

# Predator-prey feedback in a gyrfalcon-ptarmigan system?

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## Abstract

Specialist predators with oscillating dynamics are often strongly affected by the population dynamics of their prey, yet they do not always participate in a predator-prey cycle. Only those that exert strong population regulation of their prey do so. Inferring the strength and direction of the predator-prey coupling from time series therefore requires contrasting models with top-down versus bottom-up predator-prey dynamics. We examine such population-level coupling using multivariate autoregressive models. The models translate several hypotheses for the joint dynamics of population densities of the Icelandic gyrfalcon *Falco rusticolus*, and its prey, the rock ptarmigan *Lagopus muta*. The dynamics of both species are likely not only linked to each other but also to stochastic weather variables acting as confounding factors on the joint dynamics. The classical MAR(1) model, used most often in ecology, predicts that the times series exhibit predator-prey feedback (i.e., Granger causality): the predator helps to explain prey dynamics and the prey helps to explain predator dynamics. Weather, in the form of spring temperature, influences gyrfalcon population growth but not ptarmigan population growth, despite individual-level evidence that ptarmigan chicks can be strongly affected by weather. MAR(2) models, allowing for species to cycle independently from each other, further suggests alternative scenarios where a cyclic prey influence its predator but not the other way around; such bottom-up models produce a better fit but less realistic cross-correlation patterns. Simulations of MAR(1) and MAR(2) models further demonstrate that the top-down MAR(1) models are most likely to be misidentified as bottom-up dynamics than vice-versa. We therefore conclude that predator-prey feedback in the gyrfalcon-ptarmigan system is very likely, though bottom-up dynamics cannot be excluded with certainty. We finally discuss what sort of information is needed to advance the characterization of joint predator-prey dynamics in birds and other vertebrates.

**Keywords:** population cycles, MAR, VAR, *Falco rusticolus*, *Lagopus muta*

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## 1 Introduction

2 Theoretical ecology predicts that among predators, specialists are the most likely to shape  
3 the dynamics of their prey (e.g. Andersson and Erlinge, 1977; Turchin and Hanski, 1997;  
4 Gilg et al., 2003). It has even been suggested that only specialist predators do exhibit  
5 multi-generation predator-prey population cycles (Murdoch et al., 2002), based on cycle  
6 periods in specialist versus generalist predators. Mechanistic modeling, however, disputes  
7 this particular point (Erbach et al., 2013). Testing more thoroughly this working theory with  
8 empirical data - the more specialized the predator, the higher the likelihood of a predator-prey  
9 cycle or more generally top-down prey regulation - would require to estimate the strength of  
10 predator-prey coupling in a number of real predator-prey systems, for which time series of  
11 both predator(s) and prey are available, preferably in the field. While the task may appear  
12 straightforward in theory, it is surprisingly difficult in practice. Cases of long-term monitoring  
13 including *both* specialized predators and their main prey, through extended periods of time,  
14 are indeed quite rare, especially in vertebrates. Two famous exceptions to the rule include  
15 the wolf-moose (*Canis lupus* - *Alces Alces*) system of Isle Royale (Vucetich et al., 2011),  
16 that has been followed for a century (although this study area is somewhat restricted for  
17 such wide-ranging species), and the celebrated cycle of the Canada snowshoe hare *Lepus*  
18 *americanus*, which interacts with the Canada lynx *Lynx canadensis* and other predators  
19 (Vik et al., 2008; Krebs et al., 2001). While there is a convincing array of evidence showing  
20 that lynx have a dynamical impact on hares (Vik et al., 2008), and wolf has an impact  
21 on moose (Vucetich et al., 2011), there is also evidence that weather and other drivers  
22 have often a strong forcing influence on prey dynamics (Vucetich and Peterson, 2004; Yan  
23 et al., 2013). Even in such strongly interacting systems that fascinate the imagination by  
24 demonstrating strong oscillations, it has been suggested that the presence of an ubiquitous  
25 external forcing hardly warrants to view such systems as a pair of autonomous coupled  
26 differential equations (Nisbet and Gurney, 1976; Barraquand et al., 2017), despite the pivotal  
27 role of autonomous and deterministic dynamical systems in ecological theory (McCann, 2011;

28 Arditi and Ginzburg, 2012). Rather, real predator-prey systems are constantly buffeted by  
29 outside forces, be those climatic or biotic variables unaccounted for (i.e, other players in the  
30 interaction web). The study of Vik et al. (2008) reports at best around 55% of prey variance  
31 in log-densities explained by both prey and predator densities; this therefore leaves ample  
32 room for other factors to influence hare dynamics (Barraquand et al., 2017; see also Vucetich  
33 et al. 2011 on ungulate-wolf systems). In birds, contrasted feedback structures (bottom-up  
34 or top-down) were found in goshawk (*Accipiter gentilis*) - grouse dynamics, depending on  
35 the grouse species considered (Tornberg et al., 2013), with marked effects of weather forces.  
36 To gain a better appraisal of the strength of top-down regulation in the field, compared  
37 to other drivers of herbivore dynamics (see Sinclair, 2003, for a discussion in mammals),  
38 the list of predator-prey systems to which stochastic models of interacting populations are  
39 fitted to time series needs to increase. Having a number of reference predator-prey systems,  
40 whose dynamical structure (e.g., top-down vs. bottom-up) have been vetted by time series  
41 analysis, will also help evaluating how future food web models should be structured to obtain  
42 reliable quantitative predictions: what is the percentage of top-down links that should be  
43 allowed? Should bottom-up interaction coefficients generally be higher or lower than top-  
44 down? Should intra-specific density-dependence dominate? Should there be strong weather  
45 effects, strong or weak noise?

