Nuisance Effects and the Limitations of Nuisance Regression in Dynamic Functional Connectivity fMRI

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Abstract

In resting-state fMRI, dynamic functional connectivity (DFC) measures are used to characterize temporal changes in the brain's intrinsic functional connectivity. A widely used approach for DFC estimation is the computation of the sliding window correlation between blood oxygenation level dependent (BOLD) signals from different brain regions. Although the source of temporal fluctuations in DFC estimates remains largely unknown, there is growing evidence that they may reflect dynamic shifts between functional brain networks. At the same time, recent findings suggest that DFC estimates might be prone to the influence of nuisance factors such as the physiological modulation of the BOLD signal. Therefore, nuisance regression is used in many DFC studies to regress out the effects of nuisance terms prior to the computation of DFC estimates. In this work we examined the relationship between DFC estimates and nuisance factors. We found that DFC estimates were significantly correlated with temporal fluctuations in the magnitude (norm) of various nuisance regressors, with significant correlations observed in the majority (76%) of the cases examined. Significant correlations between the DFC estimates and nuisance regressor norms were found even when the underlying correlations between the nuisance and fMRI time courses were relatively small. We then show that nuisance regression does not eliminate the relationship between DFC estimates and nuisance norms, with significant correlations observed in the majority (71%) of the cases examined after nuisance regression. We present theoretical bounds on the difference between DFC estimates obtained before and after nuisance regression and relate these bounds to limitations in the

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efficacy of nuisance regression with regards to DFC estimates.

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1. Introduction

In resting-state functional magnetic resonance imaging (fMRI), the correlation between the BOLD time courses from different brain regions is used to estimate the functional connectivity (FC) of the brain in the absence of an explicit task. This approach has revealed a number of resting-state networks, where each network consists of brain regions that exhibit a high degree of mutual correlation (Fox et al., 2005). For the most part, FC estimates have assumed temporal stationarity of the underlying BOLD time courses and are obtained by computing correlations over the entire scan duration. As the resulting estimates represent a *temporally averaged* measure of brain connectivity over a typical scan duration of 5-10 minutes, they can miss important dynamic temporal changes in FC (Allen et al., 2014; Preti et al., 2017; Hutchison et al., 2013).

An increasing number of studies have focused on the dynamics of functional brain connectivity by considering dynamic FC (DFC) measures that are computed over time scales typically on the order of tens of seconds and thus much shorter than the scan duration (Allen et al., 2014; Preti et al., 2017; Hutchison et al., 2013). Approaches for estimating DFC include sliding window correlation method (Hutchison et al., 2013; Calhoun et al., 2014), time-frequency methods such as wavelet transform coherence (Chang and Glover, 2010; Yaesoubi et al., 2015), and probabilistic inference methods such as hidden Markov modeling (Vidaurre et al., 2017). To date, the sliding window correlation approach is the most widely used DFC estimation method (Hutchison et al., 2013). In this approach, the correlations between BOLD time courses from different brain regions are computed over sliding windows with durations typically ranging from 30-60 seconds (Preti et al., 2017).

Regardless of the analysis method, non-neural processes that affect the BOLD time series can also contaminate the DFC estimates (Murphy et al., 2013; Preti et al., 2017; Hutchison et al., 2013). These confounds are often referred to as *nuisance terms* and include the effects of motion, cardiac and respiratory activity, and fluctuations in arterial CO_2 concentration. Hutchison et al. (2013) noted that non-neuronal sources that introduce spatial correlations into the time series can also give rise to spurious dynamics in FC measures. Recently, Nikolaou et al. (2016) reported that temporal fluctuations in network degree were related to fluctuations in both heart rate and end-tidal CO_2. Glomb et al. (2017) found that

temporal fluctuations in FC were related to temporal variations in a global measure of average BOLD signal magnitude.

A common step in most fMRI analyses is the use of nuisance regression to minimize the contributions of nuisance terms in BOLD time courses (Liu, 2016). Nuisance regressors include cardiac and respiratory activity derived time courses (Birn et al., 2008; Chang et al., 2009), head motion parameters, Legendre polynomials to model scanner drift, signals from white-matter (WM) and cerebrospinal fluid (CSF) regions, and a whole brain global signal (GS) (Liu et al., 2017). Although nuisance regression is widely employed prior to the computation of DFC estimates (Murphy et al., 2013), efforts to examine its efficacy have been limited. Hutchison et al. (2013) noted that residual nuisance effects "inevitably remain" in the BOLD time series and must be considered in the interpretation of DFC measures. In support of this view, Nikolaou et al. (2016) found that nuisance regression diminished but did not completely remove the relationship between network degree and measures of cardiac and respiratory activity. On the other hand, Xu et al. (2018) reported that global signal regression (GSR) had a spatially heterogeneous impact on DFC estimates, but did not assess whether GSR removed GS contributions from the DFC estimates.

In this paper, we take a closer look at (1) the role of nuisance terms in DFC estimates and (2) the efficacy of nuisance regression for DFC studies. In particular, we use two independent datasets to examine the relationship between sliding window correlation-based DFC estimates and the norms of various nuisance regressors (WM and CSF signals, GS, cardiac and respiratory measurements, and head motion). We then assess the effect of nuisance regression on this relation, considering both regression applied to the entire scan and regression applied on a sliding window basis. To interpret the empirical findings, we derive mathematical expressions to describe the effect of regression on DFC estimates and compare the experimental results with the theoretical predictions. Preliminary versions of this work have been presented in (Nalci et al., 2017a; Nalci and Liu, 2018).

2. Methods

2.1. Datasets

In this work, we analyzed two datasets in order to show the generality of our results and to experiment with different types of nuisance measurements. First, to understand the effect of nuisance measurements derived directly from the MRI images such as the GS, WM and CSF, and head motion (HM) time courses on the DFC estimates, we used a publicly available dataset originally analyzed by Fox et al. (2007), which

we will refer to as the BS002 dataset. Second, to understand the effect of physiological measurements such as changes in the respiration and cardiac rate on the DFC estimates, we used the dataset analyzed by Wong et al. (2012), which we will refer to as the CFMRI dataset.

BS002 data were acquired from 17 young adults (9 females) using a 3 T Siemens Allegra MR scanner. Each subject underwent 4 BOLD-EPI fixation runs (32 slices, TR=2.16 s, TE=25 ms, 4×4×4 mm), each lasting 7 minutes (194 frames). The subjects were instructed to look at a cross-hair and asked to remain still and awake. High-resolution T1-weighted anatomical images were acquired for the purpose of anatomical registration (TR=2.1 s, TE=3.93 ms, flip angle=7 deg, 1×1×1.25 mm).

CFMRI data were acquired from 10 healthy volunteers (4 males and 6 females) using a 3 Tesla GE Discovery MR750 system. Each subject underwent four separate resting-state scans (5 minutes per scan with either their eyes open or closed). High resolution anatomical data were collected using a magnetization prepared 3D fast spoiled gradient (FSPGR) sequence (TI=600 ms, TE=3.1 ms, flip angle = 8 degrees, slice thickness = 1 mm, FOV = 25.6 cm, matrix size = 256×256×176). Whole brain BOLD data were acquired over thirty axial slices using an echo planar imaging (EPI) sequence (flip angle = 70 degrees, slice thickness = 4 mm, slice gap = 1 mm, FOV = 24 cm, TE = 30 ms, TR = 1.8 s, matrix size = 64×64). Cardiac pulse and respiratory data were monitored using a pulse oximeter (InVivo) and a respiratory effort transducer (BIOPAC), respectively. The pulse oximeter was placed on each subject's index finger while the respiratory effort belt was placed around the subject's abdomen. Physiological data were sampled at 40 Hz using a multi-channel data acquisition board (National Instruments).

2.2. Preprocessing steps for the BS002 dataset

Standard pre-processing steps were conducted with the AFNI software package (Cox, 1996). The initial 9 frames from each EPI run were discarded to minimize longitudinal relaxation effects. Images were then slice-time corrected and co-registered, and the 6 head motion parameter time series were retained. The resultant images were converted to coordinates of Talairach and Tournoux (TT), resampled to 3 mm cubic voxels, and spatially smoothed using a 6 mm full-width-at-half-maximum isotropic Gaussian kernel. The 1st and 2nd order Legendre polynomials (a constant term to model the temporal mean and a linear trend) were projected out from each voxel's time course. Each voxel time series was then converted into a percent change BOLD time series through demeaning and division by its mean value.

To analyze the effect of nuisance regression on DFC estimates, we used both (1) full linear regression in which regression is performed over the whole scan duration and (2) block regression (also referred to

as sliding window regression) in which regression is performed for each window separately (more details on block regression are provided in Section 2.4). For each type of regression, we performed a separate regression on the data using one of the following types of regressor: (1) the set of all 6 head motion parameters, (2) nuisance signals from WM and CSF regions, and (3) the global signal (GS), which was calculated as the average of the percent change time series across all voxels within the brain.

For DFC analysis, we used seed signals derived from the posterior cingulate cortex (PCC), intraparietal sulcus (IPS), and motor network (MOT). These seed signals were obtained by averaging time series selected over spheres of radius 6 mm (2 voxels) centered about their corresponding TT coordinates (He and Liu, 2012). The sphere centers were obtained by converting the MNI coordinates from Van Dijk et al. (2010) to TT coordinates (Lacadie et al., 2008). For the PCC, left MOT, and right MOT seeds we used the coordinates [0,-51,26], [-36,-22,52] and [37,-21,52], respectively. A combined MOT seed was obtained by using the left and right MOT coordinates to define two spheres and by merging them. For the IPS and FEF seeds, we used the coordinates [27,-58,49] and [24,-13,51] from (Fox et al., 2006). Finally, for the WM and CSF nuisance signals, we defined the sphere centers as [12,-36,27] and [9,-9, 15], respectively.

2.3. Preprocessing steps for the CFMRI dataset

We used FSL and AFNI packages to preprocess the CFMRI dataset (Woolrich et al., 2009; Smith et al., 2004). First, high-resolution anatomical data were skull stripped and segmentation was applied to estimate WM, gray matter, and CSF partial volume fractions. Images were then slice-time corrected and co-registered and the 6 head motion parameter time series were retained. The anatomical data were then aligned to the functionals. Each subject's native space data were transferred to TT coordinates and spatially smoothed using a 6 mm full-width-at-half-maximum isotropic Gaussian kernel. A binary brain mask was created for each subject using the transferred data and was eroded by 2 voxels along the brain edges to eliminate possible inclusion of non-brain areas (Rack-Gomer and Liu, 2012). The first 6 frames (10.8 seconds) from each functional data were discarded to minimize longitudinal relaxation effects. For each run, the 1st and 2nd order Legendre polynomials and the 6 head motion parameters were projected out to obtain the 'baseline' CFMRI data.

