

Effect of migration and environmental heterogeneity on the maintenance of quantitative variation: a simulation study

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1 **Abstract**

2 The paradox of high genetic variation observed in traits under stabilizing selection is a
3 longstanding problem in evolutionary theory, as mutation rates are 10-100 times too low to
4 explain observed levels of standing genetic variation under classic models of mutation-selection
5 balance. Here, we use individual-based simulations to explore the effect of various types of
6 environmental heterogeneity on the maintenance of genetic variation (V_A) for a quantitative trait
7 under stabilizing selection. We find that V_A is maximized at intermediate migration rates in
8 spatially heterogeneous environments, and that the observed patterns are robust to changes in
9 population size. Spatial environmental heterogeneity increased variation by as much as 10-fold
10 over mutation-selection-balance alone, whereas pure temporal environmental heterogeneity
11 increased variance by only 45% at max. Our results show that some combinations of spatial
12 heterogeneity and migration can maintain considerably more variation than mutation-selection
13 balance, potentially reconciling the discrepancy between theoretical predictions and empirical
14 observations. However, given the narrow regions of parameter space required for this effect, this
15 is unlikely to provide a general explanation for the maintenance of variation. Nonetheless, our
16 results suggest that habitat fragmentation may affect the maintenance of V_A and thereby reduce
17 the adaptive capacity of populations.

18 Introduction

19 As genetic variation is the fundamental basis upon which evolution acts, it is important to
20 understand how variation is maintained in order to provide a foundation for answering various
21 questions in biology and related fields, such as missing heritability (Maher, 2008), conservation
22 of biodiversity (Cook & Sgrò, 2017), and population potential to respond to change (Houle,
23 1992). And yet, the relative importance of factors that influence variation and the mechanism(s)
24 under which it is maintained are not wholly understood (Barton & Turelli, 1989; Mackay *et al.*,
25 2009). The majority of quantitative traits experience stabilizing selection, which in theory should
26 erode genetic variation. However, high levels of standing variation and heritability of
27 quantitative traits are consistently observed in nature (Johnson & Barton, 2005). This paradox – a
28 high degree of genetic variation maintained in the face of stabilizing selection – remains a
29 longstanding, unsolved problem in evolutionary biology and quantitative genetics theory.

30
31 The most widely studied explanation for this paradox is mutation-selection balance (henceforth
32 referred to as MSB), the appeal of which lies in its intuitive logic: mutation, as the ultimate
33 source of genetic variation, provides enough input to offset the eroding effect of selection,
34 leading to a state of equilibrium. Under such models, stabilizing selection is assumed according
35 to a Gaussian fitness function with parameter V_S setting the strength of selection on genotypes
36 (where large values result in weaker selection). Multiple MSB models have been proposed, most
37 notably the continuum-of-alleles model from Kimura and Crow (1964) of which two main
38 approximations have been put forth: the Gaussian approximation (Kimura 1965; later expanded
39 by Lande 1976), and the House-of-cards approximation (Turelli, 1984; Bürger *et al.*, 1989).
40 The continuum-of-alleles model makes the basic assumption that for a locus with a continuous
41 distribution of possible alleles, mutation can result in a new allele with an effect different from
42 the pre-existing one. In general, the two approximations differ in the way each handles this
43 assumption. For example, for an arbitrary diploid locus i with an allele of effect x_i and a mutation
44 with effect y_i , under the Gaussian approximation the mutated allele would take on a new value
45 which is conditional on the previous state ($x_i + y_i$). This results in a predicted equilibrium genetic
46 variance $V_G(G) \cong 2n\sqrt{\mu\alpha^2V_S}$, where n is the total number of loci that compose the quantitative
47 trait, μ the per locus mutation rate, and α^2 the variance of the distribution of mutational effects.

48 Under the House-of-Cards approximation, x_i may take on any effect independent from $x_{i+1} \dots x_n$,
49 and mutation results in the replacement of x_i by y_i . In this case, the predicted equilibrium
50 variance, $V_G(HC) \cong 4n\mu V_S$ is independent of the distribution of mutation effects. In contrast with
51 the continuum-of-alleles model, there is the diallelic model (Bulmer, 1972; Barton, 1986), which
52 assumes only two possible values at a locus with equal forward and backward mutation rates,
53 such that mutation causes a flip between states (*e.g.*, $x_i = \{0,1\}$). In this case, the resulting
54 equilibrium variance has been shown to be generally comparable to $V_G(HC)$.

55 Although these models have been extensively studied, conflicting evidence over their
56 applicability in relation to realistic biologic parameters has left debate open (Barton & Turelli,
57 1989; Johnson & Barton, 2005; Zhang & Hill, 2005). The assumptions and requirements of these
58 models are unrealistic: primarily, they require extreme mutation rates or too many loci per trait
59 (Johnson & Barton, 2005). This can be most simply illustrated by considering two empirical
60 observations: heritability, h^2 , for the majority of traits are relatively high ($h^2 = 0.2 \sim 0.6$;
61 Mousseau and Roff 1987), and stabilizing selection is typically relatively strong in nature, as
62 evidenced by the median value of estimates of reported quadratic selection gradient ($\gamma = -0.1$;
63 Kingsolver *et al.*, 2001). The value $\gamma = -0.1$ implies that the ratio of selection to phenotypic
64 variation is $V_S / V_P = 5$ (Kingsolver *et al.*, 2001; Johnson & Barton, 2005), which can be
65 rearranged to account for typical estimates of heritability, yielding an expected empirical range of
66 values for variation maintained in a natural population of approximately $0.04 < V_A / V_S < 0.12$.
67 Putting this in the context of Turelli's (1984) $V_G(HC)$ model under the assumption that $V_G \sim V_A$,
68 then $0.01 < n\mu < 0.03$. If there are 100 loci underlying a given trait, then this would require per-
69 locus mutation rates on the order of $\sim 10^{-4}$, which is 10 to 100 times higher than most estimates
70 from quantitative genetic studies, typically thought to be in the range of 10^{-6} to 10^{-5} (Barton &
71 Turelli, 1989; Bürger, 2000).

72 More complex extensions of these models have been investigated (Bürger *et al.*, 1989; Zhang
73 & Hill, 2002; Zhang *et al.*, 2002, 2004), namely by extending to multiple traits, such that
74 'apparent stabilizing selection' is generated through pleiotropic effects. Pure pleiotropy and joint
75 effects models have been studied, as well as other extensions to include factors such as
76 dominance or balancing selection. Nonetheless, these models have yet to provide a sufficient

77 explanation for the patterns of variation maintained as suggested by empirical data (Johnson &
78 Barton, 2005; but see Zhang and Hill 2005).

