Asynchronous population dynamics induced by higher-order and negative asymmetric ecological interactions

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Phase synchronized population dynamics of various species constituting a complex ecosystem elevates the risk of their extinction due to both environmental stochasticity and simulateneous low density fluctuations. Therefore, an extremely efficient approach to measure the extinction risk of an ecosystem as a whole is to quantify the phase synchrony among the co-habiting species populations. Generally, in models describing population dynamics of ecosystems, both trophic and non-trophic inter-species interactions are classified as two-species interactions. This approach contradicts the fact that several species living in close proximity must partake in the same interaction, influencing the population dynamics of each other. To address this, higher-order interactions need to be incorporated in the models describing population dynamics of an ecosystem and their effect on phase synchronization of populations need to be investigated. In this study, we model a species-rich ecosystem as a multi-node complex ecological network and examine the phase dynamics of the total species population. Each node of this network represents a constituent species, modelled as a Sakugachi-Kuramoto phase oscillator coupled non-linearly to the other nodes through both first-order and higher-order inter-species interactions. These interactions can be both commensalist/mutualist (positive) and antagonistic (negative) in nature. Along with the higher order interactions, we also incorporate inherent asymmetry among the nodes to account for habitat heterogeneity. We investigate the effects of both higher-order coupling and asymmetry on the phase synchronization of the ecological network as a whole. Our findings demonstrate that higher-order interactions above a threshold amplitude result in a transition from synchronous to asynchronous dynamics of the ecosystem. Further, we find that increase in the size and diversity of the ecosystem lead to increased threshold higher order coupling required to reach asynchronous dynamics, though eventually it happens. We also demonstrate that the asynchrony in dynamics induced by higher-order interactions, is further promoted by high asymmetry among the individual nodes. Notably, negative inter-species interactions, if existing to a high degree also induce asynchrony in the system. However, the size of the network also plays a role in deciding the threshold value of higher order coupling required to induce asynchrony.

Keywords: Sakaguchi-Kuramoto model, higher-order interaction, mutual and antagonsitic interactions, asynchronous dynamics, persistence of ecological networks

I. INTRODUCTION

Ecosystems consist of populations of multiple interacting species living in close proximity. A paradigm of modelling these populations are prey-predator models[1, 2]. Often in prey-predator models, the prey and predator populations exhibit oscillatory dynamics as a single oscillator [3]. For a spatially extended ecosystem, such oscillators are coupled with one another linearly through processes like dispersal [4], or non-linearly like cross-trophic interactions [5] or other competitive and/or mutual interactions [6, 7]. Ecosystems are thus accurately described as complex networks formed by coupled oscillators [8], where the total population dynamics of the network is governed by non-linear, coupled differential equations. As species-richness (i.e., number of species residing an ecosystem) increases, the number of coupled non-linear equations increase and the complex parameterisation and limited analytical scope of these equations pose a serious challenge to accuracy of predictions. To address this, modelling of population of large ecological networks formed by various species needs to be simplified without losing the essential dynamical features of these models, with specific focus on measures quantifying extinction risk of a large population forming a large ecological network. It is well established that when all species residing in an ecosystem have phase synchronized population dynamics, the persistence of the ecosystem is lowered, while asynchronous population dynamics of the constituent species denote greater resilience of an ecosystem[9, 10]. Therefore, an essential approach to quantify the extinction risk of the population of a large ecological network is to investigate the phase synchronization of the total population dynamics of the network. For investigating phase phase dynamics of large populations of coupled oscillators, the Sakaguchi-Kuramoto model, focusing on the phases of oscillators and the synchronisation of these phases, offers an extremely powerful framework [11].

Indeed, recent studies have used the Sakaguchi-Kuramoto model, to illustrate significant ecological effects [12]. [6, 12, 13] have demonstrated that there exists strong similarities in dynamical behavior of a population described by the paradigmatic prey-predator Rozensweig-McArthur model and the dynamics described by the Sakaguchi-Kuramoto model. Thus, prey-

predator pairs can be effectively modelled using the Sakaguchi-Kuramoto framework, with special focus on the phase dynamics of the population. [13] has reported that weak cross-feeding leads to in-phase synchrony among the oscillators, while competitive interactions result in anti-phase synchrony, both of which are phase synchronizations nevertheless. Empirical confirmation of oscillatory dynamics in such qualitative coupling arrangements has also been observed [14], cementing the importance of the phase-oscillator approach in modelling populations of large ecological networks. Therefore, an ecosystem can be described as a large ecological network, which in turn consists of non-linearly coupled Sakaguchi-Kuramoto phase oscillators. This is the approach we take in this work.

As evidenced by numerous studies, close proximity of mutltiple species in a species-rich ecosystem ensures that various inter-species interactions are essentially not first-order in nature, but rather involve more than two species simultaneously [15–18]. Therefore, in this study we consider the inclusion of higher-order (second-order) inter-species interactions both trophic and non-trophic in nature. An ecologically suitable network model must also account for habitat heterogeneity found in a natural ecosystem. Therefore, we incorporate asymmetry [19] in the dynamics of the constituent oscillators or nodes forming the ecological network, i.e., we allow each of the oscillators or nodes to possess varying phases even as they tend to synchronise. An adjacency matrix is defined and introduced to detail the interactions (couplings) between the constituent nodes of the network. The interaction between two species in an ecosystem are not always exclusively trophic in nature. Interactions like cross-feeding, resource competition, mutualism, commensalism and others involving species from different pairs are prevalent in ecosystems [5, 6]. These interactions either benifits each of the species involved mutually (i.e., postive) or antagonizes all the involved species (i.e., negative).

Our study demonstrates that incorporating the aforementioned modifications to the phase oscillator model leads to overall asynchronous dynamics for the whole network. Asynchronous dynamics denotes beneficial conditions within the ecosystem, often due to creation of a source-sink metacommunity [4]. Therefore, the continued existence of numerous ecological networks may quite possibly be due to the overall network dynamics of an ecological network being asynchronous, which in turn very strongly indicates the presence of large magnitude of higher-order interactions, as illustrated by our results.

