Article

Source Reconstruction of Resting-State MEG and EEG Activity: A Technical Note on the Choice of Noise Covariance

Alexander Moiseev [1*], Sam M. Doesburg [1], George Medvedev[2] and Vasily A. Vakorin [1,2]

[1] Department of Biomedical Physiology and Kinesiology, Simon Fraser University, Burnaby, Canada
[2] Royal Columbian Hospital, Fraser Health Authority, New Westminster, Canada
* Correspondence: amoiseev@sfu.ca

Abstract: Minimum variance beamforming is widely used to reconstruct neural sources from MEG and EEG data, but results critically depend on the choice of noise covariance. In task-based studies, this is often defined from pre-stimulus baselines, but for resting-state data the problem presents a fundamental challenge. Conventional solutions, such as empty-room recordings or diagonal white sensor noise, are not optimal. They either ignore brain-generated noise or yield artificial, non-uniform source-level baselines that can distort results. Our proposed approach is to define a baseline at the source level as a uniform distribution of uncorrelated, randomly oriented neural dipoles, representing a maximum-entropy "ground state" of brain activity. Projecting this source model through the electromagnetic lead fields yields a sensor-level covariance that captures realistic spatial correlations. A data-driven constraint scales the model to match measured data, ensuring a physically admissible solution. Applied to real human resting-state data, the method produces a structured, non-uniform sensor covariance dictated by subject's anatomy, source reconstructions that are smooth and plausible, and free from the artificial peaks induced by diagonal noise models. This source-level approach provides a principled and physiologically grounded baseline for beamforming and improves the reliability of resting-state analyses and interpretation.

Keywords: MEG; EEG; resting state; inverse solutions; beamformers

1. Introduction

Neuroimaging techniques such as magnetoencephalography (MEG) and electroencephalography (EEG) provide an unparalleled non-invasive window into the temporal dynamics of human brain function [1,2]. By deploying an array of sensors around the head, these methods capture the weak electromagnetic fields generated by synchronous neural activity. However, the data recorded at the scalp or just outside it are a complex superposition of signals from multiple underlying neural sources. The fundamental challenge is to reconstruct the spatial distribution and temporal evolution of electromagnetic activations within the brain volume from these external measurements. This is known as the bioelectromagnetic inverse problem, a classic ill-posed problem for which no unique solution exists without the imposition of additional constraints and modeling assumptions [1,3].

Among the various techniques developed to provide an approximate solution, the minimum variance beamformer, that was known for quite some time in radar field and acoustics [4], over the last two decades has emerged as a potent and popular tool within the neuroimaging community [5–7]. A beamformer is a linear spatial filter designed to reconstruct the time course of activity from a specific target location in the brain while optimally suppressing signals originating from all other locations. This paper will focus on the scalar formulation of the minimum variance beamformer, which models individual brain sources as current dipoles with fixed positions and orientations. By design, the scalar beamformer solution is optimal in a statistical sense, ensuring the minimal mean squared error of the reconstructed source time course, provided that the precise location and orientation of the source are

known in advance. In nearly all practical applications, this is not the case; therefore, the application of beamforming necessitates a preceding source localization step.

The nature of this localization problem differs significantly between stimulus-driven and resting-state paradigms. In stimulus-based studies, the goal is to identify focal and spatially compact brain activations time-locked to a task. Localization is typically achieved by scanning the source parameter space with a "localizer function". Several such functions have been developed [1,7–11], which, when unbiased, produce maxima corresponding to true source parameters by optimizing a metric such as the signal-to-noise ratio (SNR) of the reconstructed time course. In the simplest situation all activations of interest are treated as independent, which constitutes a so-called "single source" approach [7,8]. In this case the search for extrema boils down to a single scan of a three-dimensional brain volume. In more other scenarios several simultaneous and possibly correlated activations must be considered together. Corresponding parameter space becomes highly dimensional and single-source localizers stop working. In this case more complex multi-source localizers must be applied [1,9–11], and the search for extrema becomes much more difficult. However in both those situations the definition of "signal" (the task-related response) and "noise" (all other activity) is relatively straightforward. The noise covariance matrix, a critical component of the beamformer equations, can be empirically estimated from pre-stimulus or "baseline" intervals where the signal of interest is absent.

In this paper, however, our focus is on resting-state activity, which is the spontaneous, ongoing neural dynamics of the brain in the absence of any explicit task. This presents a profound methodological challenge for beamforming. In the resting state, it is assumed that continuously active sources are distributed across a predefined grid spanning the brain volume or the cortical surface. The localization problem thus shifts from finding the position of a few focal sources to determining the optimal orientation for a vast number of distributed sources to ensure an optimal time course reconstruction for each one. While the same localizer functions can be applied for this purpose, the fundamental distinction between signal and noise collapses. When analyzing resting-state data, there is no "control" or "baseline" interval from which to estimate a noise covariance matrix, although attempts to identify such intervals were made [12]. In general, the entire recording consists of the signal of interest.

This ambiguity lies at the heart of the problem we address. The localizer expressions are derived under the assumption that the measured sensor covariance matrix $\boldsymbol{R}$ can be decomposed into a sum of the source signal covariance and the noise covariance, $\boldsymbol{N}$. Since $\boldsymbol{R}$ is measured directly from the data, the choice of $\boldsymbol{N}$ implicitly defines the signal. Therefore, the selection of a noise model is not a mere technicality but a fundamental modeling decision that directly shapes the resulting inverse solution. Our choice of what constitutes "noise" dictates what will be reconstructed as a "signal." In principle, to reconstruct the activity of a single brain source, the "noise" should encompass all other yet unknown brain sources, in addition to environmental and instrumental noise. This ideal is difficult to achieve, although such attempts have been made [13]. Using simulated or surrogate resting state data as a baseline was also suggested [14]. However in practice simplified, and often flawed, noise models are most commonly used.

The most popular approaches currently employed in resting state studies are twofold, and in our view neither is adequate. The first approach is to use an "empty room" covariance matrix, recorded with no subject present. While this method correctly captures the instrumental and environmental noise characteristics of the M/EEG system, these factors constitute only a minor part of the total interference at the source level. The dominant source of interference for any single brain location is the collective activity of all other brain sources. By using an empty-room covariance, this major contribution to the noise field is completely ignored.