79 The above models all assume stabilizing selection to a constant environment, and yet
80 environments varying in space and time can also affect the maintenance of variation. If
81 environmental heterogeneity can maintain sufficient differences in allele frequencies within a
82 subdivided population, and migrant individuals introduce novel variants that can be maintained
83 for some time, then migration can result in an increase in genetic variation within a population.
84 Structured populations with limited amounts of gene flow have the potential to increase within-
85 population variance (Lythgoe, 1997; Tufto, 2000; Spichtig & Kawecki, 2004), and a temporally
86 fluctuating environment has been shown in some cases to increase variance under particular
87 regions of parameter space (Kondrashov & Yampolsky, 1996; Bürger & Gimelfarb, 2002;
88 Gulisija & Kim, 2015). However, these studies have not explicitly framed results in terms of the
89 relative increase in variance over mutation-selection to address the problem of the discrepancy
90 between MSB predictions and empirical observations, and the effect(s) of time and space have
91 not been investigated simultaneously.

92 Here, we assess how different kinds of environmental heterogeneity affect the maintenance of
93 genetic variation. We simulate spatial and temporal heterogeneity both independently and
94 simultaneously to explore how the maintenance of variation is affected by variation in migration
95 rate, population size, mutation rate, strength of selection, and pattern of environmental variability.
96 In all cases, we explicitly focus on comparing heterogeneous vs. homogeneous environments to
97 represent the increase in variance relative to that expected under mutation-selection balance. This
98 provides some indication of how much more variance can be maintained under heterogeneous
99 environments, which can be compared to the discrepancy between the mutation-selection balance
100 predictions and empirical observations described above.

101 **Methods**

102 **Simulation Setup**

103 A diploid Wright-Fisher two-population scenario was modeled using the stochastic, individual-
104 based simulation program *Nemo* (v2.3.45; Guillaume and Rougemont 2006) under various
105 parameter combinations of population size N (1000, 10000), strength of selection V_S (2, 5, 10),
106 mutation rate μ (10^{-6} , 10^{-5} , 10^{-4}), and backwards migration m (rate ranging from 10^{-5} to 0.1 per

107 generation) between two equal sized patches with some local optima θ . See Table 1 for detailed
108 reference of all parameters.

109 In each simulation, relative fitness is determined by Gaussian stabilizing selection acting on a
110 single quantitative trait of 50 unlinked loci, each with an independent allelic value a ($n = 50$; $r =$
111 0.5). Nemo defines individual fitness of a quantitative trait according to the equation:

$$W(z) = \exp \left[-\frac{(z - \theta)^2}{2w^2} \right]$$

112 Where z is the phenotypic value of the individual, equal to $\sum a_i$, θ the local optimum, and w^2 the
113 strength of selection on the phenotype ($V_S = w^2 + V_E$). As no effect of environmental variance
114 was included ($V_E = 0$), hereafter we use V_S to represent the strength of selection on the genotype
115 to simplify comparison to theoretical models. The allelic value (a) at each locus is randomly
116 drawn from a gamma distribution (mean = 0.05, shape = 1), with mutation causing a change of
117 states $\pm a$ (this follows a pseudo-diallelic model with a House-of-Cards mutation scheme). Loci
118 have additive effects on the phenotype, with no dominance or epistasis and free recombination
119 between adjacent loci ($r = 0.5$). Thus, genetic architecture cannot evolve through addition of
120 multiple successive mutations at a given locus or through competition among alleles with
121 different linkage relationships.

122 A total of six scenarios were run, each differing in the relation of local optima between patches:

- 123 (1) **Homogenous patches** ($\theta_1 = \theta_2 = 1$). The control set used for comparison to the other
124 scenarios; the local optimum is the same in each patch such that any differences that arise are
125 due to drift rather than selection.
- 126 (2) **Pure spatial heterogeneity** ($\theta_1 = +1$; $\theta_2 = -1$). This set of simulations introduced spatial
127 environmental heterogeneity with opposite local optima between each patch.
- 128 (3) **Pure temporal heterogeneity** ($\theta_{t,1} = \theta_{t,2}$). To reflect a population inhabiting a changing
129 environment, temporal fluctuations were modelled using an oscillating optimum defined by a
130 sine wave centered about zero with amplitude of 1; $\theta(t) = \sin \left(\frac{2\pi t}{P} \right)$. Simulations were run
131 with varying periodicity ($P = 4, 16, 40, 100, 1000$). The main focus of the analysis considers a
132 temporal oscillation over 100 generations, unless otherwise specified.

133 (4) **Combined (Spatial & Temporal) heterogeneity.** The last set of simulations combined the
134 setup of (2) and (3). Three subsets were used to represent differing degrees of spatial
135 variation (see *SI*): (*A*, $\theta_{t,1} = -\theta_{t,2}$) opposing optima reflected about the horizontal axis; (*B*, $\theta_{t,1}$
136 $= \theta_{t,2} + 2$) in phase temporal fluctuations with one patch oscillating about -1 and the other
137 about +1; (*C*, $\theta_{t,1} = \theta_{t+(P/4),2}$) a partial phase shift between optima functions, such that θ_2 lags
138 behind θ_1 .

139 All simulations were run for an initial period of 40,000 generations under these conditions,
140 allowing the trait value to stabilize at an approximate equilibrium (assessed visually for the
141 absence of change in trajectory of the trait value over time). A census of the population was taken
142 every t generations ($t=100$, or $t=5$ for temporal simulations with $P \leq 100$ generations) for the
143 mean trait value and genetic variation (in terms of V_A , the additive genetic variation) present in
144 each patch. This data was averaged over the stable period of the last 10,000 generations (1000 for
145 temporal simulations with $P \leq 100$) for 40 replicates. For consistency, a 40x50 matrix of allele
146 values was generated such that each replicate had a different complement of mutation effect
147 sizes, but that this complement was held constant across all simulation scenarios. Under the
148 simulations that included temporal heterogeneity, preliminary results showed a cycling pattern
149 emerge for both the trait value and V_A over time; to use an estimate as representative as possible,
150 the V_A maintained within a cycle was sampled at 20 evenly spaced time-points within a cycle,
151 and the mean calculated over the last ten cycles of the simulation. To obtain a measure of
152 variation maintained within the simulated population that is comparable to empirical evidence,
153 the ratio V_A / V_S was calculated from the final mean V_A estimate of each simulation set. All data
154 processing and analysis was done with R v3.3 (R Development Core Team, 2016).

