Mental calculation: the fastest high-level cognitive ability of human beings?

Bastian Wiederhold
bastian.wiederhold@lmu.de

Faculty of Biology
LMU Munich
May 28, 2025

Abstract

High-level cognitive processing in human beings is thought to be slow. To test just how fast humans can process abstract knowledge, we analyze information rates associated with performances of top competitors in mental calculation. Even without taking the algorithms to perform the calculations into account, reading the numbers alone reaches information rates of around 90 bit/s in short tasks, eclipsing, to our knowledge, any previously reported value for high-level cognitive abilities. The execution speed of the underlying algorithms is as high as 70 bit/s, which in combination suggests that humans can process around 160 bit/s for a short time. Even for tasks of longer duration information rates remain remarkably high.

1 Introduction

Two recent studies have tried to estimate the speed of high-level information processing in humans. One study argues that the human mind processes around 10 bit/s [ZM24], based on examples as diverse as speaking and playing Tetris. In contrast, a recent analysis of memory competitions has shown that around 70 bit/s are possible, at least for short duration tasks [Wie25b]. For tasks longer than a few minutes, the processing rates in memory fall far below 10 bit/s with the decline following a power-law in the task duration. Investigating mental calculation has the advantage that competitive algorithms for certain tasks are known, which gives us a direct handle on the amount of internal processing involved. Yet, reliable information rates for experts in mental calculation remain to be clarified.

Evidence suggests that expert mental calculation is a learned skill supported by sophisticated strategies [Sta88; Bre90; MS13; Bel+20]. Indeed, with the exception of mental calculation, other forms of information processing by expert human "calculators" are normal [Jen90; LA22]. Hence, mental calculation records might tell us about the limits of human cognition under training, rather than just the abilities of singularly gifted individuals.

We, therefore, analyze the information rates in bit/s corresponding to performances of top competitors in mental calculation competitions, focusing on the following categories that are frequently featured: addition, multiplication, calendar dates and square roots. Experiments relating the response time of mental calculations to the entropy of the task have long been known [Tho63]. To capture different cognitive dimensions, we calculate the information rates associated with the visual processing, the mental algorithm and the output of the final result. Adding the three rates, we find combined values at over 160 bit/s eclipsing the speed of any other previously reported high-level cognitive ability.

2 Results

2.1 The record times are well-explained by the combined, strategy-adjusted entropy

Mental calculations consist of three stages: transforming a visual input into a mental representation, executing the computational algorithm itself and reporting the output. Our aim was to capture the different dimensions and calculate the processed entropy for vision, algorithm and output of top mental calculation performances (calculations in Section 4, performances in Section 4.2).

We first calculated input and output entropy independent of strategy (Fig. 1). In most tasks, the digits in the numbers are uniformly and randomly distributed, making it straightforward to calculate the input entropy. The output entropy, on the other hand, is calculated on the basis of the digits of the final result that are no longer uniformly distributed. The completion time includes typing the result, except for the two shortest addition disciplines. The 15 or 30 to-be-added, three-digit numbers are flashed in below 2 respectively 4 s for top competitors. In these two cases, we have excluded the output entropy from the following figures and the combined entropy of the disciplines. If input and output entropy were reliable metrics for processing, one would expect a strong association between the entropy and the time required for the tasks. Time is correlated with input and output entropy, but exhibits significant scatter. Further insight could be gained by analyzing the algorithms competitors employed.

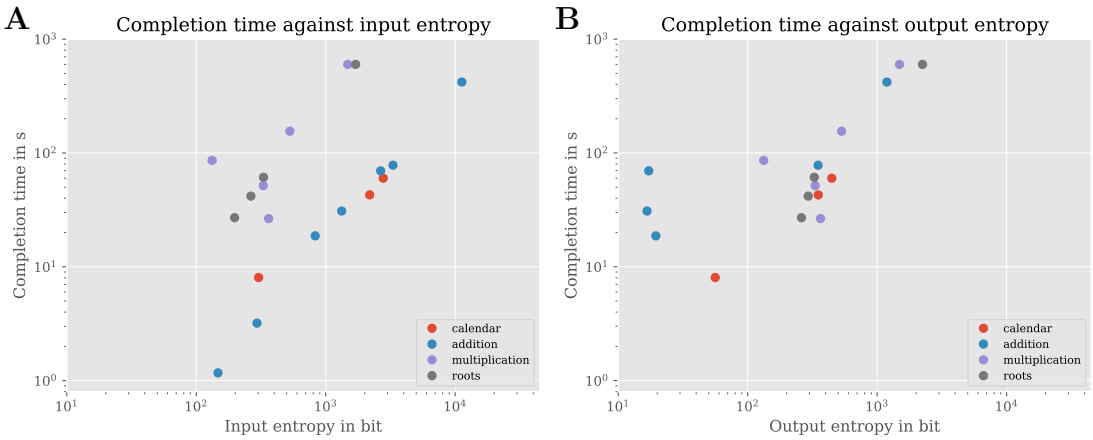

Figure 1: Time required for top performances in mental calculation in dependence of the entropy of the task, measured in several ways. **A** Time as a function of input entropy, i.e. the entropy of all numbers given in the task and essential for the solution without any assumption on underlying strategies. **C** Time as a function of output entropy calculated based on the final result of the discipline. If competitors indicate the result after the discipline ends, the output entropy is set to zero.

The algorithm, represented by a sequence of decisions, on the other hand, is a matter of strategy. In many cases, strategies of winning competitors are known and we can, therefore, analyze the algorithmic steps. In addition and calendar, each digit read is used only once, while in multiplication and square root tasks digits are "reread". So we defined a visual processing entropy by the number of times digits, including already computed parts of the result, are read multiplied by the entropy of the digits (Fig. 2 **A**). Additionally, inspired by Hick's law [PS18], which postulates that the reaction time for decisions is proportional to the entropy of the number of options, we took another, contrasting approach. We added the entropy of the decisions, which competitors make in the course of executing the algorithms; the so-called algorithmic entropy (Fig. 2 **B**). With these entropies at hand, we formed the combined entropy by adding visual processing, algorithmic and output entropy (Fig. 2 **C**).

In general, the time competitors take increases with the complexity of the mental calculation task, whether measured in terms input, visual, algorithmic, output or combined entropy. The closest link was found with the combined entropy (Fig. 2). Notably, the scatter of response times is greater, when plotted either against the input or output entropy than against the

visual, algorithmic or combined entropy. As a function of the latter, response times of different categories such as addition and square roots align and increase as an approximate power law. This indicates that completion times have a strong relationship to the cumulative entropy of the steps of the employed strategy.

