# Quenching of Chaos in externally driven metacommunities

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The chaotic population dynamics of various natural ecosystems with widely varying spatial scales has been extensively investigated in the last three decades. Chaotic fluctuations in population, barring a very few specific conditions, have generally been established to be more prone to extinction through cascading effects, environmental noise, and regular occurrence of low species density. Thus, chaotic population dynamics has been largely associated with low persistence in ecosystems. Contrastingly, ecosystems with steady-state dynamics have been established to be extremely stable. Nevertheless, the continnued existence of chaotic ecosystems have been suggested to be a result of internal and external control of chaos engineered by either the ecosystems themselves, or some external factor. However, the question of chaos control, specifically, the quenching of chaos in metacommunities arising due to internal habitat heterogeneity has not been investigated. In this study, we analyze the dynamics of a metacommunity of ecosystems, where one patch is the drive, and the other patches are responses. Each patch consists of an inherently chaotic tritrophic food web, with habitat heterogeneity among themselves. We report that in a drive-response metacommunity, dynamical chaos in both drive and response patches is quenched under a strong influence of the drive, subsequently leading to steady states in most cases. We report the occurrence of this phenomenon for 2 metacommunity network structures. We discover that the heterogeneity of the response systems and the dissimilarity of the drive and response systems play a major role in quenching the chaos. Thus, it is extremely possible that the existence of an inherently chaotic metacommunity is strongly an interplay of chaos, habitat heterogeneity, and the structure of the network created by dispersal. Notably, metacommunities consisting of inherent chaotic patches also face the risk of extinction due to dispersal-induced synchronization. Therefore, we have investigated dispersal-induced complete, synchronization between the constituent patches of these drive-response metacommunity.

## I. INTRODUCTION

Ecological metacommunities exhibiting chaotic oscillation for their population density have been observed in various ecosystems, and the importance of chaos in ecology has also been the subject of extensive research [1–3]. Chaotic dynamics have been reported in several population models describing the evolution of a plethora of natural ecosystems [1–3]. Most importantly, the extinction risk and, consequently, the highly relevant question of the persistence of such inherently chaotic ecosystems has been investigated in some detail. Several studies have associated the chaotic fluctuation in population density to a low degree of persistence, whereas some studies revealed that ecosystems in dynamically periodic or chaotic states has a high probability of extinction through cascading effect or synchronized response to external environmental noise [4, 5]. In particular, it has been shown that low density populations have elevated chance of extinctions in the chaotic domain of phase space [6], chaotic fluctuations in the population densities can lead to population extinctions under stochasticity [7–10], and chaotic dynamics might mask early warning signals and indirectly leading to increase of extinction risk [11]. Further, Dietze [12] reported that 'truly chaotic populations would fluctuate wildly, resulting in frequent population crashes, genetic bottlenecks, and high risk of stochastic extinction'.

Interestingly, some studies [6, 13, 14] have strongly claimed that natural selection prevents ecosystems from allowing parameter values that renders the population dynamics chaotic and hence such ecosystem intrinsically refrain from chaotic dynamics for their population density. Nevertheless, Gamarra and Solé [15] reported that the chaotic domain is large for a metacommunity of chaotic ecosystems connected by dispersal, wherein one part of the chaotic domain has population fluctuations low enough for an elevated extinction risk, while the other part has species density fluctuating with high amplitudes with relatively much lower extinction risk. Consequently, 'strong' dispersal can elevate extinction risk of the metacommunity by bringing it into the chaotic domain with low fluctuations [15]. However, it is to be noted that chaotic population has been shown to decrease extinction risk in metapopulations [16] and in habitats with scarcity of resources [17]. Specifically, Allen et. al [16] reported the decorrelating effect of chaos in a 'weakly' coupled metapopulation resulting in increased persistence. Thus, the role of dispersal on the persistence of a metacommunity of chaotic population deserves more detailed investigations.

Further, inherent ecological mechanisms like toxin release [18] or omnivory [19], or environmental preconditions like habitat heterogeneity [11, 20], or external intervention have all been reported to control or impede chaos [14, 21]. Furthermore, metapopulation dynamics have been known to undergo transition to steady state under the influence of both internal mechanism like dispersal [22] or external mechanism like predation turnoff [23].

As explained above, chaotic dynamics can clearly indicate low persistence depending on how 'weak' or 'strong' the dispersal (or coupling) is. Understandably then, in

a metacommunity formned by dispersal, controlling of chaos is of utmost importance. haotic dynamics is governed by parameter values, which implies that chaos can be controlled by change in parameter values [18, 24]. Chaos control in ecosystems have been reported in various aspects. A real-life example of the drive-response metacommunity is a river-distributary ecosystem. It has been reported that rivers control the water level, nutrient level, and soil level of the distributaries ([25]). Since all of this contributes to the habitat quality of the distributary ecosystem, it is but logical to conclude that the ecological dynamics of the distributaries are 'driven' by the corresponding rivers ([26, 27]).

In this study, we consider a drive-response metacommunity with inherently chaotic foodwebs describing the species dynamics of each patch. We consider different levels of heterogeneity between drive and response patches and demonstrate how habitat heterogeneity in such metacommunities quenches the chaos in each patch. We also demonstrate that the complete synchronization between homogeneous drive and response systems is absolute, but decreases considerably with increasing dispersal in case of heterogenous drive and response.

## II. MODELS

The food web of the drive patch is described by the Blasius model [2] and governed by the equations

$$\dot{X}_d = aX_d - \alpha_1 \frac{X_d Y_d}{1 + kX_d} + \frac{e}{N} \sum_{i=1}^{N} (X_i - 4X_d), \quad (1a)$$

$$\dot{Y}_d = -b_d Y_d + \alpha_1 \frac{X_d Y_d}{1 + kX_d} - \alpha_2 f_2(Y_d, Z_d), \quad (1b)$$

$$\dot{Z}_d = -c(Z_d - Z_s) + \alpha_2 f_2(Y_d, Z_d), \quad (1c)$$

, where $f_2 = Y_d Z_d$. The parameter $a$ represents the growth rate of the resource in absence of inter-species interactions, while the parameters $b$ and $c$ represnts the mortality rate of the predator and consumer. The parameter $\alpha_1$ represents the predation rate of the predator and $\alpha_2$ represents the predation rate of the consumer. The predator mortality rate of the drive patch is given by $b_d$. The amplitude of drive dispersal is given by $e$. The parameter $N$ represents the number of response patches. For this entire study, $N = 4$.