II. MODELS

We construct an ecological network, where each node of the ecological network behaves as a phase-oscillator governed by the higher order Sakaguchi-Kuramoto model. To account for multi-species interactions and inbuilt heterogeneity of a spatially extended natural ecosystem, we introduce asymmetry and higher-order interactions. In this study, we have considered two kinds of topology to describe ecological networks. First, we consider globally coupled networks (i.e., all nodes are interconnected), and then we extend our results for a random network structure, created using Erdös–Rényi random network formalism with finite connectivity probability. Both these network structures are quantified by an adjacency matrix incorporated into the dynamics. Factoring in all this, the combined phase dynamics of the total ecological network formed by coupled phase oscillators, is governed by the equation [20, 21],

$$\dot{\theta}_i = \omega_i + \frac{K_1}{N}\sum_{j=1}^{N}\Gamma_{i,j}\sin\left(\theta_j - \theta_i - \alpha_1\right)$$
$$+\frac{K_2}{N^2}\sum_{j=1}^{N}\sum_{k=1}^{N}\Gamma_{i,j}\Gamma_{i,k}\sin\left(\theta_j + \theta_k - 2\theta_i - \alpha_2\right), \quad (1a)$$

where, $0 \leq \alpha \leq \pi/2$ is the asymmetric phase lag parameter which introduces a phase delay among the interacting oscillators, K_1 and K_2 are the 1st and second-order coupling terms and the adjacency matrices γ_i, j and γ_i, k captures the interactions between oscillators i,j and i,k respectively. By adjusting the value of α_1 and α_2, we can introduce varying degrees of asymmetry in the coupling. All ω_is were chosen at random from a normal distribution with standard deviation of 0.001 and initial condition for $\vec{\theta}$ were generated randomly. The degree of phase synchrony of the entire ecological phase-oscillator network is quantified by the order parameter measure, which is given by,

$$z = Re^{i\theta} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \quad (2)$$

, where, R is the vector sum of the individual vectors and Θ is the average phase. When $\mod z = 1$, the oscillators are fully phase synchronised and when $\mod z = 0$, the oscillators are anti-phase synchronised or completely asynchronous. Throughout this work, ω_i

A. Positive and negative interactions

Previously, we highlighted the presence of various interactions within the same network. These interactions can lead to either synchrony or asynchrony. Mutualistic interactions are considered positive interactions (+) which have a synchronizing effect between the nodes, while antagonistic interactions are considered negative interactions (-) have an asynchronous effect on the network dynamics[22].[23] states that stable equilibrium is maintained through positively coupld oscillators synchronizing with one another while avoiding proximity to negatively coupled counterparts. To account for such various

possible inter-species interactions (couplings) naturally occuring in an ecosystem, we further incorporate both positive and negative interactions, manifested through both (+ve) and (-ve) K_1 and K_2 values in the network dynamics.

III. RESULTS

A. Effect of increasing asymmetry on the network dynamics

Figure. 1 demonstrates the effect of increasing asymmetry (α_1) on a large network of 50 nodes linked through exclusively first-order coupling. Fixing the first-order coupling value, we vary the first-order asymmetry parameter α_1 across the range $0-\pi/2$. With increasing coupling strength, the network exhibits strong phase synchrony. However, with increasing asymmetry (α_1), the collective transition of oscillators to phase synchrony starts to dissipate. For high value of asymmetry, perfect phase synchronization is lost and partial phase synchronization is exhibited. Figure. 1(b) demonstrates that inclusion of only asymmetry in the dynamics can lower the degree of phase synchrony of the network but cannot give rise to the more persistent asynchronous dynamics. Next, we investigate the effect of increasing network size and varied network structure on this trend.

Figure. 2(a) demonstrates the behaviour of the phases of a globally coupled network of 20 nodes, for even higher first-order asymmetry. When asymmetry is very high, the oscillators form 2 clusters. Even though the oscillators in a single cluster are phase synchronized (with phases varying over a limited range), the 2 clusters are individually anti-phase synchronized,lowering the phase synchorny of the dynamics. Notably, there are also few oscillators which do not cluster with any of the groups. Such elements are called chimeras. As demonstrated by figs. 2(b), with increasing size of globally coupled networks, the distribution of both the phase synchronized cluster begins to increase, but the size of the network also implies such distribution should occur. A much more prominent change in noticed in case of random networks. In case of high asymmetry, the clusters are numerous in number, smaller in size and in fact, slightly increases with increasing network size. In case of random networks, figs. 2(c) and 2(d) demonstrate an intriguing relationship between high value of α_1 and the synchronization patterns of the network, suggesting the existence of a critical threshold, after which higher values of α_1 leads to a transition of the network dynamics from synchrony to complete asynchrony. Notably however, the asymmtery required for these networks to exhibit asynchronous dynamics is pretty high, and we will elaborate more on that later. Our results demonstrate the emergence of anti-phase group dynamics in the network when the network topology is global, and the emergence of chimeric elements and singular splay states when the network topology is represented by a random graph. Figure. 2 also showcases the distribution and arrangement of clusters and chimeric elements within the networks.

B. Effect of higher-order coupling on network dynamics

As discussed before, to model an ecological system realistically, it is imperative to consider more than 2 species as part of the same interaction. Accordingly, we turn on higher-order (second-order) interaction between different oscillators on the dynamics of the ecological network with the amplitude of both first-order (K_1) and higher-order coupling (K_2) being comparable and investigate its impact. The introduction of second-order interactions drastically disrupts the phase synchrony within the network, leading to overall asynchronous network dynamics when the coupling altitude reaches a value higher than a threshold. Figure. 3(a) demonstrates for a realistic value of K_1, the decrease of order parameter Z of a globally coupled 20 node network with increasing higher order coupling. There is a transition of order parameter from high value (1.0, signifying perfect phase synchrony between the nodes) to very low value (≈ 0.2, signifying very low phase synchrony between the nodes) and eventually approaching zero when the higher-order coupling increases beyond a threshold value of higher-order coupling. This also corresponds to formation of distinct clusters (and chimeric elements) of phase synchronized oscillators, but much smaller compared to the previous case where no higher-order coupling existed.

