Improved Source Localization of Auditory Evoked Fields using Reciprocal BEM-FMM

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May 2025

Abstract

We apply our recently introduced technique of reciprocal magnetoencephalographic (MEG) source estimation via the boundary element fast multipole method (reciprocal BEM-FMM) to localize auditory evoked fields (AEFs). We compare our results with the source estimates of MNE-Python against simulated N1m components of AEFs, as well as experimental data for a cohort of 7 participants subjected to binaural auditory stimulation. Previous comparisons of reciprocal BEM-FMM with MNE focused on evoked somatosensory fields, which produced results of similar quality. In this work we show that the localization of AEFs using high-resolution reciprocal BEM-FMM is significantly better in terms of accuracy and focality than those estimates of the low resolution 3-layer BEM of MNE. Our findings suggest that the use of high-resolution models plays a significant role in the quality of source estimates and that these may provide improvements in a wide range of applications.

1 Introduction

The objective of source localization (Knösche and Haueisen 2022) is to estimate the location of neural activation from non-invasive recordings such as *electroencephalography* (EEG) or *magnetoencephalography*

(MEG) (Hämäläinen et al. 1993). In Nuñez Ponasso et al. 2025 we introduced a novel forward solver for MEG source localization based on Lorentz's reciprocity theorem and the *boundary-element fast multipole method* (BEM-FMM) (Makarov et al. 2018; Makarov et al. 2020). This new method is based on the computation of cortical basis functions by means of a fictitious current injected through each MEG sensor coil, similar to forward solutions in transcranial magnetic stimulation (TMS). The use of the fast multipole method (Greengard and Rokhlin 1987) coupled with the charge-based formulation of the BEM (Nuñez Ponasso 2024) allows us to avoid computing the dense system matrices of the traditional surface-potential BEM (Geselowitz 1967; Geselowitz 1970), and as a result our method enables source localization using multiple high resolution meshes. In Nuñez Ponasso et al. 2025 we localized somatosensory evoked fields and obtained good results compared to the classical MNE-Python with 3-layer models (Gramfort et al. 2013; Gramfort et al. 2014). In this work, we apply our methods to auditory evoked fields, which are more challenging from a source reconstruction perspective than the somatosensory evoked fields: on the one hand, the binaural auditory stimulation elicits activation on both hemispheres, with two approximately symmetric sources at the far-apart left and right temporal lobes; on the other hand, the head anatomy near the temporal lobes is more complicated than near the parietal lobes, and it is harder to represent accurately using downsampled 3-layer head models. As a result, high-resolution models will be able to provide more accurate source localizations—this was already suggested in the error maps of Nuñez Ponasso et al. 2024.

Auditory evoked fields (AEFs) are magnetic signals elicited by auditory stimuli. They are localized in the auditory cortex, having components in the Heschl gyri (primary auditory cortex) and in the planum temporale as a secondary area (Pelizzone et al. 1987; Liégeois-Chauvel et al. 1994; Yoshiura et al. 1995; Godey et al. 2001). There are three main components of auditory evoked fields: 1) Middle-latency components, e.g. Pam (< 50 ms) which originates in regions medial and lateral to the Heschl gyri; 2) N1m (~ 100 ms), consistently localized in the lateral Heschel gyri and planum temporale; and 3) P2m (~ 200 ms), which has been found to have origins near the N1m source locations although with much greater interindividual variability (Lütkenhöner and Steinsträter 1998).

The source localization of AEFs is relevant from an applications standpoint as they can be used clinically for presurgical mapping in the temporal lobe epilepsy (Godey et al. 2001), in the identification of functional reorganization in patients with lesions (Nakasato and Yoshimoto 2000), and as a biomarker for language and developmental disorders (Oram Cardy et al. 2008; Park et al. 2022). The sources of AEF are also very important due to their correlations with mental health and psychological disorders (Kim et al. 2021; Fitzgerald 2024; Dib et al. 2024).

In this study, we analyze the source localization of auditory evoked fields elicited by binaural auditory stimuli for a cohort of 7 patients with drug-resistant depression, whose data were provided by the authors of the study of Dib et al. 2024. Using each patient's BEM models, we simulate noiseless N1m peaks of binaural auditory stimulation. We compute the source localization results for both the simulated and experimental N1m peaks. Additionally, we compare the source localization results from our own solver to that of MNE-Python.

2 Materials and methods

2.1 Forward modeling using reciprocal BEM-FMM

We construct forward models using the reciprocal BEM-FMM approach (Nuñez Ponasso et al. 2025), which hinges on the Lorentz reciprocity theorem relating the primary electric field $\mathbf{E}\ [\mathrm{V\,m^{-1}}]$ induced by an induction coil outside the cortical surface and the magnetic flux density $\mathbf{B}\ [\mathrm{T}]$ induced by a

Table 1: Parameter values used in the computation of the forward model using BEM-FMM. The symbol μ_A denotes the average triangle area in the mesh.

Parameter	Value
FMM precision for ρ	1×10^{-4}
FMM precision for $\mathbf{E}^i$	1×10^{-4}
GMRES relative residual tolerance	1×10^{-4}
Region of interest (ROI) radius	$7 \times 10^{-2}[\mathrm{m}]$
Neighbor radius for local integral correction	$2\sqrt{\mu_A}\ [\mathrm{m}]$
Characteristic of the coil pulse waveform: $\partial i_j/\partial t$	$1[\mathrm{A\,s^{-1}}]$

current dipole source at location $\mathbf{r}$ $[\mathrm{m}]$ within the cortical surface (Plonsey 1972; Heller and Hulsteyn 1992; Nummenmaa et al. 2013; Nuñez Ponasso et al. 2025):

$$\mathbf{p} \cdot \mathbf{E}(\mathbf{r}) = I_0 \int_S \mathbf{B}(\mathbf{s}) \cdot \mathbf{n}(\mathbf{s}) d\mathbf{s}, \tag{1}$$

where $\mathbf{p}$ $[\mathrm{A\,m}]$ is the dipole moment of the current dipole source located at $\mathbf{r}$, and I_0 $[\mathrm{A}]$ is the amplitude of the current flowing through the induction coil S. Assuming the dipole moment is oriented normal to the cortical surface, we can extend Equation 1 to $i = 1, \ldots, M$ induction coils and $j = 1, \ldots, N$ current dipoles, yielding

$$Lp = b, \tag{2}$$

where L denotes the lead-field matrix whose entries are given by

$$L_{ij} = \mathbf{n}_j \cdot \mathbf{E}_i(\mathbf{r}_j), \tag{3}$$

where $\mathbf{E}_i$ is the induced electric field of the i-th induction coil, p_j is the strength of the j-th current dipole, and b_i is the total magnetic flux through the i-th induction coil. Equation 2 represents the reciprocal BEM-FMM forward model, with each of the i coils acting analogously to TMS coils. However, in the context of MEG source analysis, each of the i coils can now be considered as MEG sensors, with the entries of b indicating the total measured output of each sensor. In this sense, Equation 2 represents a typical lead-field model of source reconstruction, with p denoting the unknown strengths of each of the current source dipoles (Knösche and Haueisen 2022).

Table 1 shows the parameter values used in the BEM-FMM computation of the forward model. Details regarding the forward model computation are found in Nuñez Ponasso et al. 2025.

2.2 Inverse solutions using minimum norm estimation and dSPM

We compute the maximum a posteriori solution to the inverse problem of solving for source strengths p given noisy MEG data b (Knösche and Haueisen 2022). Such solutions are found by using minimum norm estimation to invert Equation 3. This is done using the inverse operator (Hämäläinen and Ilmoniemi 1994)

$$M = RL^{\mathrm{T}}(LRL^{\mathrm{T}} + \lambda^2\Sigma)^{-1}, \tag{4}$$

where R is the prior source covariance, Σ is the noise covariance, and λ is a regularization parameter chosen as the inverse of the signal-to-noise ration, $\lambda = 1/\mathrm{SNR}$. In this way, the minimum norm estimated solution to Equation 2 is $p = Mb$.

To correct for superficial sources appearing on the cortical gyri nearest the MEG sensors, we use a depth weighted source covariance matrix which is a diagonal matrix whose diagonals are the norms of the corresponding lead-field matrix column (Knösche and Haueisen 2022),

$$R_{i,i} = \left(L_{1,i}^2 + L_{2,i}^2 + \cdots + L_{M,i}^2\right)^{-1/2}. \tag{5}$$

We can further bias our solutions towards the most statistically relevant dipoles using dynamic statistical parametric mapping (dSPM) (Dale et al. 2000). The dSPM solution is obtained from normalizing the reconstructed sources by

$$p^* = \frac{p^{\mathrm{T}} p}{\mathrm{tr}(M \Sigma M^{\mathrm{T}})}. \tag{6}$$

Then p^* is the vector of depth-corrected reconstructed source strengths.

2.3 Generation of synthetic data for the evoked N1m peak

We simulated noiseless data for the N1m peak of binaural auditory stimulation. This is similar to the simulation of evoked somatosensory data described in Nuñez Ponasso et al. 2025, with the difference that to model activation on the primary auditory cortex we used multiple dipoles distributed over a patch of the region rather than a single dipole. We generate this simulated data in two manners, the first using direct BEM-FMM and the second using MNE-Python's `SourceSimulator` functionality.

To generate the BEM-FMM simulated data, we manually segmented the Heschl gyri of each participant from their T1-weighed MRI data by selecting nearby points in MRI coordinates. The nearest WM mesh triangles were tagged, and a dipole was placed directly above each of the tagged triangle centers between the WM and GM interfaces. The orientations of each of these dipoles were chosen to be the average normal to the nearest GM triangles. Once the dipoles were placed, we used the method of BEM-FMM with b-refinement described in Wartman et al. 2025, to calculate the electric potential on each tissue interface; see also Nuñez Ponasso et al. 2024. Once the potential is calculated, we computed the B-field on 64 observation points for each magnetometer coil and each individual coil the gradiometer pairs. With this, the integral of the normal component of the B-field is computed to generate the sensor outputs of the magnetometers, and in the case of gradiometers we oriented the unit normals in opposite ways for each gradiometer coil and divided the integral by the distance separating the coils of each gradiometer pair, cf. Knösche and Haueisen 2022. This process of simulation of the evoked N1m is illustrated in Figure 1.

The sensor outputs were simulated independently for the dipoles on the Heschl gyri of each hemisphere. The strength of the dipoles was normalized by the number of placed dipoles to make the total strength of the dipoles placed on each hemisphere exactly $1 \times 10^{-5}\ \mathrm{A\,m}$. To obtain the sensor outputs for the binaural N1m peak, we simply added the outputs of the activation of the left and right hemisphere — this process is valid by the quasi-static approximation to Maxwell's equations (Plonsey 1972; Nuñez Ponasso 2024).

For the simulated source outputs using MNE-Python, the regions of interest were chosen according to the FreeSurfer `aparc.a2009s` atlas (Destrieux et al. 2010) using the label `G_temp_supG_T_transv` across both the left and right hemispheres. From the centroids of these regions, we select all sources within a $10\mathrm{mm}$ extent. These sources are then activated with a sinusoidal waveform time series via MNE-Python's `mne.simulation.SourceSimulator` protocol (Gramfort et al. 2014). Figure 2 illustrates the activated sources on both hemispheres and the corresponding sensor outputs. We performed source reconstruction with these simulated sensor outputs at time $t = 90\mathrm{ms}$.