The food web of the response patch is governed by the following equations (Blasius Model):

$$\dot{X}_i = aX_i - \alpha_1 \frac{X_i Y_i}{1 + kX_i} + e(X_d - X_i) + e_i \sum_{j=1, j\neq i}^{N} \gamma_{ij}(X_j - X_i), \quad (2a)$$

$$\dot{Y}_i = -b_i Y_i + \alpha_1 \frac{X_i Y_i}{1 + kX_i} - \alpha_2 f_2(Y_i, Z_i), \quad (2b)$$

$$\dot{Z}_i = -c(Z_i - Z_s) + \alpha_2 f_2(Y_i, Z_i), \quad (2c)$$

$\Gamma_{ij}$ is the adjacency matrix detailing the network structure created through dispersal between patches. When the response patches are unconnected, all elements of the matrix $\gamma_{ij} \epsilon \Gamma_{ij} = 0$. For nearest neighbor coupling between the responses, elements of the matrix $\gamma_{ij} \epsilon \Gamma_{ij} \neq 0$, only if $i = j$. When the response patches are randomly coupled, elements of the matrix $\gamma_{ij} \epsilon \Gamma_{ij} \neq 0$ randomly. For the response patch $i$, the parameter $b_i$ describes the mortality of the predator of the drive patch. The other parameters are given by $\alpha_1 = 0.2$, $\alpha_2 = 1, k = 0.05$, $Z_s = 0.006$, $c = 10.0$, $a = 1$ and $f = 0.25$. The magnitude of simple diffusive dispersal between the $i^{th}$ resource patch and its nearest neighbors is given by the parameter $e_i$. Note that when we consider the scenario for **unconnected** response patches $\varepsilon = 0$. For the $i^{th}$ response patch, the parameter $b_i$ describes the mortality of the predator of the $i^{th}$ patch. The other parameters are same as above.

## III. RESULTS

Figure 1 shows the bifurcation diagram of the Blasius model food web with respect to the mortality of predator $b$. We have chosen both $b_d$ and all $b_r$ values such that our investigated food webs all exhibit chaotic dynamics, when unperturbed ($b \epsilon (0.8 - 1)$). The structure of our metacommunity is represented in 1(b), where the circle filled with red represents the drive patch, the arrows from it represent the drive-response diseprsal. The circles filled with green represent the response patches. The dotted lines represent inter-response dispersals, whose magnitude is 0 when we consider unconnected responses. As stated above, the habitat heterogeneity across various patches in the metacommnunity comes from the variation in predator mortality rate $b$ from one patch to another.

### A. Locally connected response patches

#### 1. *Homogeneous drive and response patches*

First, we consider a group of locally connected, completely identical responses that are linked to an identical drive through dispersal, where we have fixed the predator mortality rate for both drive and response patches as $b_{drive} = b_{responses} = 0.9$. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 2(a) and 2(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 2(a) and 2(b). It is evident from these figures that the time series of $Z_d$ and $Z_r$ evolve identically corroborating the complete synchronization among the drive and uncoupled response

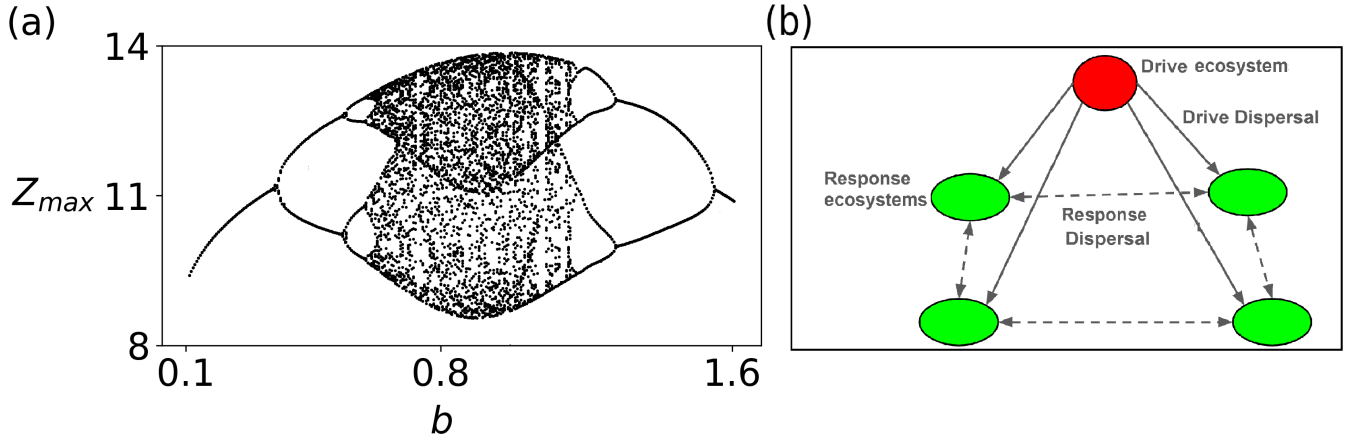


FIG. 1. (a) Bifurcation diagram of one single food web model described by the Blasius model, plotted as a function of predator mortality rate $b$.

