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POINT OF VIEW

Running head: RETHINKING PCMS

Rethinking phylogenetic comparative methods

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Keywords: Macroevolution, Causality, Graphical Models, Phylogenetic Natural History

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Abstract

As a result of the process of descent with modification, closely related species tend to be similar to one another in a myriad different ways. In statistical terms, this means that traits measured on one species will not be independent of traits measured on others. Since their introduction in the 1980s, phylogenetic comparative methods (PCMs) have been framed as a solution to this problem. In this paper, we argue that this way of thinking about PCMs is deeply misleading. Not only has this sowed widespread confusion in the literature about what PCMs are doing but has led us to develop methods that are susceptible to the very thing we sought to build defenses against — unreplicated evolutionary events. Through three Case Studies, we demonstrate that the susceptibility to singular events indeed a recurring problem in comparative biology that links several seemingly unrelated controversies. In each Case Study we propose a potential solution to the problem. While the details of our proposed solutions differ, they share a common theme: unifying hypothesis testing with data-driven approaches (which we term "phylogenetic natural history") to disentangle the impact of singular evolutionary events from that of the factors we are investigating. More broadly, we argue that our field has, at times, been sloppy when weighing evidence in support of causal hypotheses. We suggest that one way to refine our inferences is to re-imagine phylogenies as probabilistic graphical models; adopting this way of thinking will help clarify precisely what we are testing and what evidence supports our claims.

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Introduction

Every so often, evolution comes up with something totally new and unexpected, a so-crazy-it-just-might-work set of adaptations that is the stuff of nature documentaries. Many biologists likely have a favorite example of a lineage that has evolved something spectacular such as devilishly horned lizards that squirt blood from their eye sockets, marine sloths that grazed ancient seabeds, or that ancient lineage of therapsid reptile that became covered in hair and filled with warm blood and milk.

As macroevolutionary researchers, it is hard to know what to do with these types of events. Their singular and unreplicated nature seems incompatible with models that we typically use to model change over time, such as Brownian motion (BM; Felsenstein, 1973). Such models presume continuity, whereas rare events, such as the evolution of novel nutritive function in milk-producing glands, have no clear precedent in history. The evolution of such traits may set in motion a cascade of changes across an organism, such that descendant lineages may look very different in many ways from their more distant relatives. Or alternatively, a suite of traits may just happen to change at the same time. In either case, it is these sorts of idiosyncratic and unreplicated events that we often think of when we think of the need to consider phylogeny in analyses of comparative data. And this is not an abstract concern; a wide breadth of macroevolutionary data suggest that abrupt shifts and discontinuities have been a major feature of life on Earth (Uyeda et al., 2011, 2017; Landis and Schraiber, 2017; Jablonski, 2017). But as recent controversies in phylogenetic comparative biology have highlighted, our methods may not be up to this task.

As examples, we highlight two recent controversies in phylogenetic comparative methods (PCMs; for recent reviews, see Pennell and Harmon, 2013; O'Meara, 2012; Garamszegi, 2014). First, Maddison and FitzJohn (2015) demonstrated that common statistical tests (e.g., Pagel, 1994; Maddison, 1990) for the evolutionary correlation of discrete characters are prone to reporting a significant association even when the pattern is driven by a single (or, very few) independent evolutionary event(s). Maddison and FitzJohn (2015) referred to such scenarios as cases of 'phylogenetic pseudoreplication' (see also Read and Nee, 1995; Nee et al., 1996). We will argue that this unresolved problem permeates not just tests for discrete character correlations, but nearly every method of finding associations in comparative methods (Figure 1), including those involved in our second example: the unacceptably high type-1 error rates (Rabosky and Goldberg, 2015) of methods used to infer trait-dependent diversification (e.g., BiSSE; Maddison et al., 2007). Specifically, Rabosky and Goldberg (2015) show that applying BiSSE to real-world phylogenies, which are usually not shaped liked the birth-death trees assumed by our models (Mooers and Heard, 1997), often leads in erroneous support for trait-dependent diversification models even when diversification dynamics are unrelated to the traits being considered. The work of Beaulieu and O'Meara (Beaulieu et al., 2013; Beaulieu and O'Meara, 2014, 2016) has illuminated the underlying reasons behind Rabosky and Goldberg's findings: the failure to consider biologicaly-plausible alternative models. To address this shortcoming, Beaulieu et al. (2013)

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borrowed an idea from molecular phylogenetics (Penny et al., 2001; Galtier, 2001), and developed a Hidden Markov Model (HMM) for describing the evolution of a binary character. In their HMM the transition rates between character states depend on the 'hidden' state of another, unobserved, trait also evolving along the tree (also see Price, 1997, who explored a related model). Applying the same principle to trait-dependent diversification models, they showed how models that included background heterogeneity in diversification rates provide a fairer comparison to the hypothesis of genuine state-dependent diversification (Beaulieu and O'Meara, 2016). Rather than considering a biologically unrealistic constant-rate null hypothesis, Beaulieu and colleagues built models that allowed traits and diversification to vary in biologically plausible ways (also see Zenil-Ferguson and Pennell, 2017, on this point).

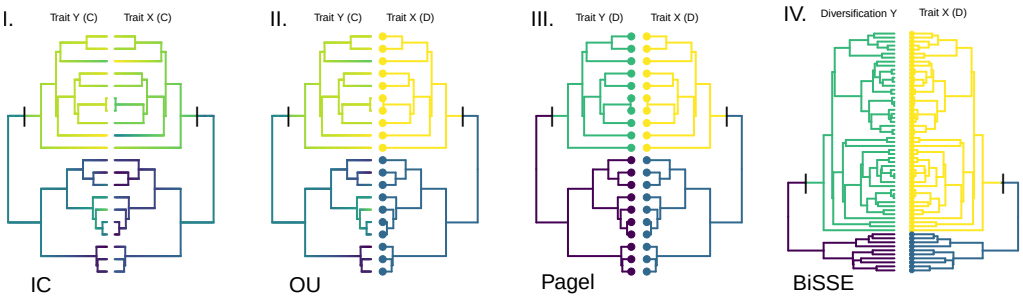

Figure 1: Singular, unreplicated events (vertical dashes) can drive significant results across several types of comparative analyses. Case Studies I–III are indicated in panels I–III, and though we do not consider diversification models such as BiSSE in our examples, they are similarly affected (panel IV). In each case, we map (in some cases, arbitrarily) the dependent variable (Y) on the phylogeny on the left and the predictor trait on the same phylogeny to the right (X), and indicate whether the trait is a continuous trait (C), a discrete trait (D) or a diversification rate. We also suggest a common method used to analyze such associations: IC - Independent Contrasts (Felsenstein, 1985); OU - Ornstein-Uhlenbeck models (Butler and King, 2004); Pagel - Pagel's correlation test (Pagel, 1994). Colors on the branches indicate the state of the character on the phylogeny — either continuous trait value, discrete character state, or diversification rate regime. Panels I and III correspond to variations of "Felsenstein's worst-case scenario" and "Darwin's scenario", respectively.

We think that the type of solution suggested by Beaulieu and O'Meara (2016) is general and applies across comparative biology. In this paper we develop this argument through a series of three Case Studies, depicted in panels I–III of Figure 1. We will show in each Case Study that rare evolutionary events may deceive our methods and distort our interpretation. For each study, we will then sketch out possible solutions for making causal inferences from comparative data. Each of these approaches share a common philosophy but may differ in their details. We do not have a one-size-fits-all solution and think that a diverse set of solutions are

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worth considering.

More specifically, all three Case Studies revolve around the problem of how to discover plausible histories of rare, evolutionary events — a practice we call "phylogenetic natural history" — and how to disentangle the impact of these events from that of the hypothesized effects we are investigating. But as we argue throughout this paper, the inference problems stemming from singular events are not actually specific to these cases. Rather they are only especially clear examples of broader challenges in comparative biology. By working through the singular events cases, we develop two ideas that we think will help move PCMs forward. First, we advocate for unifying hypothesis-testing and data-driven approaches. Rather than being alternative methods of investigating macroevolutionary processes and patterns, they are complementary, and in our view, essential, to one another. Second, we propose that comparative biologists need to be more careful about how we draw causal inference from phylogenetic data. One particular solution is to render comparative analyses as graphical models. These graphical models can help clarify exactly what causal statements we are making and what the limits of these inferences are.

Case Study I: Felsenstein's Worst-Case Scenario

More than anything else, it was the famous series of figures depicting his "worst case scenario" (Figures 5, 6, and 7 in the original; our Figure 2) from Felsenstein's iconic 1985 paper "Phylogenies and the comparative method" that really grabbed biologists by their Chacos and got the ball rolling on modern comparative thinking. The idea is simple: as a result of shared ancestry, measurements taken on one species will not be independent from those collected on another and especially so, if the two species are closely related. While other researchers had hit upon similar notions throughout the early 1980s (e.g., Clutton-Brock and Harvey, 1980; Mace et al., 1981; Ridley, 1983; Stearns, 1983; Cheverud et al., 1985), none of these had the pervasive impact that Felsenstein's presentation did (see for example, Losos, 2011, who reproduces the figures and the accompanying reasoning in his presidential address for the American Society of Naturalists). The problem is just so obvious; all you have to do is look. And while of course his proposed solution, "independent contrasts" (IC), was widely adopted, we suspect it is the clarity with which Felsenstein articulated the problem that has kept his paper a hallmark of biological education and a testament to the importance of tree-thinking, even as his method has largely been largely superseded by the related least squares (Grafen, 1989) and mixed model (Lynch, 1991; Housworth et al., 2004; Hadfield and Nakagawa, 2010) approaches.

However, an important part of this story is often missed: Felsenstein also noted that the problem of non-independence does not occur if "characters respond essentially instantaneously to natural selection in the current environment, so that phylogenetic inertia is essentially absent" (p. 6). Despite this comment, a frequent misunderstanding of his argument is that the problem inherent in a non-phylogenetic regression of phylogenetically structured data is that species are not

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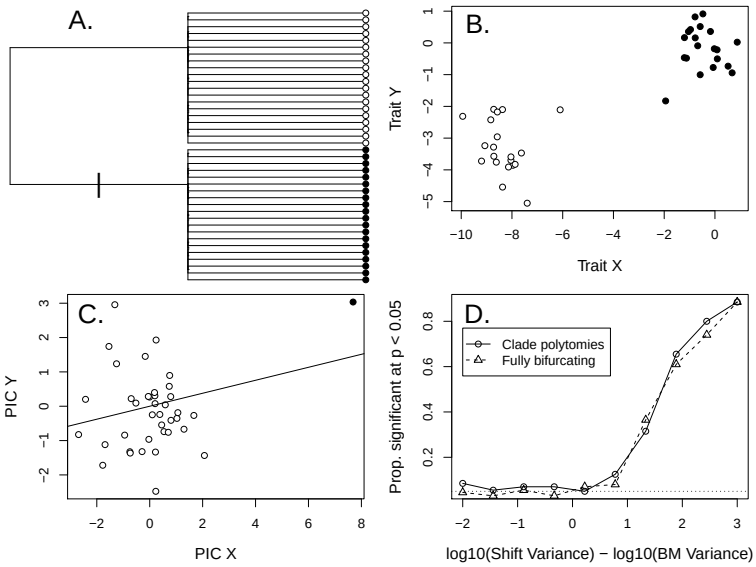

Figure 2: Felsenstein's scenario (Felsenstein, 1985) illustrates a problem quite like that identified by Maddison and FitzJohn. Here we modify Felsenstein's original generating process from simple Brownian Motion, to A) Brownian Motion with a single burst occurring on the stem branch of one of the two clades (indicated by vertical dash). B) The distribution of trait values produces a figure very similar to Felsenstein's original scenario, but results in C) a single contrast (black) that is not well-explained by the estimated Brownian Motion process, and thereby generates a significant regression of PIC Y and PIC X (dotted line) despite both X and Y in the shift and BM distributions being uncorrelated. D) As the ratio of the shift variance to the BM variance increases, the proportion of contrast regressions that return a significant result increases dramatically (each point represents 200 simulations for a fixed phylogeny, with both the BM process and the random draw from the shift distribution being uncorrelated with equal variance for both traits). While IC corrects for singular events consistent with Brownian Motion, it does not correct for the more general phenomenon of dramatic singular events driving significant results in comparative analyses. Note that independence of species as data points is not the issue.

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independent. In fact, independence of data is not an assumption of standard (non-phylogenetic) linear regression at all! Rather, standard linear regression assumes that the *residuals* of the fitted model are independent and identically distributed (i.i.d.). As a result, many applications of a "phylogenetic correction" seem to be missing the point (Revell, 2010; Hansen and Bartoszek, 2012): if all of the phylogenetic signal in a dataset is present in the predictor trait and residual variation is i.i.d., then there is no need for any phylogenetic correction (Rohlf, 2001, 2006). (However, phylogenetic analyses are nearly always needed to determine this condition in the first place.)