155 **Results**

156 **Pure Spatial Homogeneity, Set 1 ($\theta_1 = \theta_2 = 1$)**

157 We first describe patterns in the homogeneous environment, which acts as a control
158 demonstrating the effect of finite population MSB processes in a two-patch model, to study the
159 effects of mutation rate (μ), selection strength (V_S), and population size (N), on the maintenance
160 of variation (Figure 1A). As expected from quantitative genetic theory (Falconer and Mackay
161 1996), variance was somewhat higher in larger populations, although the difference is minimal
162 under most parameters (Figure S1). Also following expectations, V_A increases with μ , with the

163 effect of mutation rate approximately linear, and independent of selection. Similarly, V_A
164 increases with V_S (selection weakens), which qualitatively matches predictions of Turelli (1984),
165 Burger et al. (1989), and as observed by Yeaman and Guillaume (2009), although some
166 deviations occur due to differences in the mutation model. While from Figure 1 there appears to
167 be an interaction between selection and mutation rate, the relative effect of a ten-fold increase in
168 mutation (from $\mu = 10^{-5}$ to $\mu = 10^{-4}$) is similar for $V_S = 2$ versus $V_S = 10$ (V_A changes by a factor
169 of 8.87 vs 8.59 for $V_S = 2$ vs $V_S = 10$; $m = 0.1$).

170 Notably, these patterns are qualitatively independent of N (Figure S1), and so the results below
171 are presented for the subset of $N = 1000$ only. There is minimal response to rate of migration
172 between the two patches, except for a small increase at low to intermediate m , which is consistent
173 with predictions of Goldstein and Holsinger (1992), and arises due to an interaction between
174 effects of genetic redundancy and genetic drift (hereafter, referred to as the “redundancy-drift
175 effect”; see Discussion).

176 **Pure Spatial Heterogeneity, Set 2 ($\theta_1 = +1$; $\theta_2 = -1$)**

177 With spatial environmental heterogeneity, two key differences arise when compared to the
178 control case (Set 1): an overall large increase in the amount of V_A maintained, and the appearance
179 of a threshold at high migration, above which the divergence among populations and amount of
180 V_A maintained both decrease (which is qualitatively consistent with the critical migration
181 threshold predicted by Yeaman and Otto, 2011). While the migration rate at which V_A peaks
182 varies as a function of V_S , the maximum V_A reached under these parameters ($\cong 0.32$) is similar
183 among levels of V_S . Given the number of loci, mutation effect sizes, and mutation rates that we
184 used, spatial environmental heterogeneity generates up to a ~10-fold increase in V_A compared to
185 the homogenous case (for the same parameters, $V_A[\text{homogeneous}] = 0.033$), as shown in Figure
186 1B. This demonstrates that the redundancy-drift effect described previously by Goldstein and
187 Holsinger (1992) is small relative to the effect of migration and spatial heterogeneity, at least
188 under these conditions. Additionally, the effect of mutation becomes relatively less pronounced
189 under spatial heterogeneity, as most of the variation is maintained by migration in this case.
190 Whereas a ten-fold change in μ (from $\mu = 10^{-5}$ to $\mu = 10^{-4}$) results in an increase by a factor of
191 8.87 in Figure 1A, this increase only changes V_A by a factor of 1.09 under spatial heterogeneity
192 ($V_S = 2$, $m = 0.1$).

193 **Pure Temporal heterogeneity, Set 3 ($\theta_{t,1} = \theta_{t,2}$)**

194 In contrast to the case with spatial heterogeneity, under a temporally fluctuating optimum there is
195 in general little change in V_A in comparison to Set 1, the homogenous case. As shown in Figure
196 2A, a similar qualitative pattern is observed in response to migration rate, with an increase at low
197 to intermediate m and rapid drop with $m > 0.001$, akin to the redundancy-drift effect observed in
198 the homogeneous case. The resulting maximum peak in V_A however is approximately 45%
199 greater than the homogeneous case ($V_S = 5$, $\mu = 10^{-4}$, $m = 2.5 \times 10^{-4}$), suggesting that temporal
200 heterogeneity can only marginally increase the importance of this effect.

201 Multiple lengths for the temporal oscillation of the environment (denoted as P) were simulated to
202 test the influence of the period length on the qualitative patterns observed, and complex
203 interactions were found (Figure S2.1). Setting $P \leq 100$ resulted in a response to migration similar
204 to the homogeneous case and an overall increase in V_A with μ , as expected. However, the
205 behavior for $P = 100$ deviates under very strong selection ($V_S = 2$), with the redundancy-drift
206 effect causing a peak in V_A at lower migration rates than for shorter periodicities (see Figure 2A
207 and S2.1). Across changes in all parameter combinations, the redundancy-drift effect causes a
208 maximum increase in V_A of 28% relative to the V_A maintained under $m = 0.1$, which was
209 observed for $P = 100$ and $V_S = 10$. Under a very long cycle ($P = 1000$ generations), there is
210 minimal response to migration rate (i.e., no effect of redundancy-drift) and very little increase in
211 variance when $\mu = 10^{-6}$ relative to simulations with shorter periods, but much more increase in
212 variance under long periods at $\mu = 10^{-4}$ (see Figure S2.1; V_A under $P = 1000$ is 1.3 ~ 2.5-fold
213 higher than $P = 100$ under $\mu = 10^{-4}$). The degree of increase in V_A under temporal heterogeneity
214 relative to homogeneity therefore depends on mutation, period, and selection, but is typically
215 considerably less than under spatial heterogeneity with intermediate migration.

216 **Combination of Spatial and Temporal heterogeneity, Set 4**

217 For spatial and temporal heterogeneity combined ($\theta_{t,1} = \theta_{t,2} + 2$; see *SI* for illustration), the
218 amount of V_A and patterns that arise are qualitatively similar to those in the case of pure spatial
219 heterogeneity. Similar threshold behavior is observed, and similar levels of V_A are maintained,
220 peaking between migration rates of 0.01 ~ 0.1, again depending on V_S (Figure 2B). Under this
221 form of combined spatial and temporal heterogeneity, there is a max 31% increase in V_A
222 compared to pure spatial heterogeneity (at $V_S = 5$, $\mu = 10^{-4}$, $m = 0.001$), which is very small when

223 compared to the 9.75-fold difference in V_A for the pure spatial heterogeneity vs. homogeneity
224 under the same parameters V_S and μ .

225 In contrast, the complex patterns involving periodicity described above that were observed in the
226 purely temporal set do not appear to carry over under combined spatial and temporal
227 heterogeneity, likely because they are swamped out by the relatively larger effect of migration,
228 which is relatively consistent across different period lengths (see S2.2). There is an overall
229 increase in V_A maintained, which appears more similar to the case of pure spatial heterogeneity
230 than pure temporal heterogeneity or homogeneity. This result is perhaps unexpected, as spatial
231 and temporal effects of heterogeneity might be expected to work synergistically.

