

1 **Full title: Inferring causal pathways among three or more variables from steady-state**  
2 **correlations in a homeostatic system**

3 **Short title: Inferring causality from steady-state correlations**

4

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## 14 **Abstract**

15 Cross-sectional correlations between two variables have limited implications for causality. We  
16 show here that in a homeostatic system with three or more inter-correlated variables, it is  
17 possible to make causal inferences from steady-state data. Every putative pathway between  
18 three variables makes a set of differential predictions that can be tested with steady state data.  
19 For example, among 3 variables, A, B and C, the coefficient of determination,  $r_{AC}^2$  is predicted  
20 by the product of  $r_{AB}^2$  and  $r_{BC}^2$  for some pathways, but not for others. Residuals from a  
21 regression line are independent of residuals from another regression for some pathways, but  
22 positively or negatively correlated for certain other pathways. Different pathways therefore  
23 have different prediction signatures, which can be used to accept or reject plausible pathways.  
24 We apply these principles to test the classical pathway leading to a hyperinsulinemic  
25 normoglycemic insulin-resistant, or pre-diabetic state using four different sets of  
26 epidemiological data. Currently, a set of indices called HOMA-IR and HOMA- $\beta$  are used to  
27 represent insulin resistance and glucose-stimulated insulin response by  $\beta$  cells respectively.  
28 Our analysis shows that if we assume the HOMA indices to be faithful indicators, the classical  
29 pathway must in turn, be rejected. Among the populations sampled, the classical pathway and  
30 faithfulness of the HOMA indices cannot be simultaneously true. The principles and tools  
31 described here can find wide application in inferring plausible regulatory mechanisms in  
32 homeostatic systems based on epidemiological data.

33

34 **Keywords:** Causal inference; correlation, homeostasis, insulin resistance, regression, steady-  
35 state, type 2 diabetes.

## 36 **Introduction**

37 In the field of biomedicine, the nature of causality, and the use of correlations as an evidence  
38 for causality are much debated (1–6). There have been many attempts to develop sound  
39 methods to address questions of causal inference from correlational data which include Hill  
40 criteria (7), path analysis (8–11) the use of instrumental variables (12), Granger causality (13),  
41 Rubin causal model (14), or additive noise models (15). Hill criteria are a set of common  
42 sense criteria useful to avoid making misguided inferences. Path analysis generally assumes a  
43 direction of causality, and is useful in determining the contributions of different causal  
44 pathways to a process or a resultant variable. It generally assumes directed acyclic paths and  
45 its application to pathways with loops and cycles is difficult. Methods like Granger causality  
46 depend upon the assumption that the cause always precedes effect and that the variables show  
47 some degree of chaos or turbulence, so that there are notable events like sudden peaks in the  
48 variables, which can be tracked using longitudinal data. In evolved systems in which  
49 predictive adaptive responses are possible, the assumption that cause always precedes effect is  
50 questionable. Another class of methods like Propensity Score matching based on the Rubin  
51 causal model works well to estimate the effect of a causal factor, but does not take into  
52 account unobserved factors. The Rubin Causal Model also incorporates the structural  
53 equations model as it includes non-parametric forms as well (16,17). Models like additive  
54 noise can suggest the direction of the arrow of causality between two variables, but they  
55 require the assumption that either A causes B, or B causes A, without any confounding,  
56 looping or circularity (18,19).

57 More specifically, here we look at homeostatic systems which are extremely common in fields  
58 such as physiology. Homeostatic systems have a unique problem for causal inferences. Causal

59 inference can be based on time-series analysis with longitudinal data (19,20). Longitudinal  
60 data are of little use however, if the time taken to reach equilibrium is smaller than the  
61 observational window, or if the system is already in a steady-state. Most homeostatic systems  
62 have negative feedback or some loop structures, because of which methods assuming acyclic  
63 causal paths or freedom from confounding are not applicable.

64 Although the use of correlations to infer causality is doubted, intervention experiments are  
65 generally taken as a convincing evidence of causality. However, causality in steady-state can  
66 be substantially different than causality in a perturbed-state and inferences from a perturbation  
67 experiment may not be applicable to steady-state causality. This necessitates a set of tools to  
68 infer causality in a steady state which is independent of perturbing interventions. We argue in  
69 this paper that it is possible to infer causal relationships among three or more variables from  
70 cross-sectional data in a homeostatic system in which the variables and their relationships are  
71 stable in time.

## 72 **Motivation**

73 Our motivation and the need for this tool came from some debated causal pathways in the  
74 pathophysiology of type 2 diabetes (T2D). According to the classical view, obesity-induced  
75 insulin resistance is primary, and rise in insulin levels is a compensatory response to insulin  
76 resistance, mediated by raised levels of glucose (21,22). This is contested (23), with  
77 increasing evidence suggesting that hyperinsulinemia precedes insulin resistance (24–  
78 28). Therefore the causal pathways between insulin levels, insulin resistance and plasma  
79 glucose are uncertain. There is also evidence of neuronal signals affecting insulin production  
80 on the one hand, and controlling glucose production by the liver partly independent of insulin  
81 on the other. Therefore, the causal relationship between insulin resistance, hyperinsulinemia  
82 and hyperglycemia needs to be re-examined (reviewed by (23)).

83 Elucidation of the causal pathway for a pre-diabetic or diabetic state is critical at the clinical  
84 level because the current approaches to medication are designed assuming one pathway but  
85 have largely failed to cure diabetes. If it is possible to determine causality reliably, it can  
86 potentially change diabetes medicine. It has long been recognized that levels of glucose and  
87 insulin are under homeostatic control, and that fasting is a steady state (29–32). With  
88 substantial data available on fasting levels of glucose and insulin from different populations,  
89 along with many other variables, a tool for inferring causality from a set of inter-correlated  
90 steady-state variables would help understand, and thereby better control T2D.

91 Beyond the specific problem of causality in pre-diabetes, a set of methods that can infer  
92 causality from steady-state data will find a large number of applications, not only in  
93 physiology and disease, but in many other areas of science. Although our investigations began

- 94 with the pathophysiology of diabetes, the emerging principles are generalizable and valuable
- 95 for inferential statistics in general.

## 96 **Methods**

97 We show here that when three inter-correlated variables are considered together with two or  
98 more causal arrows connecting them to make a causal pathway, each of the possible pathways  
99 makes a set of differential predictions by which the pathways can be differentiated from each  
100 other. Our approach to develop a method of inferring causality from cross sectional regression  
101 correlation parameters comprises following steps.

- 102 1. We first list the perceived possible hypothetical causal pathways among three  
103 variables.
- 104 2. For each pathway, we write a set of causal equations arising out of the hypothetical  
105 pathways. Steady-state solutions of these equations lead to a set of four general, and a  
106 few pathway-specific predictions. Each pathway therefore has a unique combination of  
107 such predictions or a prediction signature by which it can be differentiated from other  
108 pathways.
- 109 3. We test, using simulated data generated from assumed causal pathways, the conditions  
110 under which the predictions can be used to accept or reject a pathway reliably.
- 111 4. Based on these results, we suggest ways of handling multivariate data and infer causal  
112 networks among them.
- 113 5. We apply this logic to the specific case of pre-diabetes to examine the pathway  
114 classically thought to give rise to this condition.

115

## 116 **Baseline assumptions and nomenclature**

117 We consider three variables labelled A, B and C. Additional variables if needed to describe a  
118 pathway will be labelled X, Y and so on. All causal relationships represented by a single  
119 arrow are assumed to be linear, and all primary input variables are assumed to be normally  
120 distributed. In a given operation, the slopes of causal pathways are assumed to be constant; the  
121 errors in causal pathways are assumed to be distributed normally, with a mean zero and a  
122 constant standard deviation, and no covariance with each other. We assume that the errors are  
123 caused by variation in individual responses, and that a given individual's response is  
124 consistent in time sufficiently long to reach a steady state. So the errors are randomized over  
125 the population, but for a given individual, they are constant in time. We assume no  
126 measurement errors in the baseline models. Since all our predictions are related to correlation  
127 coefficients and regression slopes, we will ignore the intercepts for the sake of simplicity in  
128 deriving many of the predictions.

129

## 130 **The possible pathways**

131 A variety of cyclic and acyclic pathways can exist in three variables. Fig 1 shows the simple  
132 primary pathways that can exist. More can certainly be constructed by combinations of the  
133 primary pathways. It is also possible to consider permutations of the three variables. For  
134 example, the linear pathway among three variables can itself be written in six different ways.  
135 Here we restrict to the primary pathways assuming a fixed sequence of the three variables  
136 denoted by A, B and C. The principles that we derive from this set of primary pathways can  
137 be extended to more complex pathways.

138



139 **Fig 1. Possible primary causal pathways between three variables.** More complex pathways  
140 can be visualized by combinations of the primary ones.

141

## 142 **Causal equations versus regression equations**

143 Based on hypothesized pathways, we can write specific causal equations for each. The causal  
144 equations are derived from the hypothesized pathway, while the regression equations can be  
145 obtained from the given cross-sectional data using regression and correlation analysis. Our  
146 causal equations are similar to the structural equations of (17). However, they differ in their  
147 interpretation and treatment. In structural equations, the left hand terms are effects and right  
148 hand terms are causes, and the two cannot be algebraically transferred without changing  
149 causal interpretations. In our approach, after finding equilibrium solutions, we can carry out  
150 algebraic operations freely in order to obtain testable predictions. The parameters of the  
151 regression equation are not necessarily identical to those of the causal equations (Table 1). For  
152 example, for a hypothesized pathway  $Y = mX + C$ ,  $m$  is the causal slope, while the regression  
153 slope would be underestimated if there is post-effect variability in  $X$  (33). Such a bias in the  
154 slope is important in making and testing predictions. In the following section, we show that  
155 the parameters of causal equations hold pathway-specific relationships with the parameters of  
156 regression equations based on which, pathway-specific predictions about the regression  
157 correlation parameters can be made.

158

159

160 **Table 1. List of abbreviations used.**

Abbreviation	Term	Remarks
A, B, C	Test variables	We have access from data
X, Y	Unknown variables	Which affect test variables. We do not have access to these from data.
$M_{ij}$	Slopes of regression of i on j	e.g. $M_{ba}$ is calculated as $M_{ba} = \frac{cov(B,A)}{var(A)}$
$K_{ij}$	Intercept of regression of i on j	e.g. $K_{ba}$ is calculated as $K_{ba} = \hat{B} - M_{ba}\hat{A}$
$E_{ij}$	Residuals of regression of i on j	e.g. $E_{ba}$ is calculated as $E_{ba} = B - M_{ba}A - K_{ba}$
$m_1m_2m_3$	Slopes in causal equations	We do not have access to these in data
$e_1e_2e_3$	Error distribution in causal equations, assumed normal with mean zero and standard deviations $sde_1$ etc.	These are post-effect errors of the causal relationships which may get incorporated in pre-effect errors of a subsequent effect
$e_ae_be_c$	Net variability in A, B and C	e.g. $e_a$ is calculated as $e_a = A - \hat{A}$
$k_1k_2$	Intercepts in causal equations	
$d_1d_2$	Degradation /destruction rate constants in causal equations	Especially necessary to use in case of cyclic pathways

161

162 Table 1 legend: Parameters of causal equations are denoted by small letters and those of

163 regression equations by capital letters.

164

165 For ensuring steady-state, we assume that a given variable has a rate of formation/increase and  
166 a rate of degradation/decrease. If the rate of degradation is positively dependent, or the rate of  
167 formation negatively dependent on the standing level, then the variable invariably reaches a  
168 steady state determined by the set of input parameters. Such steady states are characteristic of  
169 homeostatic systems, and this principle is central to our methods.

170

171 For example, in a linear pathway  $A \rightarrow B \rightarrow C$ ,  $\frac{dB}{dt} = m'_1A - d_1B$  and  $\frac{dC}{dt} = m'_2B - d_2C$ . At a  
172 steady-state the net change in any variable is zero. Therefore, the steady-state levels of  $B$  and  
173  $C$  will be  $B = \frac{m'_1}{d_1}A = m_1A$  and  $C = \frac{m'_2}{d_2}B = m_2B$  respectively.

174 In simple cases, we need not explicitly include the rates of degradation in the equations but  
175 directly use parameters  $m_1$  and  $m_2$ . For pathways involving loops and feedbacks the  
176 relationships between variables are more complex and for such cases we will explicitly use  
177 the degradation constants in the causal equations for ensuring steady-states.