We performed RVHRCOR by simultaneously projecting out the physiological nuisance regressors derived from cardiac and respiratory signals using both full linear regression and block regression (Chang et al., 2009; Birn et al., 2008). Specifically, a respiratory variation (RV) signal was computed as the standard deviation of the respiratory signal using a 7.2 second sliding window and was then convolved

with the respiration response function (RRF) to obtain the respiration regressor (RVf). Similarly, a heart rate (HR) signal was computed as the inverse of average peak to peak time interval between two consecutive heartbeats over a 7.2 second sliding window. The HR was then convolved with a cardiac response function (CRF) to obtain the cardiac regressor (HRf). In this process, we discarded 4 scans out of 40 due to the poor quality of physiological signals, leaving a total of 36 scans for analysis. For DFC analysis, we used the same seed coordinates as mentioned for the BS002 dataset.

2.4. Calculation of the DFC estimates and nuisance metrics

The DFC estimates were obtained by computing the sliding window correlations between the PCC and IPS, PCC and FEF and PCC and MOT seeds. Denoting x_1 and x_2 as two seed time series and $x_{1,k}$ and $x_{2,k}$ as the windows taken from those signals (where k denotes the window index), we computed the window correlation value as $r_k = \dfrac{x_{1,k}^T x_{2,k}}{|x_{1,k}||x_{2,k}|} = \cos\theta_k$. Here, θ_k is the observed angle between those time courses and $|.|$ is the vector norm. This process was repeated by temporally shifting the window index until all sliding window correlations were carried out over the entire scan duration. The final aggregated set of correlation values $r_{DFC} = [r_1, r_2, \cdots, r_T]$ was used as an estimate for the DFC. To maximize the number of available DFC samples per scan, the sliding window duration in this paper was fixed at 30 TRs and a window shift of 1 TR was used (Hutchison et al., 2013; Xu et al., 2018). This duration corresponds to a window length of 63 seconds for the BS002 dataset and 54 seconds for the CFMRI dataset. These window lengths lie within the range of durations typically used in the existing DFC literature (Preti et al., 2017).

To examine the relationship between the nuisance measurements and the DFC estimates, we used the norm of the regressor as our nuisance metric. For a single regressor, this was done by first demeaning the nuisance regressor for each window. Denoting n_k as the demeaned nuisance measurement in the kth window, the window norm was computed as $|n_k| = \sqrt{\sum_{t=t_k}^{t_k+29} n_k(t)^2}$, where t indicates time in TR units. When multiple regressors were used in the regression (e.g. for HM), we computed the total norm as $|n_k|_{\text{Total}} = \sqrt{\sum_{i=1}^{N_R} |n_{k,i}|^2}$, where i denotes the index for different regressors and N_R is the number of regressors. In the text we use the term norm and the notation $|n_k|$ to denote both $|n_k|$ and $|n_k|_{\text{Total}}$. The norm was computed for all windows by temporally shifting the window index. The final set $\{|n_1|, |n_2|, ..., |n_T|\}$ of nuisance norms comprised the nuisance norm time course. To examine the relationship between the nuisance norm and DFC estimates, we computed the correlation between the DFC estimates and the nuisance norm time courses.

As mentioned above, we applied two types of linear regression, which we refer to as full regression and block regression. In full regression, the nuisance terms were projected out of the voxel time series over the entire scan duration. For example, for a single regressor, the clean time course after full regression was obtained as $\tilde{x} = x - n(n^T n)^{-1} n^T x = x - bn$, where $b = (n^T n)^{-1} n^T x$ is the scalar fit coefficient for the entire scan duration. In block regression, the nuisance measurement was projected out of each window separately, such that the clean time course for the kth window was $\tilde{x}_k = x_k - n_k(n_k^T n_k)^{-1} n_k^T x_k = x_k - b_k n_k$, where $b_k = (n_k^T n_k)^{-1} n_k^T x_k$ is the window-specific scalar fit coefficient.

Note that we will refer to the "clean" DFC estimates obtained after the application of regression as Post FullReg DFC and Post BlockReg DFC estimates for full and block regression, respectively. To simplify the presentation, we will also use the shorter term Post DFC to refer to Post FullReg DFC, since full regression is the method that has typically been used in the literature.

3. Results

In this section, we first show that the DFC estimates obtained between pairs of seed time courses can be significantly correlated with the norms of various nuisance measurements. We demonstrate that significant correlations between the DFC estimates and nuisance norms exist even when the correlations between the nuisance and seed time courses are small. We then show that performing nuisance regression prior to the computation of the DFC estimates does not eliminate the presence of significant correlations between the nuisance norms and DFC estimates.

3.1. DFC estimates are significantly correlated with nuisance norms

In Figure 1, we present DFC estimates and nuisance norms from 9 representative scans from the BS002 dataset. This figure serves as a qualitative demonstration of the relationship between the DFC estimates and nuisance norms. The column labels indicate the seed region pair (e.g. PCC and IPS, PCC and FEF, and PCC and MOT) and the row labels indicate the type of nuisance norm. The solid blue line in each panel shows the DFC estimate before nuisance regression (labeled as Pre DFC) and the solid black line shows the respective nuisance norm. In these scans, the correlations between the nuisance norms and the Pre DFC estimates were significant ($p < 10^{-6}$) and varied from $r = 0.68$ to $r = 0.94$ with a mean correlation value of $r = 0.79$.

To provide examples with physiological measurements, Figure 2 shows 6 representative scans from the CFMRI dataset. The first row shows 3 scans using the HRf norm as the nuisance measure and the

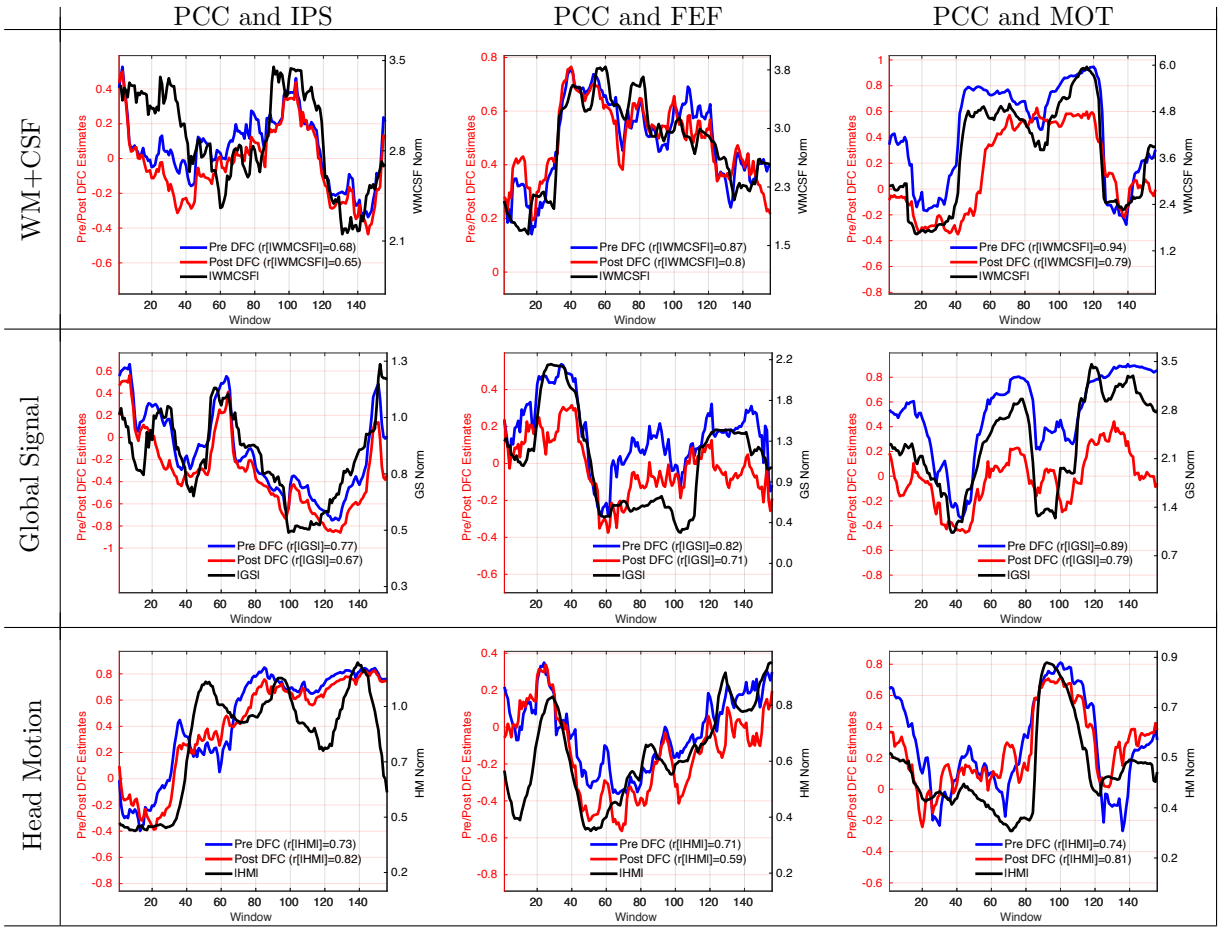

Figure 1: 9 representative scans that demonstrate significant correlations between the nuisance norms and the DFC estimates. The type of nuisance regressor is indicated by the row label. The seed pair for the DFC estimate is indicated by the column label. The solid blue line in each panel shows the DFC estimate prior to nuisance regression (labeled as Pre DFC on the legend) and the solid black line shows the corresponding nuisance norm. Correlation values between the DFC estimates and nuisance norms are indicated in the legend labels. The correlations between the nuisance norms and the Pre DFC estimates varied from $r = 0.68$ to $r = 0.94$ with a mean value of $r = 0.79$. The DFC estimates after performing full linear regression are shown with solid red lines (labeled as Post DFC on the legend). The correlations between the nuisance norms and the Post DFC estimates ranged from $r = 0.59$ to $r = 0.82$ with a mean value of $r = 0.74$.

second row shows 3 scans using the RVf norm. Each column shows a different seed pair for the DFC estimates. The correlation between the Pre DFC estimates and the nuisance norms in these scans were significant ($p < 10^{-6}$) and ranged from $r = 0.63$ to $r = 0.88$ with a mean correlation of $r = 0.78$.