The second common approach is to assume a diagonal white noise model, where $\boldsymbol{N}$ is proportional to an identity matrix (see for example [15–17]). Often this is a built-in default option in popular software toolboxes. At first glance, diagonal white noise might appear to be a reasonable choice, representing a "uniform, uninformed prior" at the sensor level. The assumption is that any deviation from this simple, uncorrelated sensor noise must represent true brain activity. However, this intuition is deeply

misleading. A diagonal covariance matrix at the sensor level does not correspond to a uniform state at the source level. Due to the physics of volume conduction and the complex geometry of the head and sensor array, a diagonal sensor-level noise profile can only be produced by a highly specific, non-uniform, and spatially correlated distribution of underlying neural sources. The exact pattern of this source distribution is determined by the subject's unique head shape, the precise spatial positions of the sensors, and the parameters of the electromagnetic forward model. There is no physiological or theoretical reason to assume that such a highly structured and subject-specific pattern of brain activity should serve as a "control" or baseline condition for resting-state analysis. It is an arbitrary and physiologically implausible null hypothesis.

In this technical note, we argue that in the absence of any *a priori* information, a more natural and principled baseline condition must be defined at the source level, not the sensor level. We propose that the most logical null hypothesis for resting-state brain activity is a uniform distribution of uncorrelated and randomly oriented brain sources. This represents a state of maximum entropy, a "ground state" from which organized, synchronous network activity can emerge as a statistically significant deviation. From this source-level model, the corresponding sensor-level noise covariance matrix can be derived by projecting the electromagnetic fields of these sources to the sensor array. As we demonstrate, the resulting matrix is in general neither diagonal nor resembles the covariance of the white noise. We show that this noise covariance matrix can constructed based on a simple expression and then properly scaled using a data-driven constraint to ensure the mathematical and physical consistency of the beamformer solution. Finally, we demonstrate the application of this approach to a real example of human resting-state MEG data.

2. Methods

We begin by formalizing the relationship between the underlying neural sources and the signals measured by the MEG/EEG sensor array, formulate the scalar minimum variance beamformer solution, and proceed with source localization and a critical role of the noise covariance. We then derive the expression for the proposed noise covariance matrix and explain how the "ground state" source power should be set based on the recorded data.

2.1. The M/EEG Measurement Model and Beamformer Solution

We assume that a recording is performed by an array of m sensors. The data at any given time point t can be represented by an m-dimensional column vector of sensor readings, $\boldsymbol{b}(t)$. This measured field is modeled as a linear superposition of signals generated by n distinct neural sources and an additive noise term, $\boldsymbol{\nu}(t)$. Each source i is characterized by a set of parameters $\boldsymbol{\theta}^i$ and a time-varying amplitude $s_i(t)$. For the case of a current dipole model, these parameters consist of the source's 3D location $\boldsymbol{r}^i$ and its orientation, a unit vector $\boldsymbol{o}^i$, such that $\boldsymbol{\theta}^i = \{\boldsymbol{r}^i, \boldsymbol{o}^i\}$.

By combining the time courses of all n sources into a single n-dimensional column vector $\boldsymbol{s}(t) = \{s_1(t), s_2(t), ..., s_n(t)\}^T$, where superscript T denotes transposition, and corresponding forward solutions into an $m \times n$ lead field matrix $\boldsymbol{H}(\boldsymbol{\Theta}) = \{\boldsymbol{h}(\boldsymbol{\theta}^1), ..., \boldsymbol{h}(\boldsymbol{\theta}^n)\}$, we can express the standard bioelectromagnetic forward model as:

$$\boldsymbol{b}(t) = \boldsymbol{H}(\boldsymbol{\Theta})\boldsymbol{s}(t) + \boldsymbol{\nu}(t) \tag{1}$$

In Eq. (1), $(m \times n)$ matrix $\boldsymbol{H}$ is comprised of m-dimensional column vectors $\boldsymbol{h}^i = \boldsymbol{h}(\boldsymbol{\theta}^i)$ of the source lead fields. We assume that $\boldsymbol{h}^i$ are known, and are static over the duration of the recording. Furthermore we assume that $\langle \boldsymbol{\nu}(t) \rangle$, $\langle \boldsymbol{b}(t) \rangle = 0$, where the angle brackets $\langle . \rangle$ denote statistical averaging, and that the processes $\boldsymbol{s}(t), \boldsymbol{\nu}(t)$ are stationary and uncorrelated with each other. The noise term $\boldsymbol{\nu}(t)$ conceptually encompasses all components of the measured field not produced by the n sources of interest, including instrumental noise and environmental interference.

The goal of the inverse problem is to reconstruct the source time courses $\boldsymbol{s}(t)$ from the measurements $\boldsymbol{b}(t)$ [1,3,7]. The beamformer approach achieves this by constructing a linear spatial filter for

each source. Assuming the source parameters $\boldsymbol{\Theta}$ are known, the linear filter estimate for the time courses of a subset of $k \leq m$ sources $\hat{\boldsymbol{s}}(t)$ is given by applying a $m \times k$ weight matrix $\boldsymbol{W}$ to the data:

$$\hat{\boldsymbol{s}}(t) = \boldsymbol{W}^T \boldsymbol{b}(t)$$

For the Multiple Constrained Minimum Variance (MCMV) beamformer, the weight matrix $\boldsymbol{W}$ is derived by minimizing the total output power of all k sources subject to the constraint $\boldsymbol{W}^T \boldsymbol{H} = \boldsymbol{I}_k$, where $\boldsymbol{H} = \{\boldsymbol{h}^j\}$ is the $(m \times k)$ lead field matrix for the selected sources and $\boldsymbol{I}_k$ denotes k-dimensional identity matrix [1,4,9]. This yields the well-known multi-source beamformer solution:

$$\boldsymbol{W} = \boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1} \tag{2}$$

In Eq.(2) $\boldsymbol{R} = \langle \boldsymbol{b}\boldsymbol{b}^T \rangle$ is the $(m \times m)$ data covariance matrix, and columns $\boldsymbol{w}^j$, $j = 1, ..., k$ of matrix $\boldsymbol{W}$ represent the weight vectors of individual sources.