But for many researchers, applying non-phylogenetic methods to phylogenetically structured data is deeply unsettling; it just seems wrong somehow, even if we cannot quite put our finger on why (a problem that we revisit below). We suggest that what made Felsenstein's *prima facie* argument so compelling was that it appealed to biologists' intuition that many large clades of organisms are just different in many potentially idiosyncratic ways. In other words, singular events are a common feature of evolution across the tree of life (Uyeda et al., 2011; Landis and Schraiber, 2017; Uyeda et al., 2017; Jablonski, 2017) and we do not want to infer a causal relationship from unreplicated data (Nee et al., 1996). To illustrate the effect of non-independence of characters, Felsenstein simulated a "worst-case scenario" (our Figure 2) in which two clades are separated by long branches. He then evolved traits according to a BM process along the phylogeny; he recovered a significant regression slope using Ordinary Least Squares (OLS) despite there being no evolutionary covariance between the traits.

Here we revisit Felsenstein's worst case scenario in order to demonstrate that IC and PGLS (which is identical to IC when the residuals are assumed to covary according to a BM model; Blomberg et al., 2012) do not completely address the problem that we tend to think they do — these methods are still susceptible to singular evolutionary events. In our first scenario, we used a phylogeny with two clades, each of which is internally unresolved, similar to that of Felsenstein's original example. We emphasize that the only phylogenetic structure is that stemming from the deepest split. We then simulated two traits under independent BM processes, each with an evolutionary rate (σ^2) of 1. So far, this is an identical procedure to Felsenstein's initial presentation. However, at some point on a stem branch of one of the two clades we introduce a singular evolutionary "event" drawn from a multivariate normal distribution with uncorrelated divergences and equal variances that are a scalar multiple of σ^2.

The resulting distribution of the data suggests a situation very similar to Felsenstein's original worst-case scenario, and what we argue is the type of problem envisioned by most biologists when they warn their students of the dangers of ignoring phylogeny. To take a more concrete example, consider birds and mammals. Lots of things have happened since these groups diverged from their common ancestor and these have happened for many idiosyncratic reasons that are not well described by our models. For example, milk evolved somewhere along the mammalian lineage and surely this affected the evolution of other traits. Yet it would be nonsensical to describe the evolution of milk as a Brownian process, starting in some ancient reptile and merrily continuing on its way from Aardvarks to Zebra

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Finches.

One would hope that our tools for "correcting for phylogeny" would recognize that the apparently strong relationship between the two traits in our example was driven by only a single contrast. However, this is not the case. That single contrast results in a very high-leverage statistical outlier that drives significance as the size of the shift increases (Figure 2). We can repeat the same exercise with more phylogenetically structured data (where the two clades of interest are fully bifurcating following a Yule process) and obtain identical results (Figure 2, see Supplementary Material). This is disconcerting since our intuition suggests that we do not have compelling evidence for a causal relationship between these two traits (i.e., there is very little reason for us to believe from this correlation alone that one trait is an adaptation to the other).

How can we formulate a better set of models that can account for what our intuition tells us is a dangerous situation for causal inference? We can do so by including another phylogenetically plausible model: a singular shift driving differences between clades. Let us consider a scenario quite distinct from Felsenstein's multivariate BM (mvBM) scenario. Instead, traits do not evolve by mvBM, but rather undergo a shift at a single point (e.g., perhaps ancient dispersal event where one clade invaded a new environment or the evolution of a novel key innovation). In such a scenario, we only need to consider the phylogeny in as much as a given species exists on either side of the event in question; except for this difference, the traits have no phylogenetic signal and the residuals are otherwise i.i.d. We can then erect two models: a linear regression model and a singular event model.

Linear regression model:

$$\begin{aligned} Y &= \beta_X X + \beta_0 + \epsilon; \\ X &= \psi(X) \end{aligned} \tag{1}$$

where β_X and β_0 are the slope and intercept to the regression of Y on X, ϵ is a vector containing i.i.d. random variables describing the error, and the predictor X is generated by some stochastic process $\psi(\cdot)$ on the phylogeny (e.g., a random variable describing a single burst in X on the stem branch of one of the two clades). Alternatively, X and Y may not be related to one another at all. Rather, they may be the products of singular random evolutionary events, $E1$ and $E2$, that just so happened to occur on the branch separating two clades:

Singular events model:

$$\begin{aligned} Y &= \beta_Y I_{E1} + \beta_{Y0} + \epsilon_Y; \\ X &= \beta_X I_{E2} + \beta_{X0} + \epsilon_X \end{aligned} \tag{2}$$

where the variables I_{E1} and I_{E2} are indicator random variables that take the value of 1 if an observation is from a lineage that experienced a phylogenetic event, or otherwise they are 0. Furthermore, β_{Y0} and β_{X0} are the parameters that describe the trait means had they not experienced the singular evolutionary event in ques-

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tion. Thus, under the laws of conditional probability, the bivariate probability $P(X, Y)$ under the liner model is:

$$P(X, Y) = P(Y|X, \beta_X, \beta_0, \sigma_Y)P(X|\theta_\psi, \sigma_X)P(\beta_X)P(\beta_0)P(\theta_\psi)P(\sigma_Y) \quad (3)$$

where θ_ψ are the parameters of the process for X on the phylogeny, and σ_Y^2 and σ_X^2 are the residual variances. This equation is derived from the assumed path of causation between X and Y, since the likelihood function of trait X, denoted by $P(X|\theta_\psi, \sigma_X)$, is independent of Y, while the likelihood function of Y, denoted by $P(Y|X, \beta_X, \beta_0, \sigma_Y)$ depends on X. The remaining terms in the probability statement are interpreted as prior distributions for the parameters in a Bayesian inferential framework. For the singular event model, a similar exercise results in:

$$\begin{aligned} P(X, Y) = {} & P(\beta_Y)P(\beta_{Y0})P(\beta_X)P)(\beta_{X0})P(\sigma_X)P(\sigma_Y) \\ & \times P(N_{E1} = 1)P(N_{E2} = 1)P(L_{E1}|N_{E1})P(L_{E2}|N_{E2}) \\ & \times P(Y|L_{E1}, \beta_Y, \beta_{Y0}, \sigma_Y)P(X|L_{E2}, \beta_X, \beta_{X0}, \sigma_X) \quad (4) \end{aligned}$$

where $P(N_{E1} = 1)$ and $P(N_{E2} = 1)$ are the probabilities of observing a single shift on the phylogeny, and $P(L_{E1}|N_{E1})$ and $P(L_{E2}|N_{E2})$ are the probabilities of observing these singular shifts in locations L_{E1} and L_{E2}, respectively. The linear regression and singular events models lead to potentially very different distributions of trait data at the tips. For example, the singular event model, the distribution of Y is conditionally independent of X after accounting for $L_{E1}, \beta_Y, \beta_{Y0}$ — a testable empirical prediction that will often result in these two models being easily distinguishable with model selection. But failing to consider the singular event model as a possibility is a problem: even for the simple case of two continuous traits, we have shown how easily data simulated under the singular event model can result in highly significant regressions for OLS, PGLS and IC regressions, regardless if the residuals are simulated as independent or phylogenetically correlated with respect to the model and phylogeny. We also note that estimating a λ transformation for the residuals (Pagel, 1999; Freckleton et al., 2002) will not rescue the analysis; the estimated value of λ will lie between 0 and 1 and we have found both these more extreme cases (OLS and IC, respectively) to be susceptible.

One might argue that the situation we describe is simply a violation of a BM model of evolution — and this would of course be correct (see also Maddison and FitzJohn, 2015). Indeed, for decades it has been common practice (but unfortunately, not universally so) to test whether contrasts are i.i.d. after conducting an analysis using IC (Garland et al., 1992; Purvis and Rambaut, 1995; Slater and Pennell, 2013; Pennell et al., 2015). Of course, Felsenstein recognized this particular vulnerability in his method, and correctly predicted that the underlying model was an "obvious point for future development" (p. 14). While today we have a much wider range of comparative models to choose from, most continuous trait models are Gaussian (e.g., Pagel, 1999; Blomberg et al., 2003; Butler and King, 2004; O'Meara et al., 2006; Eastman et al., 2011; Beaulieu et al., 2012; Uyeda and Harmon, 2014). It is only recently that alternative classes of models have been con-

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sidered (Landis et al., 2012; Elliot and Mooers, 2014; Schraiber and Landis, 2015; Boucher et al., 2017; Duchen et al., 2017). Whether or not these types of models can sufficiently account for these types of singular events will be examined in the next section. However, our primary point here is to suggest that the phenomenon that made Felsenstein's argument so intuitive is not the violation of i.i.d. residuals but rather the biologically intuitive realization that unreplicated differences co-localized on a single branch provide only weak evidence of a causal relationship between traits. However, this alternative model is rarely included in comparative analyses. Even for continuous traits, such unreplicated events can cause similar problems as those outlined by Maddison and FitzJohn (2015) in the case of discrete character correlations (as we will further elaborate in Case Study III).

Case Study II: Adaptive hypotheses and singular shifts

As stated above, the IC method is based on the BM model of trait evolution. While this model is useful (and has often been used) for testing for adaptation, it is inconsistent with how we think of the *process of adapting* to an optimal state (Lande, 1976; Hansen, 1997; Hansen and Orzack, 2005; Hansen et al., 2008; Hansen and Bartoszek, 2012). Hansen's introduction of the Ornstein-Uhlenbeck (OU) process to comparative biology and the suite of methods built on his approach have been the only real attempts to actually try and capture the basic dynamics of adaptive trait evolution on phylogenies. While it is formally equivalent to a model of stabilizing selection within a population with a fixed additive genetic variance (Lande, 1976; Hansen and Martins, 1996), we agree with other researchers (Hansen, 2012) that the OU model is usually best thought of as a phenomenological descriptor of the long-term movement of adaptive peaks or adaptive zones rather than that of a population climbing along a fixed adaptive landscape.

While an OU model with a single stationary peak is often matched up against BM and other alternatives (Harmon et al., 2010; Slater et al., 2012; Pennell et al., 2015; Cooper et al., 2016), multi-peak OU models have been widely used to test for the presence of shifts in evolutionary regimes (i.e., parts of the phylogeny with their own optima, or less commonly, their own strength of selection parameters). Tests of adaptive evolution come in two flavors: those with an *a priori* hypothesis (or hypotheses) regarding which lineages belong to which distinct regimes based on ancestral state reconstruction of explanatory factors (Butler and King, 2004; Beaulieu et al., 2012) and those where the locations of regime changes are themselves estimated along with the parameters of the OU process (Ingram and Mahler, 2013; Uyeda and Harmon, 2014; Khabbazian et al., 2016).

These two types approaches represent two different philosophies of data analysis that follow a schism that cuts through comparative methods. For example, there are two major ways to investigate the dynamics of lineage diversification: test specific hypotheses about the drivers of diversification rate shifts (for example, the 'SSE' family of models; Maddison et al., 2007; FitzJohn, 2012) or search for the most-supported number and configuration of shifts (Alfaro et al., 2009; Stadler, 2011; Rabosky, 2014). The former (hypothesis-testing) seeks to understand

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the causes of evolutionary shifts, while the latter is a descriptive and exploratory approach to understanding evolutionary patterns. As we alluded to above, we refer to these data-driven approaches as "phylogenetic natural history" due to their similarity to the practice of natural history observations in nature but projected backwards through phylogenetic space and time (Maddison and FitzJohn, 2015)

Of course, the types of inferences we can make will be limited by our choice of approach. For example, it may be tempting to use exploratory approaches such as *BAMM* (Rabosky, 2014) or *bayou* (Uyeda and Harmon, 2014) to search a vast range of model space to find a particularly well-supported statistical hypothesis, observe the shifts identified, and then come up with post hoc explanations for why that particular configuration fits an adaptive story that the researcher can suddenly construct with great precision. (Comparative biologists are of course not unique in succumbing to such temptations; see for example Pavlidis et al., 2012). However, good scientists recognize that such a practice can easily become a form of data snooping. In fact, discovering the location of well-supported shifts on the phylogeny does not say anything about causation; it is merely a descriptive technique to find major features of the data where there is evidence that the parameters governing the dynamics of trait evolution have shifted on the phylogeny. It is nonetheless useful — and we argue essential — that a researcher know where these shifts occur. The reasons for this are covered in Case Study I: these major shifts are likely to drown out any biological signal in a dataset if they are unaccounted for by our hypothesis-driven models. While it is dangerous to come up with your hypothesis after viewing the data, it is equally dangerous to apply and interpret a model fit to your data without plotting and visualizing the signal in your data. We argue that hypothesis-driven and phylogenetic natural history approaches are complementary: we must pit our particular causal hypotheses against a "stuff-happens" model built on idiosyncratic singular evolutionary events.

To illustrate how we might go about uniting these two modes of inference to disentangle the support for causal models of evolution from that attributable to singular events, we reanalyze a dataset introduced by Scales et al. (2009) on lizard muscle fiber proportions (hereafter, the 'Scales' dataset). (An expanded dataset was re-analyzed by Scales and Butler (2016) with slightly modified hypotheses; but the original 2009 paper serves as a clearer illustration of our perspective and since we are using it only for rhetorical purposes, we will not delve into differences between the two.)