46 Our goal here is to contribute, using large-scale field data, to improving the understanding  
47 of predator-prey dynamics. We do this by fitting stochastic, statistically-driven predator-  
48 prey models to a presumably tightly coupled predator-prey pair, gyrfalcon *Falco rusticolus*  
49 and rock ptarmigan *Lagopus muta* in North-East (NE) Iceland. The gyrfalcon is a predator  
50 specialized on ptarmigan (rock ptarmigan and willow ptarmigan *Lagopus lagopus*) (Nielsen  
51 and Cade, 2017). In Iceland, the rock ptarmigan amounts to on average 72% by biomass of  
52 the gyrfalcon summer diet (range 52-86%) (Nielsen 1999). We combine detailed monitoring  
53 data from Iceland with multivariate autoregressive (MAR) modeling to infer the strength of  
54 trophic coupling in this system. Previous studies have computed autocorrelation functions

55 to infer periodicity (Nielsen, 1999) and fitted autoregressive models on each species (Bryn-  
56 jarsdóttir et al., 2003), but this is to our knowledge the first time a MAR model is fitted  
57 to this dataset. MAR modeling, now an established technique within ecology (Ives et al.,  
58 2003; Hampton et al., 2013), has been largely developed in econometrics (Granger, 1969;  
59 Lütkepohl, 2005), where it is primarily used to establish causal relationships in the sense of  
60 prediction (i.e., a variable has causal influence if it helps improving predictions about the  
61 future, Granger 1969), which is the statistical philosophy that we adopt here.

## 62 **Material and methods**

### 63 **Study area and design**

64 The study area (5327 km<sup>2</sup>) in NE Iceland and survey methods used have been extensively  
65 detailed elsewhere (Nielsen, 1999, 2011) so we will remain brief. The gyrfalcon population  
66 is censused annually by visiting all known territories within the study area to determine  
67 predator occupancy ( $n = 83$  territories). The number of territorial rock ptarmigan males is  
68 surveyed every spring (mostly in May) on 6 plots (total area 26.8 km<sup>2</sup>) within the general  
69 study area. The study started in 1981 and we use data for the period 1981-2014.

### 70 **Ecological variables**

71 We consider two main variables, the occupancy rate of gyrfalcon territories, that was con-  
72 sidered a good proxy for gyrfalcon population density, and mean density of territorial rock  
73 ptarmigan cocks on the 6 plots. Both variables are standardized in the statistical models.

74 We also consider weather variables that are known to potentially affect the dynamics of  
75 the two populations. We have selected 3 stations for the temperature (Akureyri, Mánárbakki,  
76 Grímsstaðir), and 6 stations for log-precipitation (Lerkihlíð, Mýri, Staðarhóll, Reykjahlíð,  
77 Mánárbakki, Grímsstaðir), all within or at the border of the study area and that have  
78 recordings from 1975 to now. The weather data was retrieved from the web site of the

79 Icelandic Met Office (<http://www.vedur.is/>).

## 80 Statistical models

81 Multivariate AutoRegressive (MAR) models have been used to assess the strength of predator-  
82 prey coupling (Ives et al., 2003; Vik et al., 2008). Let us denote the ln-transformed predator  
83 density  $p_t = \ln(P_t)$  and  $n_t = \ln(N_t)$  the ln-transformed density of the prey; the log transfor-  
84 mation is useful to transform log-normal into Gaussian noise. These ln-densities are then  
85 centered and stacked into a vector  $\mathbf{x}_t = (x_{1t}, x_{2t})' = (n_t, p_t)'$ . The dynamics of the MAR(1)  
86 model, with one timelag, are then written as a forced recurrence equation (eq. 1),

$$\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + \mathbf{C}\mathbf{u}_t + \mathbf{e}_t, \mathbf{e}_t \sim \mathcal{N}_2(0, \Sigma) \quad (1)$$

87 where  $\mathbf{B}$  is an interaction matrix that characterizes the effects of net interactions on popu-  
88 lation growth of the 2 species,  $\mathbf{C}$  describes the effect of environmental covariates  $\mathbf{u}_t$  on the  
89 population growth rates of predator and prey, and  $\mathbf{e}_t$  is a Gaussian bivariate noise term.

90 We considered both a model without interactions where  $\mathbf{B} = \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$ , hereafter  
91 referred to as the null MAR(1) model, and a model with full interaction matrix,  $\mathbf{B} =$   
92  $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ . Both models were considered without ( $\mathbf{C} = \mathbf{0}$ ) and with environmental forc-  
93 ing ( $\mathbf{C} \neq \mathbf{0}$ ). The weather variables that we stacked within  $\mathbf{u}_t$  are delayed: the predator  
94 population is believed to be affected by weather 5 years before, because recruits enter the  
95 adult population at the age of 4 years (average time to maturity), while the prey population  
96 is affected by the weather of the year (between  $t$  and  $t + 1$ ) or that of the preceding year  
97 (between  $t - 1$  and  $t$ ). We considered models with temperature effects, log(precipitation)  
98 effects, or both.

99 Several model fitting techniques have been considered in preliminary explorations (MCMC  
100 using JAGS within R, least squares for vector autoregressive models in R package `vars`, simple

101 independent linear autoregressive models using `lm()` in R). Maximum likelihood estimation  
102 using the MARSS package (Holmes et al., 2012) and the EM algorithm was finally chosen  
103 because it allowed to easily perform model selection for contrasted interactions matrices (i.e.,  
104 setting some interactions to zero). All algorithms gave however similar model estimates (see  
105 Appendix S1).

106 We then considered more complex MAR(2) models that are able to allow for both pop-  
107 ulations to cycle independently, because each univariate AR(2) component can model long  
108 cycles ( $\approx 7$  to 10 years cycles, like those observed in the field). Selection of the optimal  
109 lag  $p$  in MAR( $p$ ) model using a variety of model information theoretic criteria (see code  
110 in [https://github.com/fbarraquand/GyrfalconPtarmigan\\_MAR](https://github.com/fbarraquand/GyrfalconPtarmigan_MAR)) in suggested an optimal  
111 lag order of 2 (BIC, HQ) or 3 (AIC, FPE). Because 2 lags are enough to model indepen-  
112 dently cycling populations of period up to 10 years and more (Royama, 1992), and MAR(2)  
113 models are already parameter-rich, we considered a maximum of 2 lags in MAR models. The  
114 MAR(2) model can be written as

$$\mathbf{x}_{t+1} = \mathbf{B}^{(1)}\mathbf{x}_t + \mathbf{B}^{(2)}\mathbf{x}_{t-1} + \mathbf{C}\mathbf{u}_t + \mathbf{e}_t, \mathbf{e}_t \sim \mathcal{N}_2(0, \Sigma) \quad (2)$$