Further, we investigated the impact of the topology and network size on this emerging trend. Figures. 3(b) and 3(c) demonstrates that with increasing network size, the threshold value required for the network to transition unto asynchronous dynamics increase. However, with increasing higher order coupling, the transition to asynchronous dynamics is inevitable. Both the trends of transition of network dynamics from phase synchrony to asynchrony when K_2 values reach higher than a threshold and the increase in this threshold with increasing network size is mirrored by random networks. It is non-trivial that random networks undergo this transition as it implies that these trends are robust to network structure and network size. To solidify this claim, we plot the critical threshold value of K_2 required for the system to transition into asynchronous dynamics against increasing network sizes in fig. 4. Figure. 4(b) demonstrates that even though the threshold K_2 value might vary even for a fixed network size, it is due to randomness in the network and the trend of increasing in threshold values carry on nevertheless.

Since very high asymmetry and higher-order coupling above a threshold can both induce asynchrony in the network dynamics, we investigate which one is the more dominant factor in that aspect. Figure. 5(a) demonstrates the order parameter with simultaneous variation

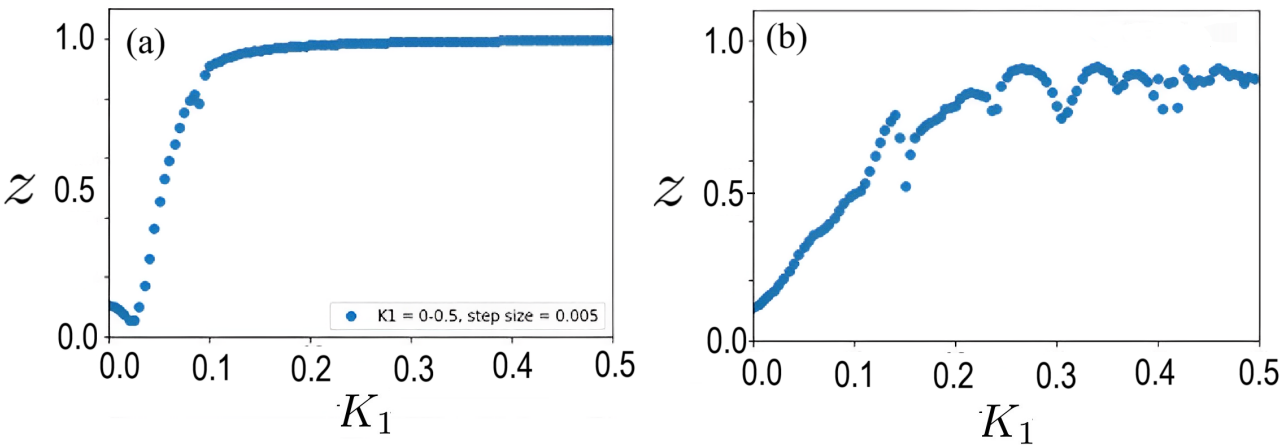

FIG. 1. The effect of different values of α_1 on phase synchronization of the ecological network. a) $\alpha_1 = 0.1$, b) $\alpha = 0.8$. Higher α values break the symmetry of synchronisation

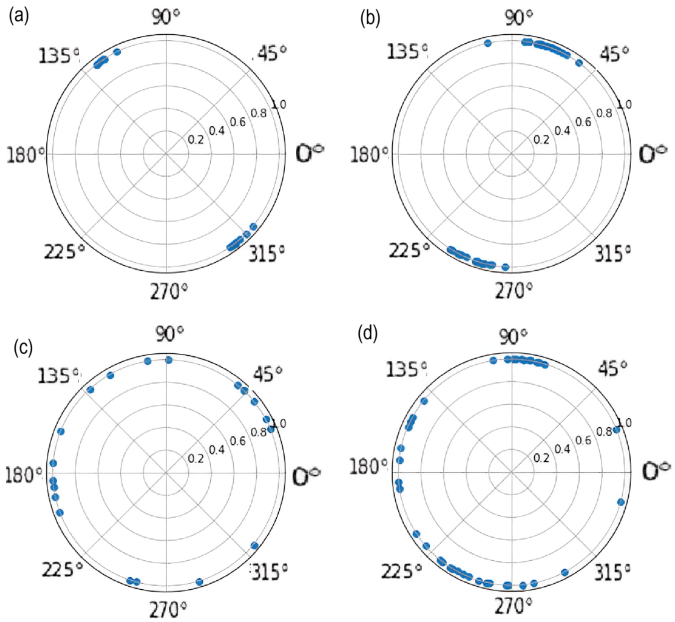

FIG. 2. Clusters of phase synchronized oscillators, i.e., synchrony groups are formed for high asymmetry ($\alpha_1 = 0.8$) and existence of chimeric elements found for global network with (a) 20, (b) 40 nodes. More prominent asynchronization, for same asymmetry is found for random network with (c) 20, (d) 40 nodes. Asynchrony is more prominent in case of random network.

in first-order asymmetry α_1 and higher order aysmmetry α_2 for a network of 20 nodes in absence of higher-order coupling. Evidently, only very high-level of asymmetry can induce asynchrony in network dynamics. It is unlikely that real ecosystems naturally and inherently select a narrow range of very high asymmetry, irrespective of other conditions. However, figs. 5(b) and 5(c) demonstrates that the interplay of increasing K_2, very high α_1 and high α_2 induces asynchrony in the system. For comparable $K1$ and $K2$ values, asynchrony starts increasing. For increasing K_2 values, there is a transition to complete asynchrony for high α_1 and α_2 (refer fig. 5(d)). For intermediary α_1 and α_2, there is also low levcel of synchrony and for low α_1 and α_2, limited partial synchrony exists. For Large K_2, almost for all range of α_1 and α_2, the phase synchrony in network dynamics is extremely limited, and even for low α_1 and α_2, phase synchronization is half of that of perfect synchrony. Hence, the presence of higher-order interactions is necessary for asynchronyous dynamics to exist.