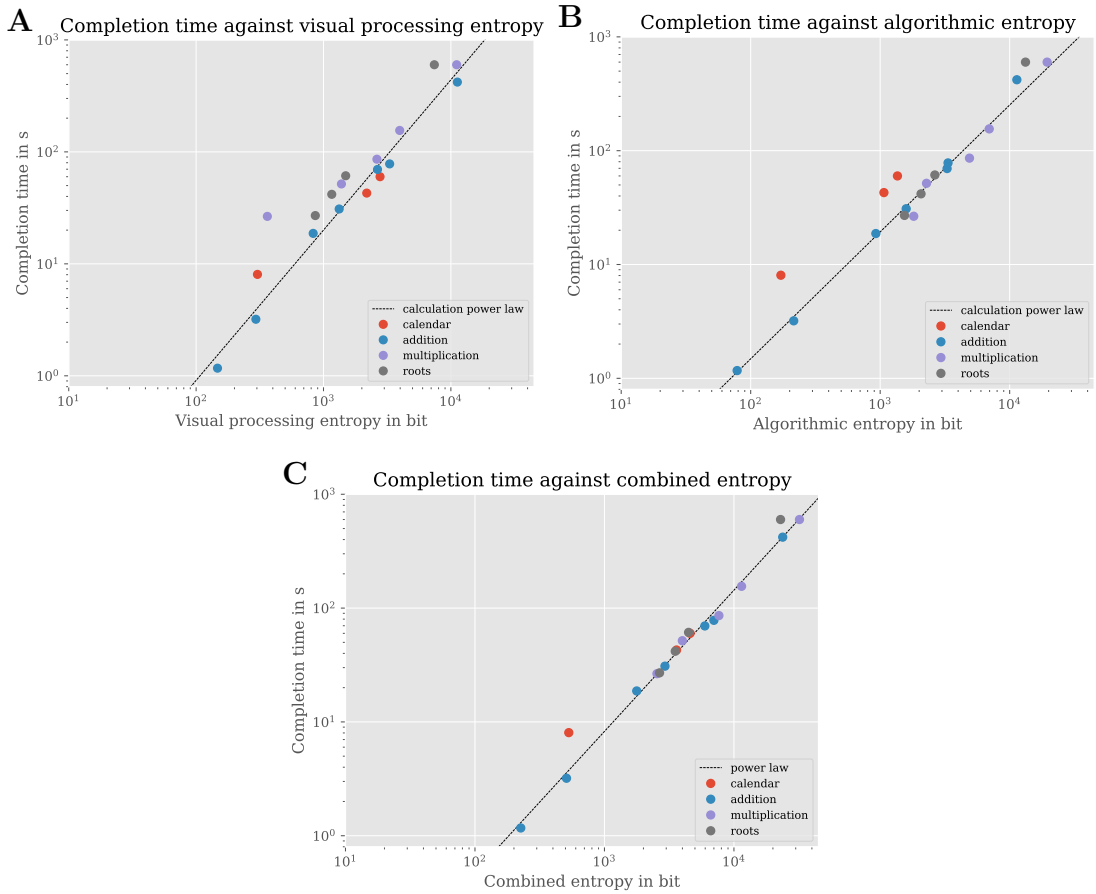

Figure 2: Time required for top performances in mental calculation in dependence of the entropy of the task, measured in several ways. **A** Time T_1 as a function of visual processing entropy E_1, i.e. the entropy of all numbers read in the task, whether part of the exercise or an already typed partial result. The black line shows the function $T_1(E_1) \approx (18.8 \pm 10.4) \cdot 10^{-4} E_1^{1.34 \pm 0.07}$ s fitted by least squares based on relative residuals. The mean squared relative error is 0.128. **B** Time T_2 as a function of algorithmic entropy E_2 estimated based on the entropy of the to-be-made decisions of competitive strategies for the different tasks. The black line shows the function $T_2(E_2) \approx (8.7 \pm 4) \cdot 10^{-4} E_2^{1.11 \pm 0.06}$ s fitted by least squares based on relative residuals. The mean squared relative error is 0.086. **C** Time T_3 in seconds as a function of the combined entropy E_3, the sum of visual, algorithmic and output entropy. The black line shows the function $T_3(E_3) \approx (15.6 \pm 4.4) \cdot 10^{-4} E_3^{1.24 \pm 0.03}$ s fitted by least squares based on relative residuals. The mean squared relative error is 0.03.

2.2 Upper limit on cognitive processing rate

The entropies for the different aspects of mental calculation tasks allow us to do a fine-grained analysis of processing rates. The visual processing rate alone reaches around 90 bit/s for short addition disciplines of a few seconds duration (Fig. 3 A), eclipsing previously reported values for human information processing [ZM24]. The algorithmic speed reaches around 70 bit/s for the shortest duration disciplines and remains above 20 bit/s even for 10-min disciplines (Fig. 3 B). Output rates are the slowest falling below 15 bit/s across all tasks (Fig. 3 **C**). Note that the processing of input, algorithm and output might be unevenly distributed over time and the rates represent averages.

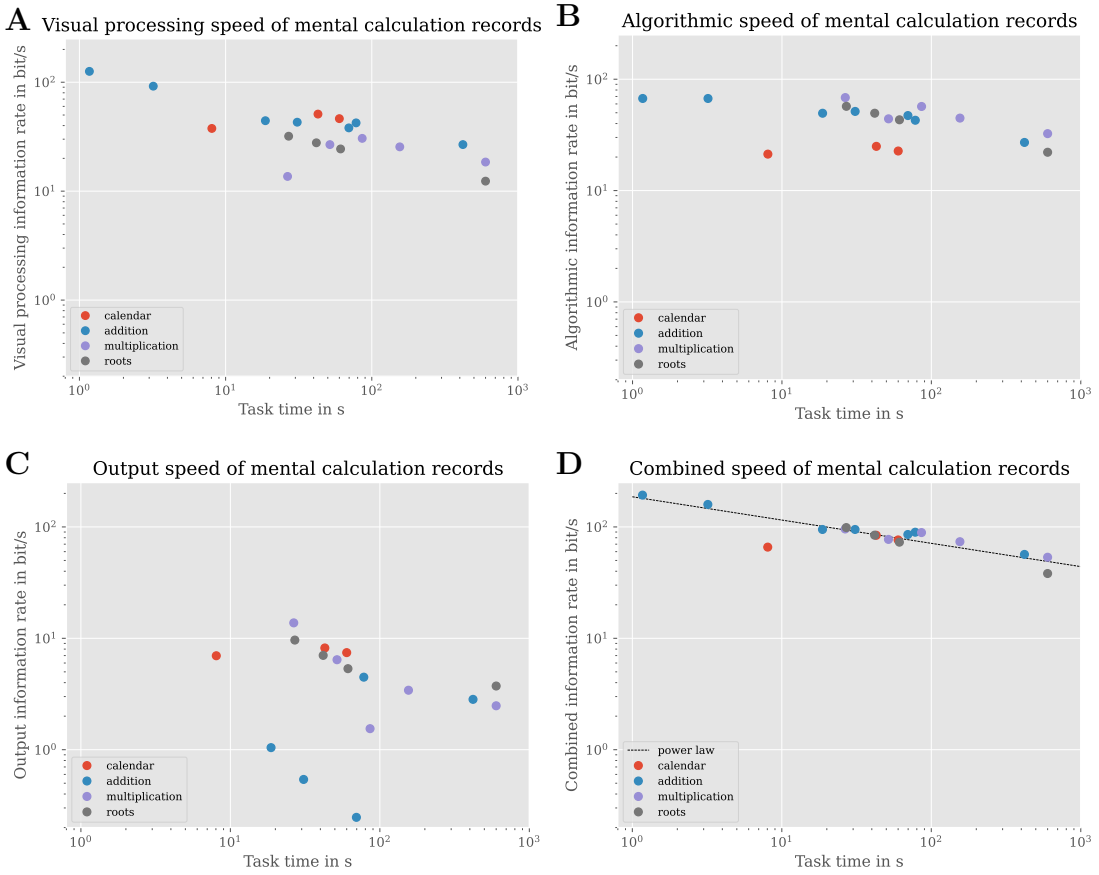

Figure 3: Information processing rates for top performances in mental calculation as a function of the length of tasks, measured in several ways. **A** Visual information rates. **B** Algorithm information rates. **C** Output information rates. If competitors indicate the result after the discipline ends, the output speed is set to zero. **D** Combined information rate R as a function of the task time T of the discipline. The black line shows the function $R(T) \approx (186.54 \pm 13.38) \cdot T^{-0.21 \pm 0.02}\ \frac{\text{bit}}{s}$ fitted by least squares.

Compared to a recent systematic study of the information processing at memory competitions [Wie25b], the response times are lower and the processing rates are, in most cases, several times higher than in memory tasks of similar duration (Fig. 4).

