August 7, 2025

Relative Timescale of Channel Voltage-Dependence and Channel Density Regulation Impacts Assembly and Recovery of Activity

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Running title: Consequences of Activity-Dependent Channel Voltage-Dependence

Keywords: activation curves, half-activation, activity-dependent regulation, voltage-gated ion channels, homeostasis, intrinsic excitability

ABSTRACT

Neurons can maintain stable activity profiles over their lifetimes despite ion

channel turnover over minutes to weeks. Neuronal activity is also influenced by

regulating the voltage-dependence of ion channels. How do these two forms of plasticity

work together to maintain a stable activity profile? We augment a classical model of

activity-dependent ion channel density regulation with a mechanism that adjusts

channel voltage-dependence based on activity. These findings reveal that the timescale

of these mechanisms shape the specific electrical activity patterns that achieve a target

activity profile. Moreover, alterations in channel voltage-dependence can impact a

neuron's ability to recover from perturbations. These results highlight a potentially

distinct role for activity-dependent regulation of channel voltage-dependence in

maintaining stable neuronal activity profiles.

INTRODUCTION

Neurons maintain stable activity patterns over years despite continual turnover in their ion channel composition, which occurs on timescales from days to weeks. Experimental studies have shown that neurons homeostatically regulate both channel number (Desai et al 1999, Haedo & Golowasch 2006, Mizrahi et al 2001, Santin & Schulz 2019, Thoby-Brisson & Simmers 2002, Turrigiano et al 1994, Turrigiano et al 1995, Viteri & Schulz 2023) and channel voltage-dependence (Gasselin et al 2015, Haedo & Golowasch 2006, Thoby-Brisson & Simmers 2002) to stabilize their activity.

These mechanisms have been widely studied through computational modeling, but most of this work has focused on changes in channel number (Abbott & LeMasson 1993, Alonso et al 2023, Golowasch et al 1999, Gorur-Shandilya et al 2020, LeMasson et al 1993, O'Leary et al 2014, Siegel et al 1994, Srikanth & Narayanan 2015). This is largely due to the plethora of work indicating such changes occur (Desai et al 1999, Haedo & Golowasch 2006, Mizrahi et al 2001, Santin & Schulz 2019, Thoby-Brisson & Simmers 2002, Turrigiano et al 1994, Turrigiano et al 1995, Viteri & Schulz 2023). However, this emphasis also reflects the challenges associated with precisely measuring changes in channel voltage-dependence. Inter-neuronal variability in activation and inactivation curves can obscure clear patterns of activity-mediated modifications, especially when analyzed at the population level (Turrigiano et al 1995). As a result, our understanding of how modifications to channel voltage-dependence contributes to activity stability remains limited.

This gap is important to address given the ubiquity of channel voltage-dependence regulation. Neurons modulate voltage-dependence through mechanisms

ranging from rapid post-translational modifications (Fraser & Scott 1999, Johnson et al 1994) or slower processes like subunit synthesis and degradation (Wu et al 2023). These processes are known to be associated with calcium-mediated second messengers. Intracellular calcium is also thought to play an important role in maintaining neuronal activity set points (LeMasson et al 1993, Turrigiano et al 1994). As such, calcium-mediated homeostasis may operate, at least in part, by tuning channel voltage-dependence—yet the consequences of this form of regulation are largely uncharacterized.

To address this, we extended a state-of-the-art computational homeostatic model. This model stabilizes neuronal activity by monitoring intracellular Ca^{2+} concentrations and inserting and deleting ion channels (Alonso et al 2023). To this model, we added a mechanism that alters channel voltage-dependence. Importantly, this model enabled independent control over the timescales of channel number and channel voltage-dependence alterations. This organization of the model let us simulate fast changes in voltage-dependence, such as those driven by phosphorylation, or slower changes, like those resulting from synthesis or degradation of channel subunits. Our central finding is that the timescale of voltage-dependence regulation influences the specific channel configurations a neuron adopts—both while attaining a prespecified activity target and while maintaining it during perturbation.

RESULTS

The Model

To explore how modifications in channel voltage-dependence influence a

neuron's capacity to achieve an activity profile as changes in ion channel density occur,

we created a computational model. This model was built around a single compartment,

Hodgkin-Huxley type conductance-based neuron with seven intrinsic currents (see

Turrigiano et al (1995) or Box 1, Supplement 1). The neuron model consisted of a fast

Na^+ current, two Ca^{2+} currents, three K^+ currents, a hyperpolarization-activated inward

current (I_h), and a leak current. Each current was described by its maximal conductance

(number of ion channels) and activation curves. Some currents also possessed

inactivation curves as well. The (in)activation curves described the effects of voltage on

the opening and closing of the membrane channels.

The activity-dependent regulation of channel number and (in)activation curves

employed a strategy that uses intracellular Ca^{2+} concentrations as a proxy for the

neuron's activity (Abbott & LeMasson 1993, Alonso et al 2023, Golowasch et al 1999,

Gorur-Shandilya et al 2020, LeMasson et al 1993, O'Leary et al 2014, Siegel et al 1994,

Srikanth & Narayanan 2015). Specifically, we followed the approach used in Liu et al

(1998) and Alonso et al (2023), for altering channel density to guide our approach to

altering (in)activation curves. In those models, the intracellular Ca^{2+} concentration and

three filters (also called sensors) of the Ca^{2+} concentration were computed. The three

sensors were employed to better differentiate between the activity of tonic spiking and

bursting neurons (Alonso et al 2023, Liu et al 1998). A target activity profile was chosen

by prespecifying the target values of these sensors. These values were selected through

trial and error to consistently produce regular bursting across a specific range of initial

conditions (see Methods). When deviations from these Ca²⁺ sensor targets were

detected, channel densities were altered to up or down regulate the number of channels

in the membrane (Alonso et al 2023, Liu et al 1998). Details on how the sensors were

computed and how the target activity—defined by the target sensor values—was

specified are provided in the Methods.

We expanded on these models by incorporating a mechanism that shifts the

(in)activation curves in response to deviations from the Ca²⁺ sensor targets. This was

accomplished by changing the half-(in)activation voltages of the (in)activation curves,

mirroring strategies employed in Liu et al (1998) and Alonso et al (2023). The details of

the Model, with their equations and assumptions are described in the Methods section.

A brief overview is provided in Box 1.

Importantly, the response times of the mechanisms responsible for ion channel

insertion and deletion, as well as those for shifts in (in)activation curves, were

independently controlled. In the model, these response times were initially set two

orders of magnitude apart. This reflected the slower processes of protein synthesis and

degradation associated with channel insertion and deletion. This contrasted with faster

time scales of post-translational modifications such as channel phosphorylation.

Specifically, the timescale for changes in maximal conductance was set at $\tau_g = 600$ s,

and the timescale for half-(in)activation shifts at $\tau_{half} = 6$ s. Later, we adjusted the

response times of the (in)activation curve shifts. In prior computational studies of

activity-dependent regulation, τ_g was set to be less than 10 seconds, substantially faster

than the timescales used in our model (Alonso et al 2023, Liu et al 1998). This study

lengthens this timescale and explores the impact of varying the relative timescales of

maximal conductance changes (τ_g) and half-(in)activation shifts (τ_{half}).

Bursters Assembled by Activity-Dependent Alterations in Ion Channel Density and
Channel Voltage-Dependence

Figure 1 shows the full model, with the two regulation mechanisms in operation.

From a set of starting randomly chosen parameters (maximal conductances and half-

(in)activations), referred to as Starting Parameters 1 (SP1), the activity-dependent

regulation mechanism altered the ion channel density and voltage-dependences to

assemble an electrical activity pattern that satisfied the Ca^{2+} targets. The targets were

chosen so that a burster self-assembled (see Methods). The final ion channel density and

voltage-dependences are shown in the bar graphs (Figure 1A). The first two rows in

Figure 1A show the electrical activity of the neuron as the model seeks to satisfy the Ca^{2+}

targets. As it does, the model assembled a burster. The second row shows the activity

over the entire trace and the first row shows blows up at: the start (a), about 100

minutes (b) and at the end of the simulation (c). At the time shown in (b), the neuron

was bursting, but the Ca^{2+} targets had not been achieved. There were two indications

that this burster did not satisfy the specified Ca^{2+} targets: (1) the model still continued to

adjust the properties of the ion channel repertoire here and (2) α was not yet zero.

Briefly, α measures whether the model has successfully tuned the neuron's ion

channel densities and half-(in)activations to create an activity pattern that matches the

preset sensor targets; values near zero indicate successful tuning (more in Methods).

