Mapping Dynamic Metabolic Energy Distribution in Brain Networks using fMRI: A Novel Dynamic Time Warping Framework

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Abstract

Understanding how metabolic energy is distributed across brain networks is essential for elucidating healthy brain function and neurological disorders. Research has established the link between blood flow changes and glucose metabolic processes that fuel neural activity. Here, we introduce a novel framework based on the normalized dynamic time warping algorithm robust to neural temporal variability, enabling reliable insights into metabolic energy demands using functional magnetic resonance imaging data. Our findings indicate that healthy brains maintain balanced energy distribution, whereas imbalances are more pronounced in schizophrenia with links to both positive and negative symptoms, particularly during rapid neural processes. Additionally, we identified a dynamic state that supports the brain criticality theory and is associated with higher-order cognitive abilities, demonstrating our framework's functional and clinical relevance. By linking metabolic energy distribution to neural dynamics, this framework provides a novel way to estimate and quantify the brain's maintenance of functional balance in a broadly applicable manner for studying brain health and disorders.

Keywords: brain energetics, dynamic time warping, high-frequency BOLD, brain criticality, schizophrenia

1. INTRODUCTION.

The human brain, despite constituting about 2% of the body's mass, consumes roughly 20% of its total energy[1]—a cost driven by intensive neuronal processing and synaptic activity essential for complex cognitive functions[2]. Functional magnetic resonance imaging (fMRI) leverages this metabolic demand by measuring the blood-oxygenation-level-dependent (BOLD) signal, which indirectly reflects neural activity through changes in cerebral blood flow and oxygenation[2,3]. The detectability of the BOLD signal arises from the retention of oxygen in the increased blood flow, underscoring glucose as the primary energy source for neurons[2]. Prior studies have shown that changes in blood flow are accompanied by comparable changes in glucose metabolism[4,5], making BOLD signal amplitude a reliable proxy for energy demand[4].

In contrast, techniques like positron emission tomography (PET) and magnetic resonance spectroscopy provide more direct insights into metabolism, but they lack the temporal resolution to capture rapid neural dynamics. Moreover, BOLD signal interpretation is complicated by variability in hemodynamics and neurovascular coupling (NVC), which introduce temporal lags and distortions[6], as well as the varying temporal scales of neural processing[7]. Commonly used measures such as the amplitude of low-frequency fluctuations (ALFF)[8], fractional ALFF[9], and dynamic ALFF[10] quantify BOLD signal power but do not account for temporal deformations, which may mask amplitude variations that are critical to assessing neural energy consumption. Similarly, prevalent dynamic functional connectivity measures—such as sliding window Pearson correlation[11], Hidden Markov Models[12], phase synchronization[13], wavelet coherence[14], and lagged cross-correlation[15]— primarily focus on synchronization and are amplitude scale invariant due to normalization procedures. Even many advanced deep learning models[16,17], including spatiotemporal models[18] struggle to fully capture these temporal dynamics.

There is therefore a pressing need for methodologies that can address both temporal misalignments and amplitude variations in BOLD signals to accurately reflect energy demand mismatches between neural networks. To this end, we propose leveraging dynamic time warping (DTW) to realign BOLD signal pairs, effectively mitigating temporal misalignments caused by hemodynamic variability and variable temporal scales. DTW is a time-series alignment technique that non-linearly warps the temporal dimension to align signals[19], allowing us to focus on the amplitude differences between the aligned signals. By analyzing BOLD signals in a way that compensates for temporal variability and hemodynamic delays, these amplitude differences indirectly estimate metabolic energy mismatch between neural networks. Specifically, DTW sums the aligned amplitude differences, and our recently proposed normalized DTW (nDTW)[20] computes the average of these differences, aligning with traditional signal energy and power computations respectively, as demonstrated in our study.

DTW has been introduced in fMRI studies as an alternative measure of functional connectivity[21], demonstrating benefits such as increased sensitivity to motor brain function[22], robustness to noise[21], higher test-retest reliability[21,22], and global signal

regression robustness[21,22] compared to conventional correlation methods. DTW has also
been effective in detecting group differences, including sex differences[21] and clinical
disorders like schizophrenia[20] and autism spectrum disorders[22]. In our previous work, we
expanded DTW's application by developing a novel warp elasticity measure, which
leverages the alignment function from DTW to capture relative BOLD signal activation
speeds (stretching and shrinking temporal dynamics) between networks[23,24]. These
findings underscore the value of DTW in neuroimaging analyses.

In this study, we build on these findings by demonstrating that DTW-aligned amplitude
differences serve as an effective metric for capturing activation amplitude discrepancies
in the presence of temporal deformations. We incorporate a parameterization of the DTW
cost function with a gamma parameter that emphasizes different scales of amplitude
disparity—where higher gamma values accentuate larger amplitude differences and
lower values highlight smaller ones[25]—to create a flexible framework for assessing
metabolic energy demand mismatches across varying amplitude scales in the BOLD
signal. Our focus is on resting-state fMRI (rs-fMRI) data, recognizing that significant
energy is consumed even at rest and that resting-state activity is linked to metabolic
processes[26].

The proposed two-cell model presents a unifying hypothesis that links schizophrenia to
dysregulated glucose metabolism in astrocytes and neurons, suggesting that cognitive
and psychiatric symptoms arise through a two-hit mechanism[27]. This hypothesis is
supported by numerous studies connecting schizophrenia to disruptions in metabolic
energy[28,29]. Based on this and recognizing that blood flow—carrying glucose—increases
in response to higher energy demand, we investigate imbalances in energy demand
between neural networks in individuals with schizophrenia compared to healthy controls
using blood flow fMRI data. Consistent with previous findings, we focus on nDTW due to
its increased sensitivity to group differences[20]. We hypothesize that energy mismatches
between networks are more pronounced in broadband signals; therefore, we extend our
analysis beyond the classic low-frequency BOLD signals to include higher frequency
bands—specifically slow-5 (0.01–0.027 Hz), slow-4 (0.027–0.073 Hz), and slow-3
(0.073–0.198 Hz)—which have been shown to exhibit functional relevance[30]. We refer to
the broader frequency ranges of 0.01–0.198 Hz and 0.01–0.15 Hz as F1 and F2,
respectively. In addition, we develop a time-resolved nDTW measure to track how brain
networks converge or diverge in their metabolic energy demands over time during rest.

By capturing these nuances, our approach offers a novel framework to understand neural
network specialization and energy demand dynamics, with the potential to advance the
characterization of neuropsychiatric disorders linked to energy metabolism abnormalities.
This methodology not only enhances our ability to detect subtle energy imbalances within
and between neural networks but also provides new insights into the pathophysiology of
conditions such as schizophrenia with the potential to inform more targeted therapeutic
interventions.

2. RESULTS

2.1. Simulation of DTW sensitivity non-stationarity and signal bandwidth

We evaluated DTW as a potential measure of metabolic energy demand mismatch between brain networks through simulations assessing its ability to detect amplitude disparities and its invariance to phase shifts. Sinusoids with varying amplitude and phase modulations were generated, and the performance of DTW, nDTW, and correlation was compared across different γ values and signal bandwidths (Supplementary Material).

DTW aligned original and temporally warped signals, demonstrating its capability to capture temporal deformations (Fig. 1a). Both DTW and nDTW remained invariant to non-stationary phase perturbations across increasing modulation indexes. In contrast, correlation decreased with higher phase modulation (Fig. 1b). In contrast, under non-stationary amplitude perturbations, correlation remained stable, while DTW and nDTW detected increasing amplitude disparities as modulation indexes rose (Fig. 1b).

The sensitivity of DTW and nDTW to amplitude scales was further examined by varying the γ parameter. Simulations involving amplitude non-stationarity across several modulation indexes revealed that smaller γ values resulted in higher rates of change in DTW and nDTW similarity scores at lower modulation indexes, indicating enhanced detection of minor amplitude disparities (Fig. 1c). Conversely, larger γ values exhibited higher rates of change at higher modulation indexes, reflecting improved tracking of substantial amplitude disparities (Fig. 1c). Additionally, while DTW generally maintains phase invariance, nDTW demonstrated slight sensitivity to phase differences when $\gamma \geq 1$ (Fig. 1c). This suggests that nDTW is better capable of responding to phase variations alongside amplitude tracking, offering a more nuanced assessment when absolute phase invariance is not essential.

In bandwidth analyses, DTW and nDTW sensitivity to randomness increased with broader full frequency spectrum, approaching the Nyquist frequency, unlike correlation, which remained unaffected (Fig. 1d). This indicates that amplitude differences become more prominent in broader full frequency bands.

Figure 1e illustrates nDTW ($\gamma = 1$) in capturing average aligned amplitude differences between brain networks. Lower nDTW values indicate convergence, potentially reflecting balanced metabolic energy demands, while higher values signify divergence, potentially highlighting greater energy demand discrepancies between networks (Fig. 1e).