178

## 179 **Making predictions from steady-state solutions**

180 We make four general predictions across all pathways and then formulate a null hypothesis  
181 for each. In addition, there are certain pathway specific predictions that will be discussed  
182 along with the description of the corresponding pathway. The four general predictions are:

- 183 1. Whether  $r_{AC}^2$  can be estimated from the product of  $r_{AB}^2$  and  $r_{BC}^2$ .
- 184 2. Whether slope  $M_{ca}$  can be estimated from the product of the slopes  $M_{ba}$  and  $M_{cb}$ .

185 3. Whether the residuals of the regression of  $B$  on  $A$  ( $E_{ba}$ ) are correlated with those of  $C$  on  $B$   
186 ( $E_{cb}$ ): The errors or residuals in a regression are assumed to be random independent  
187 errors. However, we will show below that if there are loops, convergent or confounding  
188 elements in a pathway,  $E_{ba}$  and  $E_{cb}$  do not remain independent. Based on the nature of  
189 dependence between  $E_{ba}$  and  $E_{cb}$ , presence of, and possible nature of the loops and  
190 convergence can be inferred.

191 4. a. Whether correction for  $A$  improves or reduces the correlation of  $B$  with  $C$ , i.e. whether  
192  $r_{E_{ba}C}^2$  is greater or lesser than  $r_{BC}^2$ .

193 b. Whether the extent to which  $r_{E_{ba}C}^2$  is greater or lesser than  $r_{BC}^2$  can be predicted by  $r_{AB}^2$ .

194 We will now state how each of the pathways makes specific predictions. For detailed formal  
195 proofs and derivations refer to S1 Text.

## 196 Making and testing analytical predictions

### 197 Acyclic pathways

#### 198 Linear Pathway (P1)

199 The causal equations for a linear pathway are:

$$A = \text{input} = \dot{A} + e_a$$

$$B = m_1 A + e_1 + k_1$$

$$C = m_2 B + e_2 + k_2$$

200 Where  $e_a, e_1, e_2$  are not correlated.

201 Regression parameters can be derived from the causal equations as follows. Since in

202 regression of  $B$  on  $A$ , the slope =  $\text{cov}(A, B)/\text{var} A$ ,

$$M_{ba} = \frac{\sum e_a e_b}{\sum e_a^2} = \frac{\sum e_a (m_1 e_a + e_1)}{\sum e_a^2} = \frac{m_1 \sum e_a^2}{\sum e_a^2} = m_1$$

$$M_{cb} = \frac{\sum e_c e_b}{\sum e_b^2} = \frac{\sum (m_2 e_b + e_2) e_b}{\sum e_b^2} = \frac{m_2 \sum e_b^2}{\sum e_b^2} = m_2$$

$$M_{ca} = \frac{\sum e_c e_a}{\sum e_a^2} = \frac{\sum (m_2 m_1 e_a + m_2 e_1 + e_2) e_a}{\sum e_a^2} = \frac{m_2 m_1 \sum e_a^2}{\sum e_a^2} = m_2 m_1$$

$$E_{ba} = e_b - M_{ba} e_a = m_1 e_a + e_1 - m_1 e_a = e_1$$

$$E_{cb} = e_c - M_{cb} e_b = m_2 e_b + e_2 - m_2 e_b = e_2$$

$$E_{ca} = e_c - M_{ca} e_a = m_2 e_b + e_2 - m_2 m_1 e_a = m_2 e_1 + e_2$$

203 For linear equations, there is little difference between the causal equations and regression

204 equations (Table 2). The regression equations therefore become

$$B = M_{ba} A + E_{ba} + K_{ba} = m_1 A + e_1 + k_1$$

$$C = M_{ca} A + E_{ca} + K_{ca} = m_2 B + e_2 + k_2$$

$$C = M_{CA}A + E_{CA} + K_{CA} = m_1m_2A + (m_2e_1 + e_2) + (m_2k_1 + k_2)$$

205 Prediction R1: Based on the equations above and Table 2, it can be shown that

206  $r_{AC} - r_{AB}r_{BC} = 0$  (See S1 Text ‘Linear pathway: Prediction R1: Proof 1’ for formal proof).

207 Prediction R2: From Table 2, it is obvious that the slope  $M_{ca}$  can be predicted from the  
 208 product  $M_{cb}M_{ba}$  ;  $M_{ca} - M_{cb}M_{ba} = m_1m_2 - m_2m_1 = 0$ .

209 Prediction R3: From Table 2, as there is no covariance between  $e_1$  and  $e_2$ ,

210  $r_{E_{ba}E_{cb}}^2 = r_{e_1e_2}^2 = 0$

211 Prediction R4: For a linear pathway, it can be shown that

212 (a)  $r_{BC} > r_{E_{ba}C}$  and further, (b)  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} = r_{AB}^2$

213 **Table 2. Relationship between the causal and regression equations for linear pathway.**

Slopes	Errors
$M_{ba} = m_1$	$E_{ba} = e_1$
$M_{cb} = m_2$	$E_{cb} = e_2$
$M_{ca} = m_1 \cdot m_2$	$E_{ca} = m_2e_1 + e_2$

214

215 **Radiating pathway (P2)**

216 The causal equations for this model would be

$$A = m_1B + e_1 + k_1$$

$$B = input = \dot{B} + e_b$$

$$C = m_2B + e_2 + k_2$$

217 Note that the relationship between causal parameters and regression parameters is

218 substantially different in this pathway than the linear pathway (Table 3). For example, the

219 causal slope is  $\frac{1}{m_1}$ , but there is an underestimation of the slope during regression which is

220 predicted exactly by  $r_{AB}^2$ .

221 However, this difference is not detectable from cross-sectional data alone. Therefore, the

222 standard four testable predictions of this pathway remain similar to the linear pathway. We

223 will describe later that differentiating between pathways P1 and P2 is possible using a

224 different strategy.

225 Prediction R1: From Table 3, it can be shown that  $r_{AC} - r_{AB} \cdot r_{BC} = 0$ .

226 Prediction R2: From Table 3; we see slope  $M_{ca}$  can be predicted from the product  $M_{cb}M_{ba}$

227 Prediction R3: From Table 3,  $\text{cov}(e_1, e_2) = 0 \therefore r_{E_{ba}E_{cb}}^2 = r_{e_1e_2}^2 = 0$

228 Prediction R4: As formally shown in S1 Text ('Radiating pathway: Prediction R4: Proof 2'),

229  $r_{E_{ba}C} < r_{BC}$  and  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} = r_{AB}^2$

230 **Table 3. Relationship between the causal and regression equations for radiating pathway**

Slopes	Errors
$M_{ba} = \frac{1}{m_1} r_{AB}^2$	$E_{ba} = e_b (1 - r_{AB}^2) - \frac{1}{m_1} r_{AB}^2 e_1$
$M_{cb} = m_2$	$E_{cb} = e_2$
$M_{ca} = \frac{m_2}{m_1} r_{AB}^2$	$E_{ca} = m_2 e_b (1 - r_{AB}^2) - \frac{m_2}{m_1} r_{AB}^2 e_1 + e_2$

231

232

233 **Convergent pathway (P3)**

234 The causal equations for this model would be

$$A = \text{input} = \dot{A} + e_a$$

235 
$$B = m_1A + m_2C + e_1 + k_1$$

$$C = input = \dot{C} + e_c$$

236 where  $e_a, e_1,$  and  $e_c$  are uncorrelated.

237 Regression parameters derived from the causal equations are given in Table 4.

238 There are two pathway specific predictions for the convergent pathway, shared only by the

239 different cause pathway. Firstly, we expect no correlation between  $A$  and  $C$  from this pathway,

240 unless there are additional external pathways linking the two. The other unique feature of this

241 pathway is that both  $A$  and  $C$  have independent causal influence on  $B$ . As a result, the effect of

242  $A$  adds to the error in the correlation between  $B$  and  $C$  and similarly, the effect of  $C$

243 contributes to the error in the correlation between  $A$  and  $B$ . As a result,  $r_{AB}^2 + r_{BC}^2$  cannot be

244 greater than 1, as shown below:

$$r_{AB}^2 + r_{BC}^2 = \frac{m_1^2 \sum e_a^2}{\sum e_b^2} + \frac{m_2^2 \sum e_c^2}{\sum e_b^2} = \frac{m_1^2 \sum e_a^2 + m_2^2 \sum e_c^2}{\sum e_b^2}$$

245  $\sum e_b^2 = m_1^2 \sum e_a^2 + m_2^2 \sum e_c^2 + \sum e_1^2$ , so

$$r_{AB}^2 + r_{BC}^2 < 1$$

246

247 This prediction is so robust that if  $r_{AB}^2 + r_{BC}^2 > 1$ , the convergent pathway can be rejected

248 right away. Since we assume  $A$  and  $C$  to be independent input variables we assume no

249 correlation between them. However, if they are correlated due to some cause other than this

250 pathway, only then  $r_{AB}^2 + r_{BC}^2$  can be greater than 1.

251

252 Prediction R1: Unlike pathways P1 and P2, for the convergent pathway, it can be seen that

253 
$$r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 < 0.$$



254 Prediction R2: Since the expected slope  $M_{ca}$  is zero, and both  $M_{ba}$  and  $M_{cb}$  are non-zero, their  
 255 product is not a predictor of  $M_{ca}$ .

256  $|M_{ca}| - |M_{cb}M_{ba}| < 0$  as  $|M_{ca}| = 0$

257 Prediction R3: The correlation  $r_{E_{ba}E_{cb}}$  is predicted to have the same sign as  $M_{cb}$ .

258 Prediction R4: It can be shown that (a)  $r_{BC} < r_{E_{ba}C}$  and further, (b)  $\frac{r_{E_{ba}C}^2 - r_{BC}^2}{r_{E_{ba}C}^2} = r_{AB}^2$ .

259 Because this expression differs from R4 (b) of the earlier pathways, we can use a more

260 generalized form for R4 (b) as  $\frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} = r_{AB}^2$

261 **Table 4. Relationship between the causal and regression equations for Convergent**  
 262 **pathway**

Slopes	Errors
$M_{ba} = m_1$	$E_{ba} = e_1 + m_2 e_c$
$M_{cb} = \frac{1}{m_2} r_{BC}^2$	$E_{cb} = e_c (1 - r_{BC}^2) - \frac{1}{m_2} r_{BC}^2 (m_1 e_a + e_1)$
$M_{ca} = 0$	$E_{ca} = e_c$

263

264

265 **Common cause pathway (P4)**

266 The causal equations for this model would be

$$X = input = \dot{X} + e_x$$

$$A = m_1 X + e_1 + k_1$$

$$B = m_2 X + e_2 + k_2$$

$$C = m_3 X + e_3 + k_3$$

267 where  $e_x e_1 e_2 e_3$  are not correlated.

268 It needs to be noted that  $e_1 e_2 e_3$  are important in defining this pathway. If  $e_2$  is negligible the  
 269 pathway approximates to the radiating pathway with  $B$  being the mediator between  $A$  and  $C$ .  
 270 Similarly, at small  $e_1$ ,  $A$  becomes the mediator and at small  $e_3$ ,  $C$  becomes the mediator in a  
 271 radiating pathway. For the way we have defined our predictions,  $e_2$  is the most important error  
 272 term in this pathway (Table 5).

273 One very special feature of this pathway is that qualitatively it is highly symmetric with  
 274 respect to all the three variables  $A$ ,  $B$  and  $C$ . This means that any permutation of them does not  
 275 change the qualitative nature of any prediction. This can be used as a pathway specific  
 276 prediction and a distinct signature for this pathway.

277 Prediction R1: It can be shown that  $r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$  for this pathway.

278 Prediction R2:  $|M_{ca}| - M_{cb} M_{ba} \vee 0$

279 Prediction R3: The sign of the correlation  $r_{E_{ba}E_{cb}}$  is decided by the signs of  $m_1$  and  $m_2$ . When  
 280 both have the same signs  $r_{E_{ba}E_{cb}} = -ve$  and when they have opposing signs  $r_{E_{ba}E_{cb}} = +ve$ . In  
 281 other words the correlation multiplied by the sign of  $M_{ca}$  is always negative.

282 Prediction R4: For this pathway (a)  $r_{E_{ba}C} < r_{BC}$  and (b)  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} r_{AB}^2$

283 **Table 5. Relationship between the causal and regression equations for common cause**  
 284 **pathway**

Slopes	Errors
$M_{ba} = \frac{m_2}{m_1} r_{AX}^2$	$E_{ba} = m_2 e_x (1 - r_{AX}^2) + e_2 - r_{AX}^2 \frac{m_2}{m_1} e_1$
$M_{cb} = \frac{m_3}{m_2} r_{BX}^2$	$E_{cb} = m_3 e_x (1 - r_{BX}^2) + e_3 - r_{BX}^2 \frac{m_3}{m_2} e_2$

$M_{ca} = \frac{m_3}{m_1} r_{AX}^2$	$E_{ca} = m_3 e_x (1 - r_{AX}^2) + e_3 - r_{AX}^2 \frac{m_3}{m_1} e_1$
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285

286 **Single different cause pathway (P5a)**

287 The causal equations for this model would be

$$X = input = \dot{X} + e_x$$

$$A = input = \dot{A} + e_a$$

$$B = m_1 A + m_2 X + e_1 + k_1$$

$$C = m_3 X + e_2 + k_2$$

288 where  $e_x e_a e_1 e_2$  are not correlated. Regression parameters derived are in Table 6.