Even after the model assembles a channel repertoire that satisfies the targets, brief

excursions of α away from zero can occur. These excursions exist because the neuron's

electrical activity is quasistable—meaning it may lose/add a spike or experience a slight

drop in slow wave amplitude, temporarily causing the sensors to be unsatisfied. When

this happens, the model quickly makes minor adjustments to the channel densities and

half-(in)activations to restore alignment with the targets (Figure 1—Supplement, Row 2,

between minutes 300 and 400). These adjustments are minimal (Figure 1, Rows 3−5).

The third and fourth traces in Figure 1A show how the model altered half-

activation and half-inactivation values to meet the Ca^{2+} target. Very early on there were

large changes in the activation curves, which then slowly moved to a steady-state value.

The fifth trace shows the evolution of maximal conductances. These started to change

early on but did so slowly. As the maximal conductances slowly changed, the half-

activation curves followed suit, adapting to the new repertoire of ion channels to close

the distance to the Ca^{2+} target. Figure 1 Supplement shows the assembly of a burster

from a different set of Starting Parameters (maximal conductances and half-(in)

activations (SP4)), with qualitatively similar results to those obtained with SP1 but

quantitatively different final parameters.

The results shown in Figure 1A require activity-dependent regulation of the

maximal conductances. When activity-dependent regulation of the maximal

conductances is turned off, the model failed to assemble SP1 into a burster (Figure 1B).

This was seen in the other 19 Starting Parameters (SP2-SP20), as well.

In general, when the model was initialized with different Starting Parameters, it

produced bursters with different final intrinsic properties. Twenty degenerate bursters

(SP1-SP20), conforming to standards specified in Methods, were selected from over 100

bursters produced with different Starting Parameters. Figure 2A shows the activity

characteristics of these 20 bursters (Period, Burst Duration, Spike Height, Maximum

Hyperpolarization and Slow Wave Amplitude). The left panel of Figure 2B shows the

range of conductances in this group. The right panel illustrates the variation in half-

(in)activation curves. The asterisks indicate the baseline half-(in)activation values; all

neuron models were assembled from starting parameters that lay within ±0.5 mV of

these baseline values (see Methods for details).

Half-(in)activation Mechanism Timescale Impacts Activity Pattern

In the previous the section, we studied the types of bursters that arose from

slowly changing the density of the channel repertoire and quickly altering the half-

(in)activations of the resulting repertoire. We next changed the timescale over which we

altered half-(in)activations and assessed how the burst characteristics of the population

changed (the timescale of channel density alterations remains the same). Figure 3A

shows that alterations in the timescale of half-(in)activation alterations significantly

impacted the period, interburst interval, spike height, and maximum hyperpolarization

of the entire bursting population.

The timescale of half-(in)activation alterations also impacted some intrinsic

properties of the population. Figure 3B plots the burster assembled with the timescale of

half-(in)activation alterations at two extremes: fast and infinitely slow (i.e. off). This

visualization is known as a currentscape (Alonso & Marder 2019). In both cases, the

resulting burster appeared to terminate using the same mechanism: hyperpolarization

via the calcium-activated potassium current. This is seen by the relative contribution the

calcium-activated potassium current makes at the point of maximum hyperpolarization

(see at the dotted line in Figure 3B). However, the manner in which this current came to

dominate the burster termination differs. Note that the burster constructed without

half-(in)activation alterations possessed a higher maximal conductance for I_{KCa} than the

burster constructed with half-(in)activation alterations (Figure 3C, Mauve dot).

Moreover, the latter burster had more depolarized half-activations (Figure 3C, Mauve

dot). In fact, this observation held true for SP2-SP20 as well (Figure 3C).

Potential Mechanism

To explore why the properties of the resulting bursters depend on the timescale

of half-(in)activation adjustments, we examined what happens when SP1 is assembled

under different half-(in)activation timescales: (1) fast, (2) intermediate (matching the

timescale of ion channel density changes), and (3) infinitely slow (i.e., effectively turned

off). The effects of these timescales can be seen by comparing the zoomed-in views of

the SP1 activity profiles under each condition (Figure 4).

When half-(in)activations are fast, the time evolution of α— which tracks how far

the activity pattern is from its targets (see Methods)—shows an abrupt jump as it

searches for a voltage-dependence configuration that meets calcium targets (Figure 4A).

As this happens, the channel densities are slightly altered, and this process continues

again. Slowing the half-(in)activations alterations reduces these abrupt fluctuations

(Figure 4B). Making the alterations infinitely slow effectively removes half-(in)activation

changes altogether, leaving the system reliant solely on slower alterations in maximal

conductances (Figure 4C). Because each timescale of half-(in)activation produces a

different channel repertoire at each time step, different timescales of half-(in)activation

alteration led the model through a different path in the space of activity profiles and

intrinsic properties. Ultimately, this resulted in distinct final activity patterns – all of

which were consistent with the Ca^{2+} targets.

Half-(in)activation Alterations Contribute to Activity Homeostasis

This model also simulates how the assembled bursters recover their activity

following perturbations. In particular, we modeled an increase in extracellular

potassium concentration by altering both the potassium reversal potential and the leak

reversal potential (see Methods). For these simulations, we used the previously tuned

models SP1–SP20. They begin with the homeostatic mechanism engaged but idling

because these models already met the prespecified activity targets. However, once

extracellular potassium concentration increases, the shifts in reversal potentials cause

all 20 models to experience depolarization block. In response, the homeostatic

mechanism becomes operative and adjusts the neuron's maximal conductances and

half-(in)activations.

We anticipated that the perturbation would depolarize the membrane. To

accommodate this shift, we increased the allowable deviation for the DC calcium sensor,

Δ_D (see Methods), from 0.015 to 0.035. Figure 5A shows SP1's activity before

perturbation (blue) and after 265 minutes of recovery (red), under three conditions: fast

half-(in)activation regulation, intermediate (same timescale as conductance regulation),

and no half-(in)activation regulation. The depolarized membrane potential is evident in

the more positive maximum hyperpolarizations during perturbation. Notably, although

all three cases eventually re-stabilized to activity patterns that satisfied the calcium

targets, the resulting burst patterns were different.

SP1 was one of eleven models that successfully recovered under all three

homeostatic conditions. Eight models required some form of half-(in)activation

modulation to stabilize; in these cases, conductance regulation alone was not sufficient

for recovery. Among these eight, two only recovered when half-(in)activation changes

were fast, and another two only recovered when they were slow. Evidently, for some

models the ability to recover bursting activity depended on both the presence and speed

of half-(in)activation modulation.

Figures 5B and 5C quantify the variability in post-recovery activity patterns and

intrinsic parameters across different half-(in)activation regulation timescales.

Comparing the fast (τ_{half} = 6 s, blue) and intermediate (τ_{half} = 600 s, green) conditions

reveals significant differences in burst period, interburst interval, and spike height

(Figure 5B). Notably, these same features also showed significant variation in Figure 3A,

where timescale-dependent effects were observed during the initial assembly of

bursters. Similarly, the half-activation voltage of the KCa current differs significantly

between these two timescales (Figure 5C), paralleling the differences observed in Figure

3C. In models that successfully recovered from perturbation, the timescale of voltage-

dependence modulation influenced both the resulting activity patterns and the intrinsic

parameters of the stabilized bursters.

DISCUSSION

Timescale of Channel Voltage-Dependence Alterations

While alterations in channel voltage-dependence are known to modulate a

neuron's excitability (Levitan 1988), their role in stabilizing a neuron around an activity

target has not been extensively explored. This study explores how modifying the

timescale of voltage-dependence regulation influences both the initial attainment of a

target activity pattern and its maintenance during perturbation. While all models met

the activity target regardless of whether voltage-dependence modulation was present,

recovery after perturbation depended on both the presence and timescale of this

mechanism. Furthermore, in both scenarios, the timescale shaped the stabilized

neuron's specific activity characteristics and underlying intrinsic parameters. This

suggests a general role for the timescale of half-(in)activation modulation in sculpting

stable activity states.

Figure 6 provides a conceptual summary of this idea. The space of possible

intrinsic parameter configurations (Figure 6A) and electrical activity patterns (Figure

6B) each contain regions that satisfy a given activity target—depicted in blue for activity

characteristics and green for intrinsic parameters. The perturbations administered in

this study simply shift these target-satisfying regions. When channel gating properties

are altered quickly in response to deviations from the target activity, the neuron

ultimately settles into a stable activity pattern. The resulting electrical patterns are

shown in Figure 6 as the orange bubble labeled "$\tau_{half} = 6$ s". At a slower timescale of

alterations, the activity characteristics and model parameters of the electrical patterns

shift, as illustrated by the other orange subregions in Figure 6. This shift conceptually

highlights how changing the timescale of alterations to channel voltage-dependence can

move the neuron through different regions within the space of activity characteristics or

intrinsic parameters while still maintaining its activity target.