In the multi-source formulation any change in the parameters of one of the sources affects the entire matrix $\boldsymbol{W}$ making all weight vectors $\boldsymbol{w}^j$ interdependent. This interdependence significantly complicates the search for localizer extrema [9,10,18]. For this reason a simpler single-source approximation ($k = 1$) is most commonly used where each source is treated independently. In this case the expression for the weight vector $\boldsymbol{w}^i$ of a single source i simplifies to:

$$\boldsymbol{w}^i = \boldsymbol{R}^{-1}\boldsymbol{h}^i / (\boldsymbol{h}^{iT}\boldsymbol{R}^{-1}\boldsymbol{h}^i)$$

where now $\boldsymbol{w}^i$ only depends on this source's parameters $\boldsymbol{\theta}^i$.

The above solutions provide the time course estimates for a sources if their locations and orientations are known. The central issue addressed in this paper is the localization step that must precede this reconstruction, particularly the challenge of defining the noise model required for this step in the context of resting-state data.

2.2. The Localization Problem and the Critical Role of Noise Covariance

The expressions for the beamformer weights in Equation (2) are predicated on knowing the source parameters $\boldsymbol{\Theta}$ – that is, the locations and orientations of the neural activity. In practice, these parameters are the very unknowns we wish to find. The process of determining them is the localization step. This is typically achieved by searching the parameter space for maxima of a so-called localizer function. For a localizer to be effective, its maxima should correspond to the true source parameters, providing an unbiased estimate of the source locations and orientations regardless of the signal-to-noise ratio (SNR).

In the context of resting-state analysis, where sources are assumed to exist across a predefined grid of positions $\boldsymbol{r}^i$, the localization problem simplifies to finding the optimal orientation $\boldsymbol{o}^i$ at each location. Historically many localizer functions have been proposed that were applicable only in a single-source case; and a few – for the multi-source case [5,6,8–11,18–20]. One of the most widely used and well understood is the pseudo-Z statistic [6,7], which for a single source comes down to the ratio of the estimated source power to the power of the noise projected by the spatial filter. This can be expressed as:

$$\mathcal{Z}(\boldsymbol{r}^i, \boldsymbol{o}^i) = \boldsymbol{h}^{iT}\boldsymbol{R}^{-1}\boldsymbol{h}^i / \boldsymbol{h}^{iT}\boldsymbol{R}^{-1}\boldsymbol{N}\boldsymbol{R}^{-1}\boldsymbol{h}^i \tag{3}$$

By construction, the pseudo-Z statistic is directly related to the source-level SNR (specifically, $SNR = \mathcal{Z} - 1$). The optimal orientation $\boldsymbol{o}(\boldsymbol{r}^i)$ at a given location $\boldsymbol{r}^i$ is the one that maximizes this function. The resulting spatial map of $\mathcal{Z}(\boldsymbol{r}^i, \boldsymbol{o}(\boldsymbol{r}^i))$ is then used to characterize the distribution of brain activity.

Equation (3) makes the fundamental challenge explicit: localization requires not only the measured data covariance $\boldsymbol{R}$ but also a separate, well-defined noise covariance matrix $\boldsymbol{N}$. This is a general situation, not restricted to this particular form of the localizer or to the single-source approximation. The choice of $\boldsymbol{N}$ is crucial, as it provides the baseline against which the signal is defined and detected.

In stimulus-based paradigms, this is straightforward; $\boldsymbol{N}$ is empirically estimated from "control" or pre-stimulus time windows where the signal of interest is absent, and $\boldsymbol{R}$ is estimated from "active" or post-stimulus windows. In the resting state, however, no such control intervals exist. The entire recording is a continuous stream of the brain activity we wish to characterize, making the definition of $\boldsymbol{N}$ a non-trivial modeling decision that directly shapes the inverse solution.

2.3. A Principled Noise Model from a Source-Level Null Hypothesis

As argued in the *Introduction*, common choices for $\boldsymbol{N}$ in resting-state analysis, such as diagonal white noise or empty-room recordings, are physiologically and mathematically inconsistent. To overcome these limitations, we propose a more natural alternative by defining a baseline or null condition at the source level itself. We posit that a natural "ground state" for spontaneous brain activity is a uniform distribution of uncorrelated and randomly oriented dipolar sources distributed throughout the brain volume.

We formalize this model with the following assumptions for the baseline state:

1. **Uniform Source Distribution:** Sources $s_i(t)$ exist at all n locations $\boldsymbol{r}^i$ within the source space.
2. **Uncorrelation and Equal Variance:** The sources are uncorrelated and share a common root mean square (RMS) amplitude, σ. This is expressed as $\langle s_i s_j \rangle = \sigma^2 \delta_{ij},\ i, j = 1, ..., n$, where δ_{ij} is the Kronecker delta.
3. **Random Orientation:** The orientations $\boldsymbol{o}^i$ are uniformly and independently distributed over the unit sphere. The expectation of the orientation outer product is therefore $\langle \boldsymbol{o}^i \boldsymbol{o}^{iT} \rangle = (1/3)\boldsymbol{I}_3$, where $\boldsymbol{I}_3$ is the 3×3 identity matrix.

The observed brain field $\boldsymbol{b}(t)$ in this baseline or ground state is the superposition of the fields from all these sources, plus any additional environmental or sensor noise $\boldsymbol{\nu}(t)$:

$$\boldsymbol{b}(t) = \sum_i s_i(t) \boldsymbol{H}^i(\boldsymbol{r}^i) \boldsymbol{o}^i + \boldsymbol{\nu}(t) \tag{4}$$

Here, we have introduced the $m \times 3$ lead field matrices $\boldsymbol{H}^i(\boldsymbol{r}^i) = \{\boldsymbol{h}^{ix}, \boldsymbol{h}^{iy}, \boldsymbol{h}^{iz}\}$, which represent the sensor responses to unit dipoles at location $\boldsymbol{r}^i$ oriented along the Cartesian axes. The sensor noise $\boldsymbol{\nu}(t)$ is assumed to be a zero-mean process with covariance $\boldsymbol{\Lambda} = \langle \boldsymbol{\nu}(t) \boldsymbol{\nu}(t)^T \rangle$, which can be estimated from empty room recordings.

