Data-driven classification of tissue water populations by massively multidimensional diffusion-relaxation correlation MRI

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Abstract

Massively multidimensional diffusion-relaxation correlation MRI provides detailed information on tissue microstructure by analyzing water populations at a sub-voxel level. This method correlates frequency-dependent tensor-valued diffusion MRI with longitudinal and transverse relaxation rates, generating nonparametric $\mathbf{D}(\omega)$-R_1-R_2 distributions. Traditionally, $\mathbf{D}(\omega)$-R_1-R_2 distributions are separated using manual binning of the diffusivity and anisotropy space to differentiate white matter (WM), gray matter (GM), and free water (FW) in brain tissue. However, while effective, this approach oversimplifies complex tissue fractions and does not fully utilize all available diffusion-relaxation parameters. In this study, we implemented an unsupervised clustering approach to automatically classify WM, GM, and FW and explore additional water populations using all components in the $\mathbf{D}(\omega)$-R_1-R_2 distributions on *ex vivo* and *in vivo* rat brain, and *in vivo* human brain. Results showed that a basic separation of WM, GM, and FW is possible using unsupervised clustering even under different multidimensional diffusion-relaxation protocols of rat brain and human brain. Additionally, when there is high frequency-dependent diffusion range, it is possible to obtain a cluster characterized by restriction localized in specific high cell density regions such as the dentate gyrus and cerebellum of rat brain. These findings were compared with rat histological sections of myelin and Nissl stainings. We demonstrated that unsupervised clustering of diffusion-relaxation MRI data can reveal tissue complexity beyond traditional WM, GM, and FW segmentation in rat and human brain without parameter assumptions. The unsupervised cluster approach could be used in other body parts (*e.g.*, prostate and breast cancer) without requiring pre-defined bin limits. Furthermore, the characterization of the clusters by diffusivities, anisotropy, and relaxation rates can provide a better understanding of the subtle changes in different cellular fractions in tissue-specific pathologies.

1 Introduction

Diffusion magnetic resonance imaging (dMRI) provides a non-invasive means to access microstructural information about brain tissue by measuring the displacement of water molecules. The interaction of these water molecules with the complex cellular architecture and composition gives rise to multiple water populations associated with various cellular structures, including cell somas, neurites, and diverse lipidic and chemical compositions. The MRI signal from a single voxel reflects a distinct degree of heterogeneity, resulting from a sum of the contributions from these water populations. In pathological conditions, alterations in cellular structures—both in morphology and composition—can further increase the complexity of the dMRI signal by affecting the underlying water populations.

Pierpaoli and collaborators (Pierpaoli *et al.*, 1996) in 1996 initially demonstrated the separation of meaningful water populations using only diffusion MRI data through diffusion tensor imaging parameters, trace and anisotropy. The use of per-voxel single trace and anisotropy values allowed to produce the segmentation of WM, GM, and FW regions. Expanding on this basis, the diffusion MRI community has made significant efforts in addressing intravoxel heterogeneity by developing both biophysical models (Novikov, Kiselev, *et al.*, 2018; Jelescu *et al.*, 2020) and advanced dMRI sequences (Lundell *et al.*, 2019; Aggarwal, 2020; Reymbaut, 2020; Henriques *et al.*, 2021; Jiang *et al.*, 2023). Biophysical modeling separates water populations by defining cellular compartments. In white matter (WM), axons are often represented as sticks, extra-axonal space as an axially symmetric tensor, and free water (FW) as a spherical tensor (Zhang *et al.*, 2012; Novikov, Veraart, *et al.*, 2018; Coelho *et al.*, 2022). In gray matter (GM), additional parameters account for the soma compartment (Palombo *et al.*, 2020). Each model has specific assumptions, requiring compatible acquisition protocols for optimal application. Consequently, these models are limited to the water populations they are designed to analyze and may not be applicable to tissues outside the brain. In practice, studies focusing on WM require specific parameter estimators to capture different diffusion regimes, enhancing the fit between the models and the tissue signal. In addition to biophysical modeling, valuable information regarding water populations can also be obtained by adjusting the scanning acquisition parameters.

Recently, there has been an increased application of multidimensional acquisitions that combine b-tensor encoding diffusion and relaxation (De Almeida Martins *et al.*, 2018, 2021; Martin *et al.*, 2021). One approach is massively multidimensional diffusion-correlation MRI (Narvaez *et al.*, 2022), which facilitates the acquisition of per-voxel nonparametric distributions of diffusivities, anisotropies, restrictions, and chemical compositions ($\mathbf{D}(\omega)$-R_1-R_2-distributions). In this approach, the signal is obtained using free gradient waveforms by modifying the trace (b), normalized anisotropy (b_Δ), orientation (Θ, Φ), centroid frequency (ω_{cent}) with variable repetition (τ_R), and echo time (τ_E). Once the distributions are obtained, an arbitrary set of "bins" limits are selected on the isotropic diffusivity (D_{iso}) and squared normalized anisotropy (D_Δ^2) space to separate and visualize water populations into meaningful tissue fractions (*i.e.*, WM and GM) and free water (FW). Similarly, earlier binning approaches included 1D R_2-distributions for separating myelin water fractions (Kim *et al.*, 2017) or 2D "spectral regions of interest" for differentiating tissue types from 2D correlation experiments (Benjamini *et al.*, 2017; Slator *et al.*, 2019; Pas *et al.*, 2020). In nonparametric $\mathbf{D}(\omega)$-R_1-R_2-distributions, D_{iso} and D_Δ^2 contains homologous microstructural information to mean diffusivity (MD) (Basser *et al.*, 1994). and microscopic fractional anisotropy (µFA) (Lasič *et al.*, 2014; Shemesh *et al.*, 2016a; Topgaard, 2016). Even though the "bin" division of the per-voxel distributions is practical and yields adequate separation of WM-like, GM-like and FW-like water populations, the selection of the "bin" limits needs to be manually modified in order to provide meaningful tissue fractions under different conditions (*i.e.*, *in vivo*, *ex vivo*, type of

sample analyzed); and it misses the opportunity to exploit the multiple dimensions available.
Moreover, the "bin" selection falls short in disentangling the complexity inherent within the
voxel. In addition to well defined water population in tissue, there are potential image artifacts
and not physical plausible distributions due to under-sampling of b_Δ at low b-values (De
Almeida Martins *et al.*, 2018) that could be mixed with true water population using manual
binning approaches. This limitation highlights the need for unsupervised methods capable of
capturing biological relevant features from multidimensional diffusion-relaxation correlation
experiments.

Previous studies have utilized unsupervised approaches incorporating full tensor distribution
data to further characterize tissue heterogeneity in diffuse gliomas (Song *et al.*, 2022). Full
tensor distributions account for both the symmetric and asymmetric components of the tensor,
in contrast to diffusion tensor distribution (Topgaard, 2019) (DTD) approaches, which consider
only the axially symmetric components. Most recently, the simultaneous acquisition of
diffusion data while modifying T1 and T2 relaxation times has demonstrated the ability to
differentiate water populations and provide greater specificity in the context of astrogliosis
(Benjamini *et al.*, 2023). Kundu and collaborators (Kundu *et al.*, 2023) have introduced an
unsupervised learning approach on the optimal transpose distances (LOT) from the
multidimensional T1-T2, T1-MD and T2-MD spaces to distinguish cortical layers that could
not be distinguishable using only 1D relaxation or diffusion parameters.

In this study, we used Gaussian mixture model (GMM) to perform data-driven clustering of
nonparametric multidimensional diffusion-relaxation distributions, extending beyond the
traditional fractions of WM, GM, and FW. We explore the classification of WM, GM, and FW
tissue fractions while also identifying additional detailed tissue clusters within both WM and
GM. These additional clusters can represent shared region-specific water diffusion populations
or possible artifacts within a voxel. The data-driven clustering was applied to a comprehensive
acquisition protocol for *ex vivo* rat brain data, as well as shorter protocols for both *in vivo* rat
and human brains. This work represents a significant advancement in understanding the
efficacy of this approach compared to previous methodologies.

2   Materials and methods

2.1   Animals and data acquisition

2.1.1 *Ex vivo* healthy rat brain at 11.7-T scanner

An adult male Sprague-Dawley rat was used for the *ex vivo* study (10 weeks old, 350 g, Harlan
Netherlands B.V.). The rat was housed individually in a cage with climate-control room under
a 12h/12h light/dark cycle with *ad libitum* diet. The animal procedures were approved by the
Animal Ethics Committee of the Province Government of Southern Finland and carried out
according to the guidelines set by the European Community Council Directives 2010/63/EEC.

