Quantum Holographic Dynamics in the Brain's Proton Spin Ensembles

Christian Kerskens

November 2025

Abstract

Non-compact symmetries such as SU(1,1) govern quantum amplification and squeezing, yet have not been directly identified in macroscopic spin ensembles. Here we apply a symmetry-based analytical framework to previously published magnetic resonance data acquired from proton spin ensembles in the living human brain. By reanalyzing the zero-quantum (ZQ) signal within this framework, we isolate a non-compact SU(1,1) sector of the full SU(4) spin algebra and exclude all compact SU(2) exchange pathways. A calibrated signal-level witness links the measured ZQ amplitude to the off-diagonal density-matrix element, yielding a direct lower bound on concurrence. The observed signal exceeds the separability threshold, confirming bipartite entanglement within the SU(1,1) manifold. This non-compact symmetry defines a hyperbolic temporal geometry that can be interpreted holographically as a non-traversable wormhole connecting correlated spin sectors. Such an entangled bridge provides a natural physical analogue of working memory. Independent evidence shows that the same magnetic-resonance signal correlates with short-term memory performance, supporting this interpretation and suggesting that transient SU(1,1) entanglement in the brain's proton spin ensemble may underlie a functional holographic memory mechanism.

Introduction

Recent magnetic resonance experiments on the living human brain have revealed zero-quantum coherence (ZQC) signals [11] that correlate with short-

term memory performance [12]. The microscopic origin of these signals has remained unresolved. Within conventional nuclear-spin theory, the observed behavior appears inconsistent with compact SU(2) exchange dynamics, which govern bounded oscillatory evolution in dipolar-coupled ensembles. This raises the possibility that the measured signal may originate from a non-compact dynamical sector—one that supports hyperbolic, unbounded trajectories characteristic of $\mathrm{SU}(1,1)$ symmetry.

Non-compact symmetries such as $\mathrm{SU}(1,1)$ play a central role in quantum optics and field theory, where they govern squeezing, amplification, and pair-creation processes [15, 5, 19]. In contrast, direct observation of $\mathrm{SU}(1,1)$ dynamics in macroscopic spin ensembles has remained elusive. Here we reanalyze previously published ZQC data from human brain tissue [11] through a symmetry-based framework that identifies the active manifold as a non-compact $\mathrm{SU}(1,1)$ subsector of the full SU(4) spin algebra. This reinterpretation links the measured signal to the temporal geometry of a Schrödinger-type hologram, in which quantum entanglement encodes geometric connectivity between coupled spin subsystems.

The Holographic Brain Theory [9, 10] proposes that physiology and psychology can be unified through a holographic duality with non-relativistic Schrödinger symmetries [17, 2]. Within this framework, the brain's proton-spin ensemble may act as the physical substrate of the boundary theory, while the emergent $\mathrm{SU}(1,1)$ dynamics represent its temporal conformal sector. The identification of non-compact $\mathrm{SU}(1,1)$ behavior in brain ZQC signals therefore provides a direct empirical bridge between measurable spin physics and the proposed holographic correspondence in neural function.

Zero-quantum $\mathrm{SU}(1,1)$ coherence can be generated using standard ZQC NMR pulse sequences, but its signal signatures differ fundamentally from those of the familiar SU(2) case—allowing the two algebras to be distinguished experimentally. Our first goal is to characterize $\mathrm{SU}(1,1)$-type zero-quantum coherence and identify its specific NMR signature using a minimal sequence design: a 45° RF pulse, followed by an asymmetric gradient and a second 45° RF pulse, repeated n times with delay τ. We then apply this framework to the published human-brain data [11], which used the same sequence but, unexpectedly, displayed no SU(2) signatures. Once the observed signal is identified as $\mathrm{SU}(1,1)$ zero-quantum coherence, we derive NMR-level witnesses for $\mathrm{SU}(1,1)$ entanglement and, subsequently, a criterion for bipartite entanglement within this algebra.

These witnesses provide a direct means of testing for quantum entanglement—and, by extension, a possible holographic correspondence—in biological spin systems. Applied to the existing human-brain data, they demonstrate that the measured ZQ signal is consistent with non-compact $\mathrm{SU}(1,1)$ dynamics and genuine two-spin entanglement. Together with independent evidence that the same ZQ signal correlates with short-term memory performance, these results suggest that transient $\mathrm{SU}(1,1)$ entanglement may represent a physical mechanism for a holographically encoded working-memory process in the brain.

Isolating Zero-Quantum SU(1,1) dynamics

First, we consider all possible algebraic origins of a zero–quantum (ZQ) signal, for which any operator O contributing to the signal satisfies

$$[I_z^{\mathrm{tot}}, O] = 0.$$

For a two–spin system, the corresponding operator space is

$$\mathcal{B}_{\mathrm{ZQ}} = \{I_{1z},\, I_{2z},\, I_{1+}I_{2-},\, I_{1-}I_{2+},\, I_{1+}I_{2+},\, I_{1-}I_{2-}\}.$$

Within this space, only a few subalgebras close under commutation [15, 5, 4, 19]:

$\mathfrak{su}(2)$:	$\{I_{1+}I_{2-},\, I_{1-}I_{2+},\, \frac{1}{2}(I_{1z} - I_{2z})\}$,
$\mathfrak{su}(1,1)$:	$\{I_{1+}I_{2+},\, I_{1-}I_{2-},\, \frac{1}{2}(I_{1z} + I_{2z})\}$,
$\mathfrak{u}(1)$:	$\{I_{1z} + I_{2z}\}$,
$\mathfrak{hw}(1)$:	$\{I_{1+}I_{2+},\, I_{1z} + I_{2z},\, \nVdash\}$ (Heisenberg–Weyl) [16],
$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$:	full two–spin algebra including both ZQ and SQC sectors.

Of these, only $\mathfrak{su}(2)$ and $\mathfrak{su}(1,1)$ are semisimple and support self–contained dynamics within the ZQ manifold. The Heisenberg–Weyl algebra $\mathfrak{hw}(1)$ represents the linearized (small–amplitude) limit of $\mathfrak{su}(1,1)$, lacking a Casimir invariant and therefore unable to sustain a stable NMR signal [16, 5]. The full $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$ algebra mixes zero–quantum and single–quantum coherences and is suppressed by the gradient filter.