232 **Comparing Sets**

233 Figure 3 shows a comparison amongst the six different patterns of environmental variation for a
234 subset of the total parameter combinations used. For a given parameter set, there is a general
235 trend of increased V_A maintained with an increase in heterogeneity, particularly with the degree
236 of spatial heterogeneity between patches. Temporal heterogeneity on its own does little to
237 increase V_A compared to the homogeneous set. When temporal heterogeneity is combined with
238 spatial heterogeneity, the increase in V_A is most pronounced in sets where the temporal
239 fluctuations in the two patches are most out of phase (Figure 3; see *SI* for relation between $\theta_{t,1}$ and
240 $\theta_{t,2}$). Of the three combinations investigated, subset 4B – where $\theta_{t,1} \neq \theta_{t,2}$ for any t – maintains a
241 considerable higher level of V_A compared to subset 4A, which in turn maintained more V_A than
242 in 4C, which was most in phase between patches. This further supports the emphasis on spatial
243 heterogeneity over temporal heterogeneity as a stronger force for maintaining variation.

244 To explore whether environmental heterogeneity could yield more realistic ratios of V_A/V_S (ie,
245 $0.04 < V_A/V_S < 0.12$), we compared values for this ratio under homogeneous vs. heterogeneous
246 simulation scenarios with all other parameters held constant (Figure 4). Without any spatial
247 heterogeneity, none of the parameter combinations result in V_A that falls within this range for
248 expected typical values (see S4, S5 for simulation sets that did not fall into range). However, with
249 spatial heterogeneity, the combination of high migration ($m \geq 0.01$) and strong selection ($V_S = 2$
250 or 5) does result in V_A within the typical range (i.e. an increase of $\sim 10x$ relative to MSB). For the
251 combination of spatial and temporal heterogeneity, there is little change for most parameter
252 combinations compared to spatial heterogeneity alone (Figure 4B); for those where there is a

253 noticeable deviation, whether the parameter set falls in or out of the target range remains
254 unchanged (with an exception at $V_S = 2$, $m = 0.001$).

255 **Discussion**

256 The results of these simulations show that environmental heterogeneity can have a large impact
257 on the maintenance of additive genetic variation (V_A) under a variety of parameters. The
258 direction and magnitude of this impact depends upon the rate of migration between
259 subpopulations, with intermediate rates of dispersal providing the largest effect. This effect is
260 also dependent on the type of environmental heterogeneity – spatial, temporal, or a combination
261 of both all result in different magnitude and pattern of response to migration. Spatial
262 heterogeneity has a much larger effect than temporal, and the effect of combined heterogeneity
263 increases with the degree of spatial difference when temporal oscillations in each patch are more
264 out of phase.

265 **The Redundancy-Drift Effect**

266 For a range of migration rates (dependent on V_S and μ) spatial environmental heterogeneity
267 substantially increases the variation a population can maintain. However, migration alone, even
268 between two environmentally similar patches, can increase the variation maintained at very low
269 rates of migration. This result is due to the interaction of genetic redundancy and the stochastic
270 force of genetic drift, as described by Goldstein and Holsinger (1992). With enough population
271 structure (i.e., limited gene flow), the genotypes existing in each patch are expected to undergo
272 different histories of mutation and drift, resulting in relatively independent fluctuations in allele
273 frequency within each patch, and possibly local fixation or loss. Over time, this results in a
274 different pool of genotypes present in each patch, even if the mean phenotype is equivalent.
275 Migration between patches with different allele frequencies will then increase the variance within
276 patches; movement of individuals adapted to a given environment with one complement of
277 alleles that result in a similarly fit phenotype, such that it can be maintained in the new
278 population, then this flux of allelic variants/combinations should increase the variation
279 maintained in the population. This effect is mainly dependent on two factors: (1) the number of
280 possible redundant genotypes, and (2) the degree of population structure – patches must be
281 sufficiently connected such that new genotypes can be introduced by migration at a high enough
282 rate to affect genetic variance, but not overly connected such that drift is no longer acting

283 (relatively) independently in each patch. This genetic redundancy-drift effect is evident in the
284 control case, where there is an observable – albeit minimal – response to low migration, which
285 persists regardless of population size, rate of mutation, or strength of selection. This effect is
286 somewhat increased under temporal heterogeneity, resulting in larger peaks over the same range
287 of low dispersal as the homogeneous case, particularly under high mutation and strong selection
288 ($\mu = 0.0001$, $V_S = 2$). The increase of this effect may be due to the temporally variable selection
289 within each patch causing different alleles to change in frequency more rapidly than migration
290 homogenizes differences, resulting in a larger pool of differentiated loci than would be generated
291 by drift alone.

292 **Environmental Heterogeneity**

293 Nonetheless, this redundancy-drift effect is small relative to the effect of spatial heterogeneity. In
294 this case, selection acts to pull each subpopulation towards its optima, resulting in different allele
295 frequencies within in each patch. With migration, the alleles that would otherwise be purged by
296 selection are persistently reintroduced into the other patch, increasing the V_A maintained in the
297 population. However, there is a threshold behavior to this system – a critical migration rate
298 beyond which polymorphism can no longer be maintained (Felsenstein 1976; Yeaman and Otto
299 2001). Past this point, the allele frequencies of each patch are similar enough that migrants are
300 much less likely to re-introduce novel/differentiated alleles, and the effect of migration on V_A is
301 reduced.

302

303 Most populations experience some form of temporally variable selection, be it seasonal
304 fluctuations or some otherwise unstable environment. Due to this, there has been much interest
305 around fluctuating selection and its effect(s) on the maintenance of variation (Kondrashov &
306 Yampolsky, 1996; Bürger & Gimelfarb, 2002; Siepielski *et al.*, 2009; Morrissey & Hadfield,
307 2012; Gulisija & Kim, 2015). Difficulties arise when attempting to draw comparisons across such
308 studies due to a variety of different assumptions, such as the way in which the moving optimum
309 is modelled and the nature of the underlying genetic architecture. A simple sinusoidal wave is
310 generally used, but varying parameters of this function (amplitude, center, periodicity) appear to
311 have complex interactions with each other as well as other parameters such as mutation rate and
312 genetic redundancy.