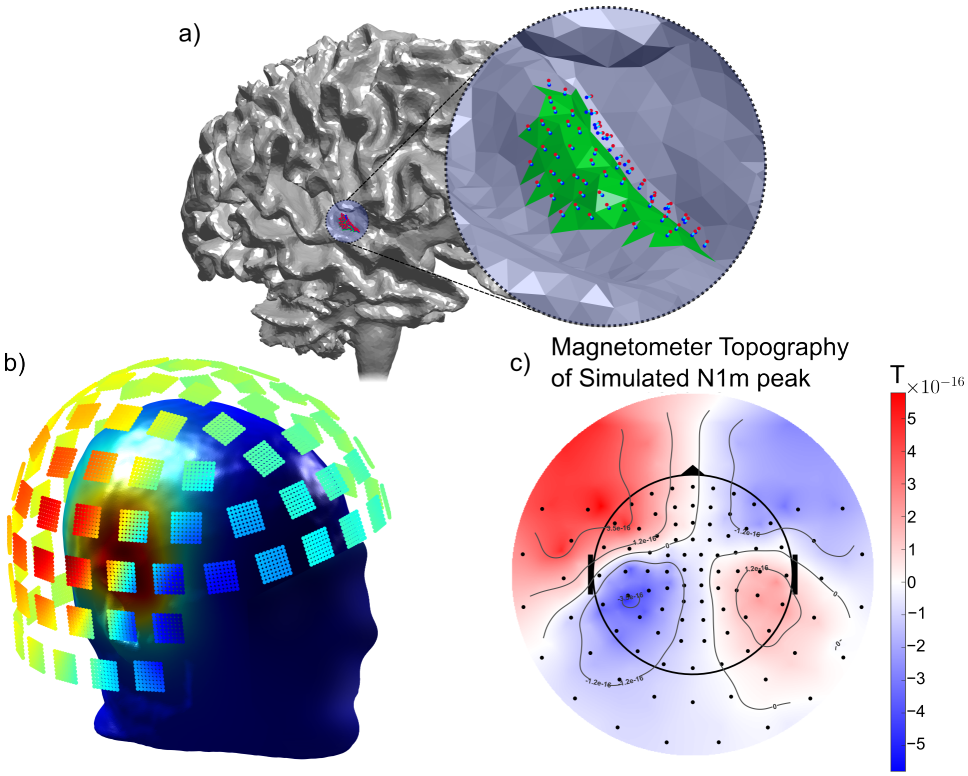

Figure 1: Simulation of the magnetometer signals for the evoked N1m peak on participant N04: a) a patch of WM points (green) at the Heschl gyrus is manually segmented from MRI data; finite-length current dipoles are placed directly above and oriented normal to the cortical surface, with the red and blue spheres denoting the isotropic current sources and sinks, respectively. As such, the direction of each dipole moment is given by the vector from the blue sphere to the red sphere; b) The potential on each tissue (skin plotted) is calculated with BEM-FMM and b-refinement, the B-field is subsequently computed at 64 observation points per magnetometer coil; c) a surface integral is computed to obtain the sensor outputs; here the interpolated and projected topography of the simulated N1m peak is shown.

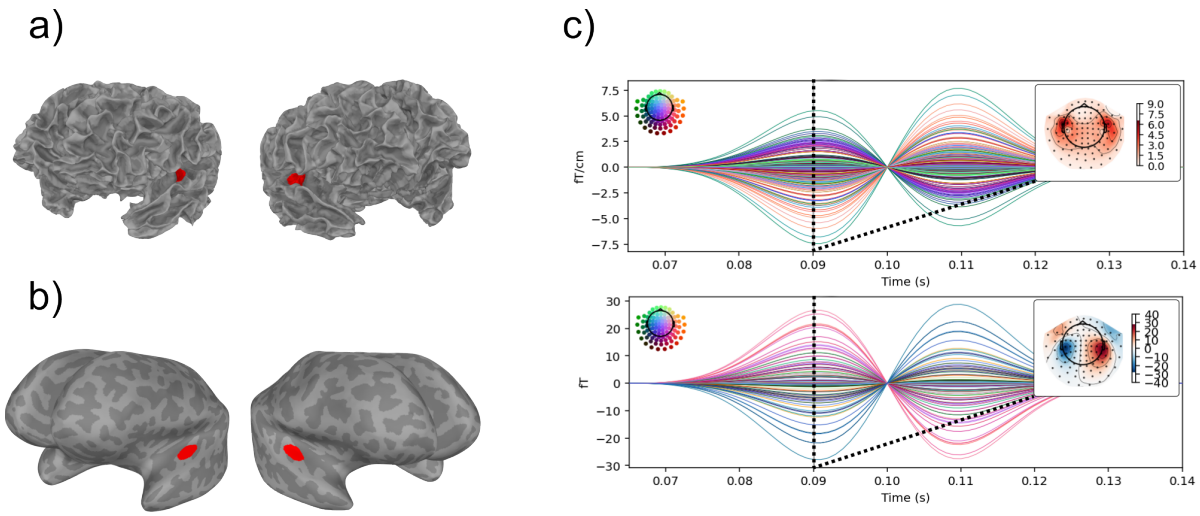

Figure 2: Simulation of sensor outputs for participant N04 using MNE-Python: a) The activated sources on the FreeSurfer white matter mesh. b) The activated sources on the inflated white matter. c) The simulated gradiometer (top) and magnetometer (bottom) sensor outputs according to the activated sources and a sinusoidal waveform, with topographies selected at the time $t = 0.09$. The waveform is constructed with oscillating frequency 15Hz, zero-measure latency 0.1ms.

2.4 Auditory response data processing

Data acquisition. Data were provided to us by the authors of Dib et al. 2024. These data were collected from 9 study participants with major depressive disorder. Out of these 9 participants (labeled N01–N09), two of them (N03 and N05) did not have MRI data available, so we restricted our study to the remaining 7. Participants were recruited from the University of New Mexico Mental Health Center to participate in a study focused on the loudness dependence of auditory evoked potentials (LDAEP) in pre- and post- electroconvulsive therapy (ECT). However, for this paper we only analyze the pre-treatment data.

T1-weighted structural MRI images were recorded using the 3-Tesla Siemens Trio MRI scanner at the Mind Research Network of Albuquerque, New Mexico. Raw MEG recordings were collected using the Elekta Neuromag VectorView 306, which possesses 102 magnetometers and 204 gradiometer pairs. Participants were stimulated with binaural auditory tones at a 2kHz frequency, with each tone lasting 50ms. The stimulation was performed using tones of varying loudness; however, for this paper we only consider the 95dB tones. Further details of patient data acquisition are described in Dib et al. 2024.

Boundary element method models. For the reciprocal BEM-FMM models, the participant MRI images were segmented into seven compartments using the SimNIBS `headreco` pipeline (Nielsen et al. 2018) to create the BEM headmodels. BEM models for MNE-Python were segmented using the Freesurfer pipeline (Fischl 2012). The conductivity values assigned to each BEM model can be seen in Table 2.

MEG signal pre-processing. MEG data pre-processing steps for the MPI pipeline were performed in MNE-Python 1.8.0 (Gramfort et al. 2013). Measurement data were run through a FIR low-pass filter with the upper pass-band edge at 85Hz, processed with Maxwell filtering —spatiotemporal signal space

Table 2: Conductivity values of each tissue layer for the reciprocal BEM-FMM and MNE-Python models.

Tissue	Conductivity $[\mathrm{S\,m^{-1}}]$	Reference
Scalp	0.25	(Truong et al. 2012)
Skull	0.01	(Wagner et al. 2004)
CSF	1.654	(Wagner et al. 2004)
GM	0.275	(Wagner et al. 2004; Thielscher and Wichmann 2009)
WM	0.126	(Wagner et al. 2004; Thielscher and Wichmann 2009)
Ventricles	1.654	(Wagner et al. 2004)†
Eyes	0.5	(Gabriel et al. 1996; Opitz et al. 2015)

(a) Tissue layers used in the reciprocal BEM-FMM forward models, their associated conductivities, and corresponding references.† The conductivity value of the ventricles is chosen to coincide with that of the CSF.

Tissue name	Conductivity $[\mathrm{S\,m^{-1}}]$	Reference
Scalp	0.25	(Truong et al. 2012)
Outer Skull	0.01	(Wagner et al. 2004)
Inner Skull	0.33	(Vorwerk et al. 2019)

(b) Tissue layers in the MNE-Python forward models and their corresponding conductivities and references.

separation (tSSS), see Taulu and Simola 2006— and finally high-pass filtered with a $1\mathrm{Hz}$ lower passband edge. Maxwell filtering was done with the default recommended expansion settings of $l_{in} = 8$ and $l_{out} = 3$, a spatiotemporal buffer duration of $20\mathrm{s}$, and a spatiotemporal correlation setting of 0.9. The applicability of the default expansion settings was evaluated for a single patient's data and found adequate. N1m peaks for each participant were determined at a mean latency of $105\mathrm{ms}\pm20\ \mathrm{ms}$.

2.5 Source reconstruction

In the reciprocal BEM-FMM source reconstruction, individual head-to-MEG coregistration was performed using an iterative closest points (ICP) algorithm (Bergström and Edlund 2014). For the MNE-Python source reconstruction pipeline, head-to-MEG coregistration was performed interactively using MNE-C 2.7.3 (Gramfort et al. 2014). All source localization solutions shown in this paper were found using dSPM. For source localization of synthetic N1m peaks, the noise covariance was set to be the identity matrix, with a noise regularization $\mathrm{SNR} = 100$. For localization using the experimental data, we created a noise covariance matrix from the pre-baseline signals at the $[-200, -5]\mathrm{ms}$ window. The signal-to-noise ratio (SNR) was set at $\mathrm{SNR} = 1.5$ for participants N06 and N08 because their averaged evoked time series were still noisy post-filtering; for all other participants we set $\mathrm{SNR} = 3$. For source reconstruction performed using MNE-Python, we used a depth weighting parameter of 0.65, as this parameter yielded the visually best results. Source reconstruction using reciprocal BEM-FMM was performed with a depth weighting of 1, which is the default parameter setting.

The source space in MNE-Python is found via the `ico-5` option of MNE-Python, which creates a uniform sampling of ~ 10242 sources on the interface between the FreeSurfer-segmented WM and GM meshes. On the other hand, the reciprocal BEM-FMM solution uses the triangle centers of the entire SimNIBS `headreco` WM mesh as a source space. To compare the source reconstructions using these two source models, we mapped the MNE-Python source space to the SimNIBS `headreco` mesh using a rotation and translation provided by ICP. For each source (i.e. triangle center) on the `headreco` WM mesh, we computed its source strength by averaging the estimated source strengths of those sources of

the MNE-Python source model within a sphere of radius $10\mathrm{mm}$ from the triangle center.

2.6 Model analysis using ROC curves

To analyze model performance, we compute the receiver operating characteristic (ROC) curve for each of the source localization solutions using the simulated data. The ROC curve is a useful statistical tool for evaluating the performance of a binary classification model (Hastie et al. 2009). The ROC curve is computed for source localization by treating the source strengths as binary classification variables across varying activation thresholds (Knösche and Haueisen 2022). Specifically, we define each of the triangle faces denoting the Heschl gyrus found in Section 2.3 as true dipoles, and all other triangle faces as false dipoles. Given the normalized dSPM solution of source localization, consider a reconstructed source or normalized strength s and a threshold T from 0 to 1. If the source lies on the tagged Heschl gyrus, we call the source a true positive (TP) if $s > T$, a false negative (FN) if $s < T$. Similarly, if the source does not lie on the Heschl gyrus, we call the source a false positive (FP) if $s > T$, a true negative (TN) if $s < T$. The ROC curve is defined as the curve plotting the false positive rate, $FP/(FP + TN)$, versus the true positive rate, $TP/(TP + FN)$, across all thresholds T.

The area under the ROC curve (AUC) can be used as a scalar metric to evaluate the performance of the solutions. In particular, if $AUC = 1$, then source localization perfectly captures the true dipoles tagged on the Heschl gyrus. If $AUC \leq 0.5$, then the source localization is no better than a random chance solution.

The smoothing that is done on the MNE-Python source estimations may negatively bias the ROC curves. Thus, when computing ROC curves for these solutions, we only consider the sparse MNE-Python solution and define the true dipoles as those on the sparse white matter mesh that are within $10\mathrm{mm}$ of the triangle centers of the tagged Heschl gyrus. The TP, FN, FP, and TN sources are then computed in the same manner as described above.

3 Results

Figure 3 shows source localization results for the simulated N1m peak data on participant N04. Figure 3a shows the solutions found from reciprocal BEM-FMM and MNE-Python using both magnetometer and gradiometer data. Figure 3b shows the sensor topography of the simulated magnetometer and gradiometer data. Figure 3c shows the ROC curves and corresponding AUC values for participant N04.