Scales et al. (2009) are interested in the composition of muscle fiber types in squamate lizards, and whether these muscle fibers evolve adaptively in response to the changing behavior and ecology of the organisms. They propose three primary adaptive hypotheses for the drivers of fast glycolytic (FG) muscle fiber proportions: i) foraging mode behavior (FM; e.g., sit-and-wait vs. active foraging vs. mixed); ii) predator escape behavior (PE; e.g., active flight vs. crypsis vs. mixed); and iii) a combined hypothesis of foraging mode and predator escape (FMPE) that assigns a unique regime to every combination of FM and PE represented in the dataset. For each hypothesis, they reconstruct a likely phylogenetic history of these behavioral modes on the phylogeny by conducting ancestral state recon-

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structions (Figure 3). After fitting the multi-optimum OU models to the muscle fiber data, they find strong support for the predator escape hypothesis, which is 13.0 AICc units better than the next closest model (FMPE). Such a finding appears quite reasonable under the "Life-Dinner Principle" (Dawkins and Krebs, 1979), which suggests that escaping a predator may have a far more direct effect on fitness than obtaining a food item (Scales et al., 2009).

However, AIC provides only relative support for a model given a set of alternatives (see Pennell et al., 2015, for more on this point in the context of comparative methods). An examination of the particular configuration of shifts in the three hypotheses may give pause to researchers familiar with squamates. For example, some may want to quibble with the suggestion that the "sit-and-wait" foraging behavior of *Phrynosoma* species, which are often ant-eating specialists that leisurely lap up passing insects, should be grouped with the "sit-and-wait" tactics of species such as *Gambelia wislizenii*, a voracious carnivore that frequently subdues and consumes other lizards close to their own size. Looking at the reconstructions, it is also apparent that the PE hypothesis is the simplest model that allows a shift on the branch leading to *Phrynosoma*, a group that any herpetologist would identify as "weird" for a multitude of reasons (indeed, these are the eyeball-socket-blood-squirters alluded to in the introduction). The question then arises: is the signal in the dataset for the PE hypothesis driven entirely by the singular evolution of different muscle fiber composition in *Phrynosoma* lizards? If so, then any number of causal factors that differ between *Phrynosoma* and other lizards could be equally as likely as predator escape — including foraging mode with a slight reclassification of character states! We want to emphasize that we are not criticizing any of the particular choices the researchers involved in this study made. Rather, we argue that such quandaries are the inexorable result whenever the primary signal in the data is due to a singular historical event.

To explore the impact of the distinctiveness of simply being a *Phrynosoma* lizard, we developed a novel Bayesian model by building on the R package *bayou* (Uyeda and Harmon, 2014). To do so, we consider the macroevolutionary optimum of a particular species to be a weighted average of past regimes, as is typical in all OU models with discrete shifts in regimes (Butler and King, 2004; Beaulieu et al., 2012), but in our case, this weighted average is itself a weighted average of two differing configurations of the locations of adaptive shifts (often referred to as "regime paintings"). One configuration assumes that shifts in the optima have occurred where a discrete character, hypothesized to shape the evolutionary dynamics of the continuous character, is reconstructed to have shifted. The other configuration is estimated directly from the data using *bayou*'s reversible-jump MCMC (RJMCMC) algorithm.

$$E[Y_i] = w(\Psi_{PE}(\alpha)\theta_{PE}) + (1-w)(\Psi_{RJ}(\alpha)\theta_{RJ}) \tag{5}$$

This equation describes the expected value of a trait for species i, Y_i as a weighted average between the expected trait value under the PE hypothesis and the expected trait value under the reversible-jump estimate of shift configurations. The vectors θ_{PE} and θ_{RJ} are the values of the trait optima for the N_{PE} and N_{RJ}

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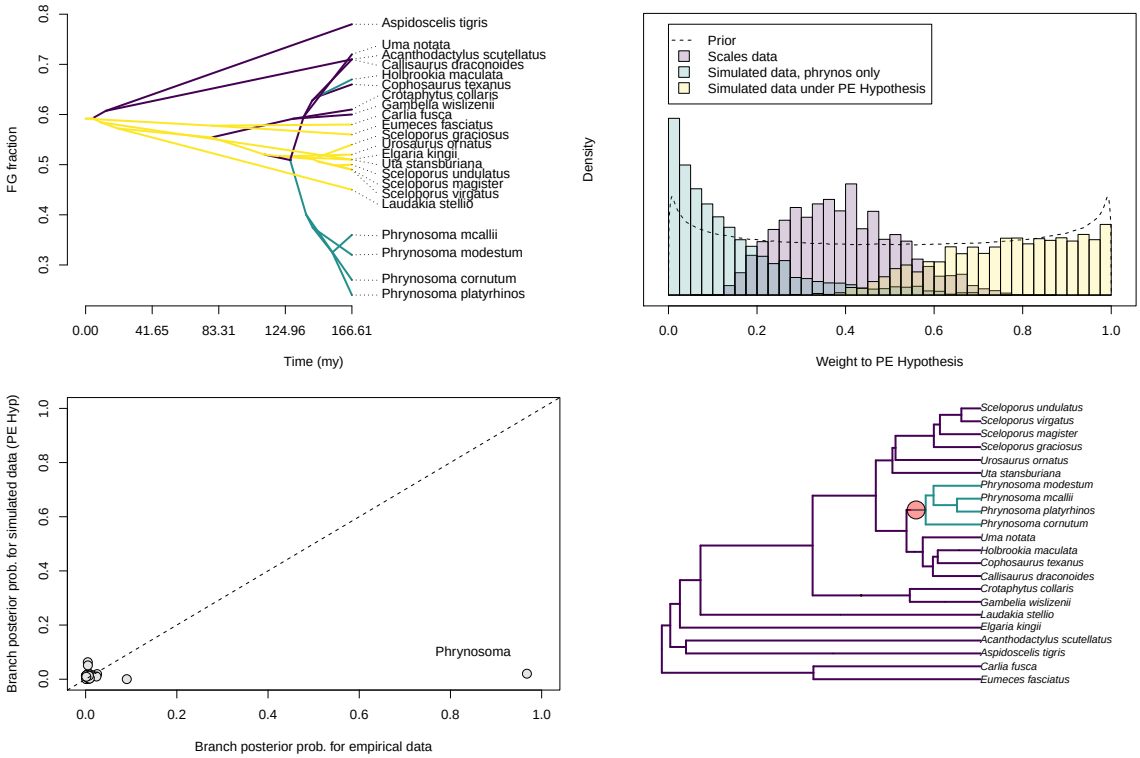

Figure 3: A reanalysis of the Scales et al. (2009) dataset of fast glycolytic muscle fiber fraction across 22 squamate lizards. A) A traitgram depicting the distribution of the data and the reconstructed regimes for the best-fitting Predator-Escape (PE) hypothesis (blue = cryptic, yellow = active flight, purple = mixed). B) Posterior distributions of weights estimated for the PE hypothesis when mixed with a RJMCMC analysis for the original empirical data (purple), data simulated under the best-fitting estimated parameters for a *Phrynosoma*-only shift model (blue), and a dataset simulated under the best-fitting estimated parameters for the full PE model (yellow). Notice that the empirical dataset has intermediate weights. C) Posterior probabilities for all branches of the phylogeny estimated for the original empirical data (X-axis) and the simulated dataset under the PE hypothesis (solid line is the 1 to 1 line). D) We estimate a high posterior probability on a shift in the genus *Phrynosoma* from the empirical data only (red circle), indicating that while the PE hypothesis explains some patterns in the data, it does not fully explain the shift present in the behaviorally and ecologically unique genus *Phrynosoma*.

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adaptive regimes, while Ψ_{PE} and Ψ_{RJ} correspond to the standard OU weight matrices that average over the history of adaptive regimes experienced by species i over the course of their evolution, with older regimes being discounted proportional to the OU parameter α (for a full description of how these weight matrices are derived, see Hansen, 1997; Butler and King, 2004).

In our model, the regime painting for our a priori hypothesis Ψ_{PE} is fixed, while we estimate the parameters the configuration of shifts for the reversible-jump component, Ψ_{RJ}, as well as the values for the optima θ_{PE} and θ_{RJ}; and standard parameters for the OU model such as α and σ^2 which are assumed constant across the phylogeny. We also estimate the weight parameter w, which determines the degree of support for the PE hypothesis against the reversible-jump regime painting. We place a truncated Poisson prior on the number of shifts for the reversible-jump analysis to be quite low, with a $\lambda = 0.5$ and a maximum of $\lambda = 10$ (meaning that we are placing a prior expectation of 0.5 shifts on the tree). Furthermore, we place a symmetric β-distributed prior on the w parameter with shape parameters of (0.8, 0.8). Additional details on the model-fitting can be found in the supplementary material.

We then fit this model to 3 different datasets: i) the original Scales data; ii) data simulated using the Maximum Likelihood estimates for the parameters of the PE model fitted to the Scales dataset; and iii) data simulated under the Maximum Likelihood estimates for a "*Phrynosoma*-only" model in which a single shift occurs leading to the genus *Phrynosoma*. We could then compare the posterior distribution of the weight parameter w to evaluate the weight of evidence for each hypothesis in each dataset.

We find that our approach places intermediate weight on the PE hypothesis for the original Scales dataset. When we simulated data under the PE hypothesis, the estimated weight given to the PE hypothesis was likewise high (Figure 3B). When data were simulated under the *Phrynosoma*-only hypothesis, the weight given to the PE hypothesis was low, as predicted (Figure 3B). Furthermore, the RJ portion of the model fit to the Scales dataset recovers only a single highly supported shift on the stem branch of the *Phrynosoma* lizards (Figure 3C and 3D). This suggests that the PE hypothesis has statistically supported explanatory power as its estimated weight is well bounded away from 0. But it does not explain everything. In particular, the PE hypothesis fails to fully explain the shift leading to the *Phrynosoma* lizards (Figure 3C and 3D), which are more extreme than they should be considering the other taxa in their regime (there is only one, *Holbrookia maculata*, which does not show such an extreme shift). Consequently, the answer to whether differences in predation escape behavior are driving the evolution of these traits is neither yes or no, but somewhere in between. This more subtle view of muscle fiber evolution conforms quite well to the conclusions drawn in the original paper and our biological intuition about the genus *Phrynosoma* — variation in predator escape behavior is a good explanation for observed patterns of muscle fiber divergence, but *Phrynosoma* are weird and other factors likely are influencing their trait evolution beyond predator escape.

We can conduct the same analysis where we test not the PE hypothesis, but the *Phrynosoma*-only hypothesis against the reversible-jump hypotheses (Figure

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4). In this case, we recover high weights for the *Phrynosoma*-only hypothesis regardless if the model is fit to the Scales dataset, or to data simulated under either the *Phrynosoma*-only hypothesis or the PE hypothesis. This is because accounting for the *Phrynosoma* shift is the primary feature of all three datasets (though weights are somewhat higher for data simulated under the *Phrynosoma*-only hypothesis than others). It may appear unsatisfying that such high weights are recovered for the a priori hypothesis when a singular event, which is easily reconstructed by the RJMCMC, explains the distribution of the data just as well. However, the analysis favors the *Phrynosoma*-only hypothesis simply because of the vague priors placed on the number and location of shifts in the reversible-jump analysis. Guessing correctly which of the 42 branches on the phylogeny has a single shift with our hypothesis is rewarded by the analysis (we will return to this issue in Case Study III). In the original Scales dataset, there are weakly supported shifts in the clades leading to the sister group of *Phrynosoma* lizards, and the branch leading to *Acanthodactylus scutellatus* and *Aspidoscelis tigris*. Finally, we can combine all three hypothesis simultaneously by placing a Dirichlet prior on the vector $w = [w_{RJ}, w_{PE}, w_{Phrynosoma}]$. Doing so recovers strongest support for the *Phrynosoma*-only model, intermediate support for the PE hypothesis, and very little weight on the reversible-jump hypothesis, which has no strongly supported shifts (Figure 5).

By combining phylogenetic natural history approaches with our a priori hypotheses, we show that we can account for rare evolutionary events that are not well-accounted for by our generating model. In the case of the PE hypothesis, we show that it does indeed have explanatory power beyond simply explaining a singular shift in *Phrynosoma* and support the original authors' conclusions. However, the intermediate result likely only occurs because the PE hypothesis places *Phrynosoma* in the same regime as *Holbrookia maculata*, which does not share the extreme shift that is found in *Phrynosoma*. Were this not the case (as in our fitting of the *Phrynosoma*-only hypothesis), it would still require visual inspection of the phylogenetic distribution of traits under the hypothesis in question to determine that a singular evolutionary event is driving support for a particular model. As discussed above, given a large enough tree such a priori hypotheses are likely to be strongly supported; if you can predict which one branch out of many will contain a shift then you may be on to something. But given the dangers of ascertainment bias and our biological intuition, we find this interpretation unsatisfying (Maddison and FitzJohn, 2015). We discuss this problem more in Case Study III.