313 Burger and Gimelfarb (2002) found that in a diallelic multi-locus model with recurrent mutation,
314 the relative genetic variance increased with period length (typically ≥ 24 generations), but the
315 magnitude of change and pattern of response to period depended on the amplitude of the
316 oscillations. They observed the highest levels of genetic variance when the amplitude was set
317 such that the optimum cycled between the most extreme possible genotypes, and that greater
318 amplitudes, where the optimum fell beyond the maximum possible genotypic values, resulted in a
319 decline in V_A at high period lengths (≥ 52 generations). However, given the small number of loci
320 (2 - 6) and scaling of allelic effects, this represents a scenario with highly non-redundant genetics.
321 By contrast, Kondrashov and Yampolsky (1996) explored the maintenance of variation under
322 temporal heterogeneity with high genetic redundancy and found that a considerable increase in
323 variance (~ 3 orders of magnitude) was observed when the amplitude of the fluctuations of the
324 optimum exceeded the width of the fitness function (amplitude, $d > \sqrt{[S^2 + V_E]}$). While the
325 assumptions they made about allele effect size and number of loci meant that the local optimum
326 never exceeded the maximum/minimum possible phenotypes, the large amplitude implied by the
327 above inequality results in unnaturally large differences in fitness for a given phenotype over
328 time. For example, an individual with a phenotype equal to $+d$ would suffer a fitness cost of 86%
329 when the optimum = $-d$, when $S = d$ and $V_E = 0$. Similarly, for the standard set of parameters used
330 in their simulations where substantial variance was maintained, the fitness cost for the same
331 difference in phenotype exceeded 99%. By comparison, the fitness cost for the same difference in
332 phenotype in our simulations was 63% (under strongest selection, $V_S = 2$; this fitness cost is
333 reduced to 18% under weaker selection, $V_S = 10$). Common garden experiments typically show
334 an overall magnitude of local adaptation of $\sim 45\%$ (Hereford 2009), so the selection regimes used
335 by Kondrashov and Yampolsky (1996) constitute much stronger selection than normally
336 observed. Thus, while they found that temporal heterogeneity could maintain much more
337 variance than observed in our simulations, this was likely a consequence of drastic changes in
338 fitness causing the underlying alleles to cycle rapidly in frequency, with increases in V_A as the
339 alleles reached intermediate frequency. Thus, the capacity for temporal heterogeneity to maintain
340 substantial amounts of variance seems to depend upon either very extreme environmental
341 fluctuations or a narrow range of rapid fluctuations with strict assumptions on the amount of
342 genetic redundancy available. As empirical data become available for quantitative analyses of a
343 population undergoing temporal variation, this will provide a realistic standard set of parameter

344 values to be used in theory studies, allowing for a more quantitative assessment of the likely
345 effects of temporal fluctuations, and further evaluation of the relative importance of the above
346 models.

347
348 **Spatial v. Temporal v. Combined heterogeneity**
349 Interestingly, it appears that spatial heterogeneity has a much stronger effect than temporal, both
350 in isolation and when combined. This is evidenced by the difference in magnitude of V_A
351 maintained in Sets 2 vs. 3 (pure spatial and pure temporal heterogeneity, respectively), as well as
352 the comparisons among the three scenarios of combined spatial and temporal heterogeneity (Set 4
353 A, B, C). The combination of spatial and temporal heterogeneity only marginally increased the
354 variation maintained by spatial heterogeneity alone, and only in the case of complete spatial
355 heterogeneity (Set 4B). Interestingly, among the three combined sets, V_A increased with the
356 spatial component (in general, $V_A[4C] \leq V_A[4A] \ll V_A[4B]$), which was robust to changes in μ
357 and V_S .

358 Thus, our results indicate that spatial heterogeneity is more important for the maintenance of
359 variation than a temporally fluctuating environment, at least for the genetic architectures and
360 assumptions about fitness used here. Additionally, from the ratio V_A/V_S (Figure 4 and S2),
361 temporal fluctuations alone fail to result in any simulation maintaining significantly more
362 variation than under homogenous conditions, indicating that it is insufficient as an additional
363 mechanism to explain the discrepancy between mutation-selection balance models and empirical
364 observations.

365 **Does heterogeneity maintain sufficient variation to explain empirical** 366 **observations?**

367 Given empirical estimates of selection and heritability, a realistic amount of variation maintained
368 in a population should fall within the range V_A/V_S 0.04 ~ 0.12, which would require either an
369 unrealistically large number of loci or very high mutation rates under models of mutation-
370 selection balance, as described in the introduction. To explore whether heterogeneity could result
371 in more realistic ratios of V_A/V_S , we compared simulations run under homogeneous vs.
372 heterogeneous conditions, and examined the degree to which heterogeneity increased this ratio.
373 The shift towards this window for simulations with spatial heterogeneity suggests that this is a

374 viable mechanism for the maintenance of variation, but is limited to populations experiencing
375 strong selection and high migration rates.

376 The results presented in Figure 4 are intended to illustrate how the maintenance of realistic levels
377 of V_A is possible under environmental heterogeneity, rather than to represent the full span of
378 parameter combinations that result in realistic behavior. Little is known about the true values for
379 many parameters involved, and changes in some values, such as the number of loci, are likely to
380 alter the quantitative results. Instead, these results illustrate that for a given set of assumptions
381 about the genetic basis of a trait, considerably more variation can be maintained by migration-
382 selection balance than mutation-selection balance. Moreover, the magnitude of this change can be
383 sufficient to explain empirical observations in some cases, indicating environmental
384 heterogeneity is a viable mechanism to consider for the maintenance of variation. However, we
385 stress that given the sensitivity of this result to the balance between selection and migration and
386 the narrowness of the region that maintains significantly more variance, it seems unlikely that
387 environmental heterogeneity will generally resolve the “paradox of variation”.

388 **Limitations**

389 A major limitation to the interpretation of this study is the difficulty in comparing results to
390 established analytic models due to differences in the mutation scheme. The majority of theory
391 and analytic models assume either a single (or few with equal effect) diallelic loci, or quantitative
392 traits with a normally distributed continuum of alleles. Our simulations attempt to combine the
393 two – varying allelic effects across loci (a_i), but with a single $|a|$ per locus, and a Gamma
394 distribution of possible values a_i . We chose this approach to reflect a (potentially) more
395 biologically realistic scheme of mutation. For example, evidence suggests that the distribution of
396 allelic effects at a locus is more likely to follow a gamma rather than a normal distribution, with a
397 small number of loci of large effect and many loci of small effect (Orr, 2003). However, this
398 leads to a less straight-forward parameterization of V_m or α^2 , which causes challenges when
399 attempting analytical comparisons.

400 Another limiting factor is the difficulty of obtaining measurements of migration rates in natural
401 populations, which is necessary to understand if the range where this effect of environmental
402 heterogeneity occurs is realistic. Evidence of migration maintaining quantitative genetic variation
403 in spatially heterogeneous environments was found in lodgepole pine (Yeaman & Jarvis, 2006),

404 but genomic signatures at individual loci may be difficult to detect. Though we did not
405 specifically quantify this here, presumably there is some effect on heterozygosity, though such
406 effects could be difficult to detect in empirical data.

407 **Implications**

408 We have shown that spatial environmental heterogeneity and migration has the potential to
409 increase genetic variance substantially, while temporal heterogeneity has a much more modest
410 effect. As local adaptation is common (Hereford, 2009), this may be an important factor affecting
411 evolvability. It has been noted that conservation management lacks proper consideration of
412 evolutionary theory, implementation of which can improve overall efforts and long-term outcome
413 (Cook & Sgrò, 2017). The described effect of migration suggests that the ecology underlying the
414 maintenance of variation may be important to consider when planning conservation efforts with a
415 focus on genetics.