First, the rat was anesthetized with 5% isoflurane in a mixture of 70% nitrogen/30 % oxygen
and transcardially perfused with 0.9% NaCl for 5 min (30 ml/min) followed by 4%
paraformaldehyde (PFA) in 0.1 M phosphate buffer (PB) pH 7.4 for 25 min (30 ml/min). After
the perfusion, the brain was removed from the skull and postfixed in 4% PFA for 4 h. Before
the *ex vivo* imaging, the brain was immersed in 0.1 M phosphate buffer saline (PBS) pH 7.4 to
remove the excess of PFA for 24 h. Only before the scan, the brain was divided into two
hemispheres. Then, the left hemisphere was placed inside a 10-mm diameter tube filled with
perfluoropolyether (Galden, Solvay, Italy) to suppress background signal in the images.

The dataset was acquired on a Bruker Avance-III HD 11.7-T spectrometer with a MIC-5 probe
giving 3 T/m maximum gradient amplitude on-axis using a 10-mm diameter coil. The images
were obtained using Bruker's multi-slice multi-echo (MSME) sequence customized to use
double-rotation gradient waveforms (Jiang *et al.*, 2023) running on Paravision 6.01. The
multidimensional acquisition of the rat brain consisted of 1,095 images varying *b*-value (0.038-
$7.13 \cdot 10^9$ sm^{-2}), centroid frequency $\omega_{cent}/2\pi$ within 10-90% (29-110 Hz), normalized anisotropy
b_Δ (–0.5, 0, 0.5, and 1), orientation (Θ,Φ), recovery time τ_R (200-7000 ms), and echo time τ_E
(6-180 ms) in a total scanning time of 87 h. The whole acquisition protocol is shown in
Supplementary Figure 1A. The images were acquired at 18 ± 1 °C. The acquisition was a
single slice in the coronal plane with a resolution of 70 × 70 × 250 μm^3 (matrix size 122 × 128
× 1) located at approximately –3.5 mm from bregma.

The acquired images were preprocessed as follows: The data was reconstructed using
Paravision 6.01 into the real and imaginary domains and denoised, using MPPCA approach
included in DESIGNER toolbox (Chen *et al.*, 2024). After denoising, the magnitude image was
calculated and Gibbs-ringing artifact removal was applied with the sub-voxel shifts (Kellner *et*
al., 2016) using MRtrix3 toolbox (Tournier *et al.*, 2019).

2.1.2 *In vivo* rat brain with at 7-T scanner

One male Sprague-Dawley rat was used for the *in vivo* experiment (10 weeks old, 350 g, Harlan
Netherlands B.V). It was housed individually in a cage with climate-control room under a
12h/12h light/dark cycle with *ad libitum* diet. The animal procedures were approved by the
Animal Ethics Committee of the Province Government of Southern Finland and carried out
according to the guidelines set by the European Community Council Directives 2010/63/EEC.

The rat was anesthetized under ~5% isoflurane in a mixture of 70% nitrogen/30% oxygen for
5 min, after which it was placed on the animal bed equipped with heated water circulation. The
anesthesia then was maintained at ~2% isoflurane with the same nitrogen/oxygen mixture. The
body temperature (36-37°C) and respiration rate (35–60 breaths per minute) were monitored
during the scans.

The dataset was acquired on a 7-T/16-cm horizontal Bruker PharmaScan 7 Tesla preclinical
system with a maximum gradient strength of 760 mT/m running on Paravision 6.01. An
actively decoupled standard volume coil was used for RF-transmission and a quadrature
surface coil for receiving. The images were obtained using Bruker's Spin Echo-Echo Planar
Imaging (SE-EPI) sequence customized to use double-rotation gradient waveforms (Jiang *et*
al., 2023; Yon *et al.*, 2025). The images were acquired in a horizontal plane with a resolution
of 250 × 250 × 700 μm^3 with a slice gap of 300 μm and matrix size of 80 × 129 × 5. Double
sampling and two segments were used. The images were acquired with two repetitions to
include the reversed phase-encode images for later preprocessing steps. The multidimensional
acquisition of the rat brain consisted of 389 pair of images varying *b*-value (0.031-$2.65 \cdot 10^9$ sm^{-}
^{2}), centroid frequency $\omega_{cent}/2\pi$ within 10-90% (18-92 Hz), normalized anisotropy b_Δ (–0.5, 0,
0.5, and 1), orientation (Θ,Φ), repetition time τ_R (1000-3500 ms), and echo time τ_E (21-77 ms)
in a total scanning time of 1 h 9 min. The whole acquisition protocol is shown in
Supplementary Figure 1B.

The preprocessing of the *in vivo* rat images was as it follows: 1) The images were reconstructed
in Paravision 6.01. 2) The images were separated into their respective phase-encoding
directions using Matlab (Natick, Massachusetts: The MathWorks Inc.) ; 3) each set of phase-
encoding directions were denoised separately using the MPPCA approach (Chen *et al.*, 2024);

4) Gibbs rings artifact removal (Kellner *et al.*, 2016; Tournier *et al.*, 2019); and 5) correction
of susceptibility-induced off-resonance field using FSL top up tool as described in (Yon *et al.*,
2025).

2.1.3 *In vivo* human brain at 3-T scanner

Six healthy volunteers were scanned (2 females, 4 males: age range 23-51 years) in compliance
with the Finnish ethical permit 134/2023, 23.05.2023 and Finnish Medicines Agency (FIMEA)
permit, FIMEA/2023/004206 in a 3-T MAGNETOM Vida Siemens scanner, equipped with a
70 cm bore size and 60/200 XT gradients. T1w images with voxel size of $1 \times 1 \times 1$ mm^3,
repetition time $\tau_R = 2.3$ s, and echo time $\tau_E = 1.9$ ms were evaluated by an experienced
radiologist, and no abnormalities were detected. The multidimensional images were obtained
with a SE-EPI sequence customized to use diffusion general gradient waveforms(Martin *et al.*,
2020). The data was acquired with a voxel size of $2 \times 2 \times 2$ mm^3 (matrix size $114 \times 114 \times 55$
mm^3). The multidimensional acquisition of the human brain consisted of 139 images varying
b-value (0-$3 \cdot 10^9$ sm^{-2}), centroid frequency $\omega_{cent}/2\pi$ within 10-90% (4-9 Hz), normalized
anisotropy b_Δ (–0.5, 0, and 1) using numerically optimized waveforms (Sjölund *et al.*, 2015),
orientation (Θ, Φ), repetition time τ_R (620-7000 ms), and echo time τ_E (40-150 ms) in a total
scanning time of 40 min. The whole acquisition protocol is shown in **Supplementary Figure**
1C.

The preprocessing of the *in vivo* human images was as it follows: 1) The images were denoised
using the MPPCA approach (Chen *et al.*, 2024); 2) Gibbs rings artifact removal (Kellner *et al.*,
2016; Tournier *et al.*, 2019); and 3) correction of susceptibility-induced off-resonance field
using FSL top up tool using a couple of non-diffusion weighted images with two different
phase-encoding directions (Andersson *et al.*, 2003).

