Optimizing strain selection for association studies under hard cost constraints

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Abstract

Quantitative genetics methods that link genotype to phenotype can be especially powerful tools in model organisms and non-human populations, where researchers can apply well-controlled perturbations to commercially-available collections of strains. However, purchasing and phenotyping large collections can be cost-prohibitive. We evaluate several approaches to select subsets of a panel of strains with the aim of maximizing power under fixed budgetary constraints. Some approaches focus solely on costs, others on genetic diversity, and some on both simultaneously. To assess how these results generalize, we simulate data across two biological and statistical settings: phylogenetic regression across bacterial isolates, and linear-mixed model regression across mouse strains. Surprisingly, we find that simply selecting the cheapest strains until the budget is exhausted (MinCost) is usually among the top-performing methods, and that algorithms that consider genetic diversity without weighting by cost tend to show much lower power overall. Methods that consider both cost and diversity are typically, though not universally, the top performers. A comparison of diversity-maximizing objectives reveals that the most commonly studied objective (MaxMin) tends to be the among the least effective at retaining power. Instead, two alternative objectives that have not been previously studied in this context — maximizing the total minor allele frequency (MaxMAF) and maximizing the total sum of pairwise genetic distances (MaxSum) — typically perform best. Finally, in addition to fully simulated data, we also test these approaches by subsampling real data on cardiovascular phenotypes from the Hybrid Mouse Diversity Panel (HMDP). The three best methods on these data are cost-aware (p-Median, MaxSum, and MinCost). Based on these results, we recommend investigators consider approaches that maximize total MAF or pairwise genetic distance weighted by strain cost, but we also conclude that a surprisingly robust approach is to simply pick the most strains one can afford.

INTRODUCTION

The cost of research has grown significantly over the past two decades. One measure is the average size of an R01 grant from the National Institutes of Health, which has increased 87% between 1999 and 2019[1], 64% faster than inflation. These cost increases come from many factors: novel techniques and technologies that supplant other, often cheaper approaches (e.g. PCR to RNA microarrays to RNAseq to single cell RNAseq), more complex study designs which necessitate additional research cohorts and endpoints, and, notably, a drive to increase the size of studies to attempt to both achieve greater statistical power and identify less penetrant drivers of disease conditions[2-4]. For example, over the past two decades, the GWAS community has been gathering larger and larger populations to try and recover the effects of polymorphisms with smaller and smaller effect sizes[5,6]. Depending on how they are funded, such large populations may be out of reach for individual investigators. Furthermore, beyond simple financial considerations, researchers who use animal populations are often mandated to follow the three Rs (Replacement, Reduction, Refinement) in their research, which states that if it is possible to achieve the same or similar result with fewer animals, then researchers should make every effort to do so; for example, the US Food and Drug administration has recently announced a roadmap to reduce animal testing in preclinical studies[7]. Taken together, the increasing costs in both money and time of doing research coupled with ethical responsibilities to animal cohorts are powerful incentives to optimize and streamline research projects, as long as this does not come at the expense of statistical power.

In comparative studies, one opportunity for streamlining concerns how we select the strains that will be phenotyped. Diverse collections of microbes have proven to be powerful tools for finding genetic variants associated with phenotypes including host range of pathogens[8,9], positive fitness effects of symbionts[10], toxin tolerance in industrially-relevant microbes[11], and gene and protein expression in model organisms[12]. When applied in diverse animal lines, GWAS, MPRA assays, and other comparative omics-scale analyses have allowed us to link genetic polymorphisms to complex disease susceptibility[13-18]. However, prices for such strains are not uniform: rather, they vary widely across strain collections, individual genotypes, and even how a particular strain is propagated and shipped. For example, a mouse from Jackson Labs costs a different amount of money depending on its strain, age, and sex, ranging from less than $30 for a 3-week-old C57BL/6J male, to over $260 for a 9 week old NZO/HILtJ male, to nearly $2,500 a litter for one of the many cryopreserved strains available for purchase. Similar factors apply to bacterial strains. Type strains are required to be preserved in at least two repositories, which may have different costs: the type strain of *Lachnospira eligens*, shipped as lyophilized cells, costs $474 from the American Type Culture Collection (ATCC) but 100 EUR (~$113) from

the German Collection of Microorganisms and Cell Cultures (DSMZ). As with mice, the
format also matters: a live, active culture of the *L. eligens* type strain from DSMZ would cost
240 EUR ($273). Finally, both ATCC and DSMZ, as well as many other repositories, have
unique strains that can only be purchased from them. Being able to optimize the selection
of strains to maximize statistical power for a given budget would be a major benefit for the
labs which use these populations.

One well-researched framing of strain selection, which Weitzman named the "Noah's Ark"
problem[19], comes from bioconservation. Solutions to this problem maximize a measure of
community diversity, typically either Faith's phylogenetic diversity or PD (here, we refer to
these as MaxPD methods)[20], or the minimum pairwise distance between all selected taxa
(which we term MaxMin)[19]. In the equal costs setting, MaxMin is equivalent to the p-
dispersion problem from operations research, first formulated in the context of locating
facilities on a network[21]. The general p-dispersion problem is NP-hard, but greedy MaxMin
algorithms to select taxa perform well[22], and under certain assumptions (distances given
by an ultrametric tree), they also maximize the PD of the selected taxa. In fact, greedy
MaxPD algorithms, where successive strains are chosen iteratively based on which
additional strain leads to the largest increase, provably reach the global optimum[23,24].
Greedy methods to maximize PD have also been applied in genotype imputation studies,
where they have been shown to improve power relative to random selection[25,26].

However, while Weitzman's original "Noah's Ark" problem does consider unequal costs,
most studies on strain selection in the context of association or genomic studies do not,
and it is less clear how well greedy methods perform in this setting. Weitzman provides one
way to integrate cost information: a greedy search where strains are ranked by genetic
distinctiveness divided by cost[27], which he terms the "myopic rule." A similar approach,
proposed by Hartmann and Steel[19], is to maximize the sum of the PD and a penalty term,
which in turn is a sum of some function of the costs of the selected strains. In this
formulation, the greedy algorithm is still guaranteed to be optimal[23] in the sense that it does
maximize this modified objective. However, this may not be the same as directly optimizing
PD under fixed costs. In the extremes, a function that multiplies all costs by zero would be
equivalent to not considering costs at all, while multiplying all the costs by a very large
number (relative to the maximum possible PD) would be equivalent to prioritizing the
cheapest strains without considering diversity. Pardi and Goldman[28] did develop an
efficient, pseudo-polynomial time algorithm for directly maximizing PD under cost
constraints, meaning that out of all sets of taxa that can be purchased with a given budget,
the set with the highest PD will be chosen. However, this algorithm involves decomposing
the problem by clade, so the taxa being considered must be related by a phylogenetic tree.

Maximizing diversity on a tree via PD has many advantages, both in terms of constraining a
more general NP-hard problem into one that can be solved exactly, and also in terms of
achieving bioconservation outcomes[29,30]. However, metrics like PD cannot capture genetic
relationships between taxa that are not described well by a tree: for example, when there is
extensive horizontal transfer between species, as with gut bacteria[31], or when there is a
complex kinship matrix, as in panels that derive from backcrosses. Because these
relationships may be particularly relevant in the context of genome-wide association
studies and comparative genomics, the current study focuses on methods that can use
more general measures of genetic distance between strains.

Furthermore, even assuming we could successfully maximize genetic divergence, doing so
may not necessarily accomplish the aim of conserving statistical power[32]. For example,
McAuliffe and colleagues [32]developed an approach for comparative genomics in which they
directly maximized power. They showed that the selected taxa were often very different
from those that maximized evolutionary divergence, having as few as one taxon in
common. Choosing power itself as the quantity to maximize has obvious advantages;
however, this method requires calculating power in a combinatorial search over possible
subtrees. While McAuliffe et al. derived an exact analytical solution for the power of their
test, making it much more tractable, the computational cost of applying this method in a
panel of hundreds or thousands of strains would be prohibitive.

