Evolutionarily Optimal Phage Life-History Traits: Burst Size *vs.* Lysis Time

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Abstract

A dynamical equation for the virus to microbe ratio (VMR) during log phase population growth shows that an optimal balance occurs between a short lysis time with low burst size *vs.* a long lysis time with large burst size. The model predicts that interventions lowering phage adsorption by killing free virus and/or limiting their access to bacteria cause the evolution of an increased lysis time and higher burst size until the intervention either drives a virulent phage extinct or, for temperate phage, drives the phage from its lytic phase into its lysogenic phase. The model also predicts that along an environmental gradient of increasing productivity the optimal lysis time shortens along the gradient, implying that the lytic life cycle goes around faster along the gradient. However, the virion productivity realized during the time step corresponding to each point along the gradient remains constant.

Introduction

Breitbart (2012) noted that "It is difficult to find a paper in marine viral ecology that does not begin with a statement about the sheer abundance of viruses". The same is true today. Even though abundance estimates for virus have been available for over twenty years, the magnitude still seems incomprehensible. The number of prokaryotes on earth is 4–6 x 10^{30} cells with a production rate of 1.7 x 10^{30} cells/yr (Whitman *et al*. 1998). The number of virus is about ten times the number of prokaryotes (Wommack and Colwell 2000) implying that the number of virus is on the order of 10^{31}. In comparison, the number of grains of sand on all the earth's beaches is only 4 x 10^{20} and the number of stars in the universe a meager 2 x 10^{19} (Plait 2025).

Moreover, marine phage infect approximately 10^{23} bacteria every second, removing 20%–40% of the standing stock of marine bacteria each day (Suttle 2007). They exert top-down control of marine microbial abundance comparable to that of zooplankton grazing (Breitbart *et al.* 2018). Reports of viral abundance also appear for soil (Srinivasiah *et al*. 2008, Williamson *et al*. 2017) and the human gut (Townsend *et al*. 2021, the "phageome").

Furthermore, virus are ancient with differences so great among them that the taxonomic rank of "realm" has been coined to reside at atop 15 lower ranks that include multiple kingdoms as well as the taxa below that (ICTV 2020, Koonin *et al.* 2020, 2021, Harris and Hill 2021). Marine phage represent 94% of the nucleic-acid-containing particles in the oceans (Breitbart *et al.* 2018). Hence, phage represent an enormous number of evolutionary agents whose phenotypes are shaped by natural selection.

What then are the phage phenotypes that natural selection is shaping? Of the many traits phage possess, including their morphology and mechanisms of inheritance and replication, the life-history

traits of the lytic/lysogeny switch, the burst size and the latency period are of particular interest because these traits underly the reproductive rate of phages. Ideally, research might offer an evolutionary explanation for such traits across various habitats. A previous paper introduces a model that predicts conditions for the lytic/lysogenic switch to occur and compares the model's predictions with ecogeographic patterns of the ratio of lytic to lysogenic life cycles (Roughgarden 2024). However, information on possible ecogeographic patterns in burst size and latency across habitats is limited. Both traits vary considerably across and within habitats, often in accordance with transient environmental conditions such as temperature (Wommack and Colwell 2000, Weinbauer 2004, Greenrod *et al.* 2024). Parada *et al.* (2006) show a correlation between burst size and bacterial productivity in coastal waters, but do not present data on latency times. Ranasinghe (2019) offers an extended review of burst size and latency across various categories of virus, but does not report the habitats where the virus live. This article extends the model in (Roughgarden 2024) to explain burst size and latency and predicts possible ecogeographic patterns whose discovery would test the model.

Interventions to control microbes with antibiotics have produced antibiotic resistance (Martinez *et al.* 2008, Davies and Davies 2010). These and other interventions might affect virus as well. The model in this article also predicts the evolutionary consequences for phage of interventions that lower the overall adsorption rate by virus onto bacteria.

This article explains burst size and latency as an evolutionarily optimal balance between reproducing early to benefit from geometric growth versus reproducing late to benefit from the time to produce a large burst size. The study of an evolutionary tradeoff between the benefits of early and late lysis time enjoys a long history (Abedon 1989, Wang *et al*. 1996, Abedon *et al*. 2001, Abedon *et al*. 2003, Bull *et al*. 2004, Bull 2006, Bull *et al*. 2006, Wang 2006, Heineman and Bull 2007, Bull *et al*. 2011, and Kannoly *et al*. 2022). The models developed in this literature are based on systems of delay-differential equations with viral colonization represented as a mass action term, that is, as $(V \times B)$ to represent collisions of virus with bacteria, and also feature density dependence in the bacteria population growth. The analysis of these equations generally involves solving for the equilibrium abundance of uninfected bacteria, infected bacteria and free phage.

This article differs from the preceding literature in being based on a new model that is defined in discrete time and features viral colonization of bacteria as a Poisson process. The model pertains to a virus-bacteria system in the log phase, where both populations grow exponentially rather than exist at equilibrium. The model envisions a bust-boom environment where bacteria frequently experience exponential growth rather than a stable environment where an equilibrium can be attained. The model's analysis rests on a single dynamical equation for the virus to microbe ratio (VMR) rather than a system of coupled equations. Thus, the situation in which the optimal life history evolves is envisioned differently here as compared to the earlier literature—log phase versus stationary phase, recursion equations in discrete time versus delay differential equations in continuous time, and Poisson colonization versus mass action infection. The Discussion section features a more detailed comparison of the differing intuition behind the earlier models compared to the present model.

Two experimental studies (Wang 2006, Kannoly *et al*. 2022) demonstrate that an optimal balance between lysis time and burst size exists. Hence, the reality of an optimal balance can now be taken as given. Instead, this article concerns how to predict *what* that optimal balance is. In nature, phage may not possess the optimal balance for their present circumstances, but knowing the optimal balance in those circumstances reveals the direction toward which evolutionary change will occur.