3 Discussion

The speeds of up to 160 bit/s of high-level information processing achieved in top performances of mental calculation are striking compared to other values reported in the literature, which usually range between 10 to 70 bit/s [ZM24; Wie25b]. A substantial part of this difference can be explained by the algorithmic processing of up to 70 bit/s. For many mental processes, such as memorization, it is usually infeasible to obtain estimates on the required internal processing. Nevertheless, even the visual processing of 90 bit/s on its own eclipses previous values.

The highest processing speeds, in particular in the addition disciplines, are achieved with the mental abacus technique. It is unclear in how far abacus training is a necessity for highest processing speeds as there are examples of non-abacus trained individuals performing exceptionally well in some competitions [Eng24]. Previous research on abacus-trained individuals has suggested that more parallel processing occurs compared to naive calculators [Bel+20] and that visuomotor and visuospatial activity is involved [Han+03]. Both features are reminiscent of participants of memory competitions, who visualize to-be-memorized information along prominent points of a familiar environment, a technique know as “memory palace" [Dre+17], and seem to reach a certain level of parallelization as well [Wie25b]. Memory experts are only slightly faster reading through the to-be-memorized information without memorizing than when they read and

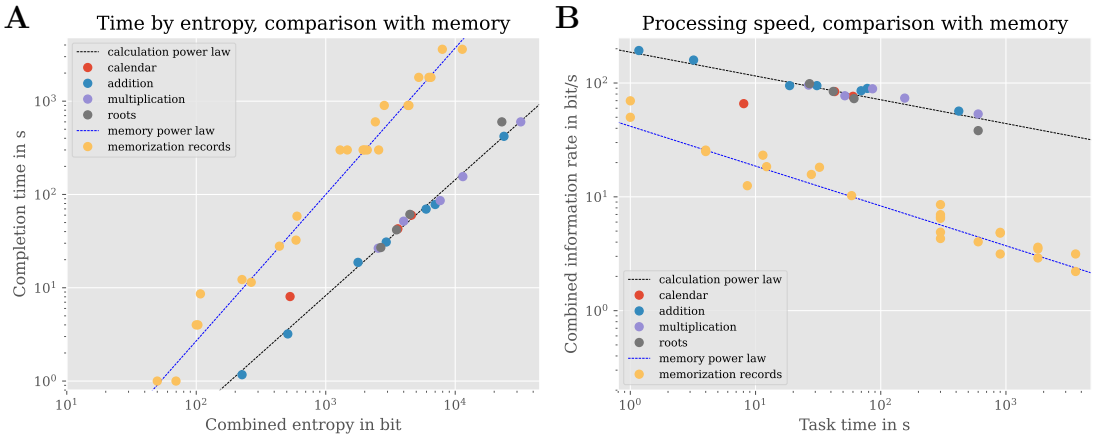

Figure 4: Comparison of response times and processing rates achieved by mental calculation and memorization experts, as given in the recent study [Wie25b]. **A** Time in seconds as a function of the combined entropy E in bit of a task. The black line shows again the function T_3 from Fig. 2. For comparison, the yellow dots show world records from memory competitions given in [Wie25b]. We have also plotted the function $T^{memo}(E) = 19.5 \cdot 10^{-4} \cdot E^{1.57}\ s$, which captured the time-entropy relationship in [Wie25b]. **B** Combined information rate as a function of the task time T of the discipline. The black line shows again the function R from Fig. 3. As a comparison, we plotted the power law $R^{memo}(T) = 41.71 \cdot T^{-0.35}\ \frac{\text{bit}}{s}$ for memorization information rates identified in [Wie25b].

memorize, suggesting that parts of the visual processing and memory palace navigation occur simultaneously.

Interestingly, compared to mental calculation, performances at memory competitions show lower information rates by a factor of five (Fig. 4). Foremost, the study [Wie25b] did not incorporate the internal processing of the memory palaces and associative mnemonic strategies, whereas the algorithms of mental calculations allow one to estimate the inner thought process. Additionally, mental calculators enjoy organized training from an early age due to local traditions [Han+03], whereas memory competitors have not yet reached the same amount of training and institutionalization [Wie25a]. The mental calculation performances, we consider, are often dedicated attempts rather than performances within competitions, leading to higher values. We conjecture that, accounting for these effects, a gap would remain: the application of memory palaces requires consolidation and limits the achievable speed. Consistent with this idea, the decline of information rates with task time is more pronounced in memory competitions. In mental calculation only updates to a current representation need to be applied and, in some disciplines, part of the solution can be continuously written down reducing the cognitive load.

Understanding the full principles of the power-law increase in response time as a function of entropy in mental calculation and memorization requires additional controlled experiments. In [Wie25b] it was argued that, as competitions reward error-free performances, a longer discipline requires a more safety-oriented approach to sample more consistent trials on shorter segments. This certainly applies to mental calculation as well, though unfortunately, sufficient data on errors is not available. Another important shared factor is the vigilance decrement as the duration of tasks increases; it becomes harder to maintain the initial cognitive readiness [Ret+19].

The input and output entropies are accurate, as we use the given numbers respectively the result distribution with only minor assumptions on human behavior. The visual processing entropy is strategy-dependent, but the calculations are still straight-forward. The estimates of the algorithmic entropy carry most uncertainty. Analysis of the response times of expert calculators implies that there is a difference between arithmetic decisions performed by retrieval from memory and actual calculation [Pes+99]. Variations of the strategies for disciplines exist and are, at times, well-guarded. We have approximated the considerable complexity of parts of the algorithms. These methodological uncertainties compound with data created under a wide range of conditions: we have mixed record attempts with results obtained at competitions. There has been no coalescence into a worldwide mental calculation community yet, preventing easily accessible, comprehensive rankings. We were still able to find a common pattern across discipline categories, which suggests that processed entropy captures a considerable part of the

difficulty of executing an algorithm.

The calendar dates disciplines shows higher visual and lower algorithmic processing, hinting that a tradeoff between the entropies, as measured here, is possible. The completion time of one calendar dates discipline is noticeably longer than expected for its entropy. The record of 8.05 s for 20 dates from one century was set by Yusnier Viera in 2010. Since then, Viera has improved the one-minute record for dates from $1600 - 2100$ several times even reaching a higher average speed for this more complex exercise [Reca].

In sum, a combined entropy accounting for the strategy seems to explain the time required by expert calculators. Visual processing rates reach around 90 bit/s and algorithmic rates 70 bit/s. The latter is a clear indication that other mental activities might require significant internal processing, which, in the absence of well-defined algorithms, remains unnoticed. Taken separately, visual and algorithmic rates are already remarkably high. Combined they eclipse previously reported values by far and shed new light on the computational limits of the human brain.

Acknowledgments Insightful comments and explanations by Samuel Engel, Marc Jornet Sanz and Daniel Timms have allowed me to improve this work substantially. I wish to thank Wenzel Grüß, Gert Mittring, Ethan Kuntz, Aaryan Shukla and Chieko Takayanagi for conversations. I am grateful to Martin Stemmler and Andreas Herz for discussions and helpful comments on earlier versions of the manuscript, which greatly improved its readability.

4 Methods

4.1 Entropy calculations

We first introduce input, visual, algorithmic and output entropy in general, before elaborating on the detailed calculations for each discipline.

We measure information through Shannon's entropy formula

$$H(\mathbb{P}) = -\sum_{x \in \Omega} \mathbb{P}(x) \log(\mathbb{P}(x))$$

for a distribution $\mathbb{P}$ on a discrete probability space Ω. A previous account on quantifying the entropy of mental calculations can be found in [Tim19]. We round to two decimal places.

Input We consider the entropy of all digits given in a task. The following formula for the entropy of a uniformly drawn k-digit number will appear several times

$$H^{k\text{-digit}} = H(k\text{-digit number}) = \log_2(10^k - 10^{k-1}) = \log_2(9) + (k-1)\log_2(10),$$

as the first digit of a k-digit number is non-zero. For example, the input entropy of the sum of two uniform single-digit numbers is $2H^{1\text{-digit}} = 2 \cdot \log_2(9)$.