Nevertheless, we show the value in combining a hypothesis testing framework with a natural history approach to identifying patterns of evolution. We show here that allowing for unaccounted shifts can provide a stronger test and more nuanced conclusions regarding the support for a particular predictor driving trait evolution across a phylogeny. Furthermore, predictors which provide additional explanatory power (if for example, regimes are convergent or if predictors vary continuously) will be even more favored over natural history models. Thus, our framework certainly does not automatically reward more complex, freely estimated models. Rather, the great uncertainty in possible models is incorporated as a prior on the arrangement of shifts and is limited in explanatory power, something that researcher-driven biological hypotheses are much more capable of ac-

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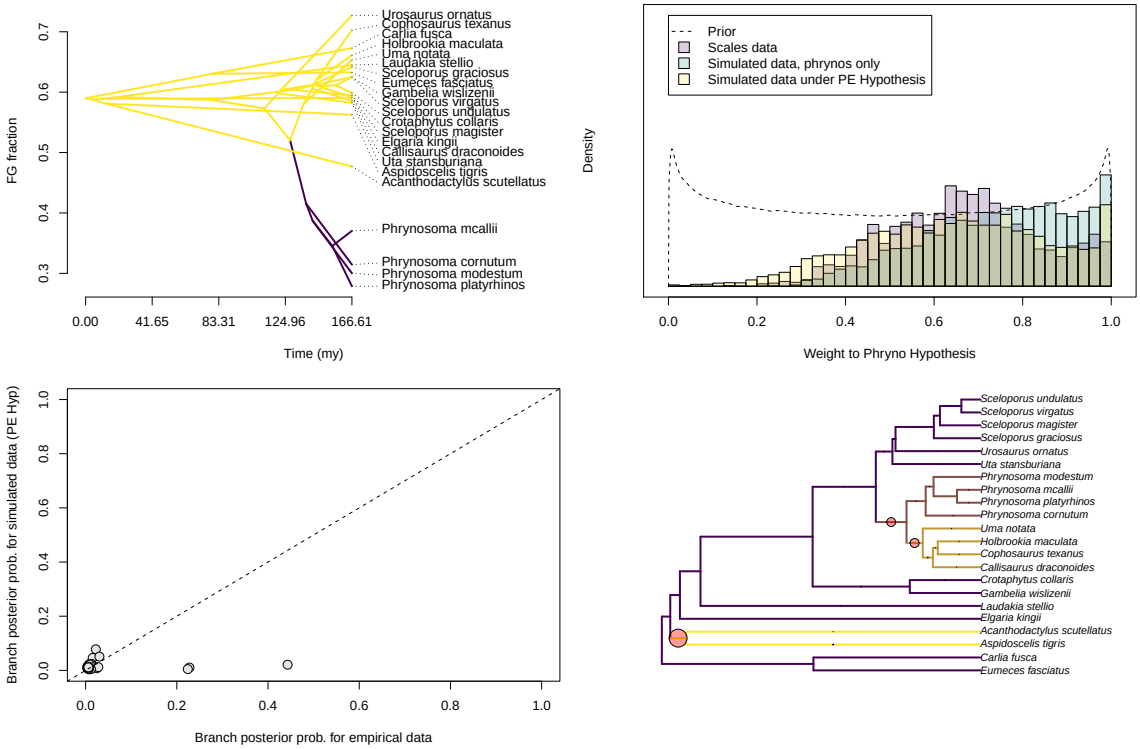

Figure 4: A reanalysis of the Scales et al. (2009) dataset of fast glycolytic muscle fiber fraction across 22 squamate lizards against the *Phyrnosoma*-only hypothesis. A) A traitgram depicting the distribution of simulated data under the *Phyrnosoma*-only hypothesis (yellow = squamates, purple = *Phyrnosoma*). B) Posterior distributions of weights estimated for the *Phyrnosoma*-only hypothesis when mixed with a RJMCMC analysis for the original empirical data (purple), data simulated under the best-fitting estimated parameters for a *Phyrnosoma*-only shift model (blue), and a dataset simulated under the best-fitting estimated parameters for the full PE model (yellow). All analysis recover high weights. C) Posterior probabilities for all branches of the phylogeny estimated for the original empirical data (X-axis) and the simulated dataset under the PE hypothesis (dotted line is the 1 to 1 line). D) Modest support for two additional shifts are recovered for the empirical data only (red circles).

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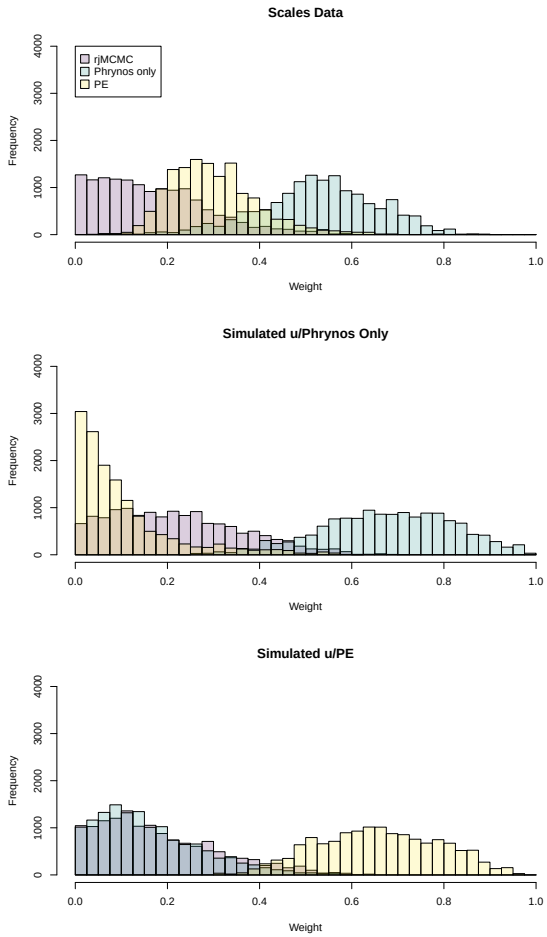

Figure 5: A reanalysis of the Scales et al. (2009) dataset of fast glycolytic muscle fiber fraction across 22 squamate lizards against with both the *Phyrnosoma*-only hypothesis and the PE hypotheses. Weights are depicted for each of the three datasets A) the original Scales dataset B) A dataset simulated under the *Phrynosoma*-only model C) A dataset simulated under the PE hypothesis. In B and C, the correct model receives highest support with neither of the alternatives being well-supported. In the original Scales dataset, the *Phrynosoma*-only hypothesis receives the most weight (indicating a singular shift best explains the patterns observed in the data), while an intermediate weight is given to the PE hypothesis (which explains a good amount of the remaining variation). In no analysis did the reversible-jump portion recover support for any additional shifts.

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complishing.

Case Study III: Darwin's scenario and unreplicated bursts

We now turn to a case where both the explanatory variable and the focal trait are discrete characters. In comparison to the continuous cases described above, we expect the signal for evolutionary covariation between such characters to be more difficult to detect. However, as we mention above, Maddison and FitzJohn (2015) recently demonstrated that commonly used methods return significant correlations all the time — and in scenarios that seem to defy our statistical intuition. For example, Pagel's (1994) correlation test would find the phylogenetic co-distribution of milk production and middle ear bones highly statistically significant even though they both are a defining characteristic of mammals (an inference so obviously dubious that even Darwin 1872 warned against it). This seems to be a clear case of phylogenetic pseudoreplication (Maddison and FitzJohn, 2015; Read and Nee, 1995). Maddison and FitzJohn describe the goal of correlation tests as finding the "weak" conclusion that "the two variables of interest appear to be part of the same adaptive/functional network, causally linked either directly, or indirectly through other variables" (p. 128). They assert that with our current approaches, we cannot even clear this (arguably low) bar. Here we delve into this idea a bit deeper. What constitutes good evidence of such a relationship? And is this a reasonable goal for comparative analyses?

Maddison and FitzJohn highlight two hypothetical situations, that they refer to as "Darwin's scenario" and an "unreplicated burst". They argue provide little evidence for an adaptive/functional relationship between two traits because the patterns of codistribution only reflect singular evolutionary events (Figure 1). In Darwin's scenario, two traits are coextensive on the phylogeny, meaning that in every lineage where one trait is in the derived character state, the other trait is as well. As an example, consider the aforementioned phylogenetic distribution of middle ear bones and milk production in animals; all mammals (and only mammals) have middle ear bones and produce milk. These traits (depending on how they are defined) have only appeared once on the tree of life and both occurred on the same branch (the stem branch of mammals). The unreplicated burst scenario is identical to Darwin's scenario except that rather than a single transition occurring in both traits, there is a single transition in the state of one trait (e.g., the gain of middle ear bones) and a sudden shift in the transition *rates* in another trait (e.g., the rates by which external testes are gained and lost across mammals). Note that these scenarios do not differ qualitatively from Felsenstein's worst-case scenario nor the *Phrynosoma*-only model scenario from Case Studies I and II (Figure 1). In all three scenarios, something rare and interesting happened on a single branch and the distribution of traits at the tips of the phylogeny reflects this.

In their paper, Maddison and FitzJohn (2015) simulated comparative data and reported a preponderance of significant results using Pagel's correlation test (1994) and Maddison's (1990) concentrated changes test. In order to hone our intuition

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of the problems they present, we dig a bit deeper and investigate the mathematical reason that Pagel's discrete correlation test (1994) returns a significant result in Darwin's scenario. (We should note here that Brookfield [1993] conducted a similar analysis that was more-or-less completely overlooked.) To make the problem tractable, we assume that the traits were selected without first looking at their phylogenetic distribution, a condition that we (as well as Maddison and FitzJohn, 2015) suspect is rarely met in practice (more on this below).

Again, under Darwin's scenario, there is a single concurrent origin of two traits leading to perfect codistribution across the phylogeny (a condition we define mathematically as event A). What is the probability that both traits X and Y undergoing a single, irreversible shift on the same branch L_i under a model where the two traits are independent (M_{ind})? And what is the probability of this occurring if the two traits are actually evolving in a correlated fashion (M_{dep})?

Under the independent model, both traits X and Y have to switch from 0 to 1 in the same branch once. We also know that there was at least one transition in each of the traits, since we would not study traits if there weren't any changes in the phylogeny. The probability of this happening is

$$\begin{aligned} P(M_{ind}) = P((X(t), Y(t)) = (1,1)|(X(0), Y(0) = (0,0), \\ N_x(t) = 1, N_y(t) = 1, N_x(T) \geq 1, N_y(T) \geq 1, L_i) \end{aligned} \tag{6}$$

where N_x and N_y are the stochastic processes that denote the number of shifts of trait X and Y at time t respectively. L_i is the branch on which both transitions occur, where L_i has a branch length of t_i. The sum of all branch lengths is T. Since X and Y are independent, the joint probability of X and Y changing at the same time is simply the product of probabilities of each event, so the above expression becomes

$$\begin{aligned} P((X(t_i), Y(t_i)) = (1,1)|(X(0), Y(0) = (0,0), N_x(t_i) = 1, N_y(t_i) = 1, N_x(T) \geq 1, N_y(T) \geq 1) = \\ = P((X(t_i), Y(t_i)) = (1,1)|(X(0), Y(0)) = (0,0)) \times \\ \times P(N_x(t_i) = 1, N_y(t_i) = 1|N_x(T) \geq 1, N_y(T) \geq 1) \times \\ \times P(N_x(T) \geq 1, N_y(T) \geq 1) \\ = P(X(t_i) = 1|X(0) = 0)P(Y(t_i) = 1|Y(0) = 0) \times \\ \times P(N_x(t_i) = 1|N_x(T) \geq 1)P(N_y(t_i) = 1|N_y(T) \geq 1) \times \\ \times P(N_x(T) \geq 1)P(N_y(T) \geq 1) = \\ = [e^{Q_x t_i}]_{(1,2)}[e^{Q_y t_i}]_{(1,2)}P(N_x(t_i) = 1|N_x(T) \geq 1)P(N_y(t_i) = 1|N_y(T) \geq 1) \times \\ \times P(N_x(T) \geq 1)P(N_y(T) \geq 1) \end{aligned}$$

where Q_x and Q_y are the infinitesimal probability matrices that describe the transition rates between states in the independent case (these Q matrices are used to conduct Pagel's correlation test, see Supplementary Material for details on matrix definitions under the independent case) and the subscripts on $[e^{Q_y t_i}]_{(1,2)}$ indicate row 1, column 2 of the resulting probability matrix. We now consider the outcome of maximizing this expression under a likelihood framework. Since there is no ev-

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idence of a transition from 1 to 0 in either trait, the maximum Likelihood estimate (MLE) for the transition rates q^x_{10} and q^y_{10} will be 0. Meanwhile, the MLEs (q^x_{01}, q^y_{01}) for the transitions from 0 to 1 in both traits will be small (because these events are so rare, occurring only once, see the small probability of a single shift occurring in the Supplementary Material) but positive since one transition does occur on L_i. Given the resulting parameter estimates of (q^x_{01}, q^y_{01}), it is likely that a great many realizations of this process would likely result in no lineages evolving the traits of interest at all — replaying the tape of life, under Markovian assumptions, will likely lead to many worlds where milk and middle ear bones don't exist at all. However, we do not study traits that don't exist. Because of this ascertainment bias, the probability of at least one switch occurring for traits that are unlikely to evolve at all (i.e. with very small q^x_{01} and q^y_{01}) should be nearly exactly one, that is $P(N_x(t) \geq 1) \approx 1$ when accounting for total branch length T of the tree (see Supplementary Material for exact derivation of this probability). The probability of exactly one transition of each trait occurring in the lineage L_i given that at least there is one transition in the tree is simply uniform $P(N_x(t_i)|N_x(T) \geq 1) = t_i/T$ (derived from a Poisson process, see Supplementary Material). Furthermore, with rare events the estimates of the probabilities of both traits changing only once in lineage L_i conditional upon observing Darwin's scenario (under the independent model M_{ind}) is also one ($e^{Q_x t_i}_{(1,2)} = \frac{\hat{q^x_{01}}}{\hat{q^x_{01}}+\hat{q^x_{10}}} - \frac{\hat{q^x_{01}}}{\hat{q^x_{01}}+\hat{q^x_{10}}} e^{-(\hat{q^x_{01}}+\hat{q^x_{10}})t_i} = 1 - e^{-\hat{q^x_{01}} t_i} \approx 1$ and $e^{Q_y t_i}_{(1,2)} = \frac{\hat{q^y_{01}}}{\hat{q^y_{01}}+\hat{q^y_{10}}} - \frac{\hat{q^y_{01}}}{\hat{q^y_{01}}+\hat{q^y_{10}}} e^{-(\hat{q^y_{01}}+\hat{q^y_{10}})t_i} = 1 - e^{-\hat{q^y_{01}} t_i} \approx 1$), meaning that at the end the probability of the independent model reduces to

$$P(M_{ind}) = P(N_x(t_i)|N_x(T) \geq 1)P(N_y(t_i)|N_y(T) \geq 1) = (t_i/T)^2 \tag{7}$$

where t_i is the branch length of branch L_i containing both shifts (Karlin and Taylor, 1981).