The article has five sections: The first defines the basic population-dynamic model. The second extends the model to predict the optimal lysis time and burst size. The third compares the predicted the optimal lysis time and burst size to data from experiments with λ-phage and *Escherichia coli* (Wang 2006). The fourth predicts the evolutionary impact of interventions that affect the adsorption rate. The fifth develops predictions for ecogeographic patterns in lysis time and burst size.

Model

Population Dynamics of Phage and Bacteria

The phage/bacteria population-dynamic model is defined in discrete time with the time step being the viral lysis time period (Roughgarden 2024). A time step begins with the state variables, $N(t)$ and $V(t)$ for the number of bacteria and free virus particles (virions) in their environmental source pools at time t. Then the virus infect the uninfected bacteria with Poisson sampling such that some bacteria remain uninfected, some have only one infection, some have two infections *etc*. (Ellis and Delbrück 1939).

The Poisson distribution describing the infection process has density parameter, $\mu(t) \equiv a(v(t))\, v(t)$ where $v(t) \equiv V(t)/N(t)$ is the ratio of virus to bacteria in their source pools (the environmental virus to microbe ratio, VMR). The $a(v(t))$ is the adsorption function—it refers to the process whereby a viral particle comes in contact with a bacterium, binds to surface molecules, and then is incorporated into the bacterial cell. Adsorption is a function of $v(t)$ because the number binding sites on the surface of the bacterium is finite and the sites may become saturated. $a(v(t))$ includes information about the binding affinity of virus particles to receptors in the bacterial surface, the number of available binding sites and the accessibility of the bacteria to virus particles. The accessibility is influenced by how dense the bacteria are as well as by how viscous the media is, or for airborne virus particles, whether air filters are installed.

According to the Poisson distribution, μ is the average total adsorption per time step for a given v. Upon measuring μ, the adsorption coefficient is calculated as $a = \mu/v$ for the given v. Repetition for several values of v yields several values of μ from which the corresponding values of a are calculated. Finally, the values of a are fitted to a curve yielding the adsorption function, $a(v)$.

According to a Poisson density function, the fraction of uninfected bacteria that remain uninfected, $P_0(\mu(t))$, is $e^{-\mu(t)}$ and the fraction of bacteria colonized by m virus, $P_m(\mu(t))$, is $\frac{e^{-\mu(t)}\mu(t)^m}{m!}$ for $m \geq 0$ where $\mu(t) = a(v(t))\, v(t)$. The m indexes the number of virus per bacterium. The fraction of bacteria that are infected by one or more virus is $1 - P_0(\mu(t))$.

The uninfected bacteria population increases by a geometric factor of increase per time step, R. If $R > 1$, the bacteria can increase in the absence of the virus. Accordingly, the uninfected bacteria resupply the bacterial source pool as

$$N(t+1) = R\, P_0(\mu(t))\, N(t) \tag{1}$$

where $P_0(\mu(t))$ is the Poisson probability that a bacterium is uninfected. Thus, the population size of bacteria at time $t+1$ equals R times the number of bacteria that are not infected. Meanwhile, the virus in the infected bacteria produce new virus particles as

$$V(t+1) = \Big(b(1)\, P_1(\mu(t)) + b(2) P_2(\mu(t)) + \dots\Big)\, N(t) \tag{2}$$

where $b(m)$ is the burst size from a bacterium infected with m virus particles (the multiplicity of infection, MOI) and $P_m(\mu(t))$ is the Poisson probability that a bacterium is infected by m virus particles. Thus, the population size of virions at time $t+1$ equals the number of virions produced from bacteria infected by one phage, plus those produced from bacteria infected by two phage, plus those from bacteria infected with three phage, and so forth. Together, Eqs. 1 and 2 constitute a dynamical system for phage and bacteria.

The viral adsorption is assumed to depend on the virus to microbe ratio, v,

$$a(v(t)) = a_o\, (1 - \frac{v(t)}{\kappa}) \tag{3}$$

where κ is the saturation coefficient—the virus to microbe ratio at which all the binding sites per bacterium are occupied and a_o is the maximum adsorption coefficient that occurs when all the binding sites are available, given the environmental conditions.

To illustrate how to measure the adsorption function, suppose a vessel of known volume is provided with 10,000 phage and 100 bacteria—this corresponds to $v = 100$. After a time step, determine how many phage are left. Say, the number phage left in one replicate is 9,990 indicating that 10 phage were adsorbed. In two more replicates, the adsorbed number was 12 and 8, respectively, so the average is 10 adsorbed—this is μ for $v = 100$. Therefore, $a(100) = \mu/v = 10/100 = 0.10$. Suppose further that this experiment is repeated for various v. If the vessel is provided with 20,000 phage and 100 bacteria, corresponding to $v = 200$, then say, 14 are adsorbed on average. Therefore, $a(200) = 14/200 = 0.07$. Continuing, with 30,000 phage and 100 bacteria, $v = 300$, the number adsorbed turns out to be 15 on average so that $a(300) = 15/300 = 0.05$. Thus, total adsorption, μ, increases with v, while the adsorption per unit virus/microbe ratio, a, decreases with v as the receptor sites fill up. The best fit of a line to the points, (100, 0.10), (200, 0.07), (300, 0.05), is $a(v) = 0.12333 - 0.00025\, v$. The Y-intercept of the line is $a_o = 0.12333$, the maximum adsorption coefficient pertaining to the limit, $v \to 0$. The X-intercept of the line is $\kappa = 493.33$, the value of v for which the adsorption coefficient is zero, reflecting saturation of all the bacterial receptor sites. (The κ is unimportant in this paper, unlike In Roughgarden 2024, because the focus here is on conditions where v is small.) Except for the section comparing the model's predictions with experimental results, the adsorption coefficient is interpreted qualitatively.

For a finite vessel, the density of the microbes (N/volume) is implicit in the value measured for a_o. For a given N, virus in a small vessel with a high density of bacteria have a higher adsorption rate than in a large vessel with low microbial density because more virus to bacteria contacts occur during the time step. If N increases through time in a finite vessel, the density increases making the absorption parameter, a_o, also increase through time. In an open infinite environment however, the local microbial density may remain relatively constant as the microbial population spreads through space so that a_o remains relatively constant.