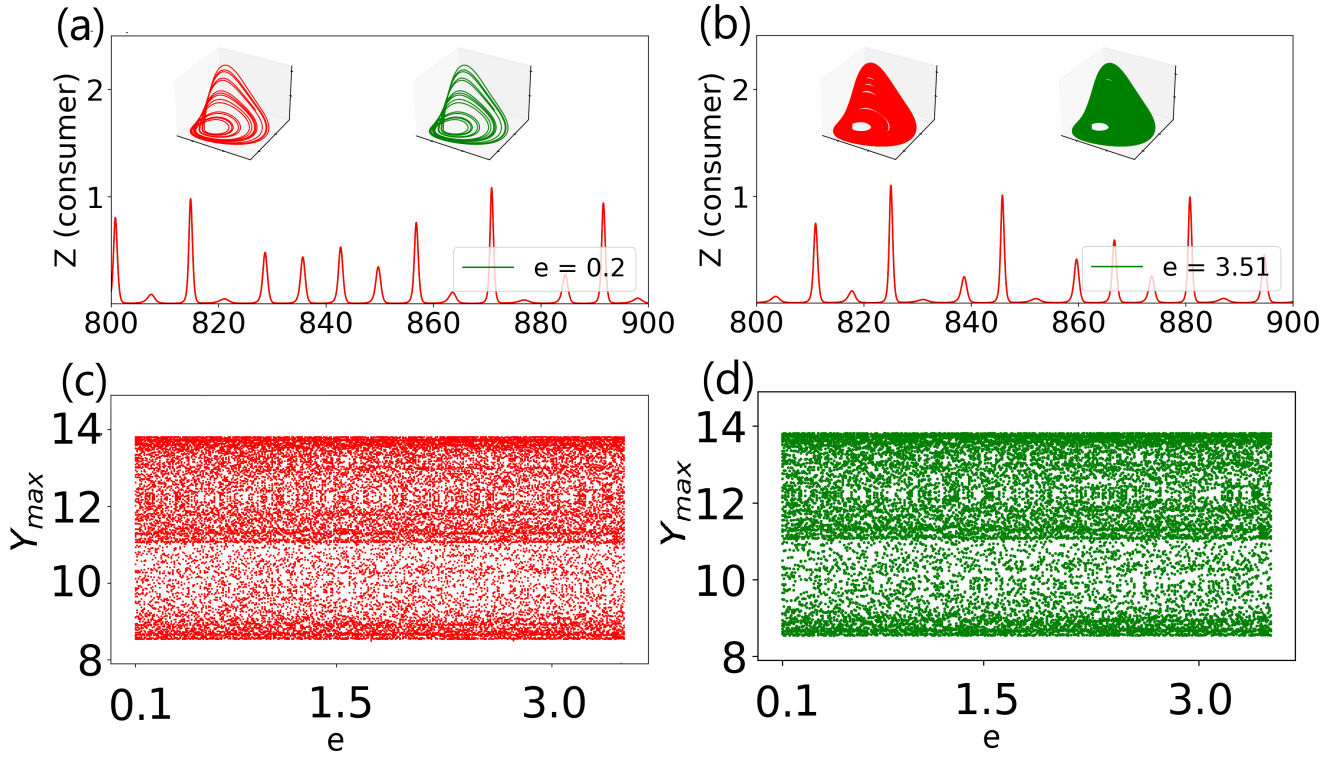


FIG. 2. Locally connected response patches: Completely identical drive and homogeneous response patches. (a) Time series of consumer dynamics of both drive patch and a response patch for the low dispersal strength $e = 0.2$, (b) same as (a) for the high dispersal strength $e = 3.5$, (c) bifurcation diagram of the drive patch as a function of $e$, and (d) bifurcation diagram of a response patch as a function of $e$. Attractors of the drive (red line) and a response (green line) patches are depicted in the inset of (a) and (b). Predator mortality rate for drive $b_d = 0.9$, and for responses $b_r = 0.8$.

systems for both low and high values of the dispersal strength. In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 2(c) and 2(d), respectively, in the range of the dispersal strength $e \in (0.0, 3.5)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive

and response patches exhibit chaotic oscillation for their population density in the entire explored range of the dispersal strength. Thus, the homogeneous drive is not capable of quenching the chaotic evolution of the population density in response patches.

### 2. *Heterogeneous drive and homogeneous response patches*

Next, we consider a group of locally connected, completely identical responses that are linked to a non-identical drive through dispersal, where we have fixed the predator mortality rate for both drive and response patches as $b_{drive} = 0.9$ and $b_{responses} = 0.8$. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 3(a) and 3(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 3(a) and 3(b). In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 3(c) and 3(d), respectively, in the range of the dispersal strength $e \in (0.0, 3.5)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive and response patches exhibit chaotic oscillation for their population density in the range of low dispersal strength. However, with increasing dispersal ($e\epsilon(0.8, 1.0)$), the dynamics of the drive patche undergo Hopf bifurcation from chaotic to a bistable state, consisting of 2 limit cycles. Upon further increase of dispersal ($e\epsilon(1.25, 3.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a bistable steady state. However, in case of the response patch, with increasing dispersal ($e\epsilon(0.8, 1.0)$) the dynamics undergoes Hopf Bifurcation from chaotic go mutli-stable limit cycles. Upon further increase of dispersal ($e\epsilon(1.25, 3.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a multi-stable steady state. Thus, the on-identical but similar drive quenches the chaotic evolution of the population density in completely identical locally connected response patches.

### 3. *Heterogeneous drive and response patches*

Next, we consider a group of locally connected, heterogeneous responses that are linked to a non-identical drive through dispersal, where we have fixed the predator mortality rate for both drive and response patches as $b_{drive} = 0.9$ and $b_{responses} = 0.77$, 0.76,0.82 and 0.85. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 4(a) and 4(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The dynamics evidently changes from chaotic to steady state on increasing drive-response dispersal. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 4(a) and 4(b). In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 4(c) and 4(d), respectively, in the range of the dispersal strength $\varepsilon_d \in (0.0, 3.0)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive and response patches exhibit chaotic oscillation for their population density in the range of low dispersal strength ($e\epsilon(0.1, 0.25)$). However, with increasing dispersal ($e\epsilon(0.3, 0.8)$), the dynamics of the drive patch undergo Hopf bifurcation from chaotic to a bistable state, consisting of 2 limit cycles. Upon further increase of dispersal ($e\epsilon(0.8, 2.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a monostable steady state, with a decreasing trend in the average population density. On further increase in dispersal, ($e > 2.5$), the mono stable state is maintained but the average population density saturates. In case of the response patch, with increasing dispersal ($e\epsilon(0.1, 0.25)$) the dynamics stays chaotic. Upon further increase of dispersal ($e\epsilon(0.25, 0.8)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a multi-stable limit cycle. Upon further increase of dispersal ($e\epsilon(0.8, 2.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a bistable steady state, with a decreasing trend in the average population density. On further increase in dispersal, ($e > 2.5$), the bistable state is maintained but the average population density saturates. Thus, the non-identical but similar drive quenches the chaotic evolution of the population density in non-identical locally connected response patches.