Vision This entropy is similar to the input entropy but accounts for competitors' strategies. Multiplication involves reading each input digit several times. In square roots competitors use their already calculated digits for subsequent iterations of the algorithm. In this case, we regard these as part of the input, as we attempt to capture the visual processing. In addition and calendar dates the visual and input entropy are equal, as the task is only parsed once. If the strategy is necessary to understand the visual processing entropy, this paragraph follows after the algorithm.

Algorithm To determine the entropy of the algorithm, we sum the entropies of the "decisions", the distributions of interim outcomes, along the steps of the algorithm. To return to the example of the addition of two uniform single-digit numbers, we identify $3+2 = 5$ with $1+4 = 5$ leading to 18, not equally distributed, combinations in the outcome space. The entropy of this distribution is 3.88 bit. In turn, the addition of three uniform single-digit numbers has two decisions along

the algorithm. First, the addition of two uniform single-digit numbers, and then the addition of the resulting number and another uniform single-digit number.

The importance of the space of outcomes for the complexity of tasks is illustrated by the questions “is this an apple, yes or no?" and “describe the apple as accurately as possible in 10 words". Both require processing of the same input, but the space of possible outcomes is binary in the first case and large in the second case.

There are limitations to the precision of the following calculations, which are discussed in Section 3. The accumulated entropy depends on the underlying strategy. Working memory is a major limitation in mental calculation [Hit78] and restricts the nature of the algorithms easily executed by human competitors. An example are multiplications, where two strategies will be presented. The first requires less entropy processing, but working memory load grows quickly with the number of digits, whereas a second requires more entropy processing, but places little demand on working memory. Consequently, the first strategy yields superior results for manageable multiplications, whereas the second is applied to large multiplications such as of two 20-digit numbers.

	$+$	$-$	$\times$
$U\{0,...,9\}$	4.03	4.03	4.81
$U\{1,...,9\}$	3.88	3.88	5.07

Table 1: Examples of entropies for operations on two uniformly distributed single-digit numbers, zero in- or excluded.

Output We call the entropy of the final result, the numbers which the calculator has to type, the output entropy.

Mathematically, the output entropy is bounded by the algorithmic entropy, since some decisions made in the course of executing the algorithm lead to the result. We still track the output entropy as a separate quantity for two reasons. First, in all disciplines, except the addition of 15 and 30 three-digit numbers, the measured response time includes typing the result, which can only happen after previous visual and algorithmic processing. Second, we capture the distinct dimension of motor output. It might be more difficult to perform a decision while executing a motor action and the longer the result the more likely are typos preventing a record performance.

In most disciplines, results are typed continuously either because many instances of the same base exercise (e.g. calendar calculations) are being solved or the algorithm successively produces new digits of the result (e.g. multiplication). In the flashed addition disciplines, for example, the addition of 100 four-digit numbers, competitors give the result at the end of the discipline.

In case of multiplications and square roots, the distribution of the final result is too complicated to be known completely. We instead calculate the range of possible results and assume that all contained numbers are equally likely. In the addition of 15 and 30 three-digit numbers the response is untimed and we set the output entropy to zero.

4.1.1 Addition

Competitors add numbers, which are either flashed at a certain rhythm or shown all at once.

Input and Vision If there are n k-digit numbers to be added, we use the formula $nH^{k\text{-digit}}$.

Algorithm Expert calculators use the mental abacus strategy to greatly accelerate speed [Bel+20], but still need to pass through all the interim sums. Up to a smaller adjustments, we therefore use a generalization of the above approach to the addition of two multi-digit numbers. We use the notation

$$H^{\text{add}}_{j,k} := H(\text{sum of j k-digit numbers}).$$

For instance, for the addition of two uniform four-digit numbers, the resulting distribution on $2000, 2001, 2002, ..., 19998$ is used to calculate the associated entropy. As an orientation, we

obtained the subsequent values for the addition of two numbers

$$\begin{aligned}
H_{2,1}^{\text{add}} &= H(\text{addition of 2 one-digit numbers}) = 3.88 \\
H_{2,2}^{\text{add}} &= H(\text{addition of 2 two-digit numbers}) = 7.21 \\
H_{2,3}^{\text{add}} &= H(\text{addition of 2 three-digit numbers}) = 10.54 \\
H_{2,4}^{\text{add}} &= H(\text{addition of 2 four-digit numbers}) = 13.86 \\
H_{2,5}^{\text{add}} &= H(\text{addition of 2 five-digit numbers}) = 17.18\ .
\end{aligned}$$

For a summation of several numbers, we add the entropies of all the intermediate distributions

$$H(\text{addition of n k-digit numbers}) = \sum_{j=2}^{n} H_{j,k}^{\text{add}}.$$

When adding 15 three-digit numbers in the shortest flashed sum discipline the result is likely between $8000 - 8999$ ($\approx 37\%$). Therefore, it is not necessary to track the first digit and the complexity of interim sums does not increase. We identify numbers of form 348 and 1348 leading to the three-digit addition

$$H_{2,3}^{\text{add, id}} = H(\text{addition of 2 three-digit numbers, first digit identified}) = 9.09,$$

which we multiply by the number of required operations. The record times in the two shortest addition disciplines do not include the time to indicate the result, but only for the flashed presentation of the digits. This makes it possible to perform a part of the addition after the timing ends. Assuming that additions are performed in a similar speed in the two disciplines, their respective best times allow us to estimate the extend of this effect. Note that the sum of 15 numbers requires 14 additions and of 30 numbers 29 additions. Let o_a be the number of addition operations performed after the flashing ends, and $o_{b,1}$ and $o_{b,2}$ before. The unofficial world bests by Rinne Tsujikubo are 1.17 and 3.2, which entails

$$\begin{aligned}
o_{b,1} + o_a &= 14 \\
o_{b,2} + o_a &= 29 \\
\frac{1.17}{o_{b,1}} &= \frac{3.2}{o_{b,2}}.
\end{aligned}$$

The solution to this system is $o_{b,1} = 15 \cdot \frac{1.17}{3.2-1.17} = 8.65, o_{b,2} = 15 \cdot \frac{3.2}{3.2-1.17} = 23.65$ and $o_a = 5.35$, hinting that slightly more than 5 additions might be carried out after the flashing stops. We thus used the formulae $8.65 \cdot H_{2,3}^{\text{add, id}}$ and $23.65 \cdot H_{2,3}^{\text{add, id}}$. The official world records of 1.61 and 3.31 seconds would suggest that the effect is smaller $o_{b,1} = 14.21, o_{b,2} = 29.21$ and $o_a = 0.79$, which would result in a higher algorithmic entropy.

In the "addition of 10 10-digit numbers" discipline competitors see all information at once. Addition of two numbers with several digits could alternatively occur by adding single-digit numbers successively. Interestingly, the above values indicate that the entropy is larger by using single-digits, as there are more decisions to be made. This explains why top competitors use columns containing several digits. For the addition of ten 10-digit numbers Aaryan Shukla can be seen to use the addition of two four-digit columns and a final two-digit column [Shu24b].