Other metrics besides PD or MaxMin diversity could potentially offer better performance.
For example, Matsen IV et al[33] developed an approach that minimizes the average
phylogenetic distance of query sequences to the closest leaf on a reference tree
("minimum-ADCL"), essentially maximizing how "representative" the selected subset is of
the whole panel. As the authors note, this is a variation of the k-medoids problem[34] on a
tree, and as elaborated by Faith, it is also equivalent to discrete p-median optimization of
diversity[35]. The p-median problem (closely related to k-medoids) can be framed as an
integer linear programming problem. Kang et al. adapted Matsen's p-median approach to
pick reference panels for genotype imputation, and showed that for this particular
problem, p-median outperformed both random and maximizing total diversity
approaches[36]. However, these p-median methods do not consider costs and would have to
be modified for the question we are interested in. Additionally, a p-median approach
should deprioritize genetic outliers[33]; in the setting of quantitative genetics, however,
outliers might actually improve the power to detect associations with rare variants.

Beyond MaxPD, MaxMin and p-median, there are also other approaches that could be used
to maximize diversity[37] that have not been explored to the same extent in the genetics
literature. One of these is maximizing the sum of the total genetic distances between all

selected strains, known as the MaxSum or (discrete) p-dispersion-sum problem[21,38].
Solutions to this problem can differ markedly from p-dispersion or p-median solutions (for
instance, MaxSum solutions may include pairs of points that are close together[21]),
potentially translating into different effects on power. Again, however, this problem is NP-
hard, and most available heuristics have been developed for a formulation that does not
include unequal costs.

Finally, because genome-wide association studies typically show the most power for
common variants, instead of maximizing distances, one might also consider choosing the
strains that maximize the minor allele frequency (or MAF, which ranges from 0 to 0.5) of all
alleles in a panel, which by analogy we call MaxMAF. A related approach from crop science,
called the "M strategy"[39,40], aims to choose strains that maximize total allele diversity
across a set of genetic markers; compared to methods based on genetic divergence, this
strategy selects taxa bearing fewer non-informative alleles[41]. To our knowledge, however,
neither MaxSum nor MaxMAF has been assessed in terms of retaining power in gene
association studies, and these methods also have yet to be adapted to the setting of
unequal costs.

In this manuscript, we set out to study different approaches for strain selection under a
hard cost constraint, and to quantify how these methods affect our power to detect genetic
associations (Figure 1). Because we wanted to apply these methods in settings where a
tree might not best describe the genetic relationships between strains, we did not further
consider methods for maximizing PD, and instead focused on approaches that allowed
more flexible measures of genetic distance. Here, we consider five families of methods for
considering genetic diversity:

• <u>None</u>: Only the number of strains is considered;
• <u>p-Median</u>: Minimize the total minimum genetic distance between non-selected and
selected strains (as with the min-ADCL method);
• <u>MaxMin</u>: Maximize the minimum genetic distance between selected strains;
• <u>MaxSum</u>: Maximize the total genetic distance between all pairs of selected strains;
• <u>MaxMAF</u>: Maximize the sum of minor allele frequencies (MAFs).

For each family, we test an algorithm that does not consider cost ("NoCost"), simply
stopping when the budget is exhausted, and at least one algorithm that does (denoted by
either "Myopic" or "Optim"). The "NoCost" algorithm is a greedy search, taking the strain
that improves the family objective the most at each step, except in the case of "p-
MedianNoCost," where greedy search is known to perform poorly and we instead use the
Partitioning Around Medoids algorithm. The "Myopic" algorithms we consider are greedy
searches that divide a diversity measure by the cost of the strain, as with the "myopic rule."

Finally, the "Optim" algorithm directly optimizes the objective via mixed-integer linear
programming, or MILP. (Note that while we did implement versions of MaxMinOptim and
MaxSumOptim, the only MILP method that was computationally tractable on our data
proved to be the p-MedianOptim algorithm.)

To our knowledge, this is the first systematic test of how different methods for cost-
constrained strain selection affect power in downstream gene association studies.
Additionally, the few studies to consider costs have focused on the MaxPD and MaxMin
objectives; we have not found any studies that have implemented and tested cost-aware
versions of the alternative MaxSum, MaxMAF, or p-Median approaches.

We evaluate these methods using simulations, which we conduct in two different settings:
phylogenetic regression in microbes, and GWAS in mice. In the first setting, we model the
effects of strain selection on power in phylogenetic regression applied to synthetic data
from genotyped bacterial isolates. We perform two tests, one where we use the real
genotypes of isolates from *Lachnospiraceae*, the most prevalent gut bacterial family
worldwide, and one where we introduce a form of study bias by simulating an additional
oversampled clade. In the second setting, we model the effects of strain selection on
GWAS results in the Hybrid Mouse Diversity Panel, a commonly used genetic reference
population for studying a number of phenotypes and diseases[42-46]. In our GWAS
simulations, we vary the minor allele frequency (MAF) and effect size cutoffs (χ^2), and also
explore the impact of reducing the variance of prices by adding extra equal, fixed costs per
strain (as might be appropriate when the cost of strain maintenance and/or phenotyping is
a non-negligible portion of a research plan's budgetary constraints). Finally, we follow up
our simulation studies with an analysis of actual data from an HMDP GWAS study in which
the beta adrenergic agonist isoproterenol was used to induce cardiac hypertrophy, which
we subsample to simulate subset selection.

METHODS

Data Sources

Mouse genotype data from the Mouse Diversity Array for the Hybrid Mouse Diversity Panel
(HMDP) was obtained from the Mouse Genomics Informatics website[17]. Mouse phenotype
data is from a previously published dataset[18] and is available at Mendeley Resources at
data.mendeley.com/datasets/y8tdm4s7nh/1[47]. Mouse costs were pulled from the Jackson
Labs website[48] as on 4/05/2024 and represent the cost of 1 male and 2 female mice or one
litter of cryopreserved animals, depending on the current status of the animals at the time
of accession.

Metadata for bacterial strains was downloaded from BacDive[49] as of 07/31/2025. Strains with a genome in NCBI's GenBank or RefSeq were retained, then matched with the metadata from the Genome Taxonomy Database (GTDB)[50] release 220 (downloaded 10/31/2024) to obtain taxonomic assignments, ignoring genome version numbers. Genomes were then filtered for GTDB annotations for the family Lachnospiraceae. The five most common strain repositories for these genomes were the German Collection of Microorganisms and Cell Cultures (DSMZ), Japan Collection of Microorganisms (JCM), American Type Culture Collection (ATCC), Culture Collection University of Gothenburg (CCUG), and Korean Collection for Type Cultures (KCTC). Costs for each repository were estimated by spot-checking five of the *Lachnospiraceae* strains on the respective websites and averaging the results; shipping costs were not included. When the exact same strain was available in multiple collections, we retained the cheapest copy. We used OrthoFinder[51] to identify orthogroups (i.e., genes descending from a common ancestor in this group of species). Orthogroups with identical distributions across genomes were merged into phylogroups, then filtered such that all remaining phylogroups were present in at least three genomes and absent in at least three genomes.

Simulation Studies

Phylogenetic regression: We performed two sets of tests: one using real *Lachnospiraceae* genotypes and the real tree ("Real"), and one using simulated genotype data on a version of the real *Lachnospiraceae* tree that had been augmented by a set of 100 very similar strains with very low cost ("Augmented"), to mimic a situation where one taxon may be highly oversampled relative to all others.