2.2 Estimation of nonparametric D(ω)-R_1-R_2-distributions and visualization

231 Nonparametric **D**(ω)-R_1-R_2-distributions are derived through the approximation of the **b**(ω)-
232 τ_R-τ_E encoded signal *S* according to the following equation:

$$S[\mathbf{b}(\omega), \tau_R, \tau_E] = \sum_i w_i \exp\left(-\int_{-\infty}^{\infty} \mathbf{b}(\omega) : \mathbf{D}_i(\omega) d\omega\right)\left[1 - \exp(-\tau_R R_{1,i})\right]\exp(-\tau_E R_{2,i}),$$

(1)

233 The inversion of **Eq. 1** allows the acquisition of the ω-dependent self-diffusion tensor **D**(ω) by
234 approximating $\mathbf{D}_i(\omega)$ as an axisymmetric tensor,

$$\mathbf{D}_i(\omega) = \mathbf{R}(\theta_i, \phi_i)\begin{pmatrix} D_{\perp,i}(\omega) & 0 & 0 \\ 0 & D_{\perp,i}(\omega) & 0 \\ 0 & 0 & D_{\parallel,i}(\omega) \end{pmatrix}\mathbf{R}^{-1}(\theta_i, \phi_i),$$

(2)

235 where $\mathbf{R}(\theta_i, \phi_i)$ is a rotation matrix and ω-dependent parallel and perpendicular eigenvalues,
236 $D_{\parallel,i}(\omega)$ and $D_{\perp,i}(\omega)$, are described by Lorentzian transitions (Narvaez *et al.*, 2024) according
237 to:

$$D_{\parallel,i/\perp,i}(\omega) = D_{0,i} - \left(D_{0,i} - D_{\parallel,i/\perp,i}\right)\frac{1}{1 + \omega^2/\Gamma^2_{\parallel,i/\perp,i}} \tag{3}$$

Using the Lorentzian approximation, each component in the $\mathbf{D}(\omega)$-R_1-R_2-distributions is
described with its weight w and parameter set [$D_\parallel$, $D_\perp$, θ, ϕ, D_0, $\Gamma_\parallel$, $\Gamma_\perp$, R_1, R_2]. We use the
Monte Carlo inversion (Prange *et al.*, 2009) described in detail in (Narvaez *et al.*, 2022) and
(Narvaez *et al.*, 2024) to retrieve the ensemble of solutions using Matlab (Natick,
Massachusetts: The MathWorks Inc.) with the *md-dmri* toolbox (Nilsson *et al.*, 2018).

The inversion limits used for the *ex vivo* rat brain at 11.7 T, *in vivo* rat at 7 T, and *in vivo* human
brain at 3 T are presented in **Table 1.**

	$\mathbf{D_{0/A/R}}$		$\mathbf{\Gamma_{A/R}}$		$\mathbf{R_1\ s^{-1}}$		$\mathbf{R_2\ s^{-1}}$	
	Min $10^{-12}\ \mathrm{m^2s^{-1}}$	Max $10^{-9}\ \mathrm{m^2s^{-1}}$	Min	Max	Min	Max	Min	Max
ex vivo rat	5	5	0.1	10^{5}	0.1	4	1	150
in vivo rat	5	5	0.1	10^{5}	0.1	4	4	100
in vivo human	5	5	0.1	10^{4}	0.1	20	1	30

Table 1. Monte Carlo inversion limits

The Monte Carlo inversion settings for the three experiments were set to 20 steps of
proliferation, 20 steps of mutation/extinction, 200 input components per step of proliferation
and mutation/extinction, 10 output components, and bootstrapping by 100 repetitions using
random sampling with replacement. As a result, the Monte Carlo inversion outputs an ensemble
of 100 independent per-voxel solutions. Each of the solutions comprised the weights w and
coordinates [$D_\parallel$, $D_\perp$, θ, ϕ, D_0, $\Gamma_\parallel$, $\Gamma_\perp$, R_1, R_2] for 10 components within the pseudo-randomly
sampled primary analysis space. We then evaluated $\mathbf{D}(\omega)$ distributions at chosen values within
10 and 90 percentiles of the ω range probed by the gradient waveforms (Narvaez *et al.*, 2022,
2024; Yon *et al.*, 2025), giving ω-dependent distributions [$D_\parallel(\omega)$, $D_\perp(\omega)$, θ, ϕ, R_1, R_2]. To
follow well stablished metrics in the field (Pierpaoli *et al.*, 1996), we obtained isotropic
diffusivity $D_{iso}(\omega)$, normalized anisotropy $D_\Delta(\omega)$, and squared normalized anisotropy $D_\Delta^2(\omega)$
through:

$$D_{iso}(\omega) = \frac{D_{\parallel,i}(\omega) + 2D_{\perp,i}(\omega)}{3}, \tag{4}$$

$$D_\Delta(\omega) = \frac{D_{\parallel,i}(\omega) - 2D_{\perp,i}(\omega)}{3D_{iso,i}(\omega)}, \tag{5}$$

and

$$D_\Delta^2(\omega) = \frac{[D_{\parallel,i}(\omega) - D_{\perp,i}(\omega)]^2}{[D_{\parallel,i}(\omega) + 2D_{\perp,i}(\omega)]^2} \tag{6}$$

respectively.

In line with previous results from oscillating gradient encoding (Aggarwal *et al.*, 2012, 2020;
Xu *et al.*, 2016; Aggarwal, 2020; Arbabi *et al.*, 2020) and frequency-dependent
multidimensional studies (Narvaez *et al.*, 2022, 2024; Johnson *et al.*, 2024; Manninen *et al.*,

2024; Yon *et al.*, 2025), restriction effect in ω-dependent D_{iso} and D_Δ^2 is quantified using the
difference within the investigated frequency window from low to high ω.

$$\Delta_{\omega/2\pi}D_{\text{iso}} = \frac{D_{\text{iso}}(\omega_{\text{max}}) - D_{\text{iso}}(\omega_{\text{min}})}{\frac{\omega_{\text{max}} - \omega_{\text{min}}}{2\pi}} \tag{7}$$

$$\Delta_{\omega/2\pi}D_\Delta^2 = \frac{D_\Delta^2(\omega_{\text{max}}) - D_\Delta^2(\omega_{\text{min}})}{\frac{\omega_{\text{max}} - \omega_{\text{min}}}{2\pi}} \tag{8}$$

264 The distributions can be visualized using contour plots in the 2D D_{iso}-D_Δ^2, D_{iso}-R_1, and D_{iso}-R_2
265 space, as well as bin-resolved signal fractions $f_{\text{bin}n}$, means $E_{\text{bin}n}[X]$, variances $V_{\text{bin}n}[X]$, and
266 covariances $C_{\text{bin}n}[X,Y]$ appropriate to create per-voxel parameter maps (Topgaard, 2019). In
267 this study, we used only per-voxel $f_{\text{bin}n}$ and $E_{\text{bin}n}[X]$ for visualization, calculated as follows:

$$f_{\text{bin}n} = \frac{1}{S_0} \sum_{i \in \text{bin}n} w_i, \ n = 1,2,3 \tag{9}$$

268 and

$$E_{\text{bin}n}[X] = \frac{1}{S_0 f_{\text{bin}n}} \sum_{i \in \text{bin}n} w_i X_i, \ n = 1,2,3 \tag{10}$$

2.3 Unsupervised clustering of D(ω)-R_1-R_2-distributions

Derived from 100 bootstraps with 10 components each, we obtain a total of 1000 per-voxel
components. To consider the weights (w), each parameter value (D_{iso}, D_Δ or D_Δ^2, R_1, R_2, and
$\Delta_{\omega/2\pi}D_{\text{iso}}$) was replicated by its associated weight normalized and discretized between 0 and
100. Then, the resulted weighted parameters D_{iso}, D_Δ or D_Δ^2, R_1, R_2, and $\Delta_{\omega/2\pi}D_{\text{iso}}$ were
organized into a 2D matrix and clustered using GMM approach, which is a probabilistic model
that assumes that the data is a result of a mixture of normally distributed processes.

Using multidimensional normal distribution,

$$\mathcal{N}(x|\mu, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{(x-\mu)^T \Sigma^{-1}(x-\mu)}, \tag{11}$$

280 where μ is the mean of the distribution, Σ is the standard deviation of the distribution, and x is
281 data, the GMM can be written as

$$p(X) = \prod_{n=1}^{N} p(x_n) = \prod_{n=1}^{N} \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k),$$

(12)

where x_n is a data sample, K is the number of clusters, N is the number of datapoints, and
$\sum_k \pi_k = 1$. The GMM was solved by an expectation maximization algorithm that maximizes
the likelihood of a model given the number of mixture components k. The different dimensions
in the bootstrapped data were normalized to the same range by dividing each variable value
with the standard deviation of the variable.