283   Our simulations show that altering the timescale of the channels' voltage-

dependent gating properties impact its activity characteristics. However, what biological

processes can implement these alterations? Fast timescale post-translational alterations

can be implemented by second messengers that are anchored next to ion channels

(Fraser & Scott 1999, Johnson et al 1994) or second messengers that diffuse across the

cytosol (Agarwal et al 2016, Zaccolo et al 2006). Phosphorylation by kinases is a

commonly studied post-translational modification, however there are other

modifications that also occur on these time scales, such as S-nitrosylation (Broillet

1999). On slower timescales, the neuron can modulate the transcription and translation

of these auxiliary subunits based on intracellular calcium levels. For instance, Potassium

Channel-Interacting Proteins are known to be controlled by CREB (Wu et al 2023).

Biological Relevance

295   One application for the simulations involving the self-assembly of activity may be

to model the initial phases of neural development, when a neuron transitions from

having little or no electrical activity to possessing it (Baccaglini & Spitzer 1977). As

shown in Figure 6, the timescale of (in)activation curve alterations define a neuron's

activity characteristics and intrinsic properties. As such, neurons may actively adjust

these timescales to achieve a specific electrical activity aligned with a developmental

phase's activity targets. Indeed, developmental phases are marked by changes in ion

channel density and voltage-dependence, leading to distinct electrical activity at each

stage (Baccaglini & Spitzer 1977, Gao & Ziskind-Conhaim 1998, Goldberg et al 2011,

Hunsberger & Mynlieff 2020, McCormick & Prince 1987, Moody & Bosma 2005,

O'Leary et al 2014, Picken Bahrey & Moody 2003).

Additionally, our results show that activity-dependent regulation of channel

voltage-dependence can play a critical role in restoring neuronal activity during

perturbations (Figure 5). Specifically, the presence and timing of half-(in)activation

modulation influenced whether the model neuron could successfully return to its target

activity pattern. Many model neurons only achieved recovery when a half-(in)activation

mechanism was present. Moreover, the speed of this modulation shaped recovery

outcomes in nuanced ways: some model neurons reached their targets only when

voltage-dependence was adjusted rapidly, while others did so only when these changes

occurred slowly. These observations all suggest that impairments in a neuron's ability to

modulate the voltage-dependence of its channels may lead to disruptions in activity-

dependent homeostasis. This may have implications for conditions such as addiction

(Kourrich et al 2015) and Alzheimer's disease (Styr & Slutsky 2018), where disruptions

in homeostatic processes are thought to contribute to pathogenesis.

In conclusion, our findings suggest that when a neuron begins from a state of no

activity and small randomly specified channel densities, changes in channel density are

essential for attaining activity. In this context, activity-dependent changes in channel

voltage-dependence alone did not assemble bursting from these low-conductance initial

states (cf Figure 1B). Based on this evidence alone, one might conclude that shifts in

(in)activation curves have negligible impact, effectively being overshadowed by the

larger changes in excitability driven by alterations in channel density. This perspective

aligns with a common assumption in systems analysis: that short- and long-timescale

processes are sufficiently distinct and can be studied independently without significant

interaction. However, our results demonstrate that fast shifts in half-(in)activation

constrain the types of activity characteristics and intrinsic properties a neuron

ultimately expresses (Figure 6). These fast alterations, while subtle, manifest over long

timescales required to assemble activity. This study exemplifies how interactions

between processes operating on different timescales can influence one another. Such

principles are particularly relevant for understanding multiscale interactions in

neuroscience, such as the connections between subcellular, cellular, and network-level

properties (Bhalla 2014, Marom 2010).

METHODS

Details of the Model and its Implementation

Box 1 presents a scheme for the equations used in the model: a Hodgkin-Huxley

type equation with 7 intrinsic currents, Ca²⁺ sensors, and an activity-dependent

regulation mechanism.

The Hodgkin-Huxley type neuron consisted of 7 intrinsic currents (Box 1,

Supplement 1, Equations 1-3) and an intracellular Ca²⁺ concentration calculation (Box 1,

Supplement 1, Equation 4). The neuron's capacitance (C) was set to 1 nF. The currents

were labeled by the index *i* and were in order: fast sodium (I_{Na}), transient calcium (I_{CaT}),

slow calcium (I_{CaS}), hyperpolarization-activated inward cation current (I_H), potassium

rectifier (I_{Kd}), calcium-dependent potassium (I_{KCa}), and fast transient potassium (I_A).

Additionally, there was a leak current (I_L). The maximal conductances were given by $\bar{g}_i$.

Activation and inactivation dynamics were given by m_i and h_i, respectively. The

exponents q_i were given by 3,3,3,1,4,4,3, respectively. The equilibrium/reversal

potentials of the sodium current, hyperpolarization activated cation current, potassium

currents, and leak current were +30 mV, -20 mV, -80 mV and -50 mV respectively. The

calcium equilibrium potential was computed via the Nernst Equation (Box 1,

Supplement 1, Equation 5) using the computed internal Ca²⁺ concentration ($[Ca^{2+}]_i$) and

an external Ca²⁺ concentration of 3 mM (temperature was 10 ºC).

We modeled perturbations in neuronal activity by a 2.5-fold increase in

extracellular potassium concentration. Using the Nernst equation, this shift corresponds

to a change in the potassium reversal potential from –80 mV to –55 mV. We assume

that leak current in our model includes a potassium component, so that the perturbation

also impacts the leak reversal potential. Additionally, assuming that the remaining leak

current comes from sodium channels, we can compute how to perturb the leak reversal

potential. The leak maximal conductance for our model is fixed at .01 μS. The new leak

reversal potential was calculated using standard computation involving the

conductance-weighted average of potassium and sodium reversal potentials, yielding a

valued of approximately -32 mV.

The activation curves and inactivation curves were given by $m_{i,\infty}$ and $h_{i,\infty}$. Their

associated time constants were τ_{m_i} and τ_{h_i}, respectively. The (in)activation curves and

their associated time constants were functions of the membrane voltage, V (see tables in

Box 1, Supplement 1). Note that I_{KCa} had an activation curve that depends on internal

Ca^{2+} concentration. The internal calcium concentration was computed using I_{CaT} and

I_{CaS} (Box 1, Supplement 1, Equation 4).

There were three Ca^{2+} sensors that responded to different measures associated

with fluctuations of $[Ca^{2+}]_i$ during a burst. They were used to determine how close these

measures are to their targets. These measures were the amount of $[Ca^{2+}]_i$ that entered

the neuron: as a result of the slow wave of a burst (S), as a result of the spiking activity

of a burst (F), and on average during a burst (D). They were computed using Equations

6-8 (Box 1, Supplement 2). Each sensor had a corresponding target value $- \bar{S}, \bar{F}, \bar{D} -$

which represented the target activity pattern.

The model computed moving averages of the mismatch between each sensor and

its target, denoted, E_S, E_F, and E_D (Equations 9–11, Box 1, Supplement 2). These

averages were taken over a two-second window ($\tau_S = 2000$ ms). allowing the model to

smooth out short-term fluctuations and capture sustained deviations from the target.

Ideally, E_S, E_F, and E_D, are zero, however, ongoing membrane fluctuations, such as

bursting or spiking still created small variations in these measures. Allowable tolerances

are specified in the model as, Δ_s, Δ_F, and Δ_D, respectively. The moving averages of the

mismatches between the calcium sensors and their targets are compared against the

allowable tolerances and combined in Equation 12 (Box 1, Supplement 2) to create a

"match score", S_F (Equation 12).

To compute this match score, we adapted a formulation from Alonso et al (2023),

who originally used a root-mean-square (RMS) or L^2 norm to combine the sensor

mismatches. In that approach, each error (E_S, E_F, and E_D) is divided by its allowable

tolerance (Δ_s, Δ_F, and Δ_D) to produce a normalized error. These normalized errors are

then squared, summed, and square-rooted to produce a single scalar score that reflects

how well the model matches the target activity pattern.

In our version, we instead used an L^8 norm, which raises each normalized error to

the 8th power before summing and taking the 1/8th root. This formulation emphasizes

large deviations in any one sensor, making it easier to pinpoint which feature of the

activity is limiting convergence. By amplifying outlier mismatches, this approach

provided a clearer view of which sensor was driving model mismatch, helping us both

interpret failure modes and tune the model's sensitivity by adjusting the tolerances for

individual sensor errors.