### 4. *Synchronization of the metacommunity*

We investigate the emergence of synchronization in a group of oscillators as a function of coupling strength, considering identical, dissimilar and heterogenous systems. Our goal is to determine what degree of heterogeneity promotoes synchrony and what level of heterogeneity is required in the network to induce asynchronous steady states. We define synchronization error for the group of oscillators. averaged over time as,

$$\eta = \frac{1}{T}\sum_t \sqrt{(\bar{Y})_t^2 - (\bar{Y^2})_t}. \tag{3a}$$

Here, $(\bar{Y})_t = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $(\bar{Y^2})_t = \frac{1}{N}\sum_{i=1}^{N} Y_i^2$ average values of the variables $Y$ and $Y^2$ of oscillators $i = 1, ...N$, at an instant of time $t$. Further, we average

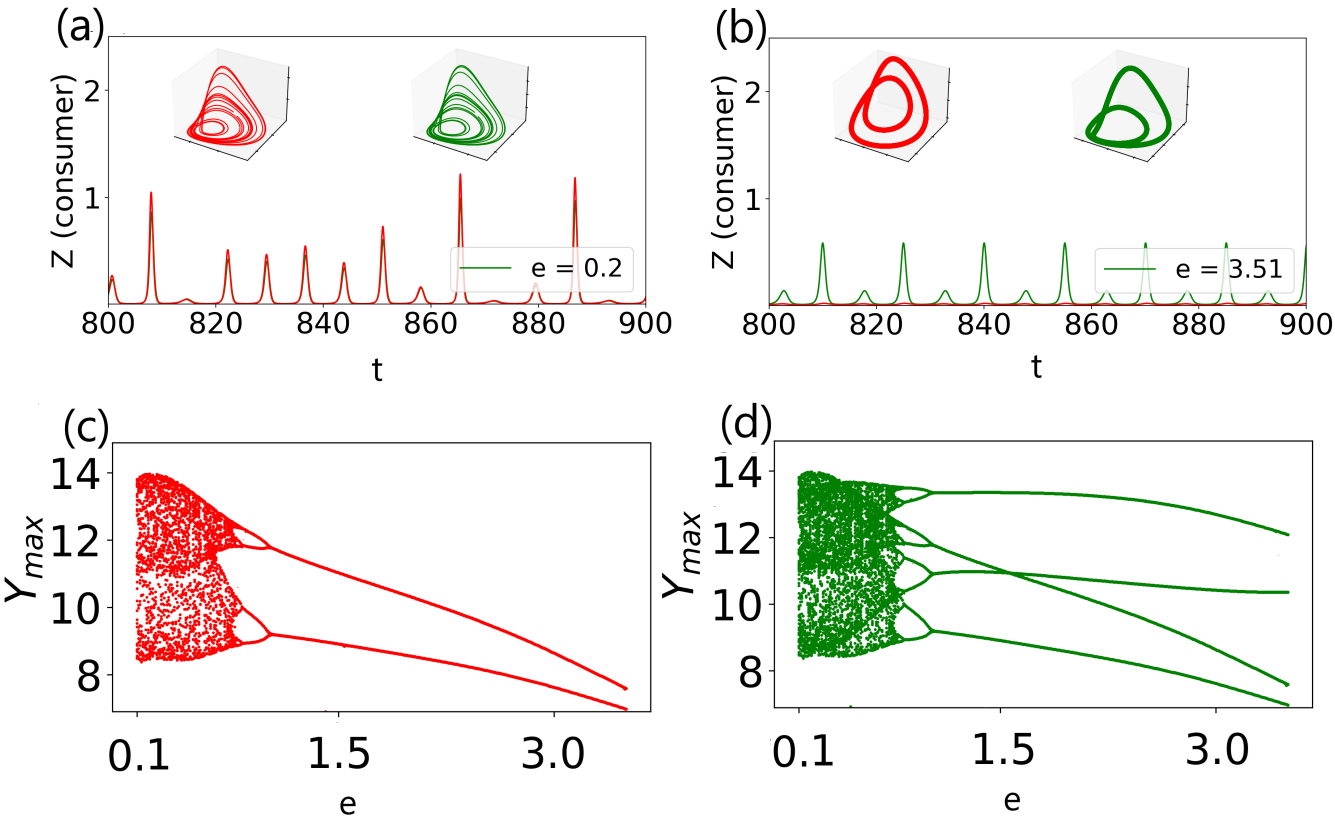


FIG. 3. Locally connected response patches: Nonidentical drive and homogeneous response patches. (a) Time series of consumer dynamics of both drive patch and a response patch for the low dispersal strength $e = 0.2$, (b) same as (a) for the high dispersal strength $\varepsilon_d = 3.5$, (c) bifurcation diagram of the drive patch as a function of $e$, and (d) bifurcation diagram of a response patch as a function of $e$. Attractors of the drive (red line) and a response (green line) patches are depicted in the inset of (a) and (b). Predator mortality rate for drive $b_d = 0.9$, and for responses $b_r = 0.8$.

$\eta$ over different initial states to obtain an ensemble averaged synchronization error $\eta$.

Figure 5 demonstrates the synchronization between all the patches in the metacommunity, when the responses are nearest neighbor coupled. When the drive and responses are completely identical, all patches are synchronized. However, with slight habitat heterogeneity in the system, the synchronization error has a low but non-zero magnitude. However, with increasing invasionary dispersal $e$, the synchronization between the patches decrease. Synchronization is detrimental to the persistence of the metacommunity. Therefore, it is clearly demonstrated that in this case, invasion increases the persistence of the metacommunity as a whole. With further increase in the heterogeneity in the network, for the range of dispersal $e0.1 - 1.5$, there is an increasing but fluctuating trend in the synchronization error $< \eta >$. However, with further increase in dispersal $e$, the synchronization error keeps steadily increasing without fluctuation.