The data was clustered in two different ways. In the first one, we only include the variables
D_{iso} and D_Δ^2 into the clustering using $k = 3$ to compare the results with the manual "3-bin"
resolved fractions as used in previous studies (De Almeida Martins *et al.*, 2020; Yon *et al.*,
2020). For the second approach, we used D_{iso}, D_Δ, R_1, R_2 and $\Delta_{\omega/2\pi}D_{\mathrm{iso}}$ variables for the *ex vivo*
and *in vivo* rat brain, and D_{iso}, D_Δ, R_1, R_2 for the *in vivo* human brain using $k = 2\text{-}10$. We
selected D_Δ for clustering to clearly differentiate the oblate distribution, facilitating the possible
identification of inversion artifacts or biologically relevant features. As it is well known,
unsupervised clustering methods rely in the selection of the k components. We calculated
Schwarz's Bayesian Information Criterion (BIC) to aid the selection of the ideal k components
using $k = 2\text{-}15$ **Supplementary Figure 2.** The spatial information from the resulting clusters,
clus$n$, can be visualized using the per-voxel normalized frequency, Fclus$n$. Using a similar
formulation as bin-resolved approach, we obtained the cluster-resolved signal fractions f_{clus}
which can be visualized using contour plots in the 2D D_{iso}-D_Δ^2, D_{iso}-R_1, and D_{iso}-R_2 space and
per-voxel means $\mathrm{E}_{\mathrm{clus}n}[X]$ parametric maps calculated as follow:

$$f_{\mathrm{clus}n} = \frac{1}{S_0} \sum_{i \in \mathrm{clus}n} w_i \,, n = A, B, C, \ldots$$

(13)

and

$$\mathrm{E}_{\mathrm{clus}n}[X] = \frac{1}{S_0 f_{\mathrm{clus}n}} \sum_{i \in \mathrm{clus}n} w_i X_i \,, n = A, B, C, \ldots$$

(14)

302 In the *in vivo* human data, we computed the clusters for every subject independently to compare
303 the cluster centroids and covariance matrices across solutions.

2.4 Tissue processing and histology

The hemisphere scanned *ex vivo* was washed 2 times for 25 min in 0.1 M PB pH 7.4 and then
cryoprotected for 48 h in a solution containing 20% glycerol in 0.02 M potassium phosphate-
buffered saline (KPBS) pH 7.4. After cryoprotection, the hemisphere was frozen in dry ice and
stored at –70°C until further processing. The hemisphere was coronally sectioned into 30-µm
thick section and 1:2 series using sliding microtome. The first series of sections were stored in
10% formalin at room temperature and used for Nissl staining using thionin. The second series
of sections were collected into tissue collection solution (30% ethylene glycerol, 25% glycerol
in 0.05 M sodium phosphate buffer) and stored at –20°C until staining with gold chloride for
myelin (Laitinen *et al.*, 2010).

2.5 MRI region-of-interest (ROI) and histology analyses

ROI analysis was performed on the *ex vivo* rat brain using the cluster fractions resulting from
the full space clustering with $k = 7$. ROIs were selected in the auditory cortex, primary
somatosensory cortex, granule cell layer of the dentate gyrus, corpus callosum and cingulum.
D_{iso}, D_Δ, R_1, and R_2 distributions of f_{clusA}, f_{clusB}, f_{clusC}, f_{clusD}, and f_{clusG} were obtained for each
ROI. The projections were normalized by the maximum intensity within the cluster fractions.
This normalization allowed to compare the cluster fraction contribution to each ROI
independently of the ROI size.

We used a Nissl-stained section and a myelin-stained section from the same level in MRI to
perform a comparison of cellular features with ROIs. Cells segmentation was performed on the
Nissl-stained section, in which the per-cell area was calculated using QuPath v.0.5.0 software
(Bankhead *et al.*, 2017). Structure tensor (ST) analysis was calculated on the myelin staining
(Budde *et al.*, 2012; Molina *et al.*, 2020), then, the photomicrograph was divided in patches of
70×70 μm^2 to match with the in-plane voxel resolution as in *ex vivo* MRI. Angular central
Gaussian (ACG) probability method was used to parametrize the results derived from the ST
(Salo *et al.*, 2021) and the coherency map ($C = \frac{\lambda_{max}-\lambda_{min}}{\lambda_{max}+\lambda_{min}}$) was calculated (Püspöki *et al.*,
2016). In addition, the optical density (OD) on the myelin staining was computed by manually
selecting a background threshold and normalizing.

3 Results

Figure 1A and B shows the division of $\mathbf{D}(\omega)$-R_1-R_2 distributions into the manual three "bin"
limits on the D_{iso}-D_Δ^2 space, as in previous studies (De Almeida Martins *et al.*, 2018, 2020,
2021; Yon *et al.*, 2020; Martin *et al.*, 2021; Reymbaut *et al.*, 2021; Narvaez *et al.*, 2022, 2024).
These bins can be considered as WM-like (bin1), GM-like (bin2) and FW-like (bin3). In the
present study, the term WM to refer to such as the corpus callosum, cingulum, internal capsule,
and optic tract (see the myelin-stained section in **Figure 6A and C**). Bin1 also included highly
myelinated regions in the deep cortical and thalamic areas. Bin2 represented the remaining GM
regions of the brain, while bin3 included the ventricles and liquid on the periphery of the brain.

Our first step was to compare the bin-resolved mean parametric maps, and the maps obtained
with our unsupervised classification approach using $k = 3$. **Figure 1A and B** shows the per-
voxel mean parametric maps of bin1 with characteristic low E[D_{iso}], high E[D_Δ^2], and both high
E[R_1] and E[R_2] values in WM. bin2 had similar E[D_{iso}] as in bin1, and lower E[D_Δ^2], E[R_1]
and E[R_2] than in bin1, as expect in GM. bin3 showed higher E[D_{iso}] than in bin1 and bin2,
while E[D_Δ^2], E[R_1], and E[R_2] values were lower than in bin1 and bin2, corresponding to FW.
Figure 1C and D display the cluster-resolved mean parametric maps using $k = 3$ in the D_{iso}-
D_Δ^2 space. The parametric maps of three cluster-fractions resemble the bin-resolved
counterparts. The major difference between these two approaches was found in the FW fraction
(**Figure 1D**). bin3 was only present in the ventricles and brain periphery, while clus3 showed
the ventricles, brain periphery but also GM. In clus3, the highest E[D_{iso}] values were found in
the ventricles and the periphery of the brain. In GM, the E[D_{iso}] values were lower than in
clus3, but not as low as in clus2. The frequency-dependent parameter, $\Delta_{\omega/2\pi}$E[D_{iso}], also showed
differences between bin- and cluster-resolved maps. While the highest $\Delta_{\omega/2\pi}$E[D_{iso}] values were
mainly found in the granule cell layer of the dentate gyrus in the bin-resolved bin1 and bin2,
these high values were found in the cluster-resolved clus2 and clus3 (white arrowhead in
Figure 1).

Figure 2 displays the resulted normalized cluster frequency by setting k from 2 to 10 when
applied the full D_{iso}, D_Δ, R_1, R_2 and $\Delta_{\omega/2\pi}D_{iso}$ space. As showed in **Figure 1D**, using the D_{iso}-
D_Δ^2 space generated biological relevant fractions (*i.e.*, a clear distinction of WM, GM, and FW
regions) using $k = 3$; however, when using the full space required at least to $k = 4$ to get such
biologically relevant fractions.

Using $k = 4$, FclusA (included 43% of the data) contains mainly GM, while major WM regions
were in FclusB (29%) including the corpus callosum, external and internal capsule or optic
tract. FclusC (16%) comprised FW regions in the ventricles and periphery of the brain, and the
granule and pyramidal cell layers. FclusD (12%) contained WM areas, especially in the
cingulum and internal capsule, as well as highly myelinated thalamic regions. The difference
between $k = 4$ and $k = 5$ clusters was that FclusC in $k = 4$ split into 2 fractions in $k = 5$: FclusC
displaying GM regions and FclusE mainly ventricles and periphery of the brain. clusD in $k = 5$
exclusively showed WM regions. With $k = 6$, an additional cluster, FclusF (2%), was identified
throughout the entire brain. Using $k = 7$, we found the separation of FclusA (30%) into FclusG
(19%) displaying GM regions. The clusters obtained with $k = 8$, 9 and 10 remained constant as
with $k = 7$, with the addition of FclusH, FclusI and FclusJ; however, these clusters contain small
percentages of data of approximately 1-7%.

