Neural representation of action symbols in primate frontal cortex

Lucas Y. Tian[1,7]*, Kedar U. Garzón[1], Daniel J. Hanuska[1], Adam G. Rouse[2], Mark A. G. Eldridge[3], Marc H. Schieber[4], Xiao-Jing Wang[5], Joshua B. Tenenbaum[6,7], Winrich A. Freiwald[1,7]*

Abstract

At the core of intelligence lies proficiency in solving new problems, including those that differ dramatically from problems seen before. Problem-solving, in turn, depends on goal-directed generation of novel thoughts and behaviors[1], which has been proposed to rely on internal representations of discrete units, or symbols, and processes that can recombine them into a large set of possible composite representations[1–11]. Although this view has been influential in formulating cognitive-level explanations of behavior, definitive evidence for a neuronal substrate of symbols has remained elusive. Here, we identify a neural population encoding action symbols—internal, recombinable representations of discrete units of motor behavior—localized to a specific area of frontal cortex. In macaque monkeys performing a drawing-like task designed to assess recombination of learned action symbols into novel sequences, we found behavioral evidence for three critical features that indicate actions have an underlying symbolic representation: (i) invariance over low-level motor parameters; (ii) categorical structure, reflecting discrete types of action; and (iii) recombination into novel sequences. In simultaneous neural recordings across motor, premotor, and prefrontal cortex, we found that planning-related population activity in ventral

[1] Laboratory of Neural Systems, The Rockefeller University, New York, NY, USA

[2] Department of Neurosurgery, Department of Cell Biology & Physiology, University of Kansas Medical Center, Kansas City, KS, USA

[3] Biosciences Institute, Newcastle University, Newcastle upon Tyne, UK

[4] Department of Neurology, University of Rochester, Rochester, NY, USA

[5] Center for Neural Science, New York University, New York, NY, USA

[6] Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA

[7] Center for Brains, Minds and Machines, MIT & Rockefeller University

* Co-corresponding authors: ltian@rockefeller.edu (L.Y.T.) and wfreiwald@rockefeller.edu (W.A.F.)

premotor cortex (PMv) encodes actions in a manner that, like behavior, reflects motor invariance, categorical structure, and recombination, three properties indicating a symbolic representation. Activity in no other recorded area exhibited this combination of properties. These findings reveal a neural representation of action symbols localized to PMv, and therefore identify a putative neural substrate for symbolic cognitive operations.

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Competing interest

The authors declare no competing interest.

Introduction

Understanding the mechanisms of intelligence requires explaining the most powerful forms of generalization, in particular, generalization to new situations or problems differing considerably from those previously encountered. For example, if children are asked to “draw an animal that does not exist”, they can generalize from prior experience to produce an imaginary animal, such as a dog-like creature with six legs, three camel humps, and three pig tails[12]. An influential hypothesis is that this seemingly unbounded generalization depends on an internal representation of discrete units (symbols) that can be recombined into composite representations, in a process called compositional generalization[1–11]. Symbols support generalization because they enable the systematic derivation of a large set of possible representations from a smaller set of components, such as new animals imagined as rule-based combinations of discrete parts (e.g., *new animal = 1 torso + 8 arms + 4 legs*). Importantly, this hypothesis is not restricted to concepts that people explicitly represent as symbol systems in language[13], computer programs[14], or mathematics[9,10], but applies broadly even to abilities, such as drawing, which are not superficially symbolic, but which evidence suggests may reflect internal symbolic representations[5,7,11]. These abilities include, in humans, geometry[7,15], perception[11], handwriting[16], drawing[17,18], dance[19], music[7], and speech[20,21], and, also in non-human animals, logical reasoning[22,23], social cognition[24], navigation[25,26], artificial grammars[27–30], communication[26], numerical cognition[31,32], and physical reasoning[11].

Despite behavioral evidence suggesting internal, symbolic representations, we lack definitive evidence for whether and how symbols are implemented in neuronal substrates. This is especially problematic in light of the uncertainty over how symbols are compatible with hypothesized mechanisms of cognition that do not presuppose symbols, including those based on distributed processing in neural networks[13,33–35], dynamical systems[36–39], and map-like representations[40,41]. Given that symbols are discrete units that are systematically recombined, a neural population representing symbols should exhibit three essential properties: (i) invariance, (ii) categorical structure, and (iii) recombination. Invariance means that activity exhibits a form of abstraction in which it is independent of variables that are irrelevant to the task goal.

Categorical structure means the neural population should express a set of distinct activity patterns, one for each symbol, and be biased towards these discrete activity states even in the setting of continuous variation in task parameters. Recombination implies that a symbol's activity pattern should occur in all contexts in which it is used in composition with other symbols.

Neural recordings during cognitive tasks have revealed a diversity of invariant representations, including of rules[42,43], actions[44,45], sequences[28,46,47], numerical concepts[48–50], perceptual categories[51,52], visual objects[53], cognitive maps[41,54,55], and other abstract concepts[56,57]. Collectively, these findings reveal a striking capacity for invariance in neural activity, and implicate critical roles for specific regions in this capacity, including prefrontal cortex[28,42,43,47,48,51,52] and medial temporal lobe[41,43,49,54,56,57]. However, it is unclear whether these representations exhibit the other properties expected for symbols. First, with a few exceptions[51,52], these prior studies did not systematically test for categorical structure by assessing whether activity varies discretely with continuous variation in task parameters. Second, evidence for recombination in neural activity is also rare, with a notable exception in the hippocampus: activity encoding spatial paths appears to reuse parts of previous paths in novel sequences[58,59]. However, whether these continuous paths reflect recombination of discrete components is unclear. Third, an important point that applies to the navigation tasks in these hippocampal studies, and generally to tasks in prior studies on invariant representations, is that they did not, to our knowledge, implement behavioral tests of compositional generalization, making it challenging to determine whether and how any identified neural representation contributes to the cognitive processes that support the ability to solve novel problems. For these reasons, we still lack evidence for a neural representation of symbols—one that jointly exhibits invariance, categorical structure, and recombination in the behavioral setting of compositional generalization.

To identify neural populations that may express these properties of symbols, we established a task paradigm involving symbol-based compositional generalization, implemented in macaque monkeys, a species with extensive abilities for abstract cognition[28,42,43,47,48,52,54,60] (**Fig. 1a, b**). This paradigm was

designed to capitalize on the brain's remarkable ability to generate novel, goal-directed actions, an ability hypothesized to involve symbolic representations in the form of discrete units of motor behavior (action symbols) that are recombined into sequences[16,17,19,21,61–70]. For example, imitating a new dance by observation may depend on internal representations of symbols corresponding to dance poses[19,71]. Action symbols also form the basis of models that capture behavioral findings in action sequencing, including in handwriting[16,61], drawing[17,62], object manipulation[63], and tool use[64], and may be related to the movement motifs (or "syllables") that have been identified in various animal behaviors[72,73]. These studies suggest that a task involving compositional generalization in action sequencing would enable the systematic investigation of action symbols. Here, we establish such a task paradigm. We then show that behavior in this task exhibits invariance, categorical structure, and recombination, indicating an internal symbolic representation of action. In multi-area neuronal recordings, we found a neural population, localized to PMv, which encodes actions in a manner reflecting these three properties of symbols. This finding therefore identifies a neural substrate of action symbols in PMv.

Results

Learned stroke primitives in a drawing-like task paradigm.

We developed a drawing-like task for macaque monkeys modeled after studies of symbolic structure in human drawing[17] and handwriting[16,61]. We trained two subjects to draw geometric figures by tracing them on a touchscreen (**Fig. 1a,c,d**; videos of behavior in **Supplementary Videos 1-10**; experimental setup in **Extended Data Fig. 1**). On each trial, subjects were presented with an image of a figure, and were rewarded for making an accurate copy of the image, quantified as the spatial similarity between the drawing and image (primarily using the Hausdorff distance; see Methods).

Once subjects understood the task's core requirements (i.e., to make accurate traces of images), they practiced drawing a diverse set of simple shapes, each using one stroke (**Fig. 1e**). Although we did not impose rules for what spatio-temporal trajectory (i.e., stroke) should be used for a given shape, each subject converged on a set of consistently reused strokes, one for each shape. We call these shape-specific strokes the subject's "primitives" (**Fig. 1e**). Analysis of stroke trajectories revealed that primitives were idiosyncratic to each subject and shape. We devised a "trajectory distance" metric, which measures the mean Euclidean distance between the velocity time series of two strokes (after normalizing scale and duration; see Methods). Within each subject, each shape was drawn consistently with the same primitive across trials (example drawings in **Fig. 1f**; quantification in **Fig. 1g**). At the same time, different shapes were drawn with distinct primitives (**Figs. 1e-g**). In a further assessment of across-trial consistency and across-shape distinctiveness using a decoding approach, we found that each shape's primitive was easily distinguishable from every other primitive based on single-trial stroke trajectories (**Extended Data Fig. 2**). Primitives were also unique to each subject. This is evident in how the two subjects used different primitives even for identical shapes (compare primitives in **Figs. 1e, f**, and quantification in **Fig. 1g**).

The finding that stroke primitives are idiosyncratic to shape and subject raised the possibility that these primitives reflect learned action symbols. To directly test this, we determined how subjects generalize to draw new figures in paradigms designed to assess three behavioral properties that, in combination,

indicate an underlying symbolic representation: motor invariance, categorical structure, and recombination (**Fig. 1b**).

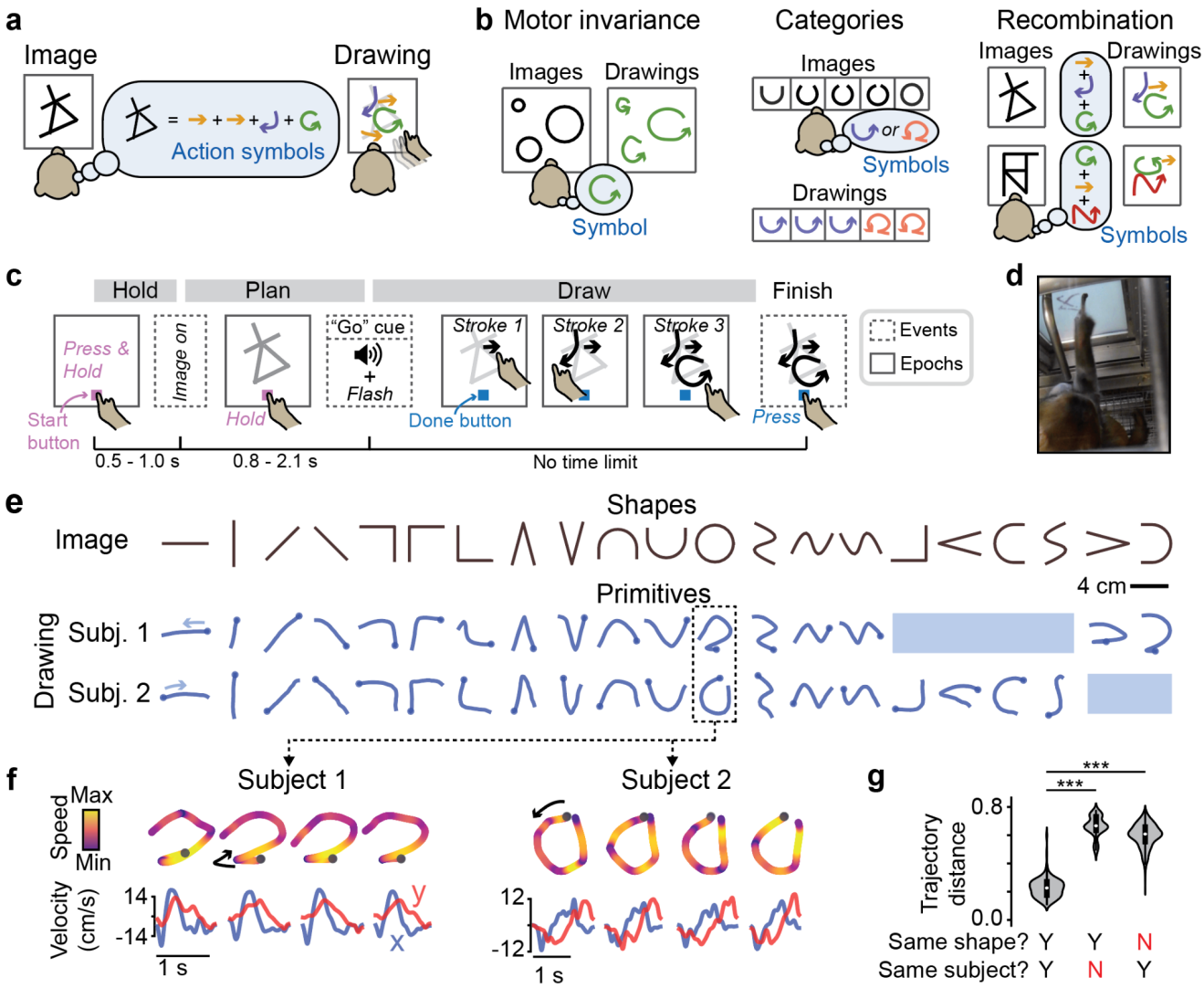

Fig. 1. Learned stroke primitives in a drawing-like task paradigm.

(a) Experimental paradigm. Given an image of a (potentially novel) figure, subjects draw it by tracing on a touchscreen, with no explicit cues for what strokes to make and in what order. We hypothesize that subjects internally represent actions as a composite of learned symbols.

(b) Three essential features of a hypothesized symbolic representation, which we test in behavior and neural activity: invariance, categorical structure, and recombination. Schematics depict predictions by the action symbol hypothesis. Predictions by alternative hypotheses are presented in **Figs. 2a, f, k**.

(c) Trial structure, showing a sequence of three discrete events (dashed boxes, centered on time points at vertical marks on the timeline) and sustained epochs (solid boxes). During the "hold" epoch, subjects press and hold a "start" button, forcing them to maintain a consistent, stable posture across trials. During the "plan" epoch, subjects see the image and must maintain their hold on the "start" button. During the "draw" epoch, subjects produce strokes. They can report completion at any time by pushing the "done" button (which appears after the "go" cue). They then receive a juice reward based on drawing performance (see Methods). The range of

epoch durations reflects the range of mean durations across task variants and sessions (further trial-by-trial jitter was also added within each session; see Methods). Note that "buttons" always refers to virtual touchscreen buttons.

(d) Photograph of a subject performing the task, showing a top-down view during in-cage training. The metal tube is the reward spout. See videos of behavior in **Supplementary Videos 1-10**.

(e) Learned stroke primitives for each subject (blue drawings), one for each practiced shape (black images). Stroke onsets are marked with a ball. Blue shading marks images for which subjects did not readily learn stereotyped stroke primitives, either because they used two strokes or they executed a single stroke variably across trials (see Methods). Drawings are averages over 15 trials.

(f) Example single-trial stroke trajectories for the "circle" shape, depicted in two ways, instantaneous speed as a heatmap on the stroke trajectory (top) or as a graph of velocity vs. time (bottom). Shown are four trials per subject.

(g) Summary of pairwise trajectory distances across trials, where pairs of trials are grouped along two dimensions: (i) whether they refer to the same shape (yes/no) and (ii) to the same subject (yes/no). Violin plots represent kernel density estimates, with overlaid medians and quartiles. Starting from a pool of 300 trials (15 shapes x 10 trials x 2 subjects), each data point represents a single trial's average distance to a specific shape, such that n = 300 for both "same shape, same subject" and "same shape, different subject" (15 shapes x 10 trials x 2 subjects), n = 4200 for "same subject, different shape" (15 shapes x 10 trials x 14 other shapes x 2 subjects). ***, $p = 6.10 \times 10^{-5}$, two-sided Wilcoxon signed-rank test (W = 0, n = 15) performed on data first averaged over trials (n = 15 shapes, each averaged across subjects and trial-pairs).

Stroke primitives exhibit motor invariance over location and size.

If subjects represent stroke primitives as symbols, then stroke primitives should exhibit motor invariance. That is, each primitive's idiosyncratic trajectory should generalize across low-level motor parameters (e.g., muscle activity patterns), as seen in handwriting and other skills[74]. To test motor invariance, we presented each shape at a size and location that varied across trials, including novel location-size conditions (using location and size parameters expected to elicit variation in motor cortical and muscle electromyographic activity based on prior studies of reaching movements[75,76]; see Methods). Motor invariance predicts similar stroke trajectories across location and size ("Symbols", **Fig. 2a**). Alternatively, if responses were memorized by rote, one for each specific stimulus, the subject would have difficulty generalizing, as seen in inflexible automatized skills[77] ("Fail" in **Fig. 2a**). A third alternative strategy could prioritize efficiency by minimizing movement from the starting position of the hand. This would predict different trajectories dependent on location and size ("Efficient" in **Fig. 2a**). We found that stroke trajectories were similar across locations and sizes (**Figs. 2b-e**), indicating that stroke primitives exhibit motor invariance.

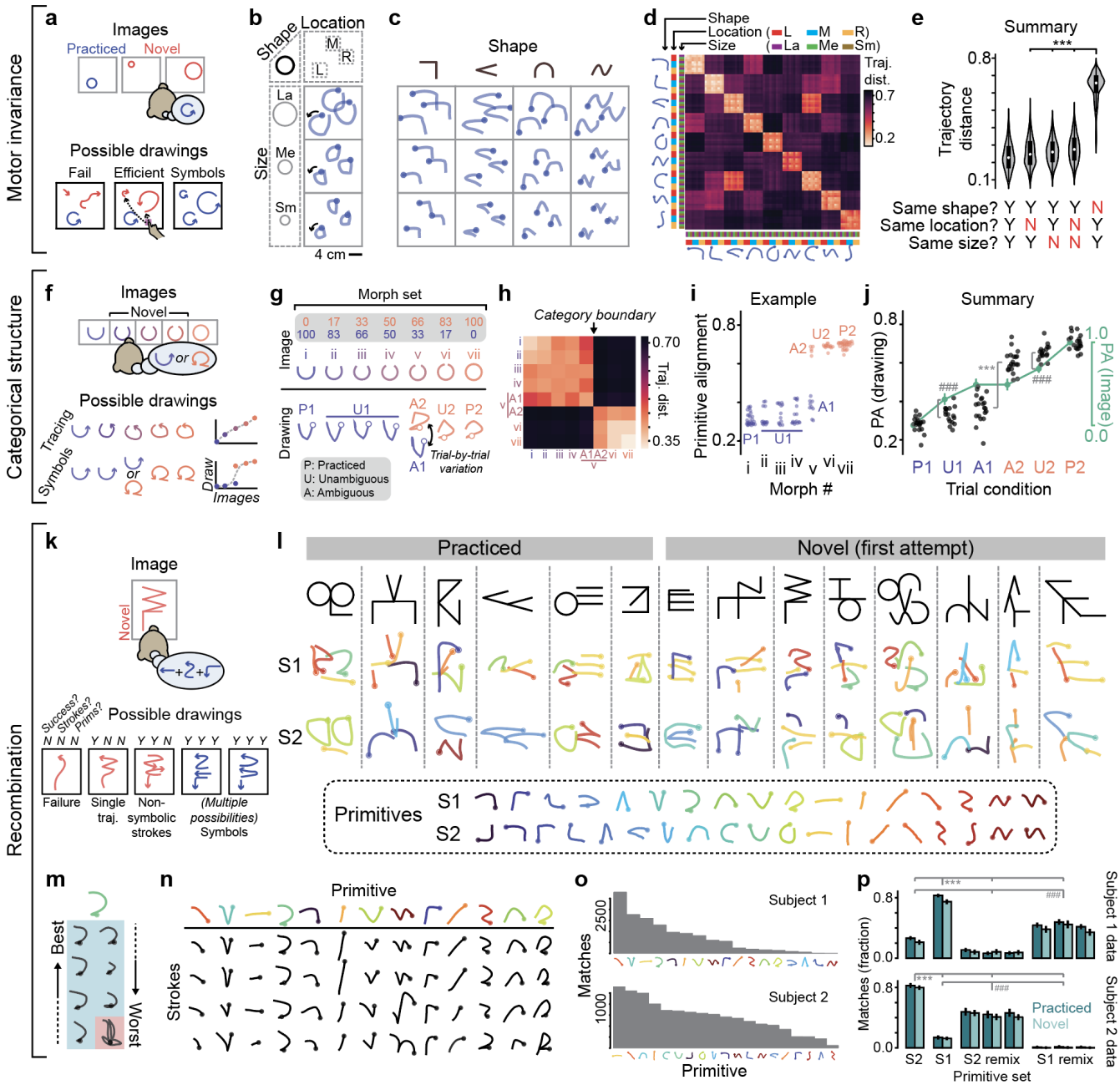

Fig. 2. Behavioral evidence for action symbols: motor invariance, categorical structure, and recombination into sequences.

(a) Schematic of experiment designed to test motor invariance over location and size. Each trial (gray rectangle) presents one shape, which varies in size and location across trials. Shown are three possibilities for how subjects will draw; only a symbolic representation predicts reuse of stroke primitives.

(b) Example drawings for a "circle" shape, across trials varying in location (left, middle, right) and size [small (Sm), medium (Me), large (La)]). Drawings from three trials are overlaid on each panel (subject 2).

(c) Example drawings for more shapes (columns), varying in location and size (convention as in panel b).

(d) Pairwise trajectory distances between each combination of shape, location, and size, plotting the mean distance across trial-pairs (n = 9 trials per shape-location-size condition).

(e) Summary of pairwise trajectory distances, where pairs of trials are grouped along three dimensions: same shape (yes/no), same size (yes/no), and same location (yes/no). Violin plots represent kernel density estimates,

with overlaid medians and quartiles. Each underlying datapoint is one trial's average distance to one shape/size/location condition. n = 648 (same shape/size/location = YYY), 1296 (YYN), 1296 (YNY), 2592 (YNN), 5184 (NYY), taken from a pool of 729 trials (81 shape-location-size conditions x 9 trials each). ***, two-sided Wilcoxon signed-rank tests comparing NYY to other manipulations: vs. YNY (W = 0, p = 5.36 x 10^{-15}), vs. YYN (W = 0, p = 5.36 x 10^{-15}), vs. YNN (W = 0, p = 5.36 x 10^{-15}); for each comparison, n = 81 (unique shape/size/location, after averaging over all its pairs).

(f) Schematic of experiment designed to test for categorical structure. Given novel images generated by morphing between two well-practiced shapes, we consider two hypotheses, "Tracing" and "Symbols". A tracing strategy predicts that drawings should closely match each interpolated figure, and thus their properties will vary linearly across the morphed images. A symbolic representation predicts drawings with two hallmarks of categorical structure: sigmoidal non-linearity and trial-by-trial discrete variation.

(g) Example experiment for one morph set. Across trials, images varied linearly (randomly sampled) between two well-practiced images (top, from left to right): morph *i* (mixture ratio of 100% "U", 0% circle) and morph *vii* (0% "U", 100% circle). Below, example drawings are shown for each morph. Two drawings are shown for morph *v*, because it elicited trial-by-trial variation between two primitives (A1, A2). We define "trial conditions" that group morphs based on whether they are well-practiced images (P), are unambiguously drawn as one primitive (U), or are ambiguously drawn as one primitive or the other across trials (A).

(h) Matrix of pairwise trajectory distances between morphs for the example morph set in panel g. The category boundary is defined as the morph that was found to elicit trial-by-trial variation in drawing (i.e., "ambiguous" morph). Thus, trials for morph *v* are split into two trial conditions based on the drawing ("A1", "A2"). n = 5 - 30 trials (range across morphs, counting A1 and A2 separately).

(i) Single-trial primitive alignment vs. morph number for the morph set in panel g. n = 5 - 30 trials (range across images, counting A1 and A2 separately). For primitive alignment using image data, instead of drawing data, see **Extended Data Fig. 3c**).

(j) Summary of primitive alignment (PA) vs. trial condition across all experiments, showing scores based on images (green, mean and 95% CI) and drawing trajectories (black). Each drawing datapoint represents a single morph set [n = 20 morph set, combining subjects 1 (13) and 2 (7)]. To facilitate comparison to the drawing-based scores, the image-based scores were rescaled so that the endpoint values (P1 and P2) match the drawing-based scores. To test for sigmoidal nonlinearity, drawing-based scores were compared to image-based scores: ###, p = 1.91 x 10^{-6}, two-sided Wilcoxon signed-rank test (W = 0, n = 20) showing that, compared to the image, primitive alignment for drawings is closer to 0, for U1, and to 1, for U2 (data were aggregated across U1 and U2); ***, p = 1.91 x 10^{-6}, two-sided Wilcoxon signed-rank test (W = 0, n = 20) of trial-by-trial switching (drawing distribution in A2 higher than in A1). Note that primitive alignments using drawing data for P1 and P2 are not exactly 0 and 1 due to trial-by-trial variation in behavior.