Figures 4-7 show dSPM source reconstruction results across all participants from simulated BEM-FMM data, with Figures 4 and 5 using the simulated magnetometer data, and in Figures 6 and 7 using the simulated gradiometer data. Figures 8-11 similarly show source reconstruction results from simulated MNE-Python data, with Figures 8 and 9 using simulated magnetometer data, and Figures 10 and 11 using simulated gradiometer data. The ROC curves corresponding theses source reconstructions for each participant are shown in Figure 12. Table 3 shows the computed AUC values corresponding to each of the ROC curves in Figure 12.

Figure 13 shows the source reconstruction results for the real N1m peak data of participant N01 using both MNE-Python and reciprocal BEM-FMM, with Figure 13a showing the evoked gradiometer signals and topography, and Figure 13b showing the dSPM solutions using each of the two solvers. Figure 14 shows detailed source reconstruction results for participant N09 using the magnetometer data, with Figures 14a and 14b showing the normalized source strengths on the left and right white matter hemispheres, respectively, and Figure 14c showing the strongest sources on the inflated white matter.

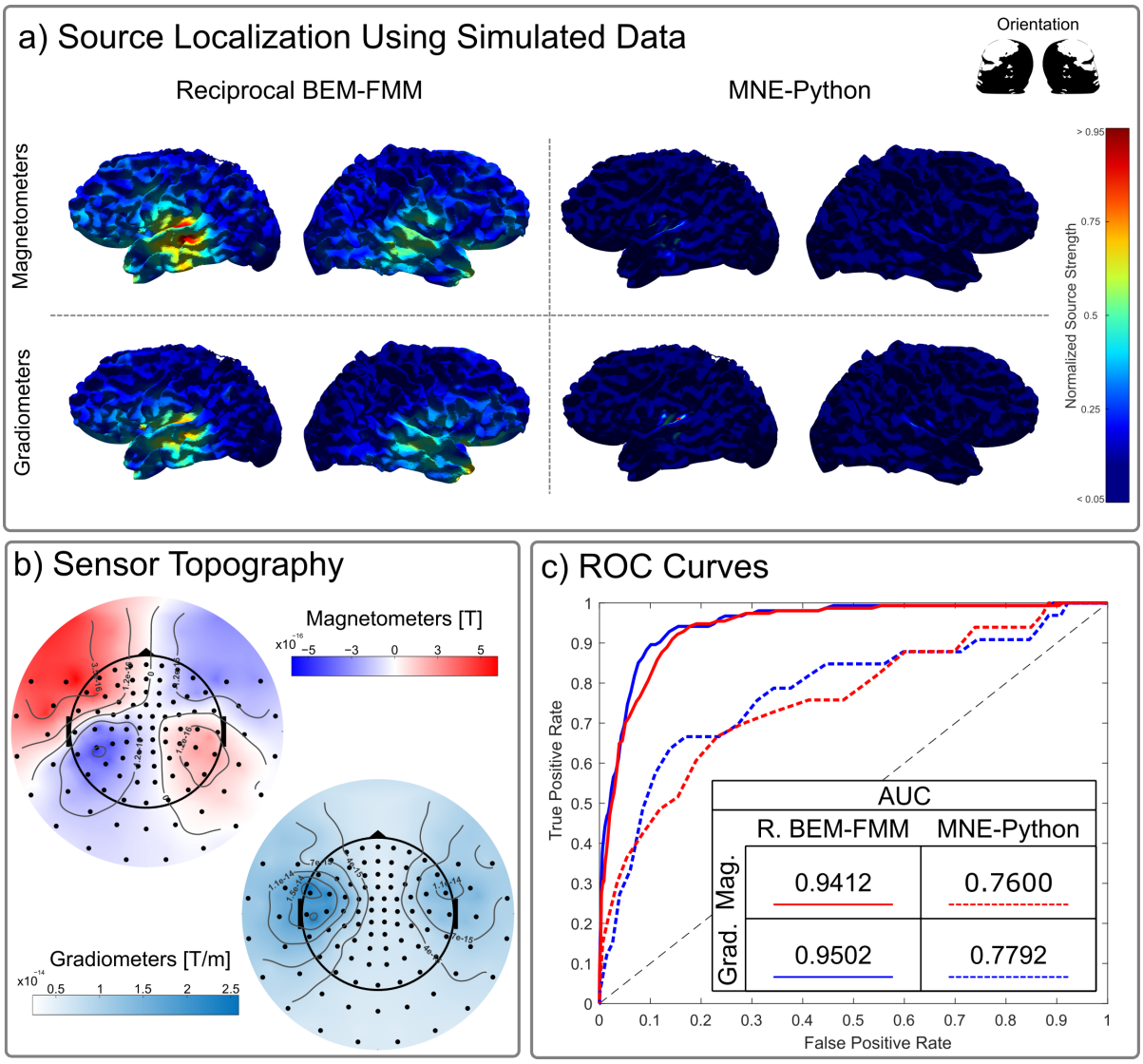

Figure 3: Source localization and ROC curve analysis for participant N04 using synthetic data. a) The source localization solutions using the simulated N1m peak data using both reciprocal BEM-FMM and MNE-Python in the left and right columns, respectively. The top row indicates solutions found using magnetometer data, and the bottom row that of gradiometer data. Lateral perspectives of the left and right hemispheres are shown from left to right, respectively. b) The sensor topography of the simulated N1m peak data is shown for both magnetometers and gradiometers. c) The ROC curves of each source reconstruction are shown, with reciprocal BEM-FMM in solid lines and MNE-Python in dashed lines. The red lines indicate solutions from magnetometer data, and the blue lines that of gradiometer data. The table of values denotes the AUC for each of the corresponding models.

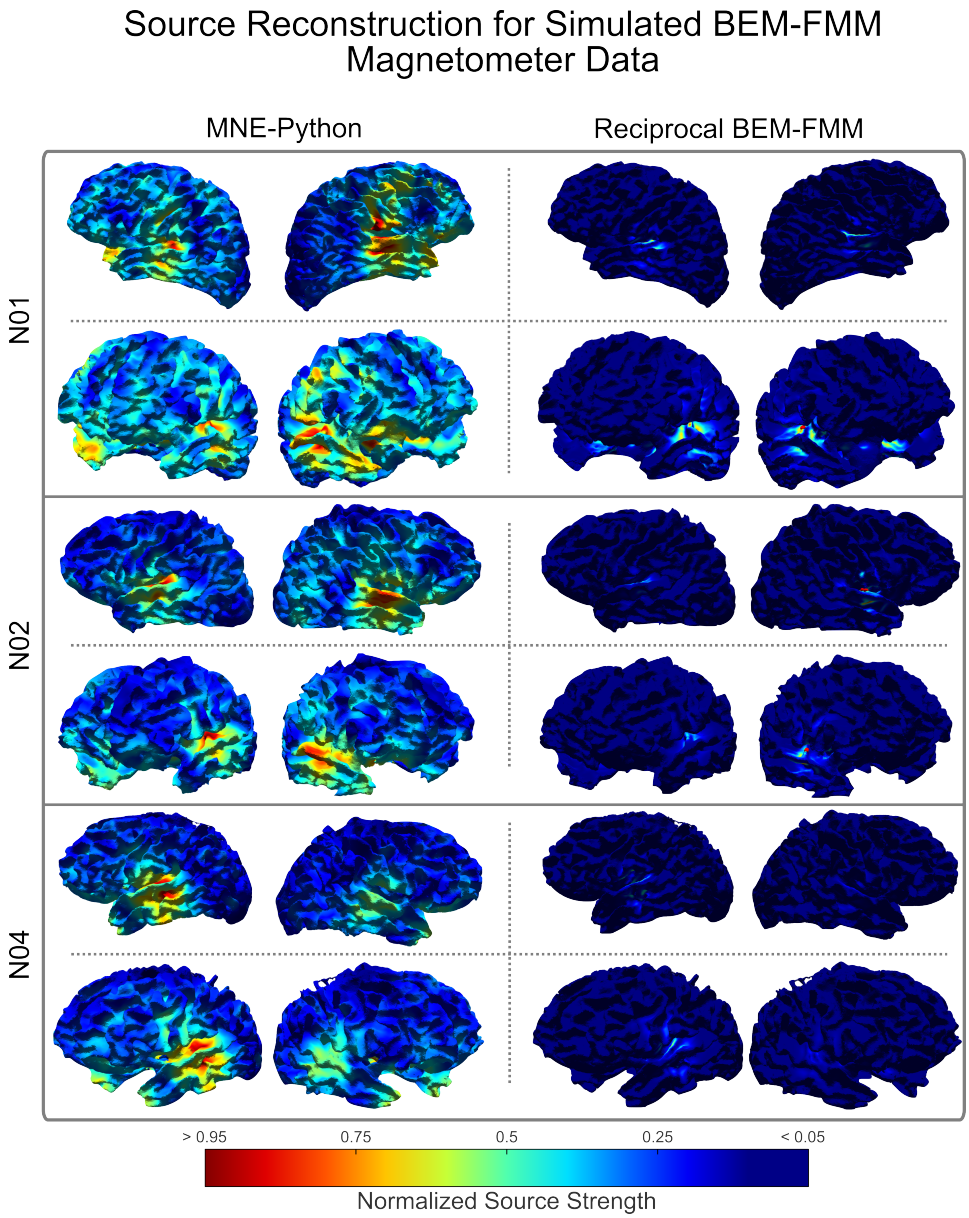

Figure 4: Source localization results of participants N01, N02, and N04 at their respective simulated magnetometer data using reciprocal BEM-FMM and MNE-Python, with the simulated data generated using BEM-FMM. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

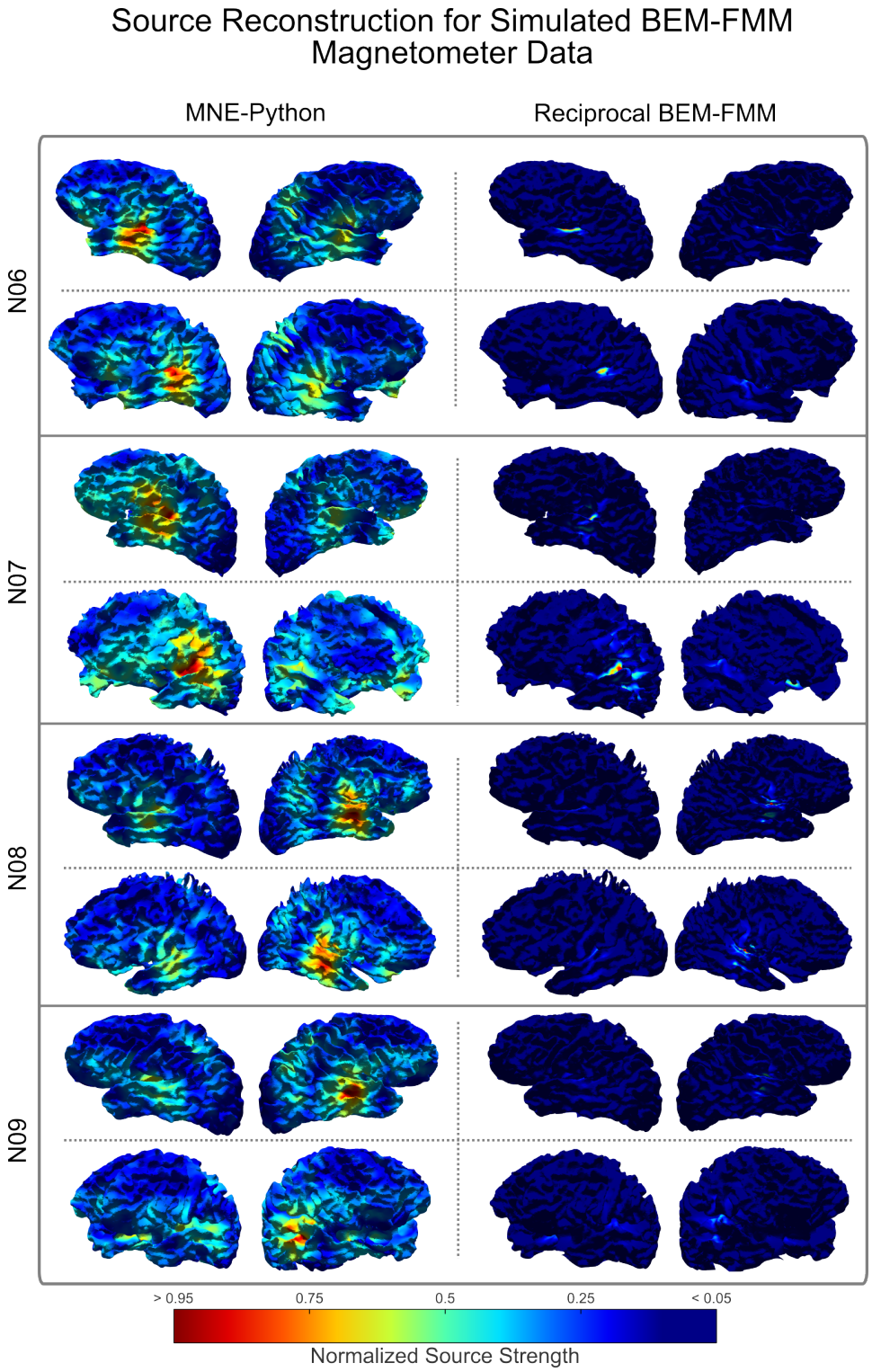

Figure 5: A continuation of the results of Figure 4 for participants N06, N07, N08, and N09.

