = B(T - \hat{x}_{\text{hand}}) \quad (17)$$

520 where the estimated hand position is computed by cue combination:

$$\hat{x}_{\text{hand}} = w_{\text{int}}T + w_v x_v = w_v x_v, \quad (18)$$

521 and the weight given to the i th cue is given by:

$$w_i = \frac{\frac{1}{\sigma_i^2}}{\sum_j \frac{1}{\sigma_j^2}} \quad i = \text{int}, v; j = \text{int}, v \quad (19)$$

522 The hand angle on the trial following the perturbation is again given by:

$$x_{h,t+1} = x_{\text{STA}, t+1} + b + \epsilon_{t+1} \quad (20)$$

523 Similar to Zhang and colleagues, we modeled σ_v as a linear function of the cursor distance from the
524 target using the bias (1.179) and slope (0.384) terms from their original work (see Table S1 from Zhang
525 et al., 2024). In total, there were 3 free parameters in this model: σ_{int} , motor bias, b , and the learning
526 rate, B .

527 **Relevance Estimation Model (REM) (Wei and Körding, 2009)**

528 In the REM model, the observer performs causal inference to compute the probability that the feedback
529 is “relevant” (p_{rel}), which in this model means the visual error is attributed to the movement made by the
530 observer rather than to the perturbation.

$$p_{\text{rel}} = S \frac{N(x_{v,t}; 0, \sigma^2)}{N(x_{v,t}; 0, \sigma^2) + \text{Const}}, \quad (21)$$

Where $x_{v,t}$ is the visual cue on trial t , S and $Const$ are scaling factors, and σ is the standard deviation of the integrated visual and proprioceptive cues, following:

$$\sigma^2 = \frac{\sigma_v^2 \sigma_p^2}{\sigma_v^2 + \sigma_p^2} \quad (22)$$

For single trial adaptation:

$$x_{STA} = B(T - p_{rel}x_v) \quad (23)$$

The next motor output is given by:

$$x_{h,t+1} = x_{STA, t+1} + b + \epsilon_{t+1} \quad (24)$$

531 There are 4 free parameters in REM: σ , S , $Const$, and b .

532 Data and code availability

533 All data and code for this study are available at the following site: <https://github.com/ccmlab-ubc/ige-ege>.

534 Acknowledgments

535 The authors thank Maurice Smith, Mike Landy, Todd Hudson, Jonathan Tsay, and members of the Cog-
536 nition and Action Lab at UC Berkeley for providing helpful feedback on this work. We also thank Lisa
537 Liu for her help with creating Fig. 3.