Next , we analyze the trend of asynchrony induced by comparable K_1 and K_2 values the order parameter

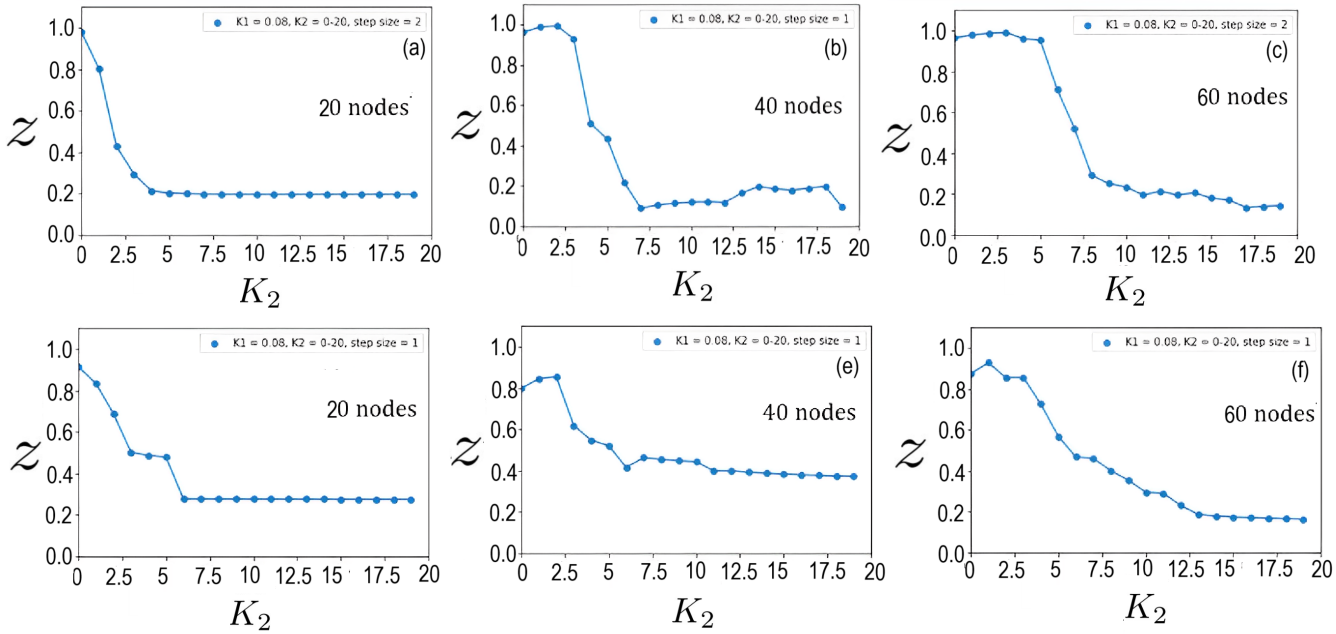

FIG. 3. Order parameter vs higher-order coupling (K_2) plot for global network of with (a) 20, (b) 40 and (c) 60 nodes. The same is done for random network with (d) 20, (e) 40 and (f) 60 nodes. The parameter values are $\alpha_1 = \pi/4 \approx 0.78$, $\alpha_2 = \pi/4 \approx 0.78$ and $K_1 = 0.08$.

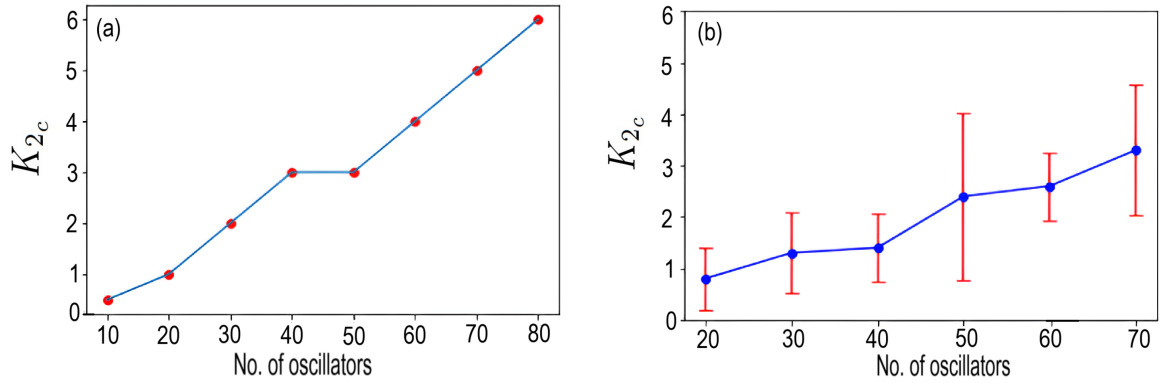

FIG. 4. The threshold higher order coupling (K_2) required for network to exhibit asynchronous dynamics plotted against increasing network sizes, once for (a) global network and once for (b) random network. The parameters are same as the previous figure.

for increasing size of network, by fixing the amplitude of first-order and higher-order coupling. Figure. 6(a) is the same as fig. 5(c). When network size increases to 40 nodes, for very high asymmetry, asynchrony is induced, but the range pf α_1 and α_2 for which asynchrony exists is decreased (refer fig. 6(b)).

C. Impact of negative interactions on phase synchrony of ecological networks

As discussed before, the inter-species interactions naturally occuring in an ecosystem may be non-trophic, and thus mutualistic or antagonistic. These interactions can be modelled as positive or negative interactions, represented by positive and negative values of K_1 and K_2.