In Figure 3a, we show the histogram of correlations between the nuisance norms and the DFC estimates across all scans, seed pairs, and nuisance norms. The histogram includes a total of 828 correlation values with 612 belonging to the BS002 dataset (68 scans $\times$ 3 seed pairs $\times$ 3 types of nuisance norm) and the remaining 216 from the CFMRI dataset (36 scans $\times$ 3 seed pairs $\times$ 2 types of nuisance norm). Overall, 629 out of 828 (76%) of the correlations between the Pre DFC estimates and nuisance norms were significant. The correlations ranged from a negative value of $r = -0.82$ to a positive value of $r = 0.96$, with a skewed distribution in which 74% of the significant correlations were positive and the remaining

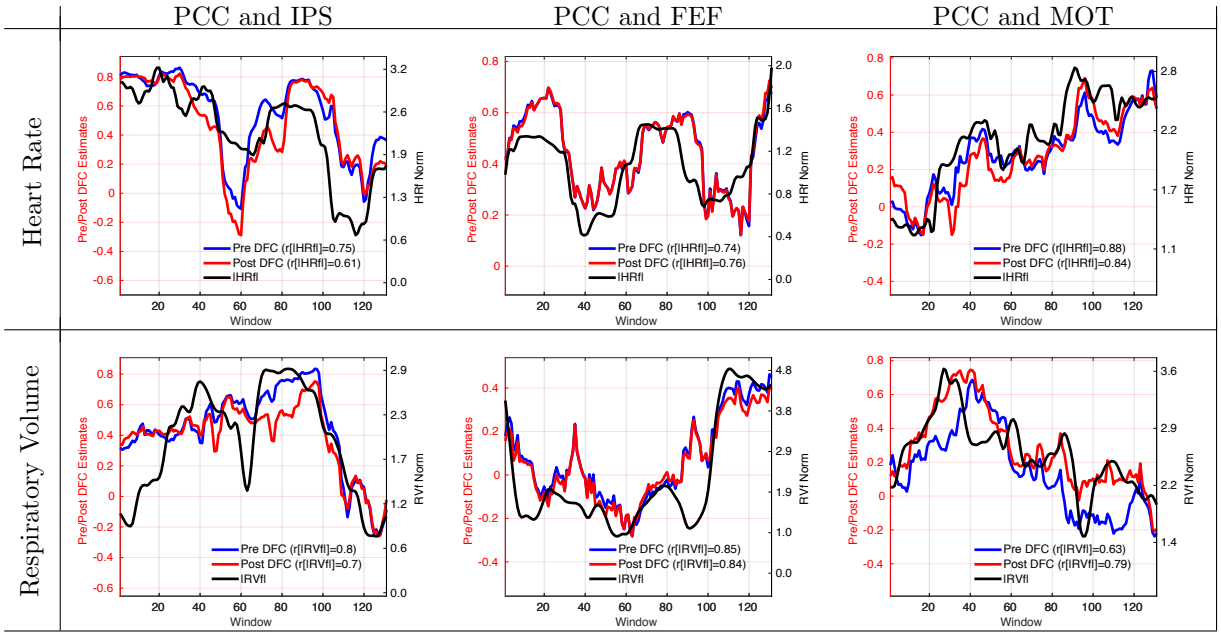

Figure 2: 6 representative scans showing that the DFC estimates are strongly correlated with the norms of physiological nuisance terms. Row labels indicate the type of physiological nuisance term considered and column labels indicate the seed pair. Correlation values between the DFC estimates and nuisance norms are indicated in the legend labels. The correlations between the Pre DFC estimates and the nuisance norms ranged from $r = 0.63$ to $r = 0.88$ with a mean correlation of $r = 0.78$. The DFC estimates obtained after performing RVHRCOR (Post DFC) are similar to the Pre DFC estimates. The correlations between the Post DFC and the nuisance norms ranged from $r = 0.61$ to $r = 0.84$ with a mean correlation of $r = 0.76$.

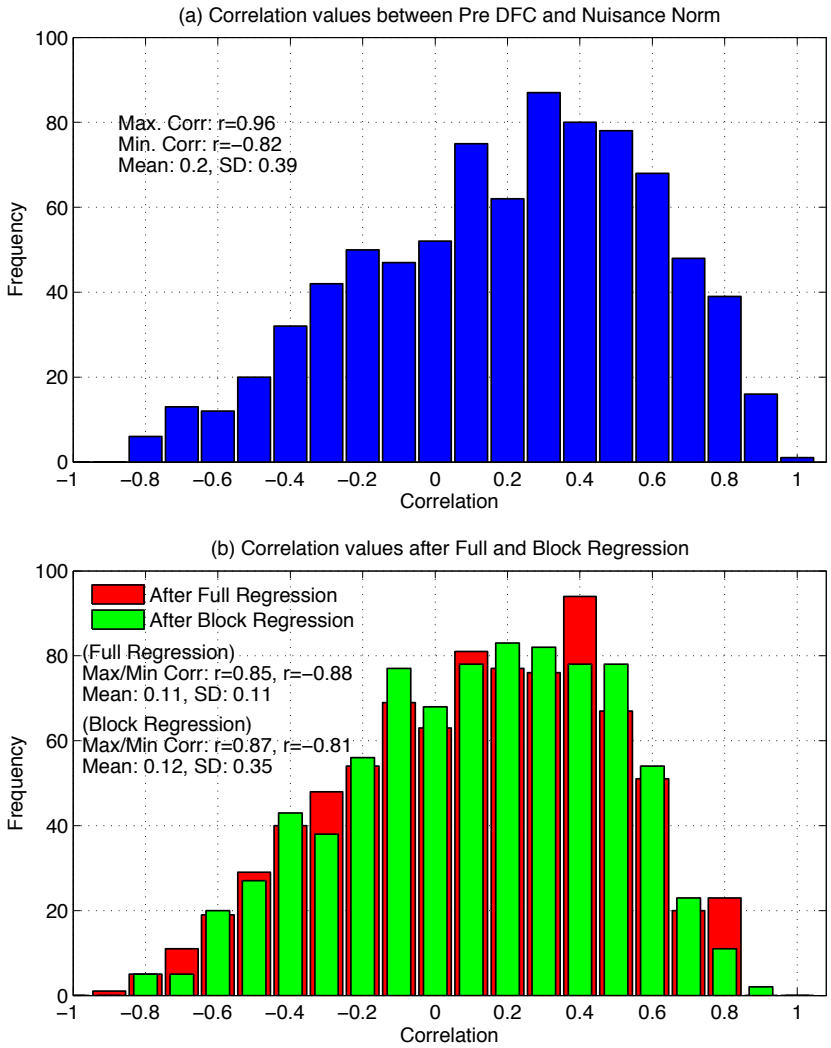

Figure 3: (a) Histogram of correlation values obtained between the Pre DFC estimates and the nuisance norm across all scans, seed pairs, and nuisance norms. Distribution of the correlation values ranged from $r = -0.82$ to $r = 0.96$. Overall, 76% of the correlations between the Pre DFC estimates and the nuisance norms were significant and 74% of the significant correlations were positive (26% were negative). (b) After regression, the correlation values ranged from a minimum of $r = -0.88$ to a maximum of $r = 0.85$ for full regression, and from a minimum of $r = -0.81$ to a maximum of $r = 0.87$ for block regression. With either full or block regression, 71% of the post regression estimates were still significantly correlated with the nuisance norm.

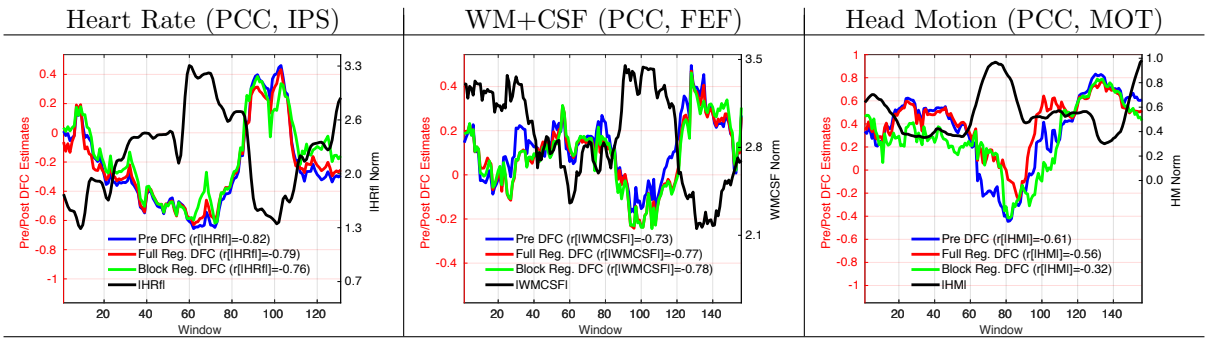

Figure 4: 3 representative scans showing examples of strong anti-correlations between the nuisance norms and the DFC estimates both before and after regression. The type of nuisance regressor and the seed region pair are indicated in the column labels. The correlation values between the DFC estimates and the nuisance norms are indicated in the legend labels.

26% were negative.

In Figure 4, we show 3 representative scans to demonstrate examples of anti-correlations between the Pre DFC estimates (solid blue lines) and various regressor norms (solid black lines). The type of nuisance regressor and the seed region pair are indicated in the column labels. For these scans, the correlation values were strongly negative (ranging from -0.82 to -0.61) with an increase in nuisance norm corresponding to a decrease in the DFC estimate.

3.2. Significant correlations between DFC estimates and nuisance norms exist even when the underlying correlations between the nuisance and seed time courses are small

In this section, we ask the question: can significant correlations between DFC estimates and nuisance norms occur even when the nuisance time course are only weakly correlated with the seed time courses? Figure 5a shows the correlations between the DFC estimates and the nuisance norms plotted against the root-mean-squared (RMS) correlations between the underlying nuisance and seed time courses. Each point corresponds to a specific scan, nuisance term, and seed region pair. For each point, the RMS correlation is computed by correlating the nuisance time course with each of the two seed time courses (from the same scan) and then calculating the RMS value (e.g. $r_{RMS} = \sqrt{(r^2_{\text{Seed1}} + r^2_{\text{Seed2}})/2}$, where r_{Seed1} and r_{Seed2} are the correlation coefficients between the nuisance time course and the first and second seed signals, respectively). We find that significant correlations (points above and below the dashed lines in Figure 5a) between the DFC estimates and nuisance norms exist even when the correlation between the nuisance and seed time courses is close to zero.