From $k = 7$ onwards, the clusters FclusA to FclusG remained similar across regions and
percentages. For this reason, we selected the $k = 7$ cluster to visualize the parameter
distributions that define each of the fractions, and obtained 2D contour plots between D_{iso}-D_Δ,
D_{iso}-R_1, and D_{iso}-R_2 (**Figure 3**). For plotting the frequency dependance, we used the frequencies
at 10, 50 and 90 % ($\omega/2\pi = 29$, 69.5 and 110 Hz). The cluster fractions were ordered by amount
of data classified into each fraction from largest to smallest.
f_{clusA}, f_{clusG}, and f_{clusC} contained GM regions; however, those fractions were defined by different
parameters (**Figure 3A**). f_{clusA} mainly consisted of D_Δ values close to zero; f_{clusG} contained
negative D_Δ with the mode (Mo) = -0.46, and f_{clusC} was characterized by variable D_Δ at different
frequencies $\omega/2\pi$ Hz. f_{clusB} and f_{clusD} were non-zero in WM regions, the main distinction between
these two fractions was that R_1 (Mo = 0.65 s^{-1}) and R_2 (Mo = 24.38 s^{-1}) values in f_{clusB} were lower
than in f_{clusD}. In f_{clusE}, non-zero values were present in FW regions with low R_1 and R_2 values
than previous cluster fractions. D_{iso} values across cluster fractions showed that f_{clusA}, f_{clusB}, and
f_{clusG} had a similar D_{iso} distributions (Mo = 0.27×10^{-9} m^{2}s^{-1}) (**Figure 3B**). f_{clusC} showed higher
D_{iso} mode values at 0.62×10^{-9} m^{2}s^{-1} at $\omega/2\pi = 110$ Hz. FW cluster (f_{clusE}) presented values over
1×10^{-9} m^{2}s^{-1}, while f_{clusD} showed the lowest D_{iso} values (Mo = 0.12 ×10^{-9} m^{2}s^{-1}) than the rest of
the fractions.

Figure 4 shows all the parameter maps using the per-voxel cluster-resolved means in the $k = 7$
cluster fractions using the full space. We obtained the color-coded WM, GM, and FW-like
fractions (**Figure 4A**) using the f_{clusB}, f_{clusA} and f_{clusE}, respectively. This visualization is a
representation of the already described distributions in **Figure 3**. Hence, we can observe in
Figure 4B WM-like regions (clusB and clusD) and GM-like regions (clusA, clusG and clusC)
in color-coded according to the per-voxel means. It is worth emphasizing that previously, in
Figure 1B and D the high $\Delta_{\omega/2\pi}E[D_{iso}]$ were located mainly in the granule cell layer of the
dentate gyrus within other tissue or FW fractions. Here (**Figure 4B**), clusC is the cluster that
contains high $\Delta_{\omega/2\pi}E[D_{iso}]$ not only in the granule cell layer but also across other GM-like
regions.

The cluster-resolved parameter maps E[D_{iso}], E[D_Δ^2], and E[R_2] from the WM-like fractions
clusB and clusD were compared with myelin staining from the same animal (**Figure 5A-C**).
Although all are major WM fiber bundles, we observed differences in organization and density

across the corpus callosum, cingulum, optic tract, and internal capsule. The optic tract
demonstrated a more compact and dense organization than the corpus callosum. In contrast,
the internal capsule exhibited complex crossing bundles, whereas to other regions like the
corpus callosum were characterized by more uniformly parallel fiber organization (**Figure 5C**).
The cingulum exhibited a rostro-caudal orientation, compared to the corpus callosum, which
displayed an in-plane orientation in coronal sections (**Figure 5C**). Notably, although less dense
than WM regions, certain GM regions—such as the thalamus, retrosplenial cortex, and deep
cortical layers—exhibit high myelin content, as shown in **Figure 5A**. Regarding the parametric
maps that can be comparable with the myelin staining by their presence in WM-like regions
(**Figure 5B**), $E[D_\Delta^2]$ in clustB showed similar values across the corpus callosum, optic tract
and internal capsule, while the $E[R_2]$ parametric map in clusD showed higher values in the
optic tract and internal capsule than the corpus callosum (**Figure 5C**).

The representative Nissl-stained section revealed regions with high cell density, such as the
granule cell layer, CA3 and medial habenular nucleus (**Figure 5D and F**), which corresponded
to high $\Delta_{\omega/2\pi}E[D_{iso}]$ values in clusC (**Figure 5F**). In contrast, clusA exhibited limited
differentiation between GM regions in the $\Delta_{\omega/2\pi}E[D_{iso}]$ and $E[D_{iso}]$ parametric maps.

Figure 6 presents the results of ROI analyses for selected brain areas using both MRI and
histology, reveling distinct distribution patterns between WM and GM regions (**Figure 6B**).
Here, the projections of the distributions were normalized by the maximum intensity within the
cluster fractions. This normalization allowed to compare the proportion of data included in the
cluster-fractions between ROIs independently of the number of voxels. f_{clusA} distributions for
all four parameters (D_{iso}, D_Δ, R_1 and R_2) displayed consistent parameter values and similar data
proportions across the auditory cortex, primary somatosensory cortex, and granule cell layer.
The most notable difference emerged in f_{clusC}, in which the proportion of data was larger in the
granule cell layer, followed by the auditory cortex and then, by the primary somatosensory
cortex. Comparison of GM regions with Nissl staining (**Figure 6C**) revealed the highest cell
density in the granule cell layer. Although segmentation of individual cell bodies in the granule
cell layer was not successful, the auditory cortex was found to have a larger cellular area
compared to the primary somatosensory cortex (**Figure 6D**), consistent with observations in
f_{clusC}. Small differences were noted between GM regions in f_{clusB} and f_{clusG}, where the auditory
cortex showed a higher data proportion than primary somatosensory cortex and granule cell
layer. As mentioned above, f_{clusB} is associated with high myelin content. Within GM regions,
the auditory cortex demonstrated higher density of myelinated axons, as showed by the myelin
staining and optical density map (**Figure 6E and F**). The coherency map further revealed
increased fiber orientation coherence toward the superficial cortical layers in both the auditory
cortex and primary somatosensory cortex (**Figure 6G**).

The proportion of data within the corpus callosum and cingulum in f_{clusA}, f_{clusG}, and f_{clusC} was
relatively small as compared to GM areas (**Figure 6B**). However, among WM regions, the
cingulum displayed a slightly higher proportion of data within those cluster-fractions than the
corpus callosum. Both the corpus callosum and cingulum demonstrated lower cell density and
smaller cellular area than GM regions (**Figure 6C and D**). In f_{clusB}, the corpus callosum and
cingulum showed consistent distributions across the D_Δ, R_1 and R_2 parameters, with only small
differences in D_{iso}. Notably, the cingulum displayed a broader D_{iso} distribution compared to the
corpus callosum. The most prominent difference was observed in f_{clusD}, where the cingulum
had a higher proportion of data and broader R_1 and R_2 distributions than the corpus callosum.
Both, the corpus callosum and cingulum contained similar high densities of myelinated axons,
as confirmed by the myelin staining and optical density map (**Figure 6E and F**). However, the

coherency map revealed lower coherence values in the cingulum compared to the corpus
callosum (**Figure 6G**).

We obtained the per-voxel cluster frequencies from the *in vivo* rat brain using $k = 2$ to $k = 10$
(**Supplementary Figure 3**). Compared to the *ex vivo* rat brain data, the *in vivo* data were
characterized by lower signal-to-noise ratio (SNR), larger voxel size, and a different acquisition
protocol. Consequently, the optimal k value for biologically relevant clustering differed
between *in vivo* and *ex vivo* datasets. Despite these differences, the resulting cluster at $k = 6$
(**Figure 7**) remained similar in terms of the overall parameter distributions.

In the *in vivo* dataset (**Figure 7A**), components belonging to clusA, clusB and clusC were
primarily located in GM with smaller proportions in WM regions (33, 19 and 18%,
respectively). clusD was observed in the ventricles and periphery of the brain (14%) while
clusF was predominantly located in WM regions (13%). clusE was distributed across the entire
brain but accounted for only a small portion of the clustered data (3%), consistent with findings
from the *ex vivo* results that clusD of the *ex vivo* rat brain (**Fig.3A**) was not present in the *in*
vivo clusters.