Compact and non–compact algebras differ fundamentally in their dynamical behavior. Compact $\mathfrak{su}(2)$ dynamics generate bounded, oscillatory exchange at the difference frequency $\Delta = \omega_1 - \omega_2$ [4], whereas the non–compact algebra $\mathfrak{su}(1,1) \cong \mathfrak{so}(2,1)$ produces hyperbolic pair–creation at the sum frequency $\Sigma = \omega_1 + \omega_2$ [15, 19, 5]. The zero–quantum condition $[I_z^{\text{tot}}, O] = 0$ defines a selection rule ($\Delta m = 0$) but not the geometry of motion, which is determined by the metric signature of the commutation algebra. Compact algebras possess a positive–definite Killing form, yielding purely imaginary adjoint eigenvalues and trigonometric evolution; non–compact algebras such as $\mathfrak{su}(1,1)$ have Lorentzian signature, giving real eigenvalues and hyperbolic (sinh/cosh) trajectories. Hence, any ZQ observable exhibiting hyperbolic time dependence cannot originate from $\mathfrak{su}(2)$ exchange (see Appendix A). As all other candidate algebras fail to close under the experimental selection rules or lead only to trivial phase evolution, excluding SU(2) dynamics is effectively sufficient to identify the underlying symmetry as SU(1,1). The observation of hyperbolic ZQ evolution thus serves as a direct symmetry witness of the active non–compact SU(1,1) sector within SU(4).

The emergent SU(1,1) zero–quantum dynamics can be interpreted as a *Schrödinger hologram* of the full SU(4) evolution. In this picture, the non–compact SU(1,1) sector represents a lower–dimensional dynamical projection that encodes the essential features of the system's time evolution within the ZQ subspace [15, 5, 19]. Although the complete two–spin system remains governed by SU(4) symmetry, its experimentally accessible ZQ dynamics unfold entirely within this embedded non–compact manifold. The term Schrödinger hologram thus emphasizes that the observed SU(1,1) behavior constitutes a faithful dynamical image of the higher–dimensional unitary evolution—capturing its symmetry, metric signature, and temporal geometry in reduced form. As proven in Appendix A, hyperbolic zero–quantum dynamics necessarily arise from a non–compact SU(1,1) sector embedded within the full SU(4) algebra. The observed SU(1,1) dynamics therefore act as a holographic projection of the underlying unitary evolution. If, in the limiting case, all spin degrees of freedom exhibit such non–compact behavior, the projection becomes complete and the entire system itself constitutes a global Schrödinger hologram of its own evolution.

Experimental Signature of SU(1,1) Dynamics

The experimental filtering isolates precisely the $\Delta m = 0$ manifold in which the SU(1,1) witness can manifest, ensuring that any detected hyperbolic evolution must arise from non-compact two-spin dynamics rather than from conventional single-quantum processes. In the experiments [11], the excitation–filter–detection block consisted of a 45° pulse, a single asymmetric gradient, and a second 45° pulse. This sequence encodes both the coherence order Δm and the single-quantum frequency difference: the gradient dephases all coherences according to their order p, while the second pulse converts only those that remain phase-coherent into detectable magnetization. Consequently, only coherence terms satisfying $\Delta m = 0$ survive refocusing; components corresponding to single-quantum coherences (SQC, $|\Delta m| = 1$) acquire a nonzero phase proportional to the gradient area and thus cancel upon spatial averaging. Residual SQC pathways can also be excluded experimentally. Their amplitudes are strictly bounded and cannot increase with time, because SQC evolution is governed by the compact $\mathfrak{su}(2)$ algebra, which conserves the norm of the Bloch vector. Moreover, SQC signals are independent of the magic angle, since their evolution depends only on local precession and not on dipolar coupling geometry. In contrast, the observed signal displays both a pronounced amplitude dependence on the dipolar coupling tensor and a complete disappearance at the magic angle, demonstrating that it originates from *two-spin* interactions within the $\Delta m = 0$ subspace. Within this manifold, a compact SU(2) zero-quantum mode would yield only bounded, oscillatory exchange and therefore cannot account for the experimentally observed signal increase. Hence, the detected growth must correspond to a non-compact SU(1,1) zero-quantum mode, whose hyperbolic dynamics enable intrinsic amplification of two-spin correlations:

$$S_{\mathrm{SU}(2)}(t) \propto \sin(\Omega t), \qquad S_{\mathrm{SU}(1,1)}(t) \propto \sinh(gt), \tag{1}$$

where Ω and g denote the effective coupling strengths for the compact and non-compact generators, respectively.

Exclusion of the SU(2) Zero-Quantum Scenario

The exclusion of SU(2) dynamics is effectively sufficient to identify the underlying symmetry as SU(1,1). Several independent features of the measured

zero-quantum (ZQ) signals [11] rule out any interpretation in terms of compact SU(2) exchange. The null interpretation proposed by Warren [20] arose from analyzing the data solely within a compact SU(2) framework, in which the observed dynamics are formally impossible. Viewed instead through the non-compact $\mathrm{SU}(1,1)$ framework, the same experimental features acquire a consistent and physically meaningful interpretation.

The following section therefore demonstrates, step by step, that the experimental observations cannot be reconciled with any compact SU(2) mechanism and instead uniquely reveal the presence of a non-compact $\mathrm{SU}(1,1)$ dynamical sector.

(i) Balanced populations. Following presaturation, the longitudinal magnetization was nearly zero, $\langle K_0 \rangle \approx 0$. Under these balanced conditions, SU(2) exchange $(I_{1+}I_{2-} + I_{1-}I_{2+})$ cannot produce a ZQ coherence, since it requires a finite population imbalance $\propto (I_{1z} - I_{2z})$. The observation of a finite ZQ signal under $\langle K_0 \rangle \approx 0$ therefore rules out a compact SU(2) pathway.

(ii) Absence of evolution delay. The signal appears immediately after the equal-phase 45°–G–45° block without any evolution interval τ. Compact SU(2) ZQ coherence requires mixing under a finite-time evolution (e.g. exchange or dipolar coupling); spontaneous appearance without delay is incompatible with that mechanism.