### B. Globally connected response patches

#### 1. *Homogeneous drive and response patches*

First, we consider a group of globally connected, completely identical responses that are linked to an identical drive through dispersal, where we have fixed the predator mortality rate for both drive and response patches as $b_{drive} = b_{responses} = 0.9$. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 6(a) and 6(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 6(a) and 6(b). It is evident from these figures that the time series of $Z_d$ and $Z_r$ evolve identically corroborating the complete synchronization among the drive and uncoupled response systems for both low and high values of the dispersal strength. In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by

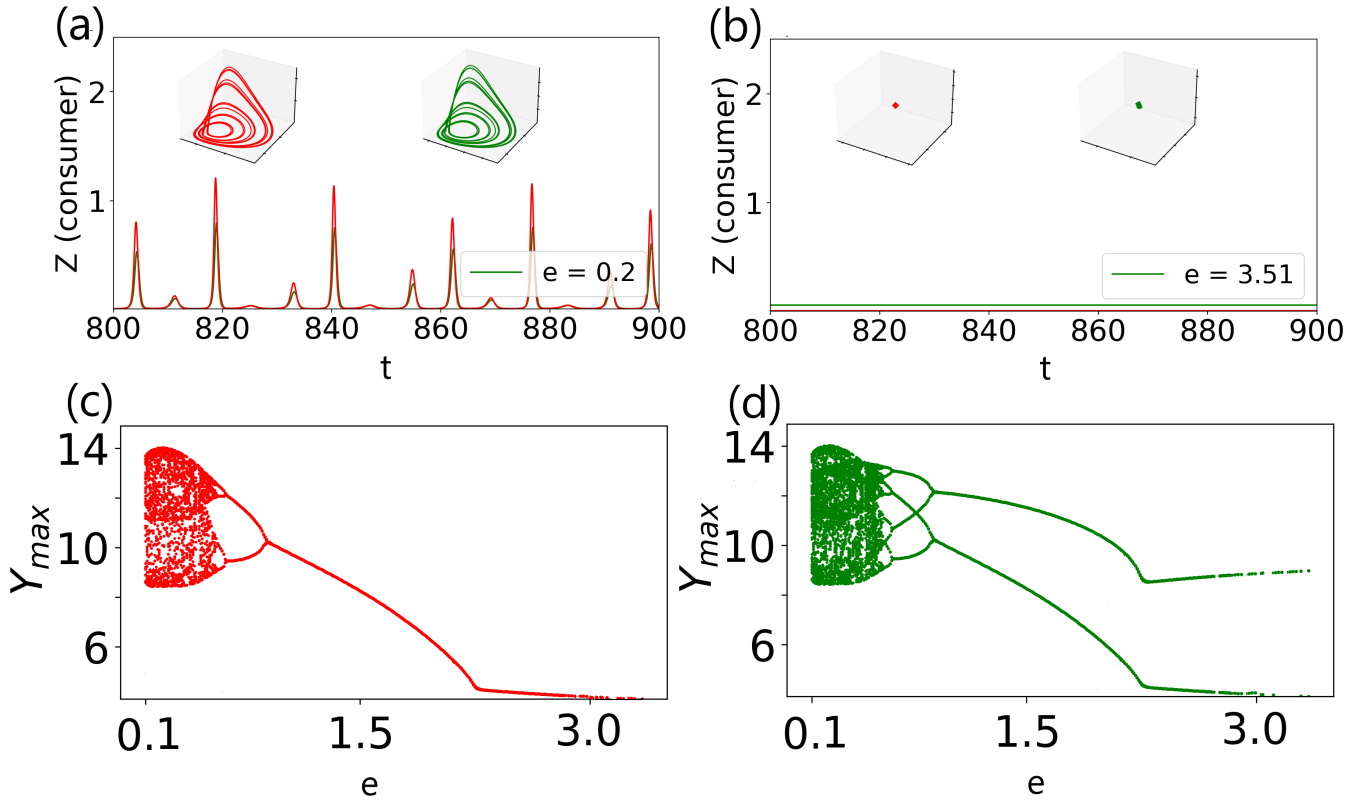


FIG. 4. Same as Fig. 3 but predator mortality rate for drive and response patches are $b_d = 0.9$ and $b_r = 0.77$, 0.76,0.82 and 0.85.

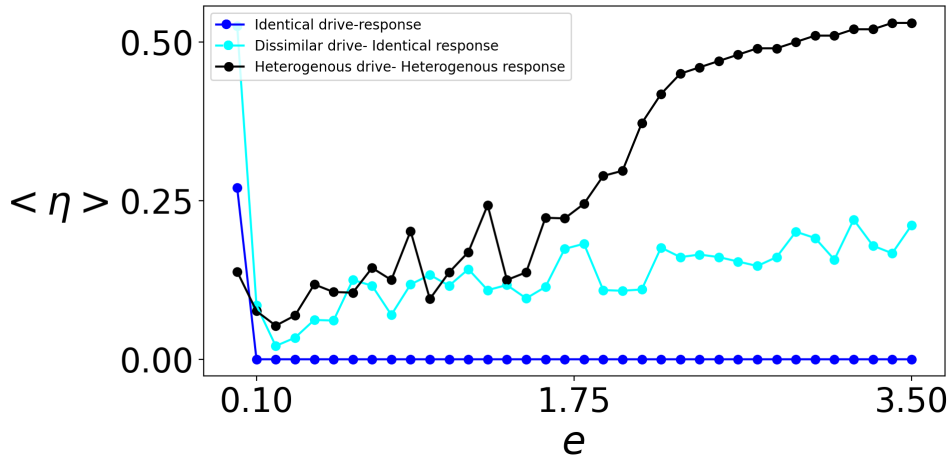


FIG. 5. N

collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 6(c) and 6(d), respectively, in the range of the dispersal strength $e \in (0.0, 3.5)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive and response patches exhibit chaotic oscillation for their population density in the entire explored range of the dispersal strength. Thus, the homogeneous drive is not capable of quenching the chaotic evolution of the population density in response patches.