538 References

- 539 Berniker, M., & Kording, K. (2008). Estimating the sources of motor errors for adaptation and generaliza-
540 tion. *Nature neuroscience*, *11*(12), 1454–1461.
- 541 Blohm, G., Kording, K. P., & Schrater, P. R. (2020). A how-to-model guide for neuroscience. *Eneuro*, *7*(1).
- 542 Bond, K. M., & Taylor, J. A. (2015). Flexible explicit but rigid implicit learning in a visuomotor adaptation
543 task. *Journal of neurophysiology*, *113*(10), 3836–3849.
- 544 Brainard, D. H. (1997). The psychophysics toolbox. *Spatial vision*, *10*(4), 433–436.
- 545 Burge, J., Ernst, M. O., & Banks, M. S. (2008). The statistical determinants of adaptation rate in human
546 reaching. *Journal of vision*, *8*(4), 20–20.
- 547 Carriot, J., Brooks, J. X., & Cullen, K. E. (2013). Multimodal integration of self-motion cues in the
548 vestibular system: Active versus passive translations. *Journal of neuroscience*, *33*(50), 19555–19566.
- 549 Chaisanguanthum, K. S., Shen, H. H., & Sabes, P. N. (2014). Motor variability arises from a slow random
550 walk in neural state. *Journal of Neuroscience*, *34*(36), 12071–12080.
- 551 Churchland, M. M., Afshar, A., & Shenoy, K. V. (2006). A central source of movement variability. *Neuron*,
552 *52*(6), 1085–1096.
- 553 Collins, T., & Wallman, J. (2012). The relative importance of retinal error and prediction in saccadic
554 adaptation. *Journal of Neurophysiology*, *107*(12), 3342–3348.
- 555 Donchin, O., Francis, J. T., & Shadmehr, R. (2003). Quantifying generalization from trial-by-trial behavior
556 of adaptive systems that learn with basis functions: Theory and experiments in human motor
557 control. *Journal of Neuroscience*, *23*(27), 9032–9045.
- 558 Ernst, M. O., & Banks, M. S. (2002). Humans integrate visual and haptic information in a statistically
559 optimal fashion. *Nature*, *415*(6870), 429–433.
- 560 Faisal, A. A., Selen, L. P., & Wolpert, D. M. (2008). Noise in the nervous system. *Nature reviews neuro-*
561 *science*, *9*(4), 292–303.
- 562 Gaffin-Cahn, E., Hudson, T. E., & Landy, M. S. (2019). Did i do that? detecting a perturbation to visual
563 feedback in a reaching task. *Journal of Vision*, *19*(1), 5.
- 564 Haith, A. M., & Krakauer, J. W. (2013). Theoretical models of motor control and motor learning. In
565 *Routledge handbook of motor control and motor learning* (pp. 16–37). Routledge.
- 566 Harris, C. R., Millman, K. J., van der Walt, S. J., Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E.,
567 Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., van Kerkwijk, M. H., Brett, M.,

- 568 Haldane, A., del Río, J. F., Wiebe, M., Peterson, P., . . . Oliphant, T. E. (2020). Array programming
569 with NumPy. *Nature*, *585*(7825), 357–362.
- 570 Hayashi, T., Kato, Y., & Nozaki, D. (2020). Divisively normalized integration of multisensory error infor-
571 mation develops motor memories specific to vision and proprioception. *Journal of Neuroscience*,
572 *40*(7), 1560–1570.
- 573 Hunter, J. D. (2007). Matplotlib: A 2d graphics environment. *Computing in Science & Engineering*, *9*(3),
574 90–95.
- 575 Kim, H. E., Morehead, J. R., Parvin, D. E., Moazzezi, R., & Ivry, R. B. (2018). Invariant errors reveal lim-
576 itations in motor correction rather than constraints on error sensitivity. *Communications Biology*,
577 *1*(1).
- 578 Kim, H. E., Parvin, D. E., & Ivry, R. B. (2019). The influence of task outcome on implicit motor learning.
579 *eLife*, *8*.
- 580 Körding, K. P., Beierholm, U., Ma, W. J., Quartz, S., Tenenbaum, J. B., & Shams, L. (2007). Causal
581 inference in multisensory perception. *PLoS one*, *2*(9), e943.
- 582 Korenberg, A. T., & Ghahramani, Z. (2002). A bayesian view of motor adaptation. *Current Psychology of*
583 *Cognition*, *21*(4/5), 537–564.
- 584 Krakauer, J. W., Ghazanfar, A. A., Gomez-Marin, A., MacIver, M. A., & Poeppel, D. (2017). Neuroscience
585 needs behavior: Correcting a reductionist bias. *Neuron*, *93*(3), 480–490.
- 586 Krakauer, J. W., Hadjiosif, A. M., Xu, J., Wong, A. L., & Haith, A. M. (2019). Motor learning. *Compr*
587 *Physiol*, *9*(2), 613–663.
- 588 Landy, M. S., Maloney, L. T., Johnston, E. B., & Young, M. (1995). Measurement and modeling of depth
589 cue combination: In defense of weak fusion. *Vision research*, *35*(3), 389–412.
- 590 Marko, M. K., Haith, A. M., Harran, M. D., & Shadmehr, R. (2012). Sensitivity to prediction error in
591 reach adaptation. *Journal of neurophysiology*, *108*(6), 1752–1763.
- 592 Marr, D. (2010). *Vision: A computational investigation into the human representation and processing of*
593 *visual information*. MIT press.
- 594 Mazzoni, P., & Krakauer, J. W. (2006). An implicit plan overrides an explicit strategy during visuomotor
595 adaptation. *Journal of neuroscience*, *26*(14), 3642–3645.
- 596 Morehead, J. R., Taylor, J. A., Parvin, D. E., & Ivry, R. B. (2017). Characteristics of implicit sensorimotor
597 adaptation revealed by task-irrelevant clamped feedback. *Journal of cognitive neuroscience*, *29*(6),
598 1061–1074.