(iii) Echo-parity behaviour. The signal alternates in sign and revives every *second* echo, corresponding to the hyperbolic reversal $r \to -r$ of $\mathrm{SU}(1,1)$ trajectories. A filter-function model of SU(1,1) echo is given in Appendix C. Compact SU(2) echoes refocus at *each* echo time and cannot exhibit this even-echo parity.

(iv) Frequency refocusing. The observed echo refocuses dephasing at the *sum frequency* $\Sigma = \omega_1 + \omega_2$, whereas SU(2) ZQ coherence dephases at the *difference frequency* $\Delta = \omega_1 - \omega_2$. Sum-frequency refocusing identifies the hyperbolic $\mathrm{SU}(1,1)$ pair mode.

(v) Amplitude scaling. The ZQ amplitude scales linearly with the equilibrium magnetization M_0 and appears instantaneously after excitation. Com-

pact SU(2) processes are bounded, sinusoidal in time, and generally scale as M_0^2 in the ensemble limit.

(vi) Coherence-pathway identity. The detected signal corresponds to the density-matrix element $\rho_{14} = \langle K_- \rangle$ linking $|\alpha\alpha\rangle$ and $|\beta\beta\rangle$, characteristic of SU(1, 1) pair correlations. Compact SU(2) ZQ exchange acts on ρ_{23} ($|\alpha\beta\rangle \leftrightarrow |\beta\alpha\rangle$) and cannot generate ρ_{14} coherence.

(vii) Magic-angle test. Although both secular dipolar SU(2) and anisotropic SU(1, 1) couplings are suppressed at the magic angle, the combination of (ii)–(vi) identifies the active term as the non-compact SU(1, 1) pair interaction.

(viii) Reservoir coupling. A purely classical reservoir can modulate intensities ($\propto M_0$) but cannot generate the phase-locked pair amplitude $\langle I_{1+} I_{2+} \rangle$. The observed coherence requires algebraic coupling to the internal reservoir mode K_0 via $[K_0, K_\pm] = \pm K_\pm$, confirming the SU(1, 1) pump structure.

From Dynamical Holography to Quantum Correlation

In the previous sections, we introduced the concept of a Schrödinger hologram to describe the emergent SU(1,1) zero-quantum (ZQ) dynamics as a reduced, non-compact projection of the full SU(4) evolution. We now make explicit that this holographic description is purely dynamical. The SU(1,1) manifold acts as an algebraic embedding that reproduces the geometry of time evolution within a lower-dimensional subspace, independent of any quantum correlations between spins. Unlike informational or entanglement-based holography [13, 18], the dynamical hologram arises solely from the structure and metric signature of the underlying Lie algebra. It therefore constitutes a *kinematic hologram*—a faithful mapping of the system's temporal geometry even in the absence of entanglement.

Having established this dynamical correspondence, we now examine whether the observed non-compact dynamics also generate genuine quantum correlations. Entanglement represents a higher-order manifestation of SU(1,1) behavior, probing correlations between subsystems that extend beyond the

purely kinematic hologram. While hyperbolic evolution signals non-compact geometry, it does not by itself imply non-separability. To address this, we analyze the time evolution of bipartite density matrices under SU(1,1) generators, employing concurrence and subsystem purity to determine whether genuine entanglement emerges.

To connect the measured ZQ signal directly to quantum correlation, we now derive an experimentally accessible signal-level entanglement witness specific to the SU(1,1) zero-quantum manifold.

Derivation of the signal–level SU(1,1) entanglement witness

Assumptions enforced by the readout. Immediately before detection, an equal-phase 45°–G–45° block with a crusher gradient G is applied. This (i) projects onto the $\Delta m = 0$ ZQ manifold, (ii) suppresses local single-quantum coherences ($\langle I_{i\pm}\rangle \simeq 0$), and (iii) symmetrizes the anti-aligned populations to excellent approximation (balanced+ε). Under these conditions, the reduced two-spin density operator in the computational basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ takes the ZQ-projected X-state form

$$\rho_{\mathrm{ZQ}} = \begin{pmatrix} p_{\uparrow\uparrow} & 0 & 0 & \rho_{14} \\ 0 & p_{\uparrow\downarrow} & 0 & 0 \\ 0 & 0 & p_{\downarrow\uparrow} & 0 \\ \rho_{41} & 0 & 0 & p_{\downarrow\downarrow} \end{pmatrix}, \qquad \rho_{14} = \langle K_- \rangle, \tag{2}$$

with $\rho_{41} = \rho_{14}^*$ and $\sum_i p_i = 1$.

What the receiver measures. For this readout, linearity of the propagator gives a fixed complex calibration factor C_{seq} such that

$$S_{\mathrm{ZQ}} = C_{\mathrm{seq}}\rho_{14}, \qquad |\rho_{14}| = \frac{|S_{\mathrm{ZQ}}|}{|C_{\mathrm{seq}}|}. \tag{3}$$

The magnitude $|C_{\mathrm{seq}}|$ is obtained from a single calibration scan using the same 45°–G–45° block.

Separable bound for X-states. For any two-qubit X-state, the concurrence obeys [21, 1, 22]

$$C(\rho_{\mathrm{ZQ}}) \geq 2\max\Big\{0,\ |\rho_{14}| - \sqrt{p_{\uparrow\downarrow}p_{\downarrow\uparrow}}\Big\}. \tag{4}$$

Hence, every separable state satisfies

$$|\rho_{14}| \leq \sqrt{p_{\uparrow\downarrow}p_{\downarrow\uparrow}}. \tag{5}$$

Defining $\mathcal{W}_{\mathrm{X}} \equiv |\rho_{14}| - \sqrt{p_{\uparrow\downarrow}p_{\downarrow\uparrow}}$, the condition $\mathcal{W}_{\mathrm{X}} > 0$ indicates entanglement.

Signal-level witness (measurable form). Combining Eqs. (3)–(5) gives the experimentally accessible signal-level witness:

$$\boxed{\mathcal{W}_{\mathrm{sig}} \equiv \frac{|S_{\mathrm{ZQ}}|}{|C_{\mathrm{seq}}|} - \sqrt{p_{\uparrow\downarrow}p_{\downarrow\uparrow}} > 0 \Longrightarrow \text{ two-spin entanglement.}} \tag{6}$$

Here $p_{\uparrow\downarrow}$ and $p_{\downarrow\uparrow}$ are obtained from an M_z readout after coherence spoiling.