To further examine this relationship, we binned the points according to the RMS correlation values using a bin width of 0.05. For each bin, we computed the percentage of significant correlations between the DFC estimates and nuisance norms by summing the number of significant correlations observed in each bin and dividing by the total number of points. In Figure 5b, the blue bars indicate the percentage of significant correlations between the DFC estimates and nuisance norms as a function of the binned RMS correlation values. We found that a large percentage of the significant correlations between the DFC estimates and nuisance norms occur for fairly small RMS correlation values between the nuisance time course and seed signals. For example, 51% of the significant correlations between the DFC estimates and nuisance norms occurred for RMS values in the interval $r = 0$ to 0.25. Note that in this interval, the nuisance terms account for less than 6.25% of the average variance of the seed time courses. If we

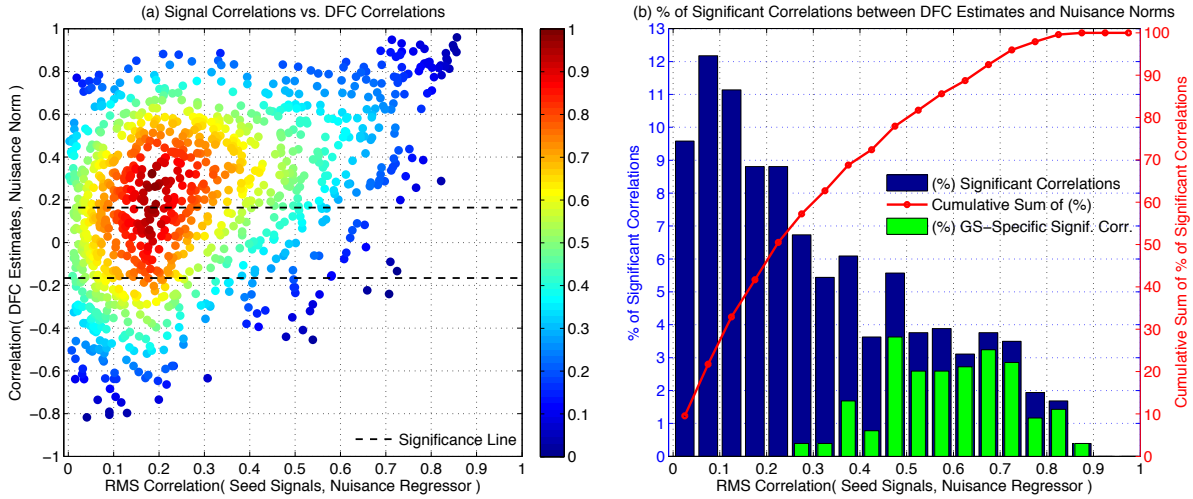

Figure 5: (a) Correlation between the DFC estimate and nuisance norm versus the RMS correlation between the raw nuisance signal and the two seed time courses. Significance thresholds ($p = 0.05$) are indicated by the black dashed lines, such that points that lie above and below these lines exhibit significant correlations between the DFC estimates and the nuisance norms. The relative density of correlations (maximum density is normalized to 1.0) is indicated by the color map on the right-hand side. The relative density is computed by summing the total number of data points in smaller sub-grids and normalizing by the total number of points. (b) The blue bars show the percentage (%) of significant correlations between the DFC estimates and the nuisance norms as a function of the RMS averaged correlation between the raw nuisance signal and the two seed time courses. The red dotted line (dots are located at bin centers) shows the cumulative percentage of significant correlations between DFC estimates and nuisance norms as a function of the RMS averaged correlations. The green bars show the contribution to the overall percentage (%) of significant correlations from the GS regressor.

expand the interval to $r = 0$ to 0.45, then 72% of the significant correlations lie within this expanded interval, corresponding to nuisance terms that account for less than 20% of the average variance of the seed time courses. The cumulative percentage of significant correlations versus RMS correlation value is shown by the red dotted line. Overall, we have the rather surprising observation that significant correlations between the DFC estimates and nuisance norms can exist even when the underlying nuisance terms account for only a small fraction of the variance of the seed time courses. We present a plausible explanation for this observation in Section 4.3 and examine its implications for nuisance regression later in the text.

3.3. Regression does not eliminate the relation between the DFC estimates and nuisance norms

In this section, we show that full regression does not necessarily eliminate the relationship between the DFC estimates and the nuisance norms. Revisiting Figures 1 and 2, our focus now is on the solid red lines (Post DFC), which show the DFC estimates obtained after projecting out the relevant nuisance regressors. The fluctuations in the Pre DFC and Post DFC estimates in both figures are similar to each other and there is very little difference in their relationships to the nuisance norm. In the scans shown in both figures, the mean correlation value between the Post DFC estimates and the nuisance norms was $r = 0.74$, which is very close to the mean correlation value of $r = 0.79$ between the Pre DFC estimates and the nuisance norms.

Viewed across the entire sample, the Post DFC correlation values shown in Figure 3b ranged from a minimum of $r = -0.88$ to a maximum $r = 0.85$. With full regression, 71% of the Post DFC estimates were still significantly correlated with the nuisance norm, when compared to the Pre DFC percentage of 76%. The correlation distribution was similar to the Pre DFC histogram in Figure 3a, with a cosine similarity of $S = 0.80$.

To further demonstrate the relationship between the Pre and Post DFC estimates, in Figure 6a we plotted the correlations between the Post DFC estimates and nuisance norms versus the corresponding correlations between the Pre DFC estimates and the nuisance norms across all scans, nuisance time courses, and seed pairs. The relative density of correlation values is indicated with the color bar on the right hand side. There was a significant linear relationship between the two correlation populations ($r = 0.79$, $p < 10^{-6}$), indicating that the Post DFC estimates largely retain the correlation with the nuisance norms that is observed in the Pre DFC estimates.

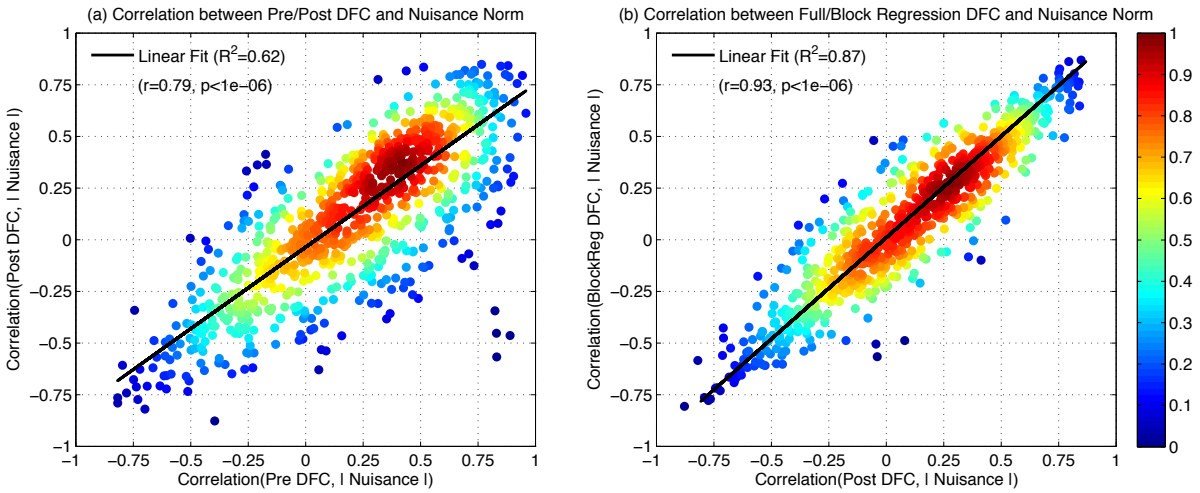

Figure 6: (a) The correlations between the Post DFC estimates and the nuisance norms (y-axis) versus the correlations between the Pre DFC estimates and the nuisance norms (x-axis). Post DFC estimates were still largely correlated with the nuisance norms and there was a strong linear relationship when compared to the correlations between the Pre DFC and nuisance norms (linear fit shown with black solid line, $R^2 = 0.62$, $r = 0.79$, $p < 10^{-6}$). The relative density of correlations (normalized to 1.0) is indicated by the color map on the right hand side. (b) The correlations between the DFC estimates and the nuisance norms after block regression (y-axis) and after full regression (x-axis). There was a strong linear relationship between the correlation populations (fit shown with black solid line, $R^2 = 0.87$, $r = 0.93$, $p < 10^{-6}$). The correlation distributions for full and block regression were not significantly different (paired two-tailed t-test $p = 0.98$). The effect size ($d = 0.0003$) and the absolute difference in correlation population means (0.0001) were negligibly small.

3.4. Block regression is similarly ineffective in removing nuisance effects from the DFC estimates

As an alternative nuisance removal approach, we performed block regression in which nuisance measurements were projected out from each window separately. In Figure 7, we superimposed plots of the DFC estimates after performing block regression (green lines; referred to as Post BlockReg DFC) on the plots previously shown in Figure 1 for the BS002 dataset. The Post BlockReg DFC estimates were significantly ($p < 10^{-6}$) correlated with the nuisance norm with correlations ranging from $r = 0.52$ to $r = 0.86$ and a mean correlation of $r = 0.69$. For the CFMRI dataset, the corresponding qualitative figures for block regression results are given in Supplementary Figure 1.

Viewed across the entire sample, the Post BlockReg DFC correlation values shown in Figure 3b ranged from a minimum of $r = -0.81$ to a maximum $r = 0.87$, with 71% of the Post BlockReg DFC estimates exhibiting a significant correlation with the nuisance norm. The distribution of correlation

values was similar to that obtained for Post FullReg DFC with a cosine similarity of $S = 0.94$.