Although the L^8 norm emphasizes large deviations more strongly than the L^2

norm, the choice of norm does not fundamentally alter which models can converge—a

model that performs well under one norm can also be made to perform well under

another by adjusting the allowable tolerances. The biophysical mechanisms by which

neurons detect deviations from target activity and convert them into changes in ion

channel properties are still not well understood. Given this uncertainty, and the fact that

using different norms ultimately shouldn't affect the convergence of a given model, the

use of different norms to combine sensor errors is consistent with the broader basic

premise of the model: that intrinsic homeostatic regulation is calcium mediated.

Equations 13 and 14 were used to evaluate whether the match score was sufficient

for the model to cease altering maximal conductances and (in)activation curves,

indicating that the target had been achieved. In Equation 14 (Box 1, Supplement 2), the

"match score" was compared to a preset threshold value the model would accept, ρ. This

equation returned 1 if the match score is below ρ, a 0 if the match score is above ρ, and

some number in between 0 and 1 if the match score was close to the threshold, ρ.

Equation 13 (Box 1, Supplement 2) then effectively computed a 2 second time average of

Equation 14. Equation 13 was an activity sensor, α, that was plotted in Figure 1. It took

the value zero when all three Ca^{2+} sensors were close to the Ca^{2+} targets but remained

close to one otherwise. The value of this activity sensor was used to slow the evolution of

maximal conductances and half-(in)activations as the Ca^{2+} sensors reached their targets.

It also served as a measure for how well the model satisfied the Ca^{2+} sensors. In this

model, $\rho = 0.3$. The "sharpness" of the threshold at $\rho = 0.3$ was set by Δ_α. We set $\Delta_\alpha =$

0.01 indicating that when the match score, S_f, was greater than ~0.32, the model

considered the calcium targets satisfied.

Using the information regarding how well the Ca^{2+} sensors were satisfied, the

model regulated conductance densities and (in)activation curves. The maximal

conductances $\bar{g}_i$, and the ion channel voltage-sensitives, as measured by activation

curve shifts, $V_{s_i^m}$, and inactivation curve shifts, $V_{s_i^h}$, from their specified locations ($V_{\hat{m}_{i,1/2}^0}$

for activation curves and $V_{\hat{h}_{i,1/2}^0}$ for inactivation curves), were altered by the model in

response to deviations from the calcium targets (Box 1, Supplement 3, Equations 15-17).

The latter were adjusted on a time scale given by τ_{half} and the former by τ_g. The index i

ran from 1 through 7 for the maximal conductances (Equation 15) and half-

(in)activations (Equation 16). In Equation 17, the index only took on a subset of values:

$i = 1,2,3,7$ – because only some intrinsic currents in this model had inactivation (the

associated currents being listed above).

Equations 16 and 17 were new equations added to the model. They were built

similarly to Equation 15. In Equation 15, the maximal conductances were altered when

the calcium sensors didn't match their targets. There are two parts to Equation 15: a part

that converts calcium sensor mismatches into changes in maximal conductances and a

cubic term. The cubic term prevents unbounded growth in the maximal conductances.

We set the constant associated to the cubic term, γ, to 10^{-7}. The other term adjusted the

maximal conductance based on fluctuations of calcium sensors from their target levels.

The parameters A_i, B_i and C_i reveal how the sensor fluctuations contribute to the

changes of each maximal conductance. These values were provided in Liu et al (1998).

Importantly, note that this term is multiplied by $\bar{g}_i$, which prevents the maximal

conductance from becoming negative and scales the speed of evolution so that smaller

values evolve more slowly and larger values evolve more quickly. As such, multiplication

by $\bar{g}_i$ effectively induced exponential changes in the maximal conductance, allowing $\bar{g}_i$

to vary over orders of magnitude.

Equations 16 and 17, introduced in this paper, enable the adjustment of the ion-

channel voltage dependence. Equations 16 and 17 also each have two components: a

component that converts calcium sensor mismatches into changes in half-

(in)activations and a cubic term. The cubic term prevents unbounded translations. The

cubic term's constant, $\hat{\gamma}$, was set to 10^{-5}. The other term adjusts the half-activations

(Equation 16) and half-inactivation (Equation 17) based on deviation of calcium sensors

from their target levels.

Finally, we discuss how the coefficients A_i, B_i and C_i in Equation 15, and the

corresponding parameters in Equation 16 and 17 were chosen. The same coefficients

used to adjust maximal conductances were used for the inactivation curve shifts, $\hat{\hat{A}}_i$, $\hat{\hat{B}}_i$,

and $\hat{\hat{C}}_i$. The coefficients of the activation curve shift are their negatives, $\hat{A}_i$, $\hat{B}_i$, and $\hat{C}_i$. To

understand this choice, consider a scenario where the neuron is bursting more rapidly

than desired. In that case, then $F > \bar{F}$, $S > \bar{S}$, and $D > \bar{D}$. To reduce the excitability of the

neuron, we made the depolarizing current, for example, I_{Na}, harder to activate, by

shifting the activation curve of I_{Na} to a more depolarizing potential. This implies $\frac{dV_{s_1}m}{dt} >$

0. Therefore, the coefficients $\hat{A}_i$, $\hat{B}_i$, and $\hat{C}_i$ must be non-positive. The coefficients for I_{Na}

in Liu et al (1998) were given as $A_1 = 1$, $B_1 = 0$ and $C_1 = 0$ and were chosen so that

$\frac{d\bar{g}_1}{dt} < 0$, thereby decreasing the maximal conductance of sodium in the same scenario.

So, we flipped the signs to give $\widehat{A_1} = -1$, $\widehat{B_1} = 0$ and $\widehat{C_1} = 0$. In this way, only the F Ca²⁺

sensor contributed to the modulation of I_{Na}, but the activation curves were now adjusted

to promote homeostasis. Furthermore, the model makes it harder to de-inactivate I_{Na} by

shifting its inactivation curve to more hyperpolarized values $\left(\frac{dV_{s_1}h}{dt} < 0\right)$. In the scenario

in which the activity sensors are greater than their activity targets, this corresponds to

setting $\hat{\hat{A}}_1 = 1$, $\hat{\hat{B}}_1 = 0$, and $\hat{\hat{C}}_1 = 0$.

A similar logic can be used to understand how the activation curves of the

outward currents were adjusted. In the scenario described above, the neuron was made

less excitable by increasing the amount of an outward current, for example I_A. This was

done by increasing its maximal conductance or moving its activation curve to more

hyperpolarized values. Liu et al (1998) assigned the coefficients $A_7 = 0$, $B_7 = -1$ and

$C_7 = -1$ to adjust the maximal conductance of I_A. In this scenario where all sensors are

above their targets, this implies $\frac{d\bar{g}_7}{dt} > 0$. Following the prescription we outlined above,

flipping the signs gives $\widehat{A_7} = 0$, $\widehat{B_7} = 1$ and $\widehat{C_7} = 1$. This, in turn, implied $\frac{dV_{s_7^m}}{dt} < 0$ –

creating hyperpolarizing shifts in I_A's activation curve when the activity is greater than

its target, as we expect. In a similar spirit, we assign the coefficients $\hat{\hat{A}}_7 = 0$, $\hat{\hat{B}}_7 = -1$,

and $\hat{\hat{C}}_7 = -1$, so that I_A's inactivation curve shifts to more depolarizing potentials

$\left(\frac{dV_{s_7^h}}{dt} > 0 \right)$ to make it easier to de-inactivate.

Note, the coefficients in Liu et al (1998) don't perfectly correspond with this

reasoning. In Liu et al (1998), some coefficients were assigned values to ensure

convergence of the model. Still, we followed the prescription outlined above (Box 1,

Supplement 3, Table 1).

__Starting Parameters__

The model has 40 ordinary differential equations: 13 equations describe the

neuron's electrical activity, 9 equations describe calcium sensor components, and 18

equations alter the maximal conductances and half-(in)activations (Box 1). This means

we needed to specify 40 starting values for the model. We defined starting parameters

(SPs), as the bursters the model assembles from a particular set of randomly chosen

starting maximal conductances and half-(in)activations. The maximal conductances

were randomly selected between 0.3 nS and 0.9 nS and half-(in)activation shifts are

randomly offset between -0.5 mV and +0.5 mV from their specified half-(in)activation

values ($V_{\hat{m}_{i,1/2}^0}$ and $V_{\hat{h}_{i,1/2}^0}$ in Box 1, Supplement 1, Table 2). Other starting values were

$V = -50$ mV and the gating variables of activation and inactivation were randomly

selected between 0.2 and 0.3. The starting value of intracellular calcium concentration

was set as $[Ca^{2+}]_i = 0.4\ \mu$M. Equations 9-11 (Sensor Errors) and Equation 13 (Activity

sensor, α) were initialized with the value 1. H_X (Equation 7) and M_X (Equation 6) were

initialized to 0 and 1, respectively.