Real: We started with the *Lachnospiraceae* genome tree generated by OrthoFinder, making it ultrametric by using relative evolutionary divergence normalization[52] using the "castor" package. For each scenario, we simulated 5,000 phenotypes on this ultrametric genome tree under a Brownian motion model using the "rTrait" function in the "phylolm" package. For each phenotype, we picked a phylogroup (without replacement), and then added an offset to that phenotype when the phylogroup was present, with the offset corresponding to 60% of that phenotype's standard deviation. Phylogroups present in fewer than three genomes or absent in fewer than three genomes were excluded. We finally performed one phylogenetic regression per phenotype and phylogroup pair, allowing for measurement error, using the phylolm function in the "phylolm" package. We estimated power as the proportion of these tests with a p-value less than 0.05, replacing any tests that returned an NA value (as might happen when, e.g., there is no variance in the phylogroup) with a p-value of 1.

Augmented: Starting with the above tree of *Lachnospiraceae* genomes, we first grafted a
random coalescent tree of 100 strains onto the immediate ancestor of *Agathobacter*
rectalis, with the random tree scaled to 1/10th the node depth of this ancestor. The
presence/absence vectors of 5,000 synthetic phylogroups were then simulated using the
rBinTrait function in phylolm[53], which uses a continuous time Markov chain approach to
model binary traits along a tree. For each synthetic phylogroup, the parameters alpha and
beta were obtained by performing Ives-Garland phylogenetic logistic regression[54] on one of
the 5,000 phylogroups selected in the "real" simulation, using the phyloglm function in the
phylolm package[54]. We used rejection sampling to make sure that all simulated
phylogroups were present in at least three genomes and absent in at least three genomes.
Finally, phenotypes were simulated and power was assessed as in the "real" simulation.

Recombinant: To assess the impact of genetic structure on the results, we first selected a
set of six "parental" *Lachnospiraceae* strains using PAM[55], then added a set of 100
simulated strains where alleles were sampled with equal probability from one of the
six parents, and costs were sampled from a discrete uniform distribution, ranging
from $20 to $200 in steps of $10. Strain picking, phenotype simulation, and power
assessment was performed as above.

GWAS Phenotype Creation: We created sets of 100 simulated phenotypes each for our
simulation studies. For each simulated phenotype, we drew a SNP at random that
matched a required Minor Allele Frequency (MAF) criterion (5-10%, 10-30% or 40-50%). We
then selected a β value for this SNP based on a desired χ^2 threshold (10, 50 or 100) and
added noise based on a multivariate normal (MVN) derived from two variance terms, the
first the proportion of variance attributable to genetic effects (σ_g^2), set at 40% consistent
with our past work[56] and the second the proportion attributable to all other sources of
variation (σ_e^2) as follows:

$$y = \beta X + MVN(0, \sigma_g^2 \mathbf{K} + \sigma_e^2 \mathbf{I})$$

where **K** is the kinship matrix of the strains of the HMDP calculated using the kinship
function of the myFastLMM R package[57].

GWAS Approach: Simulated phenotype-genotype pairs were analyzed using the linear
mixed model approach fastLMM[57]. Power was calculated based on the number of
associations that met a genome-wide significance threshold of 4.2E-6 as previously
determined in the HMDP[17,44,46,58]. At a χ^2 of 100, there was 100% power in the full
panel(Figure S1A). In the χ^2 of 50 set, power ranged from 82-93% depending on the Minor
Allele Frequency (MAF)(Figure S1A). Power in the χ^2 of 10 set of simulated phenotypes was
very low (<10%) in the whole panel and was not used going forward.

Strain Subset Selection

We selected strain subsets for our simulation study using the following algorithms.

Greedy approaches

Random: Strains were selected randomly until the budget was exhausted.

MinCost: Strains were ranked from cheapest to most expensive, breaking ties randomly,
then selected in that order until the budget was exhausted.

MaxMinNoCost: Strains were selected according to a greedy algorithm that chooses the
most diverged strain at every step, regardless of cost. This is represented by the following
pseudocode, where n is the total number of strains, x is a binary vector with x_i representing
whether strain i is selected, c_i is the cost of strain i, B is the total budget, and $d_{i,j}$ is the
genetic distance between strains i and j:

```
261         1) cheapest = which(c == min(c))
262         2) x[sample(cheapest, 1)] = 1     # random initialization
263         3) while (B > 0):
264            a) candidates = which(x == 0 & c <= B)
265            b) selected = which(x == 1)
266            c) for (a in candidates):
267               i) mindist_a = min(d_{a,s}) for s in selected)
268            d) new = which(mindist_a == max(mindist_a))[1]
269            e) x[new] = 1
270            f) B = B - c[new]
```

271 MaxMinMyopic: This method uses the same algorithm as MaxMinNoCost, except that the
272 minimum distances are scaled by cost, meaning that step 3d would read:

```
273         o  new = which(mindist_a/c_a == max(mindist_a/c_a))[1]
```

274 MaxSumNoCost: This method uses a similar approach to MaxMinNoCost, but steps 3c-d
275 are different:

```
276         1) cheapest = which(c == min(c))
277         2) x[sample(cheapest, 1)] = 1
278         3) while (B > 0):
279            a) candidates = which(x == 0 & c <= B)
280            b) selected = which(x == 1)
281            c) for (a in candidates):
282                i) sumdist_a = sum(d_x, y for x, y in union(a, selected))
283            d) new = which(sumdist_a == max(sumdist_a))[1]
284            e) x[new] = 1
285            f) B = B - c[new]
```

286 <u>MaxSumMyopic:</u> As MaxSumNoCost, but as before, the sum of distances is scaled by cost
287 in step 3d:

288   ◦   `new = which(sumdist_a/c_a == max(sumdist_a/c_a))[1]`

289 <u>MaxMAFNoCost:</u> This method uses a similar framework as the algorithms above, where we
290 choose the next strain such that it minimizes the following equation:

$$\sum_{i=1}^{m}\left(0.5 - \left|\left(\frac{\sum_{j=1}^{n}x_{i,j}}{\|x_i\|}\right) - 0.5\right|\right)$$

292 Here, x is a m x n matrix in which there are m SNPs; the first (n-1) columns are the currently
293 selected strains and the nth column is the 'applicant' strain. This algorithm acts to select
294 strains such that they push the minor allele frequency (MAF) of the selected strains as
295 close to 50% as possible, thereby attaining 'maximum MAF'. In pseudocode:

```
296        1) cheapest = which(c == min(c))
297        2) x[sample(cheapest, 1)] = 1
298        3) while (B > 0):
299           a) candidates = which(x == 0 & c < B)
300           b) selected = which(x == 1)
301           c) for (a in candidates):
302              i) sumMAFs_a = sum(0.5 - abs(rowMeans(snp_matrix[, union(selected,
303                 a)]) - 0.5))
304           d) new = which(sumMAFs_a == max(sum_MAFs_a))[1]
305           e) x[new] = 1
306           f) B = B - c[new]
```

<u>MaxMAFMyopic:</u> As MaxMafNoCost, but as before, sumMAFs_a is replaced by sumMAFs_a
/ c_a in step 3d:

◦   `new = which(mindist_a/c_a == max(mindist_a/c_a))[1]`

Other heuristic approaches:

<u>p-MedianNoCost:</u> Because greedy search does not work well for the p-Median problem, we
instead used the fast Partitioning Around (k-)Medoids method[55] implemented in the pam
function in the "cluster" package in R[59]. The PAM algorithm for p-Median problems involves
making iterative improvements to the set of selected medoids, but unlike a pure greedy
search, it allows medoids to be replaced. To handle the cost constraint, we ran PAM with k
increasing from 2 to n-1, where n is the total number of strains. We kept solutions that were
under budget, and of these kept the result that selected the most strains.

Optimization based approaches:

p-MedianOptim: We formulated this problem to be similar to minimum-ADCL[33] and k-
medoids, with an additional constraint on total cost and with no predetermined number k
of representatives. Thus, we were interested in picking k medoids or representative strains
with minimum distance to the closest non-chosen strains for each representative, with k
allowed to vary between one and the number of strains.