In Figure 6b, we plot the correlations between the Post BlockReg DFC estimates and the nuisance norms versus the correlations between the Post FullReg DFC estimates and the nuisance norms. A linear fit between the two correlation populations (shown with solid black line) revealed a significant linear relationship ($R^2 = 0.87$, $r = 0.93$, $p < 10^{-6}$). In addition, the correlation distributions for full and block regression were not significantly different from each other ($p = 0.98$, paired two-tailed t-test). The effect size ($d = 0.0003$) and the absolute difference in correlation population means (0.0001) were negligibly

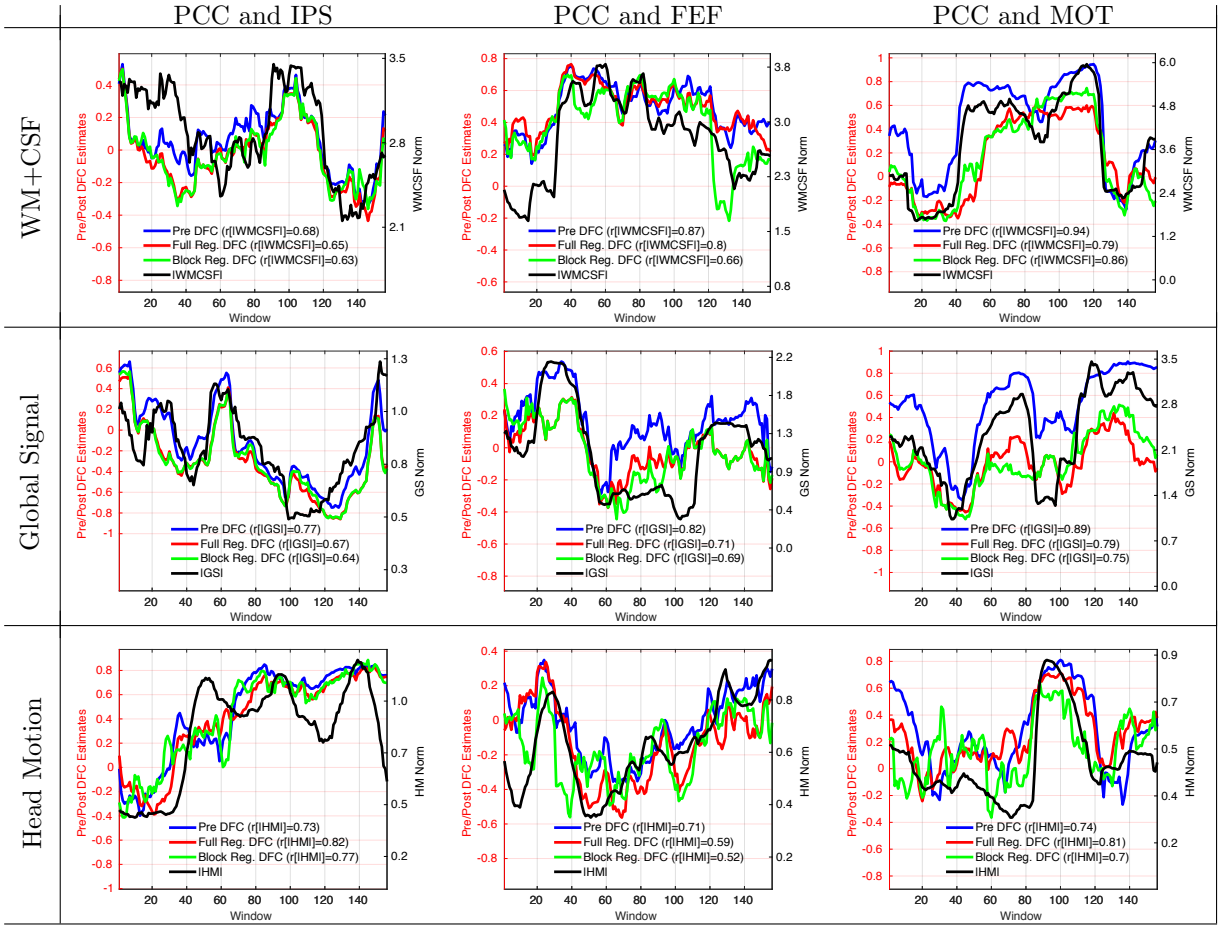

Figure 7: Same 9 representative scans from Figure 1 with the DFC estimates after block regression (solid green line) superimposed. The block regression DFC estimates were significantly correlated ($p < 10^{-6}$) with the respective nuisance norms with a mean correlation value of $r = 0.69$.

small. Thus, with respect to the relationship between the DFC estimates and the nuisance norms, block and full regression have nearly identical effects.

4. Interpretation

4.1. Nuisance effects on correlation estimates

In Section 3.1, we showed that DFC estimates can be related to the norms of various nuisance terms. Here, we aim to provide an intuitive understanding of how this relationship might arise. We use simple toy examples to demonstrate the key principles and to establish concepts that will be further developed in the Theory section. In the toy examples, we represent time series as vectors in a low-dimensional (2D or 3D) space, such that the correlation between time series is simply the cosine of the angle between the vectors. For all of the examples, we will assume that there is a set of two underlying vectors with a fixed angle across time windows, corresponding to an idealized case in which the windowed correlation between two time series is fixed across time. Then, we examine what happens when a nuisance term is added to the underlying vectors.

4.2. 2D Examples

4.2.1. Positive correlations (Aligned Case)

We begin with a simple 2D toy example in which an additive nuisance term points in the same general direction as the underlying vectors. The underlying vectors are depicted with green vectors and denoted as y_1 and y_2 in Figure 8a for 5 consecutive windows (k through $k+4$). The angle between the vectors is fixed at an angle of 135°, such that the correlation is $r = -0.70$ for each window. We consider an additive nuisance term pointing along the horizontal axis with a norm that varies across windows (indicated by the red vectors and denoted as n_k through n_{k+4}).

In this example, the nuisance vectors n_k are aligned with the underlying vectors y_1 and y_2, meaning that the inner products ($y_1^T n_k > 0$ and $y_2^T n_k > 0$) are positive. The sum of the underlying vectors and additive nuisance terms yields the observed time series as shown by the blue vectors in the second row of Figure 8a and denoted as $x_{1,k}$ and $x_{2,k}$. The angle between the observed vectors (shown with θ_k) varies with the nuisance norm, with smaller angles observed for larger nuisance norms. In the last row of Figure 8, we plot the correlation between the observed signals (cosine of the angle between observed signal vectors) and the nuisance norm across windows. There is a strong relationship ($r = 0.98$) between

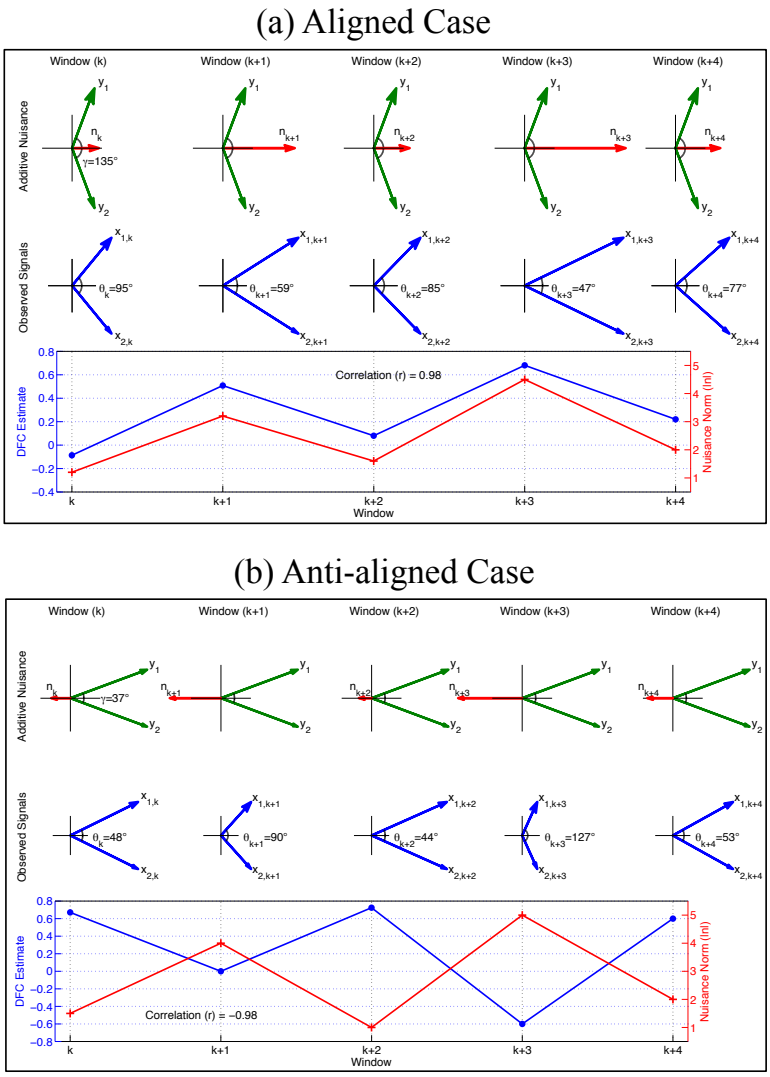

Figure 8: The relationship between the DFC estimates and the nuisance norms under an additive nuisance model. We present two cases indicated as the (a) aligned case and the (b) anti-aligned case. In the first row in (a), we simulate a set of fixed underlying vectors y_1 and y_2 across 5 windows (green arrows). The nuisance vectors n_k have different norms in each window (red arrows). The observed time courses $x_{1,k}$ and $x_{2,k}$ in the second row (shown with blue arrows) are the sum of the corresponding underlying vectors (y_1 and y_2) and the nuisance vector n_k. In the last row in (a), the simulated DFC estimates ($\cos\theta_k$, solid blue line) follows the norm of the nuisance term ($|n_k|$, solid red line). The aligned case shows that an increase in the norm of a nuisance vector that points in the direction of the underlying signals decreases the inner angle between the observed time courses and increases the value of DFC estimates. The anti-aligned case in (b) shows that an increase in the norm of a nuisance vector, that points in a direction opposite to that of the underlying signals, can decrease the DFC estimate and lead to anti-correlation between the nuisance norm and the DFC estimate.

the window correlation values and the nuisance norm. This simple example shows how variations in the norm of a nuisance term may induce variations in the windowed correlation estimates that are highly correlated with the norm of the nuisance term.