In conclusion, we obtain the following entropy formulae for the different disciplines

$$
\begin{aligned}
H(\text{addition of 15 three-digit numbers}) &\approx 8.65 \cdot H_{2,3}^{\text{add, id.}} \\
H(\text{addition of 30 three-digit numbers}) &\approx 23.65 \cdot H_{2,3}^{\text{add, id.}} \\
H(\text{addition of 50 five-digit numbers}) &= \sum_{j=2}^{50} H_{j,5}^{\text{add}} = 925.95 \\
H(\text{addition of 100 four-digit numbers}) &= \sum_{j=2}^{100} H_{j,4}^{\text{add}} = 1587.87 \\
H(\text{addition of 200 four-digit numbers}) &= \sum_{j=2}^{200} H_{j,4}^{\text{add}} = 3287.31 \\
H(\text{addition of 10 10-digit numbers}) &\approx 2\sum_{j=2}^{10} H_{j,4}^{\text{add}} + \sum_{j=1}^{10} H_{j,2}^{\text{add}} = 334.27\ .
\end{aligned}
$$

Combinatorical considerations greatly accelerate the above computations. Consider j uniform k-digit numbers $X_1, ..., X_j$. Then, by an extension of the stars-and-bars theorem [Ear19],

$$
\begin{aligned}
&\mathbb{P}\Big(\sum_{m=1}^{j} X_m = l\Big) \\
&\quad = \frac{1}{(10^k - 10^{k-1})^j} \sum_{i=0}^{\min\{j, l/10^k\}} (-1)^i \cdot \binom{j}{i} \cdot \binom{l - j \cdot 10^{k-1} - i(10^k - 10^{k-1}) + j - 1}{j-1},
\end{aligned}
$$

where $j10^{k-1} \leq l \leq j(10^k - 1)$. The total number of combinations is $(10^k - 10^{k-1})^j$, so the initial factor is a normalization, whereas the second describes the number of different options to obtain the sum l.

Output We found

$$
\begin{aligned}
H_{15,3}^{\text{add}} &= 12.02 \\
H_{30,3}^{\text{add}} &= 12.52 \\
H_{50,5}^{\text{add}} &= 19.53 \\
H_{100,4}^{\text{add}} &= 16.71 \\
H_{200,4}^{\text{add}} &= 17.21 \\
H_{10,10}^{\text{add}} &\approx 34.98\ .
\end{aligned}
$$

Since 10-digit numbers are rather large, we instead used the observation that as one adds 10 uniform single-, two-, three-, four-, ... digit numbers, the entropy increases by roughly $\log 2(10) = 3.32$ with each digit. Indeed, we determined the first seven members of this sequence to be $5.07, 8.41, 11.73, 15.05, 18.37, 21.69, 25.05, ...$ and estimated the final entropy to be $25.02 + 3 \cdot 3.32$.

4.1.2 Multiplication

Competitors multiply large, uniform, integer numbers with k_1 and k_2 digits with each other.

Input We use $H^{k_1\text{-digit}} + H^{k_2\text{-digit}}$.

Algorithm There are two main strategies used by top competitors. The first "traditional method" requires the ability to visualize several digits simultaneously, achievable by a mental abacus, whereas the second "cross-multiplication method" places little demand on visualization and working memory [Eng25b]. The traditional method is superior in terms of processed

entropy. However, as the cross-multiplication places a small load on working memory, it generalizes to almost arbitrarily large numbers, for instance the multiplication of 20-digit numbers, whereas the traditional method becomes infeasible.

Strategy 1: Traditional This strategy is used by Hiroaki Tsuchiya and other top competitors at Japanese soroban competitions [Eng25b]. The mental algorithm is equivalent to a common pen-and-paper method, one might have learned in school. For example, to multiply $323 \cdot 446$, one performs three multiplications of $4, 4, 6$ with 323 and adds the results:

$$\begin{array}{ccccccc} 3 & 2 & 3 & \cdot & 4 & 4 & 6 \\ & 1 & 2 & 9 & 2 & & \\ & & 1 & 2 & 9 & 2 & \\ & & & 1 & 9 & 3 & 8 \\ \hline & 1 & 4 & 4 & 0 & 5 & 8 \end{array} \tag{4.1}$$

Performing the calculation of the final result mentally from left to right, the strategy requires to visualize the number of digits of the left-hand factor plus one. Moreover, in the upper example one has to perform two three-digit additions.

To understand the entropy of a six-digit by five-digit multiplication, note that there would be six multiplications of a single-digit by a five-digit number and five five-digit additions. As visible from 4.1, if consecutive numbers in the right-hand factor are equal, one of the single- by six-digit multiplication repeats and we do not count its entropy. The probability is 0.9 and, as the first cannot repeat, we assume that there are 5.5 multiplications on average. We computed

$$\begin{aligned} &H(\text{multiplication single-digit } U\{0, ..., 9\} \text{ by five-digit number}) = 17.15 \\ &H(\text{multiplication single-digit } U\{1, ..., 9\} \text{ by five-digit number}) = 18.54\ . \end{aligned}$$

In total, this implies the formula

$$\begin{aligned} &H(\text{traditional multiplication of a six- and five-digit number}) \\ &\approx 18.54 + 4.5 \cdot 17.15 + 5 \cdot H^{\text{add}}_{2,5} = 181.62\ . \end{aligned}$$

Strategy 2: Cross-multiplication This method is used by Aaryan Shukla [Eng25a]. The method builds upon the idea to collect all terms which contribute to a certain power of 10. For example, the product $462 \cdot 501$ can be calculated by collecting all terms associated to $1, 10, 100, ...$

$$\begin{aligned} 2 \cdot 1 &= 2 \\ 60 \cdot 1 + 2 \cdot 0 &= 60 \\ 400 \cdot 1 + 60 \cdot 0 + 2 \cdot 500 &= 1400 \\ 400 \cdot 0 + 60 \cdot 500 &= 30000 \\ 400 \cdot 500 &= 200000 \\ 462 \cdot 501 &= 231462. \end{aligned}$$

If one visualizes the digits of $462 \cdot 501$ multiplied in each term, they represent “crosses" of the digits, which gave the method its name. By carrying out the operation from right to left, there is no need to keep the number of zeros in mind. We assumed that each of the digits of the factors is uniformly distributed on $\{0, ..., 9\}$ except the leading digit, which is uniformly distributed on $\{1, ..., 9\}$. Generalizing the above algorithm to the multiplication of k-digit numbers, there will be $2k - 1$ steps, involving the addition of $1, 2, 3, ..., k-1, k, k-1, ..., 3, 2, 1$ times the product of two uniform single-digit numbers. We carried the exact intermediate distribution along all of these steps, accumulating the entropy for each of the additions and multiplications to arrive at

$$\begin{aligned} H(\text{cross-multiplication of 2 five-digit numbers}) &= 282.86 \\ H(\text{cross-multiplication of 2 eight-digit numbers}) &= 750.66 \\ H(\text{cross-multiplication of 2 20-digit numbers}) &= 4950.78\ . \end{aligned}$$

As top competitors have memorized any multiplication of two two-digit numbers, which occur at the begin and end of the algorithm, we subtracted

$$2\Big(H(\text{cross-multiplication of 2 two-digit numbers}) - H(\text{product of two two-digit numbers})\Big)$$
$$= 2(38.63 - 11.12) = 55.02$$

from each of the above values.

Vision *Method 1: Traditional* Abacus experts can perform this computation after hearing the numbers, suggesting that no rereading is necessary.

Method 2: Cross-multiplication The leading non-zero digits are each read k times and in total $4\sum_{i=1}^{k-1} i$ reads of any of the remaining non-zero digits take place. The memorized two-digit products avoid reading each of the involved digits once. This translates to the formula

$$(2k-2)\cdot H^{\text{1-digit}} + \left(4\sum_{i=1}^{k-1} i - 6\right)\cdot \log_2(10).$$

Output For the multiplication of a k_1-digit and a k_2-digit number, we use

$$\log_2\Big((10^{k_1}-1)(10^{k_2}-1) - 10^{k_1+k_2-2}\Big),$$

reflecting the space of outcomes between the largest $(10^{k_1}-1)(10^{k_2}-1)$ and smallest $10^{k_1-1}10^{k_2-1}$ possible result.

4.1.3 Calendar calculations

Participants are given calendar dates and determine the corresponding day of the week.