The noise covariance matrix $\boldsymbol{N}$ for our model is the expected covariance of this baseline field, $\boldsymbol{N} = \langle \boldsymbol{b}(t) \boldsymbol{b}(t)^T \rangle$. Using (4), straightforward algebra yields the following expression for our proposed baseline noise covariance matrix:

$$\boldsymbol{N} = (1/3)\sigma^2 \sum_i \boldsymbol{H}^i \boldsymbol{H}^{iT} + \boldsymbol{\Lambda} \tag{5}$$

This model separates the contributions of physiology and anatomy from the overall source power. The term $\sum_i \boldsymbol{H}^i \boldsymbol{H}^{iT}$ is a purely geometric construct that depends only on the head model and sensor layout. The scalar term $(1/3)\sigma^2$ represents the uniform power of the underlying neural sources. The matrix $\boldsymbol{\Lambda}$ accounts for non-biological noise. This formulation provides a structured, non-diagonal, and physiologically motivated noise covariance that can be computed efficiently for any given source space. The final step is to determine the scalar source power σ^2 in a data-driven manner, which we address in the following section.

2.4. Data-Driven Constraint and Model Scaling

The expression for the noise covariance matrix $\boldsymbol{N}$ derived in Equation (5) contains a single, crucial free parameter: σ , the root mean square (RMS) amplitude of the underlying, uniformly distributed neural sources. This parameter cannot be chosen arbitrarily, as its value has a direct physical interpretation and fundamentally impacts the resulting source reconstruction. To complete

our model, we must establish a principled, data-driven method for determining its value based on the actual measured recordings.

To this end, we consider the relationship between the total measured sensor covariance, $\boldsymbol{R}$, and its constituent parts. $\boldsymbol{R}$ can be conceptualized as the sum of the covariance of our baseline "ground state" noise model, $\boldsymbol{N}$, and the covariance of the "activity of interest," $\boldsymbol{C}$. The matrix $\boldsymbol{C}$ represents the organized neural dynamics – the deviations from the uniform, random baseline that we aim to detect and localize. This relationship is expressed as:

$$\boldsymbol{R} = \boldsymbol{C} + \boldsymbol{N}$$

All three matrices – $\boldsymbol{R}$, $\boldsymbol{C}$, and $\boldsymbol{N}$ – are covariance matrices, and are therefore symmetric and positive semi-definite. This reflects a critical physical principle that the power of any signal component cannot be negative. This fundamental requirement allows us to place an upper bound on the noise covariance and, consequently, on the value of σ. From $\boldsymbol{C} = \boldsymbol{R} - \boldsymbol{N}$ and constraint $\boldsymbol{C} \geq 0$ follows

$$\boldsymbol{N} \leq \boldsymbol{R}$$

Here, an expression $\boldsymbol{A} \leq \boldsymbol{B}$ for two non-negative matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ means that the difference $\boldsymbol{B} - \boldsymbol{A}$ is also a non-negative matrix. Substituting our model for $\boldsymbol{N}$ from Equation (5), this constraint can be written explicitly as:

$$(\sigma^2/3) \sum_i \boldsymbol{H}^i \boldsymbol{H}^{iT} + \boldsymbol{\Lambda} \leq \boldsymbol{R} \tag{6}$$

If the value of σ is set too high, this condition will be violated. A violation implies that the resulting signal covariance matrix $\boldsymbol{C}$ would no longer be positive semi-definite, which is a physically untenable situation that can lead to unphysical results, such as the estimation of negative source powers in certain brain regions. The constraint in Equation (6) therefore ensures the physical plausibility of the source reconstruction.

It is important to note that a value of σ that satisfies this constraint can always be found. When the measured covariance $\boldsymbol{R}$ is strictly positive, a sufficiently small σ will always satisfy the inequality. In practice, however, $\boldsymbol{R}$ may be rank-deficient due to common preprocessing steps such as the application of a common average reference in EEG, Maxwell filtering for noise cancellation in MEG, or the out-projection of specific interference components. This does not invalidate our approach. These same linear operations must be consistently applied to the components of our noise model, which will result in $\boldsymbol{N}$ having the same rank deficiency and null space as $\boldsymbol{R}$. Consequently, the constraint remains well-defined and can be evaluated within the common, non-null subspace of the data, ensuring the robustness of the method to standard preprocessing pipelines.

In practice, it is not only possible but also desirable to choose the maximal value for σ that still satisfies the constraint in (6). We denote this value as σ_{max}. There are two primary reasons for this choice. First, when the environmental and instrumental noise $\boldsymbol{\Lambda}$ is not negligible, choosing σ_{max} ensures that the contribution of the brain's own baseline activity to the noise covariance is maximized relative to the non-biological noise. This aligns with the principle that the dominant source of interference in MEG/EEG is typically the brain itself. Second, in the common case where $\boldsymbol{\Lambda}$ is negligible, choosing a value $\sigma < \sigma_{max}$ merely leads to a proportional scaling of the pseudo-Z values without affecting the locations of the maxima. This increase in pseudo-Z is a purely artificial consequence of scaling down the denominator $\boldsymbol{N}$ and carries no additional physical significance. Selecting σ_{max} therefore provides a principled, data-driven approach to scaling the resulting activity maps.

The actual value of σ_{max} depends on the measured data $\boldsymbol{R}$ and the instrumental noise $\boldsymbol{\Lambda}$, and it can always be found numerically using a straightforward iterative procedure to any desired accuracy.

This completes the construction of our proposed noise model for beamformer analyses of spontaneous brain activity. The final, fully specified expression for the baseline noise covariance matrix is given by:

$$\boldsymbol{N} = (1/3)\sigma_{max}^2 \sum_i \boldsymbol{H}^i \boldsymbol{H}^{iT} + \boldsymbol{\Lambda} \tag{7}$$

3. Practical example

3.1. MEG Data and Forward Model Construction

To demonstrate our proposed approach and contrast it with conventional methods, we used real human resting-state MEG data. The data were collected from a single healthy adult subject using a 273-channel CTF/VSM whole-head scanner housed in a magnetically shielded room. A total of nine minutes of eyes-closed resting state data were acquired. The raw data were preprocessed using third-order gradient noise cancellation, filtered to a 0–100 Hz band, and downsampled to a 200 Hz sampling rate.