115 The independent cycling model has diagonal matrices  $\mathbf{B}^{(1)}$  and  $\mathbf{B}^{(2)}$ . The full model  
116 has interaction matrices  $\mathbf{B}^{(1)} = \begin{pmatrix} b_{11}^{(1)} & b_{12}^{(1)} \\ b_{21}^{(1)} & b_{22}^{(1)} \end{pmatrix}$  and  $\mathbf{B}^{(2)} = \begin{pmatrix} b_{11}^{(2)} & b_{12}^{(2)} \\ b_{21}^{(2)} & b_{22}^{(2)} \end{pmatrix}$ . To model also an  
117 asymmetric and nonreciprocal effect from the cyclic prey to its predator, we used the following  
118 interaction matrices  $\mathbf{B}^{(1)} = \begin{pmatrix} b_{11}^{(1)} & 0 \\ 0 & b_{22}^{(1)} \end{pmatrix}$  and  $\mathbf{B}^{(2)} = \begin{pmatrix} b_{11}^{(2)} & 0 \\ b_{21}^{(2)} & b_{22}^{(2)} \end{pmatrix}$ . The model was named  
119 ‘bottom-up’, in order to designate a predator dynamics driven by that of its cyclic prey.

## Time series of log-densities

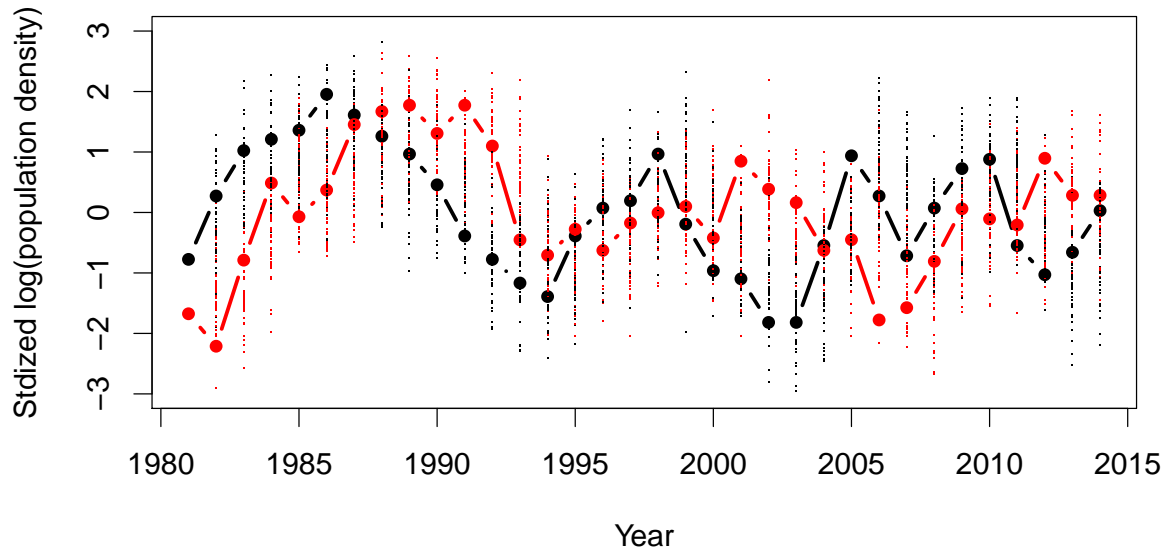


Figure 1: Time series of gyrfalcon (red) and rock ptarmigan (black) standardized log-densities in NE Iceland, and their corresponding one-step ahead predictions under the best-fitted, full interaction matrix MAR(1) model. 100 model simulations one step ahead are plotted, for each year, as small points - red for predator and black for prey.

## 120 Results

### 121 MAR(1) model results

#### 122 Models without environmental covariates

123 The predator-prey time series and the MAR(1) model one-step ahead predictions are pre-  
124 sented in Fig. 1, while Table 1 shows the MAR(1) model fitted parameters. All  $\mathbf{B}$  coefficients  
125 are found to be significantly different from zero, with commensurate strengths of predator  
126  $\rightarrow$  prey ( $b_{12}$ ) and prey  $\rightarrow$  predator ( $b_{21}$ ) interaction. There is therefore consistently negative  
127 effect of predator on prey and a consistently positive effect of prey on predator. Note that a  
128 clear-cut sign was not obligatory, given those are net interaction coefficients, blending several  
129 ecological processes, e.g., direct and indirect predation effects, into one number.

### Estimates of the MAR(1) full model without environmental covariates

Parameter	value	SE	lower 95% CI	upper 95% CI
$b_{11}$	0.7710	0.1112	0.5528	0.9890
$b_{21}$	0.2150	0.1096	0.0001	0.4298
$b_{12}$	-0.2333	0.1114	-0.4516	-0.0149
$b_{22}$	0.6601	0.1097	0.4449	0.8752
$\sigma_1^2$	0.3961	0.0774	0.2280	0.6103
$\sigma_2^2$	0.3844	0.0763	0.2212	0.5922

Table 1: The off-diagonal interaction coefficients are statistically significant at a 95% level. Similar results are obtained for non-diagonal  $\Sigma$  (not shown); for parsimony we use a diagonal error matrix.

### Comparison of model selection criteria for MAR(1) models

Model type	LogLik.	AIC	AICc	BIC
MAR(1) null	-70.01	148.0	148.7	154.1
MAR(1) full	-66.14	144.3	145.7	153.4
MAR(1) full + May temperature year $t + 1$	-63.98	144.0	146.4	156.2
MAR(1) null + May temperature year $t + 1$	-67.03	146.1	147.4	155.2
MAR(1) full + May temp. of year $t$	-63.94	143.9	146.3	156.1
MAR(1) full + May log(precipitation) of $t$	-64.89	145.8	148.2	158.0
MAR(1) full + May temp + log(precipitation)	-62.81	145.6	149.5	160.9
MAR(1) full + July temperature year $t$	-61.95	143.9	147.8	159.2
MAR(1) full + June temperature year $t$	-63.79	147.6	151.4	162.8

Table 2: MAR(1) ‘null’ indicates a diagonal  $\mathbf{B}$  matrix while MAR(1) ‘full’ indicates a full 2x2 interaction matrix. Models including temperature effects on growth rates (third row and below) take the form  $\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + \mathbf{C}\mathbf{u}_t + \mathbf{e}_t$ ,  $\mathbf{e}_t \sim \mathcal{N}_2(0, \Sigma)$ . Here the environmental vector is  $\mathbf{u}_t = (T_{t-l_P+1}, R_{t-l_P+1}, T_{t-l_G+1}, R_{t-l_G+1})^T$ , with  $T$  the temperature and  $R$  log-precipitation. There is a time-lag  $l_P$  for the ptarmigan (0 or 1 year) and  $l_G = 5$  (always) for the gyrfalcon: weather is expected to have such delayed effects on the gyrfalcon counts because of age structure. April weather is considered for gyrfalcon as it is the critical period for reproduction, and it is always included in models from row 3 and below. Models from rows 3 to 7 considered May temperature for ptarmigan, log(precipitation), or both. The models of rows 8 and 9 considered instead July and June temperatures as environmental variables for ptarmigan.