Balanced($+\varepsilon$) specialization and M_0 insensitivity. Under equal-phase 45°–G–45° preparation, $p_{\uparrow\downarrow} \simeq p_{\downarrow\uparrow} \equiv q$. A small residual longitudinal offset $M_0 = \langle I_{1z} + I_{2z}\rangle$ shifts only the aligned populations and leaves q unchanged, giving

$$\boxed{\mathcal{W}_{\mathrm{sig}} = \frac{|S_{\mathrm{ZQ}}|}{|C_{\mathrm{seq}}|} - q, \qquad \text{independent of } M_0.} \tag{7}$$

In the strictly balanced limit ($q \to 0$), any non-zero calibrated ZQ signal certifies entanglement.

Practical bounds on q. Static dephasing and RF inhomogeneity impose an upper bound on q. For a crusher gradient of first moment $k = \gamma G \delta$ over voxel extent L,

$$q \lesssim \tfrac{1}{4}\,\mathrm{sinc}^2\!\Big(\tfrac{kL}{2}\Big) + \tfrac{1}{2}\varepsilon_\theta^2,$$

where ε_θ denotes a small flip-angle mismatch. Typical experimental conditions ($kL/2 \gtrsim 8$, $\varepsilon_\theta \sim 2°$) yield $q \lesssim 5 \times 10^{-3}$, well below the observed signal ratio.

Experimental threshold. The operational condition for SU(1,1) entanglement is

$$\frac{|S_{\mathrm{ZQ}}|}{|C_{\mathrm{seq}}|} > q \equiv \sqrt{p_{\uparrow\downarrow} p_{\downarrow\uparrow}}. \tag{8}$$

For the reported data [11], with $|C_{\mathrm{seq}}| \approx 0.5 M_0$ and a measured $|S_{\mathrm{ZQ}}| \approx 0.15 M_0$ (relaxation-corrected by ≈ 2.7), the ratio is

$$\frac{|S_{\mathrm{ZQ}}|}{|C_{\mathrm{seq}}|} \approx \frac{0.15 \times 2.7 M_0}{0.5 M_0} = 0.81 \gg q, \tag{9}$$

far exceeding the separability bound (8).

Conclusion. The observed zero-quantum amplitude lies well outside the separable region and thus constitutes a robust signature of SU(1,1)-type entanglement under the equal-phase 45°–G–45° protocol. The derived witness establishes a direct, experimentally testable link between the measured ZQ signal and genuine two-spin entanglement within the SU(1,1) manifold.

In this sense, the appearance of non-zero $\mathcal{W}_{\mathrm{sig}}$ marks the conversion of a purely dynamical SU(1,1) hologram into a correlational one: the same algebraic non-compactness that governs hyperbolic evolution also produces entanglement between spins [15, 5, 19]. Thus, the SU(1,1) sector does not merely project temporal geometry—it actively encodes quantum correlations that complete the holographic correspondence between dynamical symmetry and quantum information [21, 22, 14].

Remarks. (i) The witness depends only on reduced two-spin observables and therefore remains valid in open conditions (any reservoir is traced out). (ii) In strictly collective realizations (single manifold), the same construction quantifies SU(1,1) squeezing or nonclassicality rather than bipartite entanglement; in that case, the SU(1,1) variance criterion $\Delta K_1^2 + \Delta K_2^2 < \langle K_0 \rangle$ (Nha–Kim) serves as a partition-free witness.

Sufficiency of the SU(1,1) witness for bipartite entanglement

In general, violation of the SU(1,1) separability bound

$$\Delta K_1^2 + \Delta K_2^2 \; < \; \langle K_0 \rangle$$

certifies nonclassical correlations within the non-compact SU$(1,1)$ manifold. Whether this constitutes bipartite entanglement depends on how the generators are realized.

For a single-mode (collective) embedding, $K_\pm = \frac{1}{2}a^{\dagger 2}, \frac{1}{2}a^2$, the witness signals internal squeezing but not tensor-product inseparability. For a *two-mode* realization, $K_\pm = a^\dagger b^\dagger, ab$, the same inequality is necessary and sufficient for entanglement between distinct subsystems A and B [7, 14]. In the present experiment two distinguishable spin ensembles are coupled, and their interaction vanishes at the magic angle; thus $I_{1\pm}$ and $I_{2\pm}$ act on separate Hilbert spaces. Under these conditions, violation of the SU(1,1) separability bound is direct evidence of bipartite entanglement rather than mere collective nonclassicality.

Further evidence for two distinguishable subsystems

Interpreting the measured SU(1,1) zero-quantum (ZQ) signal as a *bipartite* entanglement witness requires that $I_{1\pm}$ and $I_{2\pm}$ act on physically distinct subspaces. The following experimental and dynamical features support this two-subsystem structure:

(a) **Magic-angle dependence.** The ZQ signal disappears at the magic angle $\theta_m = \arccos(1/\sqrt{3})$, where the dipolar coupling between two spatially distinct spin groups is nulled. This identifies the coherence as inter-group (anisotropic) rather than collective intraspecies, so I_1 and I_2 refer to separate manifolds. Sufficiency once SU(2) is excluded. Although magic-angle extinction is generic for dipolar couplings, it becomes a *sufficient* diagnostic for an SU(1,1) entangling interaction when compact SU(2) exchange pathways are ruled out. In our system, balanced preparation ($\langle K_0 \rangle \simeq 0$), absence of any evolution delay, linear M_0 scaling, and sum-frequency phase evolution jointly exclude SU(2) ZQ exchange ($I_{1+}I_{2-} + I_{1-}I_{2+}$). The only remaining dipolar-mediated ZQ pathway compatible with the data is the non-compact SU(1,1) pair term $K_\pm = I_{1\pm}I_{2\pm}$.

(b) **Linear scaling with M_0.** The signal scales linearly with total magnetization M_0, not quadratically (derivation in Appendix B). Quadratic behavior ($S_{\mathrm{ZQ}} \propto M_0^2$) is characteristic of collective single-manifold SU(1,1) squeezing, whereas linear scaling indicates pairwise coupling between independent ensembles.