(k) Schematic of experiment designed to test for recombination of primitives into sequences, in the Character task. Four possible kinds of drawing responses are shown, each differing along three properties: success in copying the figure, use of multiple strokes, and reuse of a subject's own primitives. Combined evidence for all three properties is consistent with a symbolic representation. In general, images have ambiguity in that there are multiple ways to draw while still exhibiting primitive reuse ("symbols").

(l) Example drawings in the Character task by subjects 1 and 2 given the same set of images (top row). Strokes are color-coded by their best-matching primitive from the subject's own primitive set (see Methods). "Primitives" plots each subject's "ground-truth" primitives taken from the Single-shape task.

(m) Example character strokes that were assigned to one particular primitive ("reversed C") in blue, uniformly ranging from best to worst quality match. The stroke with red background failed to match any primitives. Data from subject 1.

(n) Example character strokes organized by the primitive they were assigned to (columns), showing four example matches. Strokes are ordered from high (top row) to median (bottom row) quality (based on trajectory distance to primitive), most to least frequent (columns), showing only the thirteen most frequent (see frequency distribution in panel o). Data from subject 1.

(o) Frequency histogram of strokes matched to each primitive. Note that the non-uniformity of this distribution is reminiscent of the non-uniform frequency distributions seen in language and other behaviors[78].

(p) Summary of behavioral evidence for primitive recombination, showing fraction of strokes that match a primitive. This analysis includes only characters (i.e., unique images) performed by both subjects. Results show each subject's strokes (data) tested against each subject's primitives, as well as against simulated primitive sets constructed by mixing subparts of primitives into new "remixed" primitives (plotted in **Extended Data Fig. 5**). Three simulations with independently remixed primitives were performed and plotted separately. Bars show mean and 95% CI. N (characters) = 133 (subject 1, practiced), 90 (S1, novel), 70 (S2, P), 135 (S2, N); note that a character could contribute a data point to both practiced and novel groups, thus explaining the different sample sizes between subjects. Wilcoxon signed-rank tests: ***, comparing match rate using one's own primitives vs. other primitives. For S1 data: S1 vs. S2 ($W = 0$, $p = 6.75 \times 10^{-25}$), vs S1 remix ($W = 125$, $p = 3.43 \times 10^{-22}$), vs S2 remix ($W = 0$, $p = 8.75 \times 10^{-25}$); for S2 data: S2 vs S1 ($W = 49$, $p = 6.26 \times 10^{-23}$), vs S2 remix ($W = 424.5$, $p = 3.61 \times 10^{-16}$), vs S1 remix ($W = 0$, $p = 5.82 \times 10^{-25}$). ###, comparing one's own remixed primitives vs. others. For S1 data: S1 remix vs S2 ($W = 995$, $p = 1.41 \times 10^{-11}$), vs S2 remix ($W = 351$, $p = 1.90 \times 10^{-20}$). For S2 data: S2 remix vs S1 ($W = 731.5$, $p = 1.09 \times 10^{-11}$), vs S1 remix ($W = 1$, $p = 2.85 \times 10^{-20}$). For tests involving remixed primitives, the least significant of the three simulations is shown. Sample sizes are identical for all tests (n = 141 characters, combining novel and practiced).

Stroke primitives exhibit categorical structure.

If subjects represent stroke primitives as categorically structured action symbols, we would expect them to draw preferentially using their idiosyncratic primitives when challenged with new figures that interpolate, or "morph", between the shapes the primitives were associated with during learning (**Fig. 2f,** "Symbols"). This would indicate that stroke primitives are represented as a set of discrete types of action, similar to stroke categories in Chinese characters[61] or phonemes in speech[20]. In contrast, if subjects simply trace images without interpreting them as action symbols, then we would expect drawings to closely match the interpolated figures (**Fig. 2f**, "Tracing"). We presented subjects with images randomly sampled on each trial by linearly morphing between two practiced shapes. Each "morph set" consisted of two practiced shapes and the four to five morphs between them. We tested whether the resulting drawings reflected a categorical boundary in the subject's interpretation of these images, which would manifest in two behavioral hallmarks (**Fig. 2f,** "Symbols"): a steep sigmoidal relationship between image variation and drawing variation[79,80]—such that images on the same side of the category boundary are drawn similarly, while images across are drawn differently—and trial-by-trial variation between two distinct primitives for images close to the boundary.

We found evidence for both hallmarks of categorical structure. In the example shown in **Fig. 2g** (more examples in **Extended Data Fig. 3**), the images varied in the extent to which the top was closed, ranging linearly between two practiced shapes, a "U" (morph *i* in **Fig. 2g**) and a circle (morph *vii*). We defined a category boundary as the morph that was found to elicit trial-by-trial variation between primitives 1 and 2 (morph *v*). We found that drawings for morphs to the left of that boundary (U1) were drawn similarly to primitive 1 (P1), and drawings for morphs to the right (U2) were similar to primitive 2 (P2, **Fig. 2g**). This discrete distinction across the category boundary was evident as two blocks in a matrix of the pairwise trajectory distances between all morphs (**Fig. 2h**), which implies high similarity within each block, and low similarity across blocks. To quantify this effect, we calculated a "primitive alignment" score, which quantifies the relative similarity of a given trial's drawing to primitive 1 (alignment = 0) and primitive 2 (alignment = 1), and is defined as $d_1/(d_1 + d_2)$, where d_1 (or d_2) is the average trajectory distance to primitive 1 (or 2) trials. First, we confirmed that applying this metric to score the images (using image distance instead of trajectory distance; see Methods) produced a linear relationship with morph number (**Extended Data Fig. 3c**). In contrast, when applied to score behavior, primitive alignment varied nonlinearly in a sigmoidal manner across morph number (**Fig. 2i**). This analysis also captured the trial-by-trial variation in behavior for morphs at the category boundary (morph *v*, **Fig. 2i**). These effects—nonlinearity and trial-by-trial discrete variation—were consistent across morph sets (**Fig. 2j**). Thus, subjects internally represent primitives as categorical stroke types.

Stroke primitives are recombined into sequences.

If subjects represent stroke primitives as symbols, they should readily recombine primitives to construct multi-stroke drawing sequences. We tested this using two tasks that challenge subjects with complex figures, including novel ones. The first Multi-shape task presented figures that combined multiple (two to four) disconnected shapes, which could be drawn in any order. We reasoned that one possible strategy was to use a single trajectory that efficiently traces over all shapes with appropriately timed touches and raises to produce strokes and gaps ("Single trajectory", **Extended Data Fig. 4a**). With this strategy, behavior would be biased to minimize the movement, or gap distance, between shapes, ignoring whether

this leads to reuse of primitives. A second strategy, consistent with a symbolic representation, would be to draw by recombining the learned stroke primitives, at the expense of potentially longer gap distances ("Symbols", **Extended Data Fig. 4a**). We analyzed all pairs of consecutive strokes for which these two strategies predicted different stroke trajectories (**Extended Data Fig. 4b,** top schematic). In all analyzed cases, subjects recombined primitives at the expense of longer movements in gaps. This is consistent with a symbolic representation (**Extended Data Fig. 4b**, bottom).

We then tested subjects on the Character task, which uses complex figures called characters, each designed by connecting together multiple simple shapes, such that it was often ambiguous regarding its components. We considered four possible drawing outcomes (**Fig. 2k**): subjects may fail to accurately draw these novel figures ("Failure"), they may succeed using a single unsegmented trajectory ("Single trajectory"), they may use multiple strokes that are not in their set of learned primitives ("Non-symbolic strokes"), or they may successfully draw by reusing their own primitives ("Symbols"). Note that, due to ambiguity, there are generally multiple possible behaviors that exhibit primitive reuse.

We found that drawings were consistent with a symbolic representation. Subjects successfully drew novel characters, and did so using multiple strokes (**Fig. 2l**, **Supplementary Videos 1-10**), thus contradicting "Failure" and "Single trajectory" predictions. Given the same image, the two subjects produced different drawings (**Fig. 2l**), raising the possibility that subjects reused their primitives. To directly test that, we quantified how often character strokes closely matched each subject's own primitives. We first scored each stroke's trajectory distance to each of the subject's own primitives (defined using strokes from "ground-truth" Single-shape trials). Using those scores, we classified each stroke as either a "match" to a specific primitive (i.e., the primitive with the lowest trajectory distance) or as a "non-match" (i.e., high trajectory distance to every primitive). Thresholds for deciding this match were determined empirically for each primitive based on its trial-by-trial variation on Single-shape trials (see Methods). Resulting matches are shown as example strokes in **Fig. 2m, n** and as a frequency distribution over primitives in **Fig. 2o**.

346 We found that a large majority of strokes were classified as matches (>82% for both subjects, **Fig. 2p**),
347 and this was true even for novel characters (>74%, **Fig. 2p**). We performed two control analyses to test
348 the specificity of match towards one's own primitives. First, we asked how often strokes matched the
349 other subject's primitives. We found that this was rare (<21% for novel characters, **Fig. 2p**). Second, we
350 asked how often strokes matched simulated primitives designed to resemble the actual primitives in their
351 subparts (i.e., their first and second halves) but not in their entire trajectories (see simulated primitives in
352 **Extended Data Fig. 5**). We found that strokes infrequently matched these "remixed" primitives (<43%
353 for novel characters, **Fig. 2p**). Combined, these control analyses show that character strokes specifically
354 resemble one's own primitives (first analysis) in their idiosyncratic stroke trajectories (second analysis).
355 Thus, subjects generalize to draw novel figures by recombining their primitives ("Symbols" in **Fig. 2k**).
356
357 **Multi-area neuronal recordings across frontal cortex.**
358 The findings that stroke primitives exhibit motor invariance (**Fig. 2a-e**), categorical structure (**Fig. 2f-j**)
359 and recombination (**Fig. 2k-p**) indicate they have an underlying symbolic representation. We next
360 searched for neural correlates of these putative representations. To search broadly, we recorded
361 simultaneously from neurons across multiple areas of frontal cortex, using chronically implanted multi-
362 electrode arrays (sixteen 32-channel arrays, **Fig. 3a, b**, and **Extended Data Fig. 6**). We targeted regions
363 that have been associated with motor, planning, and other cognitive functions (see Methods), including
364 (but not limited to) motor control in primary motor (M1), dorsal premotor (PMd), and ventral premotor
365 cortex (PMv)[81]; motor planning and decision-making in PMd and PMv[82]; motor abstraction in PMv[44];
366 abstract reasoning and planning in dorsolateral and ventrolateral prefrontal cortex[83,84] (dlPFC and vlPFC)
367 and frontopolar cortex[85] (FP); and action sequencing in the supplementary motor areas[46] (SMA and
368 preSMA). We recorded 48.4 +/- 19.9 units per area (mean and S.D. across sessions) for subject 1 and
369 48.0 +/- 16.0 for subject 2.
370
371 We found clear task-related activity in all recorded areas except FP. This is evident as non-zero baseline-
372 subtracted trial-aligned activity in **Fig. 3c**. A coarse inspection revealed that different areas exhibited

grossly different activity patterns in relation to the ongoing trial. For example, many units in prefrontal areas vlPFC and dlPFC and premotor areas PMd and PMv had rapid responses locked to the image onset (**Fig. 3c**, at around 0.1 s post-onset) and varied activity patterns during the planning epoch. In contrast, motor area M1 had relatively weak visual and planning-related activity, but strong movement-related activity during the stroke epoch (**Fig. 3c**). Paralleling the behavioral experiments, we tested whether activity in each of these areas encodes stroke primitives in a manner exhibiting motor invariance, categorical structure, and recombination. We found that each of these properties was strongest in a single area, ventral premotor cortex (PMv).

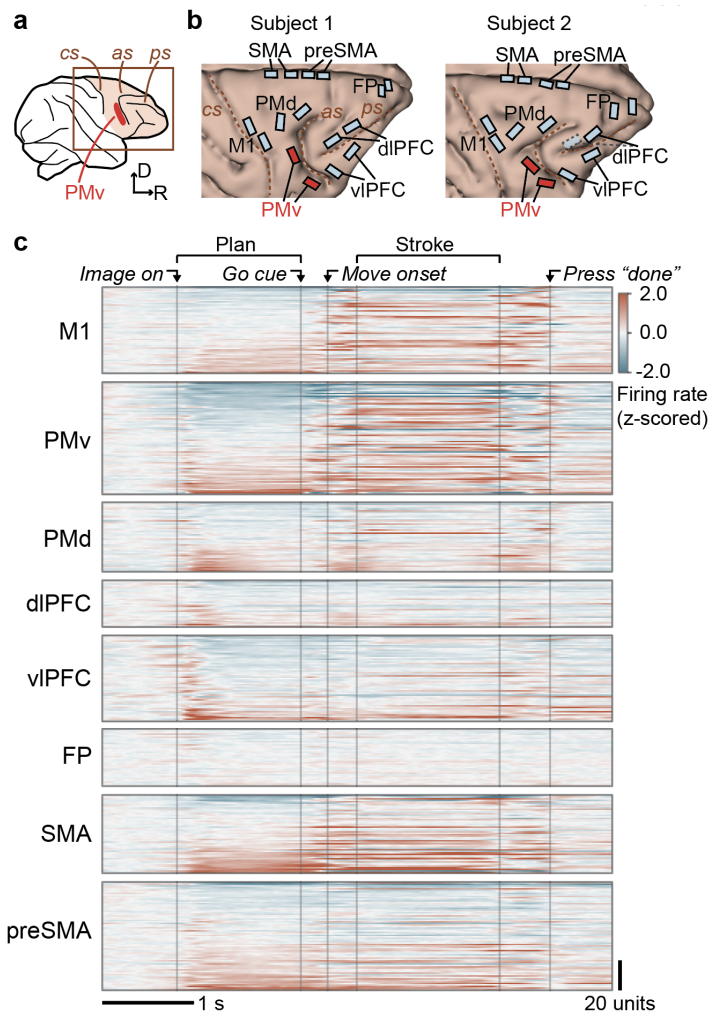

Fig. 3. Multi-area neuronal recordings across frontal cortex.

(a) Recordings were targeted to multiple areas of frontal cortex contralateral to the hand used for drawing (right hemisphere, for both subjects). Arrays were targeted stereotactically and registered to sulcal landmarks, including central sulcus (cs), arcuate sulcus (as), and principal sulcus (ps). R, rostral. D, dorsal.

(b) Final locations of arrays, depicted to scale on a 3D rendering of the cortical surface. Note that although SMA and preSMA arrays were positioned on the dorsal surface as shown here, their electrodes were angled to target the medial wall (**Extended Data Fig. 6**). The caudal dlPFC array for subject 2 was implanted but malfunctioned due to cable damage.

(c) Average activity across trials, grouped by brain area (panels) and split by unit (inner rows). Before averaging, trials were aligned by linear time-warping to a median trial, aligned at "anchor" events that consistently occurred across trials (see Methods). Both single- and multi-units are included. Each unit's activity was z-scored relative to the baseline time window preceding image onset, and sorted within each area from low to high activity in the planning epoch. Sorting was done in a cross-validated manner, by determining sort indices using one subset of data (n = 50 trials) and applying those indices to the remaining subset that is plotted (n = 235). This plot includes from a single session of Single-shape trials (subject 2).

Motor-invariant encoding of stroke primitives in PMv.

In the Single-shape task, where subjects drew primitives that varied in location across trials (**Fig. 2a-e**), we analyzed activity during the planning epoch (between image onset and the go cue). PMv activity varied strongly depending on the planned primitive, with relatively small influence of location. For example, the unit shown in **Fig. 4a, b** fired strongest for one specific primitive (black), second strongest for another (blue), and so on, and this pattern was similar across the four locations. To quantify how strongly population activity encodes primitives, we first identified a primitive-encoding population subspace using principal components analysis (i.e., a linear projection of population activity that maximizes activity variation related to primitives; see Methods). PMv activity in this subspace varied strongly depending on the primitive, and minimally depending on location (**Fig. 4c,d**). For example, in **Fig. 4d**, within each location (subpanel), primitive trajectories (each color) differed from each other after image onset ("x" icon), reflecting strong encoding of primitives; but across locations, trajectories were similar, reflecting minimal encoding of location. In contrast, in a different area known for its role in cognitive control, dlPFC, activity recorded in the same trials and analyzed the same way strongly reflected location, with minimal encoding of primitive (**Fig. 4e-h**). Encoding of primitives, invariant to location, was apparent in PMv population activity even on individual trials (see low-dimensional embedding of activity in **Extended Data Fig. 7**).

To compare the extent to which activity in each area encodes primitives vs. locations, we devised a "neural distance" metric, which quantifies the activity dissimilarity between any two sets of trials representing two conditions (e.g., two different location-primitive conditions). Neural distance is the average pairwise Euclidean distance between all across-condition trials, normalized by average within-condition pairwise Euclidean distance, such that neural distance ranges between 0 (no difference) and 1 (a ceiling, defined as the 98th percentile of pairwise Euclidean distances). We computed neural distance between each pair of primitive-location conditions. The resulting pairwise distances in PMv were consistently low for the same primitive across locations, visible as off-diagonal streaks in a matrix of pairwise distances (**Fig. 4i**). In contrast, the matrix for dlPFC had four blocks along the diagonal,

441 indicating similar activity within each location, regardless of primitive (**Fig. 4i**). Summarizing these effects,
442 we found that, in PMv, the neural distances between primitives controlling for location ("primitive
443 encoding") were consistently higher than the distances between locations controlling for primitives
444 ("location encoding") (**Fig. 4j,** left column). In dlPFC the opposite was true: primitive encoding was low
445 and location encoding was high (**Fig. 4j,** left column). A comparison of all areas (**Fig. 4j,** right column)
446 showed that PMv was the only one to exhibit this combination of strong primitive encoding and weak
447 location encoding.
448
449 This strong encoding of primitives and weak encoding of location suggests that PMv population activity
450 uses the same population code for primitives across locations. We tested this by measuring the
451 performance of a linear decoder, trained to decode primitives at one location, in generalizing to held-out
452 locations. We found that cross-location decoder generalization was strong in PMv and relatively weak in
453 other areas (**Fig. 4k**).
454
455 We performed two other tests of motor invariance. First, we tested whether location invariance was
456 present also during the initial reaching movement between the go cue and the start of drawing (**Extended**
457 **Data Fig. 9a**), an epoch during which premotor and motor cortical areas are known to strongly encode
458 reach direction[86,87]. We found strong location-invariant encoding of primitives in PMv, while M1, PMd,
459 and SMA showed a mixture of primitive- and location-encoding (**Extended Data Fig. 9**). Second, we
460 found that PMv population activity during planning encodes primitives in a manner that is also invariant
461 to size (**Extended Data Fig. 10**). Together, these findings indicate that PMv represents primitives in a
462 manner invariant to the motor parameters of location and size.
463

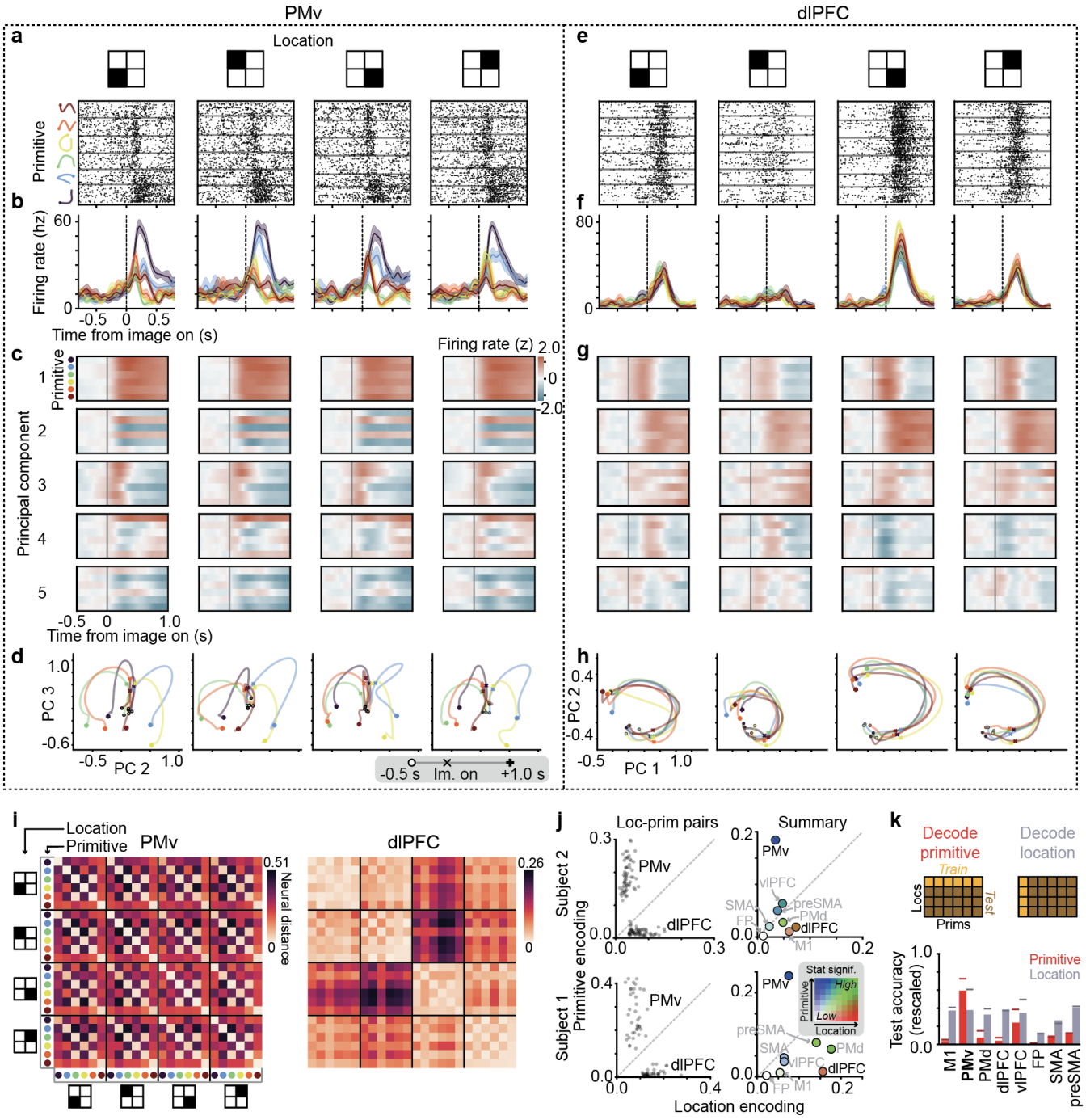

Fig. 4. Location-invariant encoding of stroke primitives in PMv (for invariance over size see Extended Data Fig. 10).

(a) Raster plot for an example PMv unit, showing single-trial spike patterns aligned to image onset. Trials (rows) are grouped by primitive and spatial location of the drawing (columns). The go cue occurred 1.1 - 1.5 seconds after image onset; therefore, the subject's finger was held still, pressing the fixation button, throughout the plotted time window.

(b) Smoothed firing rates for the example PMv unit, averaged across trials for each primitive (colors), split by location (columns). Curves show mean and SE (n = 13 - 20 trials per primitive-location combination).

(c) PMv population activity in primitive-encoding principal components (outer rows), showing average activity for each primitive (inner rows), and split by location (columns). Here, activity was z-scored and baseline subtracted relative to activity before image onset.

(d) Average time course of PMv population activity (i.e., state space trajectories) for each primitive (colors), plotted in a subspace spanned by principal components 2 and 3. Data are all from before the "go cue" and therefore before movement onset. No z-scoring or baseline subtraction was performed. Gray legend depicts trajectory, with image onset marked with an "x" icon.

(e) Analogous to panel a, but for an example dlPFC unit recorded simultaneously as the PMv unit.

(f) Analogous to panel f, but for the example dlPFC unit.

(g) Analogous to panel c, but for dlPFC population activity in the same session. Here, we use PCs 1 and 2 in order to emphasize the main effect of location in dlPFC.

(h) Analogous to panel d, but for dlPFC.

(i) Pairwise neural distances between unique primitive-location conditions, for the same session as panels a-h, averaged over time (0.05 to 0.6 s after image onset). n = 13 - 20 trials per primitive-location combination.