The random initialization of gating variables was not expected to significantly

impact the formation of an activity profile that satisfied the activity target. Bursters may

exhibit bistability if they are initialized with different gating variables. However, this is

for a fixed set of maximal conductances and half-(in)activations. Any bistability in the

initially specified profile was likely lost as the model altered the maximal conductances

and half-(in)activations.

The model was integrated using the Tsitouras 5 implementation of Runga-Kutta

4(5) in Julia's DifferentialEquations.jl package (Tsitouras 2011). The simulation was

saved every 0.5 milliseconds regardless of step size (step size of the integrator was

adaptive and left unspecified). Maximum number of iterations was set to 10^{10} and

relative tolerance was set to 0.0001.

Burster Measurements

We defined regular bursters as activity patterns that exhibited groups of spikes

separated by regular quiescent gaps in activity. To identify bursters, we measured the

following activity characteristics: period, burst duration, interburst interval, spike

height, maximal hyperpolarization, and slow wave amplitude. Peaks were identified as

local maxima in the activity pattern. We used the `findpeaks` function in MATLAB on

the last 90 secs of activity to identify local maxima in the trace, setting the minimum

'prominence' at 2. The latter parameter rejected certain local maxima from counting as

peaks. This parameter is one we arrived at by trial and error. Figures 7-B1 and 7-B2

show red arrows where this function identified peaks. Note in Figure 7-B2, panels A-C

have local maxima that are not counted as peaks (there is one local maxima following

the sequence of peaks in the burst). The choice of 'prominence' in the 'findpeaks'

function was intentionally set to exclude these. To find troughs, we took the negative of

the activity profile and found local maxima using the `findpeaks` function. For this, the

same 'prominence' parameter was used over the same time window. The arrows in

Figure 7C show an example of what was identified by the algorithm as a peak in this

case. Troughs were defined as the negative of the identified points. The red arrow in

Figure 7C shows the maximum hyperpolarization of a cycle obtained using the method

described below. The green arrows are the troughs associated with each spike in the

burst (Figure 7C).

Maximum hyperpolarization was found as follows. First all troughs were

identified in the manner described above. Then, starting at -85 mV, we checked whether

there were any troughs below this value. If there were none, we incremented by 1 mV to

-84 mV and checked again if there were any troughs below this value. This process was

repeated until troughs were detected. The troughs identified this way demarcated the

ends of a cycle in the activity pattern. They were averaged to obtain the maximum

hyperpolarization measurement for the activity pattern.

The time interval between successive maximum hyperpolarizations in the activity

was used to measure the cycle period (Figure 7D). We averaged these cycle period

measurements to give a measurement for the activity pattern's period. To obtain the

burst duration, we segmented the activity pattern into cycles using the time of

maximum hyperpolarization. The burst duration of a cycle was measured as the time

between the first and last spike in the cycle (Figure 7C). The interburst interval of the

cycle was computed by subtracting the cycle's burst duration from the cycle period.

To measure the slow wave amplitude, we identified the troughs of each cycle. The

slow wave amplitude of a cycle is the difference between the maximum

hyperpolarization (Figure 7D, red arrow) during a cycle and the average of the

membrane potential at the troughs associated with each spike in a burst (Figure 7D,

green arrows). The slow wave amplitudes across all cycles were averaged to obtain a

measurement for the activity pattern's slow wave amplitude.

The spike height was obtained by finding the peaks of the membrane potential

during each cycle. A measure for the cycle's spike height was obtained by averaging over

all peaks, subtracting the cycle's maximum hyperpolarization, then subtracting the

cycle's slow wave amplitude. This value was averaged over all cycles to obtain a measure

of the activity pattern's spike height.

We distinguished regular bursting from tonic spiking or irregular bursting using

the following procedure. We classified an activity pattern as a regular burster when: 1)

there was more than 1 spike per cycle, 2) the maximum hyperpolarization

measurements for all cycles were within 3 mV of one another, and 3) the spikes per cycle

varied by no more than 1 spike. The first condition eliminated tonic spiking, the second

condition is a crude measure to check that the average voltage wasn't increasing, and the

third checked for irregular bursting. In addition, we verified that the cycle-to-cycle

measurements exhibited low variability to ensure the appropriateness of averaging

across all cycles. The relative standard deviation for all cycle-to-cycle measurements was

less than 3% for the assembled bursters.

Activity Target Selection

We needed a set of sensor targets that *consistently* produced regular bursters,

regardless of the starting parameters selected. To determine whether a set of calcium

targets consistently produced a burster, we monitored the activity sensor α. α

approached 0 when the calcium sensors were close to their specified targets. There were

small oscillations in α that resulted from calcium fluctuations associated with the

neuron's activity. These fluctuations were smoothed out by taking a running average

over the 8 previous seconds. When a simulation fell below $\alpha = 0.02$ for the last time and

remained there for 100 minutes, we considered the simulation to have "*assembled*" a

burster that satisfied the calcium targets. For simulations involving elevated

extracellular potassium concentration, this criterion was adjusted: a model was

considered to have satisfied the calcium targets if the moving average of α fell below

0.02 and remained there for at least 5 minutes. This change was made because the

perturbation simulations were shorter.

To ensure the activity pattern obtained did not depend on feedback from the

activity regulation mechanism, the mechanism was turned off ($\tau_{half} = \infty$, $\tau_g = \infty$) and

the simulation run for an additional 9000 secs to ensure $\alpha < .02$ for the duration. This

step was taken in the initial generation of bursters (111 in total, described below), but

not for the analysis where the time constants of the half-activation alterations were

changed from $\tau_{half} = 6$ s.

'*Consistently*', in this context, meant that the model assembled bursters from

over 95% of the randomly chosen starting parameters. The remaining simulations did

not complete assembly within the given simulation period. For the calcium targets we

found ($\bar{S} = 0.03, \bar{F} = 0.25$, and $\bar{D} = 0.02$), the model 'consistently' produced bursters. To

obtain these targets, we tried different combinations of calcium targets until a suitable

set of targets consistently produced regular bursters.

Groups of Bursters

Using the obtained calcium targets, we assembled a total of 111 regular bursters

from a wide range of randomly initialized starting parameters. To create a

representative subset of this population, we selected a Group of 20 bursters through

successive rounds of restriction. The goal was to form a set of models that shared similar

burst periods but varied in other activity characteristics and intrinsic properties. This

allowed us to assess how different bursters responded to changes in the speed of half-

(in)activation alterations as the model attempted to reach or maintain its activity target.

We first filtered the 111 models by slow wave amplitude, retaining only those with

amplitudes less than 25 mV. This excluded models with unusually large oscillations and

preserved activity patterns with waveforms similar to those shown in Figure 7C,

reducing the pool to 80 models. The distribution of these bursters' periods are shown in

Figure 7A. The peak of this distribution is between 380 and 500 ms. We further

narrowed the population by selecting bursters around the peak of this distribution

(Figure 7A, highlighted in Red), resulting in 33 bursters (the "selection group"). This

range contained about 30% of the distribution.

In the next round of restriction, we obtained a Group of 20. We ordered the

bursters from shortest to longest period. For each burster from the 3rd to the 31st

position, we examined the two bursters immediately preceding and the two bursters

immediately succeeding in this sorted list and computed the range of burst periods

across these five bursters (Figure 7B). This method provided a rough estimate of the

variability in periods among neighboring bursters. The burster exhibiting the smallest

inter-instance period variability was selected as the first member of the Group of 20

(Figure 7B, Red Arrow). This first burster is referred to as SP1 in Figure 1 (also shown in

Figure 7C) and has a period of about 416 ms. In the second step, we symmetrically

expanded the inter-instance period range around SP1's period by 20%. We randomly

selected an additional 20 models from this group to create the Group of 20.

Model Assumptions

This model of channel voltage-dependence alterations is based on certain

assumptions. The concept outlined in Figure 6 is not expected to depend on these

choices. However, examining these assumptions reveals additional insights.