289 Two specific predictions of this pathway shared only by the convergent pathway (P3) are that

290  $r_{AC}^2 = 0$  and that  $r_{AB}^2 + r_{BC}^2 < 1$ .

291 Prediction R1:  $r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 < 0$

292 Prediction R2:  $|M_{ca}| < M_{cb} M_{ba} \vee$

293 Prediction R3: The correlation  $r_{E_{ba}E_{cb}}$  is predicted to have the same sign as  $M_{cb}$ .

294 Prediction R4: (a)  $r_{BC} > r_{E_{ba}C}$  and further (b)  $\frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} = r_{AB}^2$  is true.

295 **Table 6. Relationship between the causal and regression equations for Single Different**

296 **Cause pathway**

Slopes	Errors
$M_{ba} = m_1$	$E_{ba} = m_2 e_x + e_1$
$M_{cb} = \frac{m_3}{m_2} r_{BX}^2$	$E_{cb} = m_3 e_x + e_2 - m_3 r_{BX}^2 \left( \frac{m_1 e_a}{m_2} + e_x + \frac{e_1}{m_2} \right)$

$M_{ca} = 0$	$E_{ca} = m_3 e_x + e_2 - m_1 e_a$
--------------	------------------------------------

297

298 **Double different causes pathway (P5b)**

299 The causal equations for this model would be

$$X = input = \dot{X} + e_x$$

$$Y = input = \dot{Y} + e_y$$

$$A = m_1 X + e_1 + k_1$$

$$B = m_2 X + m_3 Y + e_2 + k_2$$

$$C = m_3 Y + e_3 + k_3$$

300 where  $e_x e_y e_1 e_2 e_3$  are not correlated. Regression parameters derived are in Table 7.

301 Two specific predictions of this pathway shared only by the convergent pathway (P3) are that

302  $r_{AC}^2 = 0$  and that  $r_{AB}^2 + r_{BC}^2 < 1$ .

303 Prediction R1:  $r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 < 0$

304 Prediction R2:  $|M_{ca}| < M_{cb} M_{ba} \vee$

305 Prediction R3: The correlation  $r_{E_{ba}E_{cb}}$  is predicted to have the same sign as  $M_{cb}$ .

306 Prediction R4: (a)  $r_{BC} > r_{E_{ba}C}$  and further (b)  $\frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} = r_{AB}^2$  is true.

307 It can be seen that all predictions of pathways P5a and P5b are identical and henceforth we

308 will treat both of them in a single group as pathway P5.

309

310 **Table 7. Relationship between the causal and regression equations for Double Different**  
 311 **Cause pathway.**

Slopes	Errors
$M_{ba} = \frac{m_2}{m_1} r_{AX}^2$	$E_{ba} = m_2 e_x (1 - r_{AX}^2) + m_3 e_y + e_2 - \frac{m_2}{m_1} r_{AX}^2 e_1$
$M_{cb} = \frac{m_4}{m_3} r_{BY}^2$	$E_{cb} = m_4 e_y (1 - r_{BY}^2) + e_3 - \frac{m_4}{m_3} r_{BY}^2 (m_2 e_x + e_2)$
$M_{ca} = 0$	$E_{ca} = m_4 e_y + e_3$

312

### 313 Pathways with loops

314 In pathways with loops, since there is a cyclic dependence between the variables we begin  
 315 with differential equations with variable-specific constant rates of disintegration that assure  
 316 steady states. This set of equations is then used to derive equilibrium solutions.

317

### 318 Positive or negative feedback pathway (P6)

319 The causal equations for this model would be

$$A = \text{input}$$

$$\frac{dB}{dt} = m_1 A - d_1 B + e_1 + m_f C + k_1$$

$$\frac{dC}{dt} = m_2 B - d_2 C + e_2 + k_2$$

320 where  $e_a e_1 e_2$  are not correlated, and  $d_1$  and  $d_2$  are positive. For a consistent definition of  
 321 feedback, we assume  $m_1$  and  $m_2$  to always be positive, and that the sign of  $m_f$  decides  
 322 whether it is a positive or negative feedback loop. Feedback loops depend crucially on the

323 relative strength of the forward versus backward causation. If the feedback term, i.e. the effect  
 324 of  $C$  on  $B$  is weak, it approximates to a linear pathway, and if the forward term i.e. effect of  $B$   
 325 on  $C$  is weak, it approximates to a convergent pathway. Therefore the predictions of linear or  
 326 convergent pathways can be expected if the forward or feedback links respectively are weak.  
 327 Additionally however, the negative feedback pathway is associated with a problem of  
 328 definition. If the feedback effect of  $C$  on  $B$  is stronger than the effect of  $B$  on  $C$ , the signs of  
 329 the slope in the causal and regression equations could be opposite, implying that while  $m_2$  is  
 330 positive,  $M_{cb}$  could become negative. This happens when

$$331 \quad M_{cb} = \frac{m_2}{d_2} + \frac{\sum e_1 e_2}{\sum e_b^2} < 0 \text{ i.e. } \left| m_f \frac{\sum e_2^2}{\sum e_b^2} \frac{1}{(d_1 d_2 - m_2 m_f)} \right| > \frac{m_2}{d_2}$$

332 This results in a paradoxical transformation of a causally negative feedback into an apparent  
 333 positive feedback since the sign of the slope and that of the feedback effect is the same.  
 334 Further, when the negative feedback is much stronger than the forward effect, the predictions  
 335 of convergent pathway are more applicable than the predictions of negative feedback  
 336 pathway. At equilibrium where both  $\frac{dB}{dt}$  and  $\frac{dC}{dt} = 0$ , the equilibrium concentrations of  $B$  and  $C$   
 337 are given by

$$B = \frac{m_1 d_2}{(d_1 d_2 - m_2 m_f)} A + \frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f} + \frac{m_f k_2 + k_1 d_2}{d_1 d_2 - m_2 m_f}$$

$$C = \frac{m_2}{d_2} B + \frac{e_2}{d_2} + \frac{k_2}{d_2}$$

338 For simplification we take  $m'_1 = \frac{m_1 d_2}{(d_1 d_2 - m_2 m_f)}$ ,  $e'_1 = \frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f}$  and  $k'_1 = \frac{m_f k_2 + k_1 d_2}{d_1 d_2 - m_2 m_f}$

$$B = m'_1 A + e'_1 + k'_1$$

339 Similarly  $m'_2 = \frac{m_2}{d_2}$ ,  $e'_2 = \frac{e_2}{d_2}$  and  $k'_2 = \frac{k_2}{d_2}$

$$C = m'_2 B + e'_2 + k'_2$$

340 It should be noted that  $e'_1 = \frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f}$  and  $e'_2 = \frac{e_2}{d_2}$  share  $e_2$ , and would therefore co-vary.

341 The sign of this covariance is decided by the sign of  $m_f$ , i.e. whether the feedback is  
342 positive or negative.

343 Regression parameters can be derived from the above as in Table 8.

344 Prediction R1: When the feedback is negative  $r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$

345 The reverse applies for positive feedbacks where  $r_{AC}^2 < r_{AB}^2 \cdot r_{BC}^2$

346 Prediction R2: In negative feedback  $M_{ca} > M_{cb} M_{ba}$  and in positive feedback  $|M_{ca}| <$   
347  $M_{cb} M_{ba}$ .

348 Prediction R3: The sign of this correlation will be decided by the sign of  $m_f$  which is  
349 negative for negative feedback and positive for positive feedback.

350 Prediction R4: (a) It can be shown that for negative feedbacks  $r_{E_{ba}C} < (r_{BC})$ . For positive  
351 feedback the prediction is more conditional. The inequality  $r_{E_{ba}C} < (r_{BC})$  will be true  
352 above a threshold  $r_{BC}$ . For smaller  $r_{BC}$  it is difficult to make a definite prediction. (b) for

353 negative feedback  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} > r_{AB}^2$  and for positive feedback  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} < r_{AB}^2$  is true above

354 a threshold  $r_{BC}$ .

355

356 **Table 8. Relationship between the causal and regression equations for Positive or**  
 357 **negative feedback pathway.**

Slopes	Errors
$M_{ba} = m_1 = \frac{m_1 d_2}{(d_1 d_2 - m_2 m_f)}$	$E_{ba} = e_1 = \frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f}$
$M_{cb} = m_2 + \frac{\sum e_1 e_2}{\sum e_b^2}$  $= \frac{m_2}{d_2} + \frac{\sum e_1 e_2}{\sum e_b^2}$	$E_{cb} = e_2 - \frac{\sum e_1 e_2}{\sum e_b^2} e_b$  $= \frac{e_2}{d_2} - \frac{\text{cov}\left(\frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f}\right)\left(\frac{e_2}{d_2}\right)}{\sum e_b^2} e_b$
$M_{ca} = m_2 m_1$  $= \frac{m_2}{d_2} \frac{m_1 d_2}{(d_1 d_2 - m_2 m_f)}$	$E_{ca} = m_2 e_1 + e_2 = \frac{m_2}{d_2} \frac{d_2 e_1 + m_f e_2}{d_1 d_2 - m_2 m_f} + \frac{e_2}{d_2}$

358

359 **Positive or negative feed-forward pathway (P7)**

360 The causal equations for this model would be

$$A = \text{input} = \dot{A} + e_a$$

$$A = m_1 X + e_1 + k_1$$

$$\frac{dB}{dt} = m_1 A - d_1 B + e_1 + k_1$$

$$\frac{dC}{dt} = m_2 B + m_3 A - d_2 C + e_2 + k_2$$

361 where  $e_a e_1 e_2$  are not correlated. At equilibrium

362  $B = \frac{m_1}{d_1} A + \frac{e_1}{d_1} + \frac{k_1}{d_1} = m_1 A + e_1 + k_1$  taking  $m_1 = \frac{m_1}{d_1}$ ,  $e_1 = \frac{e_1}{d_1}$  and  $k_1 = \frac{k_1}{d_1}$

$$C = \frac{m_1 m_2 + d_1 m_f}{d_1 d_2} A + \left( \frac{m_2}{d_1 d_2} e_1 + \frac{1}{d_2} e_2 \right)$$



$$C = \frac{m_1 m_2 + m_f d_1}{d_2 m_1} B - \frac{m_f}{m_1 d_2} e_1 + \frac{e_2}{d_2} + \frac{k_2}{d_2}$$

363 We will take  $m_2 = \frac{m_1 m_2 + m_f d_1}{d_2 m_1} e_2 = \frac{-m_f}{m_1 d_2} e_1 + \frac{e_2}{d_2}$

364 Note that since  $e_1$  decides both  $e_1$  and  $e_2$ , the covariance between  $e_1$  and  $e_2$  will be

365  $\frac{-m_f}{m_1 d_2} e_1$ , which will be positive when  $m_f$  is negative i.e. for negative feed-forward, and

366 negative when  $m_f$  is positive i.e. positive feed-forward.

367 For simplifying the definition of feed-forward, we assume  $m_1$  and  $m_2$  to be positive, and the

368 sign of  $m_f$  decides whether the feed-forward is positive or negative; a negative feed-forward

369 pathway is once again associated with a problem of definition. If the feed-forward effect of  $A$

370 on  $C$  is stronger than that through  $B$ , and if their signs are opposite, the signs of slope in the

371 causal and regression equations could be opposites. That is, if  $m_f d_1 > m_1 m_2$  then  $M_{cb}$  can be

372 negative although the causal relationship between  $B$  and  $C$  is positive. This results in a

373 paradoxical transformation of a causally negative feed-forward pathway into an effectively

374 positive feed-forward pathway as the product  $M_{ba} M_{cb}$  and  $M_{ca}$  both have the same sign.

375 Note that while all the expressions are the same as in feedback pathways (Table 9), the

376 differences lie in the meanings of  $\sum e'_1 e'_2$ ,  $m'_2$  etc.

377 Prediction R1: For positive feed-forward  $r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$ , and for negative feed-forward

378  $r_{AC}^2 < r_{AB}^2 \cdot r_{BC}^2$ , but under conditions in which the result mimics positive feedback,  $r_{AC}^2 >$

379  $r_{AB}^2 \cdot r_{BC}^2$ . This is the condition when a causally negative feed-forward transforms into an

380 apparent positive feed-forward.