(j) Summary of primitive encoding and location encoding across areas and sessions. In the left two panels, each point represents a unique primitive-location condition. Each condition's primitive encoding is its average neural distance to all other conditions that have the same location but different primitive. Location encoding is defined analogously. Only PMv and dlPFC are plotted, to reduce clutter. n = 37 (subject 1, aggregating across 2 sessions), 59 (subject 2, 3 sessions). In the right two panels, each point depicts the mean encoding scores. The color denotes statistical significance. It indicates the number of other brain areas that this area "beats" in pairwise statistical tests of primitive encoding and location encoding (represented in the inset heatmap, "low" to "high" ranges from no (0) to all (7) other regions "beaten"). For example, PMv is deep blue because it has higher primitive encoding than every other area, but not higher location encoding. See Methods. For statistics, each data point was a unique pair of primitive-location conditions (trial-averaged). For testing primitive encoding, these pairs were the same location, but different primitive [n = 93 (subject 1), 288 (subject 2)]. For location encoding, the pairs were the same primitive, but different location [n = 114 (S1), 132 (S2)]. Statistical tests were performed on condition pairs pooled across sessions (2 for S1, 3 for S2). See Methods for details. For exact effect sizes and p-values, see **Extended Data Fig. 8a,b**.

(k) Across-condition generalization of linear support vector machine decoders for primitive (red) and location (gray). Top: schematic of the train-test split method; the primitive decoder was trained on data from one location and tested on data from all other locations (and analogously for location decoding). Bottom: accuracy is linearly rescaled between 0 (chance) and 1 (100%). The horizontal line represents within-condition decoding accuracy (with train-test splits partitioning data at the level of trials).

Categorical encoding of stroke primitives in PMv.

In behavioral experiments on categorical structure (**Figs. 2f-j**), we had found sigmoidal variation in behavior even with linear variation in images. Combining this paradigm (**Fig. 5a**) with recordings, we found that PMv activity during planning diverges towards separate primitive-representing states matching what primitive the subject will draw (**Fig. 5**). This is illustrated in a state space plot of PMv population trajectories (**Fig. 5b**). After image onset ("x" icon), trials in which the subject is planning to draw primitive 1 (morphs *i-iv*) separated from trials for primitive 2 (morphs *vi, vii*), such that activity diverged to two states

514 representing primitives 1 and 2 (**Fig. 5b**). Similarly, for the image at the category boundary (morph *v*),
515 trajectories diverged towards these two states depending on whether the subject was planning to draw
516 primitive 1 or 2 (A1 or A2, in **Fig. 5c**).
517
518 To quantify this separation into two primitive-representing states, we computed the Euclidean distance
519 between each pair of trials, and then scored each trial with its primitive alignment score, representing its
520 relative distance to activity encoding primitives 1 and 2 (P1 and P2, **Fig. 5a**). This revealed the same two
521 hallmarks of categorical structure we had seen in behavior: sigmoidal nonlinearity and trial-by-trial
522 switching. Sigmoidal nonlinearity is evident in the matrix of pairwise neural distances as two main blocks
523 separated by the category boundary (**Fig. 5d**), and in the plot of primitive alignment versus morph number
524 (**Fig. 5e, f**). Trial-by-trial switching between primitive-representing states for ambiguous images was
525 evident in the matrix of pairwise neural distances (**Fig. 5d**, A1 is closer to morphs *i-iv*, while A2 is closer
526 to morphs *vi-vii*), and in the trial-by-trial variation in primitive alignment scores (morph *v* in **Fig. 5e;** A1 vs.
527 A2 in **Fig. 5f**). Comparing across areas, this effect was strongest in PMv (**Fig. 5g**).
528
529 We considered two possible explanations for the trial-by-trial switching in PMv's activity for the ambiguous
530 images. One explanation is that activity is biased towards whichever primitive-representing state it
531 happens to be closer to at baseline, before image onset. Arguing against this, we found that the relative
532 proximity of baseline activity to states encoding primitive 1 or primitive 2 did not predict the chosen
533 primitive (in **Fig. 5h**, before image onset, the difference A2 - A1 is not significantly larger than zero).
534 Another possible explanation is competition between states after the image turns on, with the winning
535 state determining what primitive will be drawn. Network modeling suggests that such competition would
536 manifest as slower temporal evolution of activity towards one state or the other (as has been observed
537 in PFC during perceptual categorization[88]) as well as slower behavioral reaction times[89]. Consistent with
538 this, the separation of activity after image onset was slower for ambiguous images ("A2 - A1" in **Fig. 5h**)
539 than for images unambiguously drawn as primitive 1 or primitive 2 ("P2 - P1" and "U2 - U1" in **Fig. 5h**).

Behavioral reaction times were also slower for ambiguous images (**Extended Data Fig. 11**). These two findings are suggestive of a winner-take-all decision process[89].

Categorical encoding of primitives implies that every primitive is associated with a distinct activity pattern. Consistent with this, population activity visualized in a two-dimensional embedding showed clear separation by primitive in PMv (**Extended Data Fig. 7**). Quantifying this separation, we found that each primitive was distinguishable from every other primitive based on single-trial activity (**Extended Data Fig. 12**; restricted to pairs of primitives recorded in the same session).

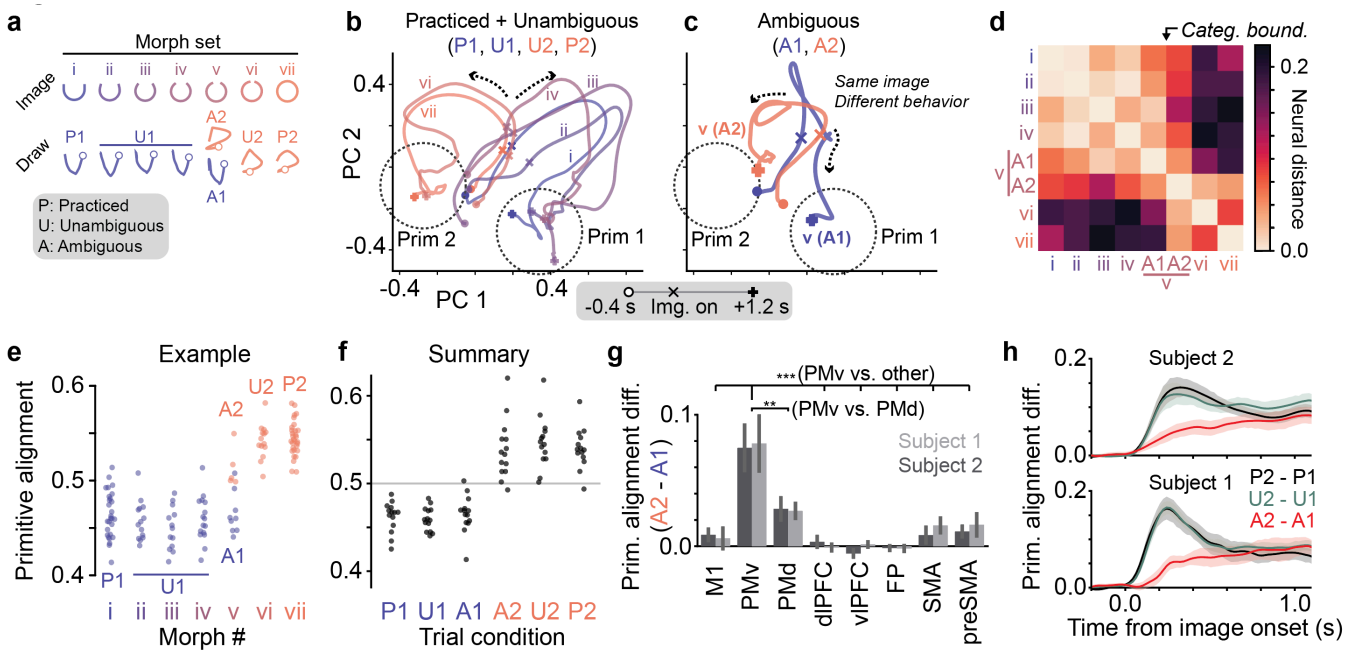

Fig. 5. Categorical encoding of stroke primitives in PMv.

(a) Example experiment showing discrete structure in drawing given continuously varying images. See also behavior in **Fig. 2f-i**.

(b) PMv mean population trajectories, showing divergence of activity after image onset (x) towards states encoding either primitive 1 or 2, depending on whether the morph was to the left (*i-iv*) or right (*vi-vii*) of the category boundary. Data are all from before the "go cue", which occurred 1.2 - 1.6 seconds after image onset; therefore, the subject's finger was held still (pressing the fixation button) throughout the plotted time window. Principal components were identified using PCA (see Methods). Trajectory legend is shown at bottom. n = 14 - 30 trials per trajectory.

(c) PMv population trajectories for morph *v*, split by whether the subject will draw primitive 1 (A1) or primitive 2 (A2), overlaid on the same subspace from panel b. n = 9 (A1) and 5 (A2) trials.

(d) Pairwise neural distance between morphs, using PMv activity. The morph on the category boundary (morph *v*) is split into two groups based on the drawing (A1 or A2). n = 5 - 30 trials (range across morph, considering A1 and A2 separately).

(e) Primitive alignment of PMv population activity as a function of morph number for the example experiment. Primitive alignment quantifies the relative similarity of each trial's neural activity to the neural activity for primitive 1 and for primitive 2, and is defined as $d_1/(d_1 + d_2)$, where d_1 and d_2 represent that trial's average Euclidean distance to primitive 1 and primitive 2. Each datapoint represents a single trial (n = 5 - 30 trials).

(f) Summary of PMv primitive alignment vs. trial condition across morph sets. Each morph set contributes one point to each condition [n = 14 morph sets (5 from S1, 9 from S2), including only morph sets that have all six trial conditions (14/20)].

(g) Summary of trial-by-trial variation in primitive alignment for ambiguous images (A2 - A1), for each brain area. This shows mean +/- SE. **, $p < 0.005$; ***, $p < 0.0005$, paired t-tests [n = 20 morph sets, combining subjects 1 (7) and 2 (13)].

(h) Average time course of difference in primitive alignment of PMv activity, split by trial condition, and aligned to image onset (mean +/- SE). [n = 14 morph sets (5 from S1, 9 from S2), including only morph sets that have all six trial conditions (14/20)].

Recombination of primitives into sequences is reflected in PMv.

When subjects recombine primitives into a multi-stroke sequence to draw characters (**Fig. 2k-p**), does this reflect a reused primitive-representing state in PMv? To answer this, we compared PMv activity when a particular primitive was used in the Single-shape task with PMv activity when the same primitive was used in the Character task. We focused on the time window immediately preceding stroke onset, because, for characters, the alternative approach of using the entire planning epoch leaves uncertain which primitive we would expect to be represented in PMv. This revealed that PMv encodes primitives similarly across Single-shape and Character tasks. This is evident in the population activity in **Fig. 6a, b**, where each primitive's activity (inner rows in **Fig. 6a**, colored trajectories in **Fig. 6b**) is similar across task types (columns in **Fig. 6a, b**). We contrasted PMv with preSMA, an area known to encode sequence-related information[46], finding that preSMA activity differed between the two tasks (**Fig. 6c, d**). A quantification of neural distance between each pair of primitive-task conditions (**Fig. 6e**, showing PMv and preSMA) confirmed that PMv encodes primitives similarly across task types. This is visible in the off-diagonal low-distance streak for PMv in the matrix of pairwise distances (**Fig. 6e**), and in the summary plot showing that in PMv the neural distance between primitives controlling for task ("primitive encoding") was high while the neural distance between task types controlling for primitive ("task-type encoding") was low (**Fig. 6f**). In contrast, preSMA activity for the same primitive differed between Single-shape or Character tasks. This is evident as a block structure in **Fig. 6e**, and in preSMA having relatively high task-

596 type encoding compared to primitive encoding (**Fig. 6f**). Across all areas, PMv most consistently exhibited
597 high primitive encoding and low task-type encoding (**Fig. 6f**).
598
599 In the preceding analysis, to account for the fact that Single-shape strokes all involve a large initial
600 reaching movement from the start button towards drawing, we included only the first stroke for the
601 Character task, which similarly involves this initial reaching movement. This was to ensure a fair
602 comparison between the two task types by removing any possible neural effects due to differences in
603 gross movements associated with the presence or absence of this reach[87,90]. In addition, we also
604 performed this analysis using all character strokes. We first applied a linear correction to neural activity
605 to account for an additive effect that we found the initial reaching movement to have on activity (see
606 explanation of additive effect and linear correction in **Extended Data Fig. 14a,b**). As in the analysis
607 restricted to the first character stroke (**Fig. 6f**), we found that, out of all the areas, PMv most consistently
608 exhibited high primitive encoding and low task-type encoding (**Extended Data Fig. 14c-e**). These
609 findings indicate that representations of primitives in PMv are reused across recombined sequences.
610

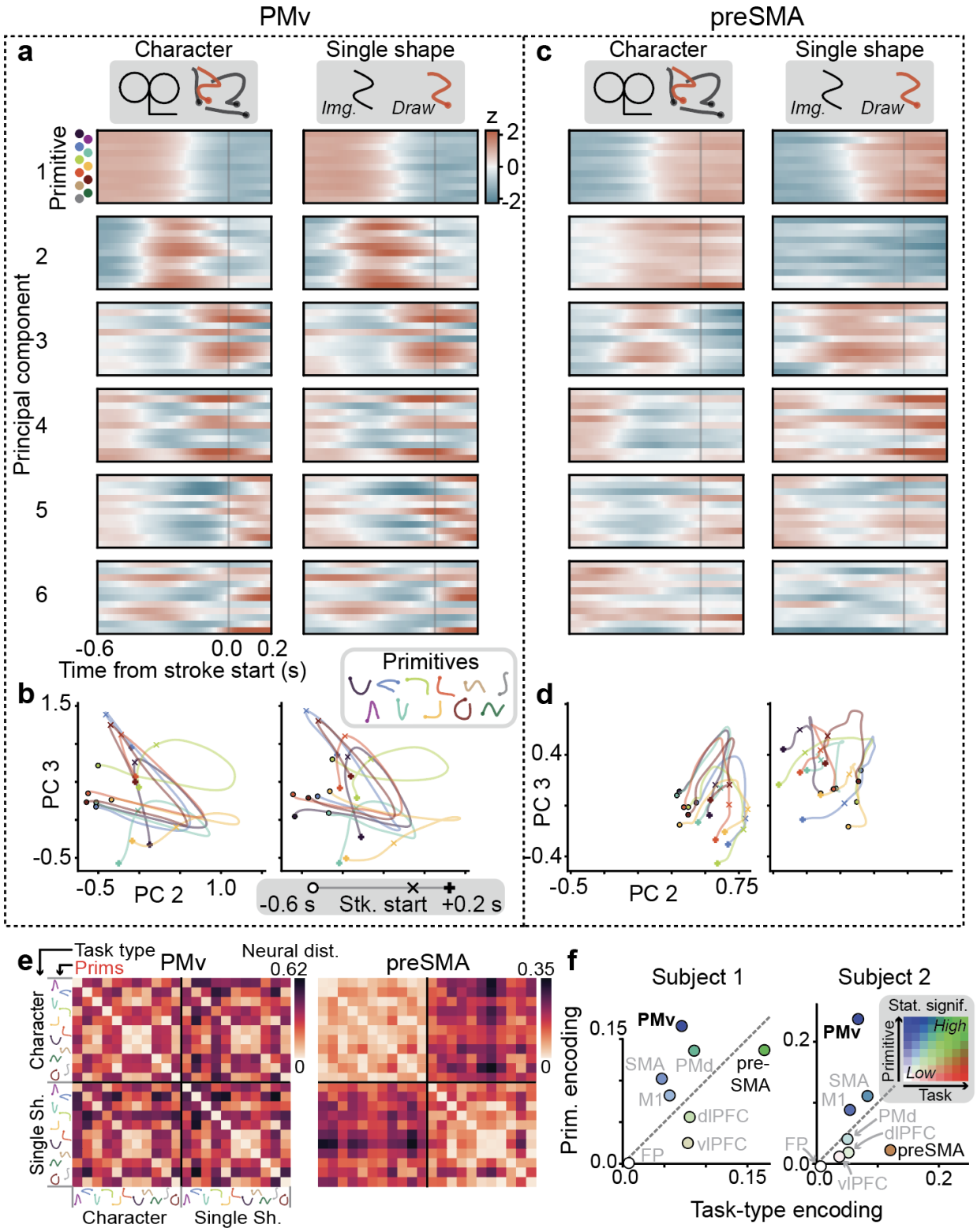

Fig. 6. Recombination of primitives into sequences is reflected in PMv.

(a) PMv population activity, projected to its first six principal components using PCA (outer rows), showing average activity for each primitive (inner rows), and split by whether primitives were performed in the Character or Single-shape task (columns). Here, activity was aligned to stroke onset and z-scored over the entire depicted window. Data from a single session for subject 2. n (trials) = 4 - 16 (Single-shape), 6 - 72 (Character). See panel b for the primitive indexed by each color.

(b) PMv population trajectories for each primitive (color), plotted in a subspace spanned by principal components 2 and 3, and aligned to stroke onset. Gray legend depicts a trajectory, with stroke onset marked with an "x" icon. This plot shows only a subset of primitives (7/11) to reduce clutter.

(c) Analogous to panel a, but for simultaneously recorded activity in preSMA.

(d) Analogous to panel b, but for simultaneously recorded activity in preSMA.

(e) Pairwise neural distance between each unique combination of primitive and task type, for the experiment in panels a-d, averaged over time (-0.5 to -0.05 s relative to stroke onset). n (trials) = 4 - 16 (Single-shape), 6 - 72 (Character).

(f) Summary of primitive encoding and task-type encoding across areas and sessions. Points show mean encoding scores. The color of each point denotes statistical significance (analogous to **Fig. 4j**). It indicates the number of other brain areas that this area beats in pairwise statistical tests of primitive encoding and task-type encoding (represented in the inset heatmap; "low" to "high" ranges from no (0) to all (7) other regions "beaten"). Statistical tests were performed on condition pairs pooled across sessions (9 for S1, 9 for S2). We used bootstrapping to balance the sample sizes across sessions, plotting the bootstrapped mean for primitive and task-type encoding values, and bootstrapped mode for the number of significant statistical comparisons (color of each datapoint). Each data point was a unique pair of primitive-task conditions (trial-averaged). For testing primitive encoding, these pairs were the same task type, but different primitive [n = 700 (subject 1), 548 (subject 2)]. For task-type encoding, the pairs were the same primitive, but different task type [n = 82 (S1), 64 (S2)]. For exact effect sizes and p-values, see **Extended Data Fig. 13**.

PMV encodes primitives in a manner dissociated from both the shape in view and from motor kinematics.

In tests of motor invariance (**Fig. 4**), categorical structure (**Fig. 5**), and recombination (**Fig. 6**), we found that all three properties were most strongly reflected in a single area, PMv. This striking conjunction is strong evidence for a representation of stroke symbols in PMv. We wished to further characterize this representation. PMv is known to respond to visual object stimuli[91]. PMv is also known to encode kinematic parameters of hand movement[92]. These prior studies raise the question of how visual and motor parameters contribute to PMv activity in the current task paradigm.

First, we considered the possibility that PMv is driven by visual properties of the shape the subject is looking at. We considered a task designed to dissociate vision from action (the Multi-shape task). Each image consists of multiple separated simple shapes, and the subject decides, with no imposed rule, what order to draw them. During the planning epoch, gaze consisted of a series of saccades and fixations on individual shapes (**Extended Data Fig. 15a-c**). Collecting fixation events from planning epochs across trials with different first-drawn primitives allowed us to dissociate the effect of visually-fixated shape from

653 the primitive the subject plans to draw first. We found that PMv strongly encodes the planned primitive
654 regardless of the visually-fixated shape (**Extended Data Fig. 15d-f**). Moreover, this encoding is
655 persistent in being largely unaffected by ongoing saccades (**Extended Data Fig. 15d)**. PMv therefore
656 maintains a representation of the planned primitive dissociated from the shape in view.
657
658 Second, we tested whether PMv encodes finger kinematics during strokes in a manner that generalizes
659 across primitives. We found that this was not the case. A linear encoding model (mapping from stroke
660 velocity to neural activity) trained on one subset of primitives did not explain any variance in PMv activity
661 for held-out primitives, and was even worse than chance for one subject (**Extended Data Fig. 16**). In
662 contrast, this same approach revealed that kinematics accounted for a significant fraction of variance in
663 M1, consistent with its known role in motor control[81,93].
664
665 Together, these results show that PMv activity in this drawing task exhibits a form of abstraction in which
666 it is independent from immediate visual input and motor output.
667

Discussion

Identification of a neural substrate of action symbols

We tested for three essential properties of symbols—invariance, categorical structure, and recombination—in both behavior and neural activity. In behavior, we found that monkeys successfully trace complex geometric figures by recombining discrete stroke primitives into new composed sequences (**Fig. 2k-p**). These primitives exhibit motor invariance (**Fig. 2a-e**) and categorical structure (**Fig. 2f-j**). These properties indicate that stroke primitives are internally represented as action symbols. In neural recordings, we tested for each of these three properties in population activity across multiple motor, premotor, and prefrontal cortical areas (**Fig. 3**). We found evidence for each of these three properties, and, strikingly, each property was strongest in a single area, ventral premotor cortex (PMv). This reveals that activity in PMv, but not elsewhere, encodes primitives in a manner that reflects the conjunction of three hallmarks of symbolic representations: invariance (**Fig. 4),** categorical structure (**Fig. 5**), and recombination (**Fig. 6**). Together, these behavioral and neural findings reveal a localized representation of action symbols in PMv.