The model makes two simplifications on how quickly channel gating properties

can be altered. First, the model assumes a single, uniform time constant for modifying

all intrinsic currents. In reality, each intrinsic current is likely influenced by a

combination of mechanisms. For example, the relative contributions of anchored second

messengers and freely diffusing kinases in phosphorylating key residues in L-Type

calcium channels are not well understood and may well involve both processes (Weiss et

al 2013). Consequently, the timescale of modulation for I_{CaS}—the current to which L-

Type calcium channels putatively contributes—likely represents a weighted average of

these mechanisms. Similarly, I_{Kr}, is modulated by PIP2 (a metabolite of calcium-

mediated second messenger pathways) and a beta subunit (Li et al 2011). While the

presence or absence of PIP2 can change on a fast timescale, the synthesis of the beta

subunit occurs over much slower timescales, resulting in mixed temporal dynamics.

These examples illustrate that the balance between fast and slow regulatory mechanisms

likely varies among intrinsic currents, complicating the assumption of a single time

constant. Second, our framework assumes post-translational modifications are removed

as quickly (or slowly) as they are added. This simplification fails to account for processes

where rapid calcium increases lead to prolonged downstream effects. For instance, the

calcium-mediated second messenger CaMKII undergoes autophosphorylation, enabling

a brief calcium signal to induce long-lasting changes in protein activity (Hudmon &

Schulman 2002, Kennedy 1989). This autophosphorylation is known to influence

intrinsic excitability of neurons (Sametsky et al 2009) and may modulate the voltage-

dependence of Ca_V2 channels (Jiang et al 2008), which putatively contribute to I_{CaS}.

Accounting for these processes would require a more complex model capable of

capturing asymmetries in modification rates.

Another assumption we make is that there are limits on the extent to which

(in)activation curves can shift. This limitation is implemented through the cubic term in

Equations 16 and 17. These constraints reflect the fact that shifts in activation curves are

not unlimited. For example, while cAMP can shift the half-activation of I_h, this effect

saturates at high cAMP concentrations (Difrancesco & Tortora 1991). In this study,

however, a uniform constraint is applied across all intrinsic currents. In reality, different

intrinsic currents likely have unique limits on how much they can shift, requiring

distinct functional forms to model their unique restrictions.

Furthermore, we assumed that activity-dependent alterations in channel voltage-

dependence are implemented by changing half-(in)activations. We have not, however,

considered the (in)activation curve's slope or the gating variable's time constant. In this

study, the gating variable's voltage-dependent time constant shift by the same amount

as the half-(in)activation. This preserves how quickly the gating variable respond to

changes in voltage – regardless of where the activation curve is centered. However, this

coupling between the time constant and the activation curve is not necessarily required.

For instance, activity-dependent shifts in voltage-dependent time constants have been

observed without concomitant shifts in activation curves (Thoby-Brisson & Simmers

2002). The voltage dependence of the time constant can, in principle, change in other

ways as well. Also, we presume that the slopes of the (in)activation curves do not

respond to changes in activity. The slope of (in)activation curves defines the steepness of

the voltage dependence for ion channel gating, thereby influencing the voltage

sensitivity of the transition between states (e.g., from closed to open or from inactivated

to deinactivated). In this model, the slopes of these curves are held constant.

Consequently, the voltage ranges remain fixed regardless of shifts in the centering of the

activation or inactivation curves. Nonetheless, with certain calcium-activated potassium

currents, the center and slope of the activation curve depends on calcium concentration

(Wang & Brenner 2006). To the extent that calcium entry during activity initiates

downstream signaling processes that alter ion channel function, this observation

supports the idea that activity could influence the slope of the I_{KCa} activation curve.

In the model, the same activity targets are used to drive changes in both channel

density and voltage dependence. This assumes extensive crosstalk between the calcium

pathways posited to be responsible for ion channel insertion, deletion, and post-

translational modifications. Nonetheless, the extent to which this crosstalk occurs is not

known, although, it is known that cells can detect the temporal patterns and specific

pathways of calcium entry, leading to specific activity-dependent changes (Bito et al

1997, Fields et al 1997, Gallin & Greenberg 1995). Therefore, it is likely that activity-

induced calcium influxes trigger changes along distinct pathways – with some impacting

only channel gating properties, some impacting channel density, and some impacting

both.

It is not necessarily true that all activity-dependent changes in the properties of

intrinsic currents are directly controlled by calcium influx. For example, neurons

coregulate I_A and I_H expression, so that a change in one current is associated with a

change in the other (MacLean et al 2005). Neuromodulatory environment may also play

a role in coordinating ion channel levels (Khorkova & Golowasch 2007). In such cases,

one may use this model for the channels that are controlled by activity and augment it

by explicitly modelling the remaining mechanisms that are independent of activity.

DATA AVAILABILITY

The code and parameters required to reproduce the results in this work are accessible

and at Zenodo. Repository:

[https://github.com/YugarshiM/fastTimescaleHomeostasis] [DOI:

10.5281/zenodo.12585617]

ACKNOWLEDGMENTS

The authors acknowledge the support of NIMH (R01MH046742), NINDS

(R35NS097343), and The Swartz Center for Theoretical Neuroscience at Brandeis

University. YM acknowledges valuable discussions with lab members. Also, YM

acknowledges assistance with code preparation from Gwendolyn Harris.

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<u>Box 1</u>

<div style="border:1px solid black; border-radius:15px; padding:20px;">

Model Components

- *Neuron Model*
  - ◦ Hodgkin-Huxley representation of a single compartment neuron - voltage- and Ca- dependent currents plus leak
  - ◦ Basic $[Ca^{2+}]_i$ handling

- *Ca^{2+} Sensors*
  - ◦ Three different sensors that reflect $[Ca^{2+}]_i$ associated with slow wave of activity (*S*), spiking activity (*F*) and the average during a burst (*D*).
  - ◦ Targets are set for each sensor independently
  - ◦ The three different sensor values are compared to their target values and computed into a single score (α) that reflects the degree to which each is satisfied.

- *Homeostatic Mechanisms*
  - ◦ α is used to adjust the maximal conductances on a time scale τ_g and shifts in half-(in)activations voltages on a time scale of τ_{half}

</div>

826　Box 1, Supplement 1

$$(1) \quad C\frac{dV}{dt} = -\sum_{i=1}^{7} \bar{g}_i m_i^{q_i} h_i (V - E_i) - g_L(V - E_L) + I_{app}$$

$$(2) \quad \frac{dm_i}{dt} = \frac{m_{i,\infty}\left(V - V_{s_i^m}\right) - m_i}{\tau_{m_i}(V - V_{s_i^m})}$$

$$(3) \quad \frac{dh_i}{dt} = \frac{h_{i,\infty}\left(V - V_{s_i^h}\right) - h_i}{\tau_{h_i}(V - V_{s_i^h})}$$

$$(4) \quad \frac{d[Ca^{2+}]_i}{dt} = \frac{1}{20}\left(-.94\,(I_{CaT} + I_{CaS}) - [Ca^{2+}]_i + .05\right)$$

$$(5) \quad E_{Ca}([Ca^{2+}]_i) = 12.197 * log\left(\frac{[Ca^{2+}]_i}{3000.0}\right)$$

834　I_{CaT} and I_{CaS} are the $i = 2$ and $i = 3$ parts of the sum in Equation 1.

836　Table 1 – Activation Curve Exponents & Equilibrium Potentials

Current	Index (i)	q_i	E (mV)
I_{Na}	1	3	30
I_{CaT}	2	3	[set by Nernst Equation – Equation 5]
I_{CaS}	3	3	[set by Nernst Equation – Equation 5]
I_H	4	1	-20
I_{Kd}	5	4	-80
I_{KCa}	6	4	-80
I_A	7	3	-80

838 Table 2 – Normative values of Half-Activation and Half-Inactivation used to generate all
839 starting parameter sets by randomized shifts. Final parameter sets are converted to the
840 normative value minus the shift.