Several choices are available for the burst size from a bacterium as a function of the number of virus in it, $b(m)$. The burst size might increase linearly with m so that $b(m) = b_o\, m$ where b_o is the contribution to the overall burst made by a single viral particle (Patel and Rao 1984, Figures 1–2). This case pertains to the absence of density dependence in burst size. Another possibility is where $b(m)$ to be a concave increasing function of m (Gadagkar and Gopinathan 1980, Figures 4–5). In this case, the burst-size function might be taken as $b(m) = b_o\, m^\alpha$ where α is an exponent between 0 and 1, say 1/2. This case shows limited density dependence in burst size. Still a third possibility is that $b(m)$ is a constant, $b(m) = b_o$, indicating strong density dependence. This situation is described in Ellis and Delbrück (1939) who write, "The bacteria which had adsorbed several phage particles behaved as if only one of these particles was effective." This case postulates that one infection disables all subsequent infections so that the burst is exactly the same with $m = 1$ as it is for all $m > 1$, a claim of uncertain plausibility. A final possibility is to allow α to be negative, say $\alpha = -1$, to indicate that burst size varies inversely with the multiplicity of infection, indicating extremely strong density dependence as a response to "superinfection" (Brown and Bidle 2014). All these relations between burst size and m may be subsumed by taking $b(m) = b_0\, m^\alpha$ with α as 1, 1/2, 0, or -1. The cases of $\alpha = 1$ and $\alpha = 0$ were presented in Roughgarden (2024) and results for other α were noted but not shown. Here, the burst size as a function of the multiplicity of infection is taken as

$$b(m) = b_o\, m \tag{4}$$

where b_o is the burst per virus infecting the bacterium. Even if $b(m)$ is not linear throughout the range of possible m, the linear form can be viewed as a Taylor approximation in the limit as $m \to 0$. This

approximation is appropriate because the analysis focusses on the conditions for the virus to increase when rare into a bacterial population that is growing in its log phase—in such a circumstance m is small.

Next, identify $v(t+1)$ with $\frac{V(t+1)}{N(t+1)}$ and $v(t)$ with $\frac{V(t)}{N(t)}$. Then doing the sum in Eq. 2 yields

$$v(t+1) = \frac{b_o\, a_o\, (1 - \frac{v(t)}{\kappa})}{R\, e^{-a_o\, (1-\frac{v(t)}{\kappa})\, v(t)}}\, v(t) \tag{5}$$

Equation 5 is a dynamical equation in one variable, $v(t)$, identical to Eq. 8 in Roughgarden (2024).

In the limit, $v(t) \to 0$, the dynamical equation in Eq. 5, upon suppressing the subscripts, reduces to

$$v(t+1) = \frac{b\, a}{R} v(t) \tag{6}$$

The virus population can infect the bacterial population if it can increase when rare, *ie.*, if $v(t) \approx 0$, then $v(t+1) > v(t)$. Inspection of Eq. 6 shows that if

$$b\, a > R \tag{7}$$

then the virus can spread into a population of increasing bacteria. Alternatively, for a virulent virus, if $b\, a/R < 1$, the virus goes extinct because the virus cannot keep up with bacterial population growth. But for a temperate virus, if $b\, a/R < 1$, the virus switches to a prophage because its population grows faster as part of the bacterial genome than by bursting new virions each time step. This criterion was introduced in Roughgarden (2025).

Optimal Balance Between Burst Size and Lysis Time

This section applies the phage-bacteria population-dynamic model of the previous section to the tradeoff between burst size and lysis time. Wang (2006) experimentally determined a functional relation between burst size and lysis time as

$$\begin{aligned} b &= \beta\, (l - \epsilon) \quad \text{for} \;\; l > \epsilon \\ &= 0 \quad \text{for} \;\; l \leq \epsilon \end{aligned} \tag{8}$$

where l is the lysis time, ϵ is the eclipse, which is the time needed for the viral replication to commence, and β is the phage production rate (also called the phage maturation rate) once viral production has begun. The burst size increases linearly with the lysis time period after the eclipse has transpired. (These symbols and all the others used in the paper are listed in Table 1.)

Because the time step in the model, Eq. 6, is the lysis time, l, to investigate how the time step itself evolves, a maximum time step, L, is stipulated that is longer than the largest feasible lysis time. Then the dynamical equation is iterated for the number of l that fit within L, given the eclipse. For example, suppose the eclipse is 0 and L is 100. If l is also 100, then only one iteration takes place within L, whereas if l is 1, then 100 iterations fit within L, and if l is 25, then four iterations fit within L, and so forth. Thus, for a given l, the v after the number of iterations that fit within L is

$$v(t+L) = \frac{(\beta\, (l - \epsilon)\, a)^{\lfloor \frac{L}{l} \rfloor}}{R^L}\, v(t) \tag{9}$$

for l = (ϵ+1, ϵ+2 ... L) where $\lfloor \frac{L}{l} \rfloor$ indicates the integer part of $\frac{L}{l}$. The exponent in Eq. 9 is the number of iterations of viral reproduction with lysis time, l, that fit into L. The numerator of Eq. 9 is the production

of a lytic virus over the interval, L, and the denominator is the bacterial production over the interval, L, which is also the production of a temperate virus residing as a prophage in the bacterial chromosome.

Further analysis is aided by taking a continuous approximation in Eq. 9, as though a fractional iteration can fit into L,

$$v(t+L) = \left(\frac{(\beta\ (l-\epsilon)\ a)^{\frac{L}{l}}}{R^L} \right) v(t) \tag{10}$$

for $l > \epsilon$. The expression in large parentheses in Eq. 10 is the fitness of a lytic phage relative to the prophage residing in the bacteria over the period, L, as a function of l

$$W_L(l) = \frac{(\beta\ (l-\epsilon)\ a)^{\frac{L}{l}}}{R^L} \tag{11}$$

The optimal lysis time, $\hat{l}$, is found by maximizing $W_L(l)$ with respect to l. The corresponding optimal burst size, $\hat{b}$, is found from Eq. 8, evaluated at $l = \hat{l}$.