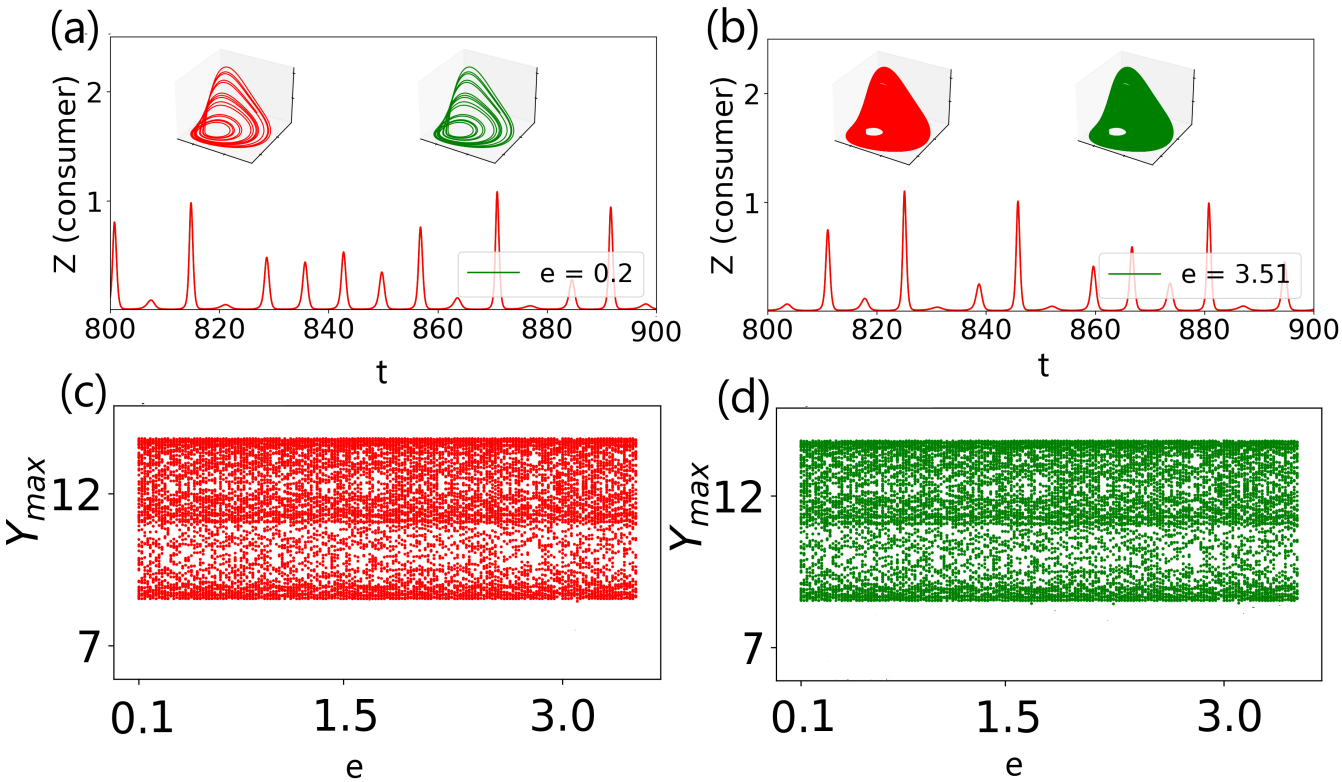


FIG. 6. Globally connected response patches: Completely identical drive and homogeneous response patches. (a) Time series of consumer dynamics of both drive patch and a response patch for the low dispersal strength $e = 0.2$, (b) same as (a) for the high dispersal strength $\varepsilon_d = 3.5$, (c) bifurcation diagram of the drive patch as a function of $e$, and (d) bifurcation diagram of a response patch as a function of $e$. Attractors of the drive (red line) and a response (green line) patches are depicted in the inset of (a) and (b). Predator mortality rate for drive $b_d = 0.9$, and for responses $b_r = 0.8$.

### 2. *Heterogeneous drive and homogeneous response patches*

Next, we consider a group of globally connected, completely identical responses that are linked to a non-identical drive through dispersal, where we have fixed the predator mortality rate for both drive and response patches as $b_{drive} = 0.9$ and $b_{responses} = 0.8$. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 7(a) and 7(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 7(a) and 7(b). In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 7(c) and 7(d), respectively, in the range of the dispersal strength $\varepsilon_d \in (0.0, 3.5)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive and response patches exhibit chaotic oscillation for their population density in the range of low dispersal strength. However, with increasing dispersal ($e\epsilon(0.8, 1.0)$), the dynamics of the drive patche undergo Hopf bifurcation from chaotic to a bistable state, consisting of 2 limit cycles. Upon further increase of dispersal ($e\epsilon(1.25, 3.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a bistable steady state. However, in case of the response patch, with increasing dispersal ($e\epsilon(0.8, 1.0)$) the dynamics undergoes Hopf Bifurcation from chaotic go mutli-stable limit cycles. Upon further increase of dispersal ($e\epsilon(1.25, 3.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a multi-stable steady state. Thus, the on-identical but similar drive quenches the chaotic evolution of the population density in completely identical locally connected response patches.

### 3. *Heterogeneous drive and homogeneous response patches*

Next, we consider a group of locally connected, heterogeneous responses that are linked to a non-identical drive through dispersal, where we have fixed the preda-