Current	Index (i)	$V_{\widehat{m}^0_{i,1/2}}$ (mV)	$V_{\widehat{h}^0_{i,1/2}}$ (mV)
I_{Na}	1	-25.5	-48.9
I_{CaT}	2	-27.1	-32.1
I_{CaS}	3	-33	-60
I_H	4	-70	
I_{Kd}	5	-12.3	
I_{KCa}	6	-28.3	
I_A	7	-27.2	-56.9

844   Table 3 – Activation Curves and Associated Time Constants

Current	$m_{i,\infty}$ (mV)	τ_{m_i} (ms)
I_{Na}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{1,1/2}^0} + V_{s_1^m}\right)}{-5.29}\right)}$	$1.32 - \dfrac{1.26}{1 + \exp\left(\dfrac{\left(V - V_{s_1^m}\right) + 120}{-25}\right)}$
I_{CaT}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{2,1/2}^0} + V_{s_2^m}\right)}{-7.2}\right)}$	$21.7 - \dfrac{21.3}{1 + \exp\left(\dfrac{\left(V - V_{s_2^m}\right) + 68.1}{-20.5}\right)}$
I_{CaS}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{3,1/2}^0} + V_{s_3^m}\right)}{-8.1}\right)}$	$1.4 + \dfrac{7}{\exp\left(\dfrac{\left(V - V_{s_3^m}\right) + 27}{10}\right) + \exp\left(\dfrac{\left(V - V_{s_3^m}\right) + 70}{-13}\right)}$
I_H	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{4,1/2}^0} + V_{s_4^m}\right)}{6}\right)}$	$272 - \dfrac{-1499}{1 + \exp\left(\dfrac{\left(V - V_{s_4^m}\right) + 42.2}{-8.73}\right)}$
I_{Kd}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{5,1/2}^0} + V_{s_5^m}\right)}{-11.8}\right)}$	$7.2 - \dfrac{6.4}{1 + \exp\left(\dfrac{\left(V - V_{s_5^m}\right) + 28.3}{-19.2}\right)}$
I_{KCa}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{6,1/2}^0} + V_{s_6^m}\right)}{-12.6}\right)} \dfrac{[Ca^{2+}]_i}{[Ca^{2+}]_i + 3}$	$90.3 - \dfrac{75.1}{1 + \exp\left(\dfrac{\left(V - V_{s_6^m}\right) + 46}{-22.7}\right)}$
I_A	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{m}_{7,1/2}^0} + V_{s_7^m}\right)}{-8.7}\right)}$	$11.6 - \dfrac{10.4}{1 + \exp\left(\dfrac{\left(V - V_{s_7^m}\right) + 32.9}{-15.2}\right)}$

847 Table 4 – Inactivation Curves and Associated Time Constants

Current	$h_{i,\infty}$ (mV)	τ_{h_i} (ms)
I_{Na}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{h}^0_{1,1/2}} + V_{s^h_1}\right)}{5.18}\right)}$	$\left(1.5 - \dfrac{-1}{1 + \exp\left(\dfrac{\left(V - V_{s^h_1}\right) + 34.9}{3.6}\right)}\right)\left(\dfrac{.67}{1 + \exp\left(\dfrac{\left(V - V_{s^h_1}\right) + 62.9}{-10}\right)}\right)$
I_{CaT}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{h}^0_{2,1/2}} + V_{s^h_2}\right)}{5.5}\right)}$	$105 - \dfrac{89.8}{1 + \exp\left(\dfrac{\left(V - V_{s^h_2}\right) + 55.0}{-16.9}\right)}$
I_{CaS}	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{h}^0_{3,1/2}} + V_{s^h_3}\right)}{6.2}\right)}$	$60 + \dfrac{150}{\exp\left(\dfrac{\left(V - V_{s^h_3}\right) + 55}{9}\right) + \exp\left(\dfrac{\left(V - V_{s^h_3}\right) + 65}{-16}\right)}$
I_H	1	1
I_{Kd}	1	1
I_{KCa}	1	1
I_A	$\dfrac{1}{1 + \exp\left(\dfrac{V - \left(V_{\hat{h}^0_{7,1/2}} + V_{s^h_7}\right)}{4.9}\right)}$	$38.6 - \dfrac{29.2}{1 + \exp\left(\dfrac{\left(V - V_{s^h_7}\right) + 38.9}{-26.5}\right)}$

849   <u>Box 1, Supplement 2</u>

$$(6) \quad \frac{dM_X}{dt} = \frac{\bar{M}_X \left(I_{CaT} + I_{CaS} \right) - M_X}{\tau_{M_X}} \; ; X = F, S, D$$

$$(7) \quad \frac{dH_X}{dt} = \frac{\bar{H}_X \left(I_{CaT} + I_{CaS} \right) - H_X}{\tau_{H_X}} \; ; X = F, S, D$$

853     $$(8) \quad F = G_F M_F^2 H_F \quad S = G_S M_S^2 H_S \quad D = G_D M_D^2$$

$$(9) \quad \frac{dE_F}{dt} = \frac{(F - \bar{F}) - E_F}{\tau_S}$$

$$(10) \quad \frac{dE_S}{dt} = \frac{(S - \bar{S}) - E_S}{\tau_S}$$

$$(11) \quad \frac{dE_D}{dt} = \frac{(D - \bar{D}) - E_D}{\tau_S}$$

$$(12) \quad S_f = e^{-\left(\left(^{E_F}/_{\Delta_F} \right)^8 + \left(^{E_D}/_{\Delta_D} \right)^8 + \left(^{E_S}/_{\Delta_S} \right)^8 \right)^{1/8}}$$

$$(13) \quad \tau_\alpha \frac{d\alpha}{dt} = \alpha_\infty (S_f) - \alpha$$

$$(14) \quad \alpha_\infty (S_f) = \frac{1}{1 + e^{\frac{-(S_f + \rho)}{\Delta_\alpha}}}$$

G_F, G_S and G_D are set to 53, 3, and 1, respectively. Also, $\Delta_F = .1$, $\Delta_S = .008$, and $\Delta_D =$
$.015$. During simulations involving increased extracellular potassium concentration Δ_D
is increased to $.035$

Table 1 – M_X Parameters and Time Constants

$\bar{M}_X$ is a sigmoid function with centering parameter Z:

$$\bar{M}_X = \frac{1}{1 + \exp \left(Z + \left(I_{CaT} + I_{CaS} \right) \right)}$$

	Z	τ_{M_X}
F	14.8	0.5
S	7.2	50

D	3	500

871 Table 2 – H_X Parameters and Time Constants

872 $\bar{H}_X$ is a sigmoid function with centering parameter Z:

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$$\bar{H}_X = \frac{1}{1 + \exp\left(-Z - (I_{CaT} + I_{CaS})\right)}$$

	Z	τ_{H_X}
F	9.8	1.5
S	2.8	60
D	-	-

Box 1, Supplement 3

$$(15) \quad \frac{\tau_g}{\alpha}\frac{d\bar{g}_i}{dt} = [A_i(\bar{F}-F) + B_i(\bar{S}-S) + C_i(\bar{D}-D)]\bar{g}_i - \gamma\bar{g}_i^3$$

$$(16) \quad \frac{\tau_{half}}{\alpha}\frac{dV_{s_i^m}}{dt} = [\hat{A}_i(\bar{F}-F) + \hat{B}_i(\bar{S}-S) + \hat{C}_i(\bar{D}-D)] \quad -\hat{\gamma}\left(V_{s_i^m}\right)^3$$

$$(17) \quad \frac{\tau_{half}}{\alpha}\frac{dV_{s_i^h}}{dt} = [\hat{\hat{A}}_i(\bar{F}-F) + \hat{\hat{B}}_i(\bar{S}-S) + \hat{\hat{C}}_i(\bar{D}-D)] \quad -\hat{\gamma}\left(V_{s_i^h}\right)^3$$

Table 1: Parameters describing Homeostatic Modulation of Intrinsic Current Properties
(Li Matrix)

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Current	Index (i)	A_i	B_i	C_i	$\hat{A}_i$	$\hat{B}_i$	$\hat{C}_i$	$\hat{\hat{A}}_i$	$\hat{\hat{B}}_i$	$\hat{\hat{C}}_i$
I_{Na}	1	+1	0	0	-1	0	0	+1	0	0
I_{CaT}	2	0	+1	0	0	-1	0	0	+1	0
I_{CaS}	3	0	+1	0	0	-1	0	0	+1	0
I_H	4	+1	-1	0	-1	+1	0	+1	-1	0
I_{Kd}	5	0	-1	-1	0	+1	+1	0	-1	-1
I_{KCa}	6	0	-1	-1	0	+1	+1	0	-1	-1
I_A	7	0	+1	+1	0	-1	-1	0	+1	+1

In this paper, the values $\hat{A}_i$, $\hat{B}_i$, and $\hat{C}_i$ are the negative of A_i, B_i, and C_i, respectively. The values $\hat{\hat{A}}_i$, $\hat{\hat{B}}_i$, and $\hat{\hat{C}}_i$ are equal to A_i, B_i, and C_i, respectively. The manner at which we arrive at these values are outlined in Methods. As outlined in Discussion, these coefficients do not need to take these values.