The p-MedianOptim method then minimizes the following sum:

$$\sum_{j=1}^{n}\sum_{i=1}^{n} d_{i,j}y_{i,j}, \ i,j \in \{1..n\}$$

subject to the following constraints:

1. $\sum_{i=1}^{n} y_{i,j} = 1, \ j \in \{1..n\}$
2. $0 \le x_i - y_{i,j} \le 1, \ i,j \in \{1..n\}$
3. $x_i, y_{i,j} \in \{0,1\}, \ i,j \in \{1..n\}$
4. $\sum_{i=1}^{n} x_i c_i \le c_{tot}, \ i \in \{1..n\}$
5. $0 \le \sum_{i=1}^{n} x_i \le n, \ i \in \{1..n\}$

The first constraint guarantees that exactly one "nearest" representative is assigned to
each of full set of strains. The second guarantees that $y_{i,j}$ is never 1 when x_i is not also 1:
this in turn means that only distances from a representative to its assigned strains will be
summed in the objective function. The third guarantees that x and y are binary, the fourth
gives the cost constraint, and the fifth allows for any number of strains k ($k = \sum x_i$) to be
selected. Strains that must be selected (for example, strains that are already part of a
researcher's collection) can also be included as additional constraints on the value of a
particular x_i.

The method was implemented in R, using the "highs" package[60] which provides bindings to
the open-source HiGHS solver[61].

We did not evaluate an ILP version of the MaxSum (or p-dispersion-sum) problem because
even in tests on very small datasets, its performance was extremely slow. We also did not
attempt to frame MaxMAF as an ILP problem because it uses the entire allele frequency
matrix instead of a distance matrix, and would therefore require orders of magnitude more
variables than the other ILP methods tested here.

Fitting and Significance Testing

We fit generalized additive models (GAMs) to the power estimates, assuming the data were
beta-distributed, using the restricted maximum-likelihood estimator the R package
"mgcv"[62]. Because all methods eventually converge given a high enough budget, for the

purpose of the model fit, we kept only budget values where the average power across every
method was less than 90% of the maximum power observed. To reduce the impact of exact
zeros at the low end, which distort the fit, we also slightly moderated power values by
adding 0.005 and divided by 1.005. Power was then modeled as a per-method smooth
function of the budget, using k=5 knots. After fitting GAMs, the estimated marginal means
over these budget ranges were calculated using the "emmeans" R package. Using the
"multcomp"[63] package, significant pairwise differences at p=0.05 were then assessed
using the Tukey adjustment, from which compact letter displays were finally generated.

Validation Approaches

HMDP: The Right Ventricular weight phenotype after isoproterenol stimulation was
selected as it had the largest number of genome-wide significant loci from its manuscript
(Figure 3A)[18]. The mice used in the selected study did not comprise the entire panel, but
instead consisted of 93 strains, which we calculated to be a total cost of $119,241. Costs
were set at $20,000 intervals from $0 to $100,000 and all selection algorithms were run ten
times per approach. Included SNPs were recalculated for each sample due to changes in
MAF across the population due to removal of strains. Phenotypes were then subset and
GWAS run through fastLMM[57] followed by analysis of how many significant SNPs were
maintained after subsetting as well as how many SNPs which had been present in the
original panel were lost due to shifts in MAF.

RESULTS

Bacterial Phylogenetic Regression Studies

We considered two scenarios for our test of strain selection methods on a bacterial
dataset. In the first of these, we used the *Lachnospiraceae* phylogenetic tree and
genotypes that we inferred from the genomes, with genotype represented as presence or
absence of "phylogroups" (sets of orthologs that had identical phylogenetic profiles). We
then simulated phenotypes correlated with a randomly selected phylogroup, such that a
phylogenetic regression on the full panel would have approximately 80% power to detect
an effect at a nominal p-value ≤ 0.05 (Figure 2).

Selecting strains randomly, as expected (Random), fared poorly in terms of power. More
interestingly, though, despite the amount of prior literature focusing on these methods, we
found that the greedy MaxMin method that did not consider costs was barely better than
random guessing, and the heuristic for the p-Median problem without costs was actually
even worse. We believe this poor performance can be explained by the fact that costs in
our panel varied over more than an order of magnitude (from $37.33 to $474.00), and so

methods that did not consider costs explicitly tended to select far fewer strains than their
cost-aware counterparts (Table 1).

Even the cost-aware versions of MaxMin or p-Median, however, were not the strongest
performers overall. In fact, even the non-cost-aware MaxSum method, which maximizes
the total sum of pairwise distances, and the MaxMAF method, which maximizes the total
sum of minor allele frequencies, had performance that approached the cost-weighted
version of MaxMin and p-Median. The cost-aware versions of MaxMAF and MaxSum
preserved even more power, and indeed, were essentially tied for the highest performance
overall. This difference was especially pronounced at lower budgets (e.g., $500-$1,000).

We were surprised to see that one of the simplest methods — picking strains in order of
cost, without considering diversity at all (MinCost) — was the third-ranked, behind
MaxMAFMyopic and MaxSumMyopic, and was not significantly worse than these across the
range of budgets. MinCost even beat the globally-optimized p-MedianOptim method in
these simulations (though not significantly). As above, a possible explanation is that
MinCost always maximizes the number of strains selected (Table 1), which, all else being
equal, should always increase power.

We reasoned that we might expect to see MinCost perform worse in settings where the
panel contained clusters of inexpensive but closely related strains, as these would make
only marginal contributions to the diversity metrics we consider. In some collections, for
example, certain individual species with high value in the wet-lab or industry, such as *E.*
coli or *B. subtilis*, have been "oversampled" relative to more distant phylogenetic
neighbors. We would not have expected to observe this phenomenon in the
Lachnospiraceae data set we considered above because this set does not contain
common model or industrial microbes (at least, not yet). To test this hypothesis, we
therefore simulated the addition of another collection that had a large number of
inexpensive, closely-related strains, which we refer to as "oversampled" strains below.

In this "oversampled" scenario, we indeed saw a much larger advantage for the methods
that consider both cost and diversity, and also saw more differentiation among these
methods. Strikingly, MaxMAFMyopic still performed the best overall, significantly
outperforming MinCost across all budgets below the very highest (where all methods
performed well). In this setting, the p-MedianOptim method was second-best overall.
Because p-MedianOptim minimizes the distance of non-selected to selected strains, it
should pick strains that are evenly distributed in terms of their genetic relatedness (see
Figure 1), which may explain why it is the most robust to oversampling a particular taxon.
The next best methods were, in order, MaxSumMyopic and MaxMinMyopic. All four of these
methods significantly outperformed MinCost across the range of budgets, again matching

the intuition that prioritizing diversity becomes more important when a low-diversity clade
is oversampled.

Unexpectedly, MaxSumNoCost and MaxMinNoCost were significantly worse than random
guessing. Because methods that considered diversity in this experiment tended to perform
so well, this was initially surprising. However, in this experiment, the highly-similar strains
we simulated were also very low cost ($20.00). This gives the Random approach an
advantage, as it would tend to run out of its budget slower than the other NoCost methods,
which are required to maximize diversity. Indeed, MaxMinNoCost and MaxSumNoCost
picked the fewest total genomes out of any method (Table 2). We found there was no
consistent relationship between performance and the number of synthetic "oversampled"
strains that were selected (Table 3). MaxMinNoCost and MaxMAFNoCost picked the least
but performed worst, while at low budgets (≤$5000), p-MedianOptim selected more than
Random and MaxMinMyopic.

Overall, failing to consider costs appears to harm performance more than failing to
consider diversity. In the presence of strongly unequal costs, maximizing diversity alone
can actually be worse than random guessing. However, the best-performing methods
consider both. The cost-aware MaxSum and MaxMAF methods were the most powerful
overall, and were joined by the cost-aware p-Median method when the panel was
expanded to include many closely-related strains.