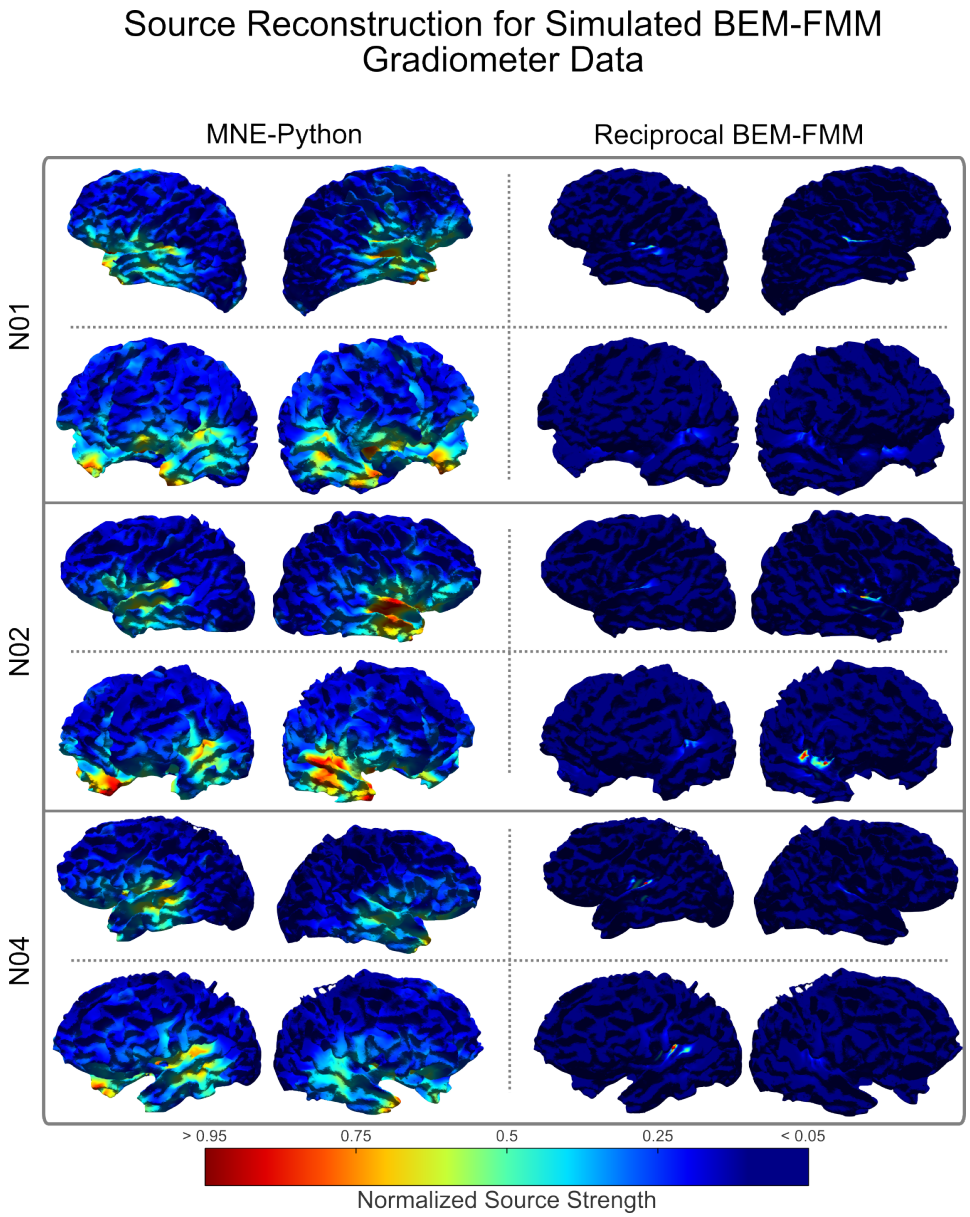

Figure 6: Source localization results of participants N01, N02, and N04 at their respective simulated gradiometer data using reciprocal BEM-FMM and MNE-Python, with the simulated data generated using BEM-FMM. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

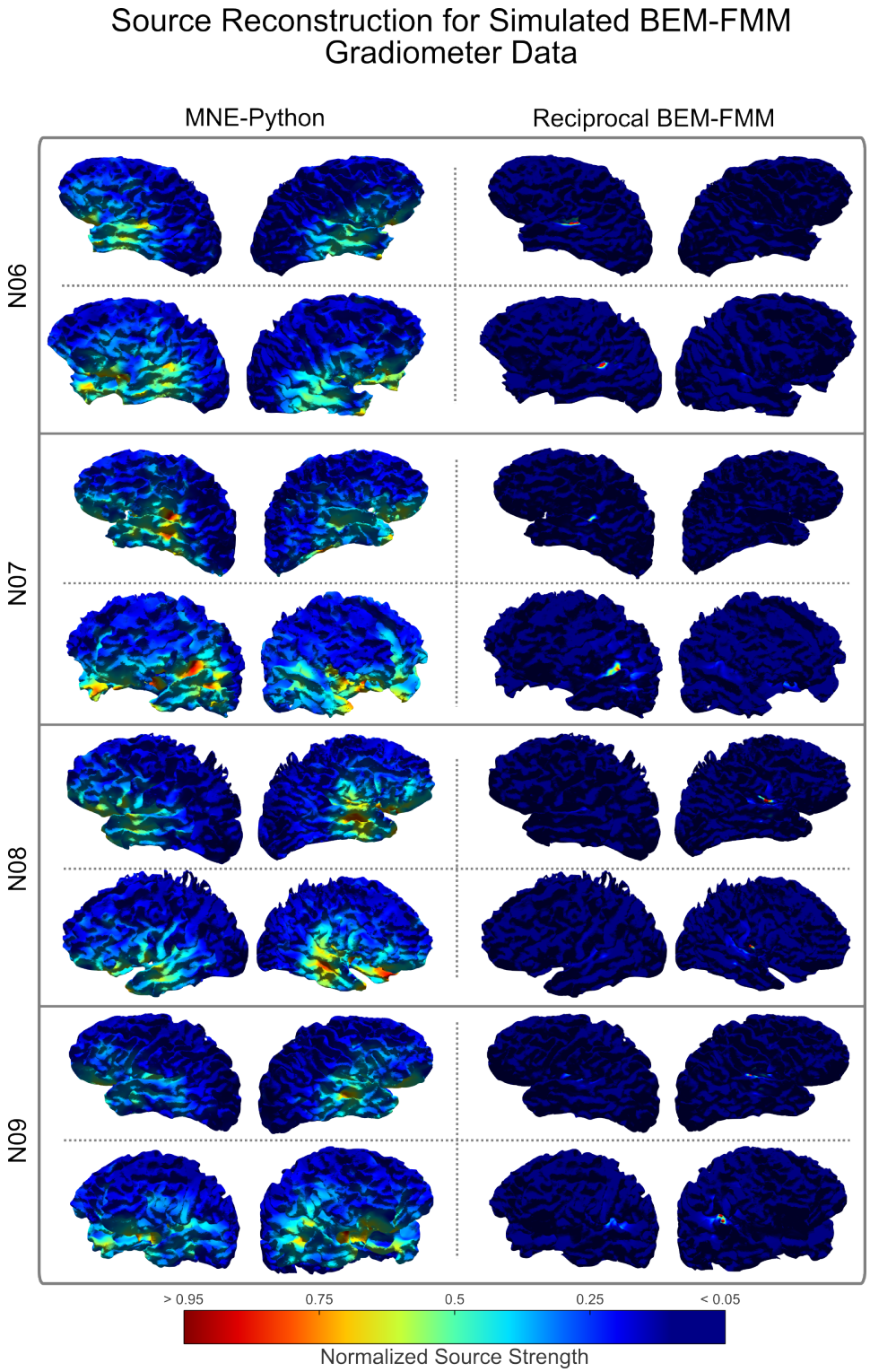

Figure 7: A continuation of the results of Figure 6 for participants N06, N07, N08, and N09.

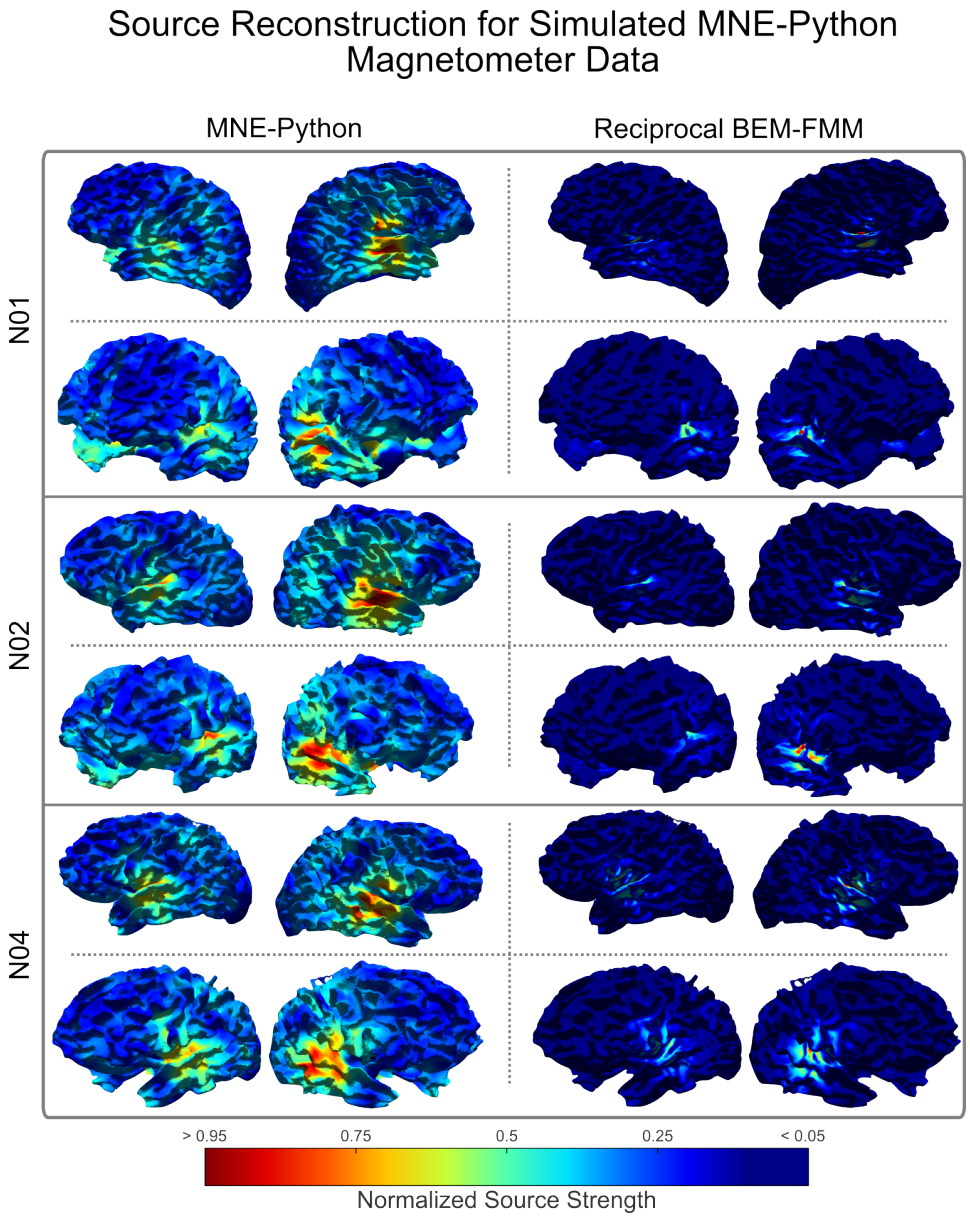

Figure 8: Source localization results of participants N01, N02, and N04 at their respective simulated magnetometer data using reciprocal BEM-FMM and MNE-Python, with the simulated data generated using MNE-Python. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