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Figure 1 – Activity-Dependent Regulation Mechanism ($\tau_{half} = 6$ s and $\tau_g = 600$ s) was

used to assemble a burster that meets the Ca²⁺ targets from 'random' starting

parameters (i.e. SP). Both this set of starting parameters and its resultant model are

called SP1. A – Row 1: blow-ups of specific points a, b, and c in the voltage trace in Row

2. Row 2: The neuron's electrical activity as its channel density and (in)activations

curves were being adjusted. α is a measure of how close the model is to satisfying Ca²⁺

targets (see Methods). It took values between 0 and 1, with 0 meaning the targets were

satisfied. Row 3: Plots the adjustments made to the half-activations as the model

attempted to satisfy the calcium targets. They are color-coded to match the intrinsic

currents shown in the left plot of Row 5. Row 4: Same as Row 3, except for (in)activation

curves. Note that not all currents in this model have inactivation curves. Row 5: Same as

Row 3, except for maximal conductances (i.e. channel density). Row 6: The final levels

of maximal conductances are shown in the left plot. The final half-(in)activations are

plotted on the right. The final half-activation of an intrinsic current, $V_{m_{i,1/2}}$, was the sum

of the initially posited level of the that current's half-activation $V_{\hat{m}_{i,1/2}^0}$ and the shifts the

model made to the activation curve, $V_{s_i^m}$ (shown in Row 3). The final half-inactivation of

an intrinsic current, $V_{h_{i,1/2}}$, was the sum of the initially posited level of the that current's

half-inactivation $V_{\hat{h}_{i,1/2}^0}$ and the shifts the model made to the inactivation curve, $V_{s_i^h}$

(shown in Row 4). [The process is illustrated in Figure 1 (supplement 1) for a different

set of starting parameters (i.e. initial half-(in)activation shifts and maximal conductance

levels).] B – This displays the neuron's electrical activity if the model was restricted to

varying only half-(in)activations ($\tau_{half} = 6$ s), with maximal conductances fixed ($\tau_g = \infty$

s). The same starting parameters as in Panel A were used for this simulation.

Figure 1 (Supplement) – This model attempts to satisfy the same Ca²⁺ targets as in

Figure 1, Panel A, but was started from a different set of 'random' initial parameters. The

rows correspond to those in Figure 1, Panel A. Note that the maximal conductances of

the two potassium currents, I_{Kd} and I_A, differ from SP1.

Figure 2 – All 20 SP's were analyzed including those illustrated in Figure 1. A) The top

left panel displays the periods for all SP's. Burst Duration, Interburst Interval, Spike

Hight, Maximum Hyperpolarization, and Slow Wave Amplitude measurements for these

SP's are provided in subsequent plots. B) B1 shows maximal conductances, and B2

displays half-(in)activation voltages for each of the 20 SP's. An asterisk on the right

indicates the initially specified level of half-(in)activation from which random deviations

were made.

Figure 3 - The speed at which the model changed half-(in)activations in response to

deviations from Ca²⁺ target impacted the type of bursters the model assembled. In all

simulation for this figure, $\tau_g = 600$. A) These plots extend the measurements from

Figure 2 to the same 20 SP's, now with half-(in)activations regulated at a different

speed. In blue, $\tau_{half} = 6$ s; recapitulating results in Figure 2. In green and orange, the

SP's were reassessed with slower half-(in)activation adjustments, $\tau_{half} = 600$ s, and

$\tau_{half} = \infty$ s (i.e. halted), respectively. Brackets indicate significant differences in

medians (p < .05). These pairwise differences were assessed using a Kruskal-Wallis test

(to assess whether any differences in medians existed between all groups) and post-hoc

Dunn tests with Bonferroni corrections to assess which pairs of groups differed

significantly in their medians. B) The model was used to stabilize SP1 to the same Ca²⁺

target but using different timescales of half-(in)activation alterations. $\tau_{half} = 6$ s is

shown on the left and $\tau_{half} = \infty$ s (representing the slowest possible response) on the

right. Rows 1-4 below each voltage trace show: the total outward current, percentage

contribution of each outward intrinsic current to the total outward current, percentage

contribution of each inward intrinsic current to the total inward current, and the total

inward current. A dashed vertical line across all rows marks the point of maximum

hyperpolarization in each activity pattern to guide focused comparison. C) When the

model stabilized around a burster, the speed at which the model changed half-

(in)activations impacted maximal conductance and half-activation location of the

calcium-activated potassium current. Shown here in blue, green, and orange are $\tau_{half} =$

6 s, $\tau_{half} = 600$ s, and $\tau_{half} = \infty$ s (i.e. halted), respectively. SP1 – SP20 are in the

colored circles. Trends are illustrated with a black line, with large circles marking

median positions. The mauve dot highlights the levels to which maximal conductance

and half-(in)activation of SP1 evolved. The asterisks mark the original position of the

KCa half-activation ($V_{\hat{m}^0_{6,1/2}}$). A Kruskal-Wallis test followed by post-hoc Dunn tests

(with Bonferroni corrections) were conducted to assess significant ($p < .05$) pairwise

differences in medians (brackets).

Figure 4 – Changes to the timescale of half-(in)activation alterations steers the model through different trajectories in intrinsic parameter space. Each row shows the behavior of Starting Parameter 1 (SP1) as the model adjusts toward a configuration that satisfies the calcium activity target, using different timescales for half-(in)activation regulation. A – $\tau_{half} = 6$ s, B – $\tau_{half} = 600$ s, C – $\tau_{half} = \infty$ s (i.e. half-(in)activations do not change). In all panels, the speed at which the model changed channel density is fixed in all rows: $\tau_g = 600$ s. Superimposed on these traces is the α parameter, which indicates whether the activity patterns were meeting the Ca²⁺ targets. Blow-ups of the activity pattern when the calcium sensors were satisfied are displayed to the right.

Figure 5 – The speed at which the model changed half-(in)activations in response to
deviations from Ca²⁺ targets impacted the type of bursters the model assembled
following perturbation. A) Voltage traces from SP1 before (blue) and 265 minutes after
(red) a simulated increase in extracellular potassium concentration. Each panel shows

recovery under a different timescale for half-(in)activation regulation: fast ($\tau_{half} = 6$ s),

intermediate ($\tau_{half} = 600$ s), and infinitely slow (i.e. off, $\tau_{half} = \infty$ s). In all condition,

alteration of maximal conductances was held fixed ($\tau_g = 600$ s). B) Summary of activity

features for all SP1–SP20 models that recovered to a stable burst pattern. The number

of models that successfully recovered varied with the homeostatic mechanism: fast (n =

17), intermediate (n = 18), and no half-(in)activation modulation (n = 20). Gray points

indicate pre-perturbation measurements of SP1-20 (from Figure 3A). Significant

pairwise differences in median values were assessed using a Kruskal–Wallis test

followed by post hoc Dunn tests with Bonferroni correction. Significant comparisons (p

< 0.05) are marked with horizontal brackets. C) The speed of half-(in)activation

regulation influenced both the maximal conductance and half-activation voltage of KCa.

Data are grouped by as they are in B). Statistical comparisons were performed as in B).

Figure 6 – The impact of adjusting the timescale of ion channel (in)activation curve

alterations can be illustrated conceptually using the space of activity characteristics

(right) and underlying intrinsic parameters (left). The regions in green and blue are the

activity patterns that are consistent with the Ca^{2+} targets in the space of activity

characteristics and underlying intrinsic parameters, respectively. As the rate of ion

channel half-(in)activation adjustments in response to deviations from Ca^{2+} targets is

changed, the regions targeted by the model also shift (shown in orange). Regardless of

998   the timescale, all targeted regions reside within a larger region encompassing all activity

999   profile measurements and intrinsic parameters consistent with the Ca^{2+} targets.

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1002 Figure 7 – We constructed the Group of 20 Bursters by selecting a putative

1003 representative of the entire population and then identifying bursters with similar

activity patterns. A) - 111 bursters were narrowed to 80 bursters by excluding certain

characteristics (see Methods). These models had a period distribution shown in A. Highlighted

in red are the models chosen for further selection. B) This plot illustrates how closely the

periods of the models chosen in A align with their two neighbors after they were ordered

from smallest to largest. The first model, SP1, was chosen as the neuron with neighbors

that also have close periods (red arrow). C) - To assemble the Group of 20, we

symmetrically expanded the inter-instance period range around SP1's period by 20%.

SP2-SP5's wave forms are shown. Depolarizing excursions (spikes) were detected using

MATLAB's findpeaks function with a prominence threshold of 2 (red arrows). D) –

Troughs were identified using the findpeaks function on the negative waveform of a

cycle period, applying the same prominence threshold. The detected troughs are

indicated by red arrows. The example displayed here represents the inverted voltage

trace of the burster assembled by the model from SP1. The period was measured as the

time difference between successive points of maximum hyperpolarization.

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