To find the optimal lysis time, differentiate W_L with respect to l, set the derivative equal to zero, and solve for l. This process is aided by noticing the expression, $Y(l)$, defined as

$$Y(l) = \frac{1}{(l-\epsilon)} - \frac{\log\left(\beta\,(l-\epsilon)\ a\right)}{l} \tag{12}$$

The optimal lysis time, $\hat{l}$ emerges as the root of $Y(l) = 0$, which is independent of L.

The critical lysis time, $l_c(L)$, at which being lytic becomes a better strategy than being a prophage over the period, L, is found by setting

$$W_L(l) = 1 \tag{13}$$

and solving for l. If the optimal lysis time, $\hat{l}$, is less than $l_c(L)$, the virus is better off not lysing at all during the period, L, and instead incorporating as a prophage into the bacterial chromosome. Alternatively, if $\hat{l}$, is greater than $l_c(L)$, the virus is better off infecting and lysing the bacteria during the period, L, and doing so with the optimal lysis time. The optimal lysis time produces a corresponding optimal burst size found from Eq. 8 evaluated at $l = \hat{I}$.

Predictions

Comparison of Model with Laboratory Data

Table 2 gives the parameters for an illustration. The β and ϵ in units of phage/min and min, respectively, are derived from isogenic λ-phages with different lysis times from Wang (2006). The bacterial geometric factor of increase, R, from Wang (2006) for *Escherichia coli* K-12, is given as 2.04/hr—this converts to the value of R in the table in minutes. In the absence of an experiment to measure the adsorption parameter, a, a nominal value of 0.1 is used corresponding to the worked example provided earlier. The adsorption parameter varies considerably depending on environmental conditions affecting the accessibility of the bacteria to the virus and the number of binding sites per bacterium varies with nutrient conditions (Schwartz 1976, Philipson 1983). The large time interval, L, is taken arbitrarily as 100 minutes, a value comfortably larger than the optimal lysis time.

Table 1: Symbols

Symbol	Description
V	Number free virus particles (virions)
N	Number of microbes (bacteria)
v	Virus to microbe ratio (VMR), V/N
$a(v)$	Adsorption function
R	Geometric factor of increase per time step for microbes
μ	Poisson density
$P_m(\mu)$	Probability microbe colonized by m virus
a_o	Maximal adsorption when viral density low
κ	Saturation viral density
$b(m)$	Burst as a function of multiplicity of infection (MOI)
b_o	Burst per infecting virus
a	a_o with subscript suppressed
b	b_o with subscript suppressed
l	Lysis time (latency), same as time step
L	Large time interval to include multiple lysis steps
ϵ	Eclipse period, time before virus production starts
β	Virus production rate once eclipse expires
W_L	Fitness of lytic to lysogenic phage over interval L
$Y(l)$	Expression whose root is the optimal lysis time
$\hat{l}$	Optimal lysis time
$\hat{b}$	Burst realized with optimal lysis time
$l_c(L)$	Critical lysis time for fitness lytic = fitness lysogenic over period L
W_{Lo}	W_L assuming ϵ is zero
$\hat{l_o}$	Optimal lysis time assuming ϵ is zero
$\hat{b_o}$	Optimal burst size assuming ϵ is zero
$\widehat{W_{Lo}}$	Optimal W_L assuming ϵ is zero

Table 2: Parameters

Name	Value	Notes
β	7.7	Wang 2006
ϵ	28	Wang 2006
R	1.035	$e^{2.04/60}$, Wang 2006
a	0.1	Nominal value
L	100	Large time interval, minutes

To find the optimal lysis time, using the NSolve[] function in Mathematica with $Y(l)$ in Eq. 12, together with the parameters from Table 2, returns the value,

$$\hat{l} = 45.5 \tag{14}$$

independent of L and R.. The corresponding optimal burst size from Eq. 8 is

$$\hat{b} = 134.7 \tag{15}$$

and the corresponding maximal relative fitness over 100 minutes from Eq. 11 is

$$\widehat{W_{100}} = 9.7 \tag{16}$$

and the critical lysis time at which being lytic becomes a better strategy than being a prophage over 100 minutes works out to be

$$l_c(100) = 31.9 \tag{17}$$

Figure 1 shows the fitness of a lytic phage relative to a prophage after a period of 100 minutes as a function of the lysis time, l, based on the parameters in Table 2. Relative lytic to prophage fitness of 1 is shown as horizontal line. If l is greater than 31.9, then the lytic phase has a higher fitness than the lysogenic phase during the 100 minute interval. If l is between ϵ and $l_c(100)$, *i.e.* between 28 and 31.9 minutes, then the phage cannot coexist during the interval with the bacteria in its lytic phase—the bacteria population grows faster than the viral population, leaving it behind, so to speak. Instead, the phage must lysogenize—incorporate into the bacterial genome to acquire the population growth rate of the bacteria.

Wang (2006) measured the fitness of 11 strains of λ phage whose latencies varied from 28.3 to 63.0 minutes. The study found that the strain whose lysis time was 51.0 minutes had the highest fitness, a value quite close to that predicted in the model here, 45.5, based on the parameters in Table 2. The burst size for the strain with the highest fitness was 170.0, also fairly close to the value predicted here, 134.7. This initial degree of agreement between theory and experiment is coincidental because the nominal value of the adsorption parameter, a, in Table 2 was not experimentally determined.

Instead, suppose a is lowered to 0.052. Then $\hat{l}$ works out to be 50.0 and $\hat{b}$ to 176.9, almost exactly the same as Wang(2006) found experimentally. Thus, lowering a from the nominal value of 0.100 to 0.052, while keeping the other parameters from Table 2 the same, brings the predicted optimal lysis time and burst size into close agreement with the experimental results. The next section considers further implications of variation in a.