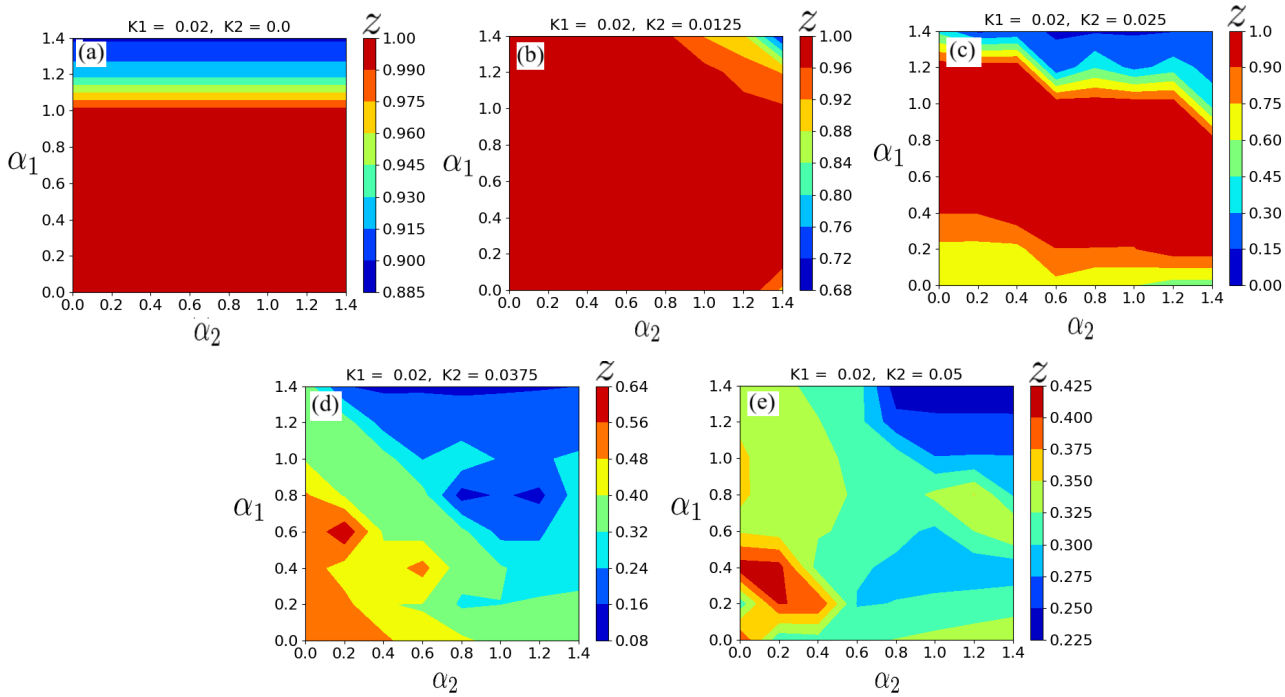

FIG. 5. The order parameter variation with simultaneous variation with α_1 and α_2 for fixed K_1 and K_2. The K_2 value increases from (a) -(e). The distribution is (a)$K_2 =$, (b)$K_2 =$, (c)$K_2 =$, (d)$K_2 =$ and (e)$K_2 =$.

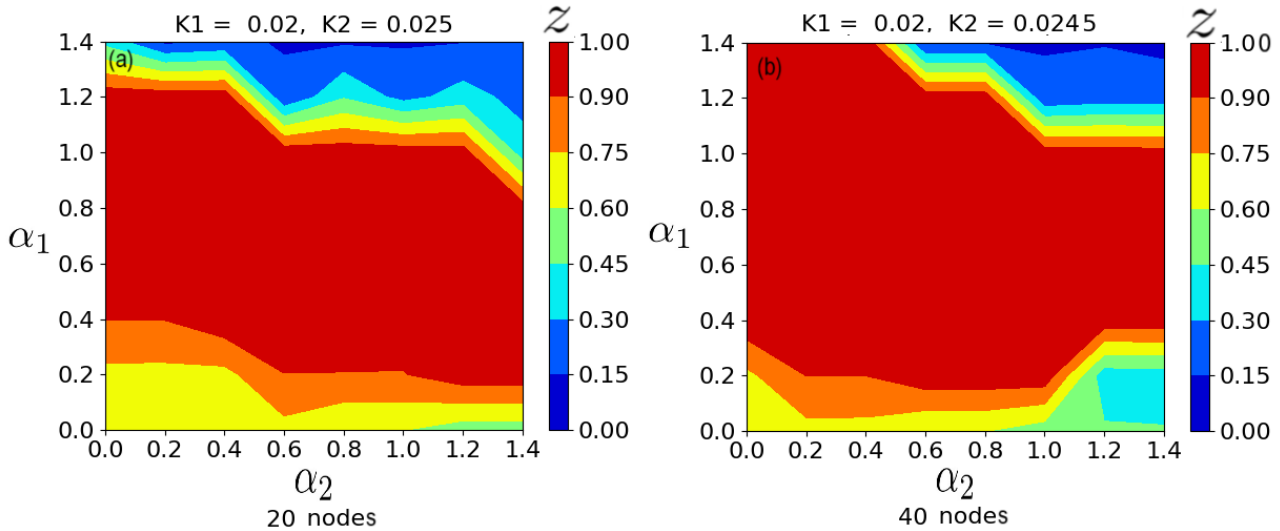

FIG. 6. The order parameter variation simultaneous variation with α_1 and α_2 for fixed K_1 and K_2, for a global network of (a) 20 nodes and (b) 40 nodes.

1. *Impact of increasing degree of negative first-order coupling*

We measure the effect of negative first-order interactions on the ecosystem. Negative first-order coupling, represented by negative values of K_1. We plot the order parameter of a 20 node network against increasing. First, we introduce 25% negative first-order couplings. As demonstrated in fig. 7(a), the oscillators formed two clusters for 75 percent positive values and 25 percent negative values, and the average order parameter value was lower than that in the case of fig.1(a). This implies that the network is partially phase synchronized. Figure. 7(b) shows that the system becomes completely asynchronous