381 Prediction R2: When  $m_f$  is positive,  $M_{cb}M_{ba} < M_{ca}$ . In the case of negative feed-forward

382  $M_{cb}M_{ba} > M_{ca}$  but under conditions in which a negative feed-forward transforms into a

383 positive feed-forward,  $M_{cb}M_{ba} < m_1 m_2 = M_{ca}$ .

384 Prediction R3: For positive feed-forward, we expect a negative correlation, and for negative

385 feed-forward, a positive correlation. Therefore, for positive feed-forward, the correlation

386  $r_{E_{ba}E_{cb}}$  will have the opposite sign of that of  $M_{cb}$ . For a negative feed-forward pathway, under

387 conditions when it transforms into an effective positive feed-forward correlation,  $r_{E_{ba}E_{cb}}$  will

388 have the opposite sign of that of  $M_{cb}$ .

389 Prediction R4: (a) In the case of positive feed-forward  $r_{E_{ba}C} < r_{BC}$  and in the case of negative

390 feed-forward prediction is conditional. Under the conditions when a causally negative feed-

391 forward becomes apparently positive feed-forward, the prediction of positive feed-forward is

392 true. When a negative feed-forward is effective  $r_{E_{ba}C} < r_{BC}$  will be true above a threshold  $r_{AB}$ .

393 (b) For positive feed-forward  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} > r_{AB}^2$ . For negative feed-forward a universal

394 prediction cannot be made. When the effect is that of a positive feedback the prediction of

395 positive feedback is true, otherwise  $\frac{r_{BC}^2 - r_{E_{ba}C}^2}{r_{BC}^2} < r_{AB}^2$ .

396

397 **Table 9. Relationship between the causal and regression equations for Positive or**  
 398 **negative feed-forward pathway.**

Slopes	Errors
$M_{ba} = m_1 = \frac{m_1}{d_1}$	$E_{ba} = e_1 = \frac{e_1}{d_1}$
$M_{cb} = m_2 + \frac{\sum e_1 e_2}{\sum e_b^2}$ $= \frac{m_1 m_2 + m_f d_1}{d_2 m_1} + \frac{\sum e_1 e_2}{\sum e_b^2}$	$E_{cb} = e_2 - \frac{\sum e_1 e_2}{\sum e_b^2} e_b$ $= \frac{-m_f}{m_1 d_2} e_1 + \frac{e_2}{d_2} - \frac{\sum e_1 e_2}{\sum e_b^2} e_b$
$M_{ca} = m_2 m_1 = \frac{m_1 m_2 + m_f d_1}{d_2 m_1} \cdot \frac{m_1}{d_1}$	$E_{ca} = m_2 e_1 + e_2$ $= \frac{m_1 m_2 + m_f d_1}{d_2 m_1} \cdot \frac{e_1}{d_1} + \frac{-m_f}{m_1 d_2} e_1$ $+ \frac{e_2}{d_2}$

399

## 400 **Testing the null hypotheses**

401 Testing in real life data needs to be different for equality and inequality predictions. The  
 402 prediction can serve as the null model wherever equality is predicted, but needs to be treated  
 403 as an alternative hypothesis wherever inequality is predicted. For pathways that predict  
 404 equality, a two-tailed probability is used, and for pathways predicting one-way inequality, a  
 405 one tailed test is used. In the results of simulations reported below, the convention  
 406 consistently followed through Figs 2 and 3 is that if  $H_0$  is true, it is indicated by green, while  
 407  $H_1$  being true is indicated by red and  $H_2$  being true by yellow.

408 R1:  $H_0: r_{AC}^2 = r_{AB}^2 \cdot r_{BC}^2$ ,  $H_1: r_{AC}^2 < r_{AB}^2 \cdot r_{BC}^2$  and  $H_2: r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$ . Since every correlation  
 409 coefficient is associated with a confidence interval, to test the null hypothesis, we check that  
 410 the confidence interval of  $r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2$  includes zero.

411 R2:  $H_0: M_{ca} = M_{cb} M_{ba}$  (green),  $H_1: M_{ca} < M_{cb} M_{ba}$  (red) and  $H_2: M_{ca} > M_{cb} M_{ba}$  (yellow).

412 R3: Since the signs of causal slopes  $m_1$  and  $m_2$  are allowed to be positive or negative in the  
 413 models, the correlation coefficients are multiplied by the sign of the slope  $M_{cb}$ .

414  $H_0: r_{E_{ba}E_{cb}}^2 \cdot \text{sign}(M_{cb}) = 0$  (green),  $H_1: r_{E_{ba}E_{cb}}^2 \cdot \text{sign}(M_{cb}) < 0$  (red) and  $H_2:$

415  $r_{E_{ba}E_{cb}}^2 \cdot \text{sign}(M_{cb}) > 0$  (yellow).

416 R4a:  $H_0: r_{BC}^2 = r_{E_{ba}C}^2$  (green),  $H_1: r_{BC}^2 < r_{E_{ba}C}^2$  (red) and  $H_2: r_{BC}^2 > r_{E_{ba}C}^2$  (yellow).

417 R4b:  $H_0: \frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} = r_{AB}^2$ ,  $H_1: \frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} < r_{AB}^2$ ,  $H_2: \frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)} > r_{AB}^2$ . Since the

418 prediction is about whether  $\frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)}$  is predicted by  $r_{AB}^2$ , in the simulations results

419 reported below we show a scatter plot between the two where good predictions lie along the

420 diagonal and failure of prediction strays away from it.

421

422 **Fig 2. Analytical signatures for each pathway.** Summarizing the analytical signature for

423 each pathway in a color code where green represents acceptance of the null hypotheses  $H_0$ ,

424 and red and yellow represent the acceptance of  $H_1$  and  $H_2$  respectively. Asterisks indicate

425 conditional prediction e.g.  $*r_{BC}^2$  above a threshold,  $**r_{AB}^2$  above a threshold.

426

427 **Fig 3. Simulation results of all pathways against all predictions.** For all acyclic pathways

428 and predictions from R1 to R4a, the result of every simulation run is plotted as a point with

429  $r_{AB}^2$  and  $r_{BC}^2$  on X and Y axes respectively. Green represents null hypothesis true, red for  $H_1$   
430 and yellow for  $H_2$ . The results match very well with the predictions in Fig 2. For converging  
431 and different cause pathways, a pathway specific prediction is that the sum of the two  
432 correlation coefficients never exceeds unity. This is also evident in the simulation results.

433 For feedback and feed-forward pathways predictions from R1 to R4a, the X axis is  $r_{BC}$  and Y  
434 is  $\frac{m_2 s d e_1}{m_f s d e_2}$  which reflects the relative strength of the feedback or feed-forward term as  
435 compared to the forward relation between B and C. The feed effect is strong when the ratio is  
436 close to zero and weak moving away from it. For negative feedback and feed-forward the Y  
437 axis goes from -1 to 0 and for positive feedback and feed-forward from 0 to 1. With negative  
438 feedback and feed-forward, there is an apparent conversion to positive feedback and feed-  
439 forward respectively under a set of conditions. When this happens  $r_{BC}$  becomes negative and  
440 the predictions of positive feedback and feed-forward respectively apply. It can also be seen  
441 that when the ratio is close to zero, predictions of converging pathway hold true.

442 Predictions from R4b for all pathways are shown as scatter plots with  $\frac{|r_{BC}^2 - r_{E_{ba}C}^2|}{\max(r_{BC}^2, r_{E_{ba}C}^2)}$  and  $r_{AB}^2$ .

443 When they are predicted equal, most points lie along the diagonal. Wherever inequality is  
444 predicted, they are on one side of the diagonal.

445 Rejection due to overfitting inequality: For all inequality predictions, overfitting is possible.

446 For example, if we expect that  $r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$ , it is also possible that  $r_{AC}^2$  is too large than what  
447 can be predicted by the pathway under consideration. It is possible to test this either  
448 analytically using equations derived for the corresponding predictions (see S1 Text), or using  
449 simulations only if the parameters of the causal equations are known. If parameter estimates  
450 for the pathways are known from independent empirical sources, it should be possible to test

451 over-fitting inequality. We will illustrate this with real life data later in the section ‘Testing  
452 specific pathways and questions: The case of pre-diabetes.’

453

## 454 **Simulations to test the sensitivity and robustness of** 455 **predictions**

456 We used simulations to test the sensitivity and reliability of the analytical predictions. The  
457 simulations were run using the causal pathway equations for each of the pathways  $P1$  to  $P7$  to  
458 generate data, assuming the errors to be distributed normally around a mean zero. Up to 10000  
459 simulations are run, with each run using randomly drawn parameters and error standard  
460 deviations from a given range (see S1 Text ‘Simulations used for testing the sensitivity and  
461 robustness of the predictions’ for details). The error standard deviation ranges were selected  
462 such that the coefficients of determination were well spread between zero and one. The  
463 generated data were then used to test the predictions of the corresponding pathway.  
464 Simulations used in this section are not based on real life data, and are mainly employed to  
465 test the reliability of the predictions over a range of regression correlation coefficients.

466

## 467 **Agreement between analytical predictions and simulation results**

468 Figs 2 and 3 show that simulations generally follow the analytical predictions quite well, but  
469 with certain limitations. Many of the predictions, particularly when  $H_1$  and  $H_2$  are expected to  
470 be true, work well above threshold values of  $r$ . When either  $r_{AB}^2$  or  $r_{BC}^2$  or both are small, the  
471 null hypothesis fails to get rejected. This threshold of sensitivity can be reduced by increasing  
472 sample size ( $n$ ) (Fig 4). In the case of cyclic pathways, many predictions are conditional as

473 described above, and that is clearly reflected in the simulations. The agreement between  
474 predictions and simulation results is weaker for a few specific pathway-prediction  
475 combinations, in the sense that they work in a narrow range of conditions. This was seen in  
476 case of P5 (different cause) prediction from R4b, and P7 (negative feed-forward) prediction  
477 from R2. The predictions become more reliable at higher  $n$ . This implies that we need to be  
478 conservative in rejecting pathways in such pathway-prediction combinations, particularly  
479 when the correlation coefficients are small. Further, wherever the predictions are themselves  
480 the null models, its rejection will naturally become conservative at low correlations. However  
481 for inequality predictions, where the null hypothesis is equality, failure of rejecting the null  
482 hypothesis should not be taken as rejection of the prediction when correlations are weak.  
483 When we take such a conservative approach, rejection of a prediction can be confidently taken  
484 to mean rejection of a pathway.

485

486 **Fig 4: Effect of  $n$  on the reliability of prediction.** Note that the parameter area over which  
487 the simulation results match the prediction increases with  $n$ .

488

489 It can be seen from Table 10 that each pathway makes a set of predictions by which some  
490 pathways can be differentiated from others. However, some have an identical set of  
491 predictions among the general predictions described so far. Table 10 shows that there are 6  
492 different signatures among 9 primary pathways. Some of the predictions are conditional, and  
493 therefore, it may not always be possible to differentiate between pathways. For example, some  
494 predictions do not work for very small  $r^2$  values, or if feedback is not distinguishable from  
495 linear pathways unless the feedback arrow is sufficiently strong. Such limitations are common

496 to all statistical tools, and they need to be used and interpreted in light of the appropriate  
497 context and conditions.