For plotting the frequency dependance, the centroid frequencies values at 10, 50 and 90% (ω
$/2\pi = 18$, 52 and 92 Hz) were used. Contour plots for each fraction (**Figure 7A**) showed that
f_{clusA} was mainly characterized by the mode on $D_\Delta = 0$. The cluster fraction f_{clusB}, had higher
D_{iso} and mode of $D_\Delta = -0.5$ compared to f_{clusB}. A clear frequency dependance in both D_{iso} and
D_Δ was displayed in f_{clusC}. The cluster fraction considered as the FW-like fraction, f_{clusD},
displayed the highest values of $D_{\text{iso}} > {\sim}1.5 \times 10^{-9}$ m^{2}s^{-1} together with low R_1 and R_2. Regarding
the WM-like fraction f_{clusE}, it contained the highest $D_\Delta > {\sim}0.5$ and high R_2. Cluster-resolved
parameter maps (**Figure 7D**) of f_{clusC} and f_{clusE} allowed to visualize that the highest values of
$\Delta_{\omega/2\pi}E[D_{\text{iso}}]$ in f_{clusC} were in the cerebellum whereas f_{clusE} exhibited high values of $E[D_\Delta]$ and
$E[R_2]$ in WM regions.

In the *in vivo* human brain, the parameters included into the clustering were D_{iso}, D_Δ, R_1 and
R_2. The parameter $\Delta_{\omega/2\pi}D_{\text{iso}}$ was excluded due to the limited frequency range available in the
dataset (4-9 Hz). Per-voxel cluster frequency was obtained using $k = 2$ to $k = 10$
(**Supplementary Figure 4**). The results from $k = 6$ were used to display the cluster fractions
contour plots and cluster-resolved maps from a representative subject (**Figure 8**). Although the
in vivo human data resulted in similar number of clusters as *in vivo* rat brain data, there is a
clear absence of a frequency-dependent fraction. However, the clustering effectively delineated
spherical and linear shapes as observed in the D_Δ parameter. f_{clusA} (22%) was primarily in GM
regions such as cortex and deep GM characterized by low D_{iso} and the mode on spherical shapes
$D_\Delta = 0$. f_{clusB} (21%) in WM regions had low D_{iso}, $D_\Delta > 0.5$, and higher R_2 than f_{clusB}. f_{clusC} (21%)
was across WM and GM regions characterized by mode on $D_\Delta = -0.5$. f_{clusD} was mainly located
in ventricles and periphery of the brain (18%) with mode in $D_{\text{iso}} \cong 3 \times 10^{-9}$ m^{2}s^{-1}. f_{clusD} was
mainly located in ventricles and periphery of the brain (18%) with mode in $D_{\text{iso}} > 3 \times 10^{-9}$ m^{2}s^{-}
^{1}, while f_{clusE} had a larger range of D_{iso} between 1×10^{-9} m^{2}s^{-1} and 4×10^{-9} m^{2}s^{-1}. Finally, f_{clusF}
(4%) was found in the periphery of the brain.

We computed the clusters for all the 6 human subjects to test repeatability (**Figure 9**). For all
the subjects the clusters were calculated independently and with $k = 2\text{-}10$. We found that at k
$= 6$, the per-voxel cluster frequencies were similar across subjects. Hence, we plot the clusters
centroids and covariance matrices in the in the normalized D_{iso}-D_Δ space for two out of the six
clusters (**Figure 9A**). The selected clusters to display were those in WM and GM regions. For
the WM clusters, normalized D_Δ across subjects were approximately on 1.6 with some variation

of D_{iso} around 0.7 and 0.9. The GM clusters displayed more consistent centroid on the D_{iso}
dimension around 0.65. We observed that the WM and GM cluster fractions (**Figure 9B**) have
a similar mode across subjects. Additionally, the D_{iso} and D_Δ^2 cluster-resolved maps of the WM
and GM clusters are displayed from subject 1 **(Figure 9C)**.

4   Discussion

In this study, we identified distinct water populations from nonparametric $\mathbf{D}(\omega)$-R_1-R_2-
distributions using unsupervised clustering. Our method was evaluated in both *in vivo* and *ex*
vivo datasets. The number of discernible clusters was influenced by the acquisition protocol,
with broader relaxation and frequency-dependent parameters enabling more detailed clustering
outcomes. Despite protocol differences, at least four biological relevant fractions—WM-like,
two GM-like, and FW-like—were consistently observed across healthy human and rat *in vivo*
brain data, as well as *ex vivo* rat brain. This flexible, data-driven approach offers broad potential
for application to other tissues, including brain tumors, heart, prostate, and breast.

Rather than depending on single per-voxel trace and anisotropy values to separate water
populations, bin division of non-parametric multidimensional diffusion-relaxation MRI uses
the per-voxel D_{iso}-D_Δ^2 distribution space to differentiate WM, GM and FW contribution to a
voxel. Additionally, it has been observed the advantages of adding dimensions such as in (De
Almeida Martins *et al.*, 2020), in which the manual binning of the D_{iso}-D_Δ^2-R_2 space allowed
the distinction of cortical GM and deep GM regions. This technique facilitates the
differentiation of various water populations in a single voxel, as well as the assessment of their
relative contributions. Nonetheless, bin limits are still selected manually and required
adjustment depending on whether the study involves *in vivo* or *ex vivo* tissue, as well as if the
data type varies from brain tissue.

Clustering methods address the limitations of manual bin selection by providing an unbiased,
data-driven approach. Unsupervised methods using tensor distribution data have improved the
characterization of diffuse gliomas (Song *et al.*, 2022). Simultaneous diffusion and relaxation
measurements enhance water population differentiation, aiding astrogliosis analysis
(Benjamini *et al.*, 2023) and cortical layers differentiation (Kundu *et al.*, 2023). In addition to
relaxation and diffusion parameters, our study incorporates restriction properties characterized
by the frequency-dependent domain in the non-parametric multidimensional distributions.
Considering variable diffusion time regimes enables a more thorough assessment of tissue
microstructure. Previous work has demonstrated that *in vivo* rat GM can contain multiple water
populations, each exhibiting distinct restriction properties, reflecting the heterogeneity of the
tissue environment (Yon *et al.*, 2024).

The incorporation of relaxation and frequency dependence significantly enhances the ability to
resolve tissue heterogeneity within a voxel. In this context, applying the unsupervised approach
to the non-parametric $\mathbf{D}(\omega)$-R_1-R_2-distributions enables the extraction of per-voxel
distributions of multiple water populations with greater biological relevance, as evidenced by
their correspondence to histological counterparts. In contrast, relying on manual D_{iso} and D_Δ^2
bin-limits in high-dimensional data can lead to the mixing of tissue fractions that are unlikely
to biologically coexist within the same spatial domain. For example, in the manual bin
approach, WM-like fraction (bin1) exhibited high $\Delta_{\omega/2\pi}D_{\text{iso}}$ values within the granule cell layer.
This ambiguity was resolved through unsupervised clustering, which correctly assigned the
granule cell layer to a separate tissue fraction. A similar challenge arises in GM, where
variations in cellular density, cell size, axonal content, and chemical composition differ across
distinct brain regions. The unsupervised clustering approach provided characteristic cluster

fraction distributions, offering a more accurate and biologically meaningful delineation of
tissue heterogeneity.

We applied the unsupervised clustering approach to both *in vivo* and *ex vivo* datasets, revealing
differences in the number of clusters identified between *ex vivo* and *in vivo* rat and *in vivo*
human brains. These differences were primarily attributed to variations in acquisition
protocols. The *ex vivo* settings allowed for higher spatial resolution and the collection of more
extensive data due to the longer acquisition times, whereas *in vivo* acquisitions were
constrained by shorter imaging sessions, MRI gradient strength, and motion-related limitations.
Despite these protocol-dependent differences, we consistently identified similar biologically
relevant fractions across both *ex vivo* and *in vivo* datasets. A recent work demonstrated the
reliability and reproducibility of non-parametric $\mathbf{D}(\omega)$-R_1-R_2-distributions in the human brain
(Manninen *et al.*, 2024). Additionally, our repeatability experiments showed that unsupervised
clustering within the same protocol and acquisition settings results in similar centroids and
covariance matrices.

Notably, certain components were relatively straightforward to interpret, such as the high D_Δ
distributions, which were predominantly associated with WM regions. In contrast, the zero and
negative anisotropic distributions without frequency-dependence, were more challenging to
interpret biologically. However, these components were notably more prominent in GM,
suggesting underlying variations in cellular and microstructural properties. When frequency
dependence data were available, as in the *ex vivo* and *in vivo* rat datasets, the clustering
approach enabled the extraction of a fraction resembling the apparent intra-soma compartment.
This fraction was characterized by increased D_{iso} at higher frequencies, consistent with
previous findings using oscillating gradient spin echo acquisitions (OGSE) (Does *et al.*, 2003;
Aggarwal *et al.*, 2012; Baron *et al.*, 2014; Oshiro *et al.*, 2024). Due to the limited frequency
range in the *in vivo* human data, this intra-soma-like feature could not be resolved, this could
be addressed by using clinical MRI scanner with stronger gradients (Tan *et al.*, 2020; Hennel
et al., 2021; Xu, 2021; Dai *et al.*, 2023).