- 599 Neggers, S. F., & Bekkering, H. (2002). Coordinated control of eye and hand movements in dynamic
600 reaching. *Human movement science*, *21*(3), 37–64.
- 601 pandas development team, T. (2020, February). *Pandas-dev/pandas: Pandas* (Version latest). Zenodo.
- 602 Pelli, D. G. (1997). The videotoolbox software for visual psychophysics: Transforming numbers into movies.
603 *Spatial vision*, *10*(4), 437–442.
- 604 Ranjan, T. (2022). *Understanding the role of internal predictions in sensorimotor adaptation* [Doctoral
605 dissertation, Harvard University].
- 606 Ranjan, T., & Smith, M. A. (2018). Cancellation of internally-generated errors from the signal driving
607 motor adaptation. *Advances in Motor Learning and Motor Control (MLMC)*.
- 608 Sato, Y., Toyoizumi, T., & Aihara, K. (2007). Bayesian inference explains perception of unity and ven-
609 triloquism aftereffect: Identification of common sources of audiovisual stimuli. *Neural computation*,
610 *19*(12), 3335–3355.
- 611 Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, *6*(2).
- 612 Shadmehr, R., Smith, M. A., & Krakauer, J. W. (2010). Error correction, sensory prediction, and adaptation
613 in motor control. *Annual Review of Neuroscience*, *33*(1), 89–108.
- 614 Smith, M. A., Ghazizadeh, A., & Shadmehr, R. (2006). Interacting adaptive processes with different
615 timescales underlie short-term motor learning. *PLoS biology*, *4*(6), e179.
- 616 Sommer, M. A., & Wurtz, R. H. (2008). Brain circuits for the internal monitoring of movements. *Annu.*
617 *Rev. Neurosci.*, *31*(1), 317–338.
- 618 Sperry, R. W. (1950). Neural basis of the spontaneous optokinetic response produced by visual inversion.
619 *Journal of comparative and physiological psychology*, *43*(6), 482.
- 620 Szarka, A., Kim, H. E., Inglis, J. T., & Chua, R. (2024). Evidence for an efferent-based prediction con-
621 tributing to implicit motor adaptation. *bioRxiv*, 2024–07.
- 622 Taylor, J. A., & Ivry, R. B. (2011). Flexible cognitive strategies during motor learning. *PLoS computational*
623 *biology*, *7*(3), e1001096.
- 624 Taylor, J. A., Krakauer, J. W., & Ivry, R. B. (2014). Explicit and implicit contributions to learning in a
625 sensorimotor adaptation task. *Journal of Neuroscience*, *34*(8), 3023–3032.
- 626 Thoroughman, K. A., & Shadmehr, R. (2000). Learning of action through adaptive combination of motor
627 primitives. *Nature*, *407*(6805), 742–747.
- 628 Trommershauser, J., Maloney, L. T., & Landy, M. S. (2003). Statistical decision theory and trade-offs in
629 the control of motor response. *Spatial vision*, *16*(3), 255–275.