Mouse Genome Wide Association Studies

We next evaluated these methods using simulations conducted in a substantially different
context: genome-wide association studies (GWAS) on subsets of the Hybrid Mouse
Diversity Panel (HMDP), which consists of approximately 175 strains of mice. At our
selected effect size (χ^2) and minor allele frequency (MAF) thresholds, we observe
approximately 50% power to detect a true positive result at ~60 strains at a χ^2 of 100 and a
MAF of 10-30%, or 100-110 strains at a χ^2 of 50 at various MAF thresholds (Fig S1A). At
χ^2=100, there is 100% power to detect phenotypes in the whole panel, while at χ^2=50, there
is 82-93% power to detect a phenotype.

With this understanding of how random strain selection is likely to affect GWAS results in
our panel, we set out to examine the behavior of each of selection approaches at price
limits ranging from $20k to $240k (approximately the cost of the entire panel). We observe
some patterns which are consistent across all χ^2 and MAF parameters (Fig. 4). The first is a
very rapid jump in power in some models at low cost thresholds (see, for example, the jump
from 10 to 50% power in Figure 4D from $20k to $40k). This reflects the specific cost
structure of these strains: there are a large number of relatively inexpensive mice that can

all be obtained at $40k total budget, while many of strains found in the Recombinant Inbred
panels are significantly more expensive.

Several results line up with our observations on the bacterial lines. The Random,
MaxSumNoCost, and MaxMinNoCost algorithms all performed significantly worse (Tukey-
adjusted $p<0.05$) than their cost-aware Myopic counterparts. Typically, MinCost also
remained in a statistical dead heat for the top rank.

However, some methods showed reversals from our bacterial simulations. We observed
that MaxMinNoCost, instead of p-MedianNoCost, was either the worst or second-worst
performer at all cost/χ^2/MAF combinations, and in fact, p-MedianNoCost was actually tied
for the top place. Most surprisingly, MaxMAFMyopic, previously in the top two methods,
performed worse than average. In fact, MaxMAFMyopic was often beaten significantly not
only by MaxSumNoCost, but by its own cost-unaware counterpart, MaxMAFNoCost (Fig. 4).

We sought to understand these differences in performance by first testing the impact of
two parameters that might differ across settings: first, effect size (χ^2), and second, the
minor allele frequencies (MAFs) of the causal SNPs (Fig. 4A-C). We found that effect size
appeared to mainly determine how much power we retain overall by subsampling, without
large method-specific differences (compare Fig. 4A and 4C). As expected, for very strong
signals ($\chi^2=100$; Fig. 4A), power rose faster with the available budget, with the best
methods reaching 90% power at a $100K budget, compared to less than 50% for weaker
signals ($\chi^2=50$; Fig. 4C). However, in both cases, methods clustered into approximately the
same three groups: the best were MaxSumMyopic, p-MedianOptim, p-MedianNoCost, and
MinCost; a second tranche included both versions of MaxMAF, MaxMinMyopic and
MaxSumMyopic; and the worst were Random and MaxMinNoCost.

In contrast, certain methods did show MAF-dependent behavior. We assessed this by
sampling three different MAF cutoffs at an effect size of $\chi^2 = 50$: 5-10%, 10-30%, and 40-
50% (Fig. 4B-D). We hypothesized that for low MAFs, methods that prioritize rare variants
should maximize power. This is because low MAFs mean that the causal SNPs are rare, and
because χ^2 was held constant, it then follows that individual rare SNPs must have larger
effects on the phenotype.

Indeed, we observed that MaxMAFNoCost, which would favor adding strains with rare
variants, was the method affected most dramatically by allele frequency. When causal
SNPs already have high MAFs (40-50%), prioritizing minor allele recovery should be
unnecessary, as many strains contain the relevant mutant allele; and, because it does not
consider costs, MaxMAFNoCost will lose power because it also selects fewer strains than
the cost aware models (Fig. S1B). Sure enough, at high MAFs, MaxMAFNoCost performs

poorly, at times worse than random selection (Fig. 4D). At moderate MAF (10-30%),
MaxMAFNoCost and MaxMAFMyopic perform similarly and in the middle of the set of
models in terms of power recovery, comfortably above Random, but significantly below
MinCost (Fig. 4C). At low MAFs (5-10%), however, MaxMAFNoCost actually becomes the
best model for budgets between $100k and $180k, at times recovering 10% more power
than the second best model (Fig. 4B). This is likely because rare SNPs carry most of the
signal in the data, and so while MaxMAFNoCost picks fewer strains (a bigger problem with
very low budgets), it also tends to pick the 'right' strains for recovering the signal.

While we have previously only considered the cost of acquiring the strains, in most real-
world scenarios, downstream phenotyping experiments require additional costs that scale
per animal or strain, placing further economic constraints on how many strains it is
feasible to purchase. To simulate these additional costs (which could include RNAseq,
histology, experimental manipulation, etc.), we added $2,500 to the cost of each mouse
strain. This also smooths out the very large cost differences seen in the mouse data, such
that now the most expensive strain would only be 2x that of the least expensive strain,
rather than >40x. Indeed, we see smaller differences in the number of mouse strains
chosen between the models (Fig S1C).

With additional costs, MaxMAFNoCost's advantage on low MAF SNPs widened even more
across the entire budget range (Fig. 5A), at times beating the next best model by 25%.
Furthermore, it became the top-ranked method at moderate MAFs as well (Fig. 5B). As we
discussed above, MaxMAFNoCost is more likely to pick more 'useful' strains, but also
fewer, leading to a power tradeoff. When strain costs become more similar, this second
factor should become less important, which explains the improvement in power across
MAF thresholds. Crucially, however, MaxMAFNoCost still performed poorly when MAFs
were between 40-50% (Fig. 5C). Thus, maximizing MAF without regard to cost is a viable
strategy when causal alleles have low frequency, and especially as cost differentials
decrease, but not when MAFs are already close to parity.

With less differentiation by cost, the other competing methods became more similar to one
another (Fig. 5A-B), except for MaxMinNoCost, which remained weaker than random
guessing. We also did not observe the same early jump in power we saw previously, which
was expected as there was less of a cost differential between the cheapest and most
expensive strains. Interestingly, MaxMAFMyopic's performance became more variable,
especially at low MAF, but still lagged behind the others. MinCost, in contrast, remained
statistically tied for the best or second-best method across these simulations.

We wished to further explore the counterintuitive result that while optimizing for either cost
(MinCost) or total MAF (MaxMAFNoCost) separately was effective for picking mouse

strains, optimizing an objective that included both factors was not, even though this was
the best approach for bacteria. We entertained two explanations. First, there could be a
bias where more expensive mouse strains contain rarer alleles, meaning that the strains
that maximize MAF were intrinsically high-cost. Second, it could be that the greedy
MaxMAFMyopic algorithm simply did not get close enough to a global optimum on the
mouse data. This failure could be because the genetic architecture of the two groups
differs: the mouse genomes include recombinant inbred lines, while the bacteria are
natural isolates from many different species.

We tested the first of these hypotheses by randomly distributing costs (pulling from a
uniform distribution from $60 to $1000 per strain) and re-running the simulations. When we
did so, we still observed that MaxMAFNoCost preserves more power than MaxMAFMyopic
(Tukey-corrected $p<0.05$, Figure S4), indicating that a correlation between rare alleles and
expensive strains does not drive this pattern. To test the second explanation, we took a set
of bacterial genomes and generated simulated "recombinant" strains, whose genotype at
each locus was randomly sampled from the "parental" strains, and then repeated the
analysis. We found that in this experiment, all of the cost-aware approaches performed
similarly, outperforming the non-cost-aware methods, with the sole exception of
MaxMAFMyopic (Figure S5). Together, these two results favor the explanation that a simple
myopic method may not effectively co-optimize MAF and cost when the full panel contains
a large number of recombinant lines, which largely contain different permutations of the
same variants, as opposed to isolates, which have more unique variants per strain.