In contrast, for the completely dependent model M_{dep}, it is enough to follow what happens in a single trait since the second will just simply change along. Therefore:

$$\begin{aligned} P(M_{dep}) = P((X(t), Y(t)) = (1,1)|(X(0), Y(0) = (0,0), N_x(t) = 1, \\ N_y(t) = 1, N_x(T) \geq 1, N_x(T) \geq 1, L_i) = (t_i/T) \end{aligned} \tag{8}$$

Thus, the test statistic used in the likelihood ratio test comparing M_{ind} and M_{dep} is simply proportional to the ratio of the length of the branch where the shift occurred to the total length of the tree (i.e., the probability of two events happening on the same branch equation (Eq. 7) vs. the probability of one event happening on the branch (Eq. 8).

$$\begin{aligned} 2(lnL(M_{dep}) - lnL(M_{ind})) &= 2(ln(t_i) - ln(T)) - 4(ln(t_i) - ln(T)) \\ &= 2(ln(T) - ln(t_i)) \end{aligned} \tag{9}$$

In other words, the results of the analysis are predetermined. Under Darwin's scenario, including additional taxa in the analysis will increase the support for the

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dependent model simply as a consequence of increasing the total length of the tree (i.e., the difference between $ln(T)$ and $ln(t_i)$ will get bigger).

The assumptions used to derive this result differ very slightly from those used in available software; however, we can use simulation to test the validity of our result and to demonstrate that this is the mathematical reason that Pagel's test returns a significant result. Using the R package *diversitree* (FitzJohn, 2012), we simulated a set of 20 taxon trees where both traits underwent a irreversible transition on a single, randomly chosen, internal branch. We then fit a Pagel model with constrained (M_{dep}) and unconstrained (M_{ind}) transition rates. We also constrained the root state in both traits to 0, rates of losses of both the traits to 0, and gain rates in the dependent model following the gain of the other trait to be extremely high. Plotting the empirically estimated differences in the MLEs against the predictions making the simplifying assumptions above reveals a strong modal correlation between them (Fig. 6). Differences likely reflect the fact that we have not explicitly made the assumption that $P(N_x(t) \geq 1) = P(N_y(t) \geq 1) \approx 1$ when we fit the model with *diversitree*. Furthermore, we compare here only fully dependent and independent models. This can be seen when calculating the probability of one switch in each trait $P(N_x(t) = 1, N(t)_y = 1)$. In the fully dependent case that simply becomes $P(N_x(t) = 1)$, in the independent case it becomes $P(N_x(t) = 1)P(N_y(t) = 1)$ but in the correlated case it becomes $P(N_y(t) = 1|N_x(t) = 1)P(N(t)_x = 1) \neq 1$ affecting the likelihood ratio test based on estimations of the correlation (see Supplementary Material). However, such intermediate cases will only introduce slight differences and may not be distinguishable from the fully dependent case under Darwin's Scenario (though they will be important in more intermediate cases, see Supplementary Material).

Maddison and FitzJohn (2015) hinted that the coincident occurrence of single events could be a way of measuring the evidence for a correlation, but did not work out the details as we have done here. The key to understanding this result is to recall Gould and Eldredge's famous dictum (1977) that "stasis is data". The remarkable coincidence is not just that the two characters happened to evolve on the same branch but that they were never subsequently lost. For even a modestly sized tree, this coincidence is so unlikely that the alternative hypothesis of correlated evolution is preferred over the null. It is therefore not completely unreasonable that Pagel's test tells us that these traits have evolved in an entirely correlated fashion.

However, one key consideration should make us suspect of this line of reasoning. As Maddison and FitzJohn (2015) point out, the traits we use in comparative analyses are not chosen independently with respect to their phylogenetic distribution (as we assumed in our analysis). Rather, researchers' prior ideas about how traits map unto trees likely inform which traits they choose to test for correlated evolution. For example, it is common practice among systematists to search for defining and diagnostic characteristics for named clades; these type of traits are of especial interest and are likely the same sorts of traits that are researchers might include in comparative analysis, thereby greatly increasing the likelihood of finding traits with independent, unrelated origins that align with Darwin's scenario. We agree with Maddison and FitzJohn (2015) that this type of ascertainment

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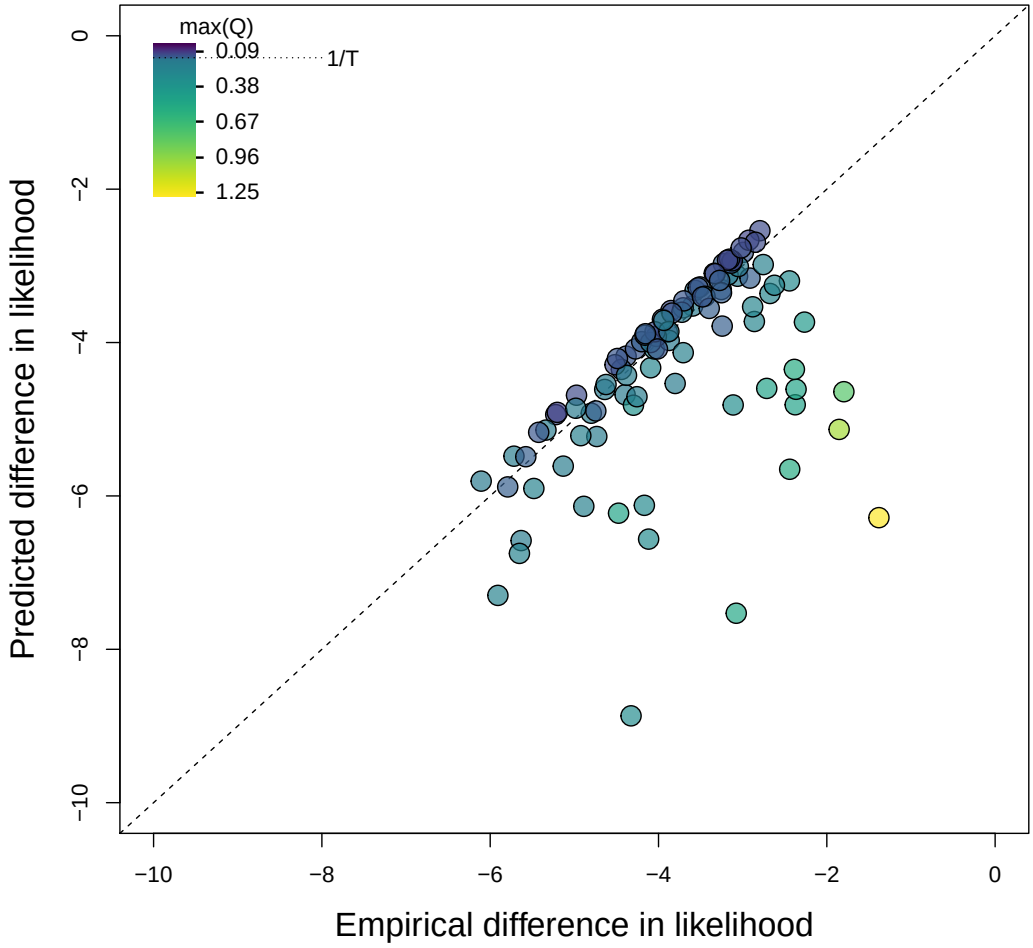

Figure 6: Darwin's scenario–the singular origin of two coextensive traits on the phylogeny–represents a boundary case to finding the correlation between discrete characters. Pagel's correlation test for Darwin's scenario can essentially be reduced to the difference in probability between choosing the same branch twice vs. choosing the branch only once. We demonstrate that here, showing our predicted differences in log likelihood between the independent and dependent trait models (y-axis) against the empirical estimates of the difference in log likelihood between models for simulated Darwin's scenarios on different phylogenies. Dotted line indicates equality. Points falling off the line represent slight violations of the assumptions we used to derive our prediction. Particularly, we assume that the rates of gain of the traits are so low that only one shift is ever observed. The color of the points indicates cases where this assumption is violated, as outlying points with max(Q) values much greater than $1/T$ (where only 1 shift is expected) are much more likely to fall off the predicted line.

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bias is likely prevalent in empirical studies, even if it is usually more subtle than testing for a correlation between milk and middle ear bones. However, we disagree with them that this renders establishing correlations in intermediate cases hopeless. Understanding the exact mathematical reasons why Pagel's test infers a significant correlation in a given case provides a clear boundary condition that can help develop quantitative corrections for ascertainment bias. Furthermore, the issues of ascertainment bias are likely to rapidly dissipate as we move away from the boundary case of Darwin's scenario. As a result, extending our analytical approach to more complicated scenarios will likely provide an even more meaningful estimate of the weight of evidence supporting a hypothesis of correlation.

The structure of a solution

We have shown in the three Case Studies that many PCMs, including those that form the bedrock of our field, are susceptible to being misled by rare or singular evolutionary events. This fundamental problem has sowed doubts about the suitability and reliability of many methods in comparative biology, even if it was not obvious that these issues were connected. But again, the fact that apparently different issues share a common root makes us hopeful that there can be a common solution.

As we illustrate through our Case Studies, we think that accounting for idiosyncratic evolutionary events will be an essential step towards such a solution. However, we will need to think hard about how best to model such events. In Case Study II, we present one solution to the problem that involves explicitly accounting for the possibility of unaccounted adaptive shifts using Bayesian Mixture modeling. We believe this approach has a great deal of promise as it provides simultaneous identification of biologically interesting shifts and the explanatory power of a particular hypothesis.

However, we do not claim that such an approach is the only solution or that it solves the problem completely. Indeed, we find that in all three Case Studies, the uniting philosophy is to consider models that account for idiosyncratic background events, rather than strict adherence to a particular methodology. For example, we highlighted in the introduction that we think HMMs (following Beaulieu et al., 2013; Beaulieu and O'Meara, 2016) are a potentially powerful, and widely applicable solution, even though we did not consider these in detail here.

And there are still other potential solutions which we have not even mentioned yet. In our own work (Uyeda et al., 2017), we have used a strategy similar to the Bayesian Mixture Modeling but instead of modeling the trait dynamics as a joint function of our hypothesized factors and background changes (represented by the RJMCMC component), we did the analyses in a two-step process: first, we used *bayou* (Uyeda and Harmon, 2014) to locate shifts points on the phylogeny, then used Bayes Factors to determine if predictors could "explain away" shifts found through exploratory analyses. For PGLS and other linear modeling approaches, modeling the residuals using fat-tailed distributions (Landis et al., 2012; Blomberg et al., 2012; Elliot and Mooers, 2014; Duchen et al., 2017) may mitigate

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the impact of singular evolutionary events on the estimation of the slope (also see Slater and Pennell, 2013, for an alternative approach using robust regression). Furthermore, we also think that rigorous examination of goodness-of-fit and model adequacy following any comparative analysis is critical for finding unforeseen singular events driving signal in the dataset (Garland et al., 1992; Boettiger et al., 2012; Slater and Pennell, 2013; Pennell et al., 2015). Which of these solutions (including those that were included in our Case Studies and those that were not) will be the most profitable to pursue will probably differ depending on the question, dataset and application — we anticipate that there will not be a one-size-fits-all solution — but we do think that any compelling solution will involve a unification of phylogenetic natural history and hypothesis testing approaches.