Interventions that Impact Adsorption

Figure 2 shows the impact of varying adsorption on phage evolution. The figure starts with a at a low value for which the phage exists as a prophage. Then increasing a induces the phage to excise from the bacterial chromosome when the ratio of lytic to prophage fitness equals 1. Thereafter, as a increases even more, the optimal latency and bust size progressively decrease while the relative fitness of the lytic to prophage progressively rises.

Many interventions to control phages, and viruses more generally, involve techniques such as air or water filters, or controlling humidity that lower the adsorption of phage by bacteria (*cf.* Sobsey *et al.* 2008, Clokie and Kropinski 2009, Wommack *et al.* 2009, Moriyama *et al.* 2020, Curtius *et al.* 2021, Van Charante *et al.* 2021, Beltrán *et al.* 2023). Overall, the model predicts that interventions lowering a cause the evolution of an increased lysis time and correspondingly higher burst size until the intervention is sufficient either to drive a virulent phage extinct, or for temperate phage, to drive the phage from its lytic phase into its lysogenic phase.

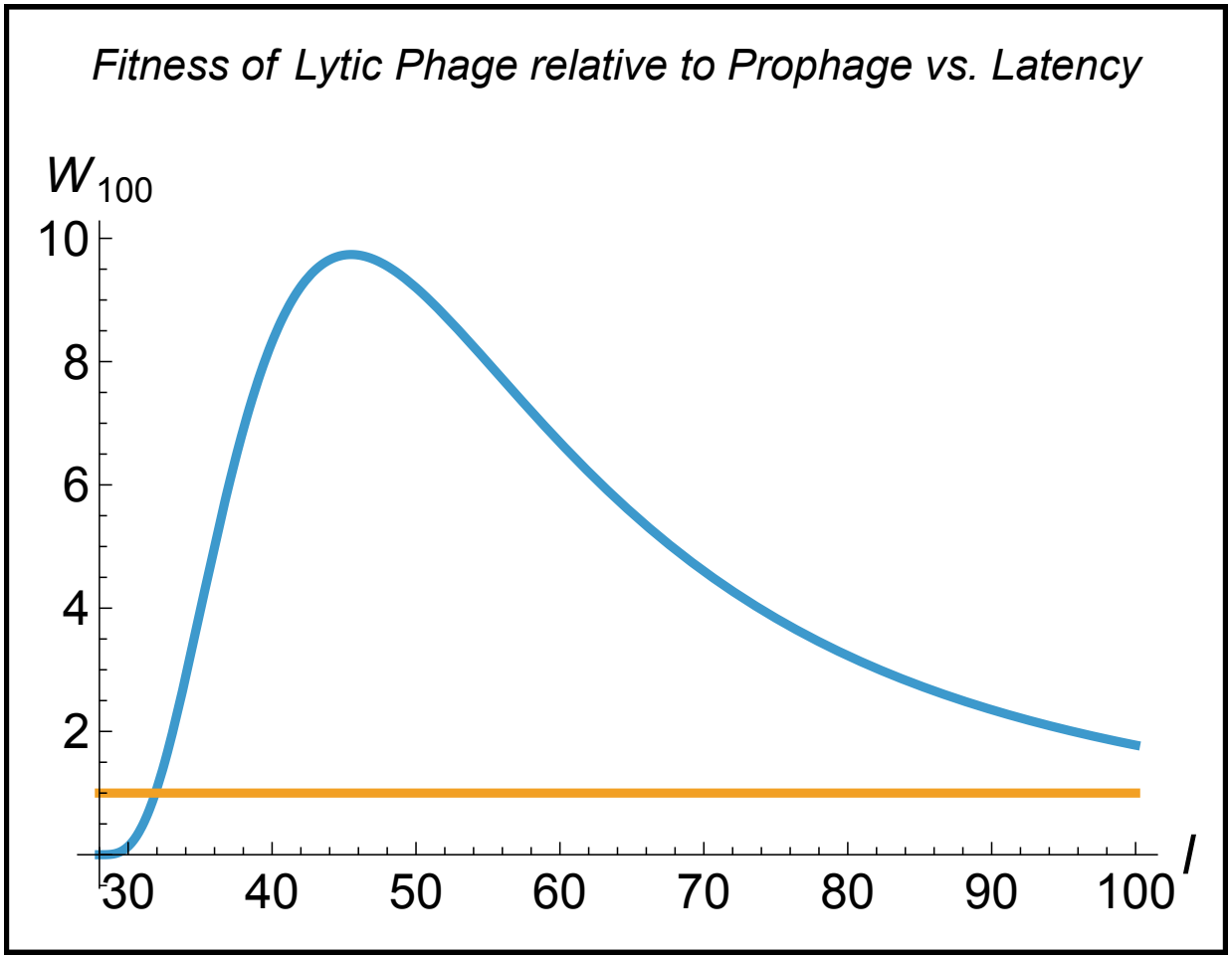

Figure 1: *Fitness of lytic phage relative to prophage after 100 minutes, W_{100}, as a function of the lysis time, l, is a humped-shaped curve starting from the eclipse period of 28 minutes. The optimal lysis time occurs at $\hat{l}$ = 45.5 which yields a maximum ratio of lytic to prophage fitness, $\widehat{W_{100}}$, of 9.7 over the 100 minute interval. Relative lytic to prophage fitness of 1 is shown as horizontal line. The hump-shaped curve for W_{100} intersects the line at $l_c(100) = 31.9$. Thus, the fitness of a lytic phage exceeds that of a prophage during the 100 minute interval when the lysis time exceeds 31.9. If the lysis time of the phage is between 28 and 31.9, the phage cannot coexist with the bacteria during the 100 minute interval unless it becomes a prophage.*

Ecogeographic Patterns

Prevalence of the lysogenic lifecycle relative to the lytic life cycle increases in conditions that promote bacterial productivity. For example, overfishing and eutrophication on coral reefs causes an increase in ungrazed fleshy algae. These release dissolved organic carbon that promotes bacterial productivity, leading to a condition called microbialization. Knowles *et al.* (2016) and Silveira *et al.* 2021) show that lysogeny increases with microbialization on coral reefs. Another example involves the guts of mammals. Here concentrated nutrients promote bacterial productivity, resulting in gut bacteria that are enriched with prophages, (López-Leal *et al.* 2022). Generally speaking, increasing bacterial productivity relative to virion productivity increases the likelihood of the lysogenic lifecycle relative to lytic lifecycle because reproduction as a prophage becomes better than reproduction through lysis.