498 **Table 10. Summary of predictions of all pathways considered.**

Prediction/ Rule → Pathway ↓	R1	R2	R3	R4 a	R4 b	Pathway specific prediction
P1 linear	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} = 0$	$ Mca  =  Mba \cdot Mcb $	$r_{Eba, Ecb} = 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} = r^2_{AB}$	
P2 radiating	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} = 0$	$ Mca  =  Mba \cdot Mcb $	$r_{Eba, Ecb} = 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} = r^2_{AB}$	
P3 convergent	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} < 0$	$ Mca  <  Mba \cdot Mcb $	$r_{Eba, Ecb} > 0$	$r_{Ebc, C} / r_{BC} > 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} = r^2_{AB}$	$r_{AC} = 0,$ $r^2_{AB} + r^2_{BC} < 1$
P4 common cause	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} > 0$	$ Mca  >  Mba \cdot Mcb $	$r_{Eba, Ecb} < 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} > r^2_{AB}$	Symmetry around A, B, C
P5 different cause	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} < 0$	$ Mca  <  Mba \cdot Mcb $	$r_{Eba, Ecb} > 0$	$r_{Ebc, C} / r_{BC} > 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} = r^2_{AB}$	$r_{AC} = 0$ $r^2_{AB} + r^2_{BC} < 1$
P6 feedback negative	$r^2_{AC} - r^2_{AB} \cdot r^2_{BC} > 0$	$ Mca  >  Mba \cdot Mcb $	$r_{Eba, Ecb} < 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r^2_{EbaC} - r^2_{BC} }{\max(r^2_{EbaC}, r^2_{BC})} > r^2_{AB}$	

P6 feedback positive	$r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 < 0$	$ M_{ca}  <  M_{ba} \cdot M_{cb} $	$r_{Eba, Ecb} > 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r_{Eba, C}^2 - r_{BC}^2 }{\max(r_{Eba, C}^2, r_{BC}^2)} \geq r_{AB}^2$	
P7 feed- forward negative	$r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 < 0$	$ M_{ca}  <  M_{ba} \cdot M_{cb} $	$r_{Eba, Ecb} > 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r_{Eba, C}^2 - r_{BC}^2 }{\max(r_{Eba, C}^2, r_{BC}^2)} < r_{AB}^2$	
P7 feed- forward positive	$r_{AC}^2 - r_{AB}^2 \cdot r_{BC}^2 > 0$	$ M_{ca}  >  M_{ba} \cdot M_{cb} $	$r_{Eba, Ecb} < 0$	$r_{Ebc, C} / r_{BC} < 1$	$\frac{ r_{Eba, C}^2 - r_{BC}^2 }{\max(r_{Eba, C}^2, r_{BC}^2)} \geq r_{AB}^2$	

499 Table 10 legend: Note that there are 6 distinct signatures among 9 pathways. Pathways with  
500 identical predictions are shaded with the same colour. There is some redundancy between the  
501 predictions. For example the results of R1, R2 and R3 are tightly correlated. We feel that the  
502 redundancy serves to validate and reinforce the predictions. Also when there are complex  
503 pathways arising through combinations of primary pathways, different predictions show  
504 differential departures from the primary pathway predictions. Therefore all predictions are useful  
505 in spite of some redundancy.

506

## 507 **Sensitivity analysis**

508 The predictions derived and tabulated above are based on the typical assumptions of mainstream  
509 statistics that the input variables are distributed normally and that all causal links are linear.  
510 However, it is important to ask how critical these assumptions are for the predictions to work. In  
511 experimental biology, the input variable is often designed to have uniform intervals and is not  
512 normally distributed. A moderate deviation from linearity is also common in physiological and  
513 other biological systems. If the predictions are too sensitive to these assumptions, they may  
514 prove to be of limited use in real-life. We used Monte-Carlo simulations to assess whether the  
515 predictions work under moderate deviations from these assumptions. When the input variables  
516 were selected randomly from a uniform rather than a Gaussian distribution, all predictions  
517 worked with nearly the same differentiating ability (data not shown). Similarly, when non-  
518 parametric Spearman ranked correlations were used instead of Pearson's correlations, the  
519 correlation related predictions (from R1 and R4) worked similarly (data not shown). This  
520 demonstrates that the tools are not too sensitive to the assumptions of normality of input variable,  
521 linearity of relationships, and parametric or non-parametric nature of correlations.

## 522 **Applications of the method**

### 523 **Accepting or rejecting pathways using real-life data**

524 Two approaches are possible by which the predictions of a pathway can be tested using real-life  
525 data. (i) Based on confidence intervals of correlations and regression slopes: The null hypotheses  
526 for every prediction can be tested using calculation of confidence intervals of regression  
527 correlation parameters. Simulations have shown that except when the underlying correlation  
528 coefficients are too low, this approach can be reliably used to test the predictions. The sensitivity  
529 of predictions depends upon the sample size as well as the position in the parameter space (Fig  
530 3). It is likely therefore that at smaller sample sizes, or at lower  $r_{AB}^2$  or  $r_{BC}^2$ , pathways that predict  
531  $H_1$  or  $H_2$  may fail to get support even if true. On the other hand, at lower  $r_{AB}^2$  or  $r_{BC}^2$  if a pathway  
532 predicts  $H_0$  to be true and the null hypothesis gets rejected, the rejection can be highly reliable.  
533 (ii) Monte-Carlo simulation approach: An alternative approach, which will be more conservative  
534 in rejecting pathways, is the Monte-Carlo approach. Assuming a specific pathway to be true, it is  
535 possible to back calculate the causal equation parameters from the regression correlation  
536 parameters obtained in the data (Tables 2 to 9). For pathways such as negative or positive  
537 feedback, it is not possible to estimate all causal parameters from regression parameters. In such  
538 cases, if empirical estimates of one or a few causal parameters can be obtained, the remaining  
539 causal parameters can be worked back. Using the estimated parameters of causal equations,  
540 Monte-Carlo simulations can be run to obtain the probabilities of getting the observed results.  
541 This approach can be particularly useful when the correlations obtained in the data are weak, and  
542 a conservative inference is preferred.

543

## 544 **Distinguishing between pathways with identical signatures**

545 From the predictions summarized in Table 10, it can be seen that some pathways share prediction  
546 signatures. For example, the linear pathway cannot be distinguished from radiating or  
547 convergent, is indistinguishable from different cause. There are three possible ways of resolving  
548 between pathways with similar signatures: (i) Swapping variables: In the generalized predictions,  
549 common cause pathway and negative feedback pathway have the same predictions. However, the  
550 predictions of the common cause pathway are symmetric around A, B and C, and flipping the  
551 positions of the three variables does not alter the predictions, which is not the case with negative  
552 feedback pathway. (ii) Involving a fourth variable whose causal relationship with at least one of  
553 the triad is already known, or (iii) involving more variables to cross validate pathways. We will  
554 discuss (ii) and (iii) in a different context below.

555

## 556 **Inferring causality between two variables**

557 It is extremely difficult to infer causal relationship between two correlated variables. Although  
558 some solutions have been suggested, their applicability is limited (15, 16). However, it is  
559 possible to infer the causal relationship between two variables if we have data on a third variable  
560 that is correlated with one or both of them with known causality. For example, in men,  
561 testosterone levels and muscle strength are correlated, but the direction of the causal arrow might  
562 be uncertain since testosterone can increase muscle mass, while (34) exercise can also induce a  
563 testosterone response (35,36). The causal relationship can be revealed in cross-sectional data if  
564 we use chronological age as a third variable. Neither testosterone nor muscle mass decides the  
565 chronological age, but age may affect one or both the variables. If age shows significant  
566 correlation with one or both the variables, the predictions from different possible pathways can

567 be tested using the set of predictions as described. By testing these predictions, it should be  
568 possible to determine the causal relationship between muscle mass and testosterone.

569

## 570 **Inferring causal pathways with three variables**

571 To infer causal pathways within three intercorrelated variables, three alternative approaches are  
572 possible. The first approach is to test and resolve between preconceived hypothetical pathways.  
573 It is likely that prior knowledge or some insights into mechanisms allow us to start with a few  
574 plausible alternative pathways. It is possible to perceive more complex pathways by  
575 combinations of the primary pathways that we considered in this paper. For example, a pathway  
576 may contain both feedback and feed-forward elements. Such complex or combinational  
577 pathways can be used to make a set of predictions by the analytical approach described above,  
578 and testing these predictions can resolve between pathways. If we do not have such preconceived  
579 pathways, it would be necessary to consider all possible combinations of pathways between the  
580 three variables, and make differential predictions from each of them. In such cases, we must also  
581 consider permutations of the variables. At the end, it may not be possible to ascertain a single  
582 unique causal pathway since the prediction signatures of some of them may be identical.  
583 Nevertheless, it would still be possible to reject some pathways based on their prediction  
584 signature. In addition, if available, we can involve a fourth variable correlated to one or more of  
585 the three, if there is some pre-existing knowledge about its causal relationship.

586

## 587 **Inferring causal networks with more than three variables**

588 In complex systems, often there are large causal networks. In such networks, combinations of 3  
589 membered motifs can be identified. Out of the possible pathways among three variables some

590 can be rejected using analysis of the three variables. Bringing in a fourth one can provide  
591 additional insights which can be used for cross checking or validating our first set of inferences.  
592 In complex causal networks, there can be many such cross check and validation possibilities. For  
593 large networks algorithms requiring massive computational power may be needed that may pin  
594 down one or a few network structures from the large number of possible ones using  
595 combinations of three member motifs and cross validation facility among the motifs.

596

## 597 **Testing specific pathways and questions: The case of pre-** 598 **diabetes**

599 Apart from some common pathways described above, it is possible that real life problems have  
600 some added complications due to which, the standard solution of testing a fixed set of predictions  
601 may not be sufficient. However, one can apply similar foundational principles to handle such  
602 pathway-specific questions. We will illustrate this using a classical hypothesis that attempts to  
603 explain a human physiological state designated as an insulin resistant, hyperinsulinemic,  
604 normoglycemic, pre-diabetic state. In this state, the plasma levels of fasting insulin (FI) are  
605 raised although fasting glucose (FG) remains normal. The classical interpretation of this state  
606 (Fig 5a) is that a rise in insulin resistance, presumably as a result of obesity, is primary. Insulin  
607 resistance interferes with insulin-induced glucose uptake by muscle and other insulin-dependent  
608 tissues. The reduced uptake raises plasma glucose levels. The raised plasma glucose induces  
609 extra insulin secretion so that plasma insulin levels go up. The extra insulin compensates for  
610 insulin resistance and normalizes glucose level. As a result, the fasting steady state of an insulin-  
611 resistant individual is characterized by raised FI and normal FG. At steady state, insulin

612 resistance is measured by the index, HOMA-IR (defined as  $\frac{Insulin(IU).Glucose(\frac{mg}{dL})}{405}$ ), and the  $\beta$   
613 cell response to glucose, by the index, HOMA  $\beta$  (defined as  $\frac{360.Insulin(IU)}{Glucose(\frac{mg}{dL})-63}$ ). Both the indices are  
614 based on the assumption of a steady state.

615

616 **Fig 5:** Possible pathways between insulin resistance, FG and FI: a) A simplified single feedback  
617 pathway that approximates the negative feedback pathway P6. b) A null model assuming FG and  
618 FI to be independent and HOMA-IR a derived construct. c) An improvised null model with an  
619 external causal factor influencing FG and FI. d) The classically perceived pathway with dual  
620 feedback from glucose and insulin.

621

622 A logical flaw in this interpretation is that, after the glucose levels return to normal, there is no  
623 reason why FI should remain high. Insulin has a short half-life of about 6 minutes (32,37) and  
624 therefore a steady state level can be achieved quite fast; 12 hour fasting should be sufficient to  
625 achieve such a steady state. Therefore, a steady state in which FI is raised but FG remains normal  
626 is not well explained by the classical theory. In spite of this flaw, the main stream thinking in this  
627 field has held on to this interpretation for over four decades, and the indices HOMA-IR and  
628 HOMA-  $\beta$  continue to be commonly used in epidemiological research. Challenges to this causal  
629 interpretation come from the arguments and evidence that rise in FI precedes insulin resistance  
630 (24–28,38). Therefore, there is a need to reexamine the classical causal pathway. We will test  
631 this pathway based on our interpretations of the interrelationships of the regression-correlation  
632 parameters.

633 The pathway in question is more complex than the basic set of pathways P1 to P7. For regulation  
634 of glucose production by the liver and glucose uptake by tissues, there is a dual negative



635 feedback. One feedback is exerted by glucose itself, which enhances tissue uptake and  
636 suppresses liver glucose production. The other feedback operates through insulin, which  
637 facilitates glucose uptake by insulin-dependent tissues and suppresses liver glucose production.  
638 If we ignore the direct glucose feedback and assume that feedback regulation operates only  
639 through insulin, then there is a single negative feedback. Thus the pathway can be simplified to  
640 the negative feedback pathway P6. If we incorporate dual feedback, as the equations show  
641 below, the relationship between insulin resistance and FG is not strictly linear. We could  
642 therefore use the standard set of predictions of a negative feedback model, assuming a single  
643 feedback. Alternatively, we can use the dual feedback model, and apply simulations to make and  
644 test predictions, since empirical estimates for most of the parameters are available from  
645 experiments (see S2 Text).

646 However, the main problem in testing these pathways is that we have no direct measure of  
647 insulin resistance. HOMA-IR and HOMA- $\beta$  are believed to measure insulin resistance and  $\beta$  cell  
648 response respectively, but they are derived from the other two variables, which makes the  
649 problem tricky and circular. We approach the problem using more than one set of assumptions.  
650 (i) First, we test the dual feedback pathway (Fig 5d) assuming HOMA-IR and HOMA- $\beta$  to  
651 faithfully represent insulin resistance and  $\beta$  cell response respectively. (ii) Then, we examine the  
652 constraints laid down by deriving these two parameters from the other two variables. (iii) In  
653 comparison, we use a null model (Fig 5b) in which the classical pathway is not true, there is no  
654 relationship between FG and FI, and HOMA-IR and HOMA- $\beta$  are artificial constructs derived  
655 from the two measured variables and may not reflect any real phenomenon. We also test the  
656 typical convergent model, in which FG and FI determine HOMA-IR. (iv) Using some  
657 oversimplification, ignoring non-linearity of the model and assuming that HOMA-IR and

658 HOMA- $\beta$  are faithful indicators, we test the classical predictions of the negative feedback  
659 pathway (Fig 5a) as described earlier. We use epidemiological data on FG and FI measurements  
660 in four populations to test the classical causal pathway using our approach.