Input and Vision If years between $[x_1, x_2)$ are featured in the discipline, we use

$$\log_2\big(365\cdot(x_2-x_1) + \#(\text{number of leap years})\big)$$

for the entropy of a single date.

Algorithm Calendar calculations come down to adding contributions from $\{0,1,2,3,4,5,6\}$ for century (e.g. 19-th), year (e.g. 84), month and day and taking the result modulo seven. One has to be subtracted for dates in January and February in leap years. The Gregorian calendar repeats every 400 years and larger years can be mapped to a known 400-year section.

There are 400 joint century/year combinations and 365 joint day/month combinations. We assume that competitors have memorized these joint contributions leading to a single modular addition [Tim]. The century/year and day/month contributions are essentially uniformly distributed, amounting each to a decision of $\log_2(7)$ entropy. The addition modulo 7 carries another $\log_2(7)$ bit. In one cycle, about every fourth year 97/400 is a leap year and January and February constitute

$$\frac{59+\frac{97}{400}}{365+\frac{97}{400}}$$

of an average year, which is why we add

$$\frac{59+\frac{97}{400}}{365+\frac{97}{400}}\cdot\Big[-\frac{97}{400}\log_2\Big(\frac{97}{400}\Big) - \frac{303}{400}\log_2\Big(\frac{303}{400}\Big)\Big] \approx 0.13$$

for the leap rule. We have checked computationally that the year ranges used in the two disciplines do not change the first two decimal places of this value.

In summary, we use the formulae

$$H(\text{calculate day of dates form the current century}) \approx 3\cdot\log_2(7) + 0.13 \approx 8.55,$$
$$H(\text{calculate day of dates between 1600-2100}) \approx 8.55,$$

as the applied algorithms are the same.

Output The output is one of the seven options Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, so $\log_2(7)$ per date.

4.1.4 Square roots

Competitors draw inexact roots of six-digit, eight-digit and ten-digit numbers to five decimal places. Here, the term inexact means that the decimal places are guaranteed to be non-zero and numbers with integer roots are excluded.

Input We use $H^{\text{6-digit}}$, $H^{\text{8-digit}}$ and $H^{\text{10-digit}}$.

Algorithm The duplex method is used by most top competitors [Eng25a; Squ]. We use $\sqrt{377463}$ as an example and indicate partial results by bold digits. One starts by considering the first two digits, which indicate **6** as $36 < 37 < 49$. Subtracting 36 from the first two digits results in the new number 17463. To generate the additional decimal places, the following steps are repeated:

1. Divide the first two digits by $2 \cdot 6$, here $17/12 = \mathbf{1}$ remainder 5. Add the next digit of the considered number to the remainder, resulting here in 54. If no digits are left add zero instead.

2. Subtract the duplex of the interim result, ignoring the first digit. The duplex of a k-digit number $a_1 a_2 ... a_k$ is defined as

$$\begin{cases} 2\sum_{j=1}^{\frac{k}{2}} a_j \cdot a_{k+1-j} & k \text{ even,} \\ 2\left(\frac{a^2_{\frac{k-1}{2}+1}}{2} + \sum_{j=1}^{\frac{k-1}{2}} a_j \cdot a_{k+1-j}\right) & k \text{ uneven.} \end{cases}$$

   So effectively multiplying the first and last digit, the second and second-to-last digit ... and adding the result of all these products. If k is uneven, the resulting middle digit is squared, halved and added to the sum. In our case $54 - 1^2 = 53$.

Indeed, iterating the above steps

$$53/12 = \mathbf{4} \text{ remainder } 5$$
$$\text{Interim result } 614.$$
$$56 - 2(1 \cdot 4) = 48$$
$$48/12 = \mathbf{3} \text{ remainder } 12$$
$$\text{Interim result } 614.3$$
$$123 - 2(1 \cdot 3 + \frac{4^2}{2}) = 101$$
$$101/12 = \mathbf{8} \text{ remainder } 5$$
$$\text{Interim result } 614.38$$
$$50 - 2(1 \cdot 8 + 4 \cdot 3) = 10$$
$$10/12 = \mathbf{0} \text{ remainder } 10$$
$$\text{Interim result } 614.380$$
$$100 - 2(1 \cdot 0 + 4 \cdot 8 + \frac{3^2}{2}) = 27$$
$$27/12 = \mathbf{1} \text{ remainder } 15$$
$$\text{Interim result } 614.3801$$
$$150 - 2(1 \cdot 1 + 4 \cdot 0 + 3 \cdot 8) = 100$$
$$100/12 = \mathbf{8},$$

which gives the result 614.38018.

Participants can memorize the square-numbers from 1 to 999 to estimate the first two digits of the result, here $\mathbf{614}^2 < 377463 < 615^2$, allowing in principal to skip the first two iterations [Pes+99]. The entropy of the distribution of the first three digits of a uniform six-digit number can be computed as 9.35. Letting X be the first digit of the square root of a uniform six-digit number, we computed $\mathbb{E}(\log_2(2 \cdot X)) = 3.67$. This value represents the average entropy of the remainder decisions assuming uniformly distributed remainders. To calculate the entropy of the duplexes, let

$$H_i^{\times} = H(\text{summation of } i \text{ uniform single-digit products})$$

and

$$H_{i,\text{square}}^{\times} = H(\text{summation of } i \text{ uniform single-digit products and a uniform half square}).$$

We call the term appearing in the duplex of a number with an uneven number of digits "half square". As a square of a single-digit is simple, we set

$$H_{0,\text{square}}^{\times} := 0$$

and computed the following values

$$H_1^{\times} = 4.81 \qquad H_2^{\times} = 16.12 \qquad H_3^{\times} = 27.97 \qquad H_4^{\times} = 40.09$$

$$H_{1,\text{square}}^{\times} = 10.9 \qquad H_{2,\text{square}}^{\times} = 22.98 \qquad H_{3,\text{square}}^{\times} = 35.17\ .$$

To compute $H^{\times}$, we counted each of the single-digit products and each of the additions similar to the calculations for the addition discipline. Duplexes have at least 0.5 probability to be a two-digit number and, if more than than two single-digit products are involved, a three-digit number. Therefore, we use subtractions of uniform two-digit and three-digit numbers with 7.21 and 10.54 bit to account for the subtractions of the duplexes.

Adding all the entropies, we arrive at

$$\begin{aligned} &H(\text{extracting the square root of a six-digit number, five decimal places}) \\ &\approx 9.35 + 5 \cdot \log_2(10) + 4 \cdot 3.67 + \sum_{i=1}^{2} H_{i,\text{square}}^{\times} + \sum_{i=2}^{3} H_i^{\times} + 2 \cdot 7.21 + 2 \cdot 10.54 = 154.11 \end{aligned}$$

The same algorithm generalizes to the square roots of eight-digit and 10-digit numbers to five-decimal places. We require one or two additional iterations, consisting of a remainder operation with 6.99 bit and additional subtractions of duplexes. This allows us to write

$$\begin{aligned} H(\text{extracting the square root of a eight-digit number, five decimal places}) &\approx 206.81 \\ H(\text{extracting the square root of a 10-digit number, five decimal places}) &\approx 264.43\ . \end{aligned}$$

Vision The algorithm shows that, after the initial step, which generates three digits of the result, competitors use one, then two, three, four... of the already calculated digits to generate the next digit. As the first iterations can be skipped through memorization, we add $\sum_{i=2}^{m} i \cdot \log_2(10)$ to the input entropy, where $m = 6, 7, 8$.

Output We assume that competitors know the upper and lower bounds of the results, but that the distribution in between is to complicated to be known for a human. The square root of a six-digit number to five decimal places is between 316.22777 and 999.99950, leading to an output entropy of

$$\log_2\left((\sqrt{999999} - \sqrt{100000}) \cdot 100000\right) = 26.03\ .$$

An equivalent formula is used for the output entropies of the square-root of a uniform eight-digit or 10-digit number.