Not as many ecogeographic patterns are known for burst size and latency time as for the lysogenic to lytic ratio. However, in one study, Parada *et al.* (2006, Fig. 5) showed a positive relationship between

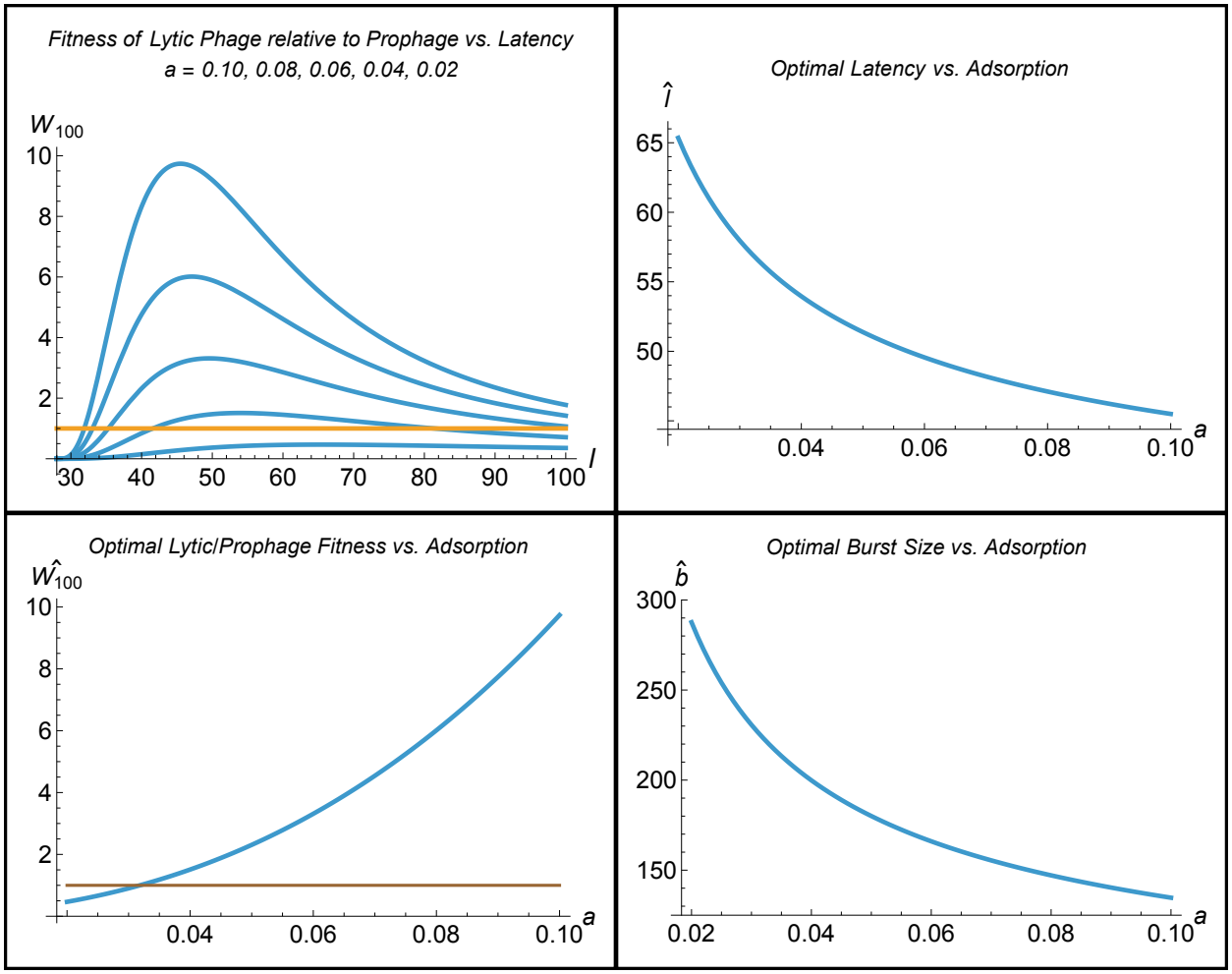

Figure 2: *Impact of varying adsorption on phage evolution. Top left: Fitness of the lytic phase* vs. *lysis time for several values of the adsorption coefficient,* a*. The top curve is for* a *= 0.10 and is identical to that shown in Figure 1. The curves below the top are successively for* a *= 0.08, 0.06, 0.04, and 0.02. The horizontal line is a relative fitness of 1 where the fitness of a lytic phage equals that of a prophage. As* a *decreases, the peaks of the curves for the lytic phase progressively become lower and shift to the right. Top right: Optimal lysis time* vs. *the adsorption parameter,* a*. As* a *increases from 0 to 0.10, the optimal lysis time decreases. Bottom right: Optimal burst size* vs. *the adsorption parameter,* a*. As* a *increases from 0 to 0.10, the optimal burst size also decreases, reflecting the decreasing lysis time. Bottom left: Optimal fitness of a lytic phage relative to the prophage* vs. *the adsorption parameter,* a*. The horizontal line is a relative fitness of 1 where the fitness of a lytic phage equals that of a prophage. As* a *increases from 0 to 0.10, the optimal lytic/prophage fitness ratio starts below 1 and increases, intersecting the horizontal line at* a *= 0.032, at which point the prophage becomes induced to excise from the bacterial chromosome and become lytic.*

bacterial productivity and viral burst size for virus in the North Sea. In terms of the model, high nutrient conditions increase both R, the bacterial geometric growth factor and perhaps β, the number of new virions produced per minute of latency. High nutrient conditions might affect a, the absorption coefficient depending on whether high nutrients increase or decrease bacterial density.