661

## 662 **Data sources**

663 We used four data sets of sample studies by two research groups. All the four sets contain  
664 individuals with and without overt type 2 diabetes. Since we are addressing the prediabetic state  
665 here we have taken the non-diabetic subset of  $n$  individuals from the four samples. (i) Coronary  
666 Risk of Insulin Sensitivity in Indian Subjects (CRISIS) Study, Pune, India (39) (n=558). (ii and  
667 iii) Newcastle Heart Project (NHP), England, (40) which has data on populations of two different  
668 ethnic origins namely European white (n=595) and south Asian (n=413). (iv) Pune Maternal  
669 Nutrition Study (PMNS), Pune, India (41) (n=299). All the predictions are tested independently  
670 in all the four data sets.

671

## 672 **The dual feedback model (Fig 5d)**

673 We assume that the standing plasma glucose level is a result of baseline rate of glucose  
674 production by the liver; suppression of this production as well as muscle glucose pickup which is  
675 proportional to the standing glucose level (direct glucose feedback); the insulin mediated  
676 suppression as well as uptake (insulin mediated feedback) and individual variability. The  
677 standing insulin levels are a result of glucose stimulated insulin secretion on the one hand and  
678 insulin degradation on the other. Thus, the causal equations can be written as

679

$$\frac{dG}{dt} = L - K_1 \cdot G - I_{SENS} \cdot K_2 \cdot I + e_1$$

680

$$\frac{dI}{dt} = K_3 \cdot G - d \cdot I + e_2$$

681

682 Where G and I are plasma levels of glucose and insulin respectively, and FG and FI are the  
683 fasting steady state levels of the same.  $K_1$  denotes the rate constant for negative feedback of  
684 glucose on liver glucose production and tissue glucose uptake;  $K_2$  denotes the rate constant for  
685 insulin-mediated feedback which is proportional to  $I_{SENS}$ , the insulin sensitivity of tissues;  $K_3$  is  
686 the rate constant for glucose-induced insulin release; and  $d$ , the rate of insulin degradation.

687

688 Steady state solution: By assuming the net change to be zero in a steady state, we get

$$FG = \frac{d(L + e_1) - I_{SENS}K_2e_2}{dK_1 + I_{SENS}K_2K_3}$$

$$FI = \frac{K_3FG + e_2}{d}$$

689 It can be seen that the steady state glucose level is a function of insulin resistance ( $IR$ ) which is a  
690 reciprocal of insulin sensitivity. Using the reciprocal, we can write

691

$$FG = \frac{IR \cdot d(L + e_1) - K_2e_2}{IR \cdot dK_1 + K_2K_3}$$

692

693 Thus the relationship between FG and IR is non-linear and follows a saturation curve.

694 Testing the pathways by the four different approaches described above:

695 (i) Assuming classical pathway and faithful indices: The following predictions of the  
696 classical pathway depicted in Fig 5d and modeled above are testable.

697 (a) HOMA-IR, FG and FI should be positively correlated to each other. This  
698 prediction is true in all the four data sets except that the correlations between FG  
699 and FI are weak in all the four data sets. In terms of the variance explained (range  
700 2.6 to 4.9 %) FG and FI are poorly related (Table 11a). The glucose homeostasis  
701 model expects a positive correlation between FG and FI. It is important to realize  
702 this since in the classical thinking, a prediabetic state is characterized by increased  
703 insulin but normal glucose levels. If the compensatory insulin response is mediated  
704 through glucose, it is impossible to have a raised FI without a proportionate rise in  
705 FG. In the pathway predictions, a positive correlation between FG and FI is  
706 expected independent of the feedback loop. However the classical thinking tries to  
707 explain a hyperinsulinemic normoglycemic state achieved through this pathway.  
708 The poor correlation between FG and FI, and a large coefficient of variation in FI  
709 compared to FG indicates that a normoglycemic hyperinsulinemic state may  
710 indeed be achieved, but whether the classical pathway offers a sound explanation  
711 for this state is the question. In an insulin resistant state, the level of FI can  
712 increase by about 10-fold the normal. However, the difference between the lower  
713 and upper limit of glucose in a pre-diabetic state is less than 1.5-fold. To achieve a  
714 tenfold increase in the effect resulting from a 1.5 fold increase in the causal  
715 variable, the slope needs to be of the order of 7 to 8. However in the data, the  
716 regression slope ranges between 0.05 and 0.2 (Table 11). Therefore the variance in  
717 FI is unlikely to be caused by variance in glucose following insulin resistance.

718 Therefore, we need to conclude that most of the variation in FI appears to be  
 719 random error independent of insulin resistance.

720

721 **Table 11. Testing the putative pathway leading to a hyperinsulinemic, normoglycemic,**  
 722 **insulin resistant prediabetic state.**

723 **a. Fasting glucose and fasting insulin in the four datasets**

Data source	FG mean (s.d.)	FI mean (s.d.)
CRISIS (n=558)	94 (18.70)	7.58 (1.82)
KEM (n=299)	86.35 (11.21)	11.84 (10.82)
NHP-SA (n=413)	98.86 (7.37)	11.11 (9.26)
NHP-EU (n=595)	99.90 (6.40)	8.41 (5.29)

724

725 **b. Correlations between FG, FI, HOMA-IR and HOMA beta in the four data sets**

726

	FG:FI		FG:HOMA IR		FG:HOMA beta		FI:HOMA IR		FI:HOMA beta		HOMA IR:HOMA beta	
	r(M)	$\rho$	r	$\rho$	r	$\rho$	r	$\rho$	r	$\rho$	r	$\rho$
CRISIS (n=558)	0.217 (0.055)	0.284	0.525	0.465	-0.209	-0.403	0.931	0.975	0.232	0.698	0.312	0.513
KEM (n=299)	0.198 (0.191)	0.241	0.317	0.352	-0.219	-0.259	0.985	0.990	0.532	0.811	0.633	0.746
NHP-SA (n=413)	0.163 (0.206)	0.282	0.236	0.464	-0.198	-0.145	0.995	0.990	0.763	0.875	0.833	0.810
NHP-EU (n=595)	0.223 (0.185)	0.261	0.309	0.464	-0.064	-0.055	0.994	0.993	0.891	0.933	0.904	0.884

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736 ay (A=Glucose, B=HOMA IR, C=Insulin)

737

Prediction R1		CI of $r^2AC$		CI of $r^2AB.r^2BC$		Accepted/Rejected
		Lower	Upper	Lower	Upper	
$r^2AC$ $<r^2AB.r^2BC$	CRISIS	0.013	0.081	0.180	0.300	Accepted
	KEM	-0.004	0.082	0.035	0.162	Not rejected
	NHP- SA	-0.004	0.057	0.013	0.098	Not rejected
	NHP- EU	0.016	0.084	0.050	0.139	Not rejected
Prediction R2		CI of Mca		CI of Mba.Mcb		Accepted/Rejected
$Mca < Mba.Mcb$		Lower	Upper	Lower	Upper	
	CRISIS	-0.053	0.163	0.104	0.146	Accepted
	KEM	-0.185	0.568	0.195	0.413	Accepted
	NHP- SA	-0.199	0.611	0.1761	0.418	Accepted
	NHP- EU	-0.178	0.548	0.190	0.320	Accepted

<b>Prediction R3</b>  <b>rEbaEcb&gt;0</b>	<b>CI of rEbaEcb</b>				<b>Accepted/Rejected</b>
	<b>Lower</b>		<b>Upper</b>		
CRISIS	0.390		0.521		Accepted
KEM	0.118		0.333		Accepted
NHP- SA	0.081		0.268		Accepted
NHP- EU	0.178		0.328		Accepted
<b>Prediction 4a</b>  <b>rEbaC&gt;rBC</b>	<b>CI rEbaC</b>		<b>CI rBC</b>		<b>Accepted/Rejected</b>
	<b>Lower</b>	<b>Upper</b>	<b>Lower</b>	<b>Upper</b>	
CRISIS	0.953	0.965	0.918	0.941	Accepted
KEM	0.966	0.978	0.982	0.988	Rejected
NHP- SA	0.980961	0.987	0.994	0.995	Rejected
NHP- EU	0.968	0.977	0.993	0.995	Rejected

738

739

**d. Negative feedback pathway (A= HOMA IR, B=Glucose, C=Insulin)**

<b>Prediction R1</b>		<b>CI <math>r^2AC</math></b>		<b>CI of <math>r^2AB.r^2BC</math></b>		<b>Accepted/Rejected</b>
		<b>Lower</b>	<b>Upper</b>	<b>Lower</b>	<b>Upper</b>	
<b><math>r^2AC</math> <math>&gt;r^2AB.r^2BC</math></b>	CRISIS	0.845	0.886	0.003	0.300	Accepted
	KEM	0.9652	0.978	-0.001	0.161	Accepted
	NHP- SA	0.988	0.99	-4.8695E-05	0.006	Accepted
	NHP- EU	0.986	0.990	0.001	0.012	Accepted
<b>Prediction R2</b>		<b>CI Mca</b>		<b>CI of Mba.Mcb</b>		<b>Accepted/Rejected</b>
		<b>Lower</b>	<b>Upper</b>	<b>Lower</b>	<b>Upper</b>	
<b><math>Mca &gt; Mba.Mcb</math></b>	CRISIS	-3.242	9.967	0.222	0.641	Accepted
	KEM	0.083	0.299	0.080	0.587	Rejected
	NHP- SA	0.085	0.326	0.038	0.334	Accepted
	NHP- EU	-3.766	11.581	0.132	0.457	Accepted
<b>Prediction R3</b>		<b>Lower <math>rEbaEcb</math></b>		<b>Upper <math>rEbaEcb</math></b>		<b>Accepted/Rejected</b>
<b><math>rEbaEcb &lt; 0</math></b>	CRISIS	-0.574		-0.452		Accepted



	KEM	-0.414		-0.210		Accepted
	NHP- SA	-0.325		-0.142		Accepted
	NHP- EU	-0.379		-0.234		Accepted
<b>Prediction R4a</b>		<b>CI rEbaC</b>		<b>CI rBC</b>		<b>Accepted/Rejected</b>
<b>rEbaC&lt;rBC</b>		<b>Lower</b>	<b>Upper</b>	<b>Lower</b>	<b>Upper</b>	
	CRISIS	-0.392	-0.243	0.136	0.294	Rejected
	KEM	-0.231	-0.008	0.087	0.305	Rejected
	NHP- SA	-0.168	0.023	0.068	0.256	Rejected
	NHP- EU	-0.167	-0.008	0.145	0.298	Rejected

740

741 Table 11 footnote: The classical pathway leading to a prediabetic state is tested using the  
 742 pathway prediction approach. (a) Means and standard deviations from the four data sets (b)  
 743 correlations obtained from empirical data (c) testing the four predictions for the null model (fig  
 744 5b) (d) testing the four predictions of a simplified classical negative feedback pathway (fig 5a).

745

746 (b) By the steady state equations, the slope of the regression of FI on FG should be  
747  $K_3/d$ . Empirical estimates for both  $K_3$  and  $d$  are available (see S2 Text) and  
748 therefore this prediction can be tested. The empirical estimates are  $K_3 = 0.08$   
749 microIU.mg/min and  $d = 0.15/\text{min}$  respectively, and thereby the expected slope is  
750 0.533. In all the four data sets, the slopes are significantly smaller than the ones  
751 predicted from the empirical estimates (0.05 to 0.2). Thus, apart from a mismatch  
752 between the slope required to cause the observed variation in FI and actual slopes,  
753 the slopes expected from the empirical estimates of parameters and those obtained  
754 in regression also do not match. The latter mismatch by itself may not be sufficient  
755 to reject the pathway since a large measurement error in the X variable, i.e. FG can  
756 lead to underestimation of regression slope, but this explanation implies that a  
757 substantial part of variation in glucose is independent of insulin resistance, and is  
758 akin to random error with respect to the hypothetical causal pathway.