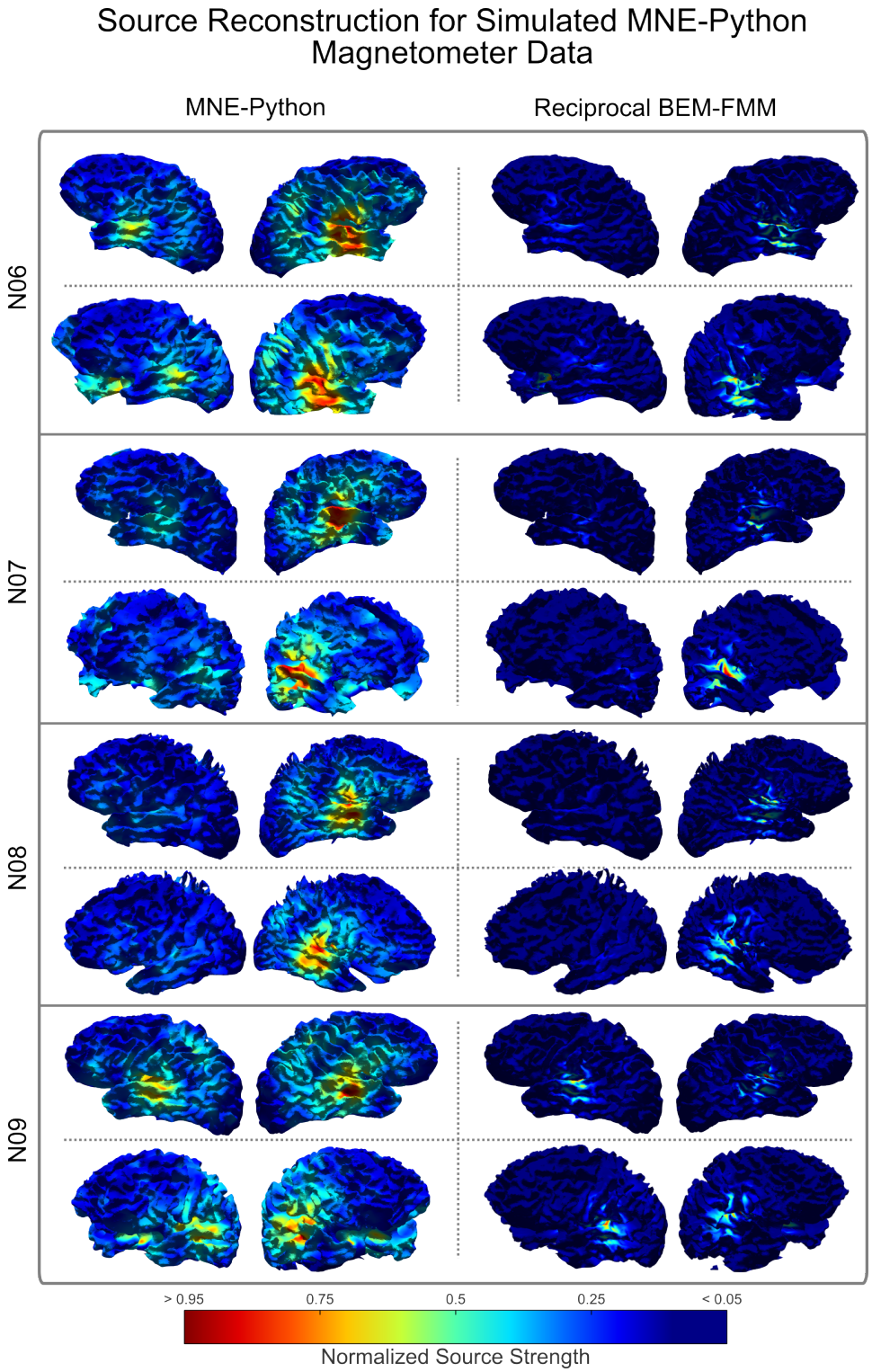

Figure 9: A continuation of the results of Figure 4 for participants N06, N07, N08, and N09.

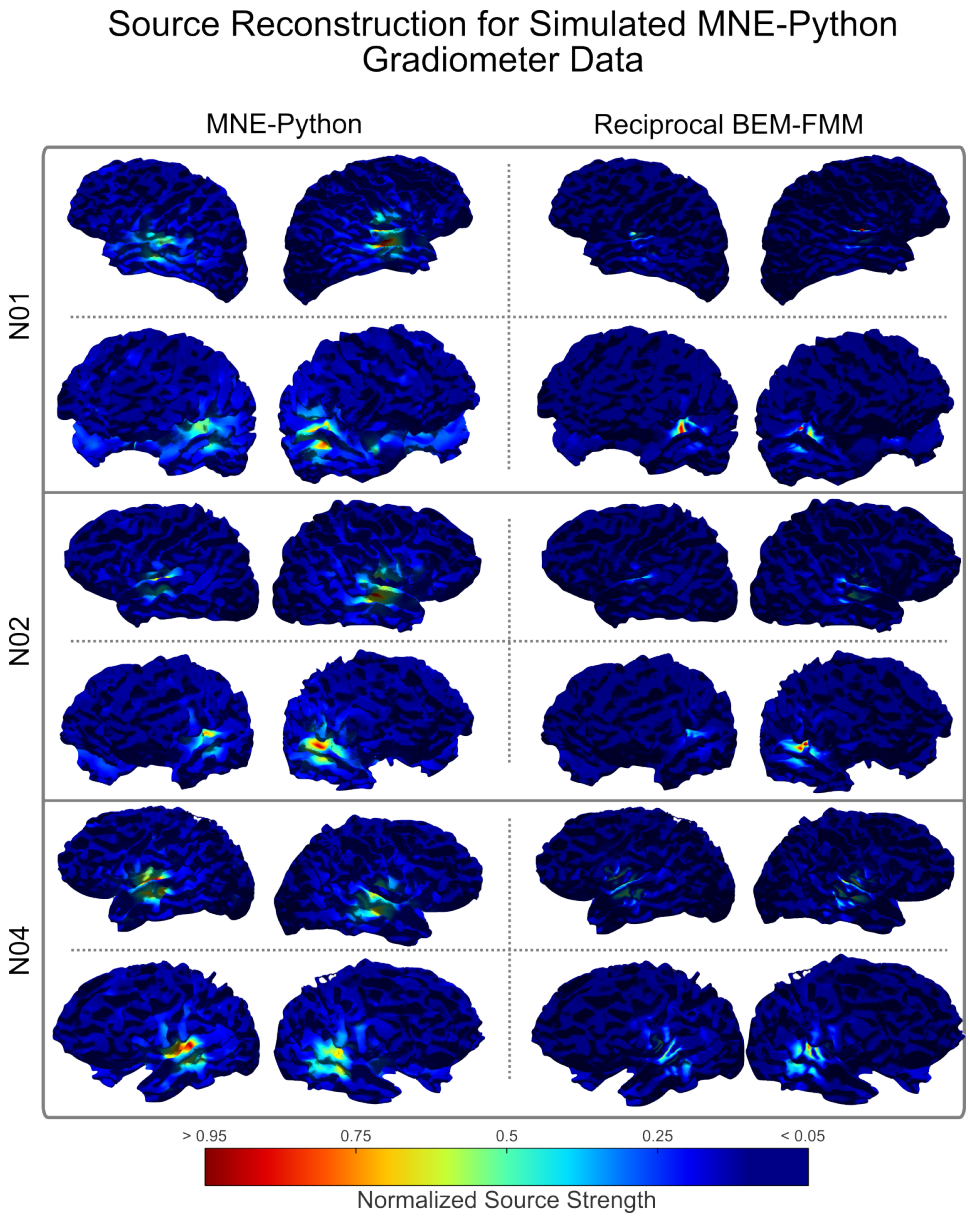

Figure 10: Source localization results of participants N01, N02, and N04 at their respective simulated gradiometer data using reciprocal BEM-FMM and MNE-Python, with the simulated data generated using MNE-Python. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

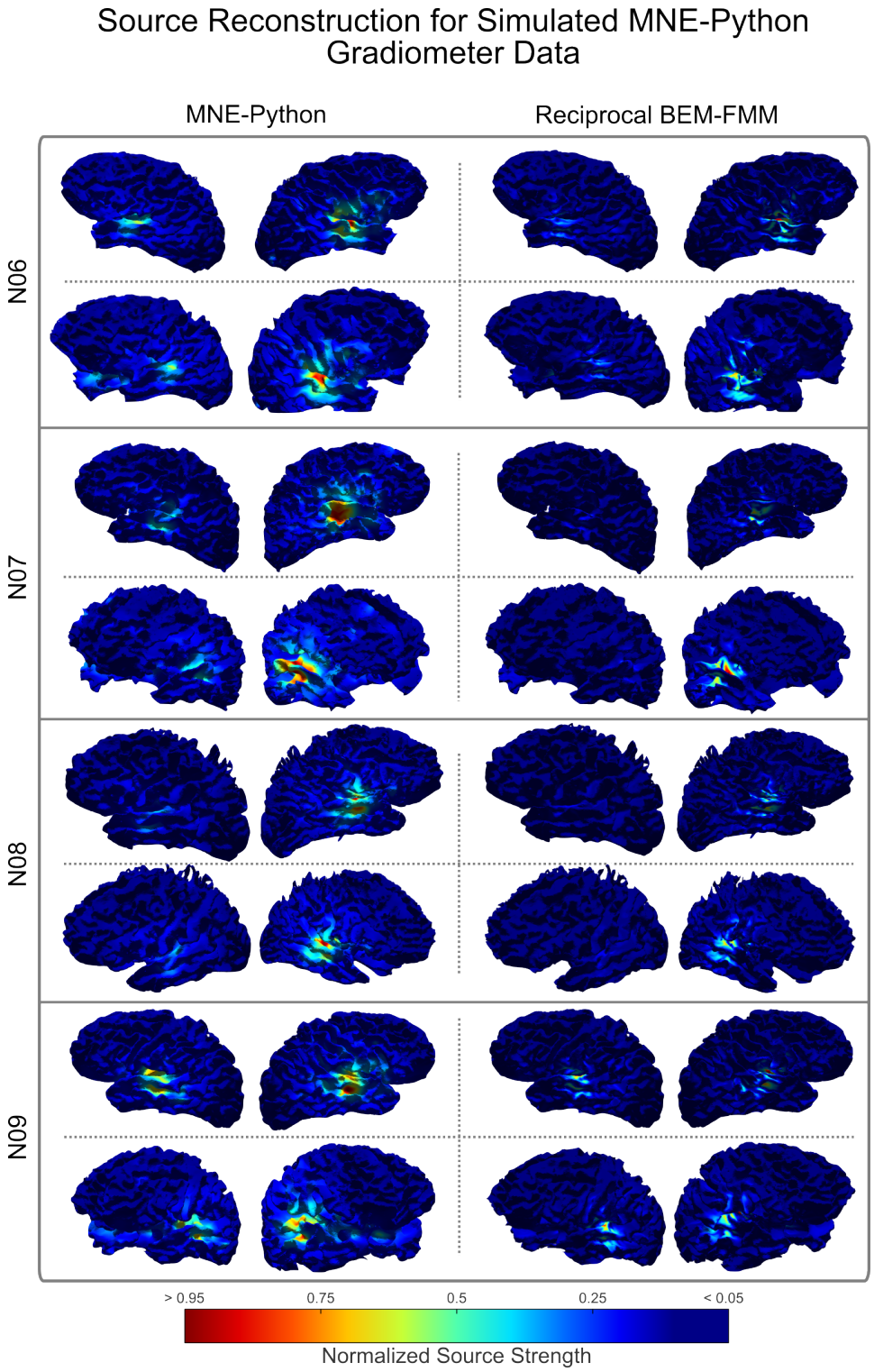

Figure 11: A continuation of the results of Figure 6 for participants N06, N07, N08, and N09.