The subject's anatomical T1-weighted MRI was used to construct the forward model. A single-layer boundary element model (BEM) was generated to define the conductor volume, and an equally spaced volumetric grid with 5 mm resolution was created within this volume, resulting in approximately 12,000 source locations. Lead fields for each source were computed using the MNE Python software suite [21,22]. Given the effectiveness of the gradient compensation, the environmental and instrumental noise were considered negligible, and corresponding term ($\boldsymbol{\Lambda}$ in Equation (7)) was omitted for this demonstration.

3.2. Characterization of the Proposed Noise Covariance Model

The baseline noise covariance matrix $\boldsymbol{N}$ was constructed according to our proposed model, as defined in Equation (7). The full data covariance $\boldsymbol{R}$ was computed from the 9-minute recording, and the σ_{max} parameter was determined numerically to satisfy the constraint $\boldsymbol{R} - \boldsymbol{N} \geq 0$. The observed effective sensor-level power signal-to-noise ratio, defined as $SNR = tr(\boldsymbol{R})/tr(\boldsymbol{N}) - 1$, was approximately 32 for this dataset.

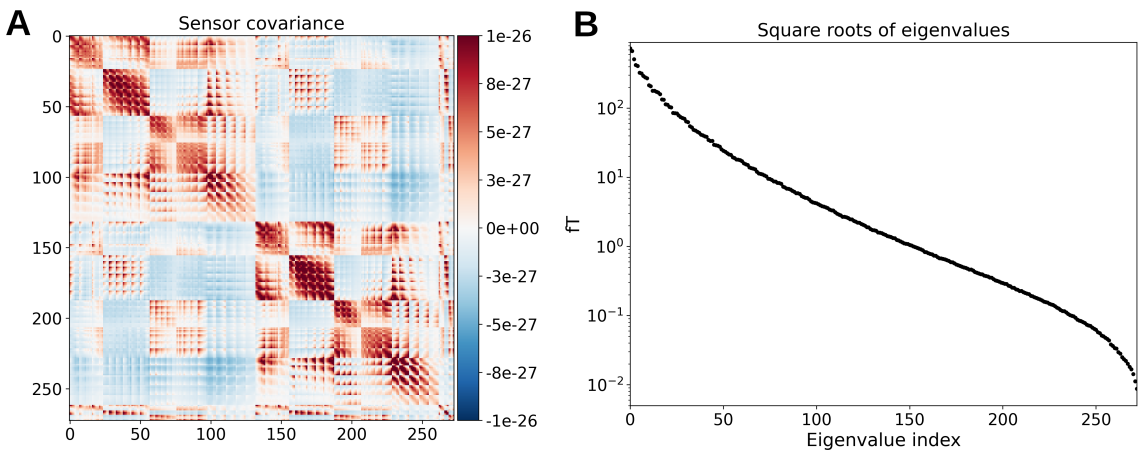

Figure 1. Noise covariance matrix constructed using the model (7). *A* – a colormap plot of the matrix elements; T^2. *B* – square roots of eigenvalues of $\boldsymbol{N}$; fT.

The resulting noise covariance matrix is visualized in Figure 1. It is immediately apparent that although the underlying source model assumes uniform and uncorrelated white noise at the brain level, the resulting sensor-level noise covariance $\boldsymbol{N}$ is far from diagonal. The colormap plot (Figure 1A) reveals strong non-diagonal elements, indicating widespread spatial correlations between sensors. This is a direct consequence of the physical reality of MEG recordings: the magnetic field from any single current dipole spreads across the sensor array, inducing correlated signals in multiple sensors. The eigenvalue spectrum of $\boldsymbol{N}$ (Figure 1B) further highlights this complexity; it is not flat, as would be

expected for white noise, but spans several orders of magnitude, reflecting the rich spatial structure of the aggregate lead fields.

Furthermore, the noise power distribution across the sensor array is highly non-uniform, as depicted in the topomap in Figure 2. This specific spatial pattern is not random; it is dictated by

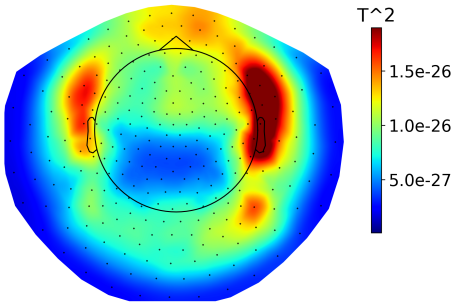

Figure 2. The topomap of the model noise power distribution over the MEG sensor array; T^2.

the subject's unique head geometry, their position within the MEG helmet, and the parameters of the electromagnetic forward model. This demonstrates a critical point: even a perfectly uniform source-level baseline produces a complex and highly structured sensor-level covariance.

3.3. Reconstructed Resting-State Activity: A Comparative Analysis

We applied our proposed model to compute the pseudo-Z distribution of brain activity corresponding to the observed sensor covariance $\boldsymbol{R}$. The resulting 3D distribution, shown in Figure 3A, reveals a pattern consistent with neurophysiological expectations for resting-state activity. The pseudo-Z values are generally moderate, typically around 3 or less, which corresponds to source-level SNR ratios of approximately 2 or lower. Critically, the overall distribution is spatially smooth, lacking the sharp gradients or distinct, localized peaks that would characterize a focal, task-related activation. This diffuse map represents a plausible source-level distribution corresponding to ongoing spontaneous, resting state brain activity.

To underscore the importance of a principled noise model, we performed a comparative analysis using the conventional diagonal white noise assumption. First, to understand the implicit baseline of this conventional approach, we inferred what brain activity would look like under the assumption that the brain's resting state is diagonal sensor noise. The resulting pseudo-Z map (Figure 3B) reveals a highly non-uniform and structured pattern, with pronounced peaks emerging in specific locations. This distribution is entirely an artifact of the interaction between the lead fields and the overly simplistic noise model; it is the artificial "activity" required at the source level to produce diagonal noise at the sensor level. Such a distribution is an unnatural and unjustifiable choice for a baseline or "control" condition.