Finding action symbols in PMv was unexpected given the substantial prior evidence for encoding of abstract cognitive variables in other brain areas, especially in PFC[28,42,43,47,48,50–52] and medial temporal lobe[41,43,49,54,56,57]. However, this may reflect a fundamental difference in the role of motor behavior between our task and commonly studied cognitive tasks. In commonly studied tasks, motor behavior is secondary—it is used for reporting a decision (e.g., press left lever for choice #1). In contrast, in our study, motor behavior is primary—it specifies the *content* of the representation, in this case an abstraction of a given primitive's stroke trajectory. More generally, various kinds of abstraction exist for motor behavior, including plans[69], hierarchies[70,94], categories[95], and action concepts[96]. Our results suggest that motor abstraction may be mechanistically distinct kind of abstraction, one in which PMv plays a central role. This privileged role may relate to a range of findings in humans, including that (i) lesions to PMv are associated with disrupted conceptual knowledge of actions[96]; (ii) PMv activation is linked to the perception[97–99], imitation[100], verbal description[101,102] and imagination[98,99,103] of action, across domains of

695 handwriting[97,98], tool use[101,102], dance[99], and others[100,103]; and (iii) PMv dysfunction is implicated in
696 apraxia[104,105], a disorder of higher-order motor skill that spares basic movements, including in drawing
697 (constructional apraxia[105]). These prior observations related to motor abstraction may turn out to reflect
698 internal operations involving action symbols in PMv. More generally, our findings highlight a kind of
699 abstraction previously underappreciated in the neuroscience of cognition (motor abstraction) and
700 establishes a potential starting point for future studies into its neural basis (action symbols in PMv).
701
702 Identifying a symbolic neural representation in monkeys may provide general insights into the neural
703 basis and evolutionary origins of symbols. Certain abstract properties—notably reversible symbol
704 reference[106], and the ability to recombine using higher-order relations[9,107] and recursive syntax[7,9,108]—are
705 thought to characterize the kinds of symbolic representations used in language, mathematics, and formal
706 reasoning in humans[7,9,106–110]. These abstract properties have not been readily apparent in animal
707 behavior[9,106,107,109,110], suggesting they may be specific to humans. At the same time, animals demonstrate
708 a variety of behaviors that appear consistent with symbols (see Introduction)[11,23–27,29–31,31], and invariant
709 neural representations have been identified in animals performing cognitive tasks, raising the possibility
710 that symbols may contribute across cognitive faculties and animal species. We discovered that at least
711 one kind of symbolic representation (action symbols) exists in macaques. It remains to be determined
712 whether and how the neural substrates identified here in motor tasks relate to other proposed kinds of
713 symbols, including those implicated in more abstract tasks, and those with properties hypothesized to be
714 unique to humans[7,9,106–110]. Nonetheless, our finding of a representation of action symbols raises the
715 possibility that symbolic operations exist across primates and taxa, and may reflect shared mechanisms
716 for generating and recombining discrete, invariant representations. Our study's criteria of invariance,
717 categorical structure, and recombination may be useful in future studies of symbolic neural
718 representations, including in reassessing prior findings of invariant activity in cognitive tasks[28,41–43,47–
719 52,54,56,57].
720
721 **Action-symbol based abstraction in ventral premotor cortex**

722 Several lines of evidence indicate that this action symbol representation in PMv is abstract, not directly
723 driven by visual features or motor parameters. Independence from visual features is indicated by the
724 finding that, in three experiments dissociating vision from the planned primitive, PMv encoded the
725 planned primitive: (i) when the visually-fixated shape was dissociated from the planned primitive
726 (**Extended Data Fig. 15**); (ii) when ambiguous shapes were drawn as different primitives (**Fig. 5f**); and
727 (iii) when different images were drawn with the same primitive (**Fig. 6f,** compare Character and Single-
728 shape tasks). Independence from motor parameters is indicated by three properties of primitive-encoding
729 activity in PMv: (i) presence during planning, temporally dissociated from drawing (by > 1 sec; e.g., **Fig.**
730 **5h**), (ii) invariance to location and size (**Fig. 4, Extended Data Fig. 9, Extended Data Fig. 10**), and (iii)
731 lack of encoding of generic motor kinematics (**Extended Data Fig. 16**). These abstract properties in PMv
732 directly contrast with motor-related activity we found in M1 (consistent with prior M1 studies[111–113]): (i)
733 lack of primitive-encoding during planning (**Fig. 4j**); (ii) lack of location-invariance during the initial reach
734 (**Extended Data Fig. 9**); and (iii) encoding of generic motor kinematics (**Extended Data Fig. 16**). Thus,
735 PMv encodes action symbols in an abstract manner.
736
737 Prior evidence for motor invariance in PMv may reflect action symbols related to those identified here.
738 Recordings in PMv have revealed a diversity of invariant firing patterns, especially during grasping and
739 object manipulation[44]. This includes activity that encodes grasp properties independent of the arm being
740 used[114], kinematics independent of muscle dynamics[92], and tool kinematics regardless of hand
741 posture[115]. Perhaps most strikingly, PMv contains so-called “mirror neurons”, which fire similarly whether
742 one observes or performs a given action[60] (but see refs[91,116,117]). These abstract properties have been
743 proposed to support a variety of functions, including object-oriented visuo-motor transformation[44,118–120],
744 action understanding[60], imitation[60], and mental imagery[71]. Perhaps most related to action symbols, PMv
745 has been proposed to encode a repertoire of action types[44,60,118,121]. There are differences between these
746 proposals and the action symbols framework, both conceptually—action symbols place stronger
747 emphasis on categorical structure and recombination—and experimentally—we tested both categorical
748 structure (in a systematic manner not previously done) and recombination into novel sequences (differing

749 from previous studies, which behaviorally tested only a few well-practiced sequences[122,123]). Also
750 essential to revealing specificity to PMv was that we recorded more areas than in prior studies. Our
751 findings raise the question of whether PMv's involvement in diverse motor behaviors—including
752 speech[124]—may be explained by the single principle that PMv encodes action symbols, a question which
753 may be addressed by testing for symbolic properties, abstraction, and involvement in compositional
754 generalization.
755
756 PMv's connectivity with other areas puts it at a unique intersection of motor, cognitive, and sensory
757 circuits. Same as the other premotor areas (including PMd and SMA), PMv is directly interconnected with
758 M1[125] and the spinal cord[126], thus affording direct control over motor output. In *contrast* to the other
759 premotor areas, PMv is strongly interconnected with prefrontal cortex (especially vlPFC)[125,127,128], thus
760 affording direct access to cognitive processes. PMv is also interconnected with preSMA, an area involved
761 in action sequencing[46]. PMv, vlPFC and preSMA are components of the "multiple-demand" system[129], a
762 network of areas implicated in abstract reasoning[130]. PMv receives input from the two main visual
763 pathways: (i) spatial- and action-related "dorsal stream" signals via inferior parietal cortex[131], and (ii)
764 shape- and object-related "ventral stream" signals via vlPFC[132]. Ventral stream signals can encode
765 shapes[133,134] in a manner exhibiting location and size invariance[135], categorical specificity[136–138], and
766 reuse of parts[137], suggesting that they contribute to the symbolic representations in PMv; however, this
767 contribution cannot fully explain PMv activity, as we showed that PMv represents the planned action, not
768 visual input (see above) and vlPFC did not exhibit similar symbolic properties to PMv). In addition to these
769 motor, cognitive, and sensory inputs, processes within PMv are likely critical, including, possibly, winner-
770 take-all competition between discrete states encoding action symbols[89], which may account for the slow
771 activity dynamics we found for ambiguous images (**Fig. 5h**), and for PMv's known contributions to
772 perceptual decision-making[139–141]. Future studies may focus on PMv to elucidate how motor, cognitive,
773 and sensory inputs interact with local mechanisms to generate symbolic representations.
774
775 **Leveraging motor behavior to understand compositional generalization**

Our study introduces a paradigm to study compositional generalization. This depended on two critical properties of the drawing task. First, visual cues (images) do not directly instruct movement details, instead requiring the subject to *infer* what strokes to make. This is because the images are generally ambiguous and memorizing a solution for each is not feasible given the large variety of (potentially novel) images. The need to construct one's own solution across many images likely incentivized subjects to learn an internal "generative model" for drawing[17]—with prior knowledge structured as action symbols—that efficiently generalizes. This learning may reflect an inductive bias to acquire "compressed" representations of action, as has been proposed for human drawing[16,17,142]. Note that this lack of direct instruction contrasts with prior drawing-like studies in macaques, where the visual cue was a moving cursor instructing how to move the finger[143] or was one of two to four highly-practiced images to which memorized joystick movements were learned[144]. A second critical property of this drawing task is flexibility in stimulus generation, with a large pool of possible images that can be varied systematically and continuously to test generalization behavior that is diagnostic of three independent hallmarks of symbolic representations: invariance, categorical structure, and recombination.

This drawing task is complementary to prior tasks in neural studies of motor behavior, which, to our knowledge, have not directly addressed compositional generalization. These studies involve the following kinds of (not mutually exclusive) behaviors: automatic, instructed, working-memory-guided, or minimally-restrained (expanding the classes from Mizes et al.[145]). Automatic movements are highly-practiced stereotyped sequences, often encoded as a single "chunk" without the need for moment-by-moment instruction[77,145–156]. Instructed movements are guided by external cues that trigger specific actions, either directly (e.g., the location of a cue instructing where to reach)[90,111,145,157–159] or indirectly from learned rules (e.g., red square means "reach left")[29,47,160]. Working-memory-guided movements are produced from short-term memory, soon (seconds to minutes) after the movement was instructed or learned[28,145,146,159,161]. Minimally-restrained movements are spontaneously produced in a naturalistic setting with minimal physical constraint or experimenter-defined goals[73,121,162,163]. Previous drawing-like

802 tasks in monkeys can be classified as either automatic[144], instructed[143,164], or unconstrained[164]. Thus,
803 compared to previous motor tasks, our drawing task opens a new direction: compositional generalization.
804
805 Prior behavioral studies suggest that continuous motor behavior can be decomposed into reused
806 segments often called "motor primitives."[165–167]. Compared to action symbols, motor primitives are at a
807 lower level of abstraction, usually defined as muscle activity "vectors" (or "synergies") derived by
808 factorization of electromyographic activity[165,166,168]. An important open question is whether motor
809 primitives correspond to distinct representational units in the brain, such as action symbols (see some
810 evidence in M1[112,158,169–171]), or instead are byproducts of biomechanical constraints, task low-
811 dimensionality, or spinal circuit properties[168]. A related line of research has identified reused motifs (or
812 "syllables") in minimally-restrained behaviors, including in birdsong[72], and spontaneous rodent
813 behavior[73]. For both motor primitives and syllables, it remains to be seen whether they reflect internal
814 representations of action symbols, a possibility which could be assessed using the criteria in this study.
815
816 **A foundation for understanding symbolic operations in neural representations and dynamics**
817 This action symbol representation in PMv may provide an entry point to bridge between two paradigms
818 that have dominated the modeling of cognition: one based on symbols and rules[2–10] and the other on
819 neural network ("connectionist") architectures and dynamical systems[33,34,36,37,39]. While these views have
820 sometimes been considered to be at odds, it is recognized that explaining cognition depends on their
821 unification, possibly by implementing symbolic program-like operations[4,7,10,14–17,47] in neural
822 representations and dynamics[25,47,65,130,172–176] (and building on insights from task-optimized network
823 models[34,35,39,177,178]). Looking forward, an important aspect of finding an action-symbol-encoding
824 population in PMv is that it could be a foundation for future studies to test for activity dynamics consistent
825 with symbolic operations in programs, including in PFC and preSMA, areas interconnected with PMv and
826 which encode abstract sequential information[28,46,47,179–181], and in hippocampal systems implicated in
827 generating compositional representations[58,182]. Identifying such activity would be a significant step
828 towards a computational explanation of intelligence that spans levels of behavior, cognition, and neural

829 mechanisms, and that could generalize beyond action to other abilities that seem to reflect program-like

830 cognition[7,10,14].

831

832 **Methods**

833 **Subjects and surgical procedures**

834 Data were acquired from two adult male macaques (*Macaca mulatta*, average weights 17 kg (S1) and 10
835 kg (S2), average ages 9 years (S1) and 7 years (S2)). All animal procedures complied with the NIH Guide
836 for Care and Use of Laboratory Animals and were approved by the Institutional Animal Care and Use
837 Committee of the Rockefeller University (protocol 24066-H).

838 After undergoing initial task training in their home cages, subjects underwent two surgeries, the
839 first to implant an acrylic head implant with a headpost, and the second to implant electrode arrays. Both
840 surgeries followed standard protocol, including for anesthetic, aseptic, and postoperative treatment. In
841 the first surgery, a custom-designed MR-compatible Ultem headpost was implanted, surrounded by a
842 bone cement cranial implant, or "headcap" (Metabond, Parkell and Palacos, Heraeus), which was
843 secured to the skull using MR-compatible ceramic screws (Rogue Research). After a six month interval,
844 to allow bone to grow around the screws and for the subject to acclimate to performing the task during
845 head fixation via the headpost, we performed a second surgery to implant 16 floating microelectrode
846 arrays (32-channel FMA, Microprobes), following standard procedures[183]. Briefly, after performing a
847 craniotomy and durotomy over the target area, arrays were inserted one by one stereotactically, held at
848 the end of a stereotaxic arm with a vacuum suction attachment (Microprobes). Using vacuum suction
849 allowed us to release the arrays, after insertion, with minimal mechanical perturbation by turning off the
850 suction. After all arrays had been implanted, the dura mater was loosely sutured and covered with
851 DuraGen (Integra LifeSciences). The craniotomy was closed with bone cement.

852 We used standard density arrays (1.8 mm x 4 mm) for all areas, except SMA and preSMA, for
853 which we used four high density arrays (1.6 mm x 2.95 mm). Four additional electrodes on each array
854 served as reference and ground. Two arrays were targeted to each of multiple areas of frontal cortex,
855 with locations identified stereotactically, and planned using brain surface reconstructions derived from
856 anatomical MRI scans. Locations were selected based on their published functional and anatomical
857 properties (see below), anatomical sulcal landmarks, and a standard macaque brain atlas[184]. During

858 surgery, locations were further adjusted based on cortical landmarks, and to avoid visible blood vessels.
859 Arrays were implanted in the right hemisphere (contralateral to the arm used for drawing).

860 Array locations are depicted in **Fig. 3b** (confirmed with intraoperative photographs). For M1, we
861 targeted hand and arm representations (F1), directly medial to the bend of the central sulcus (which
862 corresponds roughly to the intersection of the central sulcus and the arcuate spur if the latter were
863 extended caudally), based on retrograde labeling from spinal cord and microstimulation of M1[126] and M1
864 recordings[111]. For PMd, we placed both arrays lateral to the precentral dimple, with one (more caudal)
865 array directly medial to the arcuate spur (the arm representation[111,126,185], F2), and the other was placed
866 more rostrally (straddling F2 and F7). For PMv, we targeted areas caudal to the inferior arm of the arcuate
867 sulcus (F5), which are associated with hand movements based on retrograde labeling from spinal cord[126]
868 and M1[127], microstimulation[186] and functional studies[119,139,187]. These areas contain neurons
869 interconnected with PFC[127]. For SMA (F3) and preSMA (F6), we targeted the medial wall of the
870 hemisphere, with the boundary between SMA and preSMA defined as the anterior-posterior location of
871 the genu of the arcuate sulcus, consistent with prior studies finding significant differences across this
872 boundary in anatomical connectivity (e.g., direct spinal projections in SMA but not preSMA[188]) and
873 function[46,189]. SMA arrays were largely in the arm representation[188]. For dlPFC, we targeted the region
874 immediately dorsal to the principal sulcus (46d), following prior studies of action sequencing[28,47,190] and
875 other cognitive functions[191]. For vlPFC, we targeted the inferior convexity ventral to the principal sulcus,
876 with one (more rostral) array directly ventral to the principal sulcus (46v) and the other rostral to the
877 inferior arm of the arcuate sulcus (45A/B), based on evidence for encoding of abstract concepts in regions
878 broadly spanning these two locations[52,192,193], including a possible heightened role (compared to dlPFC)
879 in encoding abstract concepts in a manner invariant to temporal or spatial parameters[192,194,195]. For FP,
880 we targeted a rostral location similar to prior recording and imaging studies (one array fully in area 10,
881 the other straddling 9 and 10)[196,197]. In general, array locations targeted the cortical convexity immediately
882 next to sulci, instead of within the banks, in order to allow shorter insertion depths that minimize the risk
883 of missing the target. The exceptions were SMA and preSMA in the medial wall, for which this was not

884 possible. To avoid damaging the superior sagittal sinus, we positioned the arrays laterally (2 mm from
885 midline) and slanted the electrodes medially (**Extended Data Fig. 6**).
886 The lengths of each electrode were custom-designed to target half-way through the gray matter,
887 and to vary substantially across the array, to maximize sampling of the cortical depth. Electrode lengths
888 spanned 1.5 - 3.5 mm (M1), 1.5 - 3.1 mm (PMd, PMv), 2.8 - 5.8 mm (SMA, preSMA), 1.5 - 2.5 mm (dlPFC,
889 vlPFC), and 1.5 - 2.6 mm (FP) for subject 1, and 1.7 - 3.75 (M1), 1.5 - 3.3 mm (PMv), 1.5 - 3.1 mm (PMd),
890 2.65 - 5.95 mm (SMA, preSMA), 1.75 - 3.15 mm (dlPFC), 1.35 - 3.2 mm (vlPFC), and 1.6 - 2.9 mm (FP)
891 for subject 2. Reference electrodes were longer (6 mm) to anchor the arrays. 28 electrodes were Pt/Ir
892 (0.5 MΩ) and 4 were Ir (10 kΩ) (for microstimulation). Array connectors (Omnetics A79022) were housed
893 in custom-made Ultem pedestals (Crist), which were secured with bone cement onto the cranial implant.
894 Four pedestals were used per subject, holding 5, 5, 4, and 2 connectors each.
895
896 **Behavioral task**
897 Task overview
898 Subjects were seated comfortably in the dark with their head restrained by fixing the headpost to
899 the chair. They faced a touchscreen (Elo 1590L 15" E334335, PCAP, 768 x 1024 pixels, refresh rate 60
900 Hz, with matte screen protector to reduce finger friction) that presented images and was drawn on. The
901 touchscreen location was optimized to allow each subject to easily draw at all relevant locations on the
902 screen (23 to 26 cm away; see diagram in **Extended Data Fig. 1**). Both subjects decided on their own
903 over the course of learning to perform the task with the left hand. The chairs were designed to minimize
904 movements of the torso and legs (by using a loosely restricting "belly plate"), and of the non-drawing arm
905 (by restricting movement to within the chair). Gravity-delivered reward (water-juice mixture) was
906 controlled by the opening and closing of a solenoid pinch valve (Cole-Parmer, 1/8" ID). Subjects were
907 water-regulated, with careful monitoring that consumption met the minimum requirement per day (and
908 typically exceeded it), and weight was closely monitored to ensure good health. The task was controlled
909 with custom-written software, using the MonkeyLogic behavioral control and data acquisition MATLAB
910 package[198]. (PC: Windows 10 Pro, Intel Core i7-4790K, 32GB RAM; DAQ: National Instruments PCIe-

6343). All stimuli (images of line figures defined as point sets) were also generated with custom-written MATLAB code. Images were presented in a “workspace” area on the screen (16.6 cm x 16.9 cm, corresponding to approximately 37° by 38° visual angle). Shape components in images were on average 4.0 cm (9°) (taking the maximum of width and height).

Each neural recording session consisted of a day’s recording (2 - 3.5 hours). We collected 5-20 trials per condition (i.e., each unique image for **Figs. 4, 5** and Single-shape tasks in **Fig. 6**; each primitive stroke for the Character task in **Fig. 6**). All trials were shuffled across all conditions within the session, and presented in a randomly interleaved fashion, except for one case, the experiment in **Fig. 6**, in which Character and Single-shape tasks were switched in blocks.

<u>Early Training</u>

Before surgery, naive subjects underwent initial training on core task components (i.e., to trace images accurately on a touchscreen, using a sequence of discrete strokes). Early training took place in the home cage, using custom-built rigs that attached to an opening in the cage, using the same hardware and software described above, except for a different computer (Lenovo IdeaPad 14" laptop, Windows 10, AMD Ryzen 5 3500U, 8GB RAM) and DAQ (National Instruments USB-6001). This initial training progressed through the following stages: *(1) Touch circle*. Subjects were rewarded for touching a circle anywhere within its bounds. The circle started large, filling the entire screen, and shrank over trials to enforce more accurate touches. *(2) Touch with a single finger*. We shrank the circle until it was so small that it could only be touched with a single finger. The trial aborted if the subject touched outside the circle, or with multiple fingers simultaneously. *(3) Hold still*. Subjects were rewarded for keeping their fingertip still on a dot, with the duration of this hold increasing across trials (up to a few seconds). *(4) Track moving dot.* Subjects had to track the dot with their finger as it moved (a lag between dot and finger was allowed). *(5) Trace a line.* We increased the speed of the moving dot over trials, until eventually the dot moved so fast that the line it traced appeared immediately. We then positioned the line at locations far from the hold position to train the subject to raise its finger from the hold position and trace lines at arbitrary locations, angles, and lengths. *(6) Trace single shape.* We presented shapes of increasing difficulty (gradually

938 “morphing” across trials from a straight line), including arcs, “L”-shapes, and squiggles, and circles.
939 Across these stages, the shapes were presented at random locations. We did not enforce any particular
940 tracing strategy for each shape (e.g., which endpoint to start at), allowing subjects to choose on their
941 own. *(7) Trace multiple shapes.* We presented images composed of multiple disconnected shapes. This
942 trained the subjects to understand that they should use multiple strokes to trace multiple shapes. At this
943 point, the subject understood the basic structure of the task—to trace shapes, using multiple strokes if
944 needed. The progression across these stages was not determined by strict quantitative criteria, but
945 instead on a combination of quantitative and qualitative evaluations of how well the subjects understood
946 the task.

947 After this basic training, subjects practiced various tasks to incentivize the learning of stroke
948 primitives (consistent stroke trajectories for each shape). They practiced Single-shape trials using the set
949 of diverse simple shapes in **Fig. 1e**, varying randomly in shape and location across trials. We chose this
950 set of shapes in order to cover a variety of trajectory profiles (by varying rotation, the number of direction
951 changes, and whether shapes were curved or linear), and yet to still be simple enough to draw with one
952 stroke and to be able to combine multiple (two to six shapes) shapes into single character images. We
953 did not constrain subjects to learn specific stroke trajectories for each primitive; therefore, differences
954 between primitives reflected each subject’s own decision for how to draw each shape. We also note that
955 our study did not depend on using an “optimal” set of shapes, but instead depended on subjects learning
956 primitives associated with the basic shapes, and then demonstrating behavioral generalization using
957 these primitives, as we found that they did. For subject 2, the four “S” shapes and four “arc” shapes were
958 also sometimes presented as rectilinear versions. For “S” these resembled “Z”, and for “arc” these
959 resembled squares missing one edge. The subject drew these rectilinear versions using curved strokes
960 similar to those used to draw “S” and “arc” shapes. Therefore, instead of considering these rectilinear
961 versions as distinct primitives, we combined their data with their respective curved versions. On different
962 days, subjects also practiced Multi-shape and Character tasks. Below, we describe the trial structure,
963 followed by details for Single-shape, Multi-shape, and Character tasks.
964

965 Trial structure

966 As depicted in **Fig. 1c**, trials began when the subject pressed and held a finger fixation button (blue
967 square) at the bottom of the screen (gray background; note that in this article "button" always means a
968 virtual button; diagram of screen in **Extended Data Fig. 1c**). After a random delay (uniform, 0.2 s window,
969 earliest [0.4, 0.6] s and latest [0.8, 1.0] s across experiments for subject 1 and [0.8, 1.1] s for subject 2)
970 the image appeared (figure colored dark gray). After a random delay (uniform, ranging from [0.6, 1.0] s
971 to [1.2, 1.6] s across experiments for subject 1, and [1.1, 1.5] s to [1.8, 2.4] s for subject 2), a "go" cue
972 (400 Hz tone and image blank for 300 ms) was presented. This delay between image presentation and
973 go cue we call the "planning" epoch. The subject had to keep its finger still on the fixation button during
974 the planning epoch. After the onset of the go cue, the subject was free to raise its finger, move its hand
975 towards the image, and start drawing. During drawing, the image stayed visible, and the finger left a trail
976 of black "ink" on the screen. Immediately after the finger was raised from the fixation button after the go
977 cue, the "fixation button" disappeared and a "done button" (green square) appeared at its location and
978 stayed on. The drawing epoch ended when the subject signaled it had completed the drawing, by
979 pressing the "done button" (no time limit was imposed). This was followed by a delay (uniformly sampled,
980 0.65 to 1 s), followed by performance feedback. Feedback spanned four modalities, each signalling
981 performance: (i) screen color, (ii) sound cue, (iii) duration of a delay before getting reward, and (iv) reward
982 magnitude. First, screen color and sound were signaled, followed by the pre-reward delay and then the
983 reward. How performance was scored and converted to feedback is described below. In addition to
984 providing this feedback at the end of the trial, we also provided feedback online by immediately aborting
985 a trial if subjects made serious errors, including (i) if they touched a position far from any image points,
986 and, (ii) for Single-shape and Multi-shape trials, if they used more than one stroke per shape in the image.
987 These "online abort" modes were turned off for trials testing novel characters.

988 Screen image changes (including image presentation and trial events) were recorded using
989 photodiodes (Adafruit Light Sensor ALS-PT19) and sounds were recorded using an electret microphone
990 (Adafruit Maxim MAX4466, 20-20KHz). We performed eye tracking (ISCAN), but did not enforce eye
991 fixation.

992

993 Scoring behavioral performance

994 Behavior was scored by aggregating multiple metrics, or “factors”. There were three classes of factors.
995 The primary class measured *image similarity*, or the similarity of the final drawing to the target image
996 (ignoring its temporal trajectory). This factor had the greatest influence on the final aggregate score.
997 Additionally, we computed factors reflecting *behavioral efficiency*, and, in some cases, factors that were
998 *task-specific*. These scores were computed on behavioral data, which was represented as a sequence
999 of touched points (x-y coordinates) with gaps between strokes, and image data, represented as a set of
1000 x-y coordinates. First, we describe the factors, and then how they were aggregated into a single score.

1001 *Image similarity*. This included two factors: drawing-image overlap and Hausdorff distance.
1002 Drawing-image overlap was computed as the fraction of the image points that were “touched” (within a
1003 margin of error) by at least one of the drawn points. A subset of the image points were weighted more
1004 heavily because they captured unique features of the shape (e.g., the corners and endpoints of an “L”
1005 shape). Hausdorff distance is a distance metric between the set of drawn points and the set of image
1006 points (see definition below).

1007 *Behavioral efficiency*. To incentivize efficiency, we included a factor comparing the cumulative
1008 distance traveled in the drawing (i.e., the amount of “ink”) to the cumulative distance of the edges of the
1009 figure in the image, with its value negatively proportional to the excess of drawn ink over image ink.

1010 *Task-specific factors.* During practice trials for the Character task (see “Task types”), we also
1011 included factors capturing the extent to which drawn strokes matched the shapes used in the image
1012 figure. This included two factors: one proportional to the similarity of the number of strokes and the
1013 number of image shapes, and the other proportional to the spatial alignment of the drawn strokes to the
1014 image shapes. Importantly, these factors were included only for practice images, not for novel test
1015 images.

1016 The final score aggregated the image similarity, behavioral efficiency, and task-specific factors,
1017 with more weight on image similarity factors. We first rescaled the factors linearly between 0 and 1 (where
1018 1 means good performance), with the dynamic range set by a lower and upper bound on the factor values.