(c) **Immediate signal onset.** A finite ZQ amplitude appears directly after the equal-phase 45°–G–45° block without any evolution delay. Compact SU(2) exchange requires finite-time mixing and is therefore incompatible with this instantaneous onset; two-manifold SU(1,1) pair excitation is consistent.

(d) **Echo-parity behavior.** Under equal-area gradient inversion the ZQ coherence refocuses on every second echo (even-echo revival). This "hyperbolic parity" is a hallmark of non-compact SU(1,1) dynamics on two coupled modes; collective SU(2) or single-mode SU(1,1) would refocus at each echo.

(e) **Sum-frequency refocusing.** The echo refocuses dephasing at the sum $\Sigma = \omega_1 + \omega_2$, not at the difference $\Delta = \omega_1 - \omega_2$. Sum-frequency behavior identifies an SU(1,1) pair mode linking two distinguishable species; SU(2) ZQ exchange oscillates at Δ.

(f) **Magic-angle suppression of SQC pathways.** At $\theta = \theta_m$, single-quantum (SQC) transfer vanishes, yet ZQ coherence persists away from this angle. This rules out single-manifold SU(2) exchange and supports assignment of the coherence to the coupled two-spin subspace $\{|\alpha\alpha\rangle, |\beta\beta\rangle\}$.

(g) **Independent calibration of K_0.** Residual longitudinal magnetization $\langle K_0 \rangle$ is obtained from separate pre-saturation recovery curves of the two groups, confirming that $K_0 = \frac{1}{2}(I_{1z} + I_{2z})$ contains additive contributions from distinct subspaces.

Conclusion. Taken together, these observations rule out any single-manifold SU(2) or single-manifold SU(1,1) realization and identify a system of two distinguishable spin ensembles coupled via the non-compact $\mathfrak{su}(1,1)$ pair operators $K_\pm = I_{1\pm} I_{2\pm}$. Under these conditions, either violation of the SU(1,1) separability bound or the detection of a finite calibrated S_{ZQ} provides a genuine bipartite entanglement witness.

Discussion

The observation of a balanced, entanglement-sensitive SU(1,1) zero-quantum (ZQ) signal in human-brain proton spins provides direct experimental evi-

dence of non-compact temporal symmetry (see Appendix A for a formal proof) in a macroscopic biological system. By reanalyzing previously published magnetic-resonance data [11] within a symmetry-based framework, we identify the active manifold as a non-compact SU(1,1) sector of the full SU(4) spin algebra. This behavior realizes a Schrödinger hologram: a reduced dynamical projection of the underlying SU(4) evolution in which the spin ensemble maps the boundary SU(1,1) time algebra onto measurable hyperbolic pair correlations. The result establishes an explicit link between group-theoretical dynamics, entanglement, and emergent temporal geometry in the brain.

Several experimental signatures confirm that the measured ZQ coherence cannot originate from conventional SU(2) exchange. Under balanced conditions ($\langle K_0 \rangle \approx 0$), compact exchange pathways are dynamically forbidden, yet a finite ZQ signal persists. Its hyperbolic echo structure, magic-angle extinction, sum-frequency refocusing, and immediate onset all point to non-compact SU(1,1) pair creation rather than population transfer. Within this geometry, K_0 acts as a polarization reservoir coupled to the pair operators $K_\pm$, allowing coherence to emerge even from a balanced, vacuum-like state through the hyperbolic evolution $\langle K_- \rangle(t) \sim \sinh(\lambda t)$. Such dynamics represent the spin-ensemble analogue of two-mode squeezing.

The derived signal-level witness,

$$\mathcal{W}_{\text{sig}} = \frac{|S_{\text{ZQ}}|}{|C_{\text{seq}}|} - \sqrt{p_{\uparrow\downarrow} p_{\downarrow\uparrow}},$$

links a measurable NMR observable directly to a lower bound on bipartite entanglement between two distinguishable spin ensembles. Because the pulse sequence isolates the $\Delta m = 0$ manifold, compact SU(2) coherences cannot contribute, and $\mathcal{W}_{\text{sig}} > 0$ provides an operational certificate of entanglement within the SU(1,1) manifold. The observed violation of the separability bound thus confirms that the ZQ coherence arises from genuine quantum correlations between distinct spin reservoirs.

The coexistence of non-compact SU(1,1) symmetry and measurable entanglement supports a geometric interpretation consistent with nonrelativistic Schrödinger holography [6, 8, 3]. In this correspondence, the generators (H, D, C) that define temporal conformal transformations on the boundary are mirrored in the spin system by (K_+, K_-, K_0), which generate hyperbolic pair correlations. The ZQ coherence then acts as a non-traversable

information bridge—an analogue of the Schrödinger-type ER=EPR connection—where entanglement encodes geometric connectivity without classical transport. In this view, the correlated spin manifolds play the role of dual boundaries connected through a hyperbolic interior region, analogous to a non-traversable wormhole. Because the same data are independently correlated with short-term memory performance [12], this SU(1,1)-mediated information bridge may represent a physical correlate of transient memory storage in the brain.

More broadly, these results establish a concrete experimental route for probing non-compact dynamical symmetries and their entanglement signatures in macroscopic quantum matter. They demonstrate that measurable correlations in driven spin ensembles can serve as operational witnesses of emergent geometric structure, linking the dynamics of SU(1,1) symmetry, entanglement, and holographic duality within a biological system.

References

[1] S. J. Akhtarshenas. Concurrence vector and entanglement in two-qubit systems. *J. Phys. A: Math. Gen.*, 38:6777–6784, 2005.

[2] K. Balasubramanian and J. McGreevy. Gravity duals for non-relativistic conformal field theories. *Phys. Rev. Lett.*, 101:061601, 2008.

[3] V. K. Dobrev. Nonrelativistic holography: a group-theoretical view. *Int. J. Mod. Phys. A*, 29:1430001, 2014.

[4] R. R. Ernst, G. Bodenhausen, and A. Wokaun. *Principles of Nuclear Magnetic Resonance in One and Two Dimensions*. Clarendon Press, Oxford, 1990. Classic NMR reference for coherence orders and spin dynamics.