Finally, we performed the most direct comparison: we computed the pseudo-Z map from the actual measured data $\boldsymbol{R}$ but used a scaled diagonal sensor noise matrix as the baseline instead the one defined by Equation (7), as is common in practice (Figure 3C). The result is dramatically different from that obtained with our proposed model (Figure 3A). The map exhibits exaggerated spatial variations and a distorted shape that directly reflects the artificial structure seen in Figure 3B. The prominent peaks and troughs do not represent true underlying neural dynamics but are artifacts generated by the mismatch between the simplistic noise model and the physical reality of the data. This comparison clearly illustrates how an unprincipled choice for the noise covariance can introduce systematic biases, creating structured patterns that could easily be misinterpreted as genuine findings. In contrast, our

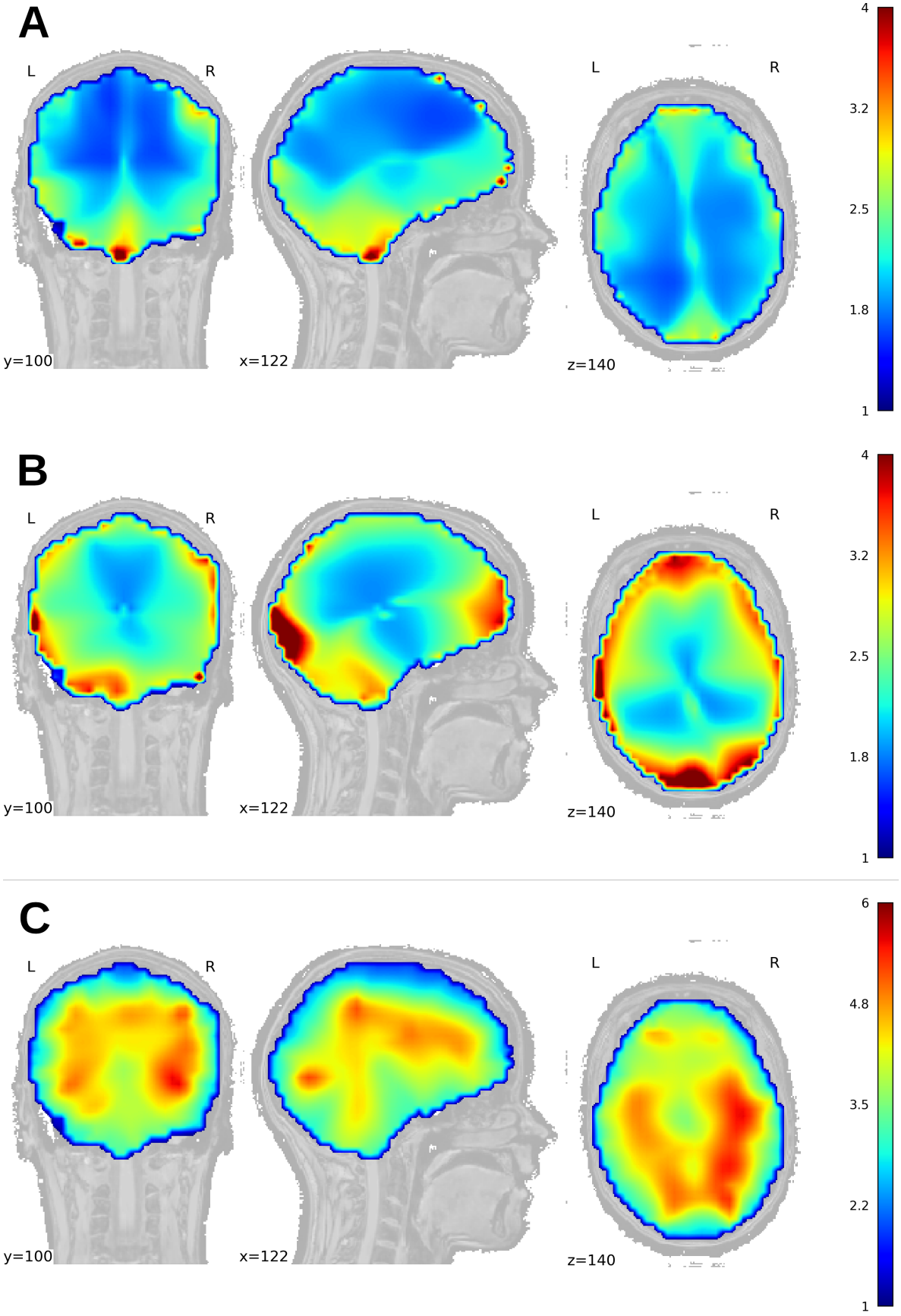

Figure 3. Single source pseudo-Z distributions of brain activity. *A* – real brain noise with $\boldsymbol{N}$ defined by Eq. (7); *B* – $\boldsymbol{R}$ replaced with the diagonal sensor noise $\boldsymbol{N}_{diag}$, scaled so that $\boldsymbol{N} \leq \boldsymbol{N}_{diag}$; *C* – real brain noise with the diagonal sensor noise $\boldsymbol{N}_{diag}$ used instead of $\boldsymbol{N}$; in this case $\boldsymbol{N}_{diag}$ was scaled to satisfy constraint $\boldsymbol{N}_{diag} \leq \boldsymbol{R}$. Note that the color bar maximum value equals 4 for panels *A* and *B*, and equals 6 for panel *C*.

physiologically grounded model provides a stable, smooth, and interpretable baseline for the analysis of resting-state brain activity.