These bounds were adaptively updated on every trial based on the distribution of factor values in the last 50 trials (lower bound at 1st percentile and upper bound at 53rd percentile), which ensured that the dynamic range of feedback matched the dynamic range of behavioral performance from recent history. We then weighted each of those factors to tune their relative contributions (using hand-tuned weights for each experiment; generally highest for image similarity), and computed the final scalar score (range 0 to 1) by taking the single factor that was worst after weighting.

$$s_{scal} = \min_i \left(1 - w_i(1 - f_i)\right)$$

where i indexes the factors, w_i are the weights (between 0 and 1) and f_1 are the factor values (rescaled between 0 and 1). We also gave each trial a categorical score:

$$s_{cat} = \begin{cases} \text{great} & s_{scal} > 0.82 \\ \text{good} & 0.65 < s_{scal} \leq 0.82 \\ \text{ok} & 0.15 < s_{scal} \leq 0.65 \\ \text{fail} & s_{scal} \leq 0.15 \end{cases}$$

The scalar and categorical scores were used to determine feedback across four different modalities. The meaning of screen color and sound were learned by the subject, whereas delay and reward had intrinsic value:

(1) Screen color. A linear mapping between two colors, such that a score of 0 was mapped to red (RBG: [1, 0.2, 0]) and a score of 1 was mapped to green ([0.2, 1, 0.2]).

(2) Sound cue. A sound determined by s_{cat}: if *great*, then three pulses (1300 Hz, 0.16 s on and off); if *good*, then a single pulse (1000 Hz, 0.4 s); if *ok*, then no sound; if *fail*, then a single pulse (120 Hz, 0.27 s).

(3) Delay until reward. A nonlinear mapping from score to delay before reward. We first applied a linear mapping to the scalar score, such that a score of 0 was mapped to a long delay (5 sec + random uniform jitter, [0, 2.5] s) and a score of 1 was mapped to 0 s delay. Further, if s_{cat} was *great*, *good*, or *ok*, then this delay was reduced by multiplying by 0.65.

(4) Reward. The open duration of the solenoid gating the juice line was defined as

$$\text{rew} = \text{C} \times m \times a \times s_{scal}$$

where C is a constant in dimensions of time (0.15 - 0.6 s, set manually depending on the difficulty of the task); m is a multiplier that gives a bonus for good performance and further penalizes bad performance, depending on the value of s_{cat}: *great* (1.3), *good* (1.0), *ok* (0.8), *fail* (0); and a is a random variable sampled from the uniform distribution $a \sim 0.75 + 0.5 \times U(0,1)$, and s_{scal} defined as above. On average, including failed trials, subjects received ~0.35 ml reward per trial. The order in which these four feedback signals were delivered is described above ("Trial structure").

Task types

Single-shape task. The Single-shape task presented one of the practiced simple shapes, or, in the "categories" experiments, sometimes a morphed shape. Subjects were allowed only a single stroke (triggering online abort if >1). In four Single-shape sessions for subject 1, the ending of the drawing epoch was triggered by completion of the stroke (i.e., on finger raise), and not on the subject pressing a "done" button, as was the case in all other sessions and experiments.

To test for motor invariance (**Figs. 2a-e, 4**), we presented images of practiced shapes, varying across trials in location, size, or both. For location variation, images spanned a distance in the x- and y-dimensions (measuring from shape centers) of 321 pixels (9.6 cm), which is 2.38 times the average size of shapes (135 pixels, 4.0 cm, maximum across width and height). For size variation, the maximum size was 2.5 times larger than the smallest (in diameter), except for two experiments for subject 1, in which the ratio was 2.0. The location and size variation in **Fig. 2b, c** is representative of all experiments.

This variation in size and location was chosen based on parameters used in prior studies of M1 and electromyographic activity (EMG) of muscles controlling the arm during reaching tasks in macaques. For reaches performed along the coronal plane around 25 cm from the subject, similar to the geometry of the touchscreen relative to subjects in our study, M1 and EMG activity are significantly affected by translating the starting and ending locations of reaches by 10 cm, similar to the location variation in our study (9.6 cm)[75], and by varying the scale of the reach by 2 fold (7 cm to 12 cm), similar to our study's variation in shape 2 to 2.5 fold[76].

To test for categorical structure (**Figs. 2f-j, 5**), we constructed morph sets (subject 1: n = 7 morph sets across 3 sessions; subject 2: n = 13 morph sets across 4 sessions), each consisting of two practiced shapes and four to five "morphed" versions of those shapes, which were constructed by linearly interpolating between the two shapes along one image parameter, such as the extent of closure of the top of the U (**Fig. 2f, g**). Across morph sets, we varied different image parameters (see examples in **Extended Data Fig. 3**).

Multi-shape task. Each image was composed of two to four shapes positioned at random, non-overlapping, locations spanning the space of the screen (four corners and center). Subjects were allowed to draw the shapes in any order, and to use any trajectory within the shapes, but were constrained to use one stroke per shape and to not trace in the gaps between shapes. In the Results of this manuscript, we present results averaged across two sessions, one from each subject (**Extended Data Fig. 4**). On each trial, an image was constructed by sampling a shape randomly without replacement. This led to n = 531 (S1) and n = 278 (S2) unique images.

Character task. Each image was generated by connecting two to six simple shapes into a single character figure. This was done by sampling characters from a generative model. A character with N shapes was defined by randomly sampling N shapes and N-1 relations, where each relation i defines the location of the attachment points on shapes i and $i+1$; these attachment points, in turn, define how these shapes will connect to each other. This is similar to a published generative model for handwritten characters[16]. Generated characters were only kept if there was minimal crossing of shapes over each other.

For experiments testing behavioral generalization to novel characters (**Fig. 2k-p**), we mixed practiced and novel characters (practiced characters: n = 189 (mean, range 22 to 491) per day; novel characters: 48 (mean, range 0 to 155) per day). For analyses, we label as "novel" only the very first trial for a given character. Because of random sampling in generating characters, it would in principle be possible that characters generated on different days are in fact identical; to avoid this, we ensured that all characters we called "novel" indeed did not match any images the subject previously encountered.

We did this by ensuring that each novel character was different from every previously encountered character across all days (quantified with the Hausdorff distance).

For neural experiments comparing Single-shape and Character tasks (**Fig. 6**), we analyzed the sessions for which we collected data from both the Single-shape and Character tasks [subject 1: n = 10 sessions, median N matching primitives between Single-shape and Character tasks = 9 (range 5 - 12); subject 2: n = 9 sessions, median N matching primitives = 10 (range 2 - 14)]. We switched between Single-shape and Character tasks using a block design (2 - 5 blocks each), except one session for subject 2, in which they were randomly interleaved across trials.

Behavioral data analysis

<u>Preprocessing of touchscreen data</u>

Touchscreen data were represented as time series of (x, y) coordinates in units of pixels (conversion: 33.6 pixels/cm) and sampled at 60 Hz, which we upsampled to 500 Hz (performed automatically in MonkeyLogic to align all behavioral signals, including trial event markers and eye tracking), and low-pass filtered below the highest frequency in the drawing movements (15 Hz). Strokes were segmented based on the time the finger first touched the screen (stroke onset) and the last time before raising off the screen (stroke offset) with 500 Hz resolution.

For some analyses—in **Fig. 1f** and as input to the "trajectory distance" below—we further computed stroke instantaneous velocity and speed in the following steps. Starting from extracted strokes, we low-pass filtered the data (12.5 Hz), and downsampled to 25 Hz. We then used the five-point stencil method to compute a finite difference approximation of the derivative:

$$f'[n] = \frac{f[n-2] - 8f[n-1] + 8f[n+1] - f[n+2]}{12\mathrm{h}}$$

where $f[n]$ is a discrete time series (i.e., the x- or y-coordinates) indexed by integer n, and h is the sampling period in seconds. This differentiation was performed separately for the x and y coordinates. The resulting velocity time series was upsampled to the original 500 Hz sampling rate with a cubic spline. Speed was computed as the norm of the (x, y) velocity at each timepoint.

Computing “trajectory distance”

To quantify the similarity between two strokes, in a way that compares their spatio-temporal trajectories, while ignoring their relative size (or scale) and location (on the screen), we devised a “trajectory distance” metric. This metric is a scalar dissimilarity score, based on the dynamic time warping distance between two strokes represented as velocity time series $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. To compute trajectory distance between two strokes, we (1) spatially rescaled each stroke (while maintaining its x-y aspect ratio), to make the diagonal of its bounding box unit length 1. (2) We then linearly interpolated each stroke to the same number of points (70) to allow direct point-by-point comparison between strokes. This was done spatially by interpolating based on the fraction of cumulative distance traveled (so that the distances between successive points were the same value over the entire stroke), in order to capture the spatio-temporal trajectory, as in a previous study modeling strokes in handwriting[16]. (3) Interpolated trajectories were then converted to velocity time series as above. (4) We then computed the dynamic time warping distance between these velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.:

$$D_{\mathrm{DTW}}(\mathbf{v}_1, \mathbf{v}_2) = \frac{\min_\pi \sum_{(i,j)\in\pi} d(i,j)}{\mathrm{N}}$$

where i and j index the two velocity trajectories, N is the number of points (70), π is a set of (i,j) pairs representing a contiguous path from (0, 0) to (N, N). The local distance metric $d(i,j)$ is the Euclidean distance plus a regularization factor that discourages excessive warping:

$$d(i,j) = \|\mathbf{v}_1[i] - \mathbf{v}_2[j]\| + \lambda \times |i-j|$$

$$\lambda = 0.045 \left\langle |\mathbf{v}_n[i]| \right\rangle_{i,n}$$

where $\langle\cdot\rangle$ is the average. For the regularization parameter, λ, the purpose of the summation term was to rescale lambda to match the magnitude of velocities. The resulting distance D_{DTW} was then rescaled to 0 and 1 to return the trajectory distance:

$$D_{traj}(\mathbf{v_1}, \mathbf{v_2}) = 1 - \frac{1}{D_{DTW}(\mathbf{v_1}, \mathbf{v_2}) + 1}$$

Computing “image distance”

To compare the similarity of two images—each a figure represented as a set of (x, y) points, with no temporal or stroke-related information—we used a modified version of the Hausdorff distance, a distance metric that has been commonly used in machine vision for comparing the similarity between two point sets based on their shape attributes[199]. There are, in principle, at least 24 variants of the Hausdorff distance based on possible variations in the formula[199]; we used a variant that is minimally susceptible to outlier points (because it is based on taking means instead of minima and maxima; variant #23 in the referenced study[199]). Image distance was computed as follows: (1) Each image was first centered so that its center of mass becomes (0,0). (2) Image distance was then computed. First, we define the distance between two points, $d(a, b)$, as the Euclidean distance. We also define the distance between a point and a set of points, $d(a, B)$, and the distance from set A to set B, $d(A, B)$, as:

$$d(a, B) = \min_{b \in B} d(a, b)$$

$$d(A, B) = \frac{1}{|A|} \sum_{a \in A} d(a, B)$$

Image distance was then defined as:

$$D_{image}(A, B) = \frac{d(A, B) + d(B, A)}{2}$$

<u>Computing “primitive alignment score”</u>

For experiments on categorical structure, we generated a set of images, with each set containing four to five novel images that morph between one primitive (P1) and another primitive (P2): a “morph set”. Across morph sets, different image parameters were morphed (see **Fig. 2g** and **Extended Data Fig. 3**). Each trial presents a single image from one morph set. We sought to quantify the relative similarity between a given trial’s data—either its behavioral, image, or neural data (see below)—and data for the two primitives, P1 and P2, in its morph set. To do so, we devised a “primitive alignment” score defined, for each individual trial, as:

$$a = \frac{d_1}{d_1 + d_2}$$

where d_1 is the average of the distances between that trial and each of the P1 trials, and d_2 the average of the distances between that trial and each of the P2 trials. A score closer to 0 implies similarity to P1, and a score close to 1 implies similarity to P2. The particular metric used for these distances depended on the analysis. For images, we used the image distance. For drawings, we used the trajectory distance. For neural activity, we used the Euclidean distance between population vectors. We confirmed that primitive alignment scores for image data varied linearly with morph number (Fig. 2j and Extended Data Fig. 3c), ensuring that any deviation from linearity in behavioral or neural data could not trivially be the consequence of how the score is defined.

<u>Classifying strokes from the Character task</u>

To assess whether subjects drew characters by reusing their own stroke primitives, we scored the fraction of character strokes that were high-quality matches to one of the subject's own primitives, and the fraction that were high-quality matches to the other subject's primitives. If the fraction of matches to a subject's own primitives was high, and to the other subject's primitives was low, then we interpreted this as evidence that subjects recombined their own primitives.

This was performed by assigning each stroke the label of its nearest primitive using the trajectory distance, and then further defining this as a high-quality match only if the trajectory distance was sufficiently low; in particular, if the distance was within the interval bounded by the 2.5th and 97.5th percentiles of the distribution of trajectory distances caused by trial-by-trial variation in behavior.

First, each stroke was assigned its best-matching primitive, p^* from a set of primitives (the choice of primitive set—same or different subject—depending on the analysis; see below):

$$p^* = \arg\min_p d(\mathbf{s}, \mu_p)$$

Where p indexes the primitives, s is the stroke trajectory, μ_p is the mean stroke for primitive p (averaged over trials from Single-shape tasks), and $d(\cdot,\cdot)$ is the trajectory distance.

Second, the quality of the stroke's assignment to its nearest primitive was scored:

$$quality = \begin{cases} \text{high} & \text{if } d(s, \mu_{p^*}) < \mathrm{D}_{\max,p^*} \\ \text{low} & \text{if } d(s, \mu_{p^*}) \geq \mathrm{D}_{\max,p^*} \end{cases}$$

Where $\mathrm{D}_{\max,p^*}$ is an upper-bound on trajectory distances that would be expected from trial-by-trial variation in primitive p^*. It is the 97.5th percentile of the distribution of trajectory distances from Single-shape trials, determined separately for each primitive, which we consider as a good—if anything, conservative—estimate of trial-by-trial variation, because Single-shape tasks present no ambiguity as to what primitive needs to be drawn.

These steps assigned each stroke a class tuple $(p^*, quality)$. In summary analyses, we pooled all cases of high-quality matches into a single "high-quality match" class (no matter the assigned primitive), and all low-quality matches into a single "no match" class (**Fig. 2o, p**). In summary analyses testing whether a given subject's character strokes aligned better with its own primitives vs. the other subject's primitives (**Fig. 2p**) we performed the above analysis separately for all four combinations of stroke data (2 subjects) x primitives (2 subjects), using only images performed by both subjects.

The control analysis testing primitive reuse using a simulated set of "remixed" primitives was performed in the following manner. We first generated remixed primitive sets by mixing subparts of different primitives. Given two actual primitives we extracted the first half of the first primitive (defined by distance traveled) and the second half of the second primitive and connected them by aligning the offset of the first half to the onset of the second half, smoothing the connection with a sigmoidal weighting function. To sample a remixed primitive set, we first generated a pool of all possible remixed primitives using every possible pair of actual primitives. We then sampled (without replacement) a set of remixed primitives from this pool, keeping only remixed primitives satisfying the following constraints: (i) no self-intersection, and (ii) no excessively sharp turns, detected as curvature at any point along the inner 80% of the stroke exceeding 0.8. Curvature was defined in a standard manner as the inverse of the radius of curvature, $(\dot{x}\ddot{y} - \dot{y}\ddot{x})/((\dot{x}^2 + \dot{y}^2)^{3/2})$, where $\dot{x}$ and $\dot{y}$ are the velocity components and $\ddot{x}$ and $\ddot{y}$ are the acceleration components. Once a candidate set of remixed primitives was sampled, it had to pass further constraints: (i) each actual primitive contributed its first half or second half to a maximum of two remixed primitives; (ii) the trajectory distance between no pair of remixed primitives was lower than the minimum trajectory distance between actual primitives; (iii) the trajectory distance between no pair of remixed primitive and actual primitive was less than the minimum trajectory distance between actual primitives.

These constraints ensured that remixed primitives in each set were different from each other and from the actual primitives (this is visually apparent in the primitive sets, **Extended Data Fig. 5**).

We then labeled strokes from character drawings using each remixed primitive set, using the same approach as for actual primitive sets, except the following. The analysis using actual primitives used trajectory distance thresholds determined empirically for each primitive based on drawings in Single-shape trials (see above). Because remixed primitives were not ever drawn, a similar approach could not be used. Instead, thresholds for remixed primitives were assigned from the pool of thresholds for the actual primitives. This was done in a manner meant to increase the match rate (and thus allow a stronger, or more conservative, test that the remixed primitives do not match the character strokes) by assigning the largest (i.e., most lenient) threshold to the "worst"-matching primitive, the second largest to the second worst, and so on. Here, "worse" is defined as having larger average trajectory distance to character strokes.

<u>Analysis of kinematic separability of primitives</u>

We tested whether each primitive was decodable from every other primitive based on single-trial kinematics (stroke trajectory) (**Extended Data Fig. 2**). Strokes were first converted from time series (x- and y-position) to 8-dimensional vectors as follows. Similar to the trajectory distance computation (above), strokes were normalized in time (linear interpolation to 50 time points) and space (rescaling each stroke, while maintaining its x-y aspect ratio, to make the diagonal of its bounding box unit length 1). The x- and y- coordinates were concatenated such that each stroke was represented by a 100-dimensional vector, and then reduced to 8 dimensions using PCA. Decoding was performed on these 8D representations using a linear support vector machine classifier (SVC) (LinearSVC, scikit-learn, regularization parameter C set to 0.1), with 10-fold cross-validation. By performing this procedure for every pair of primitives (each time returning a single decoding accuracy score) we populated the matrix in **Extended Data Fig. 2**.

Neural recordings

Recordings were acquired using a Tucker-Davis Technologies (TDT) system, including headstage (Z-Series 32 Channel Omnetics, LP32CH-Z), amplifier (PZ5M-256), processor (RZ2), and storage (RS4), sampled at 25 kHz (local reference mode), controlled with TDT Synapse software run on a Windows 10 PC (Intel Core i7-3770, 32GB RAM), and saved to disk. Analog and digital task-related signals, including behavioral events (photodiode, audio, and trial event markers) and eye tracking (ISCAN, 125 Hz), were synchronized external triggers recorded by the neural data acquisition system.

Neural data preprocessing

Spike sorting

We extracted for later analysis both isolated single-unit (SU) and multi-unit (MU) spike clusters from the stored broadband signal. MU clusters consisted of threshold crossings that were clearly spikes, but which were not isolatable into distinct SU clusters. We used a three-step approach for extracting and sorting spikes into these clusters, with a first pass using Kilosort[200] (v2.5) to extract putative spike clusters, a second pass using a custom-written program to label these clusters as either SU, MU, or noise, and a final manual curation step. Note that Kilosort classifies clusters, but we did not use those labels.

For Kilosort, we used the default parameters, except *AUCsplit* (0.90), *Th* ([6, 4]), and *lam* (10), which we optimized using parameter sweeps on data from representative sessions and by manual evaluation of results.

We next refined these cluster labels. For each cluster, we first removed outlier waveforms (any exceeding a 3 x interquartile-range threshold for any of the minima, maxima, or sum-of-squares). Waveforms were then shifted slightly in time (<1 ms) to improve their alignment by peaks (or toughs, for negative-going waveforms). For each cluster, we computed two features. *(1) Signal-to-noise ratio (SNR)*, defined as the ratio of the peak-to-trough difference (of the average spike waveform) divided by standard deviation (averaged across time bins). Before computing SNR, we checked whether the cluster contained both positive- and negative-going waveforms. If so, SNR was computed separately for these two subsets of data and then averaged. *(2) Inter-spike-interval violations (ISIV)*, defined as the fraction of inter-spike intervals less than a refractory period (1.5 ms). Based on these SNR and ISIV, we provisionally classified

clusters as SU [if either (SNR > 9.6 and ISIV < 0.05) or (SNR > 6.9 and ISIV < 0.01)], noise (SNR < 3.9), or MU (the remaining clusters).

We then manually curated these clusters. We visualized every cluster to either confirm its label (MU, SU, noise) or to manually re-assign it to a different label (including “artifact”), using a custom-written MATLAB GUI. We also manually checked whether multiple SU clusters on a single channel should be merged into a single SU cluster, if they have high waveform similarity, inversely correlated spike count frequency over the course of the session, or a negative peak close to zero lag in a cross-correlogram of spike times. Finally, for each channel, all MU clusters were merged into a single MU cluster. Combining SU and MU, this yielded the following total number of units per area (mean +/- S.D. across sessions). For subject 1: M1 (59.9 +/- 12.5), PMd (44.1 +/- 6.2), PMv (34.2 +/- 7.3), SMA (63.0 +/- 7.9), preSMA (75.4 +/- 17.7), dlPFC (47.8 +/- 17.2), vlPFC (43.3 +/- 9.9), FP (19.2 +/- 3.8). For subject 2: M1 (40.7 +/- 13.1), PMd (54.7 +/- 5.5), PMv (71.1 +/- 6.6), SMA (53.4 +/- 7.2), preSMA (57.9 +/- 11.1), dlPFC (24.6 +/- 4.8), vlPFC (38.6 +/- 13.7), FP (42.6 +/- 5.0).

<u>Converting spike times to firing rates</u>

Single-trial spike trains were converted to firing rate functions by smoothing with a 0.025 s Gaussian kernel (0.01 s slide). We removed units with very low firing rates (if the 80th percentile of their firing rates across all trials and time bins was less than 1 Hz). We also removed units whose firing rates were unstable, by first removing units with high systematic drift in firing rate over the session ($m/u > 0.2$, where m is the slope from regressing square-root-transformed firing rate vs. time (in hours), and u is the mean firing rate), and then removing units with any large fluctuations in firing rate across the session. For the latter, we excluded units satisfying $(s_{max} - s_{min})/s_{mean} > 1.15$ or $(u_{max} - u_{min})/u_{mean} > 0.65$, where the session is first split into contiguous 50-trial bins, the across-trial standard deviation in square-root-transformed firing rate is computed within each bin, and then s_{max}, s_{min}, and s_{mean} are defined as the maximum, minimum, and mean standard deviation across bins. u_{max}, u_{min}, and u_{mean} are defined similarly, except using within-bin mean firing rate instead of within-bin standard deviation. We then processed firing rates as follows. We square-root transformed activity to normalize its variance. Following

a common approach in analyses of population firing rates[180], we "soft" z-scored each unit's activity to ensure that neurons with very different firing rates contributed similarity to population analyses, but with higher-firing-rate neurons still contributing relatively more:

$$x_i^{\mathrm{norm}}(t) = \frac{x_i(t) - \mu_x}{\sigma_x + C}$$

Where $x_i(t)$ is firing rate for trial i at time bin t, μ_x and σ_x are the mean and standard deviation, respectively (across trials and time bins), and C is an additive factor to ensure "softness": $C = \min(\mathbf{m}) + 3\mathrm{Hz}$ where $\mathbf{m}$ is a vector of mean firing rates, one for each unit. All subsequent analyses used this normalized firing rate representation of the data.

Time-warping neural activity to a common trial template

For the figure showing the average firing rates over the entire trial (**Fig. 3**), we first time-warped each trial to a common trial template. We defined a set of events that occur across trials as "anchors" (fixation touch, image onset, go cue, finger raise off fixation, stroke onset, stroke offset, touch done button, reward). We included only single-stroke trials. We first generated a "median trial". For each segment (i.e., time window between a pair of successive anchor events), we found its median duration, and then concatenated these median segments to construct a median trial. We then aligned each trial to this median trial at the anchor events, warping time linearly within each segment. To avoid sharp discontinuities at anchor points, we smoothed the final firing rates at the times of the anchor points (2.5 ms Gaussian kernel). This warping did not change the firing rate values, just their timing.

Neural data analyses

Dimensionality reduction of population activity

Principal components analysis. We performed dimensionality reduction on the neural population activity, in general because high-dimensional noise can reduce the interpretability of the Euclidean distance[162], and in one case in order to identify a potential linear projection of population activity (i.e., a subspace) that preferentially encodes primitives, a standard approach[47,178]. We represent a single area's

data from a single session and within a specific within-trial time window as a matrix $\mathbf{X}$, of size N x KT, where N, K, and T are the number of units, trials, and time bins, constructed by concatenating time bins from all trials along the second dimension. Data were first binned in time (0.15 s window, 0.02 s slide) before constructing this data matrix. We used principal components analysis (PCA), but instead of applying PCA on single-trial data $\mathbf{X}$, we applied PCA on trial-averaged data $\mathbf{X}_C$, in order to minimize the influence of trial-by-trial variation (noise). $\mathbf{X}_C$ holds the mean activity for each trial condition, of size N_C x KT, where N_C is the number of unique conditions, and where the specific conditions depended on the experiment (see below). We performed PCA on $\mathbf{X}_C$, and retained the top eight principal components (PCs). The specific trial-averaged conditions used for identifying PCs were the following. For analysis of motor invariance (**Fig. 4**), the conditions were each unique primitive (averaging over location/size), which resulted in identifying PCs that preferentially encoded primitives if they exist in the dataset. For analysis of categorical structure (**Fig. 5**), PCA was performed separately for each morph set, and the conditions were the unique images (i.e., the two endpoint shapes plus the morphed shapes in between). For the analysis of primitive representational reuse in characters (**Fig. 6**), the conditions were each combination of primitive and task type (i.e., resulting in *num_primitives* x *2* conditions).