130 Based on the comparison of AICc and BIC between the full  $2 \times 2$  interaction matrix  
131 and the diagonal matrix model (null model), the full model was favored (Table 2). The  
132 model with environmental covariates did not lead to substantially better fit or very different  
133 biotic interaction parameters (Table 2) than the full model, and the weather effects were not  
134 consistent (Table 3), save for those of delayed April temperature on predator growth (see  
135 below).

136 Additional Granger causality testing using the MAR(1) model revealed a two-way recip-  
137 rocal feedback, though the Wald test was weakly statistically significant (at the 0.1 level)  
138 due to the low sample size (i.e., not by ecological standards but compared to other fields  
139 using time series analysis such as econometrics). While accounting for the relative short-  
140 ness of ecological time series, the MAR(1) model therefore strongly suggests a reciprocal  
141 predator-prey coupling of the ptarmigan and gyrfalcon populations.

## 142 **Models with environmental covariates**

143 The addition of environmental covariates did not improve significantly model fit (Table 2).  
144 The coefficients were mostly non-significant, as illustrated by the model including both  
145 temperature and log(precipitation) (0 is included within CIs for environmental  $\mathbf{C}$  matrix  
146 coefficients, Table 3). The model with both precipitation and temperature was deemed  
147 over-parameterized by the information criteria. It is likely that an effect of 5-year delayed  
148 temperature on predator growth is present as this effect was found positive, large and nearly  
149 statistically significant at 95% (Table 3). Although this weather effect on the predator does  
150 not seem to improve significantly the predictive ability of the model. The effect of tempera-  
151 ture in  $\text{May}_{t+1}$  (May of the year) on ptarmigan growth, by contrast, is both not statistically  
152 different from zero and of unexpected sign (negative here, while positive temperature usu-  
153 ally have positive effects on the ptarmigan chicks, Nielsen et al. 2004). The effect of rain in  
154  $\text{May}_{t+1}$  on ptarmigan population growth was negative and relatively strong, but not statis-  
155 tically significant at 95% (point estimate -0.1596, 95%CI: [-0.3735; 0.0541]). It is therefore

### Coefficients for biotic and abiotic effects on population growth

Parameter	value	SE	lower 95% CI	upper 95% CI
$b_{11}$	0.7475	0.1071	0.5376	0.9575
$b_{21}$	0.2031	0.1044	-0.0014	0.4077
$b_{12}$	-0.1984	0.1135	-0.4210	0.0241
$b_{22}$	0.7021	0.1043	0.4977	0.9065
temperature May $_{t+1}$	-0.0853	0.1142	-0.3092	0.1385
precipitation May $_{t+1}$	-0.1596	0.1091	-0.3735	0.0541
temperature April $_{t-4}$	0.2072	0.1061	-0.0008	0.4153
precipitation April $_{t-4}$	-0.0558	0.1068	-0.2652	0.1535
$\sigma_1^2$	0.3653	0.0743	0.2104	0.5628
$\sigma_2^2$	0.3418	0.0733	0.1943	0.5307

Table 3: Species 1 is ptarmigan and species 2 gyrfalcon. May variables only affect species 1 while April variables, delayed by 5 years (we model the effect of variables at  $t - 4$  on growth between  $t$  and  $t + 1$ ), affect only species 2's population growth.

156 possible that such a negative effect is present, but it does not appear clearly with the current  
 157 dataset.

158 We also fitted models where winter weather affects ptarmigan growth (Appendix S2), to  
 159 test the idea that the survival of first-year chick might be lower in harsher winters, but again  
 160 we did not find consistent effects of weather on ptarmigan growth.

### 161 **MAR(2) model results**

162 The MAR(2) models showed uniformly better fit than the MAR(1) models (Table 4). Note  
 163 that, in order to make this comparison, we re-fitted the MAR(1) model with one less year  
 164 to compare MAR(1) and MAR(2) models with an equal number of points, as any difference  
 165 in data can strongly affect AIC and BIC values. The MAR(2) model with independent  
 166 populations (i.e., diagonal interaction matrices  $\mathbf{B}^{(1)}$  and  $\mathbf{B}^{(2)}$ ) and the bottom-up predator-  
 167 prey model (see Methods), assuming an independently cycling prey and a predator whose  
 168 dynamics is forced by its prey, were the better-ranking models (Table 4).

169 Because AIC and BIC assess only one aspect of statistical model quality, the trade-off  
 170 between model parsimony and fit, we also present the results of simulations of the models

## Comparison of model selection for MAR(1) and MAR(2)

Model type	logLik.	AIC	AICc	BIC
MAR(1) null	-67.57	143.1	143.8	149.1
MAR(1) full	-64.41	140.8	142.3	149.8
MAR(2) full	-54.78	129.6	133.6	144.5
MAR(2) full	-57.91	129.8	131.8	140.3
MAR(2) null	-58.78	129.6	131.0	138.5
MAR(2) null + temperature	-56.95	129.9	132.4	141.9

Table 4: Comparison of model selection criteria for MAR(1) and MAR(2) models with different structures. See Methods for definitions. The MAR(2) null + temperature uses April temperature with a 5 year delay, which affects the predator only - this model adds temperature to the list of potential drivers for predator dynamics, as it was found marginally significant in previous MAR(1) analyses.

171 (Fig. 2). The examination of time series plots is however difficult because the simulated time  
172 series are relatively short and noisy.