The clusters obtained in this study should not be interpreted as isolated features corresponding
to a single microstructural property. For instance, a cluster characterized by high $\Delta_{\omega/2\pi}D_{iso}$ may
be attributed to the apparent soma fraction, analogously to the time-dependent microscopic
kurtosis (μK) (Novello *et al.*, 2022). However, it is essential to acknowledge that the
pronounced restriction properties revealed by the frequency dependence of diffusion can also
reflect underlying exchange effects. This phenomenon has been demonstrated by Chakwizira
et al., who showed that the sensitivity to restriction or exchange can be modulated by varying
the design of gradient waveforms (Chakwizira *et al.*, 2023). In the current study, we employed
a broad spectrum of gradient waveforms, which inherently introduces the potential for
confounding effects between restriction and exchange. To disentangle these contributions more
effectively, a targeted approach could be considered, such as selecting a subset of the data with
waveforms specifically optimized to enhance sensitivity to either restriction or exchange.

The inclusion of frequency dependence is crucial for probing the microstructure of GM
microstructure. Specifically, for $\Delta_{\omega/2\pi}D_\Delta$, we observed that increasing the frequency shifts the
anisotropy distribution closer to 0. A tendency for FA to slightly decrease at lower diffusion
times has been previously noted in time and frequency-dependent diffusion studies (Tan *et al.*,
2020; Tétreault *et al.*, 2020; Ba *et al.*, 2024). However, in contrast with FA and μFA, D_Δ
distributions provides a more detailed representation of tensor shapes, distinguishing between
prolate and oblate (Lawrenz *et al.*, 2010; Shemesh *et al.*, 2016b). In our observations, at low
frequencies, the $\Delta_{\omega/2\pi}D_\Delta$ values in GM regions are predominantly closer to 1 (stick-like

structures) and -0.5. As the frequency increases, a distinct shift occurs in the distribution toward
$D_\Delta = 0$ (spherical structures). This observation highlights the value of frequency dependence
in identifying and characterizing specific microstructural compartments within GM.
Investigating the soma more precisely—whether through biophysical modeling or signal
representation—requires selecting an optimal frequency or time-dependent range. Intermediate
ranges may incorporate contributions from other microstructural features, thereby complicating
the interpretation and potentially leading to suboptimal signal fitting. There are only few
studies about the time-dependent effect in the microscopic anisotropy (Ianuş *et al.*, 2018;
Narvaez *et al.*, 2022, 2024; Johnson *et al.*, 2024; Yon *et al.*, 2025).

The use of Monte Carlo inversion algorithms to resolve multidimensional parameter spaces is
a well-established method (Prange *et al.*, 2009), supported by prior studies and simulations (De
Almeida Martins *et al.*, 2018; Narvaez *et al.*, 2024). This approach explores the entire solution
space through multiple component decomposition, which, while comprehensive, can introduce
challenges such as data overfitting and the generation of highly complex distributions. To
address potential overfitting, the resulting solutions are typically weighted and summarized
into statistical descriptors, such as medians, variances, and covariances, to enable the
calculation of parametric maps (Yon *et al.*, 2020; De Almeida Martins *et al.*, 2021; Martin *et
al.*, 2021; Narvaez *et al.*, 2022, 2024). In the current study, we leveraged the weighted solutions
directly as input for clustering, acknowledging that not all data points contribute equally to the
overall biological relevance of the solution space. By doing so, we aimed to retain the
granularity of the multidimensional data while reducing the influence of less significant
components. The Gaussian Mixture Model (GMM) first iteratively determines the centroids
(μ) of the clusters, with each centroid representing a meaningful data point. In contrast, the
covariance (σ) reflects the spread and shape of the clusters, which plays a key role in preventing
data overfitting. When applied to non-parametric $\mathbf{D}(\omega)$-R_1-R_2-distributions, these GMM
parameters correspond to the mean and variance of the distributions, both of which require
careful interpretation. As the number of clusters increases within a given tissue, additional
cluster fractions with less than approximately 8% of the data start to emerge. To avoid
overfitting, we limited the number of clusters selected for each experiment. The validity of the
identified cluster fractions can only be confirmed through spatial correlation with well-
established biological structures, which necessitates histological comparisons for accurate
biological interpretation.

In this study, we utilized two histological stainings, Nissl staining for cell bodies and myelin
staining to identify myelinated fibers. While these stainings provide valuable insight into
cellular density and myelin distribution, they do not capture the full complexity of the tissue
microstructure. Brain tissue consists of diverse components, including various cell populations,
dendrites, unmyelinated axons, vasculature, and extracellular matrix, which were not
accounted for in this study, thus limiting the interpretation of the cluster fractions. Furthermore,
while light microscopy is a well-established tool, its resolution is limited to visualize certain
microstructural features, such as subcellular components and neuropil. These features are
critical for understanding the full extent of tissue heterogeneity, particularly in GM regions
with diverse cellular, axonal and synaptic characteristics. To strengthen the associations
between MRI and histology, more comprehensive analyses incorporating additional stainings
(*e.g.*, for neuronal and glial markers, vasculature, or extracellular components) and advanced
imaging techniques are needed. Electron microscopy (EM) could play a crucial role in this
process, providing much higher resolution images that allow for visualization of ultrastructural
features, such as synaptic junctions, fine axonal structures, and the organization of myelin at a
sub-micrometer scale. Such insights would significantly enhance our understanding of the

tissue microstructure and improve the biological interpretation of MRI data. Moreover, the integration of advanced image analysis techniques, including 3D reconstructions, machine learning-based segmentation, and quantitative morphometric analyses, could offer more detailed and precise evaluations of tissue organization (Abdollahzadeh *et al.*, 2021; Larsen *et al.*, 2021; Salo *et al.*, 2021). Future work should thus incorporate these complementary methods to provide a more comprehensive view of tissue organization and its relationship to multidimensional MRI-based measures.

5   Conclusions and Outlook

In conclusion, this study highlights the potential of unsupervised classification of multidimensional diffusion-relaxation correlation data to enhance tissue characterization at the voxel level. The method effectively disentangles tissue components and provides valuable insights into diffusivity, anisotropy, and chemical properties, which are crucial for identifying tissue changes in pathologies such as tumors, demyelination, Wallerian degeneration, and cell swelling. This approach offers significant advantages over traditional methods, enabling a more comprehensive understanding of tissue microstructure without relying on assumptions. While promising, further validation is needed in clinical settings, particularly with complementary histological data. Expanding this method to other tissues and diseases could broaden its applications, making it a powerful tool for both research and clinical diagnostics, especially in neuroimaging and other medical fields.

Conflict of Interest

The authors declare that the research was carried out without any commercial or financial relationship that could be interpreted as a potential conflict of interest.

Author Contributions

ON: conceptualization, methodology, data acquisition and processing, data analysis, visualization, manuscript writing, review, and editing. MY: methodology, data acquisition and processing, manuscript review and editing. RS: conceptualization, data analysis. JK: methodology, data acquisition and processing. ME: data processing and analysis. EP: acquiring license and imaging permits, data acquisition and processing. VL: acquiring license and imaging permits. JH: acquiring license and imaging permits, and radiological reviewing. FL: data acquisition. DT: methodology, manuscript review, and editing. AS: conceptualization, funding acquisition, project administration, supervision, manuscript writing, review, and revision. All authors revised and contributed to the final version of the manuscript.

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Data and Code Availability

MATLAB source code for preprocessing, Monte-Carlo data inversion and clustering is freely available at https://github.com/UEF-Multiscale-Imaging. The data used in this study is available upon request.