We performed PCA in a cross-validated manner, to ensure that it was not overfitting to noise. We partitioned trials into two subsets (in a stratified manner), one “training” set that was used only for identifying the PCs, and a “test” set that was projected onto these PCs and then used for all subsequent analyses. We performed 8 randomized train-test splits (including all downstream analyses) and averaged their results.

Representing time-varying population activity as a vector. For some analyses, we captured single-trial time-varying population activity during the planning epoch (0.05 to 0.6 sec after image onset) as a vector, which could then be visualized after dimensionality reduction (**Extended Data Fig. 7**) or used in decoding analyses (**Extended Data Fig. 12**). Starting from a region’s population data, represented as a K x N x T matrix, where K, N, and T are the numbers of trials, units, and time bins (with time binned using a 0.2 s window, 0.1 s slide), we concatenated the N channels’ length T vectors end-to-end to construct the data matrix $\mathbf{D}$, of size K x NT, where each trial is represented by a vector of length N x T.

Nonlinear dimensionality reduction. To visualize population activity in two dimensions (**Extended Data Fig. 7**), we used the nonlinear dimensionality reduction method Uniform Manifold Approximation and Projection (UMAP), performed on the D, using values of 40 and 0.1 for the parameters *n_neighbors* and *min_dist*.

Computing "neural distance"

To quantify the similarity of population activity between two sets of trials, such as trials for conditions A and B (where A and B are specific values of task-relevant variables), we devised a "neural distance" metric. This metric has the useful property of being unbiased (so that the expected value of the neural distance between two sets of trials sampled from the same distribution is zero). Inspired by the "normalized distance" in Liu et al[201], it is the average pairwise Euclidean distance across conditions A and B, minus the within-condition distances. This subtraction step ensures that the distance is unbiased (unlike the mean Euclidean distance, which is biased upwards[45]). In addition, the resulting distance is normalized by dividing by an upper-bound distance to normalize it between 0 and 1. Neural distance is defined as:

$$D^*_{AB} = D_{AB} - D_{AA} - D_{BB}$$

where the normalized Euclidean distance between sets of trial indices in conditions A and B is:

$$D_{AB} = \left\langle \frac{1}{d_{\max}(t)} \left\langle \left\| \boldsymbol{x}_i(t) - \boldsymbol{x}_j(t) \right\| \right\rangle_{i \in A, j \in B} \right\rangle_{t \in \{\mathrm{t}_1, \ldots, \mathrm{t}_\mathrm{n}\}}$$

Here, $\mathrm{x}_i(t)$ is the population activity vector at time t (in a window between times t_1 and t_n), and $d_{\max}$ is a normalization factor that is an upper-bound (98th percentile) of the distances between all pairs of different trials combined across all conditions.

Computing a variable's encoding strength

To compute how strongly a given variable is encoded in population activity (e.g., "primitive encoding" in **Fig. 4j**), we computed the mean effect of that variable on population activity, in terms of neural distance, while controlling for the other relevant variables. Consider an experiment varying two variables, primitive

and location, such that a condition is represented by the tuple (p, l), where p and l index the primitives and locations. Primitive encoding is the average neural distance across all pairs of conditions that have different primitives but same locations:

$$\text{primitive encoding} = \left\langle D^*_{(p,l),(p',l')} \right\rangle_{p \neq p', l = l'}$$

Location encoding is defined analogously:

$$\text{location encoding} = \left\langle D^*_{(p,l),(p',l')} \right\rangle_{p = p', l \neq l'}$$

This approach generalizes to any pair of variables, such as primitive and task type in **Fig. 6**.

<u>Statistically comparing brain regions in strength of variable encoding</u>

In analyses that compare the encoding strengths of a particular pair of variables (e.g. primitive vs. location in **Fig. 4j**), we performed the following statistical tests to compare each brain region with every other brain region, in terms of how strongly they encode these two variables. We used the following procedure. (1) For each variable and pair of brain regions, we performed a statistical test comparing how strongly these two regions encoded that variable. This involved first extracting a dataset of neural distances between each pair of trial conditions for each of the two brain regions. For example, if the variable was "primitive", then each of the two brain regions would contribute a dataset consisting of neural distance scores between all pairs of trial conditions that have different primitives but the same location. The datasets for these two regions would be combined into a single dataset, to which we fit a linear least-squares regression model to test for an effect of brain region on neural distance y, controlling for trial-condition pair:

$$y = \beta_0 + \beta_r X_r + \sum_{j=1}^{N_c} \gamma_j Z_j + \epsilon$$

where X_r is 0 or 1 depending on brain region, and Z_j is an indicator variable for trial-pair condition, with γ_j as their coefficients, and ϵ is a noise term. Finally, we extracted the p-value for β_r (two-sided t-test), which represents the significance of the difference between this region pair in how strongly they encode the variable being tested. (2) This procedure was performed once for each combination of variable (2) and brain region pair (28), resulting in 56 p-values. We corrected the p-values for multiple comparisons

using the Bonferroni method (28 brain region pairs x 2 variables = 56 comparisons). (3) Using these 56 values, we then summarized each region with two numbers representing the number of regions for which this region more strongly encodes these two variables. For example, for primitive x location experiments, each region was scored with a tuple (N_{prim}, N_{loc}), where N_{prim} is the number of other regions that this region beats in the pairwise statistical tests of primitive encoding (and analogous for N_{loc}, except that it scores location encoding). In summary plots (**Figs. 4j, 6f**), the results of this entire procedure were represented by mapping each region's resulting tuple to a color based on a 2D color map.

Specific analyses

Analysis of motor invariance in neural activity. Dimensionality reduction was performed as described above, using a time window of 0.05 s to 0.6 s after image onset (to avoid including data after the go cue, which, for a subset of these motor-invariance experiments, occurred 0.6 to 1.0 s after image onset) for fitting the PCs and for analyses that involve time-averaging (**Fig. 4i-k**).

To test cross-condition decoder generalization (**Fig. 4k**), we used a linear support vector machine classifier (SVC), using a one-vs-the-rest scheme for multi-class classification (LinearSVC, scikit-learn, regularization parameter C set to 0.1). We report test accuracy linearly rescaled so that chance level (inverse of the number of classes) and 1 were mapped to 0 and 1. Because decoders were trained and tested on different conditions (with non-overlapping sets of trials), there was no concern of overfitting. Decoding was performed separately for, and averaged across, time bins (0.05 to 0.6 s relative to image onset).

Analysis of categorical structure in neural activity. Dimensionality reduction was performed as described above, using a time window from 0.05 s to 0.9 s after image onset for fitting the PCs. For analyses involving time-averaging (**Fig. 5d-g**), we used a window late in the planning epoch (0.6 to 1.0 s), when separation for A1 and A2 trials was the greatest (**Fig. 5h**). Primitive bias index was computed as above, using the Euclidean distance.

To assess whether primitives were associated with distinct activity patterns, we asked whether each primitive's single-trial activity was separable from every other primitive's activity, using a decoding

1437 approach (**Extended Data Fig. 12**). We used given region's population activity during the planning epoch
1438 (0.05 to 0.6 sec after image onset) represented as a dataset D (size K x NT, where each trial is
1439 represented by a vector of length N x T; see section "Representing time-varying population activity as a
1440 vector"), with its dimensionality further reduced to K x 50 by performing PCA and keeping the top 50
1441 principal components. For each pair of primitives decoding was performed using a linear support vector
1442 machine classifier (SVC) (LinearSVC, scikit-learn, regularization parameter C set to 0.1). Decoding was
1443 performed in a cross-condition manner to assess generalizable encoding of primitives; on each iteration
1444 we trained a decoder using data for drawings at one screen location (or size, for sessions with size
1445 variation), and tested using held-out data from all other locations (or sizes), and then averaging over
1446 analyses for different locations (or sizes). Because decoding performance can be biased upwards for
1447 regions with more units, to fairly compare the regions that had the strongest primitive encoding (preSMA,
1448 SMA, PMv, PMd, and vlPFC), we first matched their numbers of units by randomly subsampling (without
1449 replacement) to match the area with the fewest units and averaged the results from repeating this 10
1450 times. By performing this overall procedure for every pair of primitives and separately for every brain
1451 region, each time returning a single decoding accuracy score, we populated the matrices in **Extended**
1452 **Data Fig. 12**. We combined results across five sessions for each subject (S1: 2 varying in location, 3
1453 varying in size; S2: 3 location, 2 size).

1454 *Analysis for recombination of primitive representations in the Character task*. For each session,
1455 we presented both Single-shape and Character tasks using a block-interleaved design (except one
1456 session using a trial-interleaved design). We analyzed primitives that were performed in both Single-
1457 shape (instructed by the shape image) and Character trials (the subject's choice). For one experiment,
1458 the subject performed Multi-shape but not Single-shape tasks—we therefore used the first stroke from
1459 Multi-shape trials instead of Single-shape trials. For Character trials, we used only strokes that were
1460 scored as high-quality primitive matches. Dimensionality reduction was performed as above, using a time
1461 window of -0.8 to 0.3 s relative to stroke onset for fitting the PCs. For analyses involving time-averaging,
1462 we used a window -0.5 to -0.05 s relative to stroke onset. Neural distance, primitive encoding, and task-

type encoding were computed as above, using primitive and task type (instead of location or size) as the two relevant variables.

For analyses comparing strokes from the Single-shape task to the non-first stroke of the Character and Multi-shape tasks (i.e., the second stroke and onwards) (**Extended Data Fig. 14**), all parameters were the same as in the main analysis comparing Single-shape strokes to the first stroke of Character drawings, except two differences. First, we used a shorter analysis time window of -0.35 to -0.05 s relative to stroke onset (instead of -0.5 to -0.05 s). This was to account for the duration of the gap between offset of one stroke and onset of the next (mean of ~0.3 sec) in Character drawings. Second, we preprocessed the neural data to account for the presence of a strong effect of the reaching movement to initiate the trial (i.e., reaching from the "Hold button" towards the image; see **Fig. 1c**), which was present for Single-shape trials, but not for the non-first strokes of Character and Multi-shape trials. This effect is clear in population trajectories (**Extended Data Fig. 14a**). To accurately assess whether primitive-encoding activity was reused across task types, it was important to correct for this effect of initial reach, because it was present for Single-shape trials, but not for the non-first strokes of Character and Multi-shape tasks. We performed this correction in the simple manner, by subtracting, for all strokes, the across-stroke mean effect of the initial reach. This correction was performed separately for each time bin as follows. For each unit, we first estimated the mean effect of initial reach by using linear least-squares regression. Activity for each stroke was modeled as:

$$y = \beta_0 + \beta_f X_f + \beta_t X_t + \sum_{j=1}^{N_p} \gamma_j Z_j + \epsilon$$

Here, is y is firing rate, X_f is 0 or 1 depending on whether the stroke is the first stroke (always 1 for Single-shape data), X_t is 0 or 1 depending on task type (Single-shape or Character), Z_j is an indicator variable for the primitive class, and ϵ is a noise term. To correct for the effect of initial reach, we subtracted β_f (the mean effect of initial reach) from the neural activity for all first strokes. We did not perform this correction for comparisons of Single-shape strokes to the first stroke in Character drawings (**Fig. 6**), because, in those cases, all strokes included the initial reaching movement.

Analysis of primitive encoding aligned to visual fixations. We assessed the extent to which PMv activity, aligned to visual fixation events during the Multi-shape task, preferentially encodes either the primitive associated with the visually-fixated shape or the first primitive the subject is planning to draw (**Extended Data Fig. 15**). We first converted time-varying eye tracking data (x- and y-position time series) to a sequence of fixation and saccade events. This was done using the Cluster Fix algorithm[202], which, briefly, uses k-means clustering on distance, velocity, acceleration, and angular velocity, and then assigns clusters to fixation and saccade events. Each fixation event was assigned a "visually-fixated" primitive based on the closest shape. If all shapes were further than 70 pixels (2.08 cm), then this fixation event was considered to be looking away from all shapes and was thus excluded from further analysis. Each fixation event was assigned a "planned primitive" based on which primitive the subject would go on to draw first on that trial. Note that choices were made freely, without experimenter-imposed rules.

We assessed the extent to which fixation-aligned neural activity encoded each primitive using a decoding approach, where the strength of representation of a given primitive was defined as the probability score returned by a decoder trained to classify that primitive vs. all other primitives. Decoders were trained using Single-shape trials from the same session. Data from the planning epoch of Single-shape trials were "cut" into multiple short-duration "snippets", each of which was a datapoint used for training the decoder. Specifically, neural data was represented as a matrix $\mathbf{X}$, of size N x KT, where N, K, and T are the number of units, trials, and time bins, constructed by concatenating time bins from all trials along the second dimension (data were first binned in time using a 0.3 s window with 0.1 s slide) before constructing this data matrix. These KT N-dimensional training data points, each with an associated "primitive" label, were used to train a multilabel, logistic regression, one-vs-rest classifier (*scikit-learn*), which pools multiple independent classifiers, one per primitive. This resulted in a multilabel decoder that could be applied to neural data at any time point and return one probability score for each primitive, where the probability can be interpreted as the strength of that primitive's representation.

Analysis of encoding of kinematic variables in neural activity. To test for encoding of generic kinematics in neural activity (**Extended Data Fig. 16**), we used a standard encoding model approach, based on a recent study of encoding of handwriting kinematics in human motor cortex[93]. We assessed

the fraction of variance in neural activity that is explained by a linear mapping from moment-by-moment finger kinematics to neural activity. As in that human study, we used neural activity in the top 10 dimensions found after performing PCA on the neural activity, which helped to reduce the effect of noise present in higher dimensions. PCA was performed as we described above, except, instead of identifying the PCs using trial-averaged data, we did so using single-trial data, in order to retain activity potentially related to trial-by-trial variation in kinematics. Activity at each time point was modeled as:

$$\mathbf{f}_t = \mathbf{E}\mathbf{v}_t + \mathbf{b}$$

Here, $\mathbf{f}_t$ is the neural activity at time bin t in the top 10 neural PCs, $\mathbf{E}$ is a 10 x 2 matrix mapping kinematics to neural activity (i.e., preferred directions), $\mathbf{v}_t$ is the 2 x 1 finger velocity and $\mathbf{b}$ is an intercept term. The fraction of variance accounted for (FVAF) was computed as follows:

$$FVAF = 1 - \frac{SS_{\text{err}}}{SS_{\text{tot}}} = 1 - \frac{\sum_{t=1}^{T}(\mathbf{E}\mathbf{v}_t - \mathbf{f}_t)^T(\mathbf{E}\mathbf{v}_t - \mathbf{f}_t)}{\sum_{t=1}^{T}\mathbf{f}_t^T\mathbf{f}_t}$$

Here,SS_{tot} is the total variance, and SS_{err} is the sum of squared errors, and T is the total number of time steps across all trials.

To test how strongly neural activity encoded kinematics in a manner that generalized across primitives, the model was trained on one subset of primitives (all except one) and tested on one held-out primitive. This was performed once for each primitive, with the final FVAF computed as the mean FVAF across all train-test splits. We used sessions performing the Single-shape task with location and size variation (S1, n = 30 primitives across five sessions; S2, n = 42 primitives across three sessions, which excludes two sessions with fewer than six primitives). We included data throughout each stroke's duration.

To allow for the possibility that kinematics better relates to neural activity at a non-zero lag, we performed this analysis multiple times with different time lags between neural and behavioral data, varying from -0.3 to 0.3 s (0.05 s increments). The final scalar summary averaged the results in a time window from -0.15 s to -0.05 s (with neural leading behavior), consistent with a peak time lag of around -0.1 s (neural leading) found in prior studies[113] and in our analysis (**Extended Data Fig. 16**).

Note that the FVAF values we found in M1 are comparable to the average value of 0.3 in the prior study of motor cortex in human handwriting[93]. Our finding of lower values (ranging from around 0.0 to 0.2, **Extended Data Fig. 16b**) is consistent with our use of trial-level, instead of trial-averaged data—which introduces more variability—and our approach of testing generalization across primitives (if we use a more traditional method of holding out trials, and not primitives, we find FVAF values are higher, with values ranging from around 0.1 to 0.3).

Data and code availability

The data and code used in this study will be made available on Zenodo upon publication.

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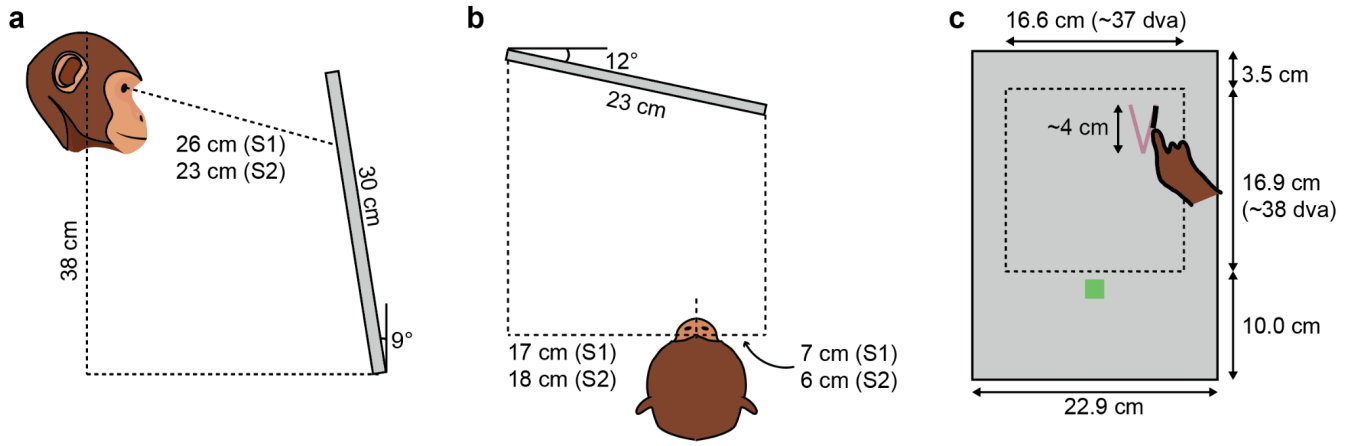
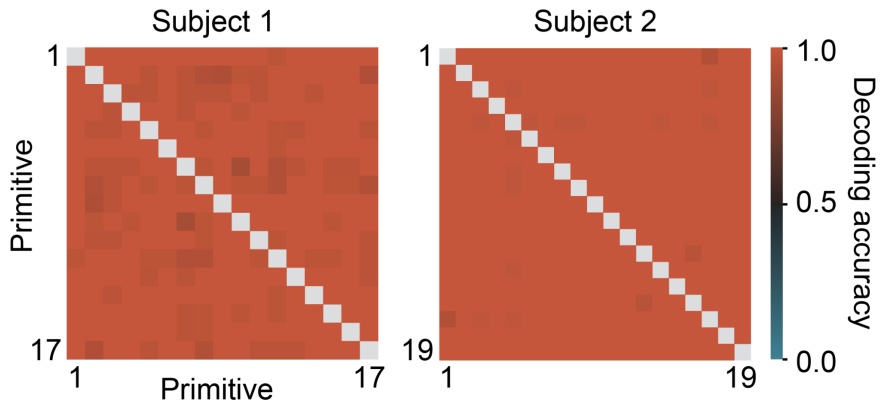


Extended Data Fig. 1. Behavioral task setup.

(a) Schematic of subject relative to screen, profile view. Subjects 1 and 2 were positioned at different distances to accommodate their individual anatomies and postures. The screen was slanted slightly to optimize the ability to see and reach to the same part of the screen (the workspace at the top of the screen).

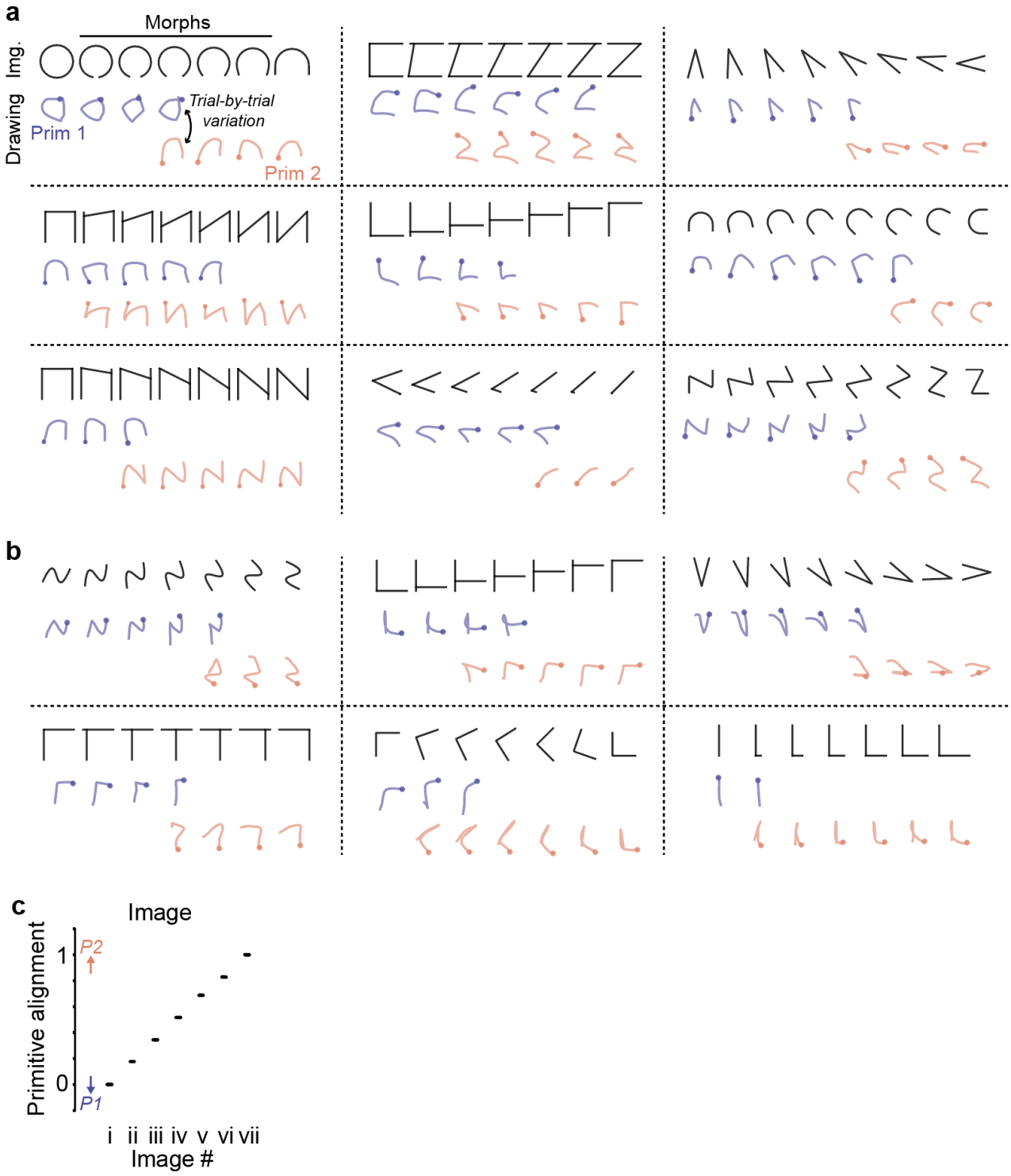
(b) Schematic of subject position relative to screen, top view. Subjects were positioned to the right, to accommodate reaching to the screen with the hand used for drawing (left).

(c) Schematic of screen during trial, with component locations and sizes to scale. The finger is tracing over the figure (purple-gray “V”), leaving a black trail of “ink” behind. The average size of shapes was 4.0 cm (maximum of width and height). The thickness and color of the figure and “ink” are to scale. The “done” button is visible (green square), and the subject can press it at any time to report completion. The dashed line indicates the workspace (not visible to the subject).



Extended Data Fig. 2. Primitives are distinct from each other in their single-trial kinematics.

Pairwise decoding accuracy between all pairs of primitives, using stroke single-trial kinematics. Strokes were represented as 8D vectors derived from their x- and y-position time series (see Methods). Cross-validated decoding was performed separately for each pair of primitives, and the results for all pairs are combined in this summary matrix. Chance decoding is 0.5, because each analysis had two primitive classes. Decoding accuracy was high for all primitive pairs (minimum accuracy was 0.90 for S1 and 0.94 for S2).