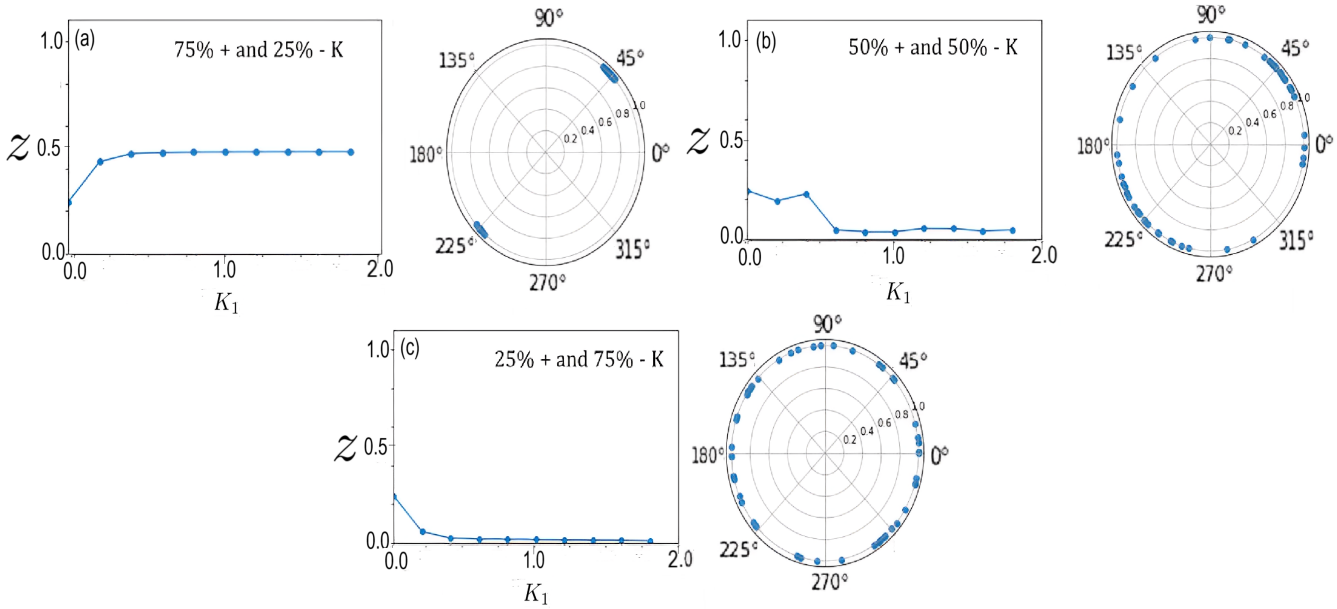

FIG. 7. Order parameter vs first-order coupling (K_1) for a global network of 20 nodes, with increasing percentage of negative coupling from (a)-(c). The negative coupling percentages are (a)25, (b) 50 and (c) 75. The parameter values are $\alpha_1 = \pi/4 \approx 0.78$.

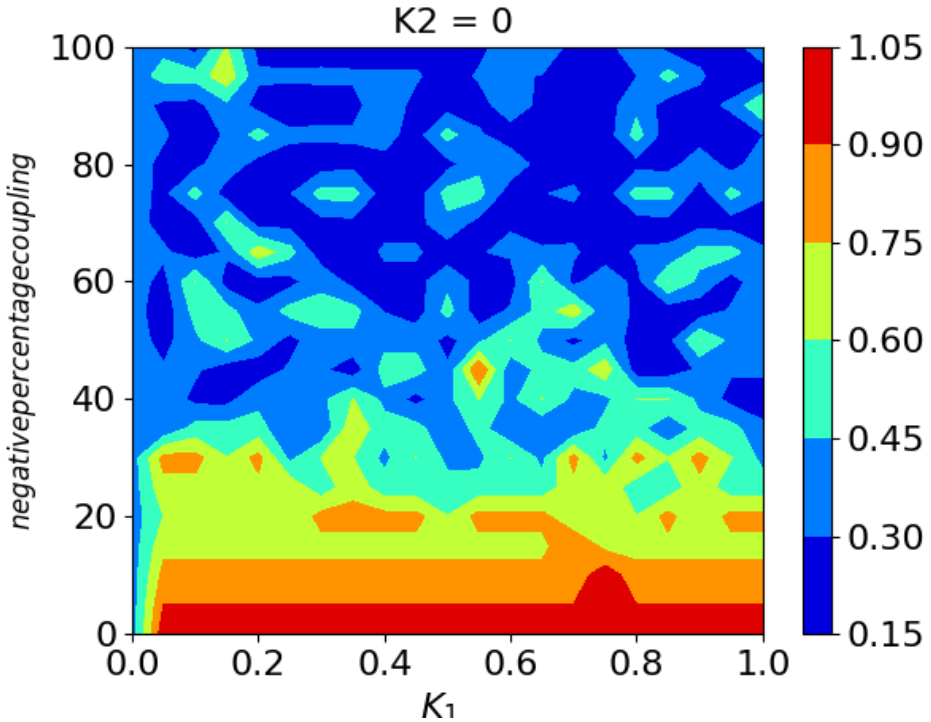

FIG. 8. Order parameter vs simultaneous increase of first-order coupling (K_1) and percentage of negative coupling for a global network of 20 nodes. The parameter values are $\alpha_1 = \pi/4 \approx 0.78$.

even for low values of K_1 when 50% K_1s are negative. On increasing the negative percentage of K_1s further, i.e., 75% negative K_1, we find that the oscillators are more asynchronously spread out. Figure. 7(c) demonstrates that presence of more positive first-order couplings than negative ones increase phase synchrony of network with increasing coupling amplitude, while the increase in percentage of negative first-order couplings leads to asynchronous dynamics in the network.

Next, we plot the effect of negative first-order couplings for increasing negative coupling percentage. We repeat this for different values of K_1. This complete picture is illustrated in fig. 8. For very low percentage of first-order coupling, there is synchrony in the network. However, with increasing percentage of negative percentage, there is steady decrease in synchrony of the system. For more than 40% negative first order coupling, the network either exhibits extremely low synchrony or complete asynchrony. This trend is repeated for all values of K_1.

2. *Impact of increasing degree of negative higher-order coupling*

We measure the effect of negative higher-order interactions on the ecosystem. Since even positive higher-order interactions induce asynchrony in the system, we directly plot the order paramater of the network corresponding to increasing percentage of negative coupling. We then repeat this for each value of K_2. Figure. 9 portrays that over a particular threshold value of K_2, there are specific ranges of K_2 for which both high and low percentages of negative coupling induces asynchrony in the system. This implies that both positive and negative K_2 induces asynchrony over a threshold value, cementing the importance of higher order interactions in increasing persistence of ecosystems.