- 630 Trommershäuser, J., Maloney, L. T., & Landy, M. S. (2008). Decision making, movement planning and
631 statistical decision theory. *Trends in cognitive sciences*, *12*(8), 291–297.
- 632 Tsay, J. S., Kim, H., Haith, A. M., & Ivry, R. B. (2022). Understanding implicit sensorimotor adaptation
633 as a process of proprioceptive re-alignment. *eLife*, *11*.
- 634 Tseng, Y.-w., Diedrichsen, J., Krakauer, J. W., Shadmehr, R., & Bastian, A. J. (2007). Sensory prediction
635 errors drive cerebellum-dependent adaptation of reaching. *Journal of neurophysiology*, *98*(1), 54–
636 62.
- 637 van Beers, R. J. (2009). Motor learning is optimally tuned to the properties of motor noise. *Neuron*, *63*(3),
638 406–417.
- 639 van Beers, R. J., Brenner, E., & Smeets, J. B. (2013). Random walk of motor planning in task-irrelevant
640 dimensions. *Journal of neurophysiology*, *109*(4), 969–977.
- 641 Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D., Burovski, E.,
642 Peterson, P., Weckesser, W., Bright, J., et al. (2020). Scipy 1.0: Fundamental algorithms for scientific
643 computing in python. *Nature methods*, *17*(3), 261–272.
- 644 Von Holst, E., & Mittelstaedt, H. (1950). Das reafferenzprinzip: Wechselwirkungen zwischen zentralner-
645 vensystem und peripherie. *Naturwissenschaften*, *37*(20), 464–476.
- 646 Wang, T., Morehead, R. J., Tsay, J. S., & Ivry, R. B. (2024). The origin of movement biases during
647 reaching. *bioRxiv*.
- 648 Waskom, M. L. (2021). Seaborn: Statistical data visualization. *Journal of Open Source Software*, *6*(60),
649 3021.
- 650 Wei, K., & Körding, K. (2009). Relevance of error: What drives motor adaptation? *Journal of Neurophys-*
651 *iology*, *101*(2), 655–664.
- 652 Wei, K., & Körding, K. (2010). Uncertainty of feedback and state estimation determines the speed of motor
653 adaptation. *Frontiers in computational neuroscience*, *4*, 1151.
- 654 Wilson, R. C., & Collins, A. G. (2019). Ten simple rules for the computational modeling of behavioral
655 data. *Elife*, *8*, e49547.
- 656 Zhang, Z., Wang, H., Zhang, T., Nie, Z., & Wei, K. (2024). Perceptual error based on bayesian cue
657 combination drives implicit motor adaptation. *eLife*, *13*.

658 Supplementary Information

659 Target jump data

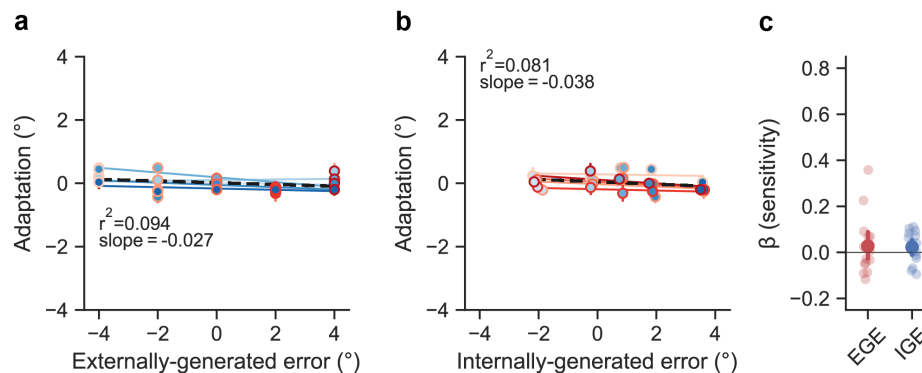


Figure 7: (a) Population-averaged adaptive responses were binned based on the level of EGE (i.e., target error; shade of red) and IGE (shade of blue) and plotted as a function of EGE in a, and as a function of IGE in b (where data are shifted right due to a small counter-clockwise reaching bias of less than 1° across participants). In direct contrast to the visuomotor rotation data, only a minimal amount of the variance in adaptive responses (9.4% here, as compared to 99.1% in the case of rotations) was explained by EGE. (e) Linear regression coefficients from an analysis of unbinned data. Error bars represent bootstrapped 95% CIs and dots represent individual participants.