[5] C. C. Gerry and P. L. Knight. *Introductory Quantum Optics*. Cambridge University Press, 2004.

[6] M. Guica, K. Skenderis, M. Taylor, and B. van Rees. Holography for schrödinger solutions of string theory. *J. High Energy Phys.*, 2011:070, 2011.

[7] M. Hillery and M. S. Zubairy. Entanglement conditions for two-mode states. *Phys. Rev. Lett.*, 96:050503, 2006.

[8] K. Jensen and A. Karch. The holographic dual of a hawking pair has a wormhole. *Phys. Rev. D*, 90:064019, 2014.

[9] C. Kerskens. Exploring holography in neuro-vascular dynamics, 2025.

[10] C. Kerskens. Exploring holography in neuro-vascular dynamics. In *Proceedings of the International Workshop 2025: Classical and Quantum Processes in the Brain: From Molecules to the Psyche, II Edition*. Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche, Matematiche e Naturali, 2025. Invited talk.

[11] C. M. Kerskens and D. L. Pérez. Experimental indications of non-classical brain functions. *Journal of Physics Communications*, 6(10):105001, 2022.

[12] D. López Pérez, A. L. Bokde, and C. M. Kerskens. Complexity analysis of heartbeat-related signals in brain mri time series as a potential biomarker for ageing and cognitive performance. *The European Physical Journal Special Topics*, 232(1):123–133, 2023.

[13] J. Maldacena. The large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.*, 2:231–252, 1998.

[14] H. Nha and J. Kim. Entanglement criteria via the uncertainty relations in su(2) and su(1,1) algebra: detection of non-gaussian entangled states. *Phys. Rev. A*, 74:012317, 2006.

[15] A. M. Perelomov. *Generalized Coherent States and Their Applications*. Springer, Berlin, 1986. Standard reference for SU(1,1) and SU(2) coherent-state theory.

[16] B. L. Schumaker. Quantum mechanical pure states with gaussian wave functions. *Physics Reports*, 135(6):317–408, 1986. Comprehensive treatment of Heisenberg–Weyl and linearized SU(1,1) algebras.

[17] D. T. Son. Toward an ads/cold atoms correspondence: a geometric realization of the schrödinger symmetry. *Phys. Rev. D*, 78:046003, 2008.

[18] M. Van Raamsdonk. Building up space–time with quantum entanglement. *International Journal of Modern Physics D*, 19(14):2429–2435, 2010.

[19] X. Wang, B. C. Sanders, and S. H. Pan. Entangled coherent states for systems with su(2) and su(1,1) symmetries. *J. Phys. A: Math. Gen.*, 33:7451–7468, 2000.

[20] W. S. Warren. Comment on: 'experimental indications of non-classical brain function' 2022 journal of physics communications 6 105001. *Journal of Physics Communications*, 7(3):038001, mar 2023.

[21] W. K. Wootters. Entanglement of formation of an arbitrary state of two qubits. *Phys. Rev. Lett.*, 80:2245–2248, 1998.

[22] T. Yu and J. H. Eberly. Evolution from entanglement to decoherence. *Quantum Inf. Comput.*, 7:459–468, 2007.

A Zero-Quantum SU(1,1) Dynamics Require a Non-Compact Algebra

This appendix provides a formal proof of the statement in the main text that hyperbolic zero-quantum (ZQ) dynamics can only arise from a non-compact SU(1,1) algebraic structure. While the condition $\Delta m = 0$ defines the coherence order of an operator, it does not by itself determine the geometry of time evolution. We distinguish the algebraic origins of compact (SU(2)-type) and non-compact (SU(1,1)-type) ZQ dynamics in two-spin systems.

Setup and notation

Consider two coupled spin-$\frac{1}{2}$ particles with total magnetization

$$I_z^{\text{tot}} = I_{1z} + I_{2z}.$$

A ZQ operator O satisfies

$$[I_z^{\text{tot}}, O] = 0,$$

which defines a selection rule ($\Delta m = 0$) but not the dynamical geometry.

Let $\mathfrak{L}$ denote the dynamical Lie algebra generated by operators in the Hamiltonian $H(t)$. For clarity we assume time-independent H (the argument extends to piecewise-constant drives). We compare two cases: the compact algebra $\mathfrak{su}(2)$ and the non-compact algebra $\mathfrak{su}(1,1)$.

Lemma 1 (ZQ condition is algebra-agnostic).

If $[I_z^{\text{tot}}, O] = 0$, then O is ZQ regardless of whether $\mathfrak{L} \cong \mathfrak{su}(2)$ or $\mathfrak{su}(1,1)$.

Proof. By definition, $[I_z^{\text{tot}}, \rho_p] = p\,\rho_p$ so $p = 0 \Leftrightarrow [I_z^{\text{tot}}, \rho_0] = 0$. No property of $\mathfrak{L}$ is required.

Lemma 2 (Compact vs. non-compact linear dynamics).

Let $\{X_a\}$ be a basis for $\mathfrak{L}$. For $H = \sum_a h_a X_a$,

$$\dot{X} = i[H, X] = \mathsf{A}_H X, \qquad (\mathsf{A}_H)_{ab} = i \sum_c h_c f^{\,c}{}_{ab},$$

where $f^{\,c}{}_{ab}$ are the structure constants.

- If $\mathfrak{L} \cong \mathfrak{su}(2)$ (compact), A_H is skew-Hermitian in the adjoint representation (positive-definite Killing form). All eigenvalues are purely imaginary and solutions are bounded rotations (trigonometric).
- If $\mathfrak{L} \cong \mathfrak{su}(1,1)$ (non-compact), the Killing form has Lorentzian signature $(+--)$. Then A_H admits real eigenvalues and solutions include hyperbolic modes (cosh / sinh).

Proof. In the adjoint representation, $(\mathrm{ad}_H)_{ab} = f^{\,b}{}_{aH}$. For compact semisimple algebras, ad_H is skew with respect to the negative-definite Killing form $\Rightarrow$ purely imaginary spectrum. For $\mathfrak{su}(1,1) \cong \mathfrak{so}(2,1)$ the form is indefinite, allowing real eigenvalues (boosts). Explicitly,

$$[J_i, J_j] = i\epsilon_{ijk} J_k \Rightarrow \dot{\mathbf{J}} = \mathbf{\Omega} \times \mathbf{J} \quad \text{(rotations, SU(2))},$$

$$[K_0, K_1] = iK_2,\ [K_0, K_2] = -iK_1,\ [K_1, K_2] = -iK_0 \Rightarrow \dot{K}_0 = \Omega K_2,\ \dot{K}_2 = \Omega K_0 \quad \text{(hyperbolic, SU(1,}$$

Lemma 3 (Casimir geometry and boundedness).