Fig. 1: Simulation to analyze non-stationarity and signal bandwidth

a. Illustration of DTW's ability to extract temporal deformations. A signal is warped by applying a modulation function (warp function) to its time index, and DTW successfully aligns the original and warped signals, extracting the induced temporal deformation. **b.** Behavior of DTW, nDTW and correlation under amplitude and phase non-stationarity. Time-varying amplitude perturbations (mi=100) and phase perturbations (mi=10) are shown. Similarity scores are derived from DTW and nDTW to highlight high distances (approaching 0) and low distances (approaching 1). Correlation is used as a benchmark. DTW and nDTW tracks amplitude disparities but remains invariant to phase shifts, unlike correlation. Both DTW and nDTW track amplitude disparities as mi increases (Time-varying amplitude sensitivity) but remain invariant to phase shifts, unlike correlation (Time-varying phase sensitivity). **c.** Sensitivity of DTW and nDTW to γ values in amplitude and phase non-stationarity. The derivative of similarity scores under amplitude non-stationarity reveals that for lower mi, smaller γ values yield higher scores compared to larger γ values, whereas for higher *mi*, larger γ values yield higher scores (Time-varying amplitude tracking). This indicates that smaller γ values are more adept at tracking minor amplitude disparities, while larger γ values are more suited to tracking larger amplitude disparities. Although DTW is inherently phase-invariant, nDTW exhibits slightly higher derivatives indication slight sensitivity to phase differences when $\gamma \geq 1$ (Time-varying phase tracking). **d.** DTW sensitivity across signal bandwidths. Random signal pairs (i.i.d Gaussian, $\mathcal{N}(0,1)$) are filtered with increasing bandwidths up to the Nyquist frequency. DTW alignment and distances demonstrate increasing sensitivity to disparities in broadband signals, unlike correlation, which remains unaffected by the increasing frequency spectrum. This highlights DTW's suitability for capturing disparities in full spectrum signals against slow narrowband ones. **e.** Illustration of convergence and divergence in brain networks. Using nDTW with $\gamma = 1$, the average amplitude difference between two pairs of aligned brain networks is shown. A small nDTW indicates convergence, while a large nDTW signifies divergence in signal power differences, providing insights into imbalances in metabolic energy demand.

2.2. Signal power imbalances between schizophrenia and control in broader fMRI bandwidth using nDTW

The intrinsic brain networks identified using the NeuroMark pipeline were categorized into seven domains: subcortical (SC), auditory (Aud), sensory motor (SM), visual (Vis), cognitive control (CC), default mode (DM), and cerebellum (Cb).

To validate the use of nDTW in the context of higher frequency fMRI data, we extended our analysis to include broader frequency bands (slow-5, slow-4, and slow-3; 0.01–0.198 Hz). We also examined the impact of varying γ values on test–retest reliability and sensitivity to group differences.

Focusing on the F1 frequency range, we observed that test–retest reliability improved with higher γ values, as indicated by lower mean ranks (Fig. 2a). γ values of ≥1.5 demonstrated significantly higher reliability compared to correlation after false discovery rate (FDR) (Wilcoxon signed-rank test, FDR corrected $p < 0.05$). Given that physiological noise correction in the fMRI data reduces within-subject variability[31], thus increasing test-retest reliability, the higher test-retest score for higher γ values may be indicative of the reduced sensitivity of higher γ values to physiological noise in the data.

Figure 2b illustrates that γ values of 1.5 and 1.25 exhibit the highest sensitivity for detecting group differences across both frequency ranges, outperforming other γ values and correlation. These γ values outperform other parameters and the correlation benchmark, as indicated by the greater number of directional arrows pointing towards them and the significant sensitivity enhancements marked by yellow asterisks. However, since $\gamma = 1.25$ has significantly lower test–retest reliability than the correlation benchmark (Fig. 2a), $\gamma = 1.5$ was selected as the optimal parameter for further analysis. Additionally, nDTW consistently detected more group differences in the broader F1 frequency range than in F2 across all γ values, whereas correlation did not. This indicates that nDTW is more sensitive to capturing clinical functional relevance at higher BOLD signal frequencies while maintaining high test–retest reliability, particularly at $\gamma = 1.5$.

Figure 2c shows that controls (lower triangle) consistently exhibit higher convergence in signal power differences, whereas individuals with schizophrenia (upper triangle) display greater divergence across several brain network pairs. This suggests that controls have more balanced metabolic energy demands between brain networks, while schizophrenia patients exhibit imbalances. Controls display significantly more convergent power differences than schizophrenia patients across multiple network pairs (positive red cells in the upper triangle of the FDR-corrected group difference matrix, Fig. 2c). Additionally, Cohen's d effect sizes (min-max $= 0.19 - 0.89$; median $= 0.49$) highlight the practical significance of these group differences.

Furthermore, significant associations between nDTW scores and PANSS scores were identified. Positive PANSS associations were primarily observed between Cb networks

178   and the DM, CC, and superior temporal gyrus (Aud), Vis and SC networks. In contrast,
179   negative PANSS was mainly associated with connections between the CC, DM, and Vis
180   brain networks.

Fig. 2: Reliability and group difference sensitivity across γ values and frequency bands

a) Test-retest reliability

F1: fMRI data (0.01-0.198Hz)

b) Group sensitivity test: McNemar's test

c) Schizophrenia group difference analysis: F1 (0.01-0.198Hz)

d) PANSS association with nDTW: F1 (0.01-0.198Hz)

a. Test-retest reliability of nDTW was evaluated across multiple γ values using the HCP dataset, with correlation serving as a benchmark. The mean ranks for all twelve γ values and correlation are presented for the F1 frequency range. Lower mean rank values indicate higher test-retest reliability. γ values ≥ 1.5 and $\gamma = 0.25$ show significantly higher test-retest than correlation, as determined by a Wilcoxon signed-rank test (FDR-corrected $p < 0.05$) on the test–retest ranks. **b.** Gamma sensitivity in distinguishing schizophrenia from controls. We applied McNemar tests across all γ values for the F1 and F2 frequency ranges, as well as between F1 and F2. In the resulting matrices, arrows point to the parameter with higher sensitivity, and yellow asterisks highlight significant results with FDR-corrected p-values < 0.05. Results showed that γ values of 1.5 and 1.25 are the most effective in distinguishing schizophrenia patients from controls compared to other γ values and correlation for both F1 and F2 frequency ranges. Furthermore, the comparison between F1 and F2 demonstrates that all γ values increase group difference sensitivity when transitioning from F1 to F2, whereas correlation (rho) does not. These findings suggest that nDTW is more sensitive to group differences when higher frequencies are incorporated. **c.** Group difference between schizophrenia and controls using a $\gamma = 1.5$. nDTW values were z-scored across both groups and multiplied by -1, converting smaller nDTW values to positive scores (indicating convergence) and larger values to negative scores (indicating divergence). Group averages of nDTW scores reveal that controls exhibit convergence (lower triangle) while individuals with schizophrenia show divergence (upper triangle) across several brain network pairs. FDR-corrected p-values from a generalized linear model, controlling for sex, age, site, and mean frame displacement, are displayed in the lower triangle, with significant results (FDR < 0.05) highlighted in the upper triangle. The findings indicate that multiple brain network pairs show significant differences, with controls consistently demonstrating greater convergence than schizophrenia patients. Additionally, Cohen's d effect sizes exceeding 0.2 underscore the practical relevance of these group differences. **d** Association of nDTW with PANSS scores. Significant associations between brain network pairs and both positive and negative PANSS scores (FDR-corrected $p < 0.05$) were identified using a generalized linear model with a Poisson distribution. The analysis controlled for sex, age, site, and mean frame displacement. These significant associations are displayed for both positive and negative PANSS, highlighting the relationship between nDTW scores and symptom severity in schizophrenia.

2.3. Signal power imbalance effects of including higher frequency fMRI

We evaluated the effect of nDTW on expanding the frequency range from F2 to F1 by conducting paired sample t-tests for controls and individuals with schizophrenia separately. Normality of nDTW scores was confirmed using Shapiro-Wilk tests with t-values close to 1 across brain network pairs (Fig. 3a). After FDR correction, controls exhibited significantly increased convergence in F1 compared to F2, whereas schizophrenia patients showed greater divergence in F1 versus F2 across specific brain network pairs. These results suggest that broader F1 frequencies enhance metabolic energy balance in controls but exacerbate imbalances in schizophrenia, indicating that higher frequency fMRI enhances the identification of schizophrenia characteristics.

Cohen's d effect sizes were modest (median d = 0.18 for controls and d = 0.15 for schizophrenia). Nonetheless, higher frequencies amplified group distinctions, as reflected by larger effect size differences between controls and schizophrenia across several brain network pairs (Fig. 3a). Upset plot analysis identified 914 significant pairs across both frequencies, with 196 unique to F1 and 10 unique to F2, highlighting the enhanced group sensitivity of nDTW in F1 (Fig. 3b).

Figure 3b displays FDR-corrected p-values for significant pairs shared by both frequencies (left panel), with F1 (lower triangle) showing higher practical relevance than

F2 (upper triangle) based on effect sizes. Pairs unique to F1 had Cohen's d > 0.2,
underscoring nDTW's increased sensitivity and clinical relevance at higher frequencies.

To validate F1-unique pairs, we conducted null hypothesis testing using 1,000 surrogate
datasets generated through phase randomization within the 0.15–0.198 Hz range
(Fig. 3c). Several network pairs remained significant, confirming that higher frequency
fMRI results are statistically robust and potentially derived from non-stationary amplitude
processes.