GWAS Validation in the HMDP

Finally, we applied our selection criteria to real data pulled from a prior HMDP study on
cardiac dysfunction[18]. In particular, we examined the effects of strain selection on GWAS
loci for right ventricle weight after catecholamine challenge, which had the largest number
of genome-wide significant SNPs in the original study (Fig 3A).

To be consistent with the original study, SNPs with a MAF of less than 5% were excluded;
because strain selection affects this calculation, we first examined the effects of removing
strains on the number of SNPs tested. As expected from our simulations, MaxMinNoCost
and Random resulted in the largest number of SNPs being lost, over 10% at $20K and 5% at
$60K. MaxMAFNoCost predictably lost the fewest number of SNPs, <3% at $20K and only
0.3% at 60K (Fig 6B). MinCost, despite not considering genetic diversity at all, did
reasonably well, with 4.3% SNP loss at 20K and 1.8% loss at 60K.

We next queried how many genome-wide significant SNPs were identified for each method
at each cutoff when compared to the full panel (Fig 6C). Strikingly, at $80K and $100K

cutoffs, MaxSumMyopic actually identified more genome-wide significant loci than the full panel, joined by MaxMinMyopic at the highest threshold. Random selection resulted in very high variance, with some permutations returning as many as 433 significant SNPs (compared to 157 found in the real data), while others at the same $60K threshold returned as few as 27. Consistent with our simulations, MaxMinNoCost recovered very few significant SNPs, returning zero significant hits at every budget other than the highest. We were surprised that the two MaxMAF algorithms also performed poorly; however, when we looked at the significant SNPs from the real panel, we found that many of the significant SNPs had MAFs >20%, a setting where MaxMAF performed less well in simulations (Fig. 4C-D).

Finally, we tested agreement with the results from the real panel. We first asked what percentage of the significant SNPs from the real panel were recovered in our sub-panels (Fig 6D); assuming that the SNPs from the real panel are valid, this is a measure of recall. Here, p-MedianOptim and MinCost performed the best, recovering about 37% of all SNPs at $60K and, in the case of p-MedianOptim, 75% at 80K. Notably, despite returning the most overall SNPs, with comparable numbers to the full panel starting at $40K and exceeding it at $80K, MaxSumMyopic did not recover a majority of the SNPs from the real panel until reaching $100K, suggesting an elevated false positive rate.

We next calculated the percentage of SNPs recovered by the sub-panels that were also found in the real panel (Figure 6E), a measure of precision. Indeed, MaxSumMyopic had fairly low precision compared to other methods. Conversely, MaxMAFNoCost, despite returning very few significant SNPs, was very faithful to the full panel, with 100% of its recovered SNPs being reproduced (other than at $100k, where it drops to 98%). Again, the best methods were p-MedianOptim, followed by MinCost, with rates between 45-80% starting at $40K onward. Thus, p-MedianOptim and MinCost optimized both precision and recall the best, while the selections made by MaxMAFNoCost strongly favored precision.

DISCUSSION

In this study, we set out to address a common problem faced by researchers: how best to select strains from an available panel in order to stay within a fixed budget while retaining the power needed to make reliable discoveries. Because such comparative approaches are used in many different fields of study within biology, we simulated experiments on phylogenetic regression in microbes and on genome wide association studies in mice, followed by subsampling of real mouse GWAS data as a validation.

We find that in both of these settings, and under most conditions we tested, explicitly accounting for cost mattered as much as accounting for diversity. In realistically-priced

strain collections, methods that ignored costs selected fewer, if potentially more diverse,

strains, driving down power. Conversely, simply maximizing sample size, by selecting

strains from cheapest to most expensive until the budget is exhausted (MinCost), was

surprisingly competitive compared to models that combined both cost and diversity

considerations. Typically, the highest-ranked methods considered both cost and diversity,

though which specific diversity-maximizing objective performed best depended on the

setting, and in many cases, MinCost statistically tied these methods. Surprisingly, even

though methods related to MaxMin have been explored much more in the genetics

literature, we found that MaxMin performed poorly, and that the best diversity objectives

were usually MaxSum or MaxMAF, and in some cases p-Median. We discuss our results in

more detail below, along with a few notable exceptions to these trends.

Why does MinCost perform better than expected?

We did not expect MinCost to outperform many methods that explicitly maximize diversity.

However, while MinCost itself does not consider genetic distance, this does not mean

diversity is irrelevant. Researchers who contribute to collections are likely motivated, at

least in part, by considerations like expanding the diversity of cultured microbes to which

we have access, or in the case of mice, breeding lines that aim to randomly shuffle

genotypes through recombination. In the context of a panel that is already diverse, "buy as

many as you can afford" becomes a viable strategy because it maximizes sample size, and

therefore the chance to sample informative genotype/phenotype pairings. Indeed, when

we changed this picture by introducing a large number of cheap, closely related bacterial

strains, we saw that MinCost's efficacy began to falter, and considering genetic diversity

became increasingly important. While methods like p-MedianOptim, and, to a lesser

extent, MaxMinMyopic were tied with or worse than MinCost on the original data, in an

oversampled panel they outperformed MinCost at many budget thresholds.

Why are MaxMin methods outperformed by alternatives?

We found that MaxSum and MaxMAF, and to a lesser extent p-Median, commonly

outperformed the more commonly-studied MaxMin method. One reason for MaxSum's

superior performance may be that MaxMin penalizes potential additions for being closely-

related to even one selected strain. In contrast, the MaxSum method maximizes the sum of

all pairwise distances. This means that being closely-related to one strain can be balanced

out by being very diverged from others in the set. Thus, MaxSum may be less likely to reject

potentially informative strains that would be deprioritized by MaxMin.

Intriguingly, MaxMAF showed even stronger performance across simulations than MaxSum.

We explain this by noting that the power of a regression analysis is typically higher when the

independent variable has a larger sample variance, and binomial distributions have the
highest variance at a mean of 0.5. Because MaxMAF pushes the average MAF towards 0.5,
it is therefore selecting for larger sample variances, which MaxMin does not consider.

Allele frequency has, in some ways, a more direct connection to genome-wide association
studies than measures of diversity based on genetic distances, as GWAS relies on the
recovery of information from individual informative SNPs or CNVs. In contrast, for example,
bioconservation studies often aim to maximize the total expected functional diversity in a
set of taxa, which may be more closely related to the overall genetic distance between
them. When causal allele frequencies are high, these two approaches may give similar
answers, but at low frequencies, ensuring more rare alleles have sufficient variation may
have a greater impact than maximizing genetic distances among selected taxa.

How does genetic architecture affect strain selection?

The two settings we tested represent different types of genetic diversity: cross-species
variation in bacterial gene family content, and within-species variation in mouse genetic
polymorphisms. We expect a high degree of nested structure in the bacterial genotypes
because many genes should be correlated to phylogeny and some may in fact be clade-
specific. In contrast, the mouse data includes many recombinant inbred lines (RILs). Unlike
the bacterial isolates, these were bred specifically with the aim of decorrelating genetic
variants from one another and thus facilitating trait mapping. In our dataset, bacterial
genes also tended to have much lower MAF on average than mouse SNPs. Comparing
these two settings can therefore give us a window into how the genetic architecture of
strains and/or traits can affect the performance of strain selection methods.

Certain trends were consistent: *most* methods that ignored prices (Random,
MaxMinNoCost, MaxSumNoCost, p-MedianNoCost) underperformed their cost-aware
counterparts in both bacteria and mice. MaxMAF was an exception, with the NoCost
version typically performing better than Myopic; as noted above, however, the Myopic
version also performed poorly when we simulate recombinant bacterial progeny from a
small set of parental strains, suggesting that this reversal is actually a failure of
optimization.