With

$$C_2^{\mathfrak{su}(2)} = J_x^2 + J_y^2 + J_z^2 > 0, \qquad C_2^{\mathfrak{su}(1,1)} = K_0^2 - K_1^2 - K_2^2,$$

we have $[H, C_2] = 0$ for $H \in \mathfrak{L}$. The quadratic-form signature fixes the trajectory geometry:

- Compact $\mathfrak{su}(2)$: motion on a sphere (bounded).
- Non-compact $\mathfrak{su}(1,1)$: motion on a two-sheet hyperboloid (unbounded/hyperbolic).

Proposition.

A ZQ SU(1,1) mode requires an underlying SU(1,1) dynamical algebra. Without that algebra, ZQ coherence ($\Delta m = 0$) is possible, but its evolution is compact (SU(2)-type), not hyperbolic.

Proof. (1) If a ZQ observable exhibits cosh / sinh time dependence, Lemma 2 implies real adjoint eigenvalues $\Rightarrow$ non-compact dynamics ($\mathfrak{su}(1,1)$). Equivalently, Lemma 3: unbounded trajectories require an indefinite Casimir. (2) If $\mathfrak{L} \cong \mathfrak{su}(2)$, adjoint eigenvalues are purely imaginary (Lemma 2), so all ZQ evolutions are bounded/trigonometric (Lemma 1 does not alter geometry).

Two-spin examples

Compact ZQ (SU(2)).

$$H_{\text{ex}} = J(I_{1+}I_{2-} + I_{1-}I_{2+}) = 2J\, J_x.$$

Within the ZQ subspace $\{|\alpha\beta\rangle, |\beta\alpha\rangle\}$, populations exhibit sinusoidal exchange (Rabi flops). Motion is confined by $\mathbf{J}^2$.

Non-compact ZQ (SU(1,1)). Define $K_\pm$ with $K_\pm = \frac{1}{2}(I_{1\pm}I_{2\pm})$ so that $[I_z^{\text{tot}}, K_\pm] = 0$. Driving with

$$H_{\text{sq}} = \Omega K_1 = \tfrac{\Omega}{2}\big(I_{1+}I_{2+} + I_{1-}I_{2-}\big)$$

yields $\dot{K}_0 = \Omega K_2$, $\dot{K}_2 = \Omega K_0$ with cosh / sinh solutions.

Diagnostic summary

- ZQ signal with sin / cos and bounded amplitude $\Rightarrow$ compact (SU(2)-type) dynamics.
- ZQ signal with sinh / cosh (or squeezing-type growth) $\Rightarrow$ presence of a non-compact SU(1,1) sector.

Thus, a "ZQ SU(1,1) mode" requires a non-compact SU(1,1) dynamical algebra; without it, only compact SU(2)-type oscillatory ZQ motion is possible.

B Scaling of ZQ SU(1,1) Signals

B.1 Collective SU(1,1): $S_{\mathrm{ZQ}}^{(\mathrm{coll})} \propto M_0^2$

Setup. High-temperature equilibrium:

$$\rho_0 \;=\; \frac{\mathbb{1}}{Z} \;+\; \epsilon \sum_{i=1}^{N} I_{iz} \;+\; \mathcal{O}(\epsilon^2), \qquad \epsilon \propto M_0 \ll 1, \tag{10}$$

with N spins. Let

$$K_+ \;=\; \sum_{i \neq j} I_{i+} I_{j+}, \qquad K_- = K_+^\dagger,$$

selected into the $\Delta m = 0$ manifold by phase-cycling/gradients. Use a small flip-angle preparation and a ZQ-preserving evolution.

Claim. $\langle K_+ \rangle = \mathcal{O}(\epsilon^2) \propto M_0^2$.

Proof. Linear response gives $S_{\mathrm{ZQ}} \sim \mathrm{Tr}\{K_+ \, \rho(t)\}$. Expanding in ϵ,

$$S_{\mathrm{ZQ}} = \epsilon \sum_k \mathrm{Tr}\{K_+ I_{kz}\} + \mathcal{O}(\epsilon^2) = 0 + \mathcal{O}(\epsilon^2),$$

since K_+ changes m by $+2$ on two spins while I_{kz} preserves m. Hence the leading nonzero term is $\mathcal{O}(\epsilon^2)$; equivalently, the pair correlator is quadratic in polarization.

B.2 Selective SU(1,1) (two subsystems): $S_{\mathrm{ZQ}}^{(\mathrm{coll})} \propto M_0$

Setup. Two distinguishable subsystems A, B:

$$K_+ = I_{1+} \otimes I_{2+}, \quad K_- = I_{1-} \otimes I_{2-}, \quad K_0 = \tfrac{1}{2}\big(I_{1z} \otimes \mathbb{1} + \mathbb{1} \otimes I_{2z}\big),$$

with $[K_0, K_\pm] = \pm K_\pm$, $[K_+, K_-] = -2K_0$. Use

$$H_{\mathrm{sq}} = 2g\, K_1, \qquad K_1 = \tfrac{1}{2}(K_+ + K_-), \quad K_2 = \tfrac{1}{2i}(K_+ - K_-).$$

Assume $\langle I_{1z} \rangle = \alpha_A M_0^{(A)}$, $\langle I_{2z} \rangle = \alpha_B M_0^{(B)}$, hence

$$\langle K_0 \rangle_0 = \tfrac{1}{2}\big(\alpha_A M_0^{(A)} + \alpha_B M_0^{(B)}\big) \propto M_0^{(A)} + M_0^{(B)}.$$

Heisenberg equations. Using $[K_1, K_+] = K_0$, $[K_1, K_0] = -iK_2$, $[K_1, K_2] = -iK_0$:

$$\dot{K}_+ = 2ig\, K_0, \qquad \dot{K}_0 = 2g\, K_2, \qquad \dot{K}_2 = 2g\, K_0. \tag{11}$$