416 For example, a major criterion for the IUCN (International Union for Conservation of Nature)
417 Red List status is population size, however there are examples of listed populations that have
418 remained relatively stable, despite small N_e (Ex. Florida Panthers, *Puma concolor coryi*; (Benson
419 *et al.*, 2011)). The genetic consequences of population size have been emphasized due to [entirely
420 legitimate] concerns about reduced diversity, which is often used as a proxy measure for the
421 ‘health’ of a population of concern in conservation biology. Here, we showed that a 10-fold
422 change in population size had less effect on V_A than a reduction in migration, which suggests that
423 migration may also be an important factor to consider when assessing and managing the genetic
424 health of populations of concern. Indeed, the related concepts of gene flow and connectivity have
425 been previously suggested for consideration and implementation in conservation management,
426 albeit with some contention, as too much migration can result in a reduction of variability (Weeks
427 *et al.*, 2011; Urban *et al.*, 2012; Cook & Sgrò, 2017). Long-term maintenance of evolvability may
428 depend in some cases on an intact and interconnected environment, rather than endogenous
429 generation of variation (mutation-selection), which would depend less on environment.
430 Therefore, a move towards improving connectivity between subpopulations and protection
431 against habitat fragmentation may be a pertinent consideration for the maintenance of
432 quantitative genetic variation, in addition to the well-recognized problems of reducing inbreeding
433 and mitigating demographic fluctuations.

434

435 In conclusion, our results demonstrate a clear effect of migration on the maintenance of variation
436 in all four investigated scenarios, and highlight the potential for environmental heterogeneity to
437 substantially increase V_A . However, it seems unlikely that a single mechanism best explains the
438 maintenance of variation that we see in nature; rather the many concepts put forth over the past
439 decades may be viable explanations in only certain scenarios, just as has been demonstrated here
440 with environmental heterogeneity. Further investigation into potential mechanisms, particularly
441 into those scenarios under which they succeed and fail, and how biologically relevant successful
442 scenarios may be, is needed in order to get a complete picture of the fundamental process that is
443 the maintenance of variation.

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529 **Tables and Figure Captions**

530 **Table 1.** Description of Parameters

Parameter	Definition	Value
N	Population size	1000, 10 000
V_s	Selection strength	2, 5, 10
μ	Mutation rate	10 ⁻⁶ , 10 ⁻⁵ , 10 ⁻⁴
m	Backwards migration rate	0.00001 ~ 0.1
θ	Local (patch) optimum	
P	Periodicity; time (as the number of generations) over which temporal oscillations complete one cycle of a sinusoidal wave	4, 16, 40, 100, 1000

531

532 **Figure Captions**

533 **Figure 1.** Effect of migration on the maintenance of genetic variation (V_A) in a sub-divided
534 population ($N=1000$) between (A) homogeneous patches, (B) spatially heterogeneous patches,
535 under combinations of different mutation rate (line colour; $\mu = \{10^{-6}, 10^{-5}, 10^{-4}\}$) and selection
536 strength (panels; $V_S = \{2, 5, 10\}$). The homogeneous environment type (broken line) represents
537 the case of mutation-selection balance.

538 **Figure 2.** Effect of migration on the maintenance of genetic variation (V_A) in a sub-divided
539 population ($N=1000$) between (A) temporal, (B) combined spatial and temporal, environmentally
540 heterogeneous patches under combinations of different mutation rate (line colour; $\mu = \{10^{-6}, 10^{-5},$
541 $10^{-4}\}$) and selection strength (panels; $V_S = \{2, 5, 10\}$). The homogeneous environment type
542 (broken line) represents the case of mutation-selection balance.

543 Period = 100 generations

544 **Figure 3.** Comparison of the amount of genetic variation (V_A) maintained in a population
545 simulated under scenarios with increasing degree of environmental heterogeneity, for various
546 rates of migration (m) and strength of selection ($V_S = \{2, 5, 10\}$; panels).

547 $N=1000$; $\mu = 10^{-4}$, Period = 100 generations

548 **Figure 4.** Comparison of the ratio of variance components, V_A/V_S , between simulations of
549 different pattern of environmental heterogeneity for various combinations of migration rate (m ;
550 point colour), mutation rate ($\mu = \{10^{-6}, 10^{-5}, 10^{-4}\}$), and selection strength (point shape; $V_S = \{2,$
551 $5, 10\}$). The grey box represents the range of predicted 'typical' values expected based on
552 empirical estimates of trait heritability (h^2 0.2 ~ 0.6). Points that fall below the 1:1 line (dashed)
553 are parameter combinations under which more V_A is maintained without (A) environmental
554 heterogeneity, or (B) addition of temporal heterogeneity (Period = 100). $N=1000$