4.2.2. Anti-correlations (Anti-aligned Case)

As noted in Section 3.1, the DFC estimates can sometimes be anti-correlated with the nuisance norms. To see how this might arise, we consider the 'anti-aligned' case shown in Figure 8b. In contrast to the aligned case discussed above, the nuisance vector points in a direction opposite to the average direction of the underlying signals, such that the corresponding dot products ($y_1^T n_k < 0$ and $y_2^T n_k < 0$) are negative. In this example, the correlation between y_1 and y_2 is fixed at a value of $r = \cos 37^\circ = 0.80$.

The addition of the anti-aligned nuisance vector leads to cancellation of vector components along the horizontal axis. As a result, the angle between the observed vectors $x_{1,k}$ and $x_{2,k}$ in the second row tends to increase as the nuisance norm increases. Taking the cosine of the angle between the observed vectors to obtain the correlation estimates, we find that the correlation values are anti-correlated ($r = -0.98$) with the nuisance norm, as shown in the third row of Figure 8b. This simple example shows that an additive nuisance term can induce variations in the windowed correlation estimates that are anti-correlated with the nuisance norm.

We should note that if the nuisance norm becomes extremely large for the anti-aligned case, the observed vectors can be dominated by the nuisance term in a manner that can cause the nuisance norm to be positively correlated with the resulting DFC estimates. In general, the extent to which a nuisance term induces positive or negative correlations between the correlation estimates and the nuisance norm depends on both the angle between the underlying vectors and the relative direction and magnitude of the nuisance vector.

4.3. Extension to 3D with addition of an orthogonal nuisance component

In the 2D examples discussed so far, the nuisance term was completely within the 2D plane spanned by the observed vectors. We now expand the example to 3D by including an additional nuisance component that is orthogonal to the 2D plane, such that the overall nuisance term is the sum $n = n_I + n_O$ of an in-plane component n_I that lies in the subspace spanned by the observed vectors x_1 and x_2 and an orthogonal component n_O that is orthogonal to the subspace.

In Figure 9, we have constructed the observed signals (blue vectors in the first row) using the fixed vectors and in-plane nuisance components previously used in the 2D example of Figure 8a. We then modify the nuisance terms by adding orthogonal components, such that the overall nuisance terms (red

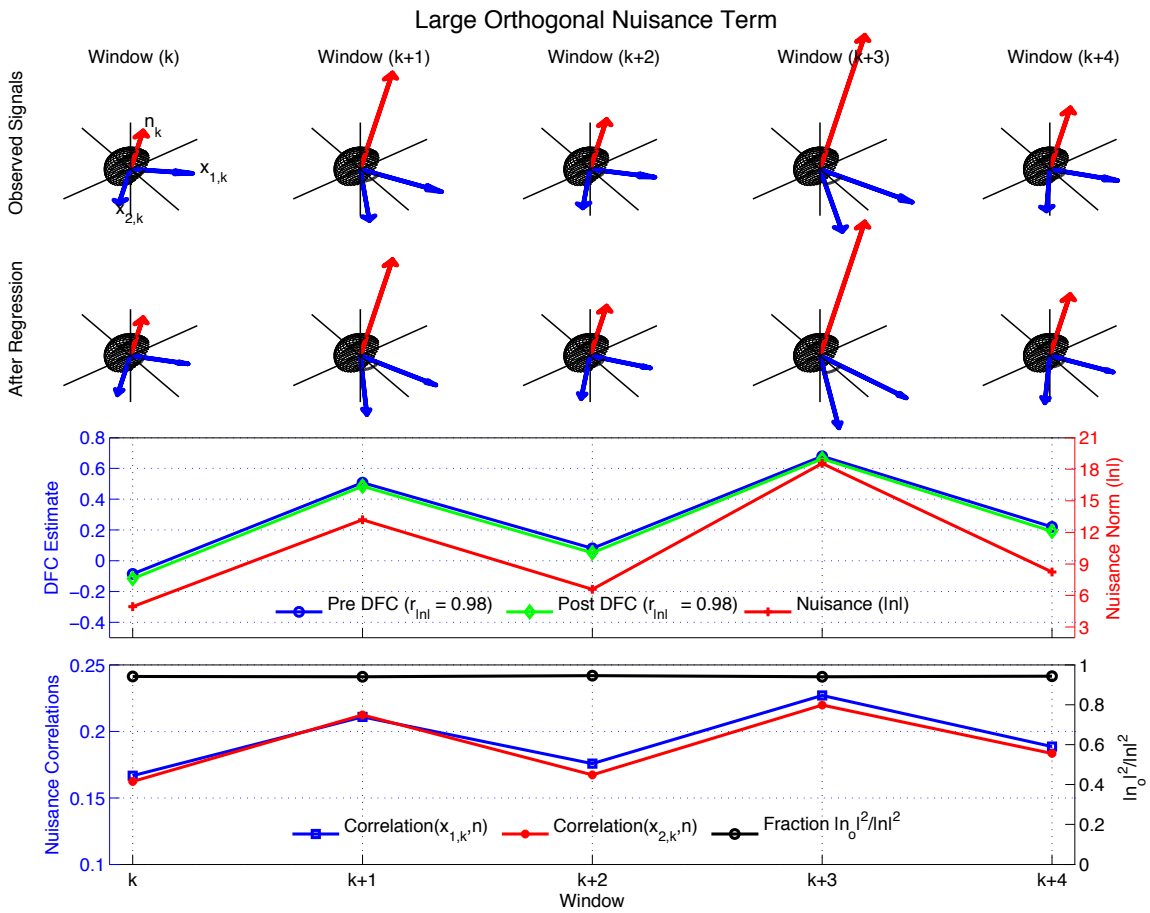

Figure 9: Linear regression can fail to eliminate the relationship between the nuisance norm and the DFC estimates due to a large degree of orthogonality between the nuisance measurement and fMRI time courses. The observed signals (blue vectors) were obtained using nuisance terms (red vectors) for which the in-plane nuisance components are the same as those previously used in Figure 8a. The Pre DFC estimate (blue line) is highly correlated ($r = 0.98$) with the nuisance norm (red line) . The nuisance term also has a large orthogonal nuisance fraction (indicated by the black dots in the fourth row) with a mean value of 0.94. Therefore, linear regression has a minimal effect and Post DFC estimates (green line) are also highly correlated ($r = 0.98$) with the nuisance norm. The correlations between the nuisance term and the observed vectors (red and blue lines in bottom row) are relatively weak. Note that for display purposes, the orthogonal axes (vertical) in the top 2 rows are compressed ($\times 0.7$) with respect to the in-plane axes.

vectors) have both in-plane and orthogonal components. We construct the orthogonal component in each window such that its norm is proportional to the norm of the in-plane component (i.e. $|n_{O,k}| \propto |n_{I,k}|$). Here, it is useful to define a metric $|n_O|^2/(|n_I|^2 + |n_O|^2) = |n_O|^2/|n|^2$ that reflects the relative fraction of nuisance energy that lies in the orthogonal component. The orthogonal nuisance fraction is equal to 1.0 when the nuisance component is completely orthogonal to the observed signal subspace and is equal to 0.0 when the nuisance component lies within the subspace. For this example, the orthogonal nuisance fraction (indicated by the black dots in the fourth row) is relatively large and fairly constant with a mean value of 0.94.

The resulting 3D example has several important properties. First, the Pre DFC estimates (blue dots in the third row) exhibit a strong correlation ($r = 0.98$) with the nuisance norm (red dots), similar to that previously shown for the 2D example in Figure 8a. This is because (1) the observed signals are the same in the 2D and 3D examples and (2) by construction the nuisance norm of the 3D nuisance term scales with the norm of the in-plane component.

Second, because the orthogonal nuisance fraction is relatively large, the underlying correlations between the nuisance terms and the observed vectors (i.e. cosine of the angle between the red and blue vectors) are relatively small as shown by the red and blue curves in the fourth row. Thus, this example is consistent with the empirical findings discussed in Section 3.2: a strong correlation between the Pre DFC estimates and the nuisance norms can exist even when the correlation between the nuisance and seed signals is small and the orthogonal fraction is high.

Third, linear regression has a minimal effect in this example, such that the signal vectors after regression (blue vectors in the second row) are similar to the original observed signals (blue vectors in the first row). Note that to simplify the presentation, we have used block regression to obtain the Post DFC estimates. The resultant Post DFC estimates (shown with solid green line and diamond markers in the third row) are also highly correlated ($r = 0.98$) with the nuisance norm. This is consistent with the empirical findings from Sections 3.3 and 3.4 that nuisance regression can have a minimal effect on the relation between the DFC estimates and the nuisance norms.

To aid in relating the orthogonal nuisance fraction to the experimental results, Figure 10 plots the orthogonal fraction values (averaged over each scan) versus the average percent variance in the seed time courses that is explained by the nuisance regressors (computed as the square of the RMS correlation values previously shown in Figure 5). As the average percent variance explained increases, the fraction of the nuisance term energy that is orthogonal to the plane spanned by the observed time series decreases.

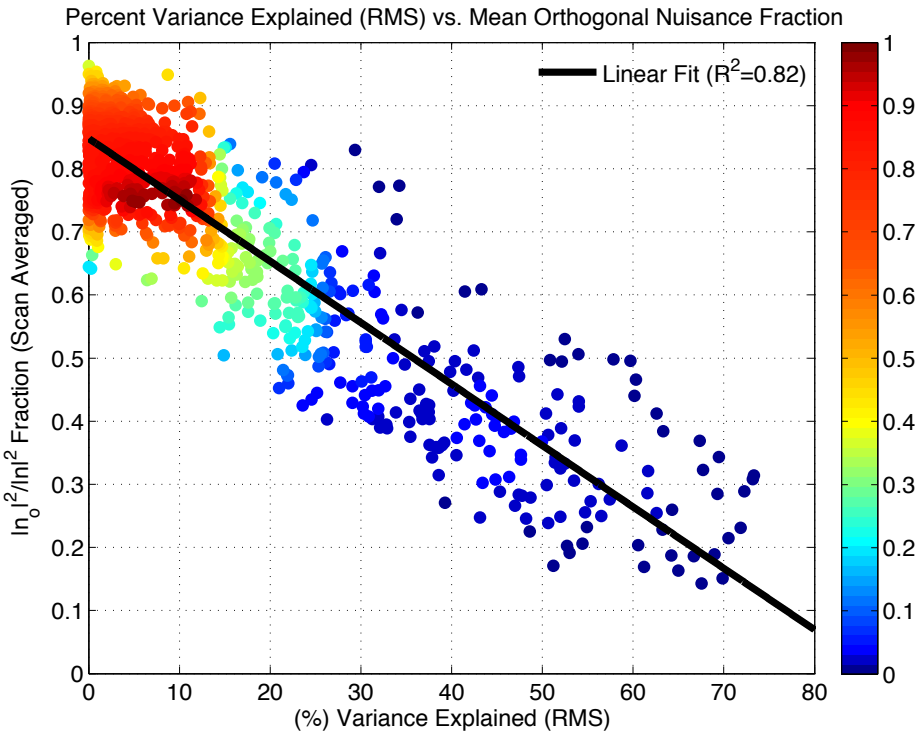

Figure 10: The scan-averaged orthogonal nuisance fraction $|n_O|^2/|n|^2$ versus the percent variance explained by the raw nuisance regressors (squared RMS correlation values from Figure 5). As shown by the black least squares line, the orthogonal fraction decreases in a linear fashion ($R^2 = 0.82$) with increasing percent variance. The relative density of values (maximum density is normalized to 1.0) is indicated by the color map on the right-hand side.