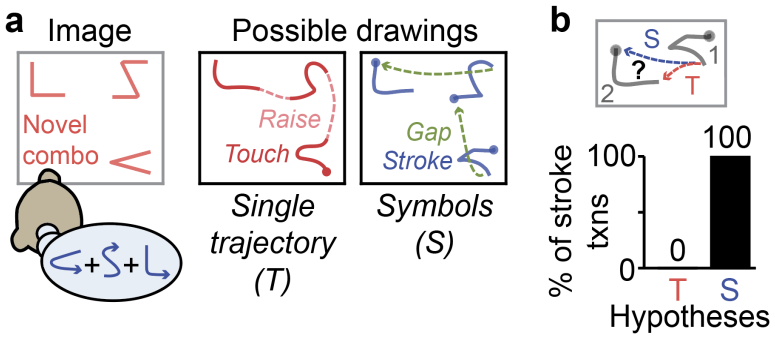


Extended Data Fig. 3. Categorical structure in behavior: more examples and quantification of primitive alignment for image data.

(a) Example single-trial drawings across nine different morph sets, for subject 2. Drawings are colored by whether they reflect use of primitive 1 (blue) or primitive 2 (orange). Images morph between two well-practiced shapes. The examples here are depicted in a similar manner to the example in **Fig. 2.**

(b) Same as panel a, but for subject 1.

(c) Primitive alignment vs. trial condition for the example experiment in panel **Fig. 2b,** performed on image data (using the image distance), and not on behavioral data as in **Fig. 2i**.

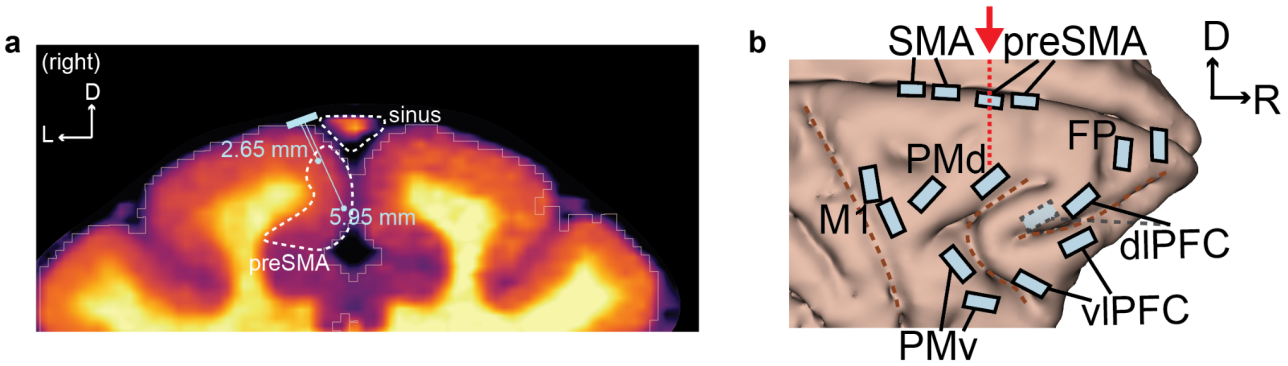


Extended Data Fig. 4. Recombination of stroke primitives into sequences, in the Multi-shape task.

(a) Experiment testing for recombination of primitives into sequences, using the Multi-shape task. Given images composed of multiple disconnected shapes, two possible drawing responses are shown, consistent with either the “Single trajectory” (T) or “Symbols” (S) hypotheses (see main text).

(b) Fraction of stroke-to-stroke transitions in which the second stroke is drawn in a manner consistent with the Single trajectory (T) or Symbols (S) strategies, restricted to transitions where the behavioral predictions of these two strategies differed [bottom bar plot, n = 391 transitions (S1), 529 (S2)]. Top, schematic of an example transition between two strokes labeled 1 and 2. Stroke 2 can be drawn either starting from the top-left, consistent with primitive reuse (“Symbols”), or from the bottom right, which would reflect the subject taking the shorter of the two gap distances (“Single trajectory”).

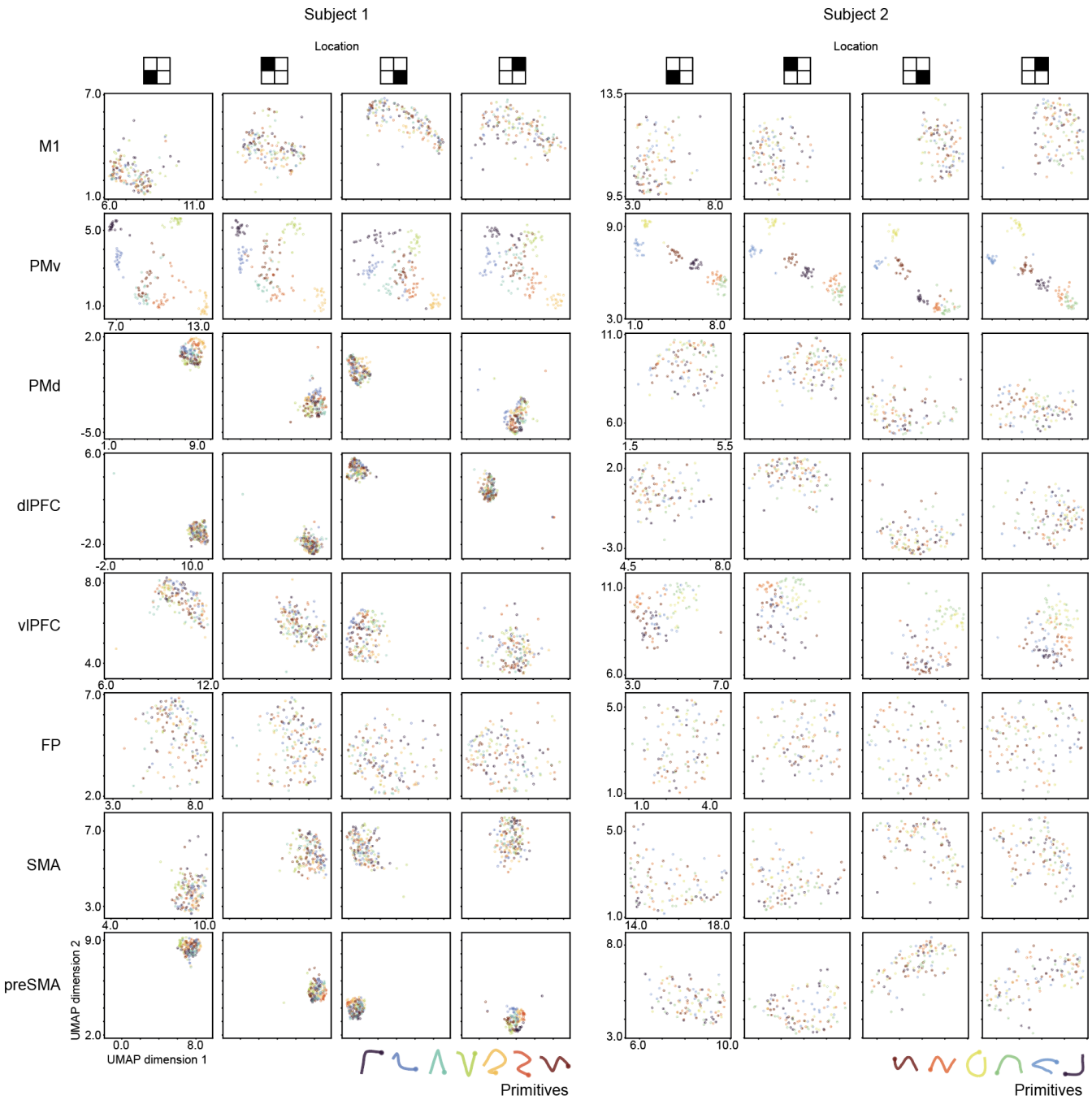
Extended Data Fig. 5. Simulated primitives created by "remixing" each subject's actual primitives. Three sets of shuffled primitives for each subject are shown. Each shuffled primitive was constructed by connecting the first half of one primitive to the second half of another primitive drawn by the same subject. Each set was optimized so that shuffled primitives within a set were sufficiently different from each other and were also sufficiently different from all actual primitives. See Methods for details. These primitives were used in **Fig. 2p**.



Extended Data Fig. 6. Angled implantation of SMA and preSMA arrays to avoid the sinus and target the medial wall.

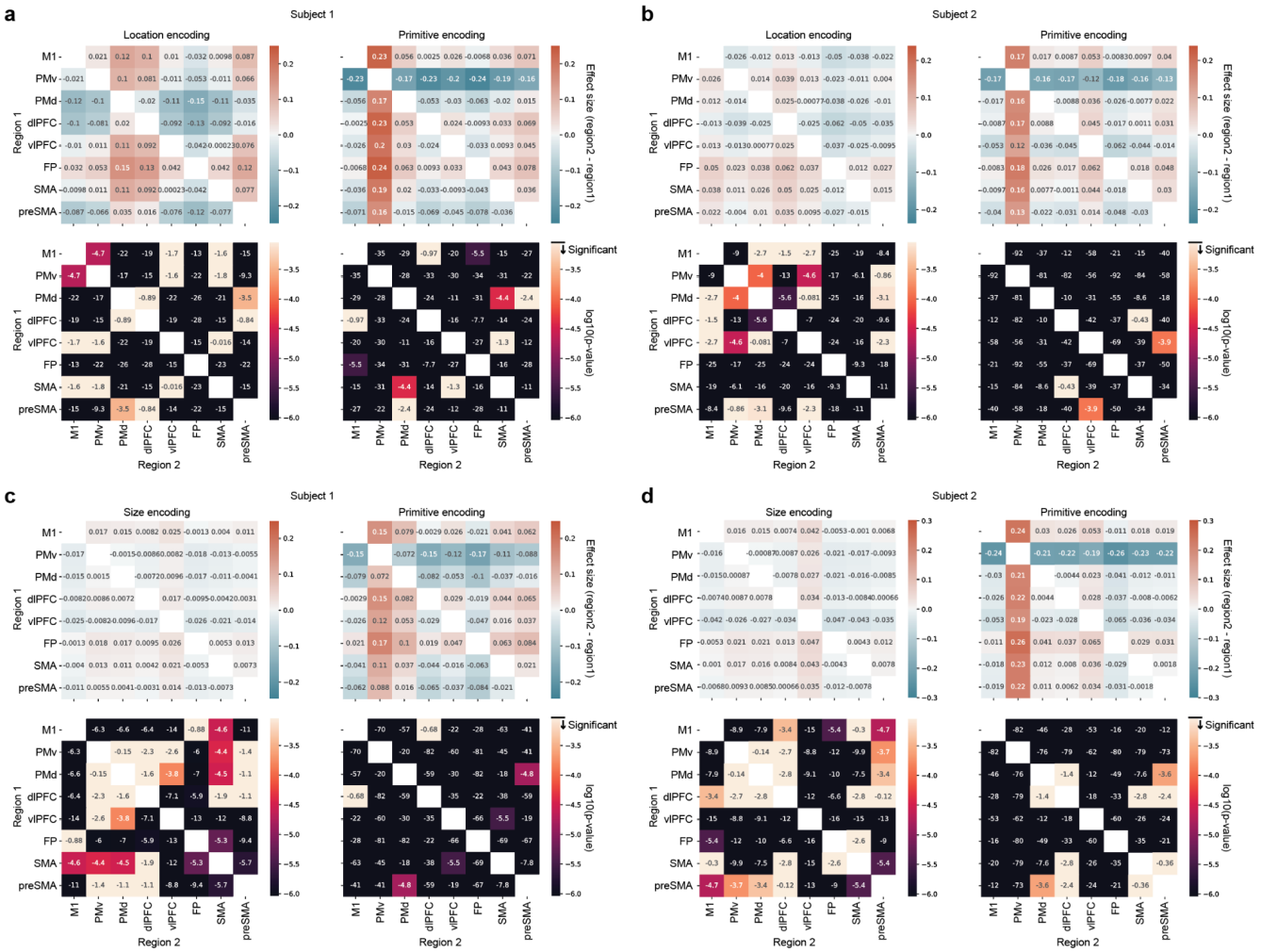
(a) Coronal MRI section showing the surgical plan for location and angle of a preSMA array relative to the superior sagittal sinus ("sinus"). SMA and preSMA arrays were implanted ~2 mm lateral to the midline, and angled medially, to target the medial wall (preSMA), while avoiding the superior sagittal sinus. The array is depicted as a rectangular blue surface and two lines representing the shortest (2.65 mm) and longest (5.95 mm) electrodes. This location and angle is our best estimate based on the stereotactic coordinates and intra-operative photographs. The other two SMA and one preSMA arrays were located and angled similarly. D, dorsal. L, lateral.

(b) Rostral-caudal location of the coronal section in panel a (red arrow and dashed line), overlaid on a brain surface model. D, dorsal. R, rostral.



Extended Data Fig. 7. Separation of single-trial activity by primitive, invariant to location.

Two-dimensional embedding of single-trial population activity using Uniform Manifold Approximation and Projection (UMAP). Each point represents a single trial, colored by primitive and split by brain region (rows) and screen location (columns). UMAP was performed on a vector representation of single-trial activity, constructed by concatenating all units' time-varying activities into a single vector (see Methods). Activity was taken from the planning epoch (0.05 to 0.6 sec after image onset), from a single session of the Single-shape task, in which primitives varied in location. It is visually apparent that PMv activity clusters by primitive and that this clustering is largely unaffected by location. In contrast, activity in other regions is either unstructured (FP), varies strongly by location with minimal effect of primitive (M1, PMd, dlPFC, SMA, preSMA), or varies somewhat by primitive but in a manner that depends strongly on location (vlPFC).



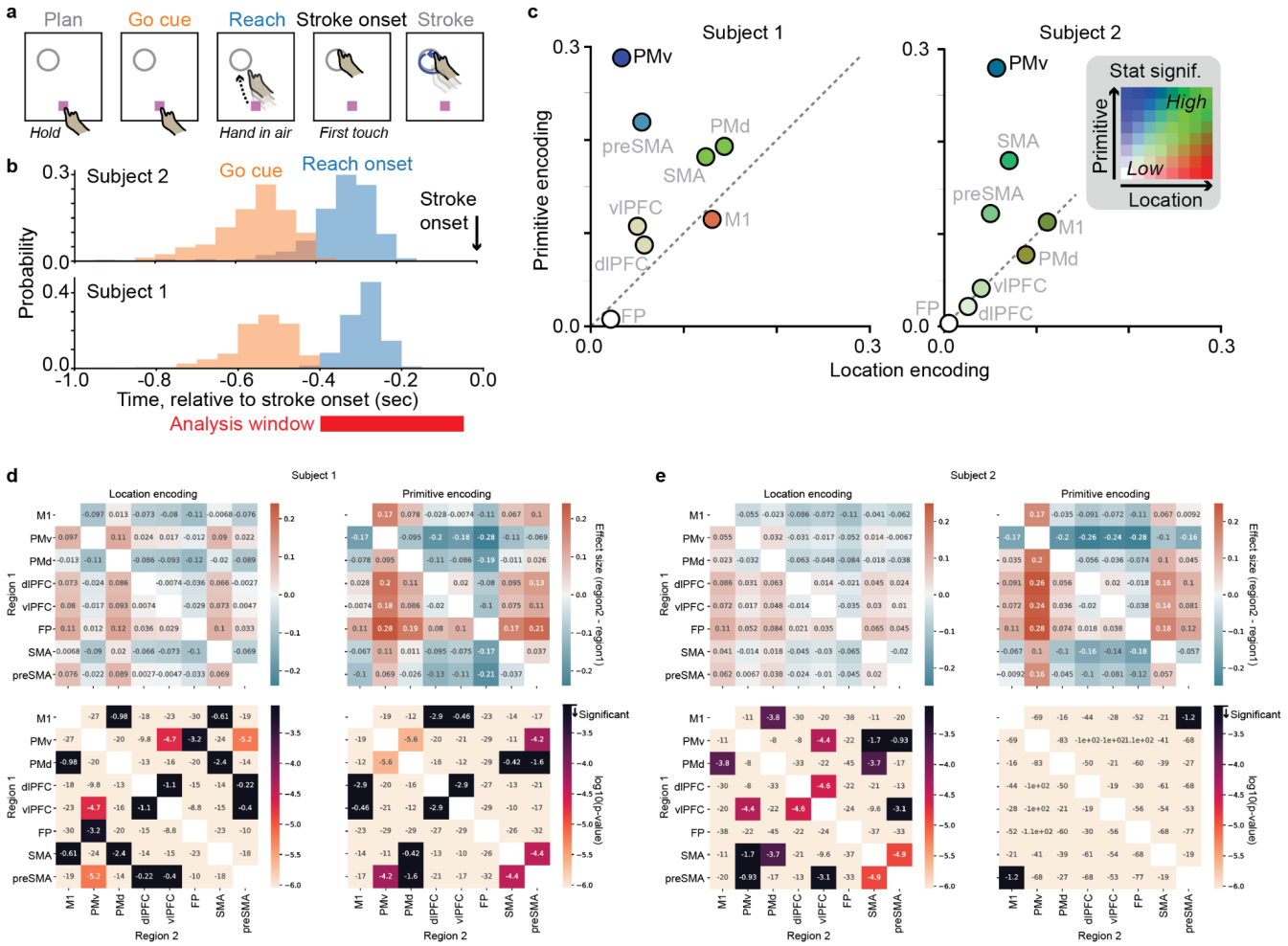
Extended Data Fig. 8. Detailed statistics for the analysis of location invariance in Fig. 4 and size invariance in Extended Data Fig. 10.

(a) Effect size (top) and p-value (bottom) for comparisons between each pair of brain regions, for location encoding (left) and primitive encoding (right), for subject 1. Corresponds to Fig. 4j. “Significant” for the p-value heatmap means that $p < 0.05$ after Bonferroni correction for 56 comparisons (28 brain region pairs and 2 variables, location and primitive).

(b) Same as panel a, but for subject 2. Corresponds to Fig. 4j

(c) Same as panel a, but for size, instead of location, encoding. Corresponds to Extended Data Fig. 10b.

(d) Same as panel c, but for subject 2. Corresponds to Extended Data Fig. 10b.



Extended Data Fig. 9. Invariant encoding of primitives in PMv during the initial reaching movement.

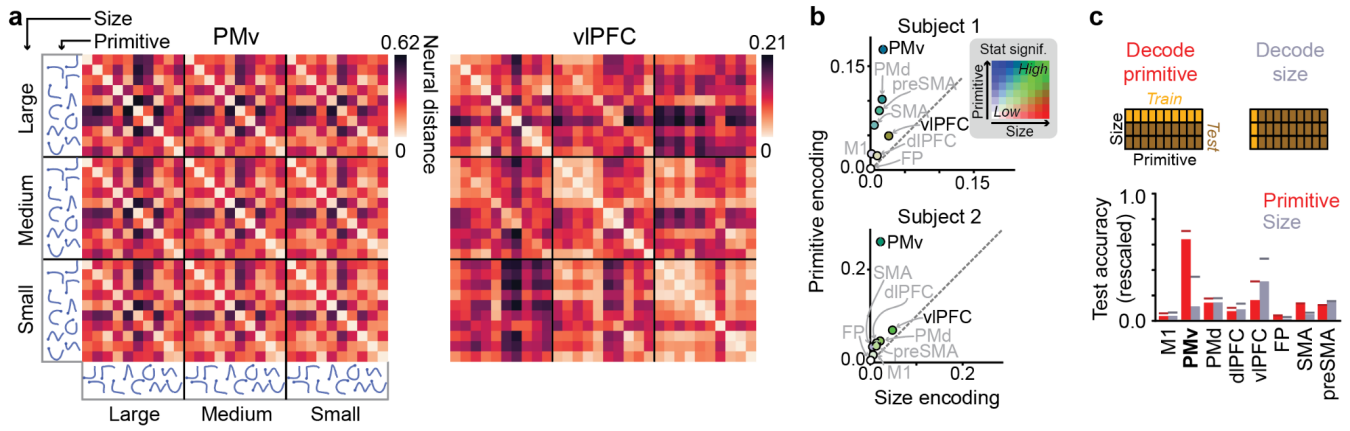
(a) Schematic of trial events, highlighting the initial reaching movement from the location of the "hold" button to the location of the stroke onset.

(b) Histogram of times of the go cue (orange) and reach onset (blue) relative to the time of stroke onset. To analyze activity during the initial reach, we used the time window -0.4 s to -0.05 s relative to stroke onset.

(c) Summary of primitive encoding and location encoding across areas and sessions. Points show mean encoding scores. The color of each point denotes statistical significance (analogous to Fig. 4j). It indicates the number of other brain areas that this area "beats" in pairwise statistical tests of primitive encoding and location encoding (represented in the inset heatmap, "low" to "high" ranges from no (0) to all (7) other regions beaten). For statistics, each data point was a unique pair of primitive-location conditions (trial-averaged). For testing primitive encoding, these pairs were the same location, but different primitive [n = 93 (subject 1), 288 (subject 2)]. For location encoding, the pairs were the same primitive, but different location [n = 114 (S1), 132 (S2)]. Statistical tests were performed on condition pairs pooled across sessions (2 for S1, 3 for S2).

(d) Statistics for primitive and location encoding in panel c. Effect size (top) and p-value (bottom) for comparisons between each pair of brain regions, for location encoding (left) and primitive encoding (right), for subject 1. "Significant" for the p-value heatmap means that $p < 0.05$ after Bonferroni correction for 56 comparisons [28 brain region pairs and 2 variables (location and primitive)].

(e) Same as panel d, but for subject 2.

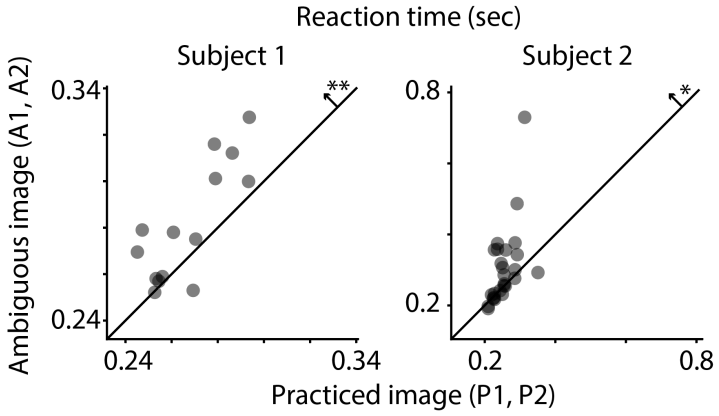


Extended Data Fig. 10. Size-invariant encoding of stroke primitives in PMv.

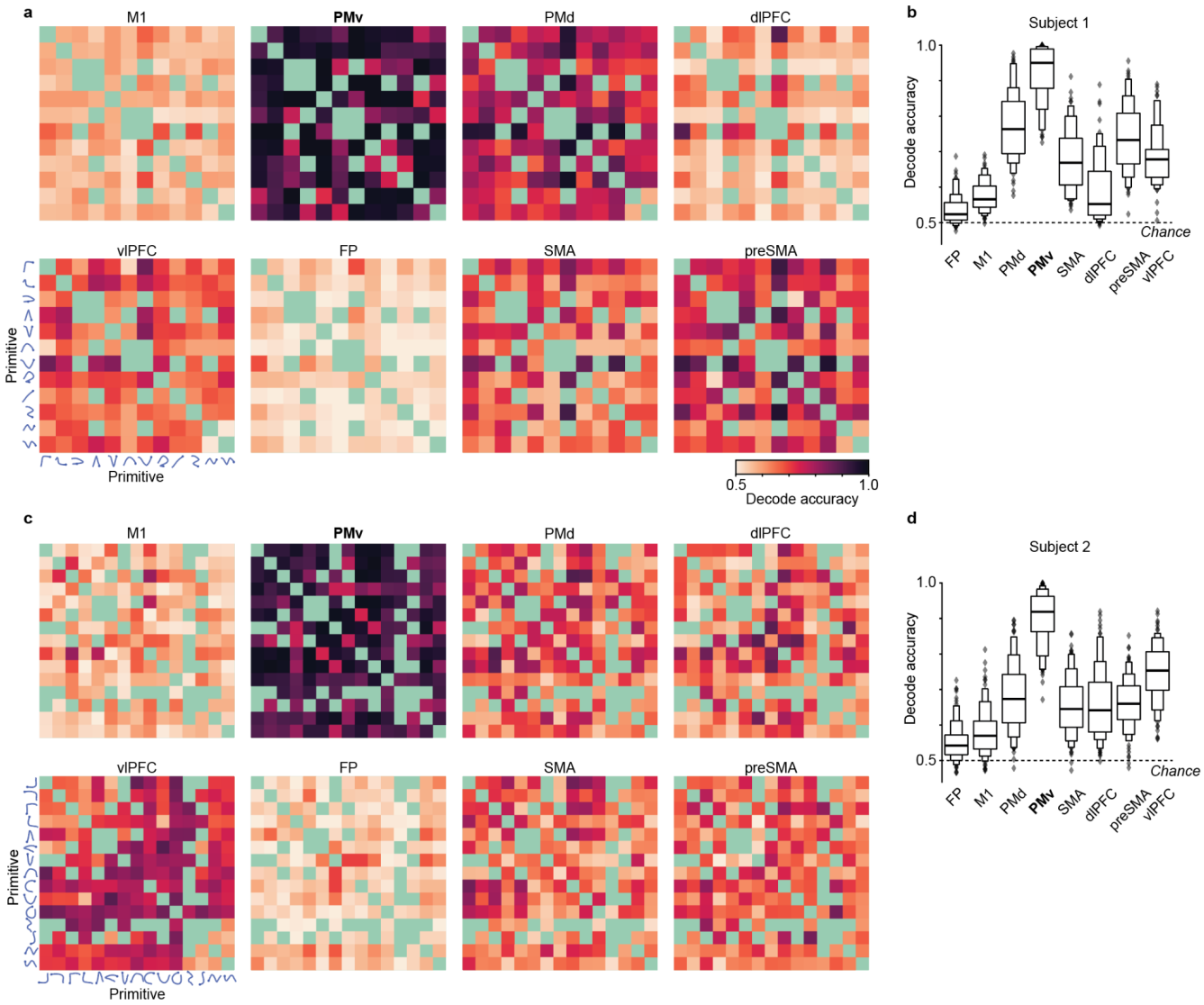
(a) Matrix of pairwise neural distances between each unique combination of primitive and size, averaged over the planning epoch (0.05 to 0.6 s relative to image onset), for PMv and vlPFC. Shown are data from a single session for subject 2. n = 17 - 23 trials per primitive-size combination.