Finally, we examined PANSS associations exclusively linked to F1 and not F2. Positive
PANSS correlated with connections between the Cb with SC, DM, CC, Aud and Vis
networks. Negative PANSS was primarily associated with CC-DM connections. These
associations were absent in F2, suggesting that higher frequency fMRI enhances
symptom severity detection.

Fig. 3: Clinical significance of high frequency fMRI data inclusion.

a) Statistical effects of including higher frequencies

F1- fMRI data (0.01-0.198hz)
F2- fMRI data (0.01 - 0.15hz)

CN- controls
SZ- schizophrenia

b) Common and Unique Clinical Relevance: F1 vs. F2

F1- fMRI data (0.01-0.198hz)
F2- fMRI data (0.01 - 0.15hz)

c) Null hypothesis test

d) PANSS association exclusive to F1

a. Statistical analysis of power imbalances between F1 and F2 frequencies. Shapiro-Wilk tests confirmed normality ($T\text{-}values \approx 1$) for all groups, validating the use of paired t-tests. FDR-corrected p-values of paired t-test indicated that controls exhibit increased signal power convergence, whereas individuals with schizophrenia show power divergence when transitioning from F2 to F1. This suggests that controls demonstrate greater convergence, while schizophrenia patients exhibit increased divergence with the inclusion of higher frequencies. Cohen's d effect sizes (~0.4) highlight enhanced separability between groups. **b.** Group differences from a generalized linear model controlling for sex, age, mean frame displacement, and site. FDR-corrected p-values across brain network pairs revealed 914 common significant pairs, with F1 showing larger effect sizes and greater practical relevance (left panel). Additionally, 196 significant pairs were uniquely identified in F1 with effect sizes above 0.2 (right panel), indicating modest practical relevance, while only 10 pairs were unique to F2. **c.** Null hypothesis testing using phase randomization confirmed that the significant brain network pairs uniquely identified in F1 are statistically relevant. **d.** Associations between normalized nDTW and PANSS scores were exclusively observed in the F1 frequency range, with no significant associations found in F2.

2.4.   Dynamics of time-resolved nDTW

To identify recurring states of power imbalance dynamics, we applied k-means clustering
with three clusters on the time-resolved nDTW, including both controls and individuals
with schizophrenia. An elbow plot (Fig. 4c) confirmed that three clusters were optimal,
determined by the derivative of the within-sum of squared distances. For visualization, we
subtracted the median of all clusters and multiplied the distribution by -1, ensuring
positive values represent convergence and negative values indicate divergence (Fig. 4a).
The clusters were classified as follows: state 1—a convergent state, state 3—a divergent
state, and state 2—a mixed state showing neither strong convergence nor divergence.

Figure 4b illustrates brain network representations for each state. In convergent state 1,
the top 5% most convergent pairs predominantly involved sensory networks, including
Vis, SM, and Aud networks. The divergent state 3 featured the top 5% most divergent
pairs, mainly between the Cb and other networks. The mixed state 2 showed the top 2.5%
most convergent and top 2.5% most divergent pairs, combining both dynamics.

A Markov chain was constructed to model state transitions, with transition probabilities
shown in Figure 4d. Mixed state 2 had the highest self-retention probability (0.9021),
followed by the convergent state 1 (0.8485) and divergent state 3 (0.8133). No direct
transitions occurred between divergent and convergent states, likely due to the
monotonicity constraint in the DTW alignment process, which prevents abrupt shifts
without intermediate states.

The Markov chain's "stable point" (stationary distribution: Fig. 4e) showed that mixed state
2 accounted for 57.36% of system dynamics, the convergent state 1 for 37.21%, and the
divergent state 3 for 5.42%. This highlights the predominance of mixed and convergent
states in stabilizing power imbalance dynamics.

We further analyzed these dynamics by calculating the entropy rate and the convergence
rate to the stationary distribution. The entropy rate was 36.6%, where 100% represents
complete randomness and 0% indicates full order. This suggests a balance between
randomness and order, with a slight preference for order (Fig. 4f). The rate of

convergence computed by the spectral gap was 0.1851, and the mixing time—defined as
the number of timesteps required to reach a tolerance level of 10^{-3} from the stationary
distribution—was 36 timesteps (72 seconds; TR=2). The mixing time was determined
from the farthest distribution point in the Markov chain's state space, specifically the state
[0, 0, 1] (completely divergent), as shown in Fig. 4e.

We explored the relationship between entropy and convergence rate and investigated
potential interventions to accelerate convergence and enhance cognitive ability by
introducing perturbations to each transition probability and measuring the relative
changes in convergence and entropy rates (Fig. 4g–h). Increasing self-retention in any
brain state led to reductions in both convergence rate and entropy rate. While adding
transitions between the mixed state and other states (excluding mixed to divergent)
enhanced both rates, transitions between divergent and convergent states were marked
with red crosses due to their impracticality as no such transitions were identified in the
dynamics (Fig. 4d).

Notably, increasing transition probability from mixed to divergent states extends
trajectories away from the stable point (Fig. 4e), reducing convergence speed. While this
increased entropy, our findings suggest that higher entropy does not universally
accelerate convergence; instead, the effect depends on the specific dynamic transitions
within the Markov chain's state space.

Fig. 4: Dynamics of time-resolved nDTW

a) Distribution of brain states

b) Brain states

c) K-means elbow plot

d) State transitions: Markov chain

e) Stationary distribution

f) Convergence & Entropy rate

g) Sensitivity of spectral gap

h) Sensitivity of entropy rate

a. Distribution of recurring brain states identified using K-means clustering on time-resolved nDTW. The median across all 3 cluster centroids were subtracted and multiplied by –1 to ensure positive values represent convergence and negative values represent divergence. Three states were identified: convergent, divergent, and mixed. **b.** Brain state matrices with corresponding axial brain thumbnails. For convergent and divergent states, the top 5% most convergent and 5% most divergent brain connections are displayed, respectively. The mixed state shows the top 2.5% most convergent and top 2.5% most divergent connections. **c.** Elbow plot illustrating the within-sum of squared distances (WSS) for different cluster numbers. The first inflection point in the first derivative of WSS indicates that three clusters are optimal. **d.** State transition probabilities between the identified brain states, highlighting the likelihood of moving from one state to another. **e.** Stationary distribution within the Markov chain state space, demonstrating that the stable point of the dynamics is predominantly influenced by the mixed and convergent states. **f.** Mixing time of the dynamics, indicating the number of timesteps required to converge to the stationary distribution within a tolerance of 10^{-3}. Additionally, the convergence rate is shown through the spectral gap and entropy rate. **g.** Effects of perturbing state transitions on the spectral gap. Small perturbations were added to each transition probability, and the relative change in spectral gap was calculated to assess changes in convergence rate. **h.** Effects of perturbing state transitions on the entropy rate.

2.5. Group differences between controls and schizophrenia

We assessed group differences in metrics derived from time-resolved nDTW, including
mean dwell time, occupancy rate, state transition probabilities, stationary distribution,
spectral gap, mixing time, and entropy rate. Schizophrenia patients showed greater
association with mixed state 2 and divergent state 3, while controls predominantly
engaged with the convergent state 1. Specifically, schizophrenia patients had longer
mean dwell times in mixed and divergent states, whereas controls spent more time in the
convergent state (Fig. 5a). Similarly, occupancy rates (Fig. 5b) reflected higher mixed and
divergent state occupancy in schizophrenia, and greater convergent state occupancy in
controls.

The stationary distribution (Fig. 5d) revealed that schizophrenia dynamics are more
influenced by mixed and divergent states than controls, while controls are more
influenced by the convergent state than schizophrenia. State transition probabilities
showed controls were more likely to transition from mixed to convergent states (Fig. 5c).
In contrast, schizophrenia patients had increased transitions from convergent to mixed,
mixed to divergent, and higher self-retention in both mixed and divergent states.

Fig. 5: Group differences and cognitive score associations

a) Mean dwell time
b) Occupancy rate
c) State transitions: Markov chain
d) Stationary distribution
e) Spectral gap
f) Mixing time
g) Entropy rate
h) CMIND score associations with controls and schizophrenia groups

a. Mean dwell time in each state for schizophrenia patients and controls, highlighting increased dwell time in mixed and divergent states for schizophrenia and convergent state for controls. **b.** Occupancy rate of each state, showing longer occupancy in mixed and divergent states for schizophrenia and in convergent state for controls. **c.** State transition probabilities between states for schizophrenia patients and controls, illustrating higher transition rates from convergent to mixed and mixed to divergent states in schizophrenia. **d.** Group difference in stationary distribution of the Markov chain. **e.** Spectral gap, reflecting the convergence rate, is higher in controls compared to schizophrenia patients, suggesting faster convergence to the stable point in controls. **f.** Mixing time, representing the number of timesteps required to reach the stationary distribution within a tolerance of 10^{-3}, is shorter in controls than in schizophrenia patients, indicating quicker convergence in controls. **g**. Entropy rate, showing the balance between order and flexibility, does not differ significantly between groups. **h.** Associations between cognitive scores and nDTW metrics, analyzed separately for each group to account for group bias. In controls, working memory and verbal learning are associated with specific state transitions and spectral gap is associated with reasoning and problem-solving. In schizophrenia, reasoning, problem-solving, and attention vigilance are linked to different transitions and state retention.