As noted in Section 3.2, 51% of the significant correlations between the DFC estimates and nuisance norms occurred when the percent variance explained was less than 6.25%. Using the linear fit shown in Figure 10, this percentage corresponds to orthogonal nuisance fractions greater than 0.78, consistent with our use of a large orthogonal fraction in the example.

4.4. Regression effects depend on the orthogonal nuisance fraction

In this section, we present a 3D example to demonstrate how the effects of regression depend on the orthogonal nuisance fraction. In each of the 5 cases shown in Figure 11 the nuisance vector n has different orientations with respect to the 2D subspace spanned by the observed signals x_1 and x_2. The orthogonal fraction starts at 1.0 for Case 1 and then decreases to 0.0 for Case 5, with intermediate values

for the other cases. The correlation between the observed time courses is fixed at $r = 0.78$ for all of the cases. In the middle column, we show the vectors (denoted as $\tilde{x}_1$ and $\tilde{x}_2$) after nuisance regression.

When the orthogonal fraction is 1.0 (Case 1), the nuisance vector is completely orthogonal to the space spanned by the observed signals, and therefore the regression coefficient between the nuisance vector and each observed signal is zero. As a result, linear regression has no effect, and the post regression signals are identical to the observed signals. Consequently, for Case 1 the post regression correlation value is equal to the pre regression correlation value, as shown in the rightmost plot in Figure 11 as the intersection between the red dot and the horizontal green line representing $r = 0.78$.

When the orthogonal fraction is 0.0 (Case 5), the nuisance vector lies completely in the plane spanned by the observed time courses. After regression, the vectors $\tilde{x}_1$ and $\tilde{x}_2$ must point in opposite directions in order to achieve orthogonality to the in-plane nuisance term. As a result, the correlation between the post regression time courses will be -1. (Note that if the nuisance term n is in-plane but lies outside of the inner angle formed by x_1 and x_2 then regression will force $\tilde{x}_1$ and $\tilde{x}_2$ to point in the same direction and the correlation will be forced to be $+1$. In the Appendix, we refer to this as the complementary case).

For the remaining cases, the post regression correlation values vary as a function of the orthogonal nuisance fraction as shown by the red circles on the right hand side of Figure 11. The green line in the plot indicates the pre-regression signal correlation values ($r = 0.78$), which are independent of the fraction. Cases with a larger orthogonal nuisance fraction have post regression correlation values that tend towards the pre regression correlation value of $r = 0.78$, whereas cases with a smaller orthogonal nuisance fraction have post regression correlation values that tend towards -1. The blue line in the plot shows the theoretical relation between the post regression correlation values and the orthogonal nuisance fraction. This relation is discussed in greater detail in the Theory section.

Overall, we see that when the orthogonal nuisance fraction is relatively high (e.g. greater than 0.5), the difference between the pre and post regression DFC estimates (distance between blue and green lines) will be relatively small. The exact bound on this difference (over all possible pre-regression correlation values) is provided in the Theory section, where it is also shown that most nuisance regressors lie within the high orthogonal fraction regime and therefore exhibit a small difference between the pre and post regression DFC estimates. The primary exception is the global signal, which provides the motivation for our next example.

4.5. DFC estimates after regression with smaller orthogonal nuisance term

In this subsection, we consider a second example in which the orthogonal nuisance fraction is relatively modest with a mean value of 0.37 across windows, as shown with the black line with circles in the fourth row of Figure 12. As discussed further in the Theory section, this value for the orthogonal nuisance fraction is consistent with what is observed for the global signal nuisance term. In this case, linear regression has a noticeable effect as can be seen by the difference between the original signal

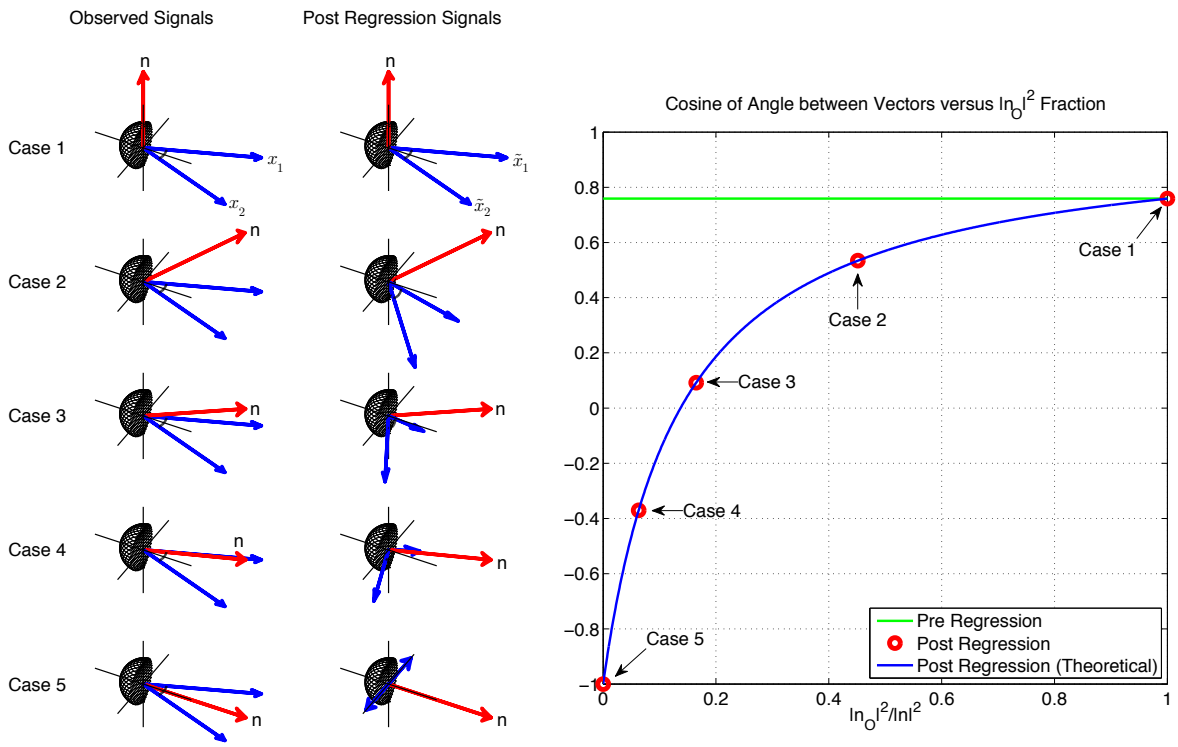

Figure 11: The effect of linear regression on cosine of angle between two vectors. We illustrate 5 cases for the same observed fMRI time courses with a fixed correlation value ($r = 0.78$; as shown with the green solid line) and a variable nuisance component. In Case 1, the nuisance vector is orthogonal to the observed vectors and the orthogonal nuisance fraction ($|n_O|^2/|n|^2$) is 1.0, in which case regression has no effect on the pre regression correlation values. Thus, the post regression correlation value is also $r = 0.78$ as shown with the red marker. In Case 5, nuisance vector lies within the observed vectors with $|n_O|^2/|n|^2 = 0$ and post regression correlation value is -1.0 regardless of the pre regression correlation value. For Cases 2, 3, and 4, the fraction $|n_O|^2/|n|^2$ takes on intermediate values, such that the difference between the pre and post values increases as the orthogonal fraction decreases. The blue line represents the theoretical values obtained from Equation 2.

vectors (blue vectors in the top row) and the post regression vectors (blue vectors in the second row). Regression moves the post regression correlation values away from the pre regression correlation value towards -1.0, similar to Cases 2 and 3 in Figure 11. However, since the orthogonal nuisance fraction