(b) Summary of primitive encoding and size encoding across areas and sessions, with each point depicting the encoding scores for a given area. Each point's color denotes statistical significance, in terms of the number of other brain areas that this area beats in pairwise statistical tests of primitive encoding and size encoding (represented in the inset heatmap; "low" to "high" ranges from no (0) to all (7) other regions "beaten"); see **Fig. 4j** and Methods for details. Each data point was a unique pair of primitive-size conditions (trial-averaged). For testing primitive encoding, these pairs were the same size, but different primitive [n = 313 (subject 1), 253 (subject 2)]. For size encoding, the pairs were the same primitive, but different size [n = 60 (S1), 56 (S2)]. Statistical tests were performed on condition pairs pooled across sessions (3 for S1, 2 for S2). For exact effect sizes and p-values, see **Extended Data Fig. 8c, d.**

(c) Across-condition generalization of linear SVM decoders for primitive (red) and size (gray). See explanation in **Fig. 4k**.



Extended Data Fig. 11. Slower reaction time for ambiguous images in tasks testing categorical structure. Average reaction time in seconds (between go cue and stroke onset), comparing ambiguous images to practiced images. Each data point represents a single primitive from one morph set, its y-value indicating reaction time when the it was drawn in response to an ambiguous image, and its x-value indicating reaction time when it was drawn in response to the practiced image. **, p = 0.0031, Wilcoxon signed-rank test (n = 14, W = 8); *, p = 0.024, Wilcoxon signed-rank test (n = 26, W = 87). Each data point is a primitive, and each morph set contributes two primitives (one at each morph endpoint).



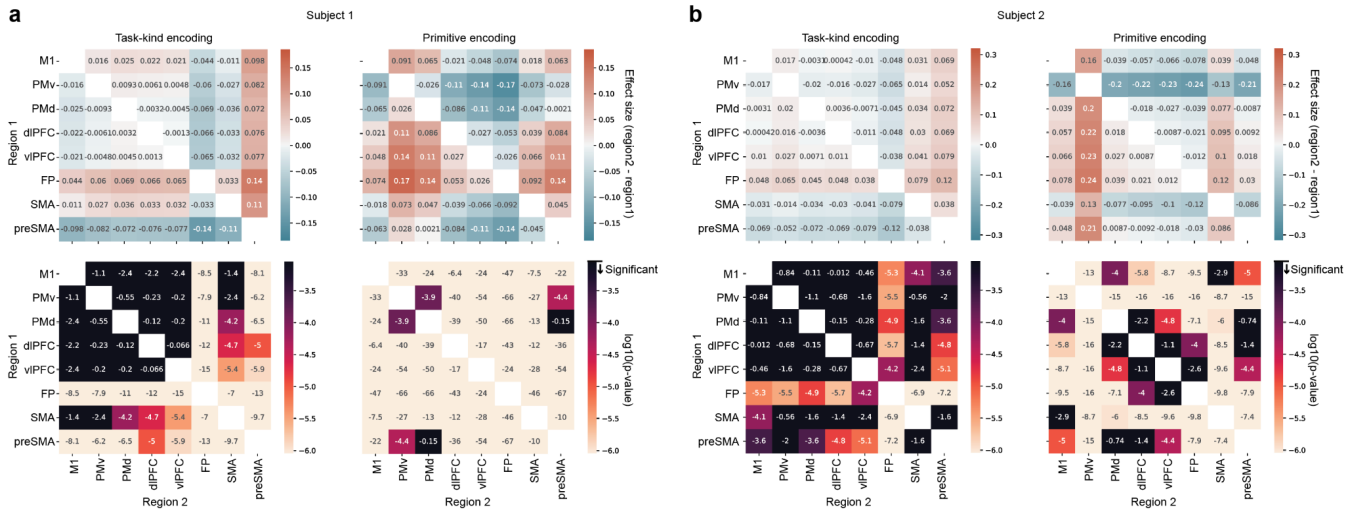
Extended Data Fig. 12. Primitives are distinct from each other in their planning-related activity in PMv.

(a) Pairwise decoding between all pairs of primitives (those performed in the same session), for subject 1, using population activity from the planning epoch (0.05 to 0.6 sec after image onset) represented as a 50-dimensional vector (see Methods). Chance decoding is 0.5. The matrices aggregate across sessions (n = 5 Single-shape sessions for each subject; S1: 2 varying in location, 3 varying in size; S2: 3 location, 2 size). The gray-green cells are pairs which lack data because the primitives were not recorded in the same session.

(b) Summary of decoding accuracy for all primitive pairs, for Subject 1, shown in a letter-value plot, a variation of a boxplot, in which each box depicts a successively decreasing quantile, from median, to quartile, to upper and lower eights, and so on. Each datapoint is one unique primitive pair.

(c) Like panel a, but for subject 2.

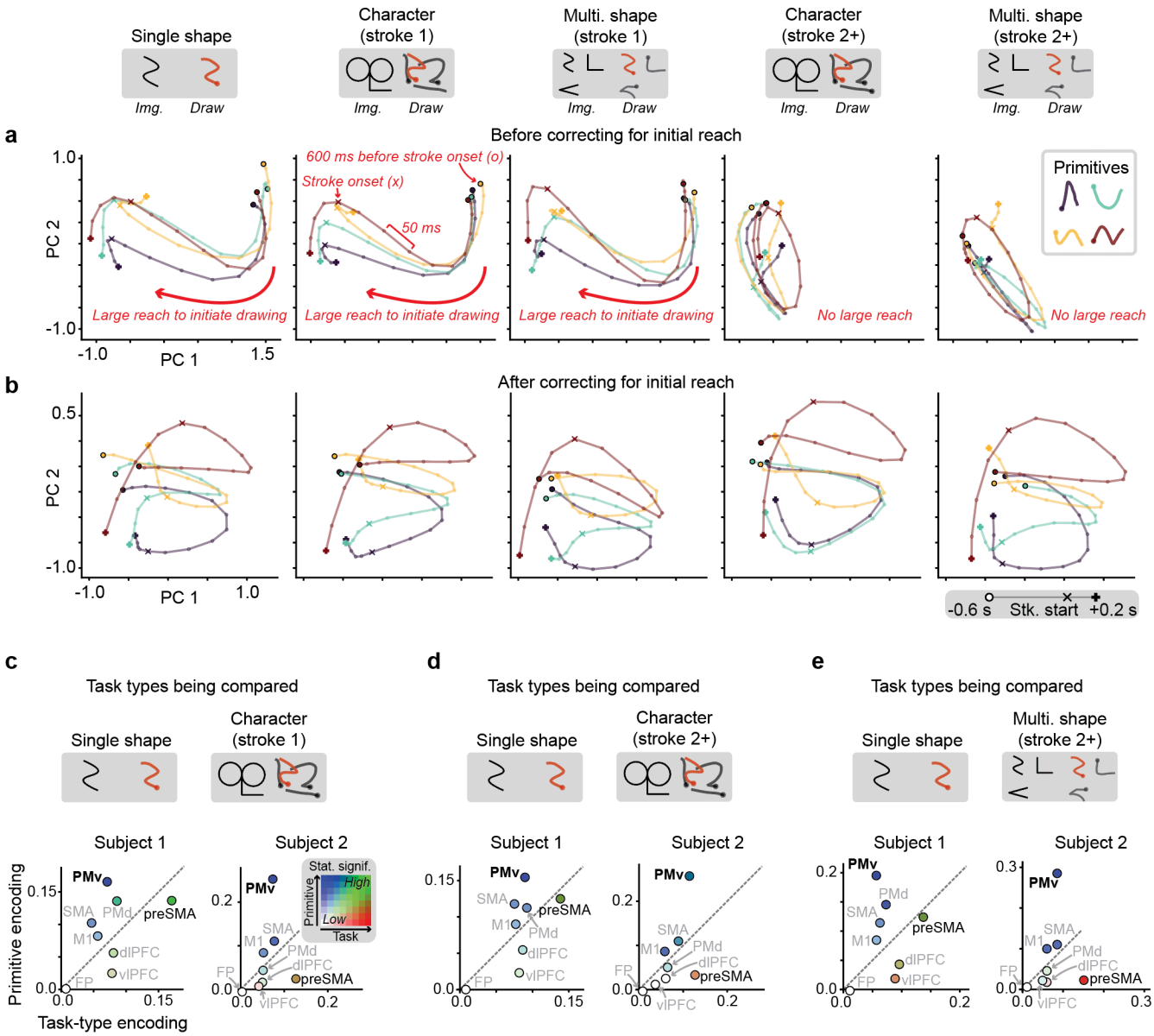
(d) Like panel b, but for subject 2.



Extended Data Fig. 13. Detailed statistics for the analysis of recombination in Fig. 6.

(a) Effect size (top) and p-value (bottom) for comparisons between each pair of brain regions, for task-type encoding (left) and primitive encoding (right), for subject 1. Corresponds to Fig. 6f. “Significant” for the p-value heatmap means that the value is less than 0.05 after Bonferroni correction for 56 comparisons [28 brain region pairs and 2 variables (task type and primitive)]. Note that because analyses involved bootstrapping to balance sample sizes across sessions (see Fig. 6f), these are statistics from a single bootstrapped sample that matches the mode.

(b) Same as panel a, but for subject 2. Corresponds to Fig. 6f.



Extended Data Fig. 14. Recombination of primitives into sequences is reflected in PMv, including for the second stroke onwards (related to Fig. 6).

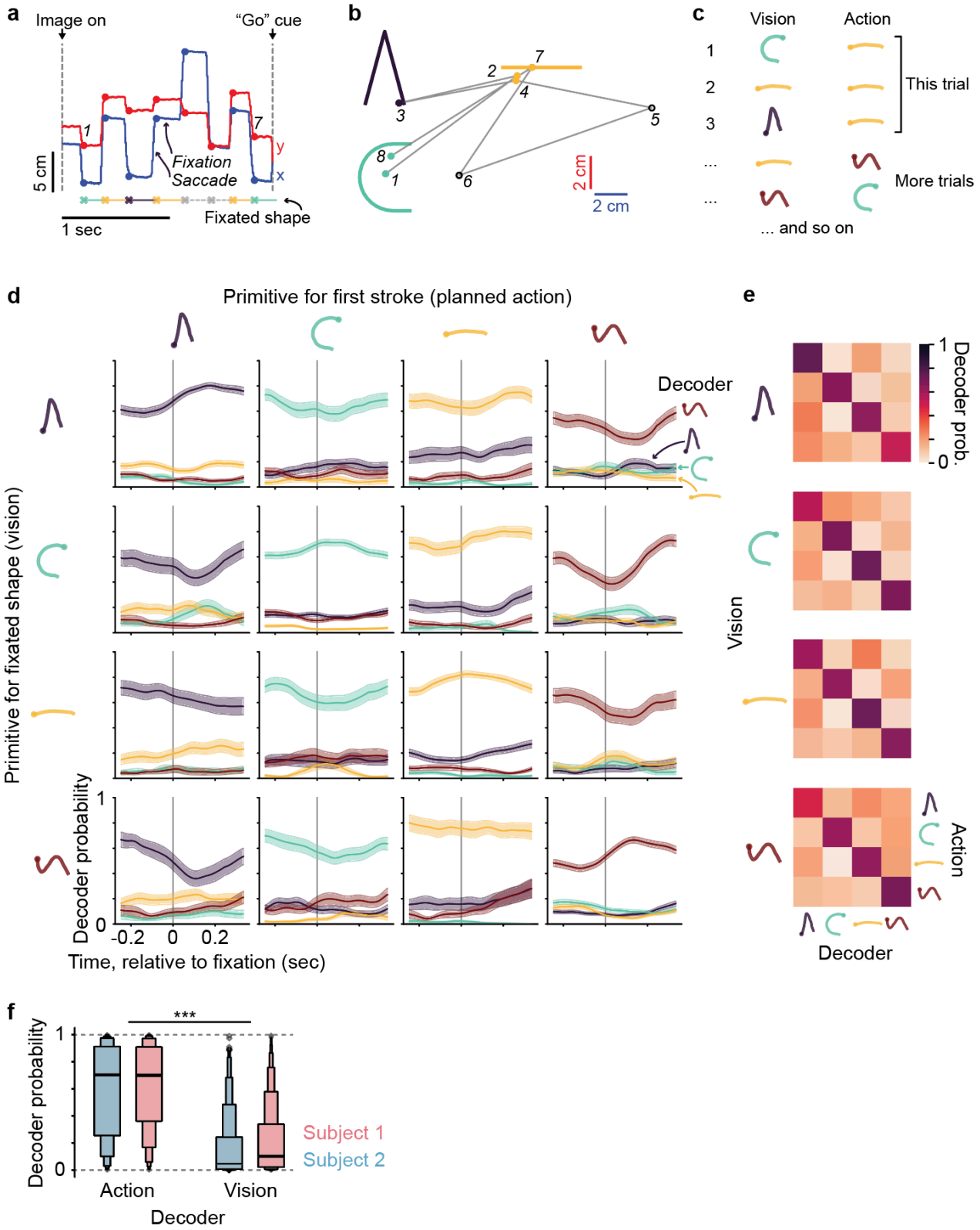
(a) PMv population trajectories for each primitive (color), plotted in a subspace spanned by principal components 1 and 2, and aligned to stroke onset ("x" icon). Each subpanel shows a different subset of trials (i.e., strokes) from this session, differing by task type (Single-shape, Character, or Multi-shape) and whether the dataset includes only the first stroke or only the second stroke onwards (stroke 1 or stroke 2+). Note that the Single-shape task by design only has one stroke. A prominent activity pattern visible in these trajectory plots is the right-to-left sweep associated with the initial reaching movement to start the drawing. This activity is visible for the conditions that involve an initial reach—Single-shape, Character (stroke 1), and Multi-shape (stroke 1)—and not for conditions that lack this reach—Character (stroke 2+) and Multi-shape (stroke 2+).

(b) The same as panel a, but after correcting for the effect of the initial reach. This correction was performed by subtracting, identically for all trials, the across-trial mean effect of the initial reach on neural activity, performed separately for each time bin (see Methods). After applying this simple linear correction, the primitive-associated trajectories are now similar across all conditions, consistent with the reach effect being an additive component (on top of the primitive-encoding component), associated with the large movement of the arm during the initial reach.

(c) Summary of primitive encoding and task-type encoding across areas and sessions, comparing Single-shape to Character (stroke 1) trials, after correcting for the mean effect of initial reach on neural activity. Apart from this correction, this is the same data as in Fig. 6f. For panels c-e, the plotting conventions are identical to those in Fig. 6f. See Fig. 6f for sample size and statistics.

(d) Same as panel c, but comparing Single-shape trials to Character (stroke 2+) trials. Each data point is a unique pair of primitive-task conditions (trial-averaged). For testing primitive encoding, these pairs were the same task type, but different primitive [n = 858 (subject 1), 624 (subject 2)]. For task-type encoding, the pairs were the same primitive, but different task type [n = 90 (S1), 63 (S2).

(e) Same as panel c, but comparing Single-shape trials to Multi-shape (stroke 2+) trials. For testing primitive encoding, these pairs were the same task type, but different primitive [n = 1458 (subject 1), 778 (subject 2)]. For task-type encoding, the pairs were the same primitive, but different task type [n = 117 (S1), 75 (S2)].



Extended Data Fig. 15. Primitive encoding in PMv reflects the first primitive the subject is planning to draw, rather than the primitive associated with the visually-fixated shape, in the Multi-shape task.

(a) Gaze location during the planning epoch (between "image on" and "go cue") for an example trial. Gaze involved interleaved saccades (rapid change in location) and fixations (stable location). The shape associated with each fixation is indicated by the colored line below (dashed gray means the fixation was too far from any shape and

therefore not assigned any shape). On this trial, the image had three shapes (see panel b), and the subject chose to draw the horizontal line first. There were no imposed rules for what order to draw the shapes.

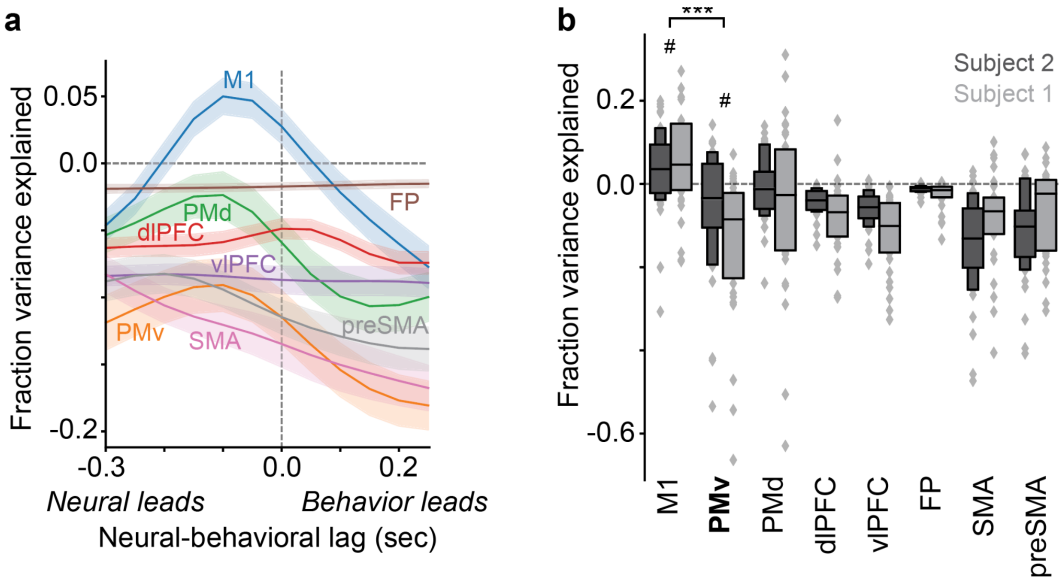
(b) Saccades (grey lines) and fixations (circles) overlaid on the image for this example trial, numbered in order, and colored by the identity of the fixated shape. Note that shapes on the screen are colored here, but were not for the monkey. Each fixation event was assigned a “vision” label based on its closest image shape, and a “planned action” label based on which shape the subject chose to draw first on that trial. Fixations that were too far from any shape (here, numbers 5 and 6) were excluded from subsequent analyses (see Methods).

(c) Table illustrating a dataset of fixation events collected across trials. Note the dissociation between “vision” and “action” labels across events.

(d) Average fixation-aligned decoder probability for each primitive class, grouping fixation events by the planned action (columns) and the fixated shape (rows). This, and the following panels, includes fixation events from the second half of the planning epoch, when the subject is expected to have decided what to draw first. Note the high probability for the decoder for the planned action, and low probability for the decoder for the fixated shape. Further, note that the decoder probability for the planned action is not strongly modulated relative to the time of fixation onset.

(e) Summary of mean decoder probability across conditions varying in vision and action. The mean is taken over the time window 0.05 to 0.3 sec after fixation onset. Note the strong diagonal band, consistent with activity encoding the planned primitive rather than the visually-fixated shape**.**

(f) Summary of decoder probability for the action vs. vision, shown in a letter-value plot (see explanation in Extended Data Fig. 12b). Each fixation event contributed one data point for the planned action and one for the visually-fixated shape. This includes only fixation events with action differing from vision. ***, $p < 0.0005$ (each subject), Mann–Whitney U test, n = 883 fixations (subject 1) and n = 446 (subject 2), each from a single session.



Extended Data Fig. 16. PMv activity does not significantly encode generalized motor kinematics.

(a) Fraction of neural activity variance (across time bins and trials) explained by stroke kinematics (x- and y-velocity) during drawing. A linear regression model, mapping from kinematics to neural activity, was trained on a subset of data consisting of every primitive except one, and then tested on the one held-out primitive, therefore testing for encoding of kinematics that generalizes across primitives. This was repeated for every primitive (S1, n = 30 primitives across five sessions; S2, n = 42 primitives across three sessions; datasets were combined in this plot). This analysis was performed separately for different time lags between the neural and behavioral data, and the resulting values were plotted as a function of time lag. Note that negative explained variance indicates failure to generalize across primitives, with the model performing worse than an intercept-only model trained on the held-out dataset (see Methods).

(b) Summary of fraction of variance explained, averaging over the time window -0.15 to -0.05 s relative to zero lag, summarized in a letter-value plot (see explanation in Extended Data Fig. 12b). Note that M1 is the only area with positive fraction variance explained. The M1 values (~0.0 to 0.2) are comparable to a prior study of motor cortex in human handwriting[93] (see Methods). Statistical tests involving M1 and PMv: ***, M1 vs. PMv: S1 ($W = 10$, $p = 8.01 \times 10^{-8}$), S2 ($W = 137$, $p = 3.36 \times 10^{-5}$), S1 and S2 combined ($W = 205$, $p = 4.87 \times 10^{-10}$). #, M1 vs. zero: S1 ($W = 117$, $p = 0.016$), S2 ($W = 230$, $p = 0.0049$), combined ($W = 667$, $p = 0.00028$), S1 does not reach significance after Bonferroni correction. #, PMv vs. zero: S1 ($W = 33$, $p = 5.97 \times 10^{-6}$), S2 ($W = 293$, $p = 0.047$), combined ($W = 559$, $p = 2.27 \times 10^{-5}$), S2 do not reach significance after Bonferroni correction. Sample size for all tests: S1, n = 30 primitives across five sessions; S2, n = 42 primitives across three sessions. Bonferroni correction considered all brain regions, thus correcting for 28 tests for PMv vs. M1, and for 8 tests for PMv/M1 vs. zero.

Supplementary Video 1. Behavior in Character task (example character 1, subject 1). This depicts the trial in **Fig. 2l**, 4th column from the right.

Supplementary Video 2. Behavior in Character task (example character 1, subject 2). This depicts the trial in **Fig. 2l**, 4th column from the right.

Supplementary Video 3. Behavior in Character task (example character 2, subject 1). This depicts the trial in **Fig. 2l**, 7th column from the left.

Supplementary Video 4. Behavior in Character task (example character 2, subject 2). This depicts the trial in **Fig. 2l**, 7th column from the left.

Supplementary Video 5. Behavior in Character task (example character 3, subject 1). This depicts the trial in **Fig. 2l**, 3rd column from the left.

Supplementary Video 6. Behavior in Character task (example character 3, subject 2). This depicts the trial in **Fig. 2l**, 3rd column from the left.

Supplementary Video 7. Behavior in Character task (example character 4, subject 1). This depicts the trial in **Fig. 2l**, 2nd column from the left.

Supplementary Video 8. Behavior in Character task (example character 4, subject 2). This depicts the trial in **Fig. 2l**, 2nd column from the left.

Supplementary Video 9. Behavior in Character task (example character 5, subject 1). This depicts the trial in **Fig. 2l**, 1st column from the left.

Supplementary Video 10. Behavior in Character task (example character 5, subject 2). This depicts the trial in **Fig. 2l**, 1st column from the left.