759 (c) HOMA- $\beta$  in our assumption represents  $K_3$ . However  $K_3$  is a constant in our model,  
760 and although it may have some variability in the population, it is uncorrelated with  
761 the three variables of concern. Therefore, HOMA- $\beta$  should show no significant  
762 correlation with FG, FI and HOMA-IR. However, in all the four data sets HOMA-  
763  $\beta$  is significantly positively correlated with FI, but negatively correlated with FG  
764 and positively correlated with HOMA-IR.

765 (d) In a negative feedback pathway  $r_{AC}^2 > r_{AB}^2 \cdot r_{BC}^2$ . Qualitatively this inequality is true  
766 for HOMA-IR, FG and FI in the data. However, simulations show that there is  
767 overfitting of the inequality.  $r_{AC}^2$  in all the four sets of data are substantially higher  
768 than the distribution obtained in the simulations (Fig 6). The correlation between

769 FI and HOMA-IR is far greater than that predicted by the simulations, leading to  
770 an overfitting rejection.

771 Thus if we assume the two HOMA indices to faithfully represent insulin resistance  
772 and beta cell response respectively, then classical pathway needs to be rejected  
773 owing to mismatches with many of its predictions.

774

775 **Fig 6:** Frequency distribution of correlation coefficients in simulations of the classical pathway  
776 leading to prediabetic state: Bars represent the distribution of Pearson's correlations obtained in  
777 10000 runs of simulations. The arrows indicate Pearson's correlations in the four sets of  
778 empirical data. The distribution generated by simulations matches well with the real life  
779 correlations for true IR-FG (grey bars and arrows), FG-FI (red bars and arrows), and the product  
780 of the two (purple bars and arrows). The correlation between true IR and FI is greater than the  
781 product as predicted by the pathway (green bars, we do not have empirical estimates of these  
782 correlations) but the correlation between HOMA-IR and FI (blue bars and arrows) is  
783 substantially greater than the predicted leading to an overfitting rejection. This indicates that  
784 either HOMA-IR as currently calculated is substantially different from true insulin resistance or  
785 the pathway get rejected based on this prediction.

786

787 e. Effects of deriving HOMA-IR and HOMA- $\beta$  from FG and FI: Since HOMA-  
788 IR and HOMA- $\beta$  are not independently measured but derived from FG and FI  
789 measurements, some correlations will follow from the derivations themselves.  
790 The overfitting anomaly observed above can be explained as an artifact  
791 coming out of the calculation of HOMA-IR. However, some other anomalies

792 remain unexplained. Here we are assuming that the classical pathway is true  
793 and therefore, FI is a linear function of FG. If FI is represented as  $m.FG + e$ ,  
794 HOMA-IR will be correlated to  $FG^2$ . Similarly, HOMA- $\beta$  should be  
795 represented as  $m.FG/(FG - 63)+e$ . Under normal physiological range,  $FG >$   
796  $63$  and therefore HOMA- $\beta$  is a decreasing function of FG. As a result both FI  
797 and HOMA-IR should be negatively correlated to HOMA- $\beta$ . Simulations of  
798 the pathway results in a negative correlation between HOMA-IR and HOMA-  
799  $\beta$  as long as the errors are small to moderate. These expectations do not match  
800 the empirical data, in which FI and HOMA-IR have significant positive  
801 correlations with HOMA- $\beta$ . Thus, accepting the classical pathway with some  
802 allowance for artifacts coming out of the derived variables is not sufficient to  
803 explain the empirical correlations.

804 f. Testing the predictions of the null model: If FG and FI are independent of  
805 each other and have some variance around a mean, HOMA-IR is expected to  
806 be positively correlated with both since it is a product of the two. FI should be  
807 positively correlated with HOMA- $\beta$ , but FG should be negatively correlated  
808 with HOMA- $\beta$ . In the HOMA-IR- HOMA- $\beta$  relationship, FI is in the  
809 numerator of both. FG is in the numerator of HOMA-IR but in the  
810 denominator of HOMA- $\beta$ . Nevertheless, since the coefficient of variation of  
811 FI is substantially greater than that of FG, FI is expected to dominate the  
812 relationship and result in a positive correlation between HOMA-IR and  
813 HOMA-  $\beta$ . All these predictions are observed in the data. The mismatch of the  
814 null model with the data is that it assumes FG and FI to be independent and

815 uncorrelated. In all the four sets of data, there is a significant but weak  
816 correlation between the two. The  $r^2$  ranges from 0.026 to 0.049, and thus not  
817 more than 5 % of variance is explained by the relationship.

818 If we consider FG and FI to be independent and HOMA-IR and HOMA- $\beta$  derived  
819 from them, they constitute a convergent pathway that can be tested by the pathway  
820 predictions. It can be seen that predictions from R1, R2 and R3 of the convergent  
821 pathway are accepted. However, prediction from R4 and the pathway-specific  
822 prediction are rejected (Table 11b). These rejections can be explained by the positive  
823 correlation between FG and FI. We have seen in the analysis of pathway P3 that if A  
824 and C are positively correlated then  $r_{AB}^2 + r_{BC}^2$  can be greater than 1. The rejection of  
825 the null model suggests that there is a relationship between FG and FI, but does not  
826 indicate whether it comes from the classical pathway or through any other source as in  
827 Fig 5c.

828

829 g. If we ignore the non-linearity of the model and assume HOMA-IR and  
830 HOMA- $\beta$  to faithfully represent insulin resistance and beta cell response, we  
831 may use the 4 predictions of the standard negative feedback pathway. It is  
832 seen that predictions from R1, R2 and R3 are accepted but the outcome of  
833 prediction from R4 is complex (Table 11c). After correcting for the effect of  
834 HOMA-IR, the FG-FI correlation should be weakened and that difference  
835 would be predicted by the correlation between HOMA-IR and FG. However  
836 instead of weakening, the FG-FI correlation becomes negative. Because of the  
837 strong positive correlation between HOMA-IR and FI, correcting for HOMA-

838 IR subtracts from every value of FG, a quantity proportionate to FI, leading to  
839 a negative correlation between the corrected FG and FI. Additionally,  
840 simulations of the pathway show that if true insulin resistance is assumed to  
841 be correlated to FG by the same order as HOMA-IR, the correlation of true  
842 insulin resistance with FI is far less than that between HOMA-IR and FI (a  
843 result similar to Fig 6 and therefore not separately shown). Thus, there is an  
844 overfitting rejection of prediction from R1 as well. Rejection of this pathway  
845 based on two predictions is due to the unrealistically strong correlation  
846 between HOMA-IR and FI, which comes from the calculation of HOMA-IR  
847 itself.

848  
849 We need to examine now to what extent HOMA-IR faithfully represents the true insulin  
850 resistance because if it does, the classical pathway certainly gets rejected. This can be examined  
851 in the simulations since the true insulin resistance is an input variable and HOMA-IR can be  
852 calculated as an outcome of the simulations. We see that HOMA-IR is correlated well with true  
853 insulin resistance when both  $e_1$  and  $e_2$  are close to zero (Fig 7). As the errors increase, the  
854 correlation becomes weaker. In the data, we do not have access to  $e_1$  and  $e_2$  but since the FG-FI  
855 correlation also becomes weaker with  $e_2$ , we can look at how HOMA-IR represents true insulin  
856 resistance at different levels of FG-FI correlation. It can be seen that as FG-FI correlation  
857 becomes weak, HOMA-IR correlation with the true insulin resistance also becomes weak (Fig  
858 7), but this relationship is affected by  $e_1$ . When  $e_1$  is close to zero, i.e. almost all the variation in  
859 FG is explained by variation in true insulin resistance, even at low FG-FI correlation, HOMA-IR  
860 represents true insulin resistance fairly well, their correlation ranging between 0.58 and 0.7. On

861 the other hand if we assume  $e_I$  to be large i.e. most of the variation in FG is due to random error  
862 or effects independent of insulin action, HOMA-IR is poorly correlated with true insulin  
863 resistance, the correlation coefficient declining to 0.2. Thus if we assume that the variance in FG  
864 is mainly caused by insulin resistance, then we have to reject the classical pathway leading to  
865 hyperinsulinemia. Alternatively, it is likely that the classical pathway is true but HOMA-IR does  
866 not represent true insulin resistance and that most of the variation in FG is not caused by insulin  
867 resistance. The substantially lower than expected slope of the FG-FI regression suggests large  
868 random errors in FG making the second interpretation more likely. In any case the classical  
869 pathway and the faithfulness of HOMA indices cannot be simultaneously true, and we have to  
870 reject at least one of them.

871 Results of the four alternative approaches to analyze the classical pathway and the null model  
872 converge on the inference that the null model is rejected on the basis of a weak but significant  
873 correlation between FG and FI. But the weak correlation in FG and FI is not adequately  
874 explained by the classical pathway owing to multiple mismatches and rejection of many of its  
875 predictions. The pathway rejection may be partially saved by saying that HOMA-IR and HOMA-  
876  $\beta$  are not good indicators of insulin resistance and beta cell response and that we do not have  
877 access to true insulin resistance to test the predictions. However the FG-FI regression slope also  
878 has a large mismatch with expectations derived from the variance in FI as well as from empirical  
879 estimates of  $K_3$  and  $d$ . Therefore, it seems more likely that FG and FI are related by causes other  
880 than the classical pathway, and HOMA-IR and HOMA- $\beta$  are derived artificial constructs that do  
881 not represent any real life phenomena.

882 There are a number of real life interpretations of the pathway in Fig 5c. Autonomic inputs from  
883 the nervous system are known to affect both insulin secretion and liver glucose production,

884 which might be represented by the common cause arrows of Fig 5c. Alternatively, a small error  
885 in data collection can also result in the observed FG-FI correlation. The fasting sampling is done  
886 by instructing the subjects to have no food or drink after the last evening meal. However, if even  
887 a small proportion of subjects happen to consume bed tea an hour or two before sampling, their  
888 glucose as well as insulin levels could be slightly elevated simultaneously. This can result in a  
889 weak positive correlation between FG and FI in the data. Since the fasting state is based on the  
890 honesty of the subjects and there is no independent monitoring, this source of error cannot be  
891 ignored. Thus, there are more than one possible reasons for external factors causing a weak  
892 correlation between FG and FI, and the correlation is not sufficient to support the classical  
893 pathway in the presence of multiple other mismatches.

894

895 **Fig 7:** The reliability of HOMA-IR as an index of true insulin resistance: The pathway  
896 simulations were carried out at a standard deviation of  $e_I=1$  (blue dots) and 10 (red dots). The  
897 FG-FI correlation weakens with increase in  $e_2$  which also affects the correlation between true IR  
898 and HOMA-IR. It can be seen that HOMA-IR is a reliable indicator of insulin resistance when  $e_I$   
899 is small, but at large  $e_I$  it is a poor indicator as suggested by a weak correlation with true insulin  
900 resistance.

901

902 It should be noted that the correlational patterns in the four data sets used are remarkably similar  
903 although they come from populations differing in location, ethnicity and culture. It would be  
904 important to see whether the same correlational patterns are observed in other populations as  
905 well, but we can be confident in rejecting the classical pathway at least in the populations  
906 sampled.



907

## 908 **What can type 2 diabetes research gain from our analysis**

909 Putting the results together, it can safely be concluded that HOMA-IR and HOMA- $\beta$  appear to be  
910 artificial constructs and reflect very marginally, if at all, the true insulin resistance and  $\beta$  cell  
911 response in a steady state. Until our approach for testing a causal pathway was available, there  
912 was no way to test whether HOMA-IR and HOMA- $\beta$  truly represent the intended states of the  
913 system. Because of this limitation, insulin resistance was a circular argument. The inability of  
914 insulin to regulate glucose was assumed to be because of insulin resistance, but insulin resistance  
915 was measured as the inability of insulin to regulate glucose. This circularity had made the  
916 hypothesis of insulin resistance and compensatory hyperinsulinemia non-falsifiable. Our  
917 approach to pathway predictions breaks the circularity, and makes it possible to test whether the  
918 insulin resistance and glucose-mediated compensatory hyperinsulinemia hypothesis is supported  
919 by epidemiological data. At least in the populations tested, many serious anomalies in the  
920 classical pathway leading to a hyperinsulinemic, normoglycemic, insulin resistance prediabetic  
921 state are exposed. Conservatively we can argue that since HOMA-IR and HOMA- $\beta$  do not  
922 represent insulin resistance and  $\beta$  cell response faithfully, and we do not have alternative  
923 measures for them, it may not be possible to clearly reject the classical pathway, but the data  
924 clearly show that even if true, the classical pathway has a very limited role in deciding FG, FI  
925 and their inter-relationship. Both the steady state levels have a large component of error or  
926 effects independent of the pathway under consideration. Although our analysis is restricted to the  
927 prediabetic state at present, establishing causality in the prediabetic state has implications for the  
928 over diabetic state. According to classical thinking, a failure of compensatory insulin response  
929 leads to diabetic hyperglycemia. Since our analysis questions the compensatory insulin response

930 itself, the pathway leading to hyperglycemia is also in question. Even a highly conservative  
931 inference would demand rethinking of the causal process leading to diabetes.