4. Discussion

The practical example in the preceding section illustrates a critical principle: the choice of the baseline noise covariance matrix $\boldsymbol{N}$ is not a peripheral step but the very foundation upon which the entire beamformer analysis is built. The influence of this choice is profound and systemic. The matrix $\boldsymbol{N}$ explicitly enters into the expressions for localizer functions [1,7], such as the pseudo-Z statistic in Equation (3), and is thus the primary determinant of the localization outcome. It is worth noting that while the final beamformer weights (Equation (2)) appear to depend only on the full data covariance matrix $\boldsymbol{R}$, they are indirectly shaped by the noise model. This is because the lead fields $\boldsymbol{h}$, which are central to the weights calculation, depend on the source orientations that are themselves estimated during the localization step. Since localization is guided by the definition of noise, the choice of $\boldsymbol{N}$ ultimately affects both the estimated source parameters and the resulting spatial filters that reconstruct the neural time courses.

Our analysis demonstrated that using a diagonal white noise covariance matrix for $\boldsymbol{N}$ in resting-state analyses is a poor choice. This approach leads to a reconstructed brain activity distribution with an artificially non-uniform shape, driven entirely by the structure of the subject-specific lead fields (as seen in Figure 3, B and C). Such a distribution, with its arbitrary peaks and troughs, is difficult to justify as a meaningful baseline condition. While our example focused on MEG, the same issue arises in EEG, where the physics of volume conduction similarly dictates that a diagonal sensor-level covariance can only arise from a complex, non-uniform source-level configuration. This fundamental flaw remains regardless of whether a volumetric or a cortical surface-based source space is used. The diagonal noise model creates an illusion of a simple, "uninformed" prior at the sensor level, while in reality imposing a highly structured and physiologically untenable baseline at the source level.

The approach we have proposed is tailored specifically for resting-state analyses, under the assumption that the most natural baseline is one of uniformly distributed, uncorrelated, and randomly oriented sources. This provides a plausible and simple-to-use null model against which true, organized network activity can be detected. However, we acknowledge that this model is a simplification and may not be optimal for capturing more complex or structured background activity that might be present in other contexts, such as during specific cognitive tasks or in certain clinical populations. Its strength lies in providing a principled and consistent starting point for the exploratory analysis of spontaneous neural dynamics.

It is important to consider the conditions under which defining a noise model $\boldsymbol{N}$ is necessary. In the resting-state case, this requirement arises because the source orientations $\boldsymbol{o}^i$ are unknown and must be estimated. The problem of defining $\boldsymbol{N}$ could be completely avoided if source orientations were predetermined. For example, physiological constraints, such as assuming cortical sources are oriented orthogonally to the cortical surface, can in principle be used. However, in practice, the high curvature of the cortex and the inherent limitations in the accuracy of anatomical reconstruction and M/EEG-MRI co-registration make it exceptionally difficult to infer source orientations with sufficient precision [23]. An alternative approach that bypasses the orientation search is the so-called vector beamformer [1,5,7], where each brain location is modeled as containing three orthogonal, fixed dipoles oriented along the coordinate axes. While this approach is robust, it comes with a significant disadvantage: it produces three time courses per location instead of one, and on average, the SNR of these time courses is two to three times lower than that obtained with a scalar beamformer that explicitly optimizes orientation to maximize SNR.

Finally, a practical note concerns the use of empty-room recordings to account for the instrumental and environmental noise component, $\boldsymbol{\Lambda}$, in our model (Equation (7)). While often negligible, if this component is included, care must be taken when estimating σ_{max} using the constraint in Equation (6). In a real-world scenario, we work with statistical estimates of $\boldsymbol{R}$ and $\boldsymbol{\Lambda}$, not their true, underlying values.

Although in theory the difference $\boldsymbol{R} - \boldsymbol{\Lambda}$ must be non-negative (since environmental and instrumental noise are components of the total measured signal), in practice this condition can be violated due to estimation errors. This is particularly likely if a significant amount of time has elapsed between the brain recording and the empty-room measurement, as the statistical properties of the environmental noise may have changed. In such cases, it is imperative to acquire updated measurements of $\boldsymbol{\Lambda}$ or to apply appropriate regularization techniques to ensure that the matrix $\boldsymbol{R} - \boldsymbol{\Lambda}$ remains positive semi-definite before proceeding to determine σ_{max}. This ensures the mathematical stability and physical plausibility of the final noise model.

5. Conclusion

In this technical note, we have addressed the fundamental problem of defining an appropriate baseline noise model for the application of minimum variance beamformers to the analysis of spontaneous brain activity. The beamformer is an inherently contrastive method; it interprets neural activity as a deviation from a predefined baseline state. Consequently, the proper definition of this baseline is not a mere technical detail but is essential for producing meaningful, interpretable, and valid scientific results. The choice of the noise model fundamentally shapes the inverse solution.

We argued that commonly used approaches for resting-state analysis are predicated on flawed assumptions. Methods such as employing empty-room recordings or assuming white, uncorrelated sensor noise lead to baseline models that, when mapped to the source space, correspond to highly non-uniform and subject-specific patterns of neural activity. Such baselines are difficult to justify, both methodologically and physiologically, as they introduce arbitrary and systematic biases into the source reconstruction.

As a more principled alternative, we proposed a model based on a physiologically plausible null hypothesis at the source level: a uniform distribution of uncorrelated, randomly oriented brain sources. This model serves as a meaningful and easily interpretable "ground state" or maximum entropy state of brain activity. True signals of interest, such as the synchronous dynamics of resting-state networks, are then identified as statistically significant deviations from this uniform baseline.

Using real resting-state MEG data, we demonstrated that the sensor-level covariance matrix associated with this source-level baseline model is profoundly different from the diagonal matrix of white noise. Our model naturally produces a highly structured covariance matrix that exhibits widespread inter-sensor correlations and a strongly non-uniform spatial power distribution, reflecting the underlying physics of MEG and the anatomy of the head. We further compared the reconstructed activity distributions obtained using our proposed model versus a conventional diagonal sensor noise model and found substantial differences in both the shape and magnitude of the results. While the ground truth in such data is inherently unknown, the smooth and diffuse activity map generated by our baseline model appears substantially more physiologically plausible than the artificially peaked and distorted map produced by the diagonal noise model.

Ultimately, this work provides a robust and theoretically grounded framework for defining the noise covariance in resting-state beamforming. By starting from a principled assumption at the source level, our approach avoids the arbitrary biases of conventional methods and yields a more accurate and interpretable view of the brain's intrinsic neural dynamics. This provides a more solid foundation for investigations into the complex functional architecture of the resting brain.