173 We therefore simulated 100 datasets using the fitted models (Fig. 3) and examined their  
174 cross-correlations, these show that MAR(1) and MAR(2) models with reciprocal predator-  
175 prey feedback (full interaction matrices) outperform both the bottom-up model (medium  
176 reproduction of the the cross-correlation pattern) and the null model (no reproduction of the  
177 cross-correlation pattern).

## 178 Discussion

179 The percentage of explained variance in log-abundances by the MAR(1) predator-prey model  
180 was about 60%; hence similar to the lynx-hare example of Vik et al. (2008). The full 2x2  
181 interaction matrix in a MAR(1) framework provided a better model than a diagonal matrix,  
182 meaning there was causality (or *feedback*) between prey and predator dynamics in the sense  
183 of Granger (1969): the addition of the predator and prey variables reduced the residual  
184 variances of the time series models for the prey and predator, respectively.

185 Weather (i.e., April temperature 5 years lagged) was found to influence predator dynam-  
186 ics, revealing an influence of weather on gyrfalcon reproduction, which takes several years to

### Simulated time series

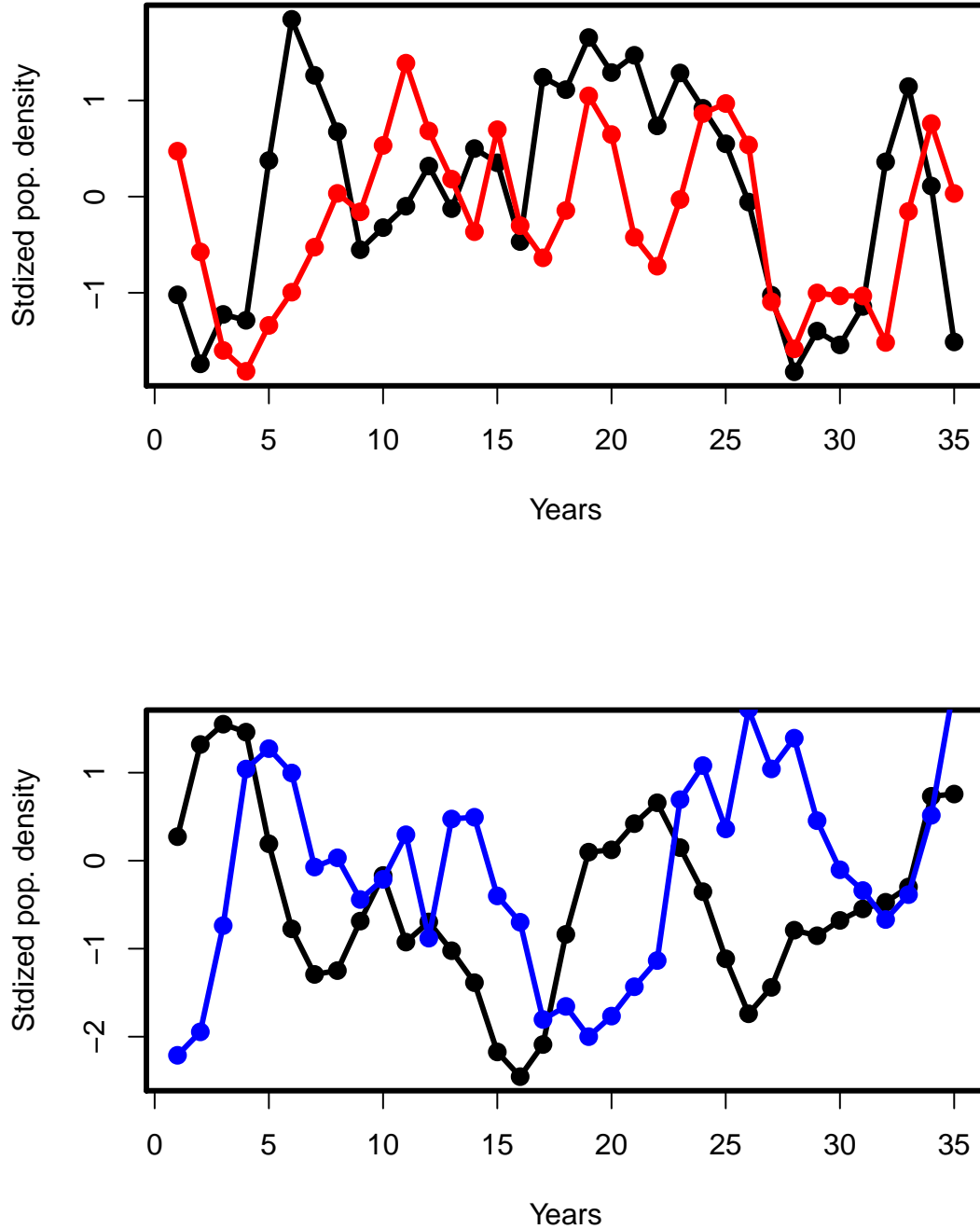


Figure 2: Time series of predator (gyrfalcon) and prey (rock ptarmigan) log-densities, simulated for 35 years from the same starting conditions as the data, for the full MAR(1) model (top panel, predator in red) and the MAR(2) ‘bottom-up’ model (bottom panel, predator in blue).