All analyses were conducted using generalized linear models controlling for sex, age, site, and mean frame displacement. Results were FDR-corrected with a significance threshold of 0.05. One asterisk (*) indicates p-values between 0.05 and 0.01, two asterisks (**) indicate p-values between 0.01 and 0.001, and three asterisks (***) indicate p-values below 0.001.

Regarding convergence to the stationary distribution, controls exhibited a higher spectral
gap, indicating a faster convergence rate (Fig. 5e). Using a tolerance of 10^{-3} and starting
from the completely divergent state ([0, 0, 1]), controls also had a lower mixing time,
reflecting quicker stabilization. No significant difference in entropy rate was observed,
suggesting that faster convergence rates in controls may stem from dynamic transition
efficiency rather than entropy.

2.6. Controls and schizophrenia cognitive score associations

We examined associations between time-resolved nDTW metrics and Computerized
Multiphasic Interactive Neurocognitive System (CMINDS) scores using generalized linear
models (GLM) for controls and schizophrenia patients separately to avoid group bias (Fig.
5h).

In controls, working memory was positively associated with transitions from convergent
to mixed states and negatively associated with convergent self-retention and mixed-to-
divergent transitions, indicating that transitions into the mixed state support memory
performance. Verbal learning was positively associated with transitions from mixed to
convergent states and negatively with convergent-to-mixed transitions, suggesting that
the convergent state facilitates verbal learning. Additionally, the spectral gap was
positively associated with reasoning and problem-solving, highlighting the link between
faster convergence rates and higher cognitive performance.

In schizophrenia, reasoning and problem-solving were negatively associated with
transitions from mixed to convergent states but positively with self-retention in the mixed
state. Attention vigilance was positively associated with transitions from divergent to
mixed states and negatively with self-retention in the divergent state. These results
suggest that specific dynamic transitions, such as moving into the mixed state, enhance

cognitive abilities in schizophrenia, whereas others, like remaining in the divergent state,
may impede them.

3. DISCUSSION

This study developed a comprehensive framework to assess power imbalances between
brain networks using fMRI data, offering insights into metabolic energy demands while
accounting for varying temporal scales of brain activity. The physiological basis of the
BOLD signal is not entirely straightforward. Studies combining fMRI with PET imaging
have shown that the magnitude of the BOLD signal can track glucose utilization in the
brain[4,5]. However, other studies indicate that even under conditions of hyperemia (excess
amount of blood) and hyperoxia (excess amount of oxygen), the BOLD signal still
accompanies neuronal activation, suggesting that it does not strictly reflect a deficiency
of glucose or oxygen[32,33]. Research further supports this by proposing that due to the
slow response of NVC, neurons initially rely on astrocytic glycogen and lactate stores
before blood flow compensates for energy use[34,35]. This indicates that blood flow does
not simply match real-time neuronal energy consumption[36]. Nevertheless, neuronal
activation consistently triggers vasoactive signaling, leading to vasodilation through NVC,
regardless of brain baseline oxygen or glucose levels[33]. Therefore, while the immediate
energy source may vary, the magnitude of the BOLD signal consistently reflects the
overall energy demand driven by neuronal activation, whether the energy is supplied by
blood-derived glucose and oxygen or by intracellular reserves. Tracking amplitude
changes in BOLD signals using DTW provides valuable insights into metabolic energy
demands, offering robustness against temporal variability introduced by neural
processing and hemodynamic modulations.

By utilizing our recently proposed nDTW metric[20], we explored functional relevance
beyond the typical 0.01–0.15 Hz range. Our time-resolved nDTW identified three intuitive
brain states: convergence, divergence, and a mixed state, indicating flexibility in energy
distribution. Simulations confirmed the robustness of nDTW in capturing amplitude and
power disparities between signals, tracking non-stationary amplitude changes, and
remaining invariant to timing and phase deformations. Importantly, our framework
demonstrated functional relevance by associating brain dynamics with clinical and
cognitive scores.

Static results revealed that control subjects exhibited stronger convergence, while
schizophrenia showed relatively greater divergence in brain network power disparities.
Our static analysis suggests that healthy controls maintain more uniform energy demand
across brain networks globally, whereas individuals with schizophrenia exhibit significant
imbalances. These imbalances reflect abnormal glucose metabolic demands and align
with existing studies demonstrating that schizophrenia is associated with glucose

metabolism abnormalities, including an increased risk of diabetes and insulin
resistance[37,38].

A key finding was the increased convergence in controls and increased divergence in
schizophrenia when higher-frequency signals were included. These results indicate that
high-frequency BOLD signals—indicating rapid glucose utilization—play a crucial role in
supporting healthy cognition and enhancing the distinction between control subjects and
those with schizophrenia. In controls, the inclusion of these frequencies appeared to
increase the balance of metabolic energy demands across brain networks, while in
schizophrenia, it exacerbates imbalances. Furthermore, several brain network pairs
showed significant differences between controls and schizophrenia exclusively when
higher frequencies were included. The phase-randomized null model confirmed that these
results were statistically significant and likely attributable to non-stationary amplitude
processes detected by nDTW. While synchronization studies have established the
functional relevance of higher-frequency BOLD signals[30], our findings provide a novel
perspective. We demonstrate that higher-frequency BOLD signals improve the distinction
between control individuals and those with schizophrenia by capturing non-stationary
energy demand disparity during rapid neural processing.

We also found positive associations between nDTW and positive symptoms, particularly
in connections involving the cerebellum with the Aud, DM, CC, SC, and Vis domains. The
cerebellum, recognized as a key predictive hub in the brain[39], not only regulates rapid
cognitive processing—including speed, capacity, and consistency[40]— but also plays a
central role in orchestrating mental coordination. This observation aligns with the
"cognitive dysmetria" theory, which attributes schizophrenia to impaired coordination
among prefrontal regions, thalamic nuclei, and the cerebellum[41].

Our findings further support this theory, revealing that altered cerebellar coordination of
neural processing is linked with positive symptoms[41]. Specifically, lower divergence—or
relatively higher convergence—in schizophrenia was correlated with more severe positive
symptoms such as hallucinations, delusions, and disorganized thinking. Considering that
our results show significantly stronger convergence in controls compared to individuals
with schizophrenia, the observed positive associations with symptom severity may
indicate maladaptive or compensatory [42] cerebellar coordination of glucose utilization,
inadvertently exacerbating positive symptomatology in the affected networks.

Negative associations were observed between brain networks and negative symptoms,
indicating that lower convergence (or higher divergence) is linked to more severe negative
symptoms. These associations were most prominent between higher-order processing
networks such as DM and CC. Given that negative symptoms—characterized by reduced
motivation, pleasure, and emotional expression—are linked to high-level cognitive
processing[43], our findings suggest that greater imbalances in metabolism demands within
and between the DM and CC are associated with increased negative symptoms in
schizophrenia. Prior studies have connected impaired metabolism to negative symptoms

like depression[44,45], and a recent study found that improvements in glucose metabolism—
achieved through prefrontal cortex stimulation—were associated with reductions in
negative symptoms[46].

Notably, most of the PANSS associations identified were due to the inclusion of higher
frequencies, emphasizing the importance of higher frequency BOLD signal. Although
higher frequency bands have been associated with various sources of noise[47], work has
also identified high frequency fluctuations as a source of non-noise variability[30,48,49].
Alterations within such frequency bands may therefore reflect alterations to rapid neural
processing and rapid glucose utilization in schizophrenia.

Our framework identified three dynamic states: convergent, divergent, and mixed. The
convergent state reflects relatively balanced energy demands, suggesting that neural
circuits are operating within optimal ranges, with consistent glucose delivery supporting
stable metabolism and neural energy homeostasis[50]. Conversely, the divergent state
represents imbalanced energy demands, where certain brain networks receive either
insufficient or excessive blood flow. This imbalance can destabilize neural activity and
disrupt the homeostatic regulation of brain energy, ultimately leading to reduced cognitive
performance[45,50,51]. The mixed state lies between these extremes, reflecting neither
strong convergence nor strong divergence. Transitions were observed only between the
mixed and either the convergent or divergent states, with no direct shifts between
convergent and divergent states (Fig. 4d). Thus, the mixed state appears to function as
an adaptive intermediary, enabling the brain to switch flexibly between functional states
as needed.

The brain criticality theory posits that the brain functions near a critical point—a delicate
balance between order and disorder[52]. The mixed state in our framework exemplifies this
balance, contributing 57.36% to the stationary distribution. Combined with the Markov
chain's entropy rate of 36.6%, this finding suggests that the brain operates closer to the
critical entropy point of 50% rather than being too ordered or too disordered, with a slight
bias toward order that remains within the "zone of criticality"[53]. Indeed, critical entropy
serves as an indicator of brain criticality[54]—a state that helps the brain maintain flexibility
and adaptability in its functions.