660 We performed the same set of binned and unbinned analyses of participants' responses to target jumps
661 as we did for visuomotor rotations (see Fig. 2). The results of these analyses are presented in Fig. 7,
662 where each point represents the average across every participant's mean response for the corresponding
663 EGE/IGE level. Here, EGE refers to the target error and is quantified as the difference between the
664 hand angle at peak velocity and the target angle post-jump. Unlike the visuomotor rotation data (see
665 Fig. 2), there was little-to-no response to either EGE (black dashed line, slope=-0.027 [-0.091, 0.026],
666 $r^2=0.094$, $p = 0.136$) or IGE (dashed black line, slope=-0.038 [-0.055, 0.014], $r^2=0.081$, $p = 0.167$). The
667 lack of an adaptive response to the target jumps was further confirmed in our unbinned analysis using a
668 bivariate regression. The sensitivities (i.e., β) to EGE (0.027[-0.025, 0.091], $t_{15} = 0.888$, $p = 0.388$) and
669 IGE (0.020[-0.013, 0.051], $t_{15} = 1.191$, $p = 0.252$) were close to zero. Furthermore, not only were the β s
670 for target jumps and IGE near zero, they were not reliably different (mean difference = 0.007, 95% CI for
671 difference scores: [-0.052, 0.066], $t_{15} = 0.254$, $p = 0.802$).

672 Combined, these results from the target jump portion of the experiment indicate that pure target error,
673 where there was no SPE, was not an effective teaching signal for the motor system, consistent with our
674 prior work and the work of others (Oza et al., 2024; Sadaphal et al., 2022; Tsay et al., 2022). Interestingly,
675 these results appear to contrast recent findings that target jumps alone do drive robust implicit adaptation

676 (Ranjan, 2022). However, in our experiment, participants were explicitly instructed to ‘continue to aim
 677 directly for the original target’, whether the target jumped or not, thus making the target jump task-
 678 irrelevant. In contrast, in the Ranjan & Smith study, quick placement of the hand within the target at
 679 the end of each trial was emphasized, thus making the target jump task-relevant. It may be that the
 680 implicit adaptation system is sensitive to task-relevant information having to do with target error (i.e.,
 681 task performance error). This is a potentially interesting dissociation with SPE, where numerous clamped
 682 visual feedback studies have shown that participants will robustly adapt to task-irrelevant cursor feedback
 683 (e.g., Kim et al., 2018; Morehead et al., 2017; Tsay et al., 2024).

684 Parameter Estimates

Model	Parameter	Mean [95% CI]
PIECE	σ_r	5.307 [4.917, 5.697]
	σ_{combined}	0.429 [0.268, 0.589]
	b	0.711 [0.391, 1.032]
PReMo	σ_u	2.939 [1.543, 4.334]
	σ_v	3.996 [2.461, 5.455]
	σ_p	3.958 [2.461, 5.456]
	B	0.575 [0.391, 0.759]
	η_p	1 [1, 1]
	b	0.729 [0.374, 1.082]
PEA	σ_{int}	7.028 [5.571, 8.485]
	B	0.297 [0.251, 0.343]
	b	0.858 [0.479, 1.238]
REM	σ_{combined}	6.538 [4.824, 8.251]
	S	0.944 [0.357, 1.531]
	$Const$	2.755 [0.904, 4.606]
	b	0.848 [0.476, 1.219]

Table 1: MLEs of each model.