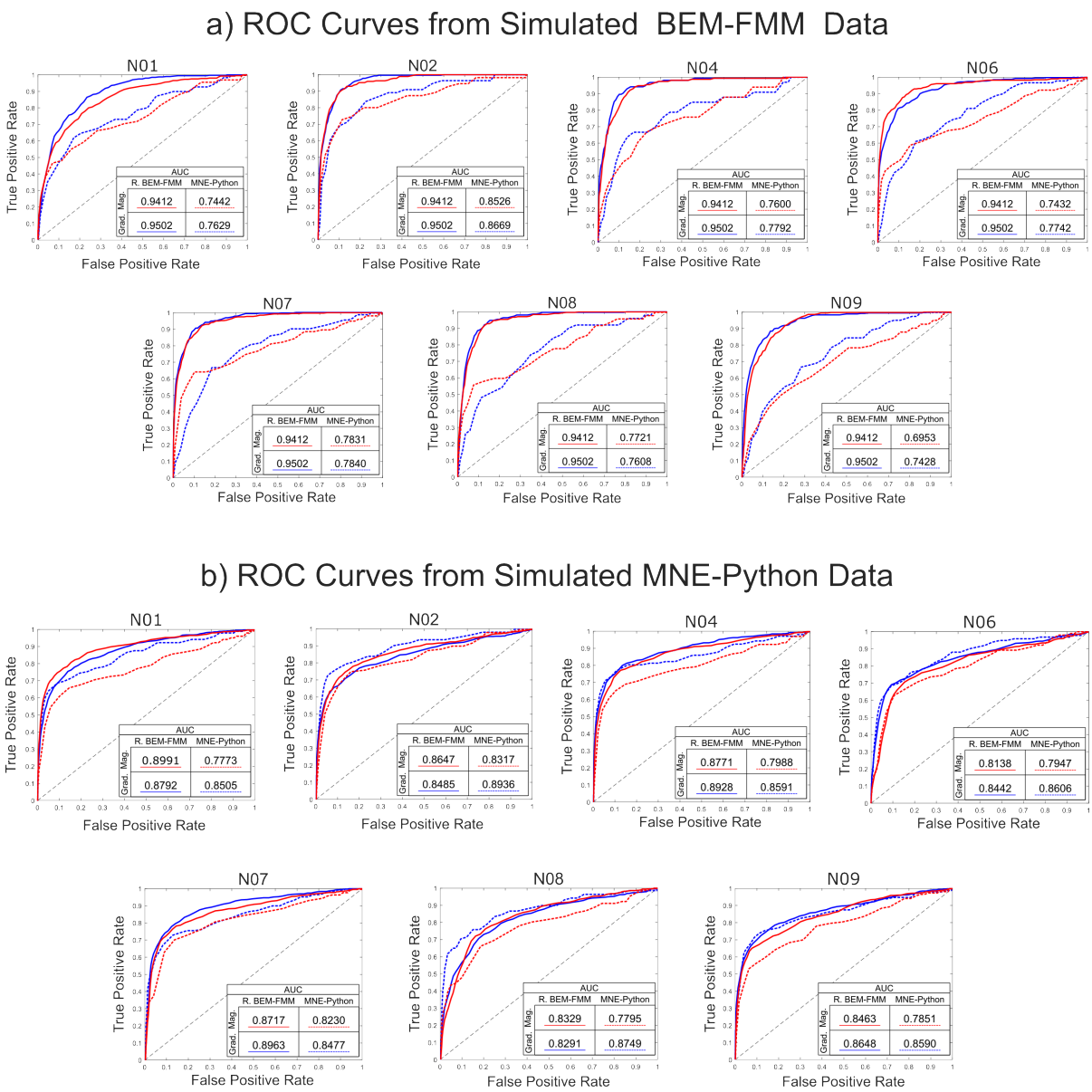

Figure 12: The ROC curves across all participants for the reciprocal BEM-FMM and MNE-Python models form simulated magnetometer and gradiometer data created from a) direct BEM-FMM, corresponding to Figures 4-7 and b) MNE-Python, corresponding to Figures 8-11.

Table 3: The AUC values of the source localization models at the simulated N1m peak across all participants using reciprocal BEM-FMM and MNE-Python.

(a) Determined AUC using the simulated BEM-FMM data, corresponding to Figure 12a.

Magnetometers	N01	N02	N04	N06	N07	N08	N09	Avg.
Reciprocal BEM-FMM	0.86	0.95	0.94	0.94	0.95	0.95	0.93	0.93
MNE-Python	0.77	0.85	0.76	0.74	0.78	0.77	0.70	0.77
Gradiometers	N01	N02	N04	N06	N07	N08	N09	Avg.
Reciprocal BEM-FMM	0.89	0.96	0.95	0.93	0.96	0.95	0.94	0.94
MNE-Python	0.76	0.87	0.78	0.77	0.78	0.76	0.74	0.78

(b) Determined AUC using only the simulated MNE-Python data, corresponding to Figure 12b.

Magnetometers	N01	N02	N04	N06	N07	N08	N09	Avg.
Reciprocal BEM-FMM	0.90	0.86	0.88	0.81	0.87	0.83	0.85	0.86
MNE-Python	0.78	0.83	0.80	0.79	0.82	0.78	0.79	0.80
Gradiometers	N01	N02	N04	N06	N07	N08	N09	Avg.
Reciprocal BEM-FMM	0.88	0.85	0.89	0.84	0.90	0.83	0.86	0.87
MNE-Python	0.85	0.90	0.86	0.86	0.85	0.87	0.86	0.86

Figures 15-18 show the dSPM source reconstructions for all participants using MNE-Python and reciprocal BEM-FMM, with Figures 15 and 16 showing the reconstructions using the N1m magnetometer peaks, and Figures 17 and 18 showing the reconstructions using the N1m gradiometer peaks.

4 Discussion

Source estimation of simulated data. The noiseless simulated data (both using 3-layer BEM using MNE-Python's forward solver, and for 5-layer BEM using direct BEM-FMM with b-refinement) was recovered significantly more focally and accurately with our method or reciprocal BEM-FMM (see Figure 3 and Figures 4–7): the estimated sources are distributed almost exclusively along the Heschl gyri, with peaks occuring on one or both of these gyri —an effect that is mostly explained by the positioning of the participant with respect to the MEG sensors. The source reconstructions using MNE-Python —see Figures 8–11— show instead spurious regions of activation in many cortical regions away from the Heschl gyri, and a much less focal distribution; nevertheless the peaks remain located near the Heschl gyri with an identical pattern of peak asymmetry observed in the reciprocal BEM-FMM reconstructions.

These conclusions can be also obtained with the statistical analyses of ROC curves shown in Figure 12. The AUC statistics in Table 3 show that when more realistic forward simulated data is involved (5-layer direct BEM-FMM models), the high-resolution reciprocal BEM-FMM significantly outperforms MNE-Python. When the simplified models are used for the simulated data (3-layer BEM models of MNE-Python), the performance of the reciprocal BEM-FMM is still superior on average, although just marginally —it should be noted that MNE-Python does not recover the sources simulated with the 3-layer BEM perfectly because we activated a cluster of dipoles rather than a single dipole on each of the Heschl gyri.

Given that real experimental data will be more similar to the high-resolution forward models, this

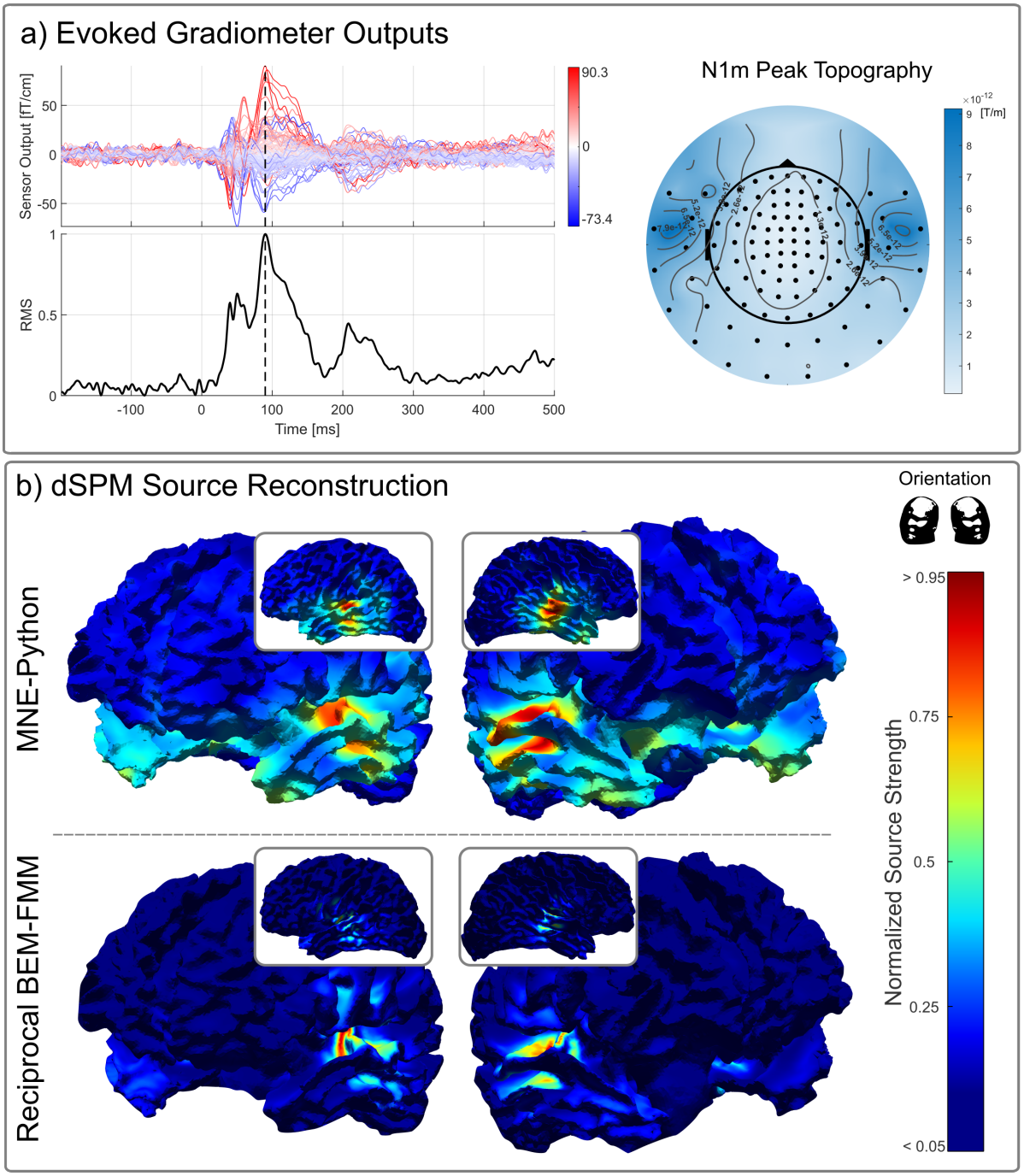

Figure 13: Source reconstruction results of participant N01 using the gradiometer data with both MNE-Python and reciprocal BEM-FMM. a) The evoked gradiometer sensor data is shown on the left, along with the signal RMS. The N1m peak is marked with the dotted black line, and the sensor topography at this peak is shown on the right. b) The normalized dSPM solutions are shown, with the smoothed MNE-Python solution shown on the top, and the reciprocal BEM-FMM solution shown on bottom. The left and right columns show focus perspectives of the left and right hemispheres, respectively. The inset images show the solution on each hemisphere from a lateral perspective.