A key dynamic metric in our study is the convergence rate, which measures how quickly
a system reaches its stable state. Our perturbation analysis revealed that increasing self-
retention probabilities reduced both convergence and entropy rates. This is because
higher self-retention enforces predictability, which runs counter to the flexibility required
by criticality theory. Generally, higher convergence rates correlated with increased
entropy, except for transitions from the mixed to divergent state—highlighting the
significant role of transition dynamics in convergence. State space analysis further
demonstrated that trajectories moving from the divergent state takes the longest to reach

the stable point (Fig. 4e), thereby validating why increasing mixed to divergent transitions
reduces the rate of convergence to the stable point. This pattern underscores the need
to balance order and disorder (i.e., to increase entropy) while optimizing the trajectory
toward stability.

Importantly, control subjects exhibited significantly higher convergence rates—evidenced
by metrics such as the spectral gap and mixing time—compared to patients with
schizophrenia, despite both groups maintaining similar entropy levels. This suggests that
although both groups operate near critical entropy, the dynamics of schizophrenia hinder
efficient convergence to stability, likely due to altered transition pathways. These findings
further imply that high entropy does not necessarily equate to enhanced information
integration or superior cognitive performance, challenging the notion that richer conscious
experiences always result from increased entropy[55].

Cognitive flexibility was further underscored by the associations observed with the mixed
state. In control subjects, working memory performance was positively linked to
transitions from convergent to mixed states and negatively associated with both self-
retention in the convergent state and transitions from mixed to divergent states. These
relationships suggest that the mixed state—reflecting critical dynamics—facilitates the
flexibility necessary for effective higher-order functions such as working memory[56]. In
individuals with schizophrenia, reasoning and problem-solving abilities were negatively
associated with transitions from mixed to convergent states, while showing positive
associations with self-retention in the mixed state and transitions from divergent to mixed
states. Additionally, attention vigilance was negatively related to self-retention in the
divergent state but positively correlated with transitions from divergent to mixed states.
Overall, these findings emphasize the pivotal role of the mixed state in supporting
cognitive flexibility and indicate that disruptions in transitions to and from this state may
contribute to the cognitive impairments observed in schizophrenia.

However, the mixed state may not be optimal for every task. For example, in control
subjects, transitions from the mixed to convergent state were positively associated with
verbal learning, whereas extended self-retention in the mixed state showed negative
associations. Given findings that verbal learning benefits from structured cognitive
training[57]—consistent with the ordered nature of the convergent state—it appears that
while the mixed state promotes flexibility, tasks that require structured cognition may rely
more on the consistency of the convergent state. Overall, these results underscore the
importance of balancing flexibility and stability to optimize cognitive performance.

Group-level analysis revealed that controls predominantly engaged in the convergent
state, while schizophrenia subjects showed greater involvement in the mixed and
divergent states. Metrics such as mean dwell time, fraction rate, state transitions and
stationary distribution confirmed these patterns. The increased presence of divergent
states in schizophrenia aligns with the disorder's characteristic disruption in neural

dynamics[58]. Furthermore, as transitions toward the mixed state were found to decrease
the convergence rate to stability, the excessive engagement of the mixed state in
schizophrenia may reflect an overly sustained flexibility that potentially undermines
cognitive processes such as working memory.

Although our study provides a unifying platform connecting various brain mechanistic
concepts, the use of BOLD fMRI inherently reflects relative rather than absolute metabolic
energy demands. Since BOLD is a hemodynamic signal rather than a direct measure of
energy consumption, its amplitude does not correspond to absolute metabolic
differences. Additionally, preprocessing steps such as z-scoring remove mean and
variance information, making the signal inherently relative. Performing ICA further
emphasizes this, as ICA focuses on maximizing statistical independence and extracting
components based on relative differences rather than absolute amplitudes. As a result,
while we cannot directly map differences in network amplitudes onto absolute
physiological energy demands, relative changes over time in ICA signals remain
meaningful and provide valuable insights into how different brain networks engage with
varying levels of neural activity and associated metabolic demand.

In summary, we provide evidence to suggest that schizophrenia is characterized by a
pronounced disparity in metabolic energy distribution across the brain, both throughout
the scanning session and dynamically, becoming more evident during rapid neural
processing. Additionally, we observed maladaptive metabolic energy demands in a hub-
like interaction between the cerebellum and other brain networks linked to positive
symptoms, while stronger heterogeneity in energy demands across high-order
processing networks, such as the DM and CC, was associated with negative symptoms.
Our findings provide a novel and intuitive perspective on brain criticality using blood flow,
revealing that cognitive flexibility—a key marker of brain criticality—is linked to improved
working memory and fluid intelligence in both controls and individuals with schizophrenia.
However, we also establish that cognitive flexibility may hinder performance on structured
tasks like verbal learning. Furthermore, transitions into a state of greater energy demand
heterogeneity are associated with declines in attention and vigilance in schizophrenia.
Finally, we propose a therapeutic intervention in Supplementary material using
photobiomodulation and the developed directional DTW. This comprehensive framework
linking blood flow to cognition and schizophrenia symptoms offers a unifying perspective
for various hypotheses of brain function in neuroscience research.

493   4. METHODS

494   *4.1. Dynamic time warping as a measure of energy/power disparity*

Dynamic time warping is a widely used signal processing algorithm that aligns signals
with nonlinear temporal deformations by applying adaptive "elastic" transformations to
achieve a meaningful similarity measure[19,59]. When temporal deformations complicate
phase or timing relationships, it becomes logical to isolate these aspects and concentrate
solely on the amplitude of the signals. DTW tackles this by resolving temporal

dissimilarities, optimally aligning signal pairs to minimize a distance metric and bring them
to a shared temporal scale, leaving amplitude as the primary source of disparity.

The algorithm computes a distance cost matrix that captures the cumulative cost of
aligning pairwise elements of the sequences and identifies the optimal warping function
to adjust their time axes, minimizing a distance metric. To enhance computational
efficiency and ensure meaningful alignments, particularly when guided by domain
knowledge, a window size constraint is applied to limit the maximum allowable warping
between indices in the time series[19].

For discrete signals x and y of lengths N and M respectively, the distance cost matrix,
$CM_{DTW(i,j)}$, is initialized as follows:

$$CM_{DTW(i,j)\epsilon(x,y)} = \begin{cases} 0, if\ (i,j) = (0,0) \\ +\infty, otherwise \end{cases}$$

511   The cells in $CM_{DTW(i,j)}$ that are within the allowable window, w, can be defined as:

$$
\begin{aligned}
CM_{DTW(i_w,j_w)\epsilon\{|i-j|\le w\}} \\
= \lambda_\gamma(x_{i_w}, y_{j_w}) \\
+ min \begin{cases} CM_{DTW(i_w-1,j_w-1)} \quad (1) \\ CM_{DTW(i_w-1,j_w)} \\ CM_{DTW(i_w,j_w-1)} \end{cases}
\end{aligned}
$$

512   Following the parameterization of the DTW algorithm recently proposed[25], let $\gamma\epsilon\mathbb{R}^+$
513   denote that γ is a positive real number. Then, the generalized distance metric is given as:

$$\lambda_\gamma(x_{i_w}, y_{j_w}) = |x_{i_w} - y_{j_w}|^\gamma \qquad (2)$$

Where $\gamma > 1$ emphasizes large amplitude disparity, $\gamma < 1$ highlights small amplitude
disparity and $\gamma = 1$ provides a balance between both[25].

The optimal warping path or function φ, with length L, derived from DTW, is the path that
belongs to the set of all possible warping functions Π, where the cumulative distance
metric between corresponding points in signals x and y is minimized. It can be expressed
as:

$$\varphi = \arg min_{\varphi\epsilon\Pi} \sum_{(i_w,j_w)\epsilon\{|i-j|\le w\}} CM_{DTW(i_w,j_w)} \qquad (3)$$

520   Where $L \ge \max(M, N)$

521   The distance cost D in DTW, which measures the phase/timing-invariant amplitude
522   disparity between the signal pair, is computed as the sum of distance metric along the
523   warping path. This cost is expressed as:

$$D = \sum_{\tau=1}^{L} CM_{DTW(\varphi_x(\tau),\varphi_y(\tau))} \qquad (4)$$

524   Where τ is the time index for the warp function, $\varphi_x(\tau)$ is the warping path along signal x,
525   and $\varphi_y(\tau)$ is the warping path along signal y.

526   Notably, when the distance metric with $\gamma = 2$ is chosen the DTW distance cost becomes:

$$D = \sum_{\tau=1}^{L} \left| x\left(\varphi_x(\tau)\right) - y\left(\varphi_y(\tau)\right) \right|^{\gamma=2} \qquad (5)$$

527   This represents the energy of the difference signal $x\left(\varphi_x(\tau)\right) - y\left(\varphi_y(\tau)\right)$, with the
528   normalized DTW [20] distance cost D_n, representing the power of the difference signal, as
529   given below:

$$D_n = \frac{1}{L}\sum_{\tau=1}^{L} \left| x\left(\varphi_x(\tau)\right) - y\left(\varphi_y(\tau)\right) \right|^{\gamma=2} \qquad (6)$$

This positions the algorithm as a measure of temporal/phase-invariant energy/power
disparity between two signals. Leveraging the $\gamma-$parameterization, DTW and nDTW
effectively capture energy and power disparities between signals, respectively. This
flexible framework adjusts to different amplitude scales based on the value of γ.