But we want to take this a step further. While it is useful to account for phylogenetic events in our statistical models, a greater goal of comparative biology should be explain why these events exist in the first place. We return to Maddison and FitzJohn's (2015) "weak" goal of finding whether or not "two variables of interest appear to be part of the same adaptive/functional network, causally linked either directly, or indirectly through other variables." We ultimately disagree with them that this constitutes a weak conclusion; the challenges of making these inferences from any comparative dataset are significant. Furthermore, we find the often repeated axiom "correlation does not mean causation" to be unhelpful. While the axiom is accurate in the strict sense, we believe that it obscures many logical and philosophical challenges to analyzing phylogenetic comparative data that are often ignored. And as is clear from reading the macroevolutionary literature, biologists do not shy away from forming causal statements from correlative data regardless. It therefore seems worthwhile to take seriously the question: "What would it take to infer causation from comparative data?" And even if we are to conclude that all the evidence for a hypothesized causal relationship stems from one or a few evolutionary events, is this finding biologically meaningful?

Phylogenies are graphical models of causation

One way to gain a foothold on the problem of causation is to build, communicate, and analyze phylogenetic comparative methods in a graphical modeling framework — a perspective that has recently been advocated by (Höhna et al., 2014). Graphical models that depict hypothesized causal links between variables make explicit key underlying assumptions that may otherwise remain obscured; indeed, the precise assumptions of PCMs were hotly debated in the early days of their development (Westoby et al., 1995b,a; Nee et al., 1996; Harvey et al., 1995; McNab, 1988) and remain poorly understood to this day (Hansen and Orzack, 2005; Hansen and Bartoszek, 2012). As examples of how using graphical models force us to be more clear in our reasoning, consider the graphs in Figure 7. We depict three different models of causation that have phylogenetic effects that each require alternative methods of analysis to estimate the effect of trait X on trait Y. In our example, a four species phylogeny provides possible pathways for causal effects, but variables may have entirely non-phylogenetic causes or may be blocked from ancestral causes by observed measurements, rendering the phylogeny irrele-

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vant (e.g. Figure 7A). Edges connect nodes and indicate the direction of causality, where the nature of phylogenies allows us to assume that ancestors are causes of descendants, and not vice versa. This asymmetry results in a what is known as a probabilistic Bayesian Network (a type of directed acyclic graph, or DAG) that predicts a specific set of conditional probabilities among the data.

Depending on the Bayesian network structure, the appropriate method of analysis can range from a non-phylogenetic regression (Figure 7A), to commonly used comparative methods such as Phylogenetic Generalized Least Squares (PGLS, Figure 7B), to methods that require modeling both the evolutionary history of interaction of both trait X and trait Y (Figure 7C) (Hansen, 1997; Butler and King, 2004; Hansen et al., 2008; Revell, 2010; Hansen and Bartoszek, 2012). We emphasize that this implies that the use of phylogeny in interspecific comparisons is an *assumption* that depends on the precise question being asked and the hypothesized causal network. It is often assumed and asserted that PCMs are simply a more rigorous version of standard regression. This is simply not true.

In cases where phylogeny does matter, we must specify the generating model for unobserved states in our causal graphs. For example, it is common to assume a BM model for residual variation in PGLS or that ancestral states are reconstructed using stochastic character mapping in OU modeling of adaptation. However, BM and other continuous Gaussian or Markov processes are only a few of the many types of processes that may generate change on a phylogeny. We have shown that discontinuous processes and rare, singular events are poorly handled in our current framework and lead to much confusion about what exactly, our statistical methods are allowing us to infer from comparative data. Such models can be similarly illustrated using graphical models (Figure 8). By making our models explicit, we see that the phylogeny is best thought of as a pathway for past factors to causally influence the present-day distribution of observed states. These "singular-event" models are alternatives to the more continuous models we typically examine. Furthermore, representing our models as graphs, we are poised to take advantage of the sophisticated approaches for causal reasoning (e.g., Pearl, 1995, 2009; Sugihara et al., 2012; Shipley, 2016) that have been embraced by fields like computer science but largely ignored by comparative biologists (a rare exception is the recent introduction of phylogenetic path analysis; Hardenberg and Gonzalez-Voyer, 2013).

One clear case where such graphical modeling would improve inference are cases where considering phylogeny reverses the sign of the relationship between two variables. This is precisely what Nee et al. (1991) found looking at the relationship between body size and abundance in British birds; depending on how they aggregated the data (means of species, means of genera, means of tribes, etc.) the direction of correlation flipped back and forth. This reversal in the sign of the relationship between two variables X and Y when conditioning on a third Z is a general, and widely studied, statistical phenomenon known as "Simpson's paradox" (Blyth, 1972). Nee and colleagues (1991; 1996) hold up their findings of the British bird study to be emblematic; in their view, the presence of Simpson's paradox in their data clearly implies that phylogeny is key to making sense of interspecific data.

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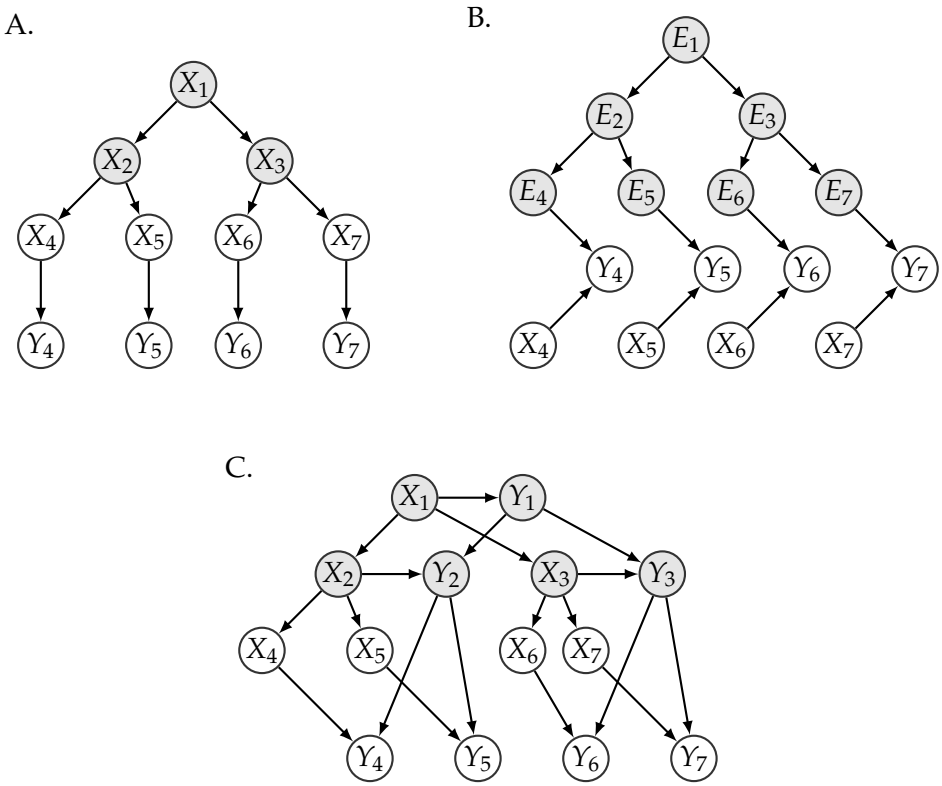

Figure 7: Graphical models of alternative causal relationships between a predictor (X) and a trait of interest (Y). Note that each node has independent, uncorrelated error as an input, but these have not been shown for clarity. A) X follows the phylogeny with observed states (white) and unobserved ancestral states (gray) and is a cause of trait Y. However, the phylogeny and pattern of evolution of X are irrelevant, and this graph can be modeled with methods such as Ordinary Least Squares regression. B) The trait Y has unobserved causes (E_i) that follow the phylogeny (gray) that can be modeled using, for example, Brownian Motion. The trait X is a cause of Y. This graph can be modeled using methods such as PGLS and PIC. C) The trait Y evolves on the phylogeny and is affected by trait X all throughout its history. Thus, the history of both X and Y must be modeled (e.g. Brownian Motion of X and Ornstein-Uhlenbeck for Y). This graph can be modeled using methods such as SLOUCH.

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However, as Pearl (2014) has convincingly demonstrated, Simpson's paradox is not really paradoxical at all when considered from the standpoint of Bayesian Networks. In fact, Pearl shows that the appropriate way to analyze the data depends crucially on what one assumes is causing what. To understand how causal inference resolves Simpson's Paradox, we now present a rather artificial, but nevertheless illustrative example (Pearl, 2009). Consider three traits: Body size (B), abundance (N) and migratory behavior (M) in birds. Given the Bayesian Networks presented in Figure 9, we have two possible hypotheses for the causal relationships between the traits. We further consider the possibility that we do not have adequate data on M, and thus only B and N are observed. Our goal is to estimate the causal effect of B on N. In Figure 9A, body size influences whether or not species become migratory, and both migratory status and body size influence species abundance (but in opposite directions). Furthermore, under this scenario, both body size and migratory status will have phylogenetic signal. We can evolve traits along the phylogeny depicted in Figure 9C and obtain a bivariate plot that looks like Figure 9D. Under the alternative Bayesian Network, migratory behavior still has a positive effect on species abundance, but also increases body size, which in turn causes decreases species abundance. These two causal structures are observationally equivalent — meaning that any distribution simulated under one can be replicated under the alternative causal structure. Therefore, both networks can produce datasets with phylogenetic signal in both body size and migratory behavior, and both can produce a dataset with the distribution in Figure 9D (see Supplementary Material for additional details on generating Figure 9).

How then should we analyze the data if we want to understand the effect of body size on species abundance? If we assume that body size influences migratory behavior, then increasing body size (for example, if natural selection leads a species to become larger), will increase the probability of that species becoming migratory — and the two opposing effects will result in relatively little change in species abundance. Therefore, we should perform Ordinary Least Squares regression to estimate the net causal effect of increasing body size. We also note that all the phylogenetic signal is coming from the evolution of body size, which becomes irrelevant once we observe body size, and thus we do not need to perform PGLS. By contrast, if migratory behavior causes changes in body size, then selecting for an increase in body size will not result in a lineage changing their migratory status at all. Therefore, we are assured that increasing body size will likewise always decrease species abundance. Consequently, we should perform PGLS to account for the phylogenetic signal in the residual variation imposed by (unobserved) migratory status.

By working through the logic of comparative analyses using graphical models we have come to essentially the same line of reasoning of Westoby et al. (1995b,a), who, in the early days of PCMs, challenged the growing consensus that phylogeny needed to be included in any interspecific comparison — a consensus which has only gotten stronger as the years passed by (also see McNab, 2003, for a related critique) . Westoby and colleagues were concerned that including phylogeny in interspecific comparisons necessarily favored some causal explanations over others. At the time, their critique was dismissed as innumerate hogwash (Harvey et al., 1995; Nee et al., 1996) and this evaluation has largely stuck. However, from our

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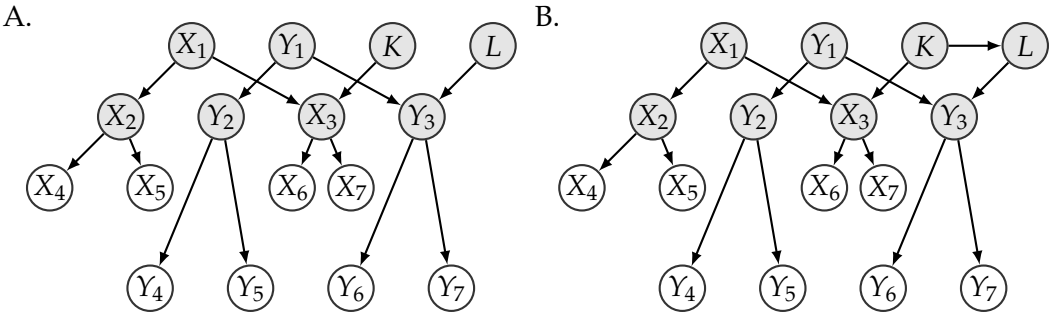

Figure 8: Graphical models of Darwin's Scenario between a predictor (X) and a trait of interest (Y). Note that each node has independent, uncorrelated error as an input, but these have not been shown for clarity. A) Singular event model. Here two independent factors cause a change on ancestral states X_3 and Y_3 (K and L respectively). However, they are independent events and coincidentally occur at the same point on the phylogeny. B) Similar to the previous model, but K and L are causally linked. Thus, whenever K occurs, it probabilistically causes L which causes a shift in Y. If only one event occurs however, this model is only distinguishable from graph (D) proportional to the probability that events K and L occur on the same branch (see Case Study III).

example of bird size and abundance, it is apparent that Westoby et al. were right all along: phylogenetic comparative methods are a powerful tools for drawing inferences from interspecific data but they necessarily imply some types of causal structures and negate others. It is too much to ask of our methods to decide what questions we ought to ask. As Westoby et al. (1995a) put it: "No statistical procedure can substitute for thinking about alternative evolutionary scenarios and their plausibility" (p. 534).

Concluding remarks: are our models valid tests of our causal hypotheses?