685 Parameter and Model Recovery

686 To validate the parameter values obtained by fitting PIECE to our behavioral data and our ability to
 687 distinguish between each of the four models, we performed parameter recovery and model recovery analyses
 688 (Wilson and Collins, 2019). We generated 100 synthetic datasets with each of the four models used
 689 in this study, totaling 400 datasets in all. The synthetic data from each model were generated using
 690 parameter values drawn from uniform distributions bounded by the minimum and maximum parameter
 691 values obtained from fits to each individual participant’s behavioral data (i.e., the range of MLEs of each
 692 model). We then fit all 400 synthetic datasets with each model using maximum likelihood estimation and

693 performed objective model selection for each dataset.

694 As shown in Fig. a, the recovered parameters for σ_{comb} and b from the PIECE model were highly
695 correlated with the parameter values used to generate the synthetic data. In the case of σ_{T} , the correlation
696 was not as strong. Future experiments that test a wider range of perturbation sizes will result in more
697 accurate recovery of this parameter, as it represents the observer's prior uncertainty about the perturbation
698 size, which in the current experiment only spanned a very narrow range.

699 More importantly, our model recovery analysis showed that we can accurately identify whether data
700 were generated by the PIECE or PEA model, and to a lesser extent with PReMo. Based on BIC scores,
701 PIECE best fit the synthetic data generated with REM more frequently than REM did. This points to
702 PIECE being able to accommodate a wide range of adaptation behaviors, and is also due to PIECE being a
703 slightly more parsimonious model than REM (3 free parameters for PIECE versus 4 for REM). Notably, the
704 results of our model recovery analysis were qualitatively similar when using Akaike Information Criterion
705 scores, with the only difference being that the diagonal values in our confusion matrix were larger for
706 PReMo and REM using AIC scores. This points to the robustness of our modeling results, especially with
707 regard to PIECE being clearly identifiable.