We define *convergence* as the scenario where the energy or power differences—
quantified by DTW and nDTW respectively—are relatively low, indicating that the signal
levels are close together. Conversely, when these differences are relatively high, we refer
to it as *divergence.*

In the context of fMRI signals, convergence between two brain networks suggests that
the metabolic energy demands, or consumption of the corresponding neural networks are
homogenous. Conversely, divergence suggests an imbalance in energy demands
between these networks. These patterns provide insights into the metabolic energy
specificity and functional interactions of brain networks.

4.2.   Allowable window size for fMRI application

The two main challenges of DTW are its computational cost, with a time complexity of
$O(M \times N)$, and the issue of singularities[19], where a single time point in one signal is
unnaturally matched to a large portion of the other signal. Both challenges are mitigated
by constraining the alignment to a window size that limits the maximum window of
allowable alignment[60]. Given the focus on temporal deformations such as leads/lags and
nonlinear dynamics like stretching and shrinking, selecting an appropriate window size to
capture relevant temporal deformations in fMRI data is crucial. In our previous study using
the proposed warp elasticity approach, we demonstrated that stretching and shrinking
dynamics could introduce anti-correlations[24]. Considering that anti-correlations may also

arise from lead/lag relationships, it is intuitive to choose a window size that spans the
length of the signal required to produce maximum anti-correlations (-1). This approach
has also been validated as optimal in prior fMRI studies[21]. The longest wavelength
yielding an anti-correlation of -1 corresponds to the lowest frequency component
$(0.01Hz)$, which equates to a 100-second window. Achieving an anti-correlation of -1,
requires a time shift of half the wavelength (50 seconds). For the DTW algorithm to
capture relevant temporal deformations in fMRI data, two 50-second windows are
sufficient to account for deformations in both directions, resulting in a total window length
of 100 seconds[21], corresponding back to the low-cutoff frequency of $0.01Hz$. However,
since the filter design for the data—commonly Butterworth—is typically nonideal, we
instead recommend selecting a window size based on the spectral characteristics of the
activity signal, determined by the $-3db$ point of the low-frequency cutoff of the fMRI
signal[13,23,24,61]. The equation for determining the window size is:

$$f_{3db} = \frac{0.88}{\sqrt{N^2 - 1}} F_s \tag{7}$$

566   Where N represents the number of points in the window, F_s denotes the sampling
567   frequency, and f_{3db} is the low cutoff frequency.

4.3.   Time-resolved nDTW

569   To derive a time-resolved evolution of power imbalances between signals, we modify the
570   nDTW measure by eliminating its summation and normalization components. This yields
571   a time-resolved power mismatch metric, $D_{tr}(\tau)$, for the aligned signals, ensuring that its
572   average corresponds to the original nDTW value. The formulations are as follows:

$$D_{tr}(\tau) = \left| x\big(\varphi_x(\tau)\big) - y\left(\varphi_y(\tau)\right) \right|^\gamma$$

$$D_n = \frac{1}{L} \sum_{\tau=1}^{L} D_{tr}(\tau) \tag{8}$$

However, the DTW algorithm results in aligned signal lengths L that vary across different
signal pairs, complicating comparisons between brain network pairs. To ensure that the
length of the time-resolved metric equals the signal length and averages to the nDTW, it
is necessary to interpolate $D_{tr}(\tau)$ with the warped time scale (τ) to have the same length
as the original time index (t), N or M (noting that $N = M$). This interpolation must preserve
the integrity of the metric, particularly the first derivatives of the warping functions, which
are crucial for capturing the varying temporal scales of fMRI time series as established in
our previous study[24].

To address this, we employed the piecewise cubic Hermite interpolating polynomial
(PCHIP) method for interpolating $D_{tr}(\tau)$ to the original signal length N. PCHIP was
selected for its ability to preserve the shape and monotonicity[62] of the original warping
functions, thereby maintaining the temporal dynamics of the fMRI signals. Unlike standard

cubic splines, PCHIP avoids introducing artificial oscillations and overshoots[62], ensuring
the fidelity of the power mismatch measurements. Additionally, PCHIP guarantees
continuous first derivatives[62], aligning with our requirement to maintain the derivatives of
the warping functions. This interpolation ensures that the interpolated metric $D_{tr}^{pchip}(\tau)$
accurately averages to the D_n, facilitating consistent and comparable analyses across all
brain network pairs.

The interpolated time-resolved nDTW metric is thus defined as:

$$D_{tr}^{pchip}(\tau) = PCHIP\big(D_{tr}(\tau)\big), \ \ where \ |\tau| = N = M$$

$$D_n \approx \frac{1}{N}\sum_{\tau=1}^{N} D_{tr}^{pchip}(\tau) \tag{9}$$

To validate our approach, we generated 10,000 independent and identically distributed
(i.i.d.) Gaussian signal pairs, each sampled from a standard normal distribution $\mathcal{N}(0,1)$.
Calculating the percentage relative difference between the nDTW and the average time-
resolved nDTW yielded a median percentage difference of 0.20% (Supplementary
Material).

This methodology ensures that the time-resolved metric retains its temporal dynamics
and derivative continuity while standardizing its length. Consequently, preserving D_n
within the time-resolved measure maintains its reliability, thereby enhancing the
robustness of our analysis.

4.4. Resting-state fMRI data

The first dataset used is from the Function Biomedical Informatics Research Network
(fBIRN) study including resting fMRI data collected from schizophrenia patients and
controls[63]. Scans were collected at a repetition time (TR) of 2 seconds. The study led to
a total of 160 controls with an average age of 37.04 ± 10.86 years, ranging from 19 to 59
years. Among these, 45 were female and 115 were male. Additionally, there were 151
patients diagnosed with schizophrenia, with an average age of 38.77 ± 11.63 years,
ranging from 18 to 62 years. In this group, 36 were female and 115 were male. The typical
controls and schizophrenic patients were meticulously matched in terms of age, gender
distribution, and mean framewise displacement during scans (age: p = 0.18; gender:
p = 0.39; mean framewise displacement: p = 0.97). Schizophrenic patients were in a
clinically stable condition during the time of their scans. The fBIRN dataset is used in all
the clinical analysis performed in this study.

The second dataset used in our study is the resting-state fMRI dataset collected from 827
subjects via the Human Connectome Project (HCP) database[64,65]. We utilized both
session scans acquired using a Siemens Skyra 3T scanner with a multiband accelerated,
gradient-echo echo-planar imaging (EPI) sequence. The scanning parameters were set
to a TR of 0.72s, 72 slices, an echo time (TE) of 58ms, and a flip angle of 90°. A voxel

size of 2 × 2 × 2 mm was used to acquire 1200 time points of fMRI data for each subject.
The HCP dataset was used for our test-retest analysis.

4.5. Resting-state fMRI pre-processing

fMRI data requires extensive preprocessing to correct for various sources of noise and
artifacts before analysis. The preprocessing steps commonly applied in fMRI studies
include slice timing correction, realignment, spatial normalization, and spatial
smoothing[66]. Following typical preprocessing[67], we implemented the NeuroMark pipeline,
a fully automated spatially constrained ICA on the preprocessed fMRI data[68]. Using the
neuromark_fMRI_1.0 template (available in GIFT
at http://trendscenter.org/software/gift or http://trendscenter.org/data) we generated 53
intrinsic connectivity networks (ICNs) for each subject. These ICNs are grouped into brain
functional domains, including subcortical, auditory, sensorimotor, visual, cognitive control,
default mode, and cerebellum.

4.6. Resting-state fMRI post-processing

To enhance the quality of these ICNs, we implemented detrending and despiking
methods to eliminate drifts, sudden fluctuations, and significant artifacts that may have
persisted after the initial preprocessing stages. The time series of the ICNs were
bandpass-filtered within two frequency ranges: F1 (0.01-0.198Hz) and F2(0.01-0.15Hz).
An infinite impulse response filter was designed using the butter function in MATLAB and
applied via the *filtfilt* function to ensure zero phase shifts and preserve phase information.
The optimal filter order was estimated using the *Buttord* function in MATLAB, which
returns the lowest order of the Butterworth filter with no more than 3dB ripple in the
passband and at least 30dB of attenuation in the stopband. Finally, we performed z-
scoring on the ICNs. The MATLAB version used was MATLAB R2022a for all steps of the
analysis.

4.7. Test-retest reliability

Typically, test-retest reliability is evaluated using the intra-class correlation coefficient
(ICC). However, the ICC relies on a linear parametric ANOVA model that assumes
separability and additivity[69]. This makes ICC unsuitable for DTW applications, as DTW
generates a distance metric based on nonlinear warping, where pairwise distances are
interdependent[20,21] and thus lack decomposable variance components. We utilize the
HCP dataset, which includes two sessions and a large sample size of 827 subjects,
facilitating robust test-retest analysis.

4.7.1. Within-subject variability

We adopt the approach of a previous study by employing a straightforward method to
assess within-subject variability: a paired sample t-test between sessions for each brain
network pair[21]. The absolute t-value is reported as a measure of how much the differences
between the two sessions deviate from zero, where lower absolute t-values indicate

higher within-subject consistency. The formula for calculating the absolute t-value
between two sessions of a single network pair across all subjects is provided below:

$$|t|_{within} = \left| \frac{\bar{d}}{s_d/\sqrt{n}} \right| \qquad (10)$$

Where $\bar{d}$ is the mean difference between two sessions, s_d is the standard deviation of the
differences, and n is the total number of subjects.