Thus,

$$\frac{d}{dt}\langle K_+\rangle = 2ig\,\langle K_0\rangle, \qquad \frac{d^2}{dt^2}\langle K_0\rangle = (2g)^2\langle K_0\rangle.$$

With $\langle K_+\rangle_0 = \langle K_2\rangle_0 = 0$,

$$\langle K_+(t)\rangle = i\,\langle K_0\rangle_0\,\sinh(2gt) \;=\; i\,\langle K_0\rangle_0\big(2gt + \mathcal{O}(t^3)\big).$$

Hence, for small t,

$$S_{\mathrm{ZQ}}^{(\mathrm{sel})}(t) \equiv |\langle K_+(t)\rangle| \;\propto\; \big(M_0^{(A)} + M_0^{(B)}\big)\, t,$$

i.e., *linear* in the initial magnetizations with immediate onset.

Remarks. (i) The scaling is algebraic and independent of readout calibration. (ii) For matched preparations $M_0^{(A)} \approx M_0^{(B)} \approx M_0$, we have $S_{\mathrm{ZQ}}^{(\mathrm{sel})} \propto M_0$. (iii) If $\langle K_0\rangle_0 = 0$ (balanced), the initial slope vanishes; yet $\langle K_+(t)\rangle = i\,\langle K_2\rangle_0(\cosh 2gt - 1)$ allows growth from a phase-imbalanced preparation, and any nonzero selective ZQ SU(1,1) signal in this limit certifies nonclassicality as discussed in the main text.

Corollary (magic-angle suppression). For secular dipolar interactions $D(\theta) \propto 1 - 3\cos^2\theta$, so at the magic angle $\theta_m = \arccos(1/\sqrt{3})$ the SU(1,1)-driving terms vanish to leading order and the ZQ SU(1,1) signal is suppressed in both collective and selective regimes. Residual isotropic J couplings generate compact SU(2) exchange (bounded ZQ dynamics), not hyperbolic SU(1,1) evolution.

C ZQ SU(1,1) Signal Time Course for the Echo Sequence

Refocusable inhomogeneous dephasing (bipartite pair). For two distinguishable groups, the ZQ SU(1,1) pair operator acquires a phase from the *sum* Larmor detuning,

$$K_-(t;\mathbf{r}) = e^{-i\int_0^t \Sigma(\mathbf{r})\,dt'}\, K_-(0), \qquad \Sigma(\mathbf{r}) = \omega_1(\mathbf{r}) + \omega_2(\mathbf{r}).$$

Over an ensemble with distribution $p(\Sigma)$ (Gaussian width σ_Σ),

$$S_{\mathrm{ZQ}}^{(\mathrm{free})}(t) = S_0 \int d\Sigma\, p(\Sigma)\, e^{-i\Sigma t} \;\approx\; S_0\, e^{-\frac{1}{2}\sigma_\Sigma^2 t^2} \;\equiv\; S_0\, e^{-t^2/{T^*_{2,\mathrm{ZQ}}}^2},$$

defining a *refocusable* inhomogeneous time $T^*_{2,\mathrm{ZQ}} = 1/\sigma_\Sigma$.

Refocusing with the SU(1,1) echo (filter-function view). Let $y(t)$ be the pair-mode toggling function (it flips sign with each equal-area crusher). The detected amplitude is the ensemble average of $\exp[-i\int_0^t \Sigma(\mathbf{r})\, y(t')\, dt']$. For static Σ, an echo occurs when the area $Y(t) = \displaystyle\int_0^t y(t')\, dt'$ vanishes, giving

$$S_{\mathrm{ZQ}}(t) = S_0\, e^{-t/T_{2,\mathrm{ZQ}}}\,,$$

i.e., inhomogeneous phase cancels and only irreversible pair-decoherence $T_{2,\mathrm{ZQ}}$ remains. For a balanced two-block echo (equal-phase 45°–G–45°–G) of duration τ per block, $Y(2\tau) = 0$, hence

$$S_{\mathrm{ZQ}}(2m\tau) = S_0\, \cosh(2r_0)\, e^{-2m\tau/T_{2,\mathrm{ZQ}}}, \qquad S_{\mathrm{ZQ}}((2m{+}1)\tau) \approx 0,$$

with $r_0 = \int_{\mathrm{block}} \Omega_{\mathrm{eff}}(t')\, dt'$ the SU(1,1) boost per block. Between echoes, the envelope includes the refocusable factor $\exp[-\frac{1}{2}\sigma_\Sigma^2 Y(t)^2]$.

Complete signal model (balanced bipartite case). Combining hyperbolic amplification, refocusable inhomogeneity, and irreversible pair dephasing,

$$\boxed{S_{\mathrm{ZQ}}(t) = S_0\, \cosh\big(2r(t)\big)\, \exp\Big[-\tfrac{1}{2}\sigma_\Sigma^2\, Y(t)^2\Big]\, \exp\Big[-t/T_{2,\mathrm{ZQ}}\Big],}$$

with $r(t) = \int_0^t \Omega_{\mathrm{eff}}(t')\, dt'$ and $Y(t_{\mathrm{echo}}) = 0$. At even echoes $t = 2m\tau$, the $T^*_{2,\mathrm{ZQ}}$ term refocuses, giving $S_{\mathrm{ZQ}}(2m\tau) = S_0\, \cosh(2r_0)\, e^{-2m\tau/T_{2,\mathrm{ZQ}}}$. The observed $\pm$ alternation follows from the fixed receiver phase acting on the SU(1,1) quadrature flip between successive blocks.

Is ordinary SQC T_2 involved? In a strictly balanced entrance ($\langle K_0\rangle = 0$), the pair channel does not receive linear pumping from SQC, so ordinary SQC T_2^* does not set the echo envelope. However, mechanisms that broaden Σ (e.g., chemical-shift dispersion, coherent dipolar shifts) contribute to σ_Σ and hence $T^*_{2,\mathrm{ZQ}}$; these are refocusable by the SU(1,1) echo, whereas the genuine pair-decoherence time $T_{2,\mathrm{ZQ}}$ is not.