While changing the effect size did not change the relative rankings of most approaches, we
did observe an effect of causal allele frequency, especially for MaxMAFNoCost and also for
MaxMAFMyopic. MaxMAFNoCost's single-minded focus on driving allele frequencies
towards 50% leads to the lowest number of selected strains of any approach (Figure S1B),
which can harm power when the causal allele is high MAF. However, as noted above,
because MaxMAF retains more rare alleles, it becomes one of the best algorithms when

traits are driven by rare SNPs, besting all others in the $160K-$240K budget window (Fig.

5A). This also partially explains why the MaxMAF methods performed so well in bacteria,

where the distribution of MAFs is shifted lower (compare, e.g., Fig. S2A, B, and D).

How does cost structure affect selection?

Strain prices can vary over orders of magnitude for both bacteria and mice. For example, a

breeding trio for the very popular C57BL/6J strain costs less than $66 at time of writing,

versus up to $2,850 for a litter of one of many cryopreserved lines[48], a 47-fold difference.

However, the expenses associated with downstream phenotyping experiments are typically

large and constant across strains, partially equalizing the total cost.

We therefore further investigated the impact of adding additional costs to the mouse

strains. This compressed a 47-fold difference between the least- and most-expensive

strains to just 2.1-fold. One impact was on the shape of the power curve: in the original

simulations, we saw a pronounced jump in power for the cost-aware methods between

$20K and $40K (Fig. 4A, 4C), as these methods could take advantage of a large number of

low-cost mouse strains (Figure S1B). After adding additional costs, though, the cost-aware

versions no longer showed this early jump (Figure S1C), and the power increase was

instead diffused across $40K-$280K (Figure 5). The most striking impact, however, was

again seen with MaxMAFNoCost, which widened its lead at low MAF and became the top-

ranked method at medium MAF (10-30%). This is likely because one of the main

disadvantages of MaxMAFNoCost is that it tends to select the fewest strains; with less

differentiation in strain costs, this disadvantage is attenuated relative to other methods

(Fig. S1B-C).

Can these methods recapitulate the results of a real GWAS study?

Applying our selection algorithms to a real HMDP GWAS (for right ventricular weight after

catecholamine challenge, Figure 6A) on 104 strains of the HMDP confirmed many of our

simulation results. Most methods which ignored costs (Random, MaxMinNoCost) lost

many SNPs from the analysis set due to shifting MAFs dropping below the minimum 5%

threshold used in the original study (Figure 6B), especially at low budgets. Again, the

exception was MaxMAFNoCost, which actually preserved the most SNPs, losing less than

1% from our second lowest budget threshold onwards (compared to over 10% for

MaxMinNoCost at this threshold).

In terms of total significant hits, p-MedianNoCost, MaxMinMyopic, and especially

MaxSumMyopic found more significant SNPs than the original study at certain budget

thresholds (Figure 6C). MaxMinNoCost, predictably, did the least well, followed by the

maxMAF models, which, as before, returned the fewest number of strains (Figure S1D).

Despite finding more significant SNPs than the full panel, however, MaxSumMyopic did *not*
recover a majority of the full panel's signals until the largest budgets, with p-MedianOptim
and minCost leading this metric at every threshold (Figure 6D). This divergence — that is,
finding many new significant SNPs that did not overlap with those found in the full dataset
— hints at the possibility of inflated false-positive rates when MaxSumMyopic is used.
Alternatively, selection via MaxSumMyopic may introduce a bias in population structure,
causing it to no longer reflect the kinship relationships in the full panel.

Finally, MaxMAFNoCost returned very few significant hits compared to the other
approaches, but with *very* high overlap with the full panel's results: 100% of its hits were
found in the original study up until the largest budget, where a single novel SNP crept in. Its
marginal performance was initially unexpected. However, a more careful analysis of the
significant SNPs suggested an explanation: many had MAFs of 20% or more, a regime
where our simulations showed weaker performance for MaxMAFNoCost. Additionally,
many SNPs had even smaller effect sizes than we had simulated, which when spread over
relatively high-frequency alleles, likely further limited the efficacy of this approach.

Future directions

We primarily considered costs by using a version of Weizman's "myopic rule," a greedy
search that penalizes a strain's genetic distinctiveness by its cost. Greedy searches often
give reasonable solutions, but are not guaranteed to reach a global optimum for every
objective. We implemented global, cost-aware optimizers using mixed-integer linear
programming (MILP) for p-Median, MaxMin, and MaxSum, but only the p-Median version
proved to be computationally tractable on our panel sizes. While these optimization
problems are NP-hard, better heuristics and more efficient search strategies have made
similar problems much more tractable[64]. It should therefore be possible to develop more
efficient algorithms for balancing cost with different diversity metrics. We expect these
efforts to be especially worthwhile in the case of MaxMAF, as our simulations indicate that
MaxMAFMyopic's reduced performance on mouse data can be at least partly attributed to
a failure of the simple greedy search to reach a global optimum.

In the current study, we modeled the effects of selecting subsets of strains from a larger
cohort based purely on cost and/or genetic diversity. Other considerations, however, can
also play a role in strain selection. For example, there is a wealth of previously generated
data on a number of these strains available through websites such as the Mouse Phenome
Database[65] or BacDive[49] that may guide researchers to select certain strains with divergent
phenotypes of interest as a 'seed' around which to grow a larger panel[65]. This approach is
compatible with the selection algorithms we described in this study, which can all

accommodate a specific set of starting strains, and so integrating prior knowledge about
phenotypic extremes may be a productive future direction for preserving power.

Future work could focus on better understanding the tradeoffs between cost, diversity and
minor allele coverage through multi-objective optimization[66], the incorporation of kinship-
aware penalties to limit potential false positives in sub-panels, and stability analyses
across budgets and phenotypes to identify optimal cohorts at any given budget point.
While some power loss is unavoidable, our results do highlight that considering both cost
and diversity may help mitigate this loss, allowing researchers to preserve more funding for
downstream analyses or personnel costs. In particular, our results identify the cost-aware
versions of MaxSum, MaxMAF, and p-Median as especially promising approaches that
merit further study. At the same time, when the full panel is already diverse, our results
indicate that simply maximizing the number of purchased strains can also be a reasonable
strategy.

Data and Code Availability

Mouse Diversity Array genotypes are available from the Mouse Genome Informatics
website (https://www.informatics.jax.org/).

HMDP Phenotype data is available through Mendeley Resources at
https://data.mendeley.com/datasets/y8tdm4s7nh/1.

All code is available at https://github.com/pbradleylab/genome_annotation_matrix.

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Figure 1: Illustration of methods considered for strain picking. Each method is illustrated with an example where selected strains are represented by black circles and unselected strains by gray circles, with cost being represented by the circle size. Methods in the same column either do not (left) or do (right) consider the cost of strains, while methods in the same row either do not consider diversity (top, Random and MinCost) or consider it in the same way (other rows). The p-Median, MaxMin, and MaxSum methods all optimize a function of distances between strains (gray lines connecting circles). The p-Median methods (purple) minimize the total distance between unselected strains and the nearest selected strain. The MaxMin methods (blue) maximize the minimum distance between selected strains, while the MaxSum methods (red) instead maximize the total distance. Instead of distances, the MaxMAF methods maximize the sum of minor allele frequencies, shown by the inset matrices. "Myopic" indicates a greedy search that prioritizes strains by diversity divided by cost, while "Optim" indicates a method that directly optimizes the objective using integer linear programming (ILP). The "NoCost" methods also use greedy search, except for "p-medianNoCost" which runs the Partitioning Around k-Medoids algorithm with successively greater k.

Figure 2. Power of phylogenetic regressions on traits simulated across 165 commercially available *Lachnospiraceae* isolate genomes. Methods are colored by how they consider genetic diversity (none: gray; p-Median: purple; MaxMin: blue; MaxSum: red; MaxMAF: green), with lighter colors corresponding to methods that consider cost (MinCost and the Myopic and Optim methods). Error bars are ±1 SD of the mean of five rounds of picking genomes. Lowercase letters (a-d) indicate significantly different groups; methods are significantly different at p=0.05 if they do not share a letter (GAM beta regression, Tukey-adjusted post-hoc pairwise tests; see Methods).