4.7.2. Between-subject variability

To assess between-subject variability, we developed a non-parametric equivalent of the
absolute t-value. First, we calculate the differences between the means of each subject's
distribution to capture variability across the group. Since the direction of these differences
is not of primary interest, we take their absolute values, resulting in a distribution skewed
toward zero. This transformation allows us to perform a non-parametric test to evaluate
how a non-parametric central tendency measure, such as the median deviates from zero.
Unlike the parametric approach, which scales the central tendency measure by the
standard deviation, our non-parametric method scales it by the interquartile range to
account for the skewed distribution. The formula for the non-parametric absolute
variability measure for a single session is as follows:

$$d_i = |s_{1,i} - \bar{s}_1|$$
$$|t|_{between} = \left| \frac{median(d)}{IQR(d)/\sqrt{n}} \right| \qquad (11)$$

Where d_i is the absolute difference between subject i's measure in session 1 ($s_{1,i}$) and
the group mean for session 1 ($\bar{s}_1$), $IQR(d)$ is the interquartile range of the differences d,
and n is the total number of subjects.

This non-parametric approach provides a robust measure of between-subject variability
that is less influenced by outliers and non-normal distributions, thereby enhancing the
reliability of our variability assessments.

4.7.3. Composite test-retest reliability score

To further summarize these two approaches to give a wholistic view of test-retest
reliability, we compute a composite score of test-retest as follows:

$$trt = 1 - \frac{|t|_{within}}{|t|_{between}} \qquad (12)$$

Scores approaching 1 indicate higher test-retest reliability.

In our study, we compared trt scores across various gamma values of nDTW against
correlation as a benchmark. To achieve this, we ranked the trt scores and performed
Wilcoxon signed-rank tests between each gamma value's trt scores and the correlation

benchmark across all brain network pairs, following a critical difference computation
approach.

4.8. Group difference and clinical association analyses

We employed generalized linear models for both group difference analyses and
associations with clinical scores. All GLM analyses controlled for sex, age, site, and mean
frame displacement. P-values were adjusted using the false discovery rate method, with
significance set at a threshold of 0.05. For group difference analyses and computerized
multiphasic interactive neurocognitive system scores, a normal distribution was assumed,
whereas a Poisson distribution was applied for PANSS score associations due to the
positive integer and skewed nature of the fBIRN dataset's PANSS scores. Effect sizes
were calculated using Cohen's d.

4.9. Group difference sensitivity test

To evaluate the sensitivity of various gamma values of nDTW and correlation in
distinguishing between controls and individuals with schizophrenia, we conducted
McNemar's sensitivity tests comparing each gamma value and correlation against each
other. Following the GLM group difference analyses, we counted the number of significant
brain network pairs (FDR corrected $p < 0.05$) for each gamma value (nDTW) and for
correlation. We then compared the unique and shared significant pairs between each
gamma and correlation using McNemar's test, with p-values adjusted via FDR correction
across all comparisons.

These sensitivity tests were performed separately for each frequency range, F1 and F2.
Additionally, we compared F1 and F2 across all gamma values and correlation metric to
assess how transitioning from the lower frequency range F2 to the broader F1 range
affects group difference sensitivity.

4.10. Validation of high fMRI frequency results.

High-frequency fMRI signals are vulnerable to various sources of noise and artifacts,
including physiological factors like respiratory and cardiac activity, as well as non-
neuronal influences such as head motion[70]. Comprehensive preprocessing steps
followed by our use of ICA help mitigate these concerns. ICA effectively separates
independent components, allowing us to identify and remove artifact-related components
from the data. Additionally, we included head motion parameters as covariates in our
GLM analyses to ensure that detected group differences in high-frequency signals are
not attributable to head motion effects.

We conducted several additional analyses further to validate our findings within the F1
frequency range.

4.10.1.   Statistical effects on groups with the inclusion of higher frequency

We aimed to evaluate the impact of transitioning from the F2 to the broader F1 frequency
range on both control and schizophrenia groups. Since F1 and F2 comprise the same

dataset differentiated only by frequency range, a paired sample t-test was appropriate for
this analysis. To validate the use of the paired t-test, we first assessed the normality of
nDTW scores for each group between F1 and F2 using the Shapiro-Wilk test, ensuring
that t-values were close to 1.

Subsequently, we conducted paired t-tests separately for the control and schizophrenia
groups to examine the effects of transitioning from F2 to F1, with p-values adjusted using
the FDR correction. Effect sizes for these comparisons were calculated using Cohen's d.
Additionally, to highlight the differential impact of transitioning from F2 to F1 between
groups, we computed the differences in Cohen's d effect sizes between controls and
schizophrenia. This comparison aimed to determine whether the transition to the broader
F1 frequency range enhances or diminishes group-specific distinctions.

Furthermore, we identified statistical group differences between schizophrenia and
controls that were present in F1 but not in F2 by analyzing the shared and unique
significant brain network pairs from our GLM analysis (see Section 3.7). We also
highlighted their corresponding effect sizes, demonstrating that significant group
differences are more pronounced in the F1 frequency range.

4.10.2.   Null hypothesis test

The phase randomization (PR) technique generates surrogate fMRI data by preserving
the correlation between brain network pairs[71]. This method is suitable for fMRI null
hypothesis testing as it assesses non-stationarity by capturing information beyond linear
and static correlations[71]. In our previous study[20], we employed PR to evaluate the
statistical relevance of both DTW and nDTW in fMRI data, demonstrating that both
metrics rejected the null hypothesis for several brain region pairs. This validation confirms
that the DTW and nDTW methods are sensitive to capturing information beyond
traditional correlation measures. Since DTW and nDTW are amplitude-sensitive methods
and the intrinsic brain network timecourses are z-scored, they likely capture amplitude
non-stationarity, potentially providing insights into temporal variations in metabolic energy
demands between networks.

In this study, we utilize PR to assess the statistical relevance of nDTW in capturing non-
stationary amplitude changes specifically due to the inclusion of higher frequency data
(0.15–0.198 Hz). The following steps were followed to generate 1,000 surrogate fMRI
datasets:

I.   Phase Randomization**:** Apply PR to the F1 frequency range data (0.01–0.198 Hz)
to create surrogate data, denoted as Z.

II.   Filtering Z: Use the designed filter applied to achieve original F2 fMRI data (see
section 3.4.) on Z to extract the 0.01–0.15 Hz frequency range of the surrogate
data, resulting in surrogate data Y.

III. Isolating Higher Frequencies: Subtract Y from Z to obtain the surrogate data
residual high frequency 0.15–0.198 Hz, denoted as X.

IV.   Full surrogate data: Add X to the original F2 fMRI data (0.01–0.15 Hz) to generate
surrogate data with phase randomization affecting only the residual 0.15–0.198 Hz
frequency range.

This process generates surrogate datasets where randomization impacts solely the
residual 0.15–0.198 Hz frequency range. A high-pass filter was not applied to Z to obtain
the 0.15–0.198 Hz surrogate data because non-ideal filters will artificially create
frequency multiples at the edges (~0.15 Hz).

We performed a Wilcoxon rank-sum test (FDR-corrected $p < 0.05$) to compare nDTW
scores from the original fMRI data with those from surrogate data. This test assessed
whether the significant brain network pairs distinguishing controls from schizophrenia,
identified exclusively in the F1 range, also rejected the null hypothesis.

4.10.3.   *PANSS association exclusive to F1*

774 We identified significant brain network pairs associated with symptom scores that were
775 exclusive to the F1 frequency range and not present in the F2 range.

4.11. Power imbalance dynamics

4.11.1.   *Brain state estimation*

In time-resolved fMRI studies, k-means clustering is frequently utilized to identify recurring
brain states within the temporal dynamics of the data[72]. The DTW algorithm inherently
anchors the first and last time points during alignment, potentially introducing bias by
disproportionately influencing brain states at the temporal extremes. To mitigate this bias,
we excluded the first and last 5 time points from the time-resolved nDTW metrics prior to
clustering. This exclusion reduces alignment-induced artifacts, enabling the clustering
algorithm to more accurately capture intrinsic transitions between brain states.

We concatenated the time-resolved nDTW metrics from all subjects in the fBIRN dataset
into a single feature matrix, where each time point from every subject serves as an
individual sample and each brain network pair constitutes a distinct feature. We
implemented the k-means algorithm with cluster numbers ranging from 1 to 10, setting
the maximum number of iterations to 10,000 and using 20 random initializations to ensure
robust convergence. The city-block (Manhattan) distance metric was selected for its
demonstrated robustness in handling high-dimensional data[73]. To determine the optimal
number of clusters, we generated an elbow plot based on the within-cluster sum of
squares (WSS). The optimal cluster number was identified as 3, corresponding to the first
significant inflection point in the WSS curve's first derivative.