555

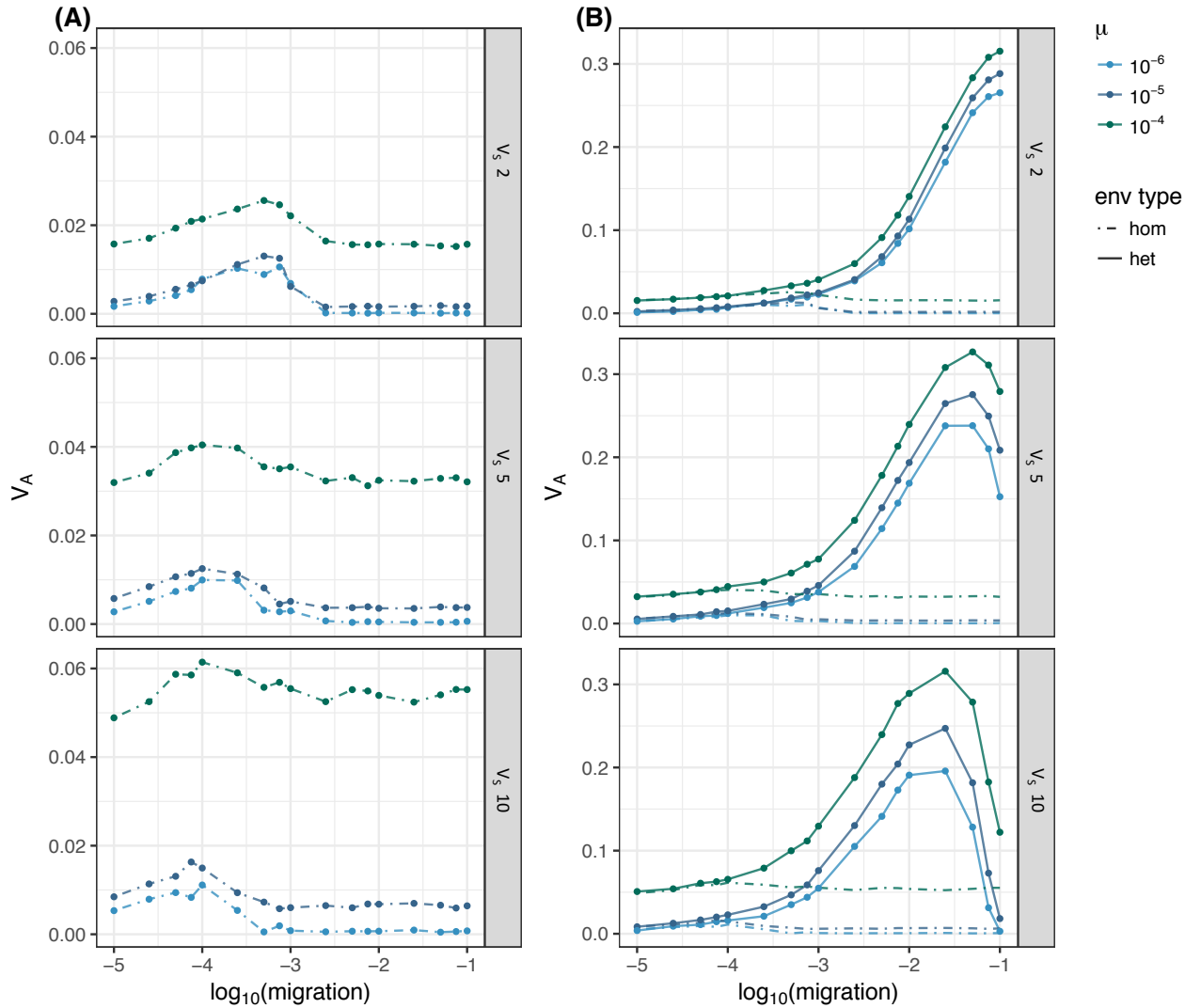


Figure 1. Effect of migration on the maintenance of genetic variation (V_A) in a sub-divided population ($N=1000$) between (A) homogeneous patches, (B) spatially heterogeneous patches, under combinations of different mutation rate (line colour; $\mu = \{10^{-6}, 10^{-5}, 10^{-4}\}$) and selection strength (panels; $V_s = \{2, 5, 10\}$). The homogeneous environment type (broken line) represents the case of mutation-selection balance

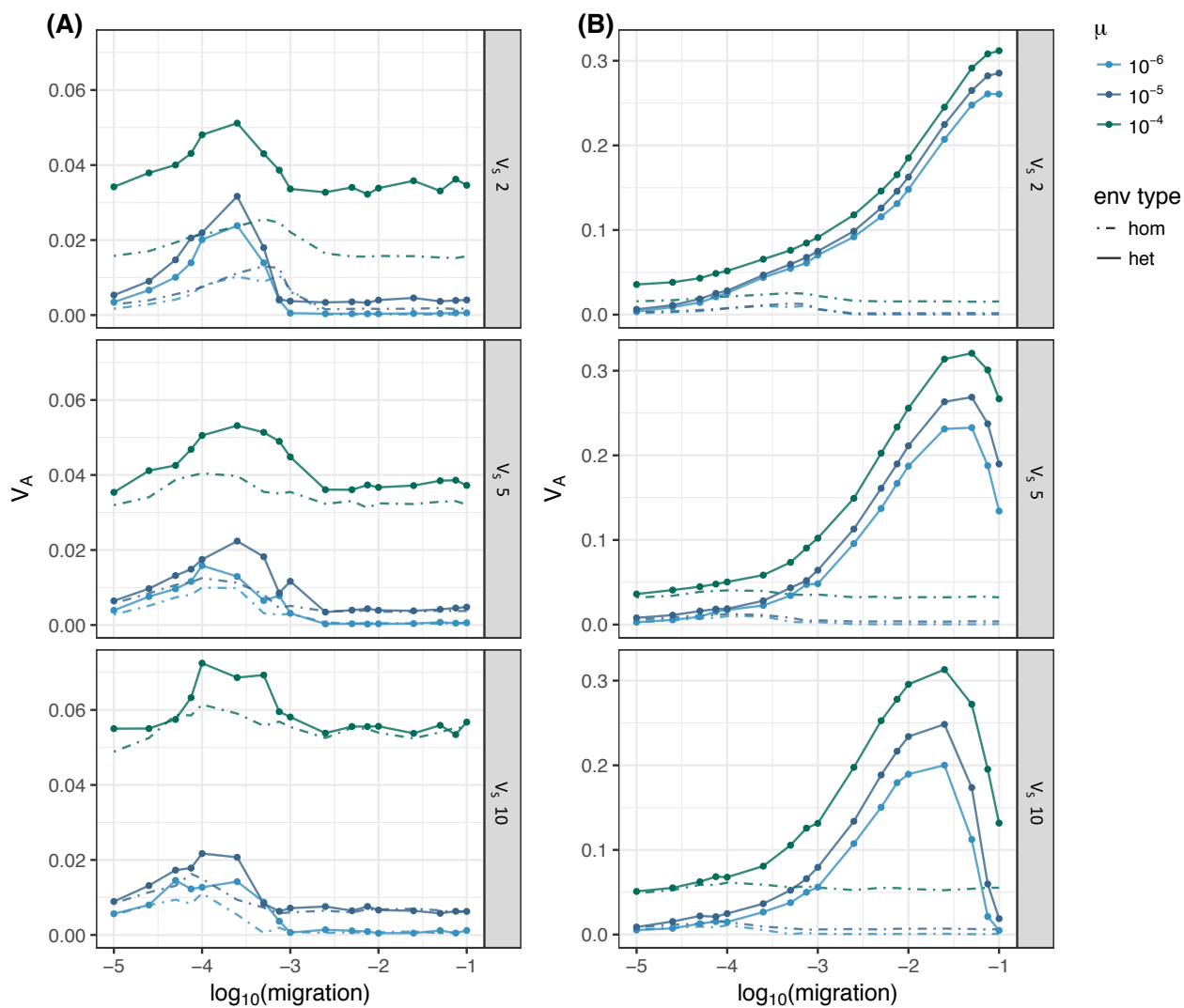


Figure 2. Effect of migration on the maintenance of genetic variation (V_A) in a sub-divided population ($N=1000$) between (A) temporal, (B) combined spatial and temporal, environmentally heterogeneous patches under combinations of different mutation rate (line colour; $\mu = \{10^{-6}, 10^{-5}, 10^{-4}\}$) and selection strength (panels; $V_S = \{2, 5, 10\}$). The homogeneous environment type (broken line) represents the case of mutation-selection balance.