References

1. Ilmoniemi, R.; Sarvas, J. *Brain Signals. Physics and Mathematics of MEG and EEG*; MIT Press, 2019.
2. Knösche, T.; Haueisen, J. *EEG/MEG Source Reconstruction*; Springer Cham, 2022.
3. Baillet, S.; Mosher, J.; Leahy, R. Electromagnetic brain mapping. *IEEE Signal Processing Magazine* **2001**, *18*, 14–30. https://doi.org/10.1109/79.962275.
4. Frost, O. An algorithm for linearly constrained adaptive array processing. *Proc. IEEE* **1972**, *60*, 926–935.
5. Van Veen, B.; van Drongelen, W.; Yuchtman, M.; Suzuki, A. Localization of brain electrical activity via linearly constrained minimum variance spatial filtering. *IEEE Trans. Biomed. Eng.* **1997**, *44*, 867–880.

6. Robinson, S.; Vrba, J. Functional neuroimaging by synthetic aperture magnetometry (SAM). *Recent Advances in Biomagnetism, Tohoku Univ. Press, Sendai, Japan* **1999**, p. 302–305.
7. Sekihara, K.; Nagarajan, S. *Adaptive spatial filters for electromagnetic brain imaging*; Springer, Berlin, 2008.
8. Sekihara, K.; Sahani, M.; Nagarajan, S.S. Localization bias and spatial resolution of adaptive and non-adaptive spatial filters for MEG source reconstruction. *NeuroImage* **2005**, *25*, 1056–1067. https://doi.org/https://doi.org/10.1016/j.neuroimage.2004.11.051.
9. Moiseev, A.; Gaspar, J.; Schneider, J.; Herdman, A. Application of multi-source minimum variance beamformers for reconstruction of correlated neural activity. *Neuroimage* **2011**, *58*, 481–496.
10. Moiseev, A.; Herdman, A. Multi-core beamformers: Derivation, limitations and improvements. *Neuroimage* **2013**, *71*, 135–146.
11. Moiseev, A.; Herdman, A.T.; Ribary, U. Subspace based Multiple Constrained Minimum Variance (SMCMV) beamformers. *Biomedical Signal Processing and Control* **2022**, *71*, 103124. https://doi.org/https://doi.org/10.1016/j.bspc.2021.103124.
12. Aydin, O.; Pellegrino, G.; Ali, O.B.K.; Abdallah, C.; Dubeau, F.; Lina, J.M.; Kobayashi, E.; Grova, C. Magnetoencephalography resting state connectivity patterns as indicatives of surgical outcome in epilepsy patients. *Journal of Neural Engineering* **2020**, *17*, 035007. https://doi.org/10.1088/1741-2552/ab8113.
13. Cai, C.; Hashemi, A.; Diwakar, M.; Haufe, S.; Sekihara, K.; Nagarajan, S.S. Robust estimation of noise for electromagnetic brain imaging with the champagne algorithm. *NeuroImage* **2021**, *225*, 117411. https://doi.org/https://doi.org/10.1016/j.neuroimage.2020.117411.
14. Afnan, J.; von Ellenrieder, N.; Lina, J.M.; Pellegrino, G.; Arcara, G.; Cai, Z.; Hedrich, T.; Abdallah, C.; Khajehpour, H.; Frauscher, B.; et al. Validating MEG source imaging of resting state oscillatory patterns with an intracranial EEG atlas. *NeuroImage* **2023**, *274*, 120158. https://doi.org/https://doi.org/10.1016/j.neuroimage.2023.120158.
15. Hillebrand, A.; Barnes, G.R.; Bosboom, J.L.; Berendse, H.W.; Stam, C.J. Frequency-dependent functional connectivity within resting-state networks: An atlas-based MEG beamformer solution. *NeuroImage* **2012**, *59*, 3909–3921. https://doi.org/https://doi.org/10.1016/j.neuroimage.2011.11.005.
16. Colclough, G.; Brookes, M.; Smith, S.; Woolrich, M. A symmetric multivariate leakage correction for MEG connectomes. *NeuroImage* **2015**, *117*, 439–448. https://doi.org/https://doi.org/10.1016/j.neuroimage.2015.03.071.
17. Nunes, A.S.; Moiseev, A.; Kozhemiako, N.; Cheung, T.; Ribary, U.; Doesburg, S.M. Multiple constrained minimum variance beamformer (MCMV) performance in connectivity analyses. *NeuroImage* **2020**, *208*, 116386. https://doi.org/https://doi.org/10.1016/j.neuroimage.2019.116386.
18. Diwakar, M.; Tal, O.; Liu, T.; Harrington, D.; Srinivasan, R.; Muzzatti, L.; Song, T.; Theilmann, R.; Lee, R.; Huang, M. Accurate reconstruction of temporal correlation for neuronal sources using the enhanced dual-core MEG beamformer. *Neuroimage* **2011**, *56*, 1918–1928.
19. Mosher, J.; Leahy, R. Source Localization Using Recursively Applied and Projected (RAP) MUSIC. *IEEE Transactions on Signal Processing* **1999**, *47*, 332–340.
20. Makela, N.; Stenroos, M.; Sarvas, J.; Ilmoniemi, R. Truncated RAP-MUSIC (TRAP-MUSIC) for MEG and EEG source localization. *Neuroimage* **2018**, *167*, 73–83.
21. Gramfort, A.; Luessi, M.; Larson, E.; Engemann, D.A.; Strohmeier, D.; Brodbeck, C.; Goj, R.; Jas, M.; Brooks, T.; Parkkonen, L.; et al. MEG and EEG Data Analysis with MNE-Python. *Frontiers in Neuroscience* **2013**, *7*, 1–13. https://doi.org/10.3389/fnins.2013.00267.
22. MNE: MEG + EEG Analysis and Visualization, 2012-2025. https://doi.org/https://doi.org/10.5281/zenodo.592483.
23. Hillebrand, A.; Barnes, G.R. The use of anatomical constraints with MEG beamformers. *NeuroImage* **2003**, *20*, 2302–2313. https://doi.org/https://doi.org/10.1016/j.neuroimage.2003.07.031.