By explicitly including phylogeny into our graphical models of causation, we are forced to reckon with the scope of the inference problem and the ability of our data to be informative. While most of the statistical assumptions of methods are often well-known (e.g., for linear models, we assume that errors have equal variance and are normally distributed, etc.), Gelman and Hill (2006) argue that there is a more fundamental assumption — validity of data — that is almost always implicit and often overlooked

> "Most importantly, the data you are analyzing should map to the research question you are trying to answer. This sounds obvious but is often overlooked or ignored because it can be inconvenient. Optimally, this means that the outcome measure should accurately reflect the phenomenon of interest, the model should include all relevant predictors, and the model should generalize to the cases to which it will

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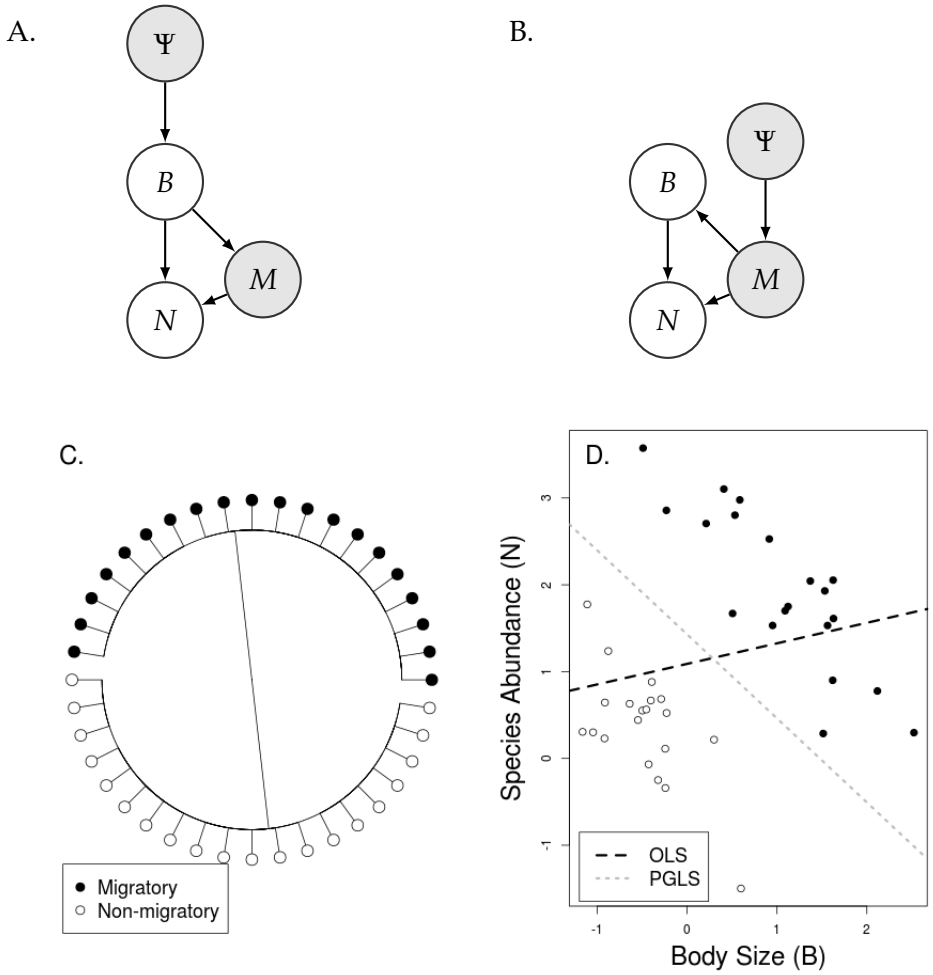

Figure 9: Simpson's paradox in phylogenetic comparative methods. Panels (A) and (B) depict two alternative Bayesian Networks. In (A), body size is a cause of both species abundance and migratory behavior, and trait B evolves on the phylogeny by a process (e.g. Brownian Motion) represented by Ψ. In (B), body size still affects species abundance, but migratory behavior itself is a cause of both body size and species abundance, but the phylogenetic effect is present in migratory behavior (in this case, we simulated with a Brownian threshold model). (C) A phylogeny similar to that of Darwin's scenario used to simulate the dataset (D), with migratory species (black) and non-migratory (white) taxa. The data in (D) can be generated by either causal structure. However, to estimate the effect of B in both networks, one must use different analytical approaches. To estimate the net effect of B on N in network (A), the appropriate method of analysis is OLS regression (black line). This is because increasing body size will simultaneously decrease species abundance and increase migratory behavior, which itself increases abundance, leading to a net slight increase in abundance. However, under network (B) the correct method is PGLS (gray line) as increasing body size will have no effect on migratory behavior, and unaccounted phylogenetic residual error is present in the observed data. Here, increasing body size will only have a direct effect of decreasing species abundance, which is reflected in the estimate of the slope. The resolution of Simpson's paradox rests entirely on causal assumptions; which are immediately apparent from graphical models but difficult to express with standard mathematical formulae.

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be applied." (Gelman and Hill, 2006)

We believe that far less discussion in comparative methods has been focused on the issue of statistical validity of the data collected to the research questions being posed by a given study. This is in large part because comparative data and the phylogeny that underly it are largely beyond the control of the researcher, but careful consideration of the data is required to understand what research questions can be reasonably answered. We find that most comparative research questions have a poorly defined scope of inference: it is unclear to what population a model or inference should generalize to. If we ask "are fur and middle ear bones correlated?", we must also specify "in what organisms?". Since no organisms other than mammals have the particular traits we define as "fur" and "middle ear bones", we actually do not need statistics at all to determine whether these traits are correlated — we have sampled nearly the entire population relevant to the question! In nature, they are perfectly collinear. If we wish to expand our scope of inference to hypothetical organisms that evolve fur and/or middle-ear bones we are free to do so. However, we have collected a very poor data sample for such a question. It is not the fault of the statistical method to demonstrate that a poorly designed experiment does not represent its scope of inference, rather it is our job as researchers and statisticians to ask whether or not such a relationship addresses our biological question and whether the sample of data collected is valid for the question being asked.

In this paper we have tried to synthesize a wide variety of statistical and philosophical concepts to lay out a roadmap for where we think comparative biology should go. We certainly do not have all the answers. Of the paths we have explored, there are many details that need to be worked out, and we fully anticipate that there are many alternative paths that we have not even considered. However, we argue that if we are going to make substantial progress in using phylogenetic data to test evolutionary hypotheses, we will need to reckon more seriously with the idiosyncratic nature of evolutionary history, and to more clearly articulate precisely what we want to test and whether our models and data are suitable for the task.

Code Availability

Data and code needed to reproduce all analyses in this manuscript are available at https://github.com/uyedaj/pnh-ms/.

Acknowledgments

We thank Luke Harmon, Daniel Caetano, Eliot Miller, Ben Freeman, Florent Mazel, Joel McGlothlin, Martha Muñoz, Barbara Neto-Bradley, Francisco Henao Diaz, and Mauro Sugawara for their critical feedback on these ideas and this manuscript. JCU would like to specially thank the insightful knowledge and teaching gleaned from conversations over the years with Thomas Hansen that inspired the bulk

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of this manuscript (though he holds no culpability for the contents and opinions therein). MWP was supported by a NSERC Discovery Grant. JCU was supported by NSF Grants to Luke Harmon (DEB-1208912) and JCU (DBI-1661516).

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Supplementary Methods

A. Case Study I-Supplementary Methods

In order to construct phylogenies that correspond to "Felsenstein's secnario", we simulated two phylogenies with a Yule process and a birth rate of 1 for 20 species and transformed them using Pagel's λ values of either 0 (polytomies) or 1 (fully bifurcating). Trees were then scaled to unit height and combined into a single, two-clade phylogeny with stem branches of equal length (again, unit height). The entire phylogeny was then scaled to unit height so that each clade begins diversifying after 0.5 units of tree height. We ran 200 simulations for each tree type and shift value (either full polytomies or fully bifurcating) where we simulated a bivariate Brownian Motion process with two uncorrelated traits with $\sigma^2 = 1$ for both traits. We then chose one of the two stem branches and simulated a shift. We tested 10 different values of the shift variance in an increasing sequence such that each value is 10 times larger than the last, ranging from $\sigma^2 = 10^{-2}$ to 10^3, from an uncorrelated bivariate Normal distribution. Thus, for each combination of shift variance (10 values) and phylogeny (2 trees) we ran 200 simulations. For each dataset, we then performed Phylogenetic Independent Contrasts (PICs) and estimated the P-value for the slope of the linear regression, forcing the intercept through the origin.

B. Case Study II-Supplementary Methods

We analyzed the dataset of Scales et al. (2009) using the trait *FG.frac* and matched to the squamate phylogeny of Pyron and Burbrink (2014). While this is a different phylogeny than was used in (Scales et al., 2009), the topology was identical and branch length differences were minimal. We implemented a novel approach for combining hypothesis-testing and exploratory reversible-jump MCMC in the software package *bayou*. To do so, we developed a customized R code (available at https://github.com/uyedaj/pnh-ms).

Priors on parameters are as follows: $\alpha \sim$ half-Cauchy(scale = 0.1); $\sigma^2 \sim$ half-Cauchy(scale = 0.1); $k \sim$ truncated Poisson($\lambda = 0.5, K_{max} = 10$); $\theta \sim$ Normal($\mu = 0.5, \sigma = 0.25$); $w \sim$ Beta(shape1 = 0.8, shape2 = 0.8). Each branch of the phylogeny was given an equal probability of a shift with a uniform probability on a given branch (i.e. shifts were allowed to occur anywhere on a given branch with equal probability).

In addition to the empirical dataset, we fit each model to two simulated datasets. First, we simulated under the a *Phrynosoma*-only model that contained a single shift at the base of the stem branch leading to the genus *Phrynosoma*. We chose parameter values close to the values estimated when fitting a *Phrynosoma*-only model model to the original Scales dataset. Specifically, we set $\alpha = 0.15$, $\sigma^2 = 0.001$, $\theta_{root} = 0.6$, and $\theta_{Phrynosoma} = 0.3$. Second, we simulated a dataset under the PE hypothesis with the same parameters of α and σ^2, but with $\theta_{mixed} = 0.5$, $\theta_{flight} = 0.7$

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and $\theta_{cryptic} = 0.3$. Again, these values are very close to the estimates we obtained from fitting the PE model to the original Scales dataset.

We ran 3 analyses in each of the 3 datasets, resulting in 9 total MCMC chains. First, we ran the PE hypothesis against the reversible-jump analysis for all 3 datasets. Next we ran the *Phrynosoma*-only hypothesis against the reversible-jump analysis for all datasets. Then we ran a model that places a Dirichlet prior on the weights of all three: PE, *Phrynosoma*-only and RJMCMC. This prior was set to $w \sim Dirichlet(0.33, 0.33, 0.33)$ with all other priors being identical to the other analyses. We ran each MCMC chain for at least 200,000 generations or until adequate effective sample sizes were obtained for all parameters (>100) and inspected each chain for evidence of poor mixing. Given the small size of the dataset and very few number of RJMCMC shifts, MCMC chains tended to converge quickly.

C. Case Study III-Supplementary Methods and Results

As described in the main text, we used a number of simplifying assumptions to generate the prediction that the difference in likelihoods between the dependent and independent cases in Darwin's scenario is a simple function of the length of branch L_i and the total length of the tree, T. To demonstrate this effect, we simulated Darwin's scenario and performed constrained Maximum Likelihood optimizations in an effort to come as close as possible to the generating assumptions we made in deriving our result (Figure 6). To do so, we simulated 100 phylogenies under a Yule process with birth rate = 1. We then scaled the tree to unit height and randomly selected an interior branch at which both traits X and Y were gained. We then estimated the likelihoods for M_{ind} and M_{dep} as described in the main text by taking the Maximum Likelihood of 5 replicated optimizations using the *optim* method in R. These replicates were performed to help minimize optimization errors.

As described in the main text, the estimates in Figure 6 were obtained by constraining the models to be either completely dependent or completely independent, for rates of gains between traits to be equal, and by constraining losses of traits to 0 (irreversibility). These assumptions were made based on the argument that under Darwin's scenario there is very little evidence to reject these assumptions. To verify that this is indeed the case, we present the results of an unconstrained model that does not impose these restrictions. Here, the only constraint on the model is that we specify that the root state of the model is state "00" (absence of both traits). Thus, under this model the independent case has 4 transition rates and the dependent case has 8 transition rates (compared to only 1 and 2 in the analysis in the main text, respectively). We used *nlminb* to maximize the likelihoods of these functions, as the increased number of parameters resulted in very flat likelihood surfaces and slow optimization—necessitating the use of bounds (all transition rates were bounded between 0 and 1000). Analyzing the same simulations as before, we obtain a very similar pattern Figure S1, although more cases of the dependent model have higher likelihoods than the independent model (as evident by more points falling above the predicted 1-to-1 line). We con-

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clude that the simplifying assumptions we used to obtain our result in Case Study III is sound, and that the primary reason the dependent model is favored over the independent model is the difference in placing one event on branch L_i and the probability of placing two events on branch L_i.