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Figures

Figure 1. Bin- and cluster-resolved parameter maps derived from D(ω)-R_1-R_2 distributions. (A) Bin fractions segmenting WM (red), GM (green) and FW (blue) regions by manually setting the "bin" limits in the D_{iso}-D_Δ^2 space. **(B)** Bin-resolved parametric maps for each "bin" fraction. The brightness represents the signal fractions, and the color scales the values of each parameter. **(C)** Color-coded unsupervised cluster-resolved normalized fractions by setting $k = 3$ using the D_{iso} and D_Δ^2 distributions. WM, GM, and FW fractions are displayed in same colors as the bin fractions. **(D)** Cluster-resolved parametric maps for each cluster fraction. Each fraction shows in percentage the amount of data belonging to the respective cluster. Arrowheads in $\Delta_{\omega/2\pi}E[D_{\mathrm{iso}}]$ maps indicate the granule cell layer of the dentate gyrus. Both bin- and cluster-resolved maps were calculated using the **D(ω)-R_1-R_2** distributions at a frequency $\omega/2\pi = 29$ Hz.

Figure 2. Normalized cluster frequency of k = 2-10 from the full distribution space (D_{iso}, D_Δ, R_1, R_2 and $\Delta_{\omega/2\pi}D_{iso}$). Per-voxel cluster frequency using k from 2 to 10 (rows show the number of clusters and columns the cluster fractions). The color scale shows the per-voxel belonging to each cluster.

Figure 3. Per-cluster distributions for $k = 7$ derived from including D_{iso}, D_Δ, R_1, R_2 and
$\Delta_{\omega/2\pi}D_{iso}$ into the clustering. (A) Per-voxel cluster frequency (columns) ordered from more
percentage of data included into the cluster to the least percentage. The contour plots display
the D_{iso}-D_Δ, D_{iso}-R_1, and D_{iso}-R_2 parameters (rows) for each cluster-fraction. Distributions at
different $\omega/2\pi$ are plotted with different color (blue = 29, yellow = 69.5 and black = 110 Hz).
The dashed red lines indicate the mode of the D_Δ, R_1, and R_2 parameters from f_{clusA}. **(B)**
Distribution amplitude of D_{iso} dimension displays the comparison across cluster-fractions.

Figure 4. Cluster-resolved ($k = 7$) mean parameter maps derived from $\mathbf{D}(\omega)$-R_1-R_2 distributions. (A) Unsupervised cluster-fractions f_{clusB}, f_{clusA} and f_{clusE} in RGB color, respectively. This representation highlights the clusters-fractions that are WM, GM and FW-like. **(B)** Cluster-resolved parametric maps for each cluster-fraction. The cluster-fractions are ordered from higher to lower percentage of data belonging to each cluster. The color scales of the parameter maps are the same as in **Figure 1**. Arrowhead in $\Delta_{\omega/2\pi}\text{E}[D_{\text{iso}}]$ map indicates the granule cell layer of the dentate gyrus. The cluster resolved maps were calculated using the $\mathbf{D}(\omega)$-R_1-R_2 distributions at a frequency $\omega/2\pi = 29$ Hz.

Figure 5. Histology and cluster-resolved mean maps comparison. **(A)** A representative myelin- stained section from the same brain as the acquired in MRI. Scale bar is 1 mm. **(B)** Representative cluster-resolved maps ($E[D_{iso}]$, $E[D_\Delta^2]$, and $E[R_2]$) from **Figure 3**. The color scale bars limits are here modified from **Figure 3** to fit the scale values of the representative cluster fractions (f_{clusB} and f_{clusD}). **(C)** Close up of two WM regions (continuous line square: corpus callosum (cc) and cingulum (cg); dashed line square: internal capsule (ic) and optic tract (opt)). Scale bar is 100 µm. **(D)** Representative Nissl-stained section from the same brain. Scale bar is 1 mm. **(E)** Representative cluster-resolved maps ($E[D_{iso}]$, $E[R_2]$, and $\Delta_{\omega/2\pi}E[D_{iso}]$). The color scale bars were modified as well from **Figure 3** to fit the scale values of the representative cluster fractions (f_{clusA} and f_{clusC}). **(F)** Close up of two different GM regions (continuous line square: medial habenular nucleus (MHb); dashed line square: granule cell layer (gcl)). Scale bar is 100 µm.

Figure 6. ROI analysis and comparison with Nissl and myelin staining. (A) selected ROIs overlayed on the S_0 MRI. ROIs in GM: auditory cortex (Au), primary somatosensory cortex (S1Tr), and granule cell layer (gcl); ROIs in WM: corpus callosum (cc) and cingulum (cg). **(B)** per-ROI distributions for f_{clusA}, f_{clusB}, f_{clusG}, f_{clusC}, and f_{clusD} ordered as in **Figure 3 and 4.** The per-ROI distributions are grouped by parameter. Upper left: D_{iso}, upper right: D_Δ, lower left: R_1 and lower right: R_2. Distributions at different $\omega/2\pi$ are plotted with different color (blue = 29, yellow = 69.5 and black = 110 Hz). Dashed red lines indicate the mode value in reference to the ROI in the auditory cortex. **(C)** Nissl-stained section from the same brain (scale bar is 1 mm) with close ups to the regions selected in the MRI slice (scale bar is 200 μm). **(D)** Cell area from the Nissl-stained section. **(E)** Myelin-stained section from the same brain (scale bar is 1 mm) with close ups to the regions selected in the MRI slice (scale bar is 200 μm). **(F)** Optical density map of the myelin-stained section. **(G)** Coherency map derived from the structure tensor analysis on the myelin-stained section. The analysis window was 70 × 70 μm² to match the MRI voxel size.

1013

Figure 7. Parametric distributions and cluster-resolved mean maps of *in vivo* rat brain derived from $k = 6$ cluster-fractions including D_{iso}, D_Δ, R_1, R_2, and $\Delta_{\omega/2\pi}D_{iso}$ into the clustering. (A) Per-voxel cluster frequency (columns) ordered from more percentage of data included into the cluster to the least. Contour plots of the D_{iso}-D_Δ, D_{iso}-R_1, and D_{iso}-R_2 projections (rows) of the $\mathbf{D}(\omega)$-R_1-R_2 distributions for each cluster. Contour plots have different color according to $\omega/2\pi$ (blue = 18, mustard = 52 and black = 92 Hz). The dashed red lines indicate the mode of the D_Δ, R_1, and R_2 parameters distributions from clusA. **(B)** Distribution amplitude of D_{iso} dimension displays the comparison across cluster-fractions. **(C)** Unsupervised cluster-fractions f_{clusE}, f_{clusA} and f_{clusD} in RGB color, respectively. This representation highlights the clusters-fractions that are WM, GM and FW-like. **(D)** Cluster-resolved mean parametric maps for clusC and clusE. Arrowhead indicates the cerebellum. The cluster resolved maps were calculated using the $\mathbf{D}(\omega)$-R_1-R_2 distributions at a frequency $\omega/2\pi = 18$ Hz.

1026

Figure 8. Parametric distributions and cluster-resolved mean maps of *in vivo* human brain derived from $k = 6$ cluster-fractions including D_{iso}, D_Δ, R_1, and R_2 into the clustering. **(A)** Per-voxel cluster frequency (columns) ordered from more percentage of data included into the cluster to the least. Contour plots of the D_{iso}-D_Δ, D_{iso}-R_1, and D_{iso}-R_2 projections (rows) of the $\mathbf{D}(\omega)$-R_1-R_2 distributions for each cluster. Contour plots have different color according to $\omega/2\pi$ (blue = 4, mustard = 6.5 and black = 9 Hz). **(B)** Distribution amplitude of D_{iso} dimension

displays the comparison across cluster-fractions. **(C)** Unsupervised cluster-fractions f_{clusB}, f_{clusA} and f_{clusD} in RGB color, respectively. **(D)** Cluster-resolved mean parametric maps. The cluster-fractions are ordered from higher to lower percentage of data belonging to each cluster. The cluster resolved maps were calculated using the $\mathbf{D}(\omega)$-R_1-R_2 distributions at a frequency $\omega/2\pi$ = 4 Hz.

Figure 9. WM- and GM-like cluster-fractions of *in vivo* human brains derived from k = 6 including D_{iso}, D_Δ, R_1, and R_2 into the clustering. (A) Centroids and covariances of the WM- and GM-like clusters for each subject derived from $k = 6$. The clusters are displayed in the normalized D_{iso}-D_Δ space. **(B)** WM- and GM-like cluster fractions of the D_{iso} and D_Δ parameters across all the subjects. The color of the lines indicates different $\omega/2\pi$ (blue = 4, mustard = 6.5 and black = 9 Hz). **(C)** Respective cluster-resolved E[D_{iso}] and E[D_Δ] maps of the WM- and GM-like fractions from sub 1.