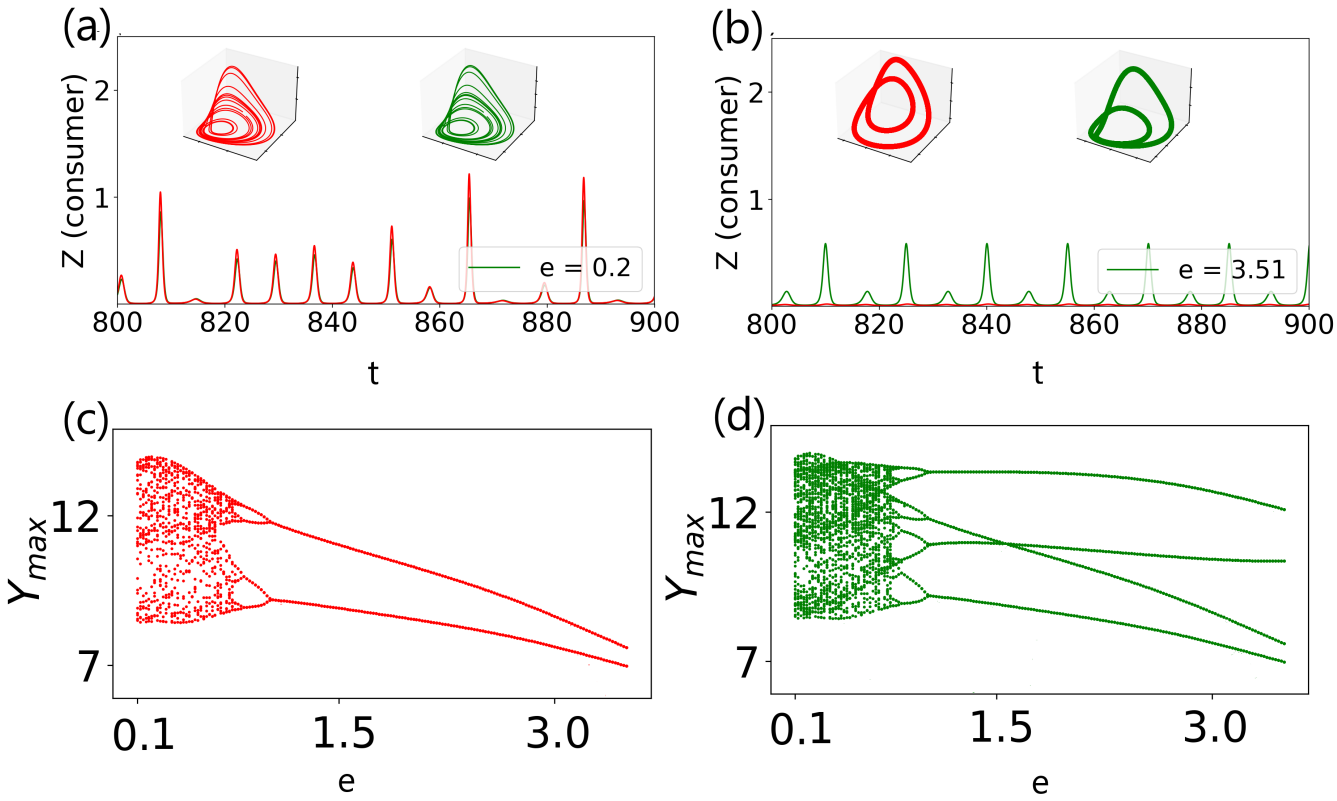


FIG. 7. Same as Fig. 6 but predator mortality rate for drive and response patches are $b_d = 0.9$ and $b_r = 0.8$.

tor mortality rate for both drive and response patches as $b_{drive} = 0.9$ and $b_{responses} = 0.77$, 0.76,0.82 and 0.85. The temporal dynamics of the consumers from the drive patch $Z_d$ and a representative response patch $Z_r$ are depicted in Fig. 8(a) and 8(b) for the dispersal strengths $e = 0.2$ and 3.5, respectively. The dynamics evidently changes from chaotic to steady state on increasing drive-response dispersal. The chaotic attractor of the both drive patch (red line) and a representative response patch (green line) are also shown in the inset of both Figs. 8(a) and 8(b). In order to elucidate the nature of the dynamics exhibited by both drive and response systems, we have plotted their bifurcation diagrams, obtained by collecting the maxima $Z_{dmax}$ and $Z_{rmax}$ of the temporal dynamics of the drive and response patches, in Figs. 8(c) and 8(d), respectively, in the range of the dispersal strength $\varepsilon_d \in (0.0, 3.0)$. It is evident from these bifurcation diagrams that the evolution of the consumer, and consequently resource $X$ and predator $Y$, of both drive and response patches exhibit chaotic oscillation for their population density in the range of low dispersal strength ($e\epsilon(0.1, 0.25)$). However, with increasing dispersal ($e\epsilon(0.4, 0.9)$), the dynamics of the drive patch undergo Hopf bifurcation from chaotic to a multistable state, consisting of 4 limit cycles to ultimately bistable states consisting of 2 limit cycles. Upon further increase of dispersal ($e\epsilon(0.8, 2.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a monostable steady state, with a decreasing trend in the average population density. On further increase in dispersal, ($e > 2.5$), the mono stable state is maintained but the average population density saturates. In case of the response patch, with increasing dispersal ($e\epsilon(0.1, 0.25)$) the dynamics stays chaotic. Upon further increase of dispersal ($e\epsilon(0.25, 0.8)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a multistable limit cycle. Upon further increase of dispersal ($e\epsilon(0.8, 2.5)$) the dynamics of the drive patch further undergoes reverse bifurcation to obtain a bistable steady state, with a decreasing trend in the average population density. On further increase in dispersal, ($e > 2.5$), the bistable state is maintained but the average population density saturates. Thus, the non-identical but similar drive quenches the chaotic evolution of the population density in non-identical locally connected response patches.

### 4. *Synchronizatiopn of the network*

Figure 9 demonstrates the synchronization between the different patches of the metacommunity. The synchronizations follow the same nature as that of Fig. 5.