3. *Impact of increasing degree of negative first-order and higher-order coupling*

We plot the order parameter of the network with simultaneously increasing values of K_1 and K_2 in fig. 10. Figure. 10(a) demonstrates that for no negative coupling, asynchrony is induced in the network only when K_2 is larger than K_1. With slight increase in negative coupling percentage (of both K_1 and K_2), the K_2 value required to induce asynchrony corresponding to each value of K_1, is slightly less as demosntrated in fig. **??**(b). On increasing the negative coupling percanetage of both K_1 and K_2 to 25%, not only does the red region signifying the phase synchronized dynamics of network decreases, the higher synchrony attainable is also not perfect but partial. This region exists only for high values of K_1 as illustratd in fig.10(c). When half (50%) of the K_1 and K_2 are negative, the region of synchrony gets decreased further, and the degree of synchrony is even lesser as illustrated in fig.10(d) . When majority of the K_1 and K_2 couplings are negative (75%), the degree of synchrony decreases further as well as the region shrinks further. Barring a very few randomly distributed pockets of large synchrony, for both high and medium K_2 and high K_1, the network exhibits asynchronous dynamics as demonstrated by fig.10(e). When all K_1 and K_2s are negative, very few very small pockets of partial synchrony exist for different values of K_1 and K_2. There is no continnuous region of asynchrony, while for low and medium values of K_2, there exists large ranges of complete asynchrony as illustrated in fig.10(f).

IV. DISCUSSION AND CONCLUSION

Population dynamics of a variety of ecosystems modelled as prey-predator systems exhibit an oscillatory behavior, with distinct phase dynamics. If such oscillations are phase synchronized, the persistence of the ecosystem is lowered as the total population reaches low density value simultaneously and is more susceptible to external factors like environmental noise, Moran effect (environmental conditions) or disease outbreaks. This could lead to the total population getting extinct through cascading effect. In contrast, if the total population dynamics is asynchronous, that implies increased persistence due to increased chance of community survival through rescue effect. Therefore, to measure extinction risk of an ecosystem, an investigation measuring the total phase dynamics of all the constituent population is needed.

Generally, species-rich ecosystems consist of numerous interacting species living in close proximity. Therefore, ecosystems can be accurately modelled as large complex networks, with each species acting as nodes and the inter-species interactions acting as links. Most of such interactions are non-linear and are incorporated into the prey-predator models which describe the population dynamics. Numerous studies establish that the phase dynamics of important prey-predator models can be fairly accurately approximated using a phase oscillator model with suitable modifications. Thus, these ecological networks can be modelled where each node is a phase oscillator, linked to each other through interactions, whose amplitude is defined by a coupling strength. In this study, we have used the paradigmatic Sakaguchi-Kuramoto phase oscillator model to represent the nodes of a species-rich ecological network. The close proximity of a multitude of species in the ecological network ensures that inter-species interactions are not pairwise. To account for numerous likely interactions involving more than two species, we include higher-order interaction term in the model of each phase oscillator. Further, to account for inherent habitat heterogeneity of an ecosystem, we include asymmetry in the dynamics of the oscillators. We study the total phase dynamics of this network, determine if the dynamic is phase synchronized quantify the degree of phase synchronization and thus commment on

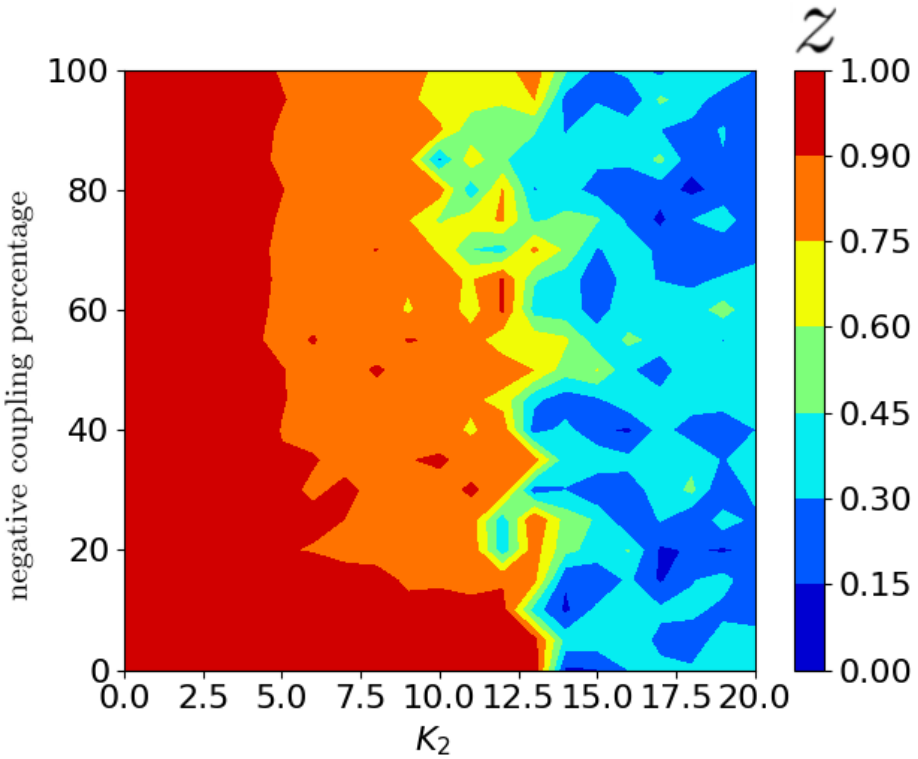

FIG. 9. Order parameter vs simultaneous increase in higher-order coupling (K_2) and increasing percentage of negative coupling for a global network of 20 nodes. The parameter values are $\alpha_1 = \pi/4 \approx 0.78$, $\alpha_2 = \pi/4 \approx 0.78$ and $K_1 = 0.04$.

the persistence of this network.