The eclipse period, ϵ, tends to be small relative to the overall latency, l, and qualitative insight can be gained by letting ϵ be zero. Accordingly, Eq. 11 becomes

$$W_{Lo}(l) = \frac{(\beta\ a\ l)^{\frac{L}{l}}}{R^L} \tag{18}$$

By differentiating $W_{Lo}(l)$ with respect to l, setting the result equal to zero and solving for l, yields an explicit result for the optimal latency

$$\hat{l_o} = \frac{e}{\beta\ a} \tag{19}$$

where e refers to Euler's constant (not the eclipse, ϵ). Thus, the optimal latency for the virus varies inversely with how good the environment is for the lytic phase. The more favorable the environment is for the lytic phase, $(\beta\ a)$, the shorter the optimal latency. That is, the better the environment is for the lytic phase, the faster the virus goes around its life cycle. Specifically, Figure 3 (Left) shows $\hat{l_o}$ as a function of $(\beta\ a)$ from Eq. 19.

The burst size realized during the optimal latency, from Eq. 8 with $\epsilon = 0$, is

$$\hat{b_o} = \beta\ \hat{l_o} = \frac{e}{a} \tag{20}$$

The optimal burst size is independent of β. To see why, suppose productive environmental conditions increase the speed at which new virions are produced, β. This leads to a shorter optimal latency, $\hat{l_o}$. Then the higher speed of virion production times the shorter latency cancel out leaving the burst unchanged. However, the optimal burst size does vary inversely with a. A lower bacterial density leading to a lower a causes a higher burst size according to Eq. 20.

The Parada *et al.* (2006) observations in which both viral burst size and bacterial productivity increase together across various sites in the North Sea can be reconciled with this model if both (1) higher nutrients increase R, as was observed, and (2) higher nutrients also decrease bacterial density, thereby lowering a, which in turn leads to higher burst according to Eq. 20. Also according to Eq. 20, any effect of nutrients on β is irrelevant. This ecological situation is where high nutrients cause high productivity and turnover associated with a low standing crop of bacteria, as is characteristic of grazed ecosystems (*eg.* Odum and Barrett 2005).

The virion productivity per the time step corresponding to the optimal latency is invariant because $\hat{b_o}$ depends inversely on a, in Eq. 20. The a therefore cancels out in

$$\hat{b_o}\ a = (\frac{e}{a})\ a = e \tag{21}$$

To see why, suppose the adsorption, a, increases. Then the optimal latency decreases. Therefore, the corresponding burst size decreases too. The decreased burst then cancels out the increase in adsorption leaving the virion productivity per the time step corresponding to the optimal latency unchanged. So, as a and β are varied across environmental gradients, the optimal latency does vary according to Eq. 19. However, the virion productivity resulting from those varying latencies remains constant for the corresponding time step.

The relative advantage of the lytic to lysogenic lifecycle can to be computed over a long period, L. Multiple iterations of the single-step model occur within this period that can be compared with bacterial population growth over the same period. The optimal virion productivity over L relative to the bacterial productivity over L is found by substituting Eq. 19 into Eq. 18 leading to

$$\widehat{W_{Lo}} = \frac{e^{(\frac{\beta\ a\ L}{e})}}{R^L} \tag{22}$$

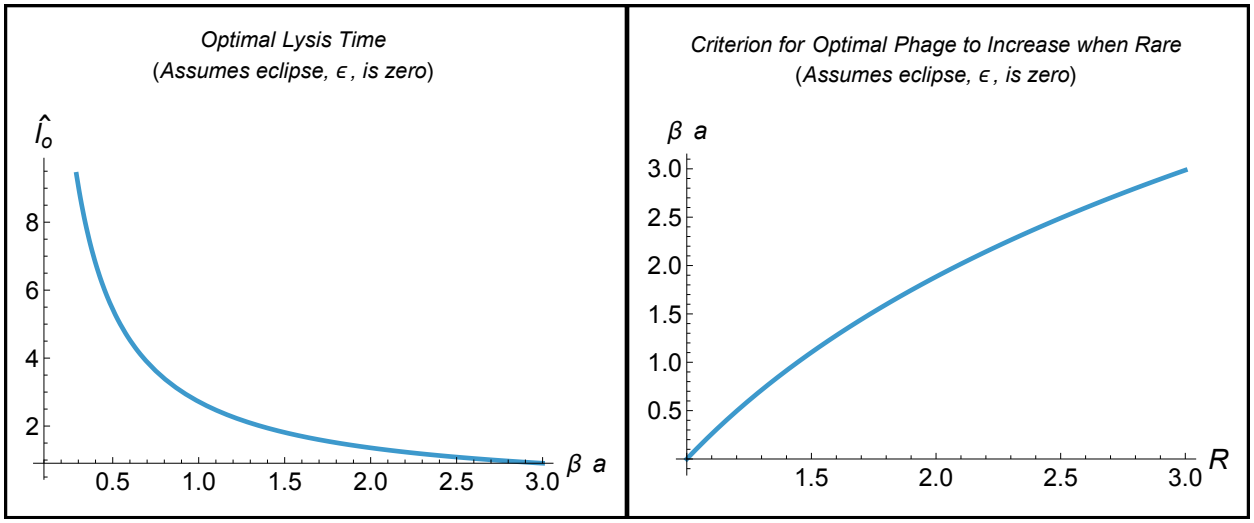

Figure 3: *Left: The optimal lysis time $\hat{l}_o$ as a function of the quantity $(\beta\ a)$, where β is the rate of virion production per minute of lysis time and a is the adsorption parameter, from Eq. 19. Right: Graph of criterion for increase when rare for an optimal lytic phage as a function of the bacterial geometric growth factor, R, from Eq. 24. If $(\beta\ a)$ is greater than the curve of $(e\ \log[R])$ then the virus can increase when rare, otherwise it is excluded from the system if virulent, or becomes lysogenic if temperate. If the lytic virus can increase when rare, the corresponding optimal lysis time can be read off the graph on the left.*