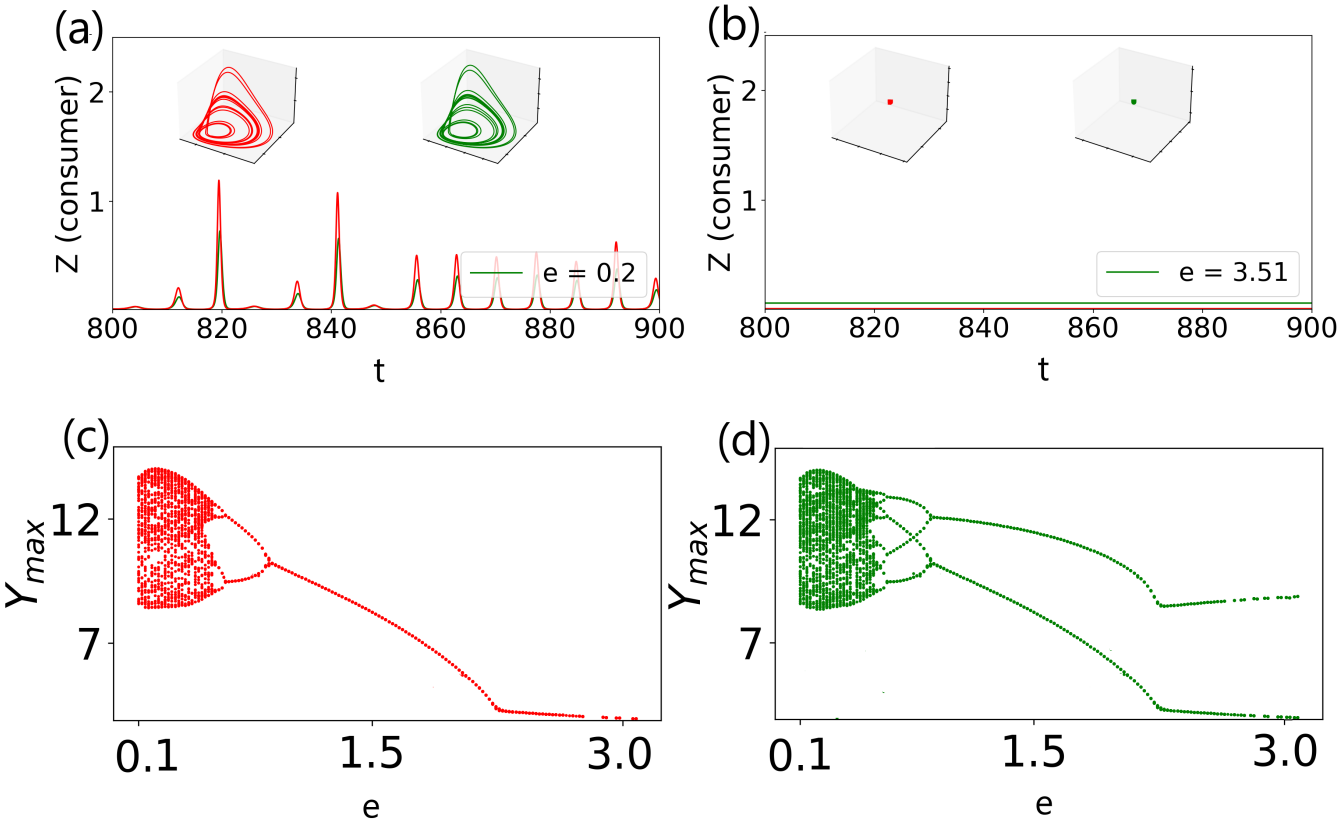


FIG. 8. Same as Fig. 7 but predator mortality rate for drive and response patches are $b_d = 0.9$ and $b_r = 0.77$, 0.76,0.82 and 0.85.

## IV. DISCUSSION AND CONCLUSION

Chaotic population dynamics in real uncoupled ecosystems has been a staple subject of studies in the recent years. Barring a specific number of conditions fulfilled, chaotic fluctuations in populations have generally been associated with low persistence. Noises and rapid, large fluctuations subject populations to genetic bottlenecks and stochastic extinctions and renders them less stable as a consequence. However, inherently chaotic ecosystems continue to exist.This can be explained by control or quenching of chaos engineered by inherent ecological mechanisms or external forcings on ecosystems. The question of chaos control or chaos quenching in ecological systems coupled through dispersal have not been investigated so far to the authors' knowledge. Such a ecological network is called a metacommunity. In this study, we looked at how a locally connected and globally network of response (ecosystems) patches behave when regulated by an external drive patch. For both network structure, we have considered the case of completely identical drives and responses and demonstrate that chaotic fluctuations in this case are too strong to quench, irrespective of drive dispersal amplitude. Then we introduce the second level of heterogeneity in drives and responses by the means of predator mortality $b$ varying across the patches. This leads to quenching of chaos in case of high levels of dispersal. The degree of drive dispersal that quenches the chaos also depends on whether the responses are themselves dispersally connected or not. This strongly signifies that hetereogenous metacommunities quench the chaos to be more persistent. Furthermore,for both kinds of network structure, we also investigate various kinds of synchronizations induced by dispersals between the drive and response patches. Even with quenching of chaos, generalized synchronization is established in some, but not all of the cases. The complete synchronization (quantified by synchronization error) and phase synchronization is non-existent in most cases. These two types of synchronization is heavily determined by network structure. However,the same investigation into synchronization between two response patches yield that in most cases, both phase and generalized synchronization exists in most cases. However, complete synchronization exists in only when two responses are completely identical. From the above results, we conclude that, barring a few idealized cases of completely identical drive and responses, dispersal primarily quenches chaos in real metacommunities consisting of inherently chaotic ecosystems and heterogeneous patches. Indeed, this study highlights

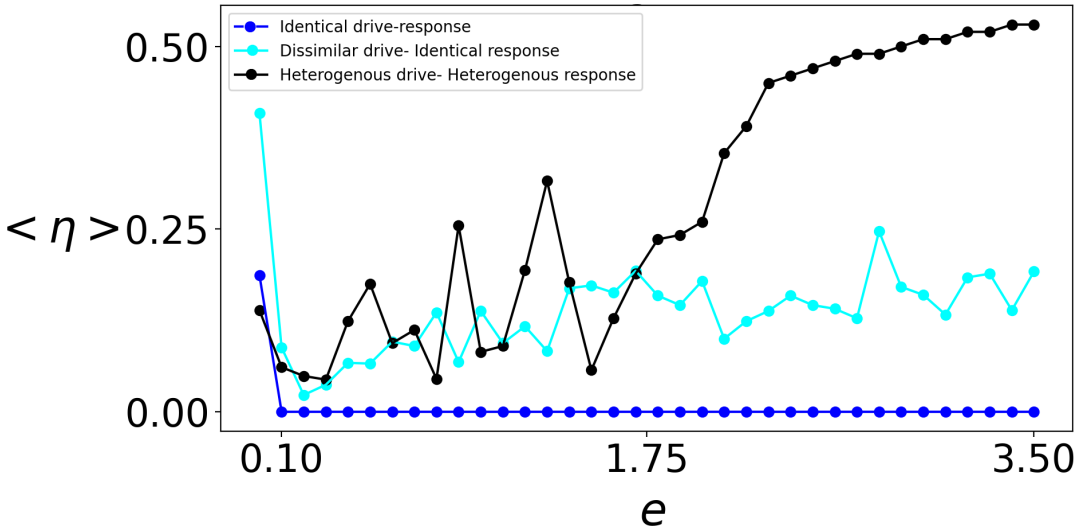


FIG. 9. N

the relevance of habitat heterogeneity in increasing persistence (or decreasing extinction risk) of metacommunities through control and quenching of chapos. This study also comments on the relevance of dispersal as one of the most fundamental mechanisms stabilizing the dynamics of metacommunities. This study quantifies the level of drive dispersal required to quench chaos and whether the effect of such dispersal on the metacommunity is stabilization or extinction. This study further proves that whether the post-quenching steady-state dynamics has high species-density or low species-density depends on degree of dispersal, habitat heterogeneity and also on the network structure of the metacommunity.

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