Our studies reveal that even with acceptable levels of asymmetry in the dynamics of each node, the network still exhibits strong phase synchrony under high coupling and introducing the higher-order interaction term changes the network dynamics drastically. Higher-order coupling over a threshold amplitude makes the network dynamics asynchronous as evident by the decrease of the order parameter to an extremely low level. With increasing network size, this threshold higher-order coupling value required to induce asynchronicity increases. This trend is mirrored by random networks of increasing size, proving that this trend, at least is robust to network structure. In the presence of a fixed higher-order coupling and fixed network size, we examine the effect of asymmetry in the dynamics. Only very high asymmetry, especially of the first order can induce some amount of asynchrony, when higher-order coupling is absent. However, with increasing higher-order coupling, even intermediate or low values of asymmetry also induces asynchrony of the dynamics. For a fixed higher-order coupling strength, the degree of asymmetry required to induce asynchrony increases with network size. For larger networks, the higher-order asymmetry becomes more important than first-order asymmetry. Various mutualistic (+) and antagonistic (-) inter-species interactions are incorporated, first through a combination of positive and negative higher order coupling, and then through a combination of positive and negative first and higher-order coupling. In presence of both negative and postive higher-order coupling only, the relative percentage of postively coupled links and negatively coupled links determine the phase synchrony of the network. Over a threshold coupling value, even though low percentage of higher-order coupling can create asynchrony, it increases with higher percentage of negative higher-order coupling. There are also occasional range of values where almost no synchrony exist in the network, especially with higher percentage of negative higher-order coupling values.

In a network of fixed size, if both first and higher order coupling have a combination of positive and negative values, asynchrony of the network is strongly dependent on the percentage of the negative coupling. In absence of any negative coupling, asynchrony is induced only when higher-order coupling is higher than first-order coupling. This trend increases directly proportional with increase in first-order coupling. For slightly higher percentage of negative coupling, this trend is maintained, even though complete asynchrony is absent. For even higher percentage of negative coupling, asynchrony is induced for a larger range of higher-order coupling. The range of first-order coupling values for which network is synchronized decreases largely. With increasing percentage of negative coupling, this trend of synchronous region decreasing is

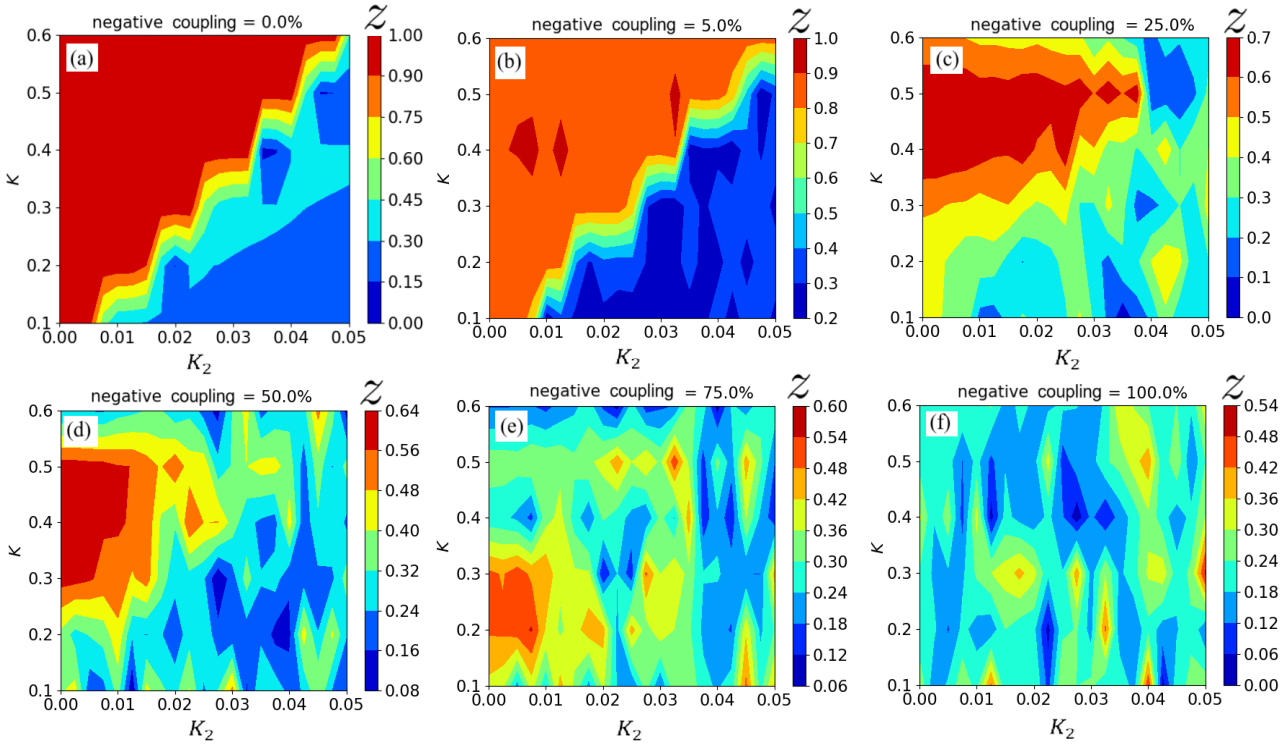

FIG. 10. Order parameter vs simulatenous increase in first-order coupling (K_1) and higher-order coupling (K_2) for a global network of 20 nodes with (a) 0% negative coupling in both K_1 and K_2, (b) 5% negative coupling in both K_1 and K_2, (c) 25% negative coupling in both K_1 and K_2 , (d) 50% negative coupling in both K_1 and K_2 , (e) 75% negative coupling in both K_1 and K_2 and (f) 100% negative coupling in both K_1 and K_2. The parameter values are $\alpha_1 = \pi/4 \approx 0.78$, $\alpha_2 = \pi/4 \approx 0.78$.

promoted. When the first and higher order couplings are completely negative, there is complete asynchrony in the system, irrespective of the value of first-order and higher-order coupling.

The continued existence of ecosystems indicate the presence of asynchrony in ecological networks, which in turn implies the applicability of this approach. A primary future goal of this research is to analyze ecological networks of increasing complexity with this approach. Environmental noises can be introduced in the dynamics of each phase oscillator and it's effect on the persistence of ecosystems can be investigated. The same could be done by incorporating further higher-order interactions and/or considering multi-layer networks. Analysis of the synchrony groups and chimeric elements that emerge from the model could provide insights into the empirical factors contributing to the formation of these groups. The relationship between chimeric elements and subgroups formed by the nodes, and its impact on persistence of the ecological network could be investigated. More heterogenous cases where parameters such as habitat quality could be considered to influence the choice of natural frequencies (ω_i s) and interaction amplitudes (K_1 and K_2 s) could also be investigated.

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