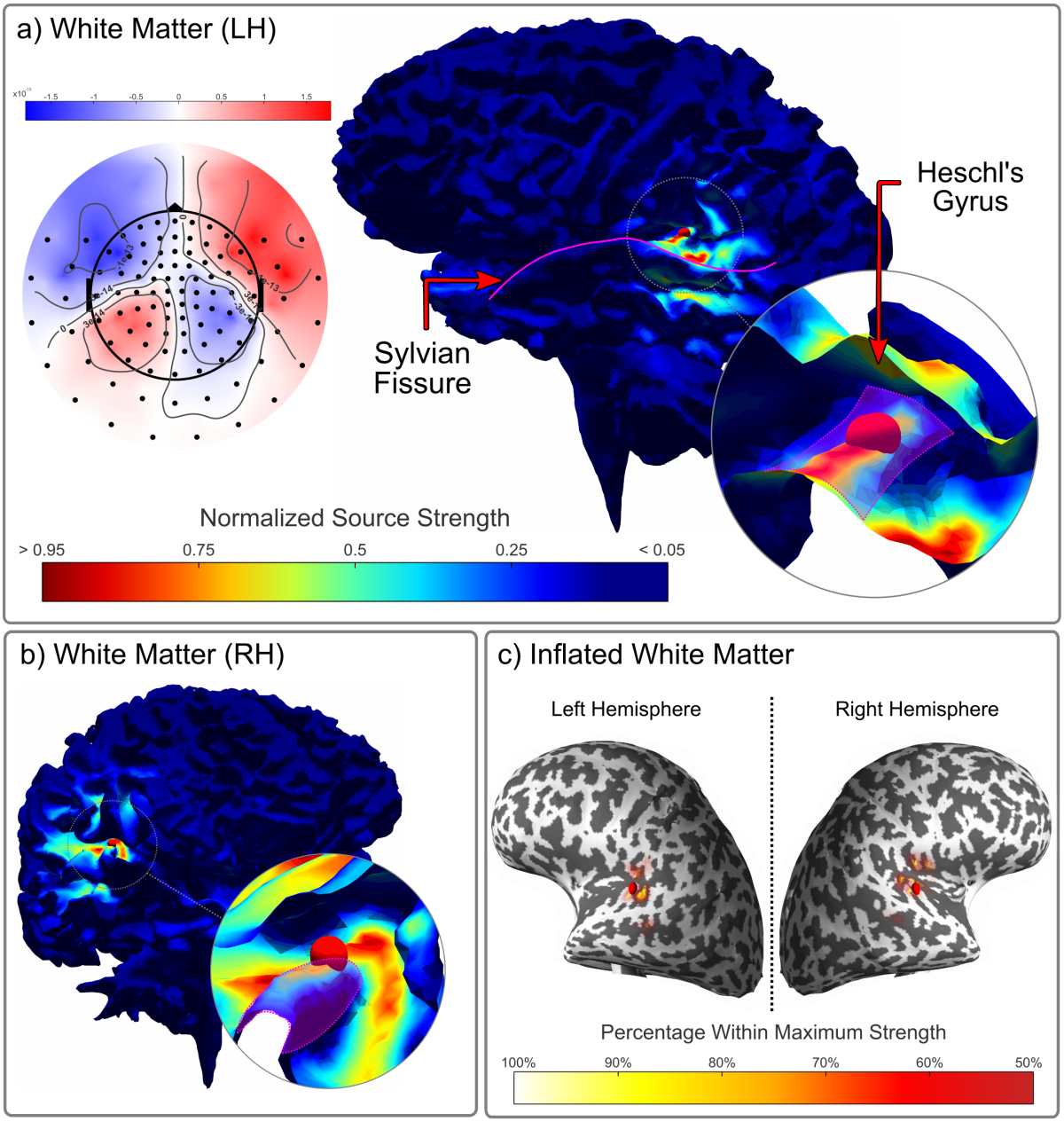

Figure 14: Detailed source reconstruction results for participant N09 using the magnetometer sensor data with reciprocal BEM-FMM. a) To the left, the magnetometer topography at the N1m peak is shown. To the right, the reconstructed source strengths are plotted on the white matter surface, with visual focus emphasizing the left hemisphere. The Sylvian fissure is outlined with the magenta curve. The source of strongest strength (activation peak) is denoted as the red sphere. In the magnified view, a portion of the Heschl gyrus is highlighted in magenta. b) The same reconstructed source strengths are shown, with visual focus now on the right hemisphere, and the Heschl gyrus highlighted again in magenta. c) The reconstructed sources whose strengths are within a percentage threshold of the maximum strength are plotted on the inflated white matter, with the left and right columns focusing on the left and right hemispheres, respectively.

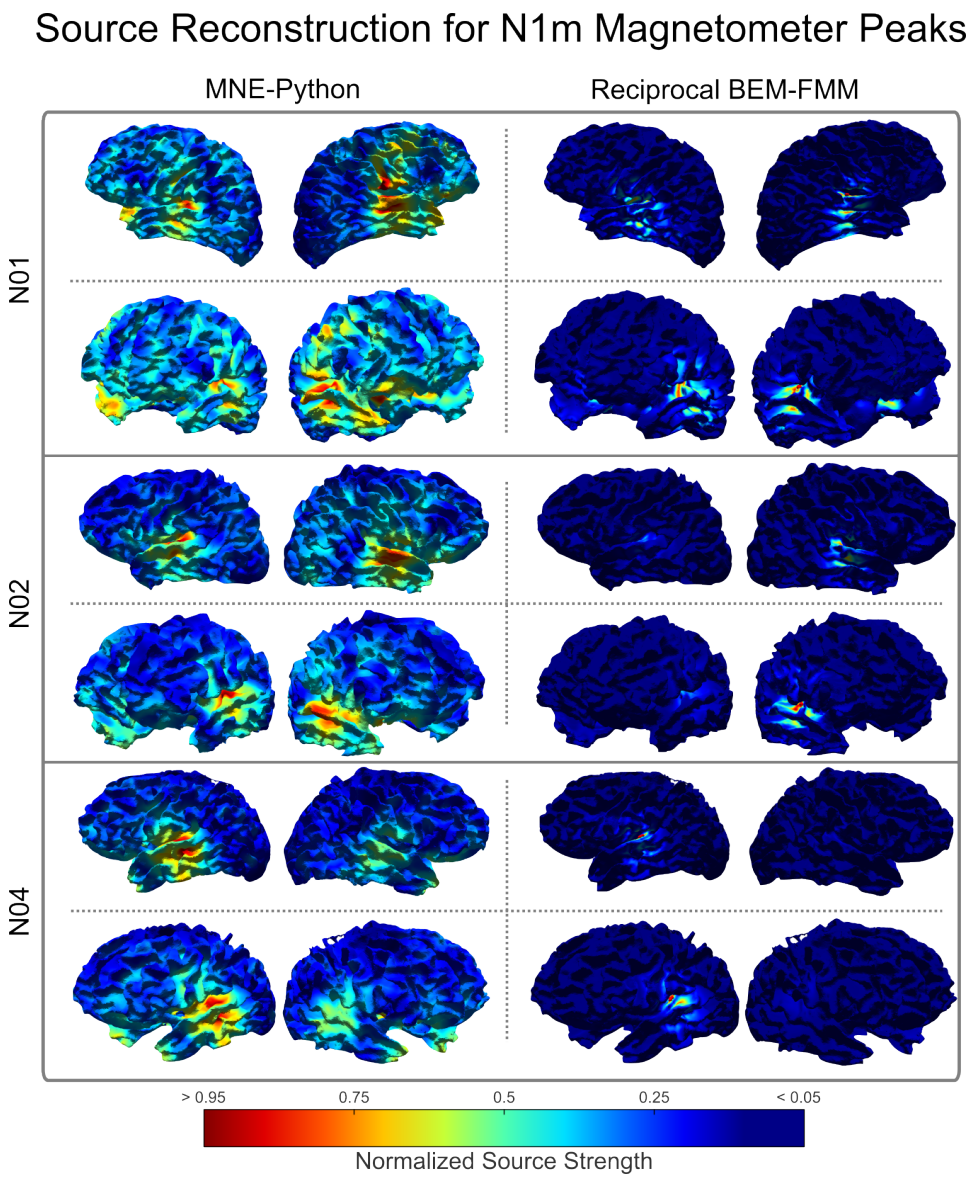

Figure 15: Source localization results of participants N01, N02, and N04 at their respective auditory evoked N1m magnetometer peaks using reciprocal BEM-FMM and MNE-Python. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

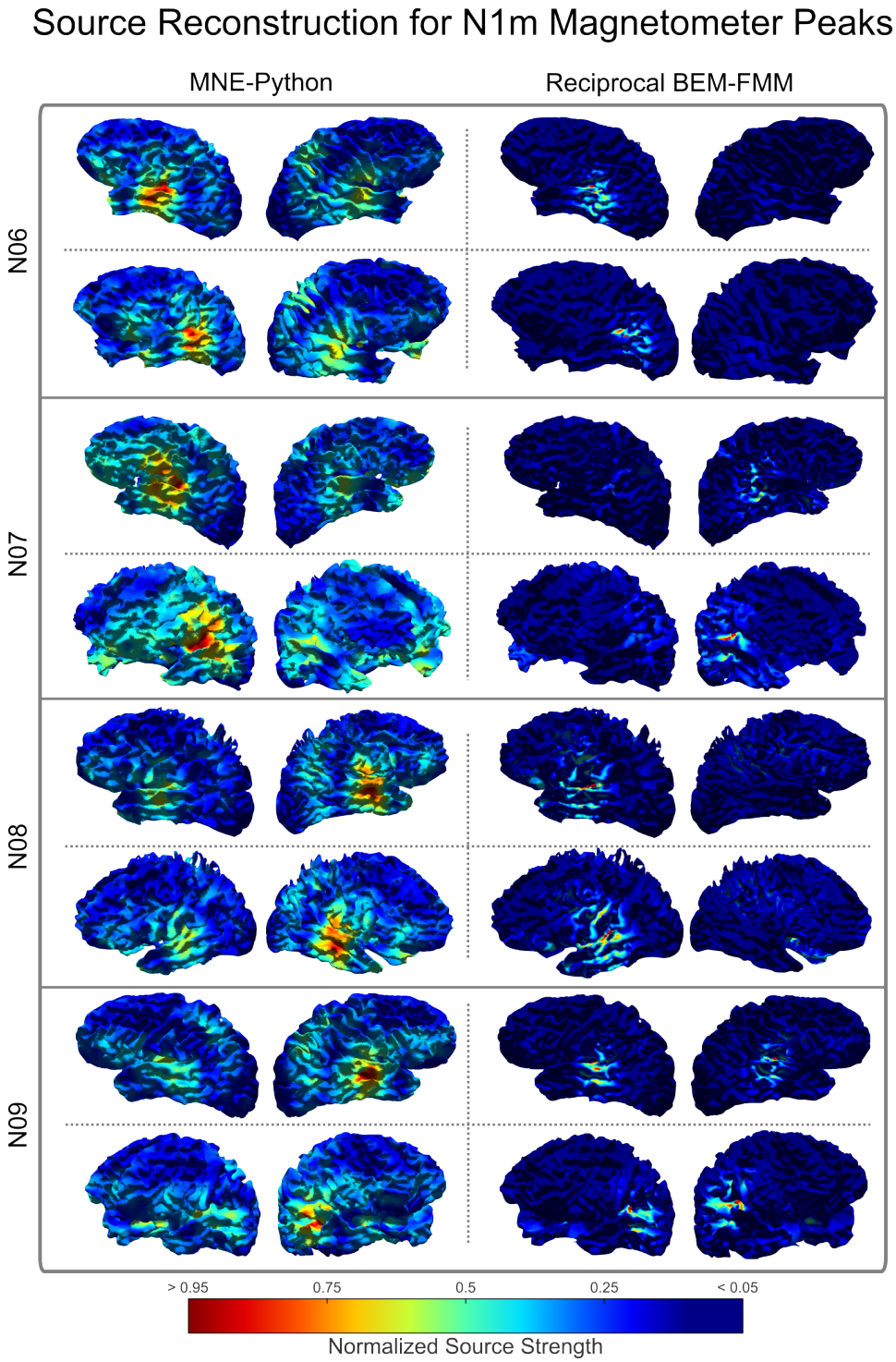

Figure 16: A continuation of the results of Figure 15 for participants N06, N07, N08, and N09.