4.11.2.   *Markov chain & stationary distribution*

We quantified the transitions between brain states by counting the number of times each
subject moved from one state to another and normalizing these counts across each state
to construct individual transition probability matrices. This represents the probability of a
subject transitioning from one state to another and further provides insights into the

Markov chain of state dynamics. To gain a comprehensive understanding of the general
transition patterns independent of specific groups, such as patients and controls, we
aggregate the probability transitions across all subjects in the fBIRN dataset.

Furthermore, the stationary distribution of the Markov chain represents the long-term
behavior of the system, indicating the proportion of time the system spends in each state
once it has converged to equilibrium[74]. This distribution serves as an indication of a
"stable point" when analyzing systems from the perspective of their Markov chains rather
than traditional state-space representations. For the stationary distribution to be unique
and ensure convergence from any initial state, the Markov chain must be ergodic[75]. A
Markov chain is considered ergodic if it is irreducible (it is possible to get from any state
to any other state, aperiodic (the system does not cycle in a fixed pattern), and positive
recurrent (the expected return time to each state is finite).

The stationary distribution π is computed by solving the following system of linear
equations:

$$\pi P = \pi$$

$$\sum_{k=1}^{K} \pi_k = 1 \tag{13}$$

814 Where $\pi = [\pi_1, \pi_2, ..., \pi_k]$ is the stationary distribution vector, P is the transition probability
815 matrix of the Markov chain, K is the number of distinct states from K-means (3 in our
816 study).

4.11.3.   Convergence analysis

818 How fast a chain takes to reach the stationary distribution gives a similar equivalent
819 meaning to how fast a system takes to reach its stable point from any given state of the
820 system. The spectral gap gives the rate of this convergence to equilibrium[74]. A larger
821 spectral gap implies faster convergence rate to the stationary distribution. It is computed
822 as follows:

$$\gamma_s = \lambda_1 - \lambda_2 \tag{14}$$

Where λ_1, and λ_2 are the largest and second largest eigen values of the Markov chain
respectively. When the chain is ergodic, $\lambda_1 = 1$.

Beyond the spectral gap, the mixing time provides a more comprehensive measure of
how quickly the chain approaches equilibrium[74]. The mixing time quantifies the number
of steps required for the chain's state distribution to approach the stationary distribution
within a specified tolerance level. A prevalent method for determining the mixing time
leverages the total variation (TV) distance, which offers a rigorous metric for assessing

the divergence between two probability distributions. The total variation distance at time
step t is defined as:

$$\|\mu P^t - \pi\|_{TV} = \frac{1}{2}\sum_{k=1}^{K}|\mu P^t(k) - \pi(k)| \qquad (14)$$

832 Where μ is the initial probability distribution, P^t represents its t-th power of transition
833 matrix, indicating the state probabilities after t transitions, $\pi(k)$ is the stationary probability
834 of state k and K is the total number of distinct states.

835 The mixing time, denoted as t_{MT}, is the smallest positive integer t for which the TV
836 distance falls below a predefined tolerance level tol:

$$t_{MT} = min\{t \in \mathbb{R}^+ \mid \|\mu P^t - \pi\|_{TV} \leq tol\} \qquad (15)$$

For our analysis, we set the tolerance level to $tol = 10^{-3}$. We also find the mixing time
using the initial probability distribution μ farthest from the stationary distribution.

These metrics—spectral gap and mixing time—are instrumental in understanding the
dynamic behavior of brain network states. The spectral gap provides insight into the
inherent convergence rate of the system, while the mixing time offers a tangible measure
of how quickly the system reaches equilibrium. Together, they facilitate a comprehensive
analysis of power imbalances and their temporal evolution within the fMRI data,
enhancing the robustness and reliability of our findings.

4.11.4.   Entropy rate

We investigated how power imbalance dynamics, indicative of the metabolic energy
demands between brain networks, inform the flexibility or rigidity of the brain's metabolic
consumption by analyzing the entropy rate. The entropy rate is defined as the limit of the
average entropy per step as the number of steps approaches infinity. For an ergodic
Markov chain, the entropy rate in bits can be mathematically expressed as:

$$H = -\sum_{i=1}^{K}\pi(i)\sum_{j=1}^{K}P(i,j)log_2 P(i,j) \qquad (16)$$

851 Where $\pi(i)$ is the stationary probability of being in state i and $P(i,j)$ is the transition
852 probability from state i to state j.

853 The maximum entropy rate for $K = 3$ states is:

$$H_{max} = log_2(3) = 1.585 \; bits \qquad (17)$$

In our study, we present entropy rate as a percentage of H_{max} to enhance interpretability.

The entropy rate provides insight into the balance between order and randomness within
the power imbalance dynamics. A low percentage entropy rate indicates strong order in
the dynamics, potentially reducing metabolic flexibility. Conversely, a high percentage
entropy rate signifies a high degree of randomness, suggesting unpredictable metabolic
energy demands between brain networks.

4.11.5. Impact of transition probabilities on Markov chain convergence and entropy rate.

To evaluate how specific transitions within the Markov chain influence the convergence
and entropy rates of the dynamics, we systematically perturbed the aggregated transition
matrix. This process involved incrementally increasing the probability of individual
transition cells and subsequently renormalizing the corresponding rows to ensure that
each row's probabilities sum to one. Consequently, the targeted transition's probability
increased while the probabilities of other transitions within the same row decreased.

For each perturbed transition, we recalculated both the spectral gap and the entropy rate
to determine their percentage relative changes compared to the original values. This
perturbation procedure was applied to every transition cell across 10,000 random
samples, with perturbation magnitudes ranging from 10^{-1} to 10^{-3} on a logarithmic scale
to ensure uniform sampling. To maintain comparability between transitions, each sample
employed a single perturbation magnitude uniformly across all transition cells.

4.12. Time-resolved nDTW group difference: schizophrenia vs controls

To assess the clinical relevance of the time-resolved nDTW measure, we conducted
group difference analyses on several derived metrics: state transitions, stationary
distribution, spectral gap, mixing time, and entropy rate (see Section 3.11). Unlike the
aggregated approach in Section 3.11, these metrics were calculated individually for each
subject. To ensure the uniqueness of these results, we excluded subjects with non-
ergodic state transitions.

Additionally, we included mean dwell time and occupancy rate in our group comparisons.
Mean dwell time quantifies the average duration a subject remains in a particular state
once entered[76], reflecting the persistence of states within the Markov chain. Occupancy
rate represents the percentage of time a subject spends in each state[76], indicating the
relative prevalence of states during the observation period.

Using these subject-specific metrics, we distinguished schizophrenia patients from
control subjects and performed group difference analyses employing a GLM, as
described in Section 3.8. FDR correction is applied to each metric across all three
identified states.

4.13. CMINDS score association

Similarly, we employed a GLM, as detailed in Section 3.8, to assess associations
between each metric and CMINDS scores—including speed of processing, attention

vigilance, working memory, verbal learning, visual learning, and reasoning/problem
solving—within the fBIRN dataset. To account for potential group effects, separate GLM
analyses were conducted for each group. For transition probabilities, we applied FDR
correction to control for multiple comparisons across all transitions.

4.14. Directional time-resolved nDTW

The derivation of DTW, nDTW, and our developed time-resolved nDTW focuses on the
absolute differences between aligned signal pairs, revealing power imbalances between
brain networks. However, using only absolute differences obscures directional
information—specifically, it does not indicate which brain network has higher metabolic
demand. Understanding this directionality is essential for designing targeted interventions
that address specific metabolic flexibilities or rigidities within neural circuits.

To retain directional information, we modified the time-resolved nDTW by modulating it
with directional information, resulting in directional time-resolved nDTW:

$$D_{tr}^{d}(\tau) = \textbf{sign}\left(\left|x(\varphi_x(\tau))\right| - \left|y\left(\varphi_y(\tau)\right)\right|\right) \times D_{tr}(\tau) \qquad (18)$$

This modification allows us to detect both the magnitude and direction of power
imbalances, identifying which brain network exerts greater metabolic demand relative to
its counterpart. We then averaged the directional time-resolved nDTW within each
identified cluster, enabling the determination of brain networks that consistently display
higher or lower power across states. This refined metric provides a more nuanced
understanding of metabolic energy demands, offering precise insights to inform
interventions aimed at modulating metabolic flexibility or addressing rigidities within
specific neural pathways, thereby enhancing the efficacy of clinical treatments.

To evaluate the similarity of directional information across states, we calculated
correlations between the directional time-resolved nDTW metrics of each state.
Additionally, we performed a Kruskal-Wallis test to determine whether the distributions of
directional information differ among the three states. This dual analysis serves as a sanity
check, as we anticipate distinct distributions even if their correlations are comparable.

919 5. DECLARATION OF COMPETING INTEREST.

920 None.

921 6. AUTHORS.

922 **Sir-Lord Wiafe:** Conceptualization, Formal analysis, Methodology, Visualization, Writing
923 –original draft. **Spencer Kinsey:** Methodology, Validation, Writing – review & editing.
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7. DATA & CODE AVAILABILITY STATEMENT.

The codes for all our analyses in MATLAB language can be accessed through GitHub ([study code](#)). The data was not collected by us and was provided in a deidentified manner. The IRB will not allow sharing of data or individual derivatives as a data reuse agreement was not signed by the subjects during the original acquisition.

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