4.2 List of records

We used publicly available information to compile the following list of mental calculation performances, focusing on calculators with successful participation in major mental calculation competitions.

Currently, a majority of the world records and unofficial best results is held by Aaryan Shukla. In many cases, performances only slightly inferior to the listed records have been demonstrated by other competitors. Therefore, if one were to remove all of his performances or of any other competitor, the regression lines would not change much and the trends identified in this document would remain in place. For example, the last world records in the shortest addition discipline were $1.62, 1.64, 1.66, 1.68, 1.7$, set by a total of four different individuals other than the current world record holder [Flad]. More performances for the addition of 30 three-digit numbers can be found at [Flab], for the world cup disciplines at [Recb] and for several other disciplines in the world rankings [Recc].

Performances set in training are labeled as (unofficial) and performances from the biennial Mental Calculation World Cup as (MCWC). All other performances have been set in more frequent competitions or dedicated record attempts. Occasionally, faster times than the official world record are known. If we found a referenced, faster time, we used the faster time for the figures, as several of the contacted mental calculation experts pointed out the difference. The shortest addition discipline illustrates also this phenomenon: the official world record is 1.61, however, for example, even at official competitions at least 1.42 s has been recorded [Flac]. In this specific case, it seems that no ratification or wider communication by a world record keeping agency was sought.

This list is not complete and records have been selected in which it seemed feasible to estimate the processed entropy with some degree of certainty. For instance, several world records involving specific exact roots of larger numbers exist such as extracting the 13-th root from a 100-digit number. These records rely on very advanced algorithms, detailed knowledge of the task and sometimes luck going beyond the scope of this document. Comprehensive reports are given in [Bre90; Doe11].

By $10\times$ we abbreviate disciplines in which the same task is completed 10 times.

Description	Record	Information rates in $[\frac{\text{bit}}{s}]$			
		Input/Vision	Algorithm	Output	
addition of 15 three-digit numbers flashed on a screen	(unofficial) Rinne Tsujikubo 1.17 s	125.82	67.17	0	[Flaa]
	Rikako Miyamoto 1.61 s	91.43	48.81	0	[Rika; Rikb]
addition of 30 three-digit numbers flashed on a screen	(unofficial) Rinne Tsujikubo 3.2 s	92	67.17	0	[Tsu24]
	Yoshiro Takakura 3.31 s	88.95	64.92	0	[Flad; Dou]
addition of 50 five-digit numbers flashed on a screen	Aaryan Shukla 18.71 s	44.35	49.49	1.04	[Bak25]
addition of 100 four-digit numbers flashed on a screen	Aaryan Shukla 30.9 s	42.96	51.39	0.54	[Bak25]
addition of 200 four-digit numbers flashed on a screen	Aaryan Shukla 69.68 s	38.1	47.18	0.25	[Bak25]
addition of 10 10-digit numbers, 10×	Aaryan Shukla 78.08 s	42.35	42.81	4.48	[Shu24b; Recc]
addition of 10 10-digit numbers, 7 minutes	(MCWC) Aaryan Shukla 34/39	26.77	27.06	2.83	[Recb; Tim24b]

Table 2: Performances of top competitors in mental addition. If the time to give the result is not included in the record, we set the output rate to 0 bit/s. The output entropy of the addition of 15 thee-digit numbers would be 12.02 and of 30 three-digit number 12.52. The last listed record is only the best result in terms of correct answers at the Mental Calculation World Cup. By official rules, the number of incorrectly solved tasks is subtracted from the number of correctly solved tasks, giving 32 by Tetsuya Ono as the best result [Recb].

Description	Record	Information rates in $[\frac{\text{bit}}{s}]$				
		Input	Vision	Alg.	Output	
multiplication of a six and a five-digit number, 10×	Hiroaki Tsuchiya 26.54 s	13.65	13.65	68.43	13.76	[Mul]
multiplication of two five-digit numbers, 10×	Aaryan Shukla 51.69 s	6.37	26.76	44.1	6.42	[Bak25; Recc]
multiplication of two eight-digit numbers, 10×	Aaryan Shukla 155.41 s	3.4	25.51	44.76	3.42	[Bak25; Recc]
multiplication of two eight-digit numbers, 10 minutes	(MCWC) Aaryan Shukla 28/31	2.47	18.5	32.46	2.48	[Recb; Tim24b]
multiplication of two 20-digit numbers	(unofficial) Aaryan Shukla 86 s	1.54	30.52	56.93	1.54	[Shu24c]
	Aaryan Shukla 105 s	1.26	25	46.62	1.27	[Tim22]

Table 3: Performances of top competitors in mental multiplication.

Description	Record	Information rates in $[\frac{\text{bit}}{s}]$			
		Input/Vision	Algorithm	Output	
20 random dates from one century	Yusnier Viera 8.05 s	37.66	21.25	6.97	[Reca]
125 random dates from $1600-2100$	(unofficial) Aaryan Shukla 42.84 s	51	24.95	8.19	[Shu24a]
from $1600-2100$, one minute	Yusnier Viera 159	46.32	22.66	7.44	[Reca; Recc]

Table 4: Performances of top competitors in mental calendar calculations. We did not include the time of 150.4 s for all dates from one year, as one cannot exclude complete memorization [Reca].

Description	Record	Information rates in $[\frac{\text{bit}}{s}]$				
		Input	Vision	Alg.	Output	
square-roots of six-digit numbers to five decimal places, 10×	(unofficial) Aaryan Shukla 26.99 s	7.33	31.94	57.1	9.64	[Shu24f]
	Shashank Jain 27.3 s	7.25	31.58	56.45	9.53	[Tim24a; Recc]
square-roots of six-digit numbers to five decimal places, 10 minutes	(MCWC) Aaryan Shukla 86/90	2.84	12.36	22.09	3.73	[Recb; Tim24b]
square-roots of eight-digit numbers to five decimal places, 10×	(unofficial) Aaryan Shukla 41.77 s	6.33	27.8	49.51	7.03	[Shu24e]
square-roots of 10-digit numbers to five decimal places, 10×	(unofficial) Aaryan Shukla 61.18 s	5.4	24.41	43.22	5.34	[Shu24d]

Table 5: Performances of top competitors in mentally extracting roots of square-roots of six-, eight- and 10-digit numbers.

References

[Bak25] V. Bakogianni. *Aaryan Shukla - six world records*. 2025. URL: `www.guinnessworldrecords.com/news/2025/2/human-calculator-kid-shatters-six-world-records-with-incredibly-speedy-mental-maths` (visited on 05/26/2025).

[Bel+20] A. N. Belkacem et al. "Neural Processing Mechanism of Mental Calculation Based on Cerebral Oscillatory Changes: A Comparison Between Abacus Experts and Novices". *Frontiers in Human Neuroscience* 14 (2020).

[Bre90] J. Bredenkamp. "Kognitionspsychologische Untersuchungen eines Rechenkünstlers". German. *Wissenschaft und Verantwortung. Festschrift für Karl Josef Klauer*. Ed. by H. Feger. Göttingen: Hogrefe, (1990), 47–70.

[Doe11] R. Doerfler. *The 13th Root of a 100-Digit Number*. 2011. URL: `www.deadreckonings.com/2011/10/05/the-13th-root-of-a-100-digit-number-part-i` (visited on 04/02/2025).

[Dou] *Yuichiro Takakura - Addition of 30 three-digit numbers WR Video*. 2022. URL: `www.youtube.com/watch?v=fN6BvGGcR0U` (visited on 05/22/2025).

[Dre+17] M. Dresler et al. "Mnemonic training reshapes brain networks to support superior memory". *Neuron* 93.5 (2017), 1227–1235.