### Cross-correlation patterns for fitted models

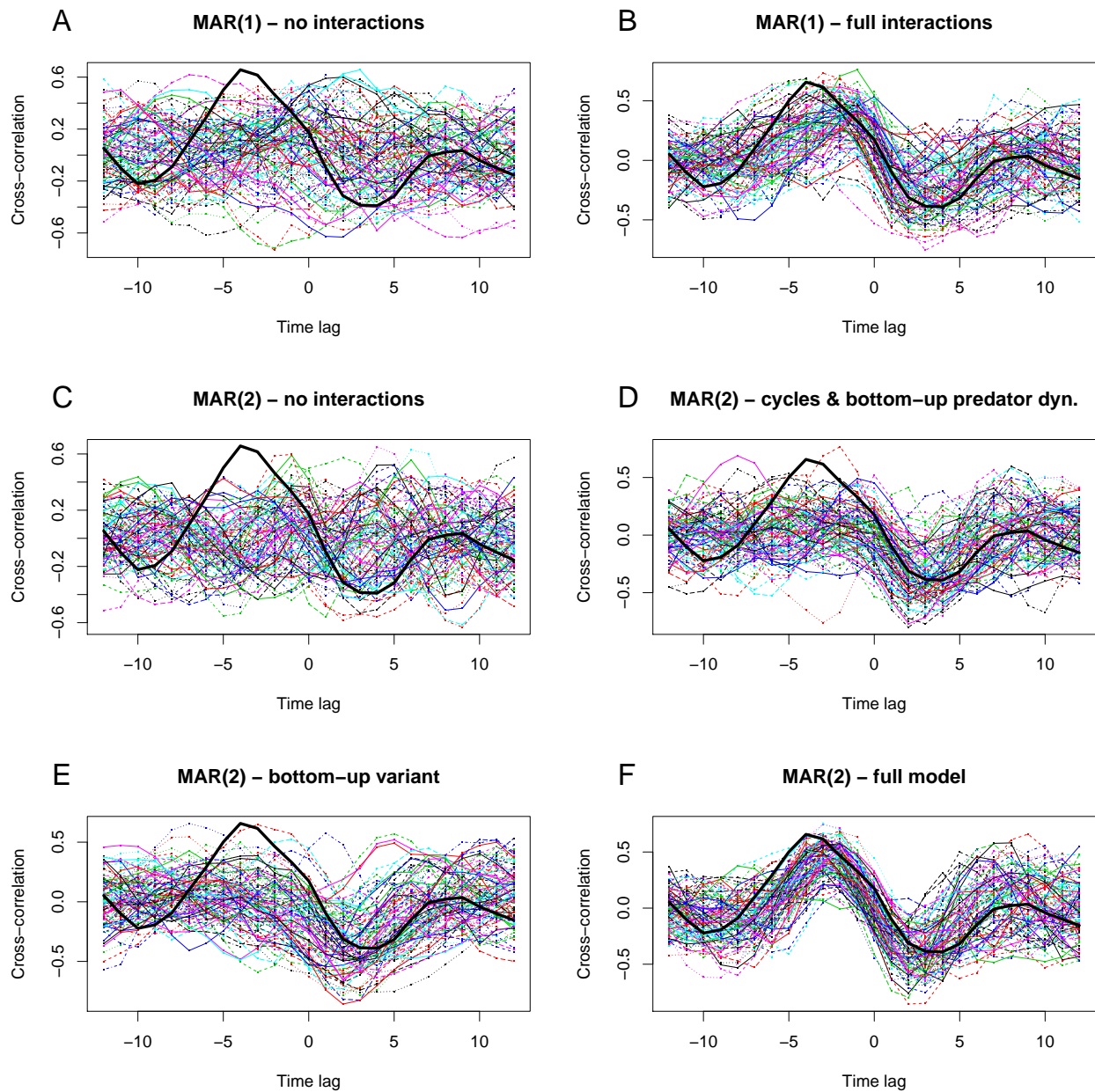


Figure 3: Cross-correlation functions (CCFs) for the fitted models (A to F). Each thin line corresponds to one simulation of the fitted model, within each panel. A and B show MAR(1) models, without and with interactions; while C to F show the CCFs of simulated MAR(2) models, without interactions (C), with only bottom-up interactions (D), bottom-up without predator regulation with a delay (E), and (D) full MAR(2) model. The cross-correlation for the real data is highlighted as a thick black line in all panels.

187 affect the growth of the adult segment of the population. However, prey population growth  
188 was not affected by any of the weather covariates considered, neither in spring nor in winter.  
189 This is a surprising find, which we discuss below.

190 If we had stopped the analyses to the MAR(1) model, which is customary in ecology (e.g.  
191 Ives et al., 2003; Hampton et al., 2013), we would have concluded unequivocally to a strong  
192 coupling between predator and prey (Table 2). However, another reasonable hypothesis  
193 was that both species - the prey especially - could cycle independently (see e.g., Dobson and  
194 Hudson, 1992, for a host-parasite modeling study in a similar prey species). Contrasting such  
195 hypotheses required to formulate a MAR( $p$ ) model with  $p = 2$  timelags (according to BIC,  
196 the optimal lag order was 2; 3 according to AIC). MAR(2) models were therefore found to  
197 realize a better trade-off between parsimony and fit than MAR(1) models (Table 4, for more  
198 information criteria see additional analyses<sup>1</sup>). While the model with independently cycling  
199 populations fitted the data well, it produced unrealistic dynamics (i.e., no cross-correlation,  
200 Fig. 3). The bottom-up predator-prey model, where the prey influences the predator but  
201 not the other way around, provided both a good fit and relatively realistic dynamics, though  
202 not as much as the models including reciprocal feedback (full-matrix MAR(1) and MAR(2)  
203 models). The bottom-up scenario could correspond, for example, to a case where the predator  
204 dynamics are driven by its prey, but prey dynamics are themselves driven by an interaction  
205 with a parasite (see Stenkewitz et al., 2016, for an appreciation of host-parasite dynamics in  
206 Iceland rock ptarmigan). The bottom-up scenario fitted the data better in terms of trade-off  
207 between parsimony and fit, but predicted the cross-correlation pattern worse. Hence with  
208 the currently available information, both scenarios must be considered plausible. Further  
209 simulation results do help, however, to interpret a little better which scenario is the most  
210 likely (see below).

211 This absence of conclusion on mechanisms, given the length of the survey, may appear at  
212 first sight distressing. However, from the perspective of time series analyses, even 35 years is

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<sup>1</sup>[https://github.com/fbarraquand/GyrfalconPtarmigan\\_MAR](https://github.com/fbarraquand/GyrfalconPtarmigan_MAR)  
The repository will be made publicly available upon acceptance.