Figure 3: Power of phylogenetic regressions on traits simulated across 165 commercially available *Lachnospiraceae* isolate genomes, augmented by 100 highly similar synthetic "oversampled" strains. Methods are colored as in Figure 2. Error bars are ±1 SD of the mean of five rounds of picking genomes. Lowercase letters (a-d) indicate significantly different groups; methods are significantly different at p=0.05 if they do not share a letter (GAM beta regression, Tukey-adjusted post-hoc pairwise tests; see Methods).

Figure 4: Mouse GWAS Power Simulations Without Additional Costs Per Strain.
Results of simulation studies at $20k intervals from 20-240k. **A)** χ^2 of 100 and MAF of 10-30%, **B)** χ^2 of 50 and MAF of 5-10%, **C)** χ^2 of 50 and MAF of 10-30%, **D)** χ^2 of 50 and MAF of 40-50%. N=100 simulated phenotypes for each of 5 replications. Values are slightly jittered on the x axis to allow for observation of each method's error. Error bars are ±1 SD. Lowercase letters (a-f) indicate significantly different groups; methods are significantly different within a panel at $p=0.05$ if they do not share a letter (GAM beta regression, Tukey-adjusted post-hoc pairwise tests; see Methods).

Figure 5: Mouse GWAS Power Simulations With Additional Cost Per Strain. Results of simulation studies after $2,500 was added to the cost of each strain at $40k intervals from 40-600k. **A)** χ^2 of 50 and MAF of 5-10%, **B)** χ^2 of 50 and MAF of 10-30%, **C)** χ^2 of 50 and MAF of 40-50%. N=100 simulated phenotypes for each of 5 replications. Values are slightly jittered on the x axis to allow for observation of each method's error. Lowercase letters (a-f) indicate significantly different groups; methods are significantly different within a panel at p=0.05 if they do not share a letter (GAM beta regression, Tukey-adjusted post-hoc pairwise tests; see Methods).

Figure 6: Mouse GWAS on Actual Data. Results of subsetting studies (N=10 permutations each) for real data pulled from the HMDP Heart Failure Study[12] at $20k cost intervals from 20-100k. **A)** Manhattan plot of the results of the whole panel, red line represents the genome-wide significance threshold of a FWER of 5%[29] **B)** Percentage of SNPs untested in the subsets because of changing MAFs. **C)** Number of significant SNPs found by each approach, the dotted red line indicates the number of significant SNPs found in the whole panel. **D)** Percentage of preserved genome-wide significant SNPs after subsetting between the subset and the full panel. **E)** The percentage of found significant SNPs which are also significant in the whole panel GWAS. Values are slightly jittered on the x axis to allow for observation of each method's error.

Table 1: Median number of *Lachnospiraceae* strains selected and range (in brackets), by method and budget ("Real" simulation).

Budget	500	1000	2000	3000	5000	7500
Random	4.6 [1-7]	10.4 [6-14]	22 [15-28]	29.2 [23-36]	54.8 [40-62]	74.8 [64-86]
p-MedianNoCost	5 [5-5]	6 [6-6]	20 [20-20]	34 [34-34]	54 [54-54]	84 [84-84]
MaxMinNoCost	8.2 [7-9]	14 [14-14]	27.6 [26-28]	36.2 [33-40]	64.8 [60-68]	90.2 [89-92]
MaxSumNoCost	9 [9-9]	14.4 [14-16]	26.4 [26-28]	39.2 [38-40]	53.8 [53-55]	94 [94-94]
MaxMAFNoCost	9 [9-9]	14 [14-14]	26 [26-26]	40 [40-40]	61 [61-61]	93 [93-93]
p-MedianOptim	13 [13-13]	26 [26-26]	51 [51-51]	78 [78-78]	97 [97-97]	119 [119-119]
MaxMinMyopic	13 [13-13]	26 [26-26]	53 [53-53]	79 [79-79]	97 [97-97]	119 [119-119]
MaxSumMyopic	13 [13-13]	26 [26-26]	53 [53-53]	79 [79-79]	97 [97-97]	119 [119-119]
MaxMAFMyopic	13 [13-13]	26 [26-26]	53 [53-53]	79 [79-79]	98 [98-98]	119 [119-119]
MinCost	13 [13-13]	26 [26-26]	53 [53-53]	79 [79-79]	98 [98-98]	119 [119-119]

Table 2: Median number of total strains selected and range (in brackets), by method and budget ("oversampled" simulation).

Budget	500	1000	2000	3000	5000	7500
Random	8.8 [5-11]	12.4 [10-18]	27.2 [24-36]	38.4 [28-47]	74 [67-82]	104.8 [85-121]
p-MedianNoCost	8 [8-8]	15 [15-15]	31 [31-31]	41 [41-41]	59 [59-59]	74 [74-74]
MaxMinNoCost	8.6 [7-9]	14 [14-14]	22.8 [21-24]	28 [28-28]	48.4 [47-52]	65.2 [64-66]
MaxSumNoCost	7 [7-7]	12.4 [12-14]	19 [19-19]	30 [28-32]	44.6 [43-45]	74 [74-74]
MaxMAFNoCost	7 [7-7]	12 [12-12]	26 [26-26]	38 [38-38]	63 [63-63]	93 [93-93]
p-MedianOptim	16 [16-16]	30.4 [30-31]	58 [58-58]	85 [85-85]	117 [117-117]	141 [141-141]
MaxMinOptim	14 [14-14]	27 [27-27]	54 [54-54]	78 [78-78]	97 [97-97]	119 [119-119]
MaxSumMyopic	17 [17-17]	34 [34-34]	68 [68-68]	100 [100-100]	165 [165-165]	198 [198-198]
MaxMAFMyopic	14 [14-14]	27 [27-27]	54 [54-54]	81 [81-81]	102 [102-102]	121 [121-121]
MinCost	20 [20-20]	40 [40-40]	80 [80-80]	113 [113-113]	166 [166-166]	198 [198-198]

Table 3: Median number of the additional oversampled strains that were selected and range (in brackets), by method and budget ("oversampled" simulation).

Budget	500	1000	2000	3000	5000	7500
Random	4.6 [2-7]	5.4 [3-9]	11.4 [8-13]	13.2 [8-17]	27.2 [23-31]	39.2 [28-49]
p-MedianNoCost	5 [5-5]	5 [5-5]	7 [7-7]	8 [8-8]	10 [10-10]	10 [10-10]
MaxMinNoCost	1.2 [1-2]	1.6 [1-2]	1.4 [1-2]	2 [2-2]	1.8 [1-2]	2 [2-2]
MaxSumNoCost	1.2 [1-2]	1.4 [1-2]	2 [2-2]	2.2 [2-3]	1.8 [1-2]	2 [2-2]
MaxMAFNoCost	2 [2-2]	2 [2-2]	4 [4-4]	5 [5-5]	10 [10-10]	16 [16-16]
p-MedianOptim	8 [8-8]	11.2 [10-13]	14 [14-14]	15 [15-15]	27 [27-27]	29 [29-29]
MaxMinMyopic	2 [2-2]	2 [2-2]	2 [2-2]	8 [8-8]	21 [21-21]	20 [20-20]
MaxSumMyopic	12 [12-12]	23 [23-23]	44 [44-44]	61 [61-61]	95 [95-95]	100 [100-100]
MaxMAFMyopic	2 [2-2]	1 [1-1]	2 [2-2]	2 [2-2]	5 [5-5]	2 [2-2]
MinCost	20 [20-20]	40 [40-40]	80 [80-80]	100 [100-100]	100 [100-100]	100 [100-100]