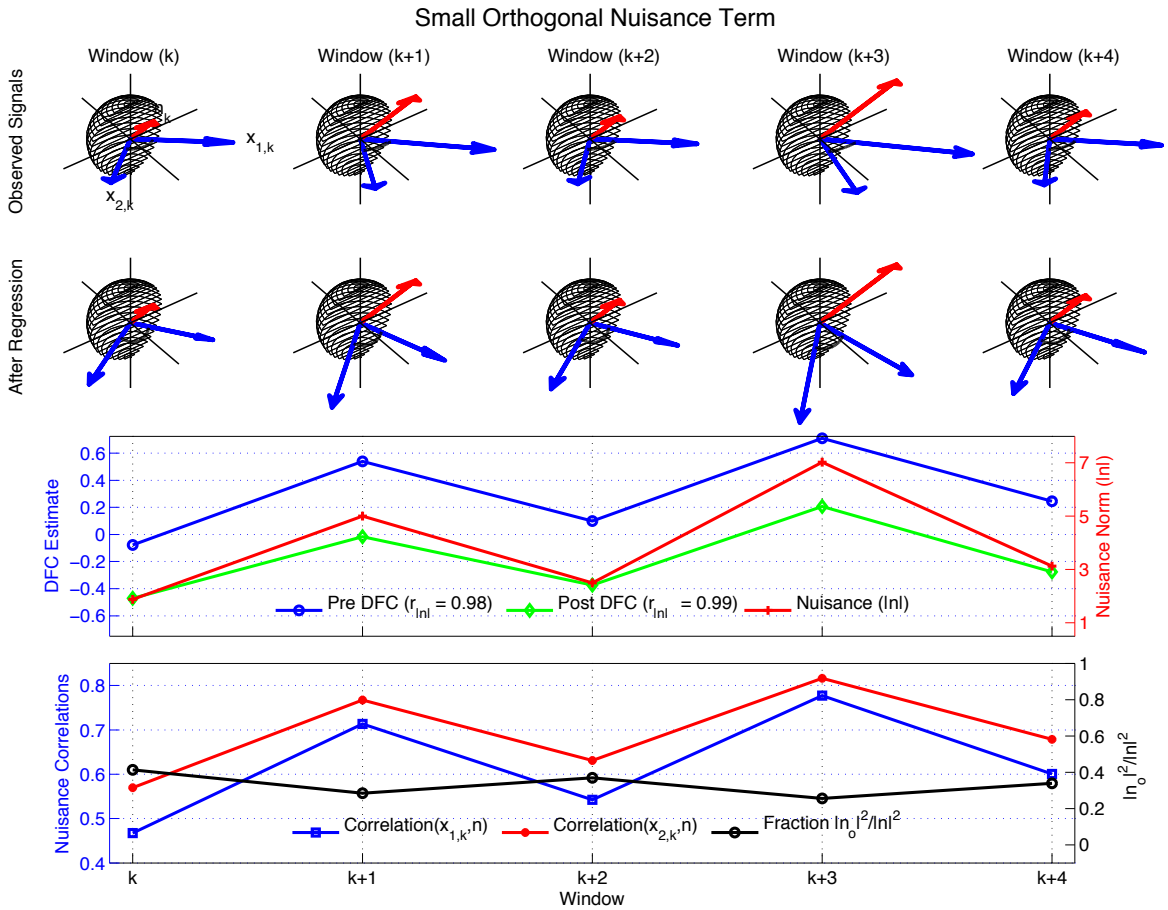

Figure 12: Toy example illustrating how linear regression can fail to eliminate the relationship between the nuisance norms and the DFC estimates when the nuisance orthogonality is weak. Similar to the first toy example in Figure 9, the in-plane nuisance components are the same as those previously used in Figure 8a, however, the orthogonal part of the nuisance term in this example is much smaller as compared to Figure 9. Therefore, linear regression has a noticeable effect on the DFC estimates. Since the orthogonal nuisance fraction is fairly constant across windows, the difference between the Pre and Post DFC estimates is also fairly constant. As the correlation coefficient is invariant to constant offsets, the correlation between the nuisance norm and Pre DFC estimate ($r = 0.98$) is essentially the same as the correlation between the nuisance norm and the Post DFC estimate ($r = 0.99$).

is relatively constant across windows, the difference between the Pre and Post DFC estimates is also fairly constant. As shown in the third row in Figure 12 the Post DFC estimates can be approximated as a shifted version of the Pre DFC estimates. Because correlation is invariant to constant offsets, the correlation ($r = 0.99$) between the Post DFC estimates with the nuisance norm is essentially the same as the correlation ($r = 0.98$) between the Pre DFC estimates and the nuisance norm. The construction of the toy example is consistent with our observation of a fairly constant difference between the Pre and Post DFC estimates obtained after GS regression. A representative scan is shown in the second row of Figure 13 for GS regression. In Section 5.3 we provide additional empirical and theoretical results regarding this effect.

The correlations between the observed signals and the nuisance terms are shown with the blue square and red dotted lines in the fourth row of Figure 12, with values ranging from $r = 0.46$ to $r = 0.81$. Consistent with the smaller orthogonal fraction in this example, these correlations are higher than those observed in the example shown in Figure 9. The range of correlation values used in this example is consistent with the range of RMS correlations that is empirically observed for scans, in which the DFC estimates and GS norm are significantly correlated. Specifically, as shown by the green bars in Figure 5b, the empirical correlations range from $r = 0.27$ to $r = 0.87$ with a mean of $r = 0.59$.

In concluding the Interpretation section, it is important to note that the toy examples considered here are designed to provide a basic level of intuition that can be helpful for understanding both the empirical findings in the Results section and the theoretical expressions which will be presented in the Theory section. While the examples shown here demonstrate behavior similar to that observed in the experimental data, they are by no means exhaustive and alternative examples might be useful to consider in future work.

5. Theory

5.1. The DFC estimate after block regression

In this section, we provide the expression for the DFC estimate after block regression. The detailed steps of this derivation can be found in Appendix A. Dropping the window index subscript k for simplicity,

the correlation coefficient after block regression for a single window is given by:

$$\tilde{r} = \frac{\tilde{x}_1^T \tilde{x}_2}{|\tilde{x}_1||\tilde{x}_2|} = \frac{x_1^T x_2 - \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_2}{|x_1||x_2| \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_1^T P_{n_I} x_1}{|x_1|^2}} \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_2^T P_{n_I} x_2}{|x_2|^2}}} \tag{1}$$

This can rewritten as

$$\tilde{r} = \lambda r + O \tag{2}$$

where $r = x_1^T x_2 / (|x_1||x_2|)$ is the correlation coefficient before regression and the scaling λ and offset O terms are defined as

$$\lambda = \frac{1}{\sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_1^T P_{n_I}^T x_1}{|x_1|^2}} \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_2^T P_{n_I} x_2}{|x_2|^2}}} \tag{3}$$

$$O = -\lambda \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_2 / \left(|x_1||x_2|\right). \tag{4}$$

As the orthogonal component n_O becomes arbitrarily large compared to the in-plane component n_I, then the orthogonal fraction $\frac{|n_O|^2}{|n|^2} \to 1$ and the term $\frac{|n_I|^2}{|n|^2} = \left(1 - \frac{|n_O|^2}{|n|^2}\right) \to 0$. In this case, the scaling term $\lambda \to 1$ and the offset term $O \to 0$, such that the correlation coefficient after regression approaches the pre-regression value $\tilde{r} \to r$. This corresponds to Case 1 in Figure 11.

On the other hand, when the orthogonal component becomes arbitrarily small and the orthogonal nuisance fraction $\frac{|n_O|^2}{|n|^2} \to 0$, it can be shown that $\tilde{r}$ approaches either -1.0 or 1.0 (see Equation (A.23) in the Appendix). When the terminal value is -1.0, this corresponds to Case 5 in Figure 11. For intermediate values $0 < \frac{|n_O|^2}{|n|^2} < 1$ of the orthogonal nuisance fraction, the correlation coefficient $\tilde{r}$ takes on values between -1 and r, corresponding to Cases 2 through 4.

5.2. A mathematical bound on the change in DFC using block regression

Here, we consider the difference $\Delta\text{DFC} = \tilde{r} - r$ between the correlation coefficients obtained before and after block regression. In Appendix B, we show that this quantity is bounded as follows:

$$|\Delta\text{DFC}| \leq 2 \left(\frac{1 - \sqrt{|n_O|^2/|n|^2}}{1 + \sqrt{|n_O|^2/|n|^2}} \right) \tag{5}$$

As an example of this bound, in the second column of Figure 13 we plot ΔDFC versus the orthogonal nuisance fraction $\frac{|n_O|^2}{|n|^2}$ for WM regression (first row) and GS regression (second row) applied separately

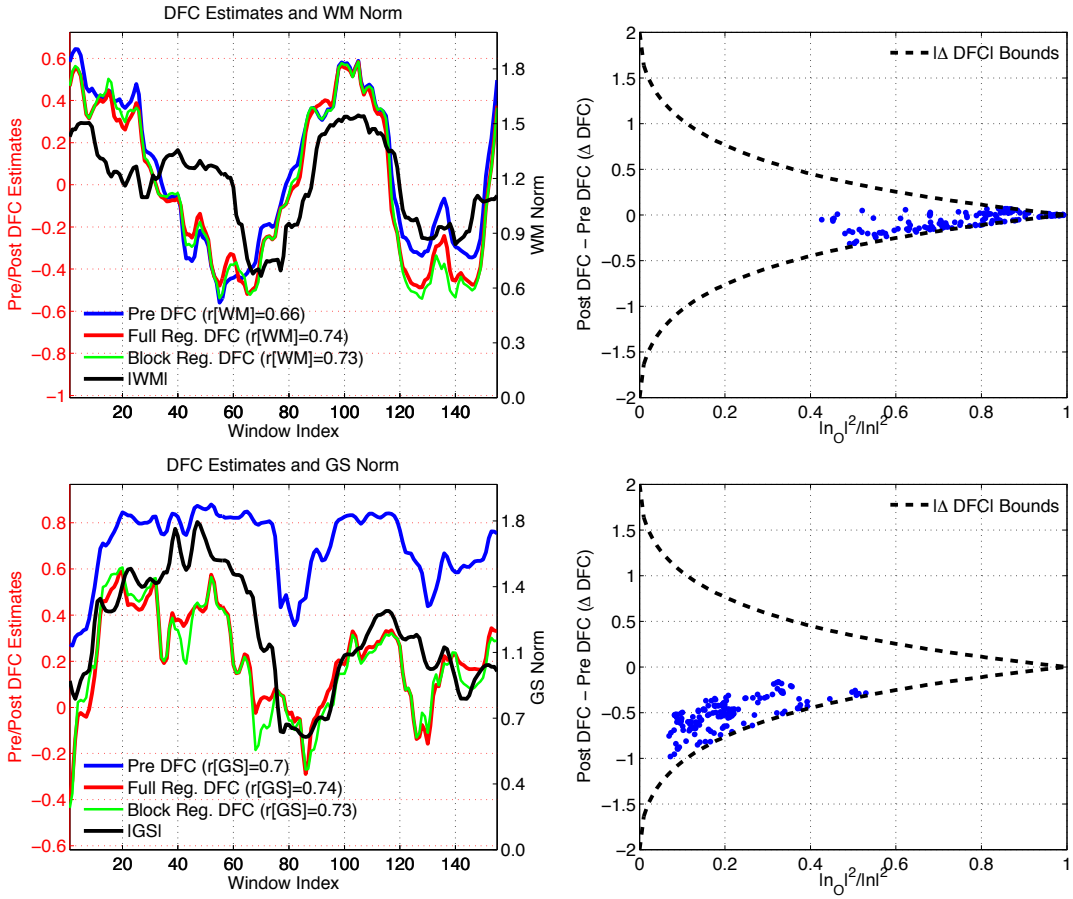

Figure