213 very short. In fact, in his authoritative book on multivariate time series modeling, Lütkepohl  
214 (2005) shows that it can be hard to recover the simulated lag order of such simple  $2 \times 2$   
215 MAR(1) and MAR(2) models. Specifically, Lütkepohl (2005) simulated a bivariate MAR(2)  
216 model with a time series length  $n = 30$ , fitted MAR( $p$ ) models up to order  $p = 6$ , and  
217 found only 32% of correctly classified simulations as  $p = 2$  using AIC, with 42% classified  
218 as  $p = 1$  (p. 155 in Lütkepohl 2005). Using BIC, he found even 80% misclassified as  
219 MAR(1). Different model selection criteria give different answers, and the BIC tends to be  
220 most conservative, but the baseline is that for  $T = 30$ , selection according to information  
221 criteria only gives inconsistent answers, while in most cases (70% for AIC) the right model  
222 order was found for  $n = 100$ . Results were overall better with simulated bivariate MAR(1)  
223 models (p. 156 in Lütkepohl 2005), where all model selection criteria were able to pinpoint  
224 the correct lag order at 90%. The results, however, are likely to be model-structure and  
225 model-parameter specific; therefore we performed such analysis for the models that we fitted.  
226 Specifically, we performed 1000 simulations of the fitted MAR(1) full (F) model and MAR(2)  
227 bottom-up (BU) model. We fitted the MAR(1) F model to both MAR(1) F and MAR(2)  
228 BU simulations, and then fitted the MAR(2) BU to both MAR(1) F and MAR(2) BU  
229 simulations. This allowed to compute the percentage of correctly ascribed scenarios, on the  
230 basis of information criteria (AIC, AICc, BIC) for each simulated model (Appendix S3).  
231 We found that for  $n = 35$ , a simulated MAR(1) F was recovered respectively 52%, 56%,  
232 64% for AIC, AICc, and BIC, respectively. By contrast, a simulated MAR(2) BU model  
233 was recovered 98%, 98% and 97%. With the current length of our dataset, the important  
234 message from these simulations is that based on AIC or BIC, we are much more likely to  
235 mistake a fully interacting predator-prey system for a bottom-up system than the reverse.  
236 These percentages were all very close to 90% and 100% in the case of  $n = 100$  (Appendix  
237 S3).

238 From our simulation experiments, we can derive three lessons. First, from an ecological  
239 viewpoint, given that the full interaction MAR(1) model both predicts the cross-correlation



240 pattern better and is the most likely to be misidentified as MAR(2) BU, we should not give  
241 too much weight to the better (lower) AIC and BIC scores of the MAR(2) BU model. It is  
242 fairly likely that top-down prey regulation and therefore reciprocal predator-prey feedback  
243 is at work here. Second, from a more statistical viewpoint, it is informative to notice that  
244 whether MAR(1) or MAR(2) models are better identified is parameter-specific: sometimes  
245 a MAR(1) model will be more likely to be correctly classified (as in the simulation study  
246 of Lütkepohl, 2005), sometimes a MAR(2) will (our case study). Therefore, whether an  
247 ecological scenario is better identified than another one from time series is likely to be  
248 context-specific, and not simply dependent on the lag order. The corollary being that new  
249 simulations from MAR(p) or other time series models will be required for each new ecological  
250 case study, in order to see which scenarios are the most likely to be misidentified. Third,  
251 we found, in agreement with Lütkepohl (2005), that time series of around 100 points are  
252 needed to allow for fairly reliable inference of top-down vs bottom-up dynamics in systems  
253 of 2 cyclic species (less points might be required for species with simpler dynamics).

254 Given that the data presented here is collected once a year for the most part, and that  
255 it is not feasible to census the population much more frequently with current means (other  
256 technologies would be necessary, such as camera traps or DNA-based evidence), it is un-  
257 likely that we will get the time series near 100 years within acceptable time frames for  
258 management of both populations (i.e., conservation of the gyrfalcon and sustainable hunting  
259 management of ptarmigan). Therefore, differentiating unequivocally between the bottom-up  
260 and predator-prey feedback scenarios will likely require other type of models and data. We  
261 still view the MAR(p) approach as useful, however, as a means to delineate likely scenarios  
262 to investigate further, and check for important abiotic drivers that need to be considered.  
263 Why the ptarmigan population growth is not affected - at the NE Iceland scale - by weather  
264 variables is also a puzzling question for further study.

265 Mechanistic modeling might help further understand the effect of drivers on ptarmigan  
266 dynamics during certain phases of the cycle. For instance, are ptarmigan declines mainly



267 driven by its predator (i.e., the gyrfalcon is responsible in large part for the declines) or  
268 mainly by other causes such as parasites? Rough estimates of predation demonstrate why  
269 this question is intrinsically difficult. Around 100 adult predator pairs can be found near peak  
270 abundance on the NE Iceland ptarmigan management zone, and to these correspond about  
271 100 000 ptarmigan individuals at best (Sturludóttir, 2015). One might think, given these  
272 numbers, that the predators are unlikely to make their prey decline. Ptarmigan, however,  
273 have a slow, long-period cycle (Fig. 1). Therefore, they decrease at worst by  $\approx 20\,000$  in a  
274 single year. A quick division indicates that about 200 would have to be eaten during the year  
275 by a predator pair for such a decrease to occur - assuming, as a first approximation, that  
276 increases in the ptarmigan population due to reproduction are offset by other causes of death  
277 than predation. This quantity, 200 kills a year, is an order of magnitude that represents a  
278 fairly high yet doable consumption by a predator pair. This might be tested further by fitting  
279 more mechanistic predator-prey models.

280 Although we currently do not possess all the information necessary to parameterize mech-  
281 anistic predator-prey or host-parasite models, we suggest a few directions. First, we have a  
282 rather imperfect knowledge of the predator population, especially its non-territorial segment  
283 (Nielsen, 2011). Non-territorial floaters can indeed be rather numerous in both real raptor  
284 populations (Katzner et al., 2011) and parameterized bird population models (Barraquand  
285 et al., 2014). Floater numbers could therefore change our perception of predator impacts on  
286 prey dynamics (i.e., the predator population might increase by half or more). Demographic  
287 modeling of the predator population and its various life stages is therefore in order - we are  
288 currently examining CMR data and hoping for DNA-based information. Second, the cur-  
289 rent models have shown that the host-parasite hypothesis for the ptarmigan dynamics (see  
290 Stenkewitz et al., 2016) needs to be examined. We therefore need to know more about the  
291 parasite loads and their potential impact on ptarmigan population growth. Third, there are  
292 spatial aspects in the dynamics of gyrfalcon and ptarmigan that we have not tackled. It is  
293 plausible, for instance, that the weather does not affect ptarmigan growth at the scale of NE

294 Iceland, using averaged state variables and covariables, and yet that weather locally affects  
295 the survival of ptarmigan chicks, as additional data seem to suggest: Nielsen et al. (2004)  
296 found that mean windspeed and mean precipitation in June-July explained a considerable  
297 part of the variance in chick production.