The $\widehat{W_{Lo}}$ increases with $(\beta\ a)$ and decreases with R. To see the long-term relative advantage of the optimal lytic lifecycle to the lysogenic lifecycle, take the limit as L tends to infinity, yielding

$$\begin{aligned} \lim_{L\to\infty} \widehat{W_{Lo}} &= \infty \quad \text{if} \quad \beta\ a > e\ \log(R) \\ \lim_{L\to\infty} \widehat{W_{Lo}} &= 0 \quad \text{if} \quad \beta\ a < e\ \log(R) \end{aligned} \tag{23}$$

Thus, the optimal virus, *i.e.*, one with the optimal balance between latency and burst size, increases when rare over the long term if

$$\beta\ a > e\ \log(R) \tag{24}$$

otherwise it goes extinct or becomes lysogenic.

Figure 3 (Right) shows the criterion for increase when rare for an optimal lytic phage as a function of the bacterial growth factor, assuming the eclipse is zero, from Eq. 24. If $(\beta\ a)$ on the vertical axis is greater than the curve for $(e\ \log(R))$ then the virus can increase when rare, otherwise it is excluded from the system if virulent, or becomes lysogenic if temperate.

To pull these results together, consider a gradient in environmental factors along which the lytic productivity, $(\beta\ a)$, increases. Assume too that the eclipse, ϵ, is small. Then (1) the optimal lysis time shortens along the gradient, implying that the lytic life cycle goes around faster along the gradient, according to Eq. 19; (2) The burst size does not vary along the gradient, according to Eq. 20, unless a changes because of bacterial density change along the gradient; and (3) the virion productivity realized during the time step corresponding to each point along the gradient remains constant, according to Eq. 21. If the bacterial productivity, R, also increases along this gradient, then the condition in Eq. 24 can flip. At the flipping point on the gradient, the virus either goes extinct or if temperate, switches to lysogeny.

Discussion

This article provides a mathematical model for the evolutionarily optimal balance between lysis time and burst size. Wang (2006) found the optimal lysis time with strains of λ phage infecting *E. coli* to be 51 minutes. Kannoly *et al*. (2022) also measured strains of λ phage infecting *E. coli* and found an optimal lysis time of 80 minutes. The value of the optimal lysis time differed between these studies because the environmental conditions in the experiments differed. Given the reality of an optimal lysis time that depends on environmental conditions, the issue is how to explain it.

This article explains the optimal lysis time by finding the optimal balance between rapid reproduction with a short lysis time at the cost of a low burst size *vs*. slow reproduction with a long lysis time and the benefit of a large burst size. The optimal balance between these two considerations is derived using a single dynamical equation in discrete time for the virus to microbe ratio (VMR) during the log phase of phage/bacteria population growth (Roughgarden 2024).

Previous models for the optimal balance use a system of delay-differential equations for the dynamics of uninfected bacteria, infected bacteria and free phage in the stationary phase of virus-bacterial population growth (*eg*. Bull 2006, Bull *et al*. 2006). The system of equations is an extension of the original delay-differential equation model introduced by Campbell (1961). The original Campbell model is critiqued in detail in Roughgarden (2024, incl. Supplementary Material).

In the present context, the relevant distinction between this paper and previous papers is the intuition behind the optimal lysis time. Eq. 4 in Bull (2006) gives a formula for the optimal lysis time, (called L). When the phage is at population-dynamic equilibrium (called $\hat{\lambda} = 0$) and when the eclipse (called E) is zero, then the optimal lysis time is $L = 1/d$, where d is the death rate of bacterial cells. This prediction from Eq. 4 in Bull (2006) for the optimal lysis time is completely different from that in this paper, Eq. 19.

The intuition behind Eq. 4 of Bull (2006) for the optimal lysis time is that a high bacterial death rate implies a short life span for the bacteria, leaving only a short time for an infecting virus to make new virions. Thus, the tradeoff is between the time needed to produce a burst and the time available within a bacterial cell to do it. This explanation for the optimal tradeoff is conceptually different from that offered by this paper's model that focusses instead on the tradeoff between rapid reproduction with low burst *vs*. slow reproduction with high burst.

Another conceptual difference between Eq. 4 of Bull (2006) for the optimal lysis time *vs*. Eq. 19 here is that, colloquially speaking, in the former the virus is trying to synchronize its lysis time with the bacterial dynamics, whereas here, the virus is trying to find the best balance between early and late lysis irrespective of what the bacteria are doing. Then once the lytic virus is doing the best it can, it compares what it can achieve by remaining lytic with what it can achieve by lysogenizing and decides to remain lytic or to switch to lysogeny according to which yields the higher fitness.

Grounds for preferring this paper's model over previous models, in addition to its simplicity and tractabilty, rest on which environmental situation is more common—log phase growth or stationary growth for the phage-bacteria pair. Previous models are directed to experimental situations such as a chemostat where a stationary phase can be realized. But in nature it is hard to imagine phage and bacteria in equilibrium. In nature the populations are invariably either recovering from or being decimated by environmental events such as storms *etc*. Natural populations of phage and bacteria are likely continually in a bust-boom regime where a log-phase model is preferable to a stationary-phase model.

The model of this paper predicts the evolutionary impact of interventions to control phage and predicts possible ecogeographic patterns of latency and burst size along environmental gradients. The model predicts that interventions lowering phage adsorption cause the evolution of an increased lysis time and correspondingly higher burst size until the intervention is sufficient either to drive a virulent phage extinct, or for temperate phage, to drive the phage from its lytic phase into its lysogenic phase.

The model predicts that along a gradient in which the viral productivity increases, the optimal lysis time shortens and the lytic life cycle goes around faster.

All in all, this paper shows that the life-history traits of virus are not arbitrary, but vary in predictable ways associated with different environments.

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