Period = 100 generations

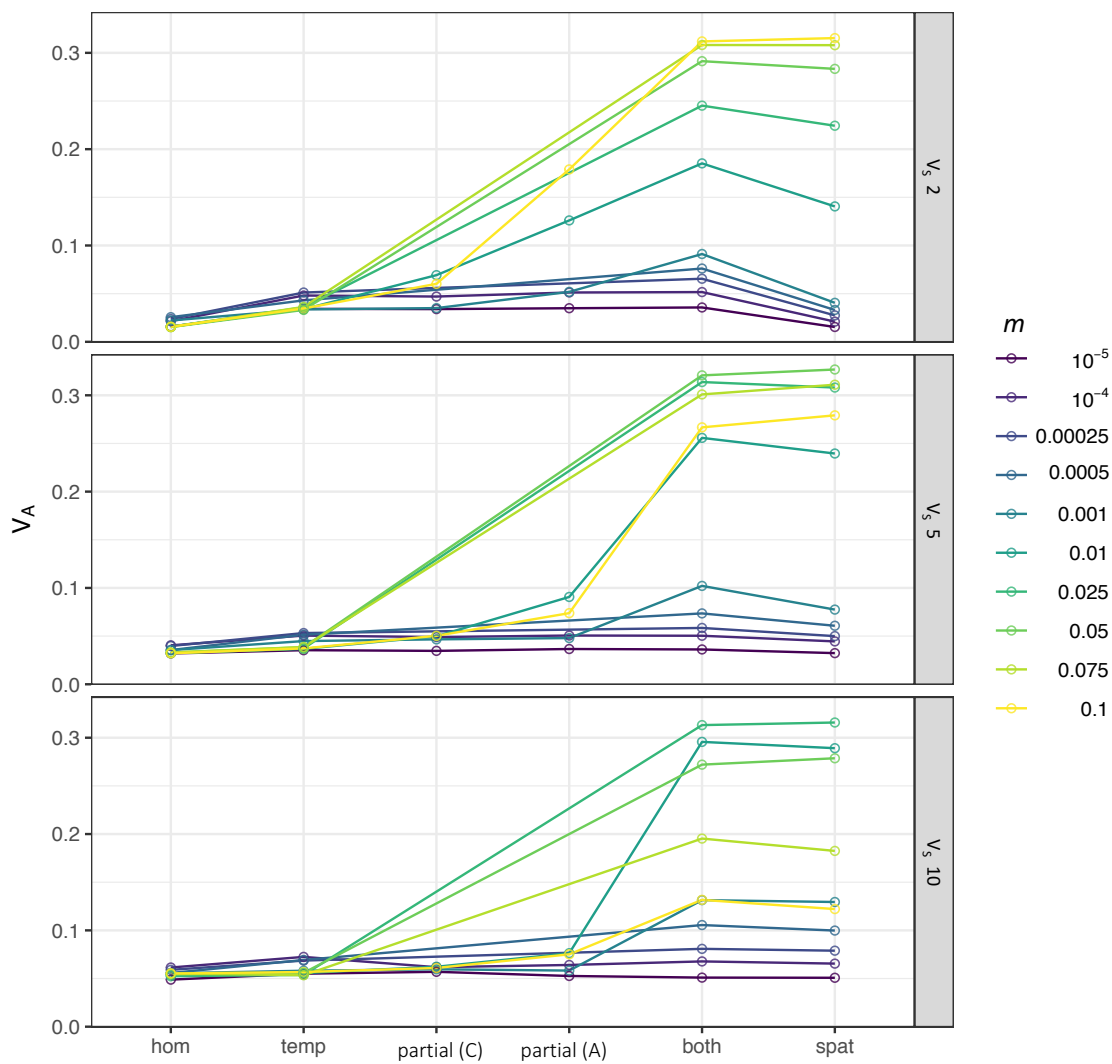


Figure 3. Comparison of the amount of genetic variation (V_A) maintained in a population simulated under scenarios with increasing degree of environmental heterogeneity, for various rates of migration (m) and strength of selection ($V_s = \{2, 5, 10\}$; panels). $N=1000$; $\mu = 10^{-4}$, Period = 100 generations

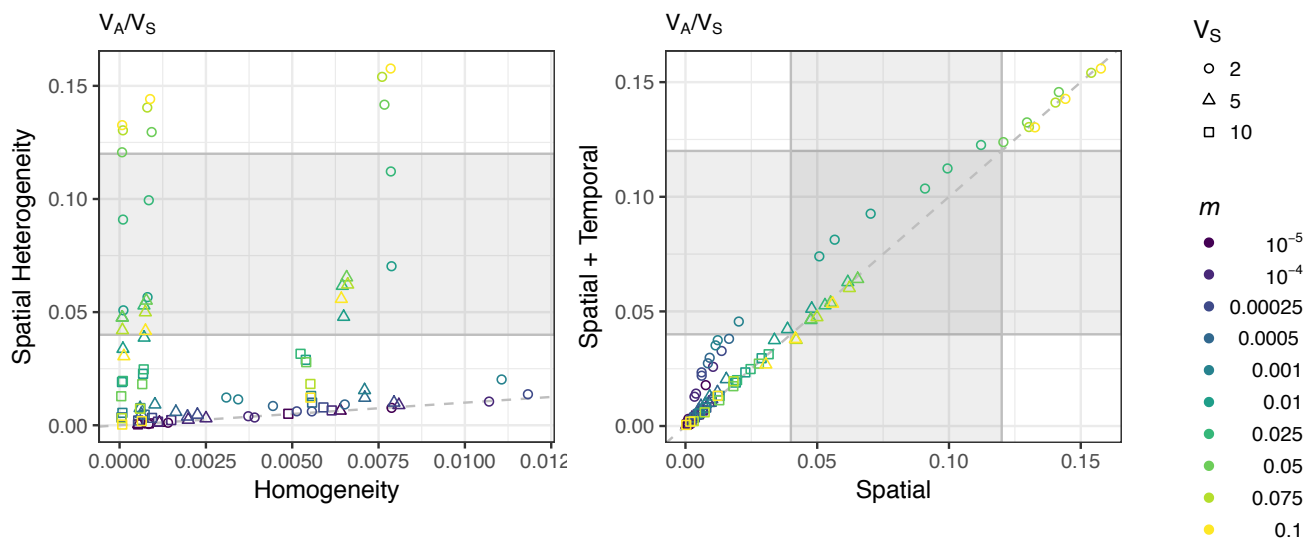


Figure 4. Comparison of the ratio of variance components, V_A/V_S , between simulations of different patterns of environmental heterogeneity for various combinations of migration rate (m ; point colour), mutation rate ($\mu = \{10^{-6}, 10^{-5}, 10^{-4}\}$), and selection strength (point shape; $V_S = \{2, 5, 10\}$). The grey box represents the range of predicted 'typical' values expected based on empirical estimates of trait heritability ($h^2 \approx 0.2 \sim 0.6$). Points that fall below the 1:1 line (dashed) are parameter combinations under which more V_A is maintained without (A) environmental heterogeneity, or (B) addition of temporal heterogeneity (Period = 100). $N=1000$