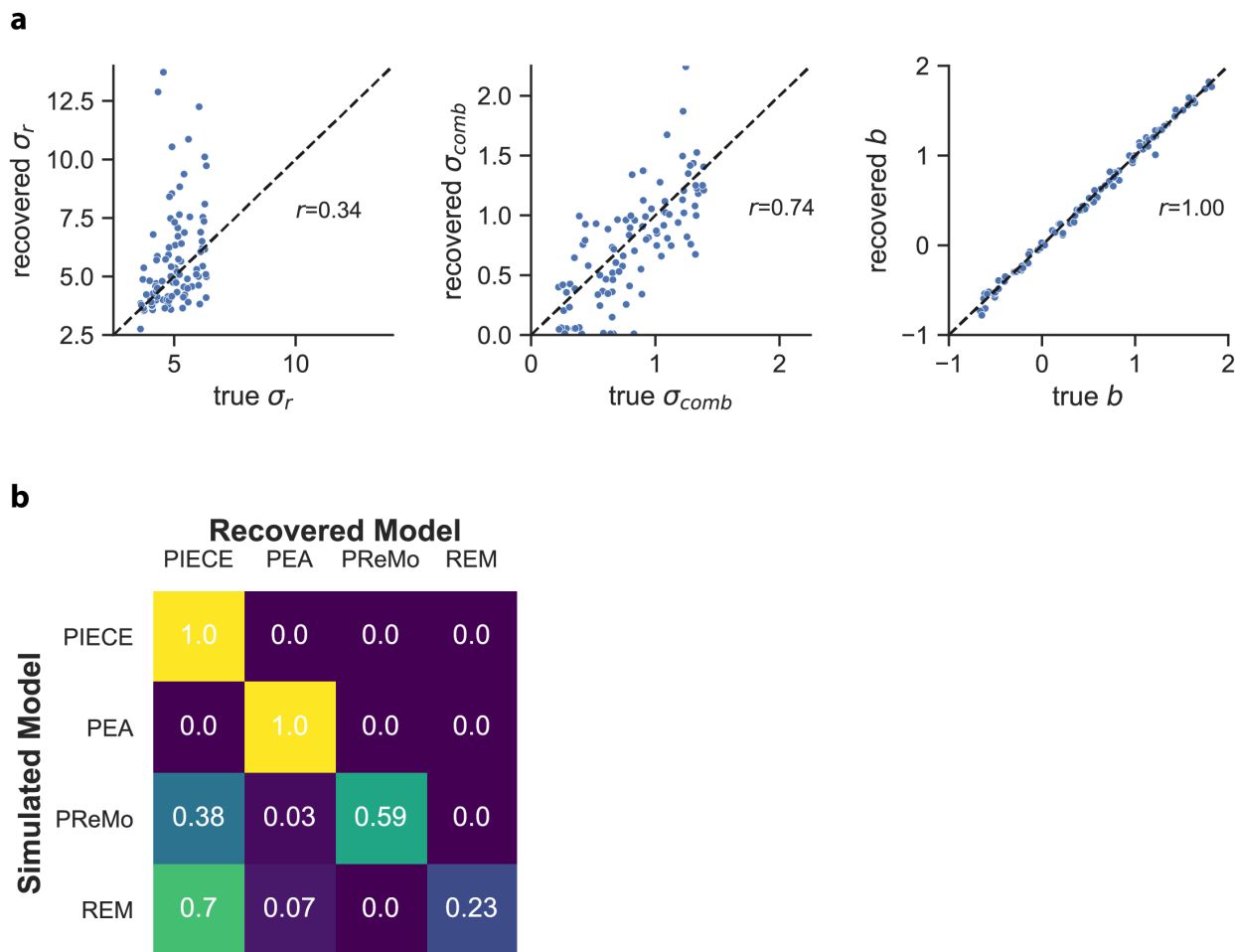


Figure 8: Parameter and model recovery analyses. (a) Plots show the PIECE model parameter values that were recovered (y-axes) from fits to 100 synthetic datasets generated with the PIECE model along with correlation coefficients. The parameter values used to generate the synthetic data are shown on the x-axes. (b) Confusion matrix for model selection. The value of each cell within the matrix indicates $p(\text{recovered model}|\text{simulated model})$.

708 **References**

- 709 Kim, H. E., Morehead, J. R., Parvin, D. E., Moazzezi, R., & Ivry, R. B. (2018). Invariant errors reveal lim-
710 itations in motor correction rather than constraints on error sensitivity. *Communications Biology*,
711 *1*(1).
- 712 Morehead, J. R., Taylor, J. A., Parvin, D. E., & Ivry, R. B. (2017). Characteristics of implicit sensorimotor
713 adaptation revealed by task-irrelevant clamped feedback. *Journal of cognitive neuroscience*, *29*(6),
714 1061–1074.
- 715 Oza, A., Kumar, A., Sharma, A., & Mutha, P. K. (2024). Limb-related sensory prediction errors and task-
716 related performance errors facilitate human sensorimotor learning through separate mechanisms.
717 *PLoS Biology*, *22*(7), e3002703.
- 718 Ranjan, T. (2022). *Understanding the role of internal predictions in sensorimotor adaptation* [Doctoral
719 dissertation, Harvard University].
- 720 Sadaphal, D. P., Kumar, A., & Mutha, P. K. (2022). Sensorimotor learning in response to errors in task
721 performance. *Eneuro*, *9*(2).
- 722 Tsay, J. S., Chandy, A. M., Chua, R., Miall, R. C., Cole, J., Farnè, A., Ivry, R. B., & Sarlegna, F. R. (2024).
723 Minimal impact of chronic proprioceptive loss on implicit sensorimotor adaptation and perceived
724 movement outcome. *Journal of Neurophysiology*, *132*(3), 770–780.
- 725 Tsay, J. S., Haith, A. M., Ivry, R. B., & Kim, H. E. (2022). Interactions between sensory prediction error
726 and task error during implicit motor learning. *PLoS computational biology*, *18*(3), e1010005.
- 727 Wilson, R. C., & Collins, A. G. (2019). Ten simple rules for the computational modeling of behavioral
728 data. *Elife*, *8*, e49547.