932 Doubts about the classical pathway are raised independently by experiments using insulin  
933 receptor knockouts or insulin suppression. Muscle-specific insulin receptor knockouts show  
934 altered glucose tolerance curves but normal fasting insulin (42). Insulin suppression experiments  
935 do not result in elevated fasting glucose (43–45). Inactivation of insulin degrading enzyme raises  
936 steady state insulin levels but does not decrease glucose levels (46,47). These experiments have  
937 already challenged the classical pathway. Thus, there are multiple reasons to doubt the classical  
938 pathway. On the other hand, a number of factors other than the mutual effects of FG and FI are  
939 known to affect insulin response as well as glucose homeostasis (48–58), but these factors have  
940 not been integrated into the mainstream glucose homeostasis models. We do not intend here to  
941 test all possible alternative pathways deciding FG and FI. But our study lays down a set of  
942 methods by which this can be done, once the pathway hypotheses are clearly spelt out and the  
943 causal variables are measured. An important contribution of our methods is that physiological  
944 causal pathways can be evaluated based on epidemiological data, which is potentially a very  
945 important tool in understanding complex disorders. Experimental biology reveals what *can*  
946 happen in a system, but what *does* happen at the population level is better revealed by  
947 epidemiological data. Therefore, discerning causal signatures of pathophysiological pathways in  
948 epidemiological data is likely to be an important breakthrough.

949

## 950 **Conclusions**

951 Making causal inferences from cross sectional correlational data is a long-standing problem. A  
952 correlation between two variables does not give reliable information about causal relations.  
953 However, we demonstrate here, in the context of steady state homeostatic systems, using  
954 mathematical proofs as well as simulations from causal pathways that, in a set of three or more  
955 correlated variables, it is possible to test causal hypotheses based on the interrelationships of  
956 regression-correlation parameters. This is potentially a highly valuable tool in making causal  
957 inferences from cross sectional data in several fields.

958 Using this set of principles, we tested the classical causal assumption behind the  
959 hyperinsulinemic, normoglycemic, insulin resistant or pre-diabetic state. The analysis showed  
960 that this causal pathway and the measures of insulin resistance and insulin response were not  
961 supported by epidemiological data. Thus, the objections raised recently to the classical causal  
962 pathway are validated and alternative causal pathways that already have substantial experimental  
963 evidence need to be integrated in the mainstream clinical thinking.

964

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971

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1160

1161 **Supporting Information**

1162 **S1 Text.** Deriving predictions for regression parameters based on causal equations and  
1163 calculation of causal parameters for simulations.

1164

1165 **S2 Text.** Empirical estimates for parameters used in simulations.

## Acyclic pathways

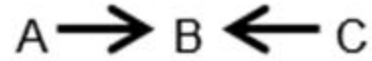
P1: Linear



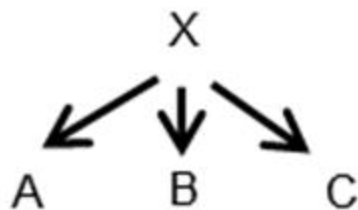
P2: Radiating



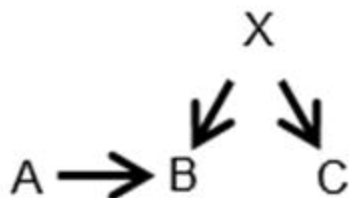
P3: Convergent



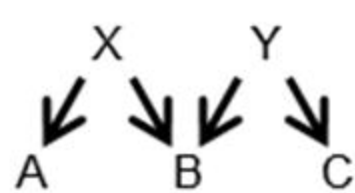
P4: Common cause



P5a: Single different cause

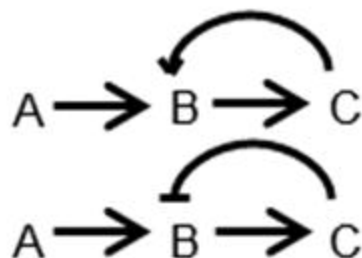


P5b: Double different causes

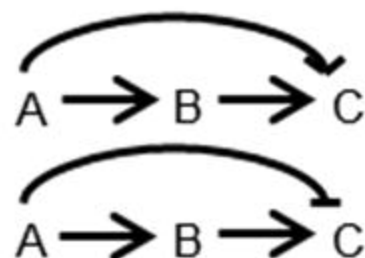


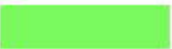
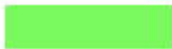











































## Cyclic pathways

P6: Positive or negative feedback

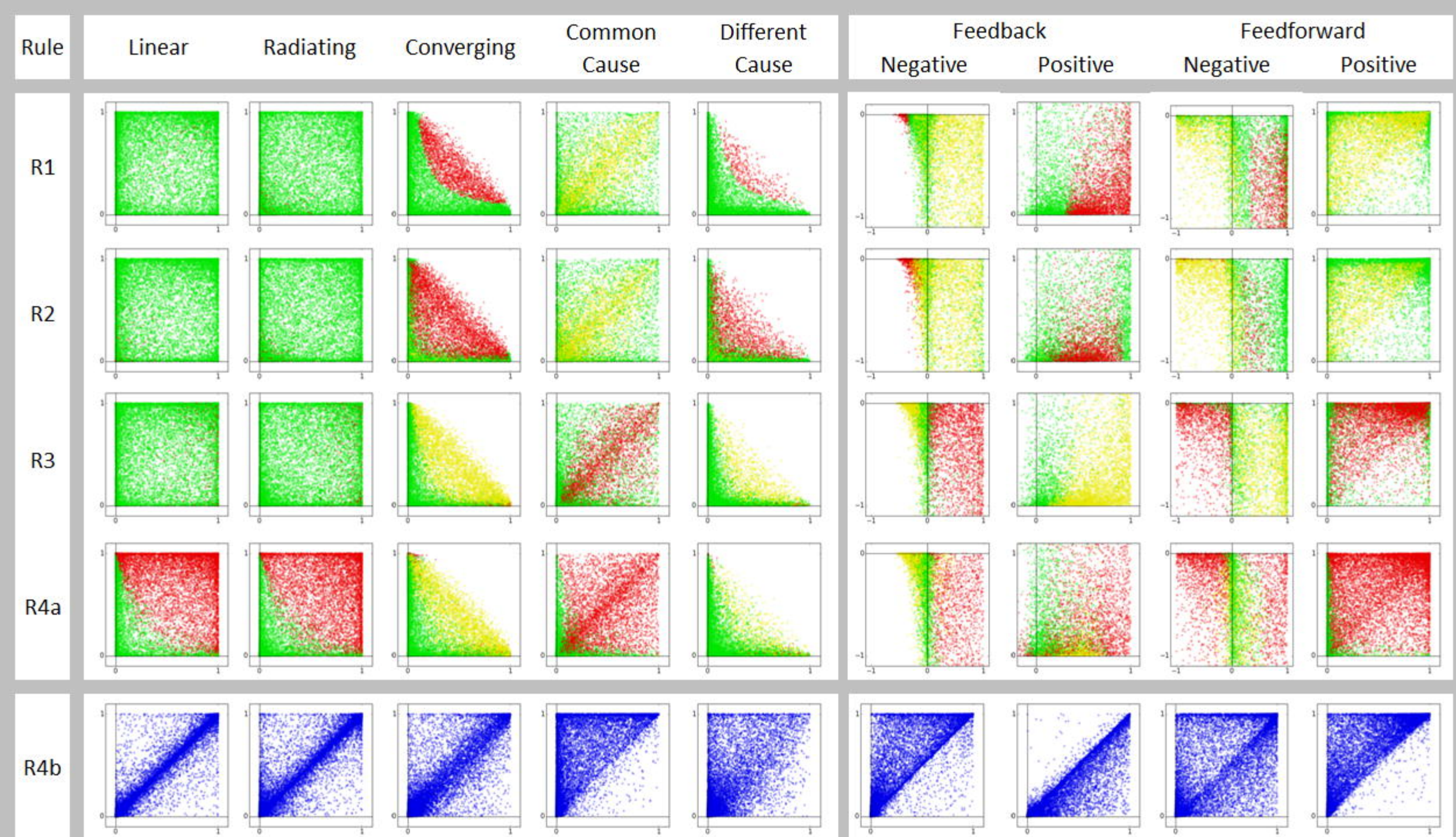


P7: Positive or negative feedforward



Rule	Linear	Radiating	Converging	Common Cause	Different Cause	Feedback		Feedforward	
						-	+	-	+
R1									
R2									
R3									
R4a							 *	 **	
R4b									



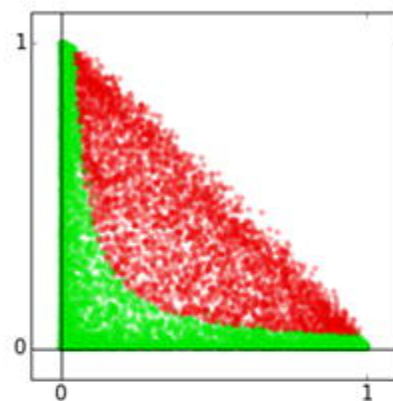
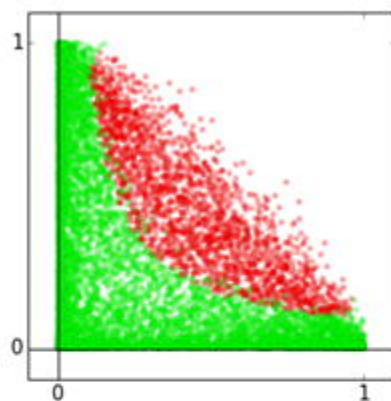


Sample Size

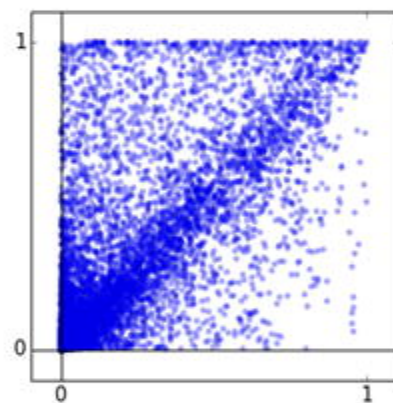
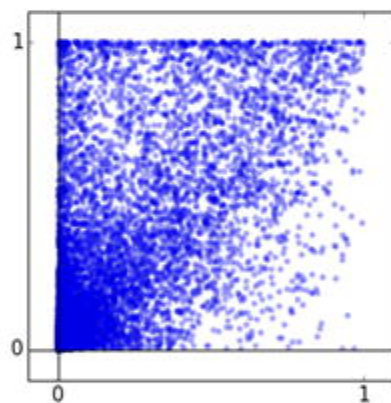
N = 100

N=500

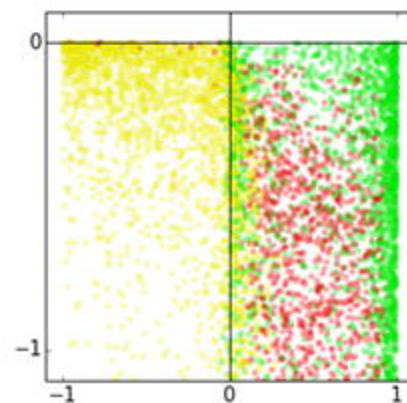
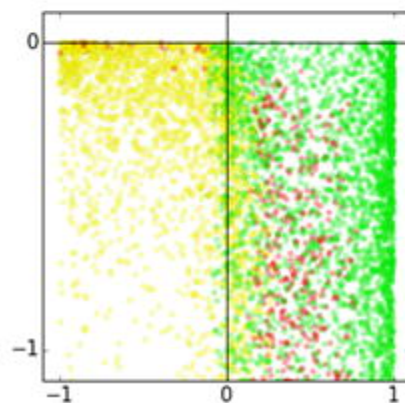
Converging, R1




Different Cause, R4b



Feedforward -ve, R2



a. IR (HOMA IR)  $\rightarrow$  Glucose  $\rightarrow$  Insulin



b. Glucose  $\rightarrow$  HOMA IR  $\leftarrow$  Insulin

c. 
$$\begin{array}{c} X \\ \swarrow \quad \searrow \\ \text{Glucose} \rightarrow \text{HOMA IR} \leftarrow \text{Insulin} \end{array}$$

d. Liver  $\rightleftharpoons$  Glucose  $\rightarrow$  Insulin