## 298 **Conclusion**

299 Our results have implications for other studies on birds and more generally vertebrates  
300 with relatively slow life histories (compared to e.g., plankton sampled many times a year).  
301 Using long time series by ecological standards (34 years), we found some evidence of re-  
302 ciprocal predator-prey feedback in this cyclic predator-prey system, without being able to  
303 exclude nonetheless more bottom-up predator-prey dynamics. MAR( $p$ ) models with  $p = 1, 2$   
304 described well this system as a forced oscillator, although the unexplained noise was gener-  
305 ally stronger than weather effects, which may point to other important biotic factors driving  
306 the dynamics, such as parasites. Simulations of the fitted models revealed than unequivocal  
307 inference of bottom-up versus reciprocal predator-prey coupling (i.e., including top-down  
308 predator influence on prey) would require about a century of time series data. We therefore  
309 think that additional demographic data (e.g., through capture-recapture, genetics,...) should  
310 always be considered in conjunction to counts taken once or twice a year, if one of the goals  
311 of a monitoring study is to infer interactions between the populations of different species.

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## 400 Supplementary Information

### 401 Appendix S1 - Alternative model fitting

Parameters	Meaning	Point estimate	LB 95%	HB 95%
$b_{11} - 1$	prey $\rightarrow$ prey	-0.24	-0.47	-0.03
$b_{12}$	predator $\rightarrow$ prey	-0.24	-0.47	-0.04
$b_{21}$	prey $\rightarrow$ predator	0.23	0.04	0.46
$b_{22} - 1$	predator $\rightarrow$ predator	-0.34	-0.57	-0.12
$\sigma_1$	noise species 1	0.67	0.53	0.87
$\sigma_2$	noise species 2	0.66	0.52	0.85

Table S1: Estimates of the full MAR(1) model using JAGS, for comparison with the results of the main text.



402 **Appendix S2 - Effects of winter weather**

403 We tested the effect of winter weather by introducing new winter weather variables into  
404 the MAR(1) models:

- 405 • The mean winter temperature from December to March
- 406 • The average of log(precipitation) over the same period

407 We also considered minimum temperature but this did not alter the following results.

408 The two above mentioned winter weather variables were inserted in place of spring  
409 weather variables for ptarmigan into a MAR(1) model. The estimated parameters are re-  
410 produced in Table S1 and the Information Criteria, with previous models for comparison, in  
411 Table S2. None of the models are able to significantly improve the fit, although it is possible  
412 that a weakly statistically significant effect of winter temperature exists.

Parameter	value	SE	low 95% CI	up 95% CI
$b_{11}$	0.6837	0.1169	0.4546	0.9129
$b_{21}$	0.2035	0.1045	-0.0013	0.4084
$b_{12}$	-0.1999	0.1077	-0.4111	0.0113
$b_{22}$	0.7018	0.1044	0.4971	0.9065
Mean winter temp $_{t+1}$	<i>-0.1466</i>	<i>0.1210</i>	<i>-0.3838</i>	<i>0.0906</i>
Winter precipitation $_{t+1}$	<i>-0.1720</i>	<i>0.1185</i>	<i>-0.4044</i>	<i>0.0602</i>
temperatureApril $_{t-4}$	0.2071	0.1062	-0.0010	0.4153
rainApril $_{t-4}$	-0.0557	0.1068	-0.2652	0.1537
$\sigma_1^2$	0.3635	0.0742	0.2092	0.5602
$\sigma_2^2$	0.3422	0.0734	0.1945	0.5314

Table S1: Coefficients for biotic and abiotic effects on population growth. Species 1 is ptarmigan and species 2 gyrfalcon. Winter variables only affect species 1 while April variables, delayed by 5 years (we model the effect of variables at  $t - 4$  on growth between  $t$  and  $t + 1$ ), affect only species 2's population growth. *Effects of winter variables are depicted in italics.*

Model type	logLik.	AIC	AICc	BIC
MAR(1) null	-70.01	148.0	148.7	154.1
MAR(1) full	-66.14	144.3	145.7	153.4
MAR(1) full + May temperature year $t$	-63.98	144.0	146.4	156.2
MAR(1) null + May temperature year $t$	-67.03	146.1	147.4	155.2
MAR(1) full + May temp. of year $t - 1$	-63.94	143.9	146.3	156.1
MAR(1) full + May log(prec.) of $t - 1$	-64.89	145.8	148.2	158.0
MAR(1) full + May temp + log(prec.)	-62.81	145.6	149.5	160.9
MAR(1) full + Mean Winter temp + mean log(prec.)	-62.72	145.4	149.3	160.7
MAR(1) full + Min Winter temp + mean log(precipitation)	-62.72	145.4	149.3	160.7

Table S2: Comparison of model selection criteria for MAR(1) models. MAR(1) ‘null’ indicates a diagonal  $\mathbf{B}$  matrix while MAR(1) ‘full’ indicates a full 2x2 interaction matrix. Models including temperature (third row and below) effects on growth rates take the form  $\mathbf{x}_{t+1} = \mathbf{a} + \mathbf{B}\mathbf{x}_t + \mathbf{C}\mathbf{u}_t + \mathbf{e}_t, \mathbf{e}_t \sim \mathcal{N}_2(0, \mathbf{\Sigma})$ . Here the environmental vector  $\mathbf{u}_t = (T_{t-l_P}, R_{t-l_P}, T_{t-l_G}, R_{t-l_G})^T$ , with  $T$  the temperature and  $R$  log-rainfall. There is a timelag  $l_P$  for the ptarmigan (0 or 1 year, depending on the month) and  $l_G = 5$  for the gyrfalcon. IC scores for the two winter models are depicted on the last two rows . prec. = precipitation (rain and snow).

413 **Appendix S3 - Simulation results**

Simulated model	AIC	AICc	BIC
$n = 35$	0.52	0.56	0.64
$n = 100$	0.98	0.98	0.97

Table S1: Frequency of correct identification of MAR(1) full and MAR(2) bottom-up models for two time series lengths.