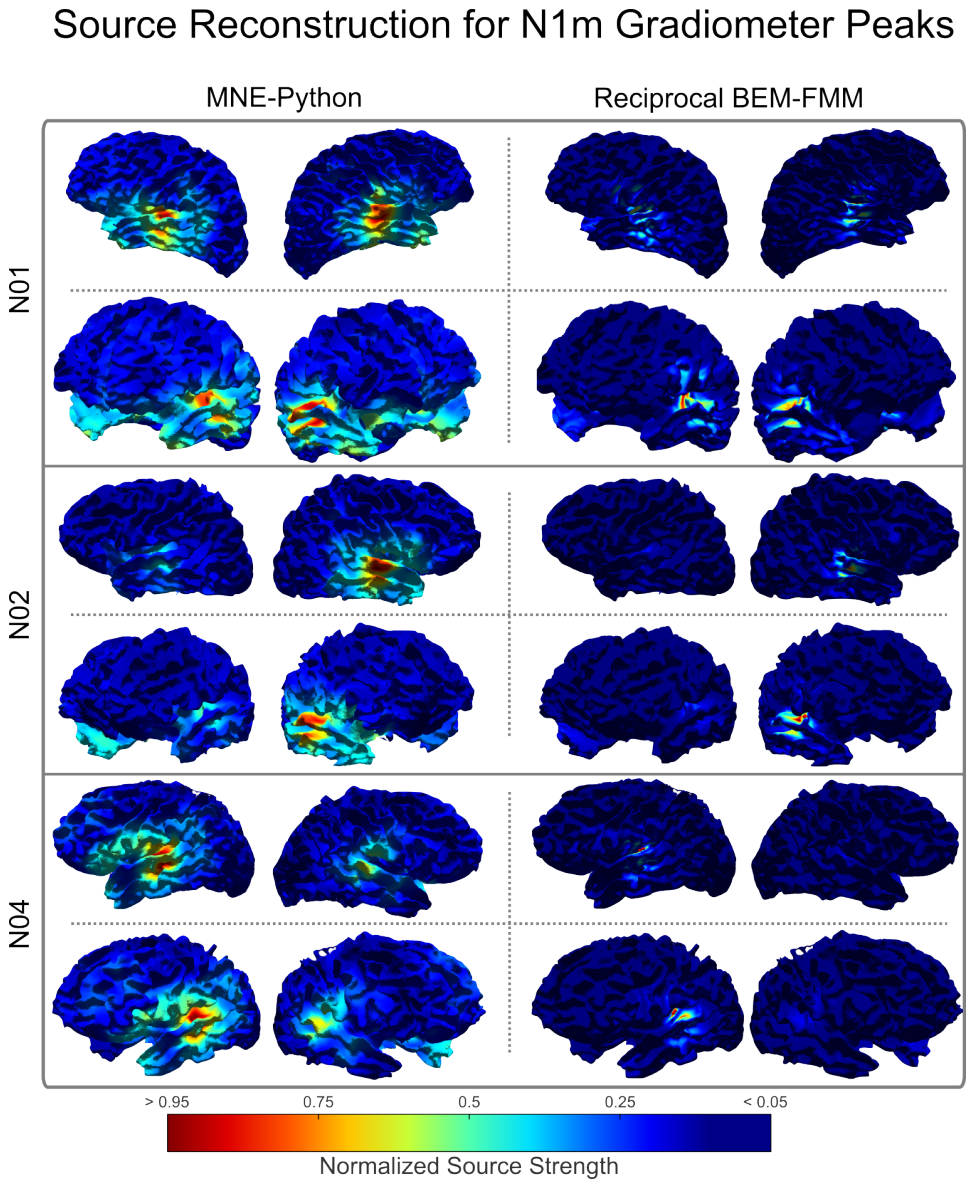

Figure 17: Source localization results of participants N01, N02, and N04 at their respective auditory evoked N1m gradiometer peaks using reciprocal BEM-FMM and MNE-Python. For each participant, white matter perspectives are chosen to focus on the temporal lobe and Heschl gyrus in the top and bottom rows, respectively, on both the left and right hemispheres. The color map indicates the normalized current density, with blue and red colors indicating 0 and 1 values, respectively.

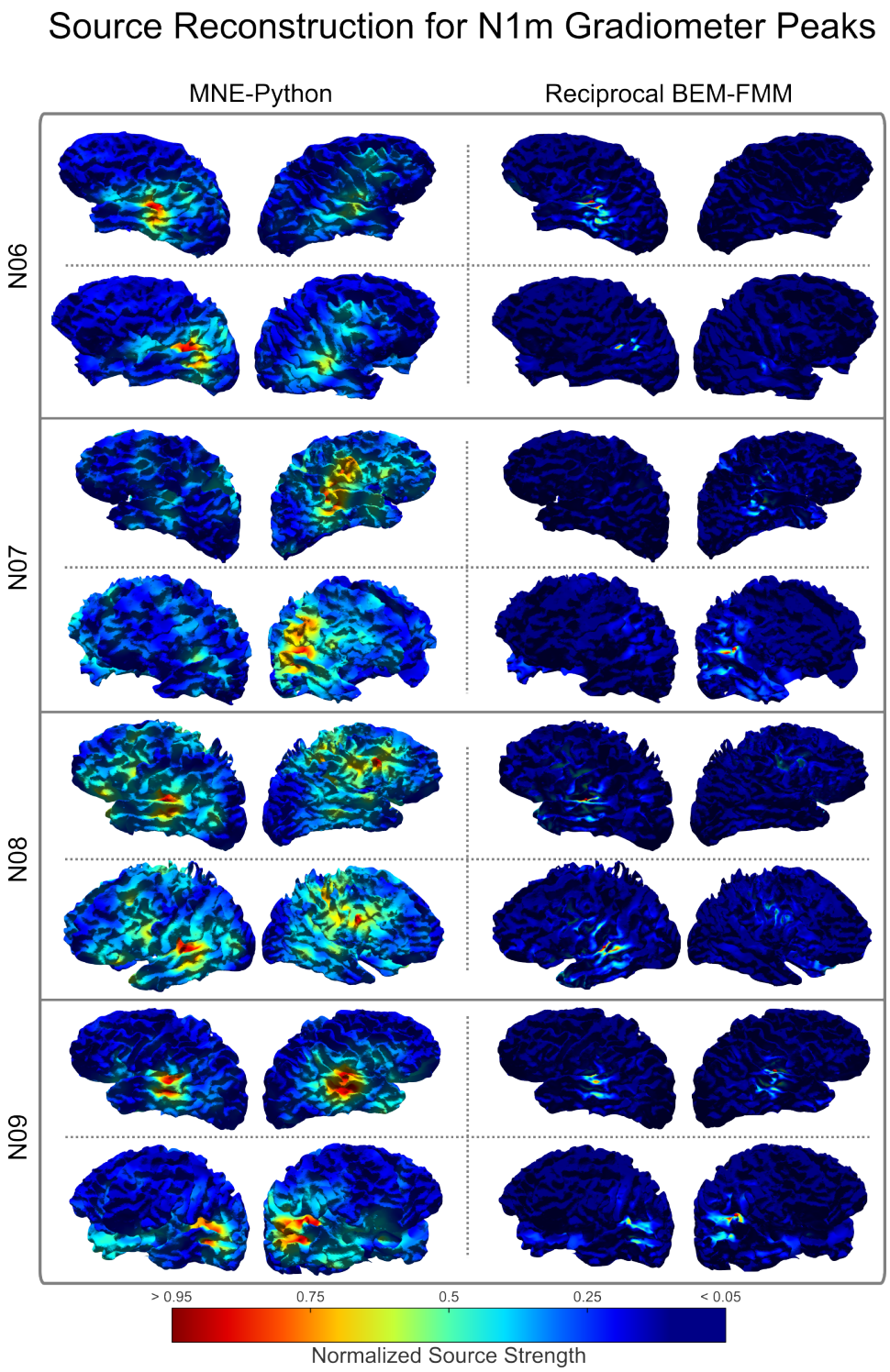

Figure 18: A continuation of the results of Figure 17 for participants N06, N07, N08, and N09.

indicates that the reciprocal BEM-FMM method will have a better performance than MNE-Python on experimental data. This is confirmed in the following subsection.

Source estimation of experimental data. The estimated sources recovered by reciprocal BEM-FMM seen in Figures 15-18 show very focal activation peaks on the Heschl gyri across all participants, as expected from a binaural auditory evoked N1m peak. In comparison, the estimated sources recovered by MNE-Python achieved activation in the Heschl gyri across all participants. Howver, these regions of activation were much less focalized than that seen in reciprocal BEM-FMM. Further, the MNE-Python solutions possessed spurious regions of activation, which are particularly pronounced in participants N07 and N08.

It is worth reiterating that, aside from the SNR values of participants N06 and N08, there was no subject-specific tweaking performed in either reciprocal BEM-FMM or MNE-Python. Both source reconstruction solvers were run, for all intents and purposes, utilizing their default settings. The only major parameter change was in the depth weighting constant, which was chosen to be visually optimal for MNE-Python. It is possible that improvements to the solutions achieved by MNE-Python could be made through additional parameter modifications. However, such subject-specific parameter tweaking may be unnecessary in reciprocal BEM-FMM based on the results found here.

In the comparative source reconstruction results of (Nuñez Ponasso et al. 2025) between our methods and MNE-Python, our methods performed only marginally better. However, for the auditory data that we considered here, our method provides a significant improvement. A possible explanation for this is that the geometry of the head near the central sulcus can be sufficiently approximated by a sphere or by a simplified classical BEM model with low-resolution shells. On the other hand, the temporal lobes cannot be accurately described with simpler models. Another possible explanation for this is that the quality of the evoked data in (Nuñez Ponasso et al. 2025) allowed for simpler low resolution models to perform well. Conversely, the experimental auditory data used here was less than ideal — it was challenging to process and, even when filtered, it still seemed subject to fair noise.

5 Conclusions

Our results indicate that high-resolution forward models including cortical layers significantly improve upon the accuracy of 3-layer BEM models for the source reconstruction of AEFs. All of our source estimations using high-resolution models indicate good quality of source localizations, with focal peaks of activation on the Heschl gyri of each participant. This performance held true across various types of data, both simulated and experimental. Using ROC measures, we find statistically superior results to MNE-Python. The high resolution source space and focalized source estimations achieved by reciprocal BEM-FMM indicates that, through some further numerical and statistical analysis, our methodology could improve the applications of source localization on clinical settings.

Acknowledgments

G.N.P., D.A.D., G.N., and S.N.M. were supported by the NIBIB Grant 1R01EB035484, and the NIMH Grant 1R01MH130490. This data collection was supported by a grant from the National Institute of Mental Health (MH128692 and MH125126, C.A.). Z.D. is supported by the National Institute of Mental Health Intramural Research Program (ZIAMH002955). A.L. and B.M. were supported by the German Federal Ministry of Education and Research (BMBF) grant DryPole (01GQ2304B). J.H. received funding from the German Federal Ministry of Education and Research (BMBF) grant DryPole (01GQ2304A) and

the Free State of Thuringia (2018 IZN 004), co-financed by the European Union under the European Regional Development Fund (ERDF).

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