[Ear19] M. Earnest. *Answer to extended stars-and-bars problem*. 2019. URL: https://math.stackexchange.com/questions/553960/extended-stars-and-bars-problemwhere-the-upper-limit-of-the-variable-is-bounded (visited on 04/14/2025).

[Eng24] S. Engel. *Interview Wenzel Grüß*. 2024. URL: https://mentalcalculation.org/wenzel-interview (visited on 05/26/2025).

[Eng25a] S. Engel. *Interview Aaryan Shukla*. 2025. URL: www.youtube.com/watch?v=NCRMF1_ncgs (visited on 05/20/2025).

[Eng25b] S. Engel. *Traditional vs. Cross Multipliation*. 2025. URL: www.youtube.com/watch?v=rNhIFVU-N94 (visited on 05/19/2025).

[Flaa] *Addition of 15 three-digit numbers ranking*. URL: https://kentei.soroban-soft.com/fcc_ranking.cgi?title=fcc2024w (visited on 05/20/2025).

[Flab] *All Japan FaST-Abacus National Convention*. URL: www.soroban-fast.com (visited on 05/26/2025).

[Flac] *Christmascup 2019*. URL: https://shuzankyokai.jp/event/christmascup/2019/2019christmas.xlsx (visited on 05/26/2025).

[Flad] *Flash Anzan*. URL: www.flash-anzan.com (visited on 05/13/2025).

[Han+03] T. Hanakawa et al. "Neural correlates underlying mental calculation in abacus experts: a functional magnetic resonance imaging study". *NeuroImage* 19.2 (2003), 296–307.

[Hit78] G. J. Hitch. "The role of short-term working memory in mental arithmetic". *Cognitive Psychology* 10.3 (1978), 302–323.

[Jen90] A. R. Jensen. "Speed of information processing in a calculating prodigy". *Intelligence* 14.3 (1990), 259–274.

[LA22] S. Lo and S. Andrews. "The Effects of Mental Abacus Expertise on Working Memory, Mental Representations and Calculation Strategies Used for Two-Digit Hindu-Arabic Numbers". *Journal of Numerical Cognition* 8.1 (2022), 89–122.

[MS13] L. Minati and N. Sigala. "Effective Connectivity Reveals Strategy Differences in an Expert Calculator". *PLOS ONE* 8.9 (Sept. 2013), 1–12.

[Mul] *Multiplication six- times five-digit numbers WR*. URL: https://youtu.be/wVbXnsp4Xcc?si=VwXgyOYL18xsBGpC&t=613 (visited on 05/20/2025).

[Pes+99] M. Pesenti et al. "Basic and Exceptional Calculation Abilities in a Calculating Prodigy: A Case Study". *Mathematical Cognition* 5.2 (1999), 97–148.

[PS18] R. W. Proctor and D. W. Schneider. "Hick's law for choice reaction time: A review". *Quarterly Journal of Experimental Psychology* 71.6 (2018), 1281–1299.

[Reca] *Calendar Dates World Ranking*. URL: www.recordholders.org/en/records/dates.html (visited on 03/14/2025).

[Recb] *Mental Calculation World Cup Results*. URL: www.recordholders.org/en/events/worldcup (visited on 04/16/2025).

[Recc] *Mental Calculation World Rankings*. URL: www.recordholders.org/en/list/mental-calculation-rankings.html (visited on 04/16/2025).

[Ret+19] L. C. Reteig et al. "Sustaining attention for a prolonged period of time increases temporal variability in cortical responses". *Cortex* 117 (2019), 16–32.

[Rika] *Rikako Miyamoto - Addition of 15 three-digit numbers WR*. 2024. URL: www.guinnessworldrecords.com/world-records/fastest-flash-mental-arithmetic (visited on 03/14/2025).

[Rikb] *Rikako Miyamoto - Addition of 15 three-digit numbers WR Video*. 2024. URL: www.youtube.com/watch?v=2hTchQIGrpc (visited on 03/14/2025).

[Shu24a] A. Shukla. *125 Calendar dates between 1600-2100*. 2024. URL: www.facebook.com/GeniusAaryan/posts/pfbid0Qsas47hW9oKHshZZMoQ1w7RUZMrRyHFRcVEjjRMoLfvMrwqjjRRf1jdMzKp5Wmwgl (visited on 05/20/2025).

[Shu24b] A. Shukla. *Aaryan Shukla - Addition of 10 ten-digit numbers WR*. 2024.

[Shu24c] A. Shukla. *Multiplication of 20-digit numbers*. 2024. URL: www.facebook.com/GeniusAaryan/posts/pfbid02AwRpq9m6nb6WE3ykAzrKqKXTLg44Cdhg6U2EiA12m4EV2be6ecGakCf93SJ9pb6Al (visited on 05/20/2025).

[Shu24d] A. Shukla. *Square roots of 10-digit numbers*. 2024. URL: www.facebook.com/GeniusAaryan/photos/mental-square-root-imperfect-10-digits-in-6118-seconds-16-november-2023-more-tha/989273123220796/?_rdr (visited on 04/16/2025).

[Shu24e] A. Shukla. *Square roots of eight-digit numbers*. 2024. URL: www.facebook.com/story.php?story_fbid=988711146610327&id=100064143515482 (visited on 04/02/2025).

[Shu24f] A. Shukla. *Square roots of six-digit numbers*. 2024. URL: www.facebook.com/GeniusAaryan/posts/pfbid0LxKRSN6WDThXVsFHNpjqTdfMdgR1xgjCt7CTi9jiaiWanwpgSoQsMtqqU8qLz6fwl (visited on 05/20/2025).

[Squ] *Duplex square root algorithm*. 2017. URL: https://forum.artofmemory.com/t/duplex-method-for-square-roots/32913 (visited on 04/15/2025).

[Sta88] J. J. Staszewski. "Skilled Memory and Expert Mental Calculation". *The Nature of Expertise*. 1st. Psychology Press, 1988, 71–129.

[Tho63] H. B. G. Thomas. "Communication theory and the constellation hypothesis of calculation". *Quarterly Journal of Experimental Psychology* 15.3 (1963), 173–191.

[Tim] D. Timms. *Calendar algorithm*. URL: www.worldmentalcalculation.com/how-to-calculate-calendar-dates (visited on 03/24/2025).

[Tim19] D. Timms. *World mental calculation - fastest possible processing speed*. 2019. URL: www.worldmentalcalculation.com/2019/06/30/fastest-possible-processing-speed-of-the-human-brain (visited on 03/05/2025).

[Tim22] D. Timms. *Mental Calculation World Cup 2022*. 2022. URL: www.worldmentalcalculation.com/2022/07/19/mental-calculation-world-cup-2022-results (visited on 03/14/2025).

[Tim24a] D. Timms. *Memoriad 2024*. 2024. URL: www.worldmentalcalculation.com/2024/12/14/memoriad-2024-results (visited on 03/25/2025).

[Tim24b] D. Timms. *Mental Calculation World Cup 2024*. 2024. URL: www.worldmentalcalculation.com/2024/09/20/mental-calculation-world-cup-2024-results (visited on 04/16/2025).

[Tsu24] R. Tsujikubo. *Addition of 30 three-digit numbers in 3.2 s*. 2024. URL: https://x.com/rinpy_0420/status/1833722591145771266 (visited on 05/20/2025).

[Wie25a] B. Wiederhold. "An introduction to memory competitions, records and techniques". *arXiv* (2025), 2504.06747.

[Wie25b] B. Wiederhold. "The speed of information processing at memory competitions: limited by reading in short tasks and declining as a power law for longer times". *bioRxiv* (2025), 10.1101/2025.03.31.646314.

[ZM24] J. Zheng and M. Meister. "The unbearable slowness of being: Why do we live at 10 bits/s?" *Neuron* 113.2 (2024), 192–204.