Probabilities of observing one transition in a single branch under independent model

Under the independent case we have that the infinitesimal probability matrix for trait X is simply defined as

$$Q = \begin{pmatrix} -q^x_{01} & q^x_{01} \\ q^x_{10} & -q^x_{10} \end{pmatrix} \tag{10}$$

Therefore the probabilities of the continuous-time Markov chain of trait X changing over time in the phylogeny are defined via $P(t) = e^{Qt}$. In fact, the full probabilities in the irreversible case (when $q^x_{10} = 0$) result in transition probability matrix

$$P(t) = \begin{pmatrix} e^{-q^x_{01}t} & 1 - e^{-q^x_{01}t} \\ 0 & 1 \end{pmatrix} \tag{11}$$

Notice that once a branch has switched to state 1 then the probability of the trait staying in state 1 is also 1 (absorbent state).If we want to calculate the number of swtiches from 0 to 1 trait X has experienced along the phylogeny we can define a new stochastic process $N_x(t)$ that follows the number of transitions from 0 to 1 in time t. This process has a binomial distribution that depends on the number of branches of the tree B but also the probability of transition from 0 to 1 in an interval t. That is,

$$P(N(t) = 1) \sim Binomial(B, P_{01}(t)) \tag{12}$$

In the case of rare traits the probability of observing a single event is small $P(N(t) = 1) = B(1 - e^{-q^x_{01}t})(e^{-q^x_{01}t})^{B-1}$ when q^x_{01} has a small value. In the main manuscript we refer this as the probability of being strike by lighting. However, the probability of someone being stroke by lighting at least once in a large group of people is 1. That is reflected in the probability of observing at least one transition across all the phylogeny, that we denote as the event $N_x(t) \geq 1$ in the main manuscript is simply one minus the probability of zero transitions occurring. Thus

$$\begin{aligned} P(N(t) \geq 1) &= 1 - P(N(t) = 0) \\ &= 1 - (1 - e^{-q^x_{01}t})(e^{-q^x_{01}t})^B \\ &\approx 1 \qquad \text{when } q^x_{01} \text{ is sufficiently small or } t \text{ is large} \end{aligned} \tag{13}$$

The binomial distribution in (12) converges to a Poisson distribution with parameter $\lambda = B * (1 - e^{-q^x_{01}t})$ when q^x_{01} is small and the number of branches B is large. Once we know there has been at least one transition from 0 to 1 we are

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only interested in the location of that single transition. Because the process $N(t)$ can be also defined via a Poisson process with parameter λ as defined above, we know that the probability of one event occurring in an specific branch L_i is simply a uniform distribution based on the length of that branch. Therefore $P(N_x(t_i)|N_x(T) \geq 1) = t_i/T$ as defined in the main manuscript (see Karlin and Taylor (1981) full derivation).

Probabilities of observing one transition in a single branch under correlated model

In the correlated full model for Pagel is described via an infinitesimal probability matrix Q with eight parameters representing the possible transitions of traits X and Y that have states 0 or 1.

$$Q = \begin{pmatrix} -(q_{(0,0),(0,1)} + q_{(0,0),(0,1)}) & q_{(0,0),(0,1)} & q_{(0,0),(1,0)} & 0 \\ q_{(0,1),(0,0)} & -(q_{(0,1),(0,0)} + q_{(0,1),(1,1)}) & 0 & q_{(0,1),(1,1)} \\ q_{(1,0),(0,0)} & 0 & -(q_{(1,0),(0,0)} + q_{(1,0),(1,1)}) & q_{(1,0),(1,1)} \\ 0 & q_{(1,1),(0,1)} & q_{(1,1),(1,0)} & -(q_{(1,1),(0,1)} + q_{(1,1),(1,0)}) \end{pmatrix} \tag{14}$$

In the irreversible case we have that $q_{(0,1),(0,0)} = q_{(1,0),(0,0)} = q_{(1,1),(0,1)} = q_{(1,1),(1,0)} = 0$, and the Q-matrix from (14) is reduced to four parameters. We are interested in calculating the probability of both traits moving to state 1. Therefore

$$\begin{aligned} P(t) &= e^{Qt} \\ &= \begin{pmatrix} e^{-(q_{(0,0),(0,1)}+q_{(0,0),(1,0)})} & \frac{q_{(0,0),(0,1)}(e^{-(q_{t(0,0),(0,1)}+q_{(0,0),(1,0)})} - e^{-tq_{(0,1),(1,1)}})}{(q_{(0,0),(0,1)}+q_{(0,0),(1,0)}-q_{(0,1),(1,1)})} & \frac{q_{(0,0),(1,0)}(e^{-(q_{t(0,0),(0,1)}+q_{(0,0),(1,0)})} - e^{-tq_{(1,0),(1,1)}})}{(q_{(0,0),(0,1)}+q_{(0,0),(1,0)}-q_{(1,0),(1,1)})} & p^{xy}_{0,1}(t) \\ 0 & e^{-tq_{(0,1),(1,1)}} & 0 & (1-e^{-tq_{(0,1),(1,1)}}) \\ 0 & 0 & e^{-tq_{(1,0),(1,1)}} & (1-e^{-tq_{(1,0),(1,1)}}) \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

So the probability that we are interested in is

$$\begin{aligned} P((X(t),Y(t)) = (1,1)|(X(0),Y(0)) = (0,0)) = p^{xy}_{01}(t) \\ = 1 - \frac{q_{(0,0),(0,1)}e^{-tq_{(0,1),(1,1)}}}{(q_{(0,0),(0,1)} + q_{(0,0),(1,0)} - q_{(0,1),(1,1)})} - \\ - \frac{q_{(0,0),(1,0)}e^{-tq_{(1,0),(1,1)}}}{(q_{(0,0),(0,1)} + q_{(0,0),(1,0)} - q_{(1,0),(1,1)})} + \\ + \frac{e^{-t(q_{(0,0),(0,1)}+q_{(0,0),(1,0)})}(q_{(0,0),(0,1)}q_{(0,1)(1,1)} + q_{(0,0),(1,0)}q_{(1,0)(1,1)} - q_{(0,1),(1,1)}q_{(1,0)(1,1)})}{(q_{(0,0),(0,1)} + q_{(0,1)(1,0)} - q_{(0,1),(1,1)})(q_{(0,0),(0,1)} + q_{(0,0)(1,0)} - q_{(1,0),(1,1)})} \end{aligned} \tag{15}$$

When the branch length t is sufficiently large we have that $\lim_{t\to\infty} p^{xy}_{01}(t) = 1$ just as the independent and fully dependent cases because $e^{-t\alpha} \to 0$ for any $\alpha > 0$.

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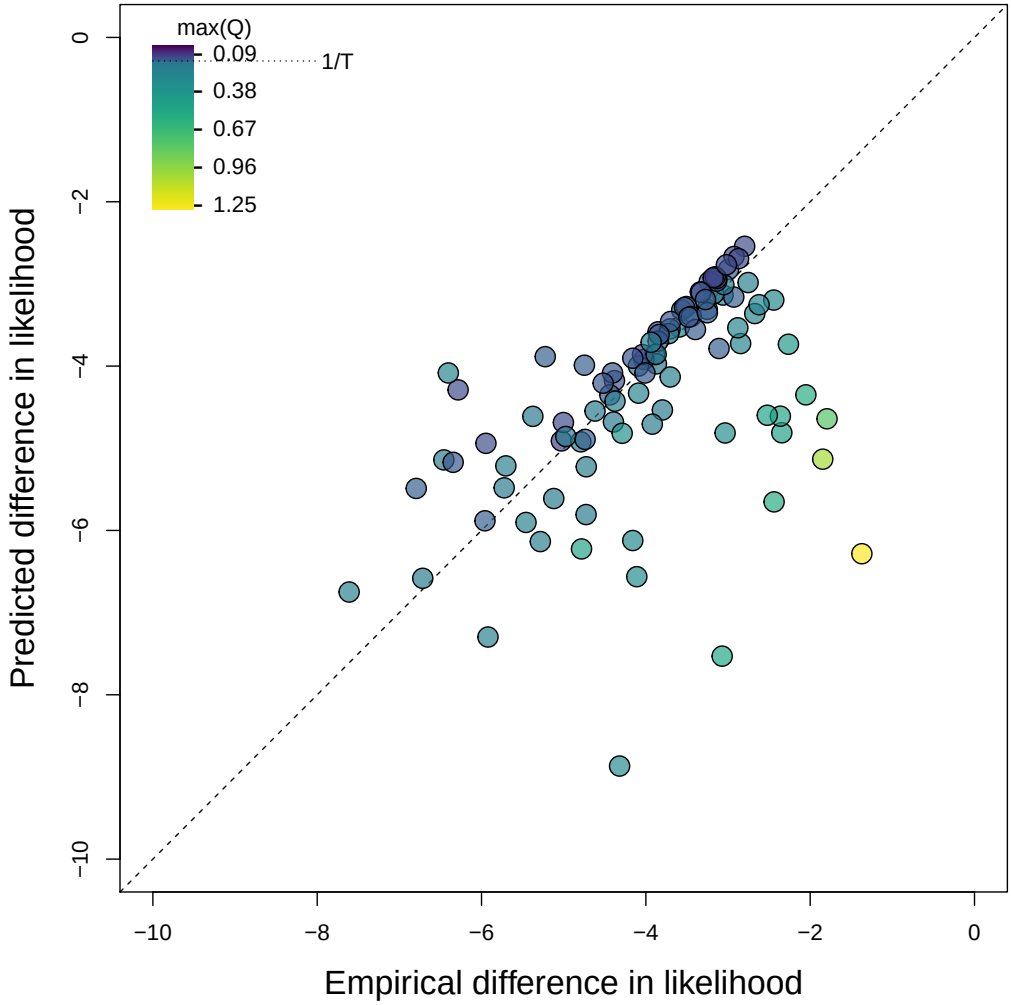

Figure S1: Darwin's scenario–the singular origin of two coextensive traits on the phylogeny–represents a boundary case to finding the correlation between discrete characters. Pagel's correlation test for Darwin's scenario can essentially be reduced to the difference in probability between choosing the same branch twice vs. choosing the branch only once. We demonstrate that here, showing our predicted differences in log likelihood between the independent and dependent trait models (y-axis) against the empirical estimates of the difference in log likelihood between models for simulated Darwin's scenarios on different phylogenies. Dotted line indicates equality. Points falling off the line represent slight violations of the assumptions we used to derive our prediction. Particularly, we assume that the rates of gain of the traits are so low that only one shift is ever observed. The color of the points indicates cases where this assumption is violated, as outlying points with max(Q) values much greater than $1/T$ (the value of q_{01} at which exactly 1 shift is expected) are much more likely to fall off the predicted line. This figure differs from Figure 6 in the main text in that estimated likelihoods are not constrained to fit the assumptions we used to derive the predicted difference in likelihood. Specifically, we do not assume irreversibility, we allow partial correlations, and we do not constrain gain rates to be equal for the independent case.

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D. Graphical Models-Supplementary Methods

To obtain the results in Figure 9 in the main text, we considered two Bayesian Networks involving 3 traits. Body size (B) and species abundance (N) are observed continuous traits, while a third trait, Migratory behavior is a threshold trait—meaning that it is a discrete trait that has an underlying continuous liability (Felsenstein, 2011). For the network in Figure 9A, we generated data by simulating Brownian Motion (root = 0, $\sigma^2 = 1$) of B on the phylogeny depicted in Figure 9C. We then simulated the liability of M as $M_{liab} = B + \epsilon$ where ϵ is a random Normal deviate with mean 0 and standard deviation of 0.5. We then discretized M_{liab} into a binary character (M) by assigning liabilities above the median value of M_{liab} 1 and below the median value 0. Finally, we simulated values of N as $N = B + -3 * M + \epsilon$, where ϵ is again a random Normal deviate with mean 0 and standard deviation of 0.5.

A similar procedure can be peformed if the network is instead what is found in Figure 9B. Here, we simulate the liability M_{liab} by Brownian Motion (root = 0, $\sigma^2 = 1$). Body size (B) is then a function of this liability using the reciprocal equation, $B = M_{liab} + \epsilon$. We then discretize migratory behavior (M) from the liability as before, and simulate values of abundance using the same equation $N = B + -3 * M + \epsilon$.

We acknowledge that the manner in which the data is generated is somewhat contrived and parameters were chosen to produce a figure that maps on to the familiar conceptual depiction of Simpson's paradox. This was done to aid visual and conceptual interpretability. Under a wider range of parameter combinations, such consistent differences between PGLS and OLS results will often break down — particularly due to the lack of robustness of PGLS results to violations of Brownian Motion and its sensitivity to singular events (which will often result from our imposition of a threshold model, see Case Study I in the main text). Thus, our primary goal in generating this figure was to choose parameter sets and networks that visually illustrated the phenomenon of Simpson's paradox in phylogenetic comparative datasets in a clearly interpretable way without substantially violating the assumptions of PGLS regression. While other parameter sets will produce less visually obvious results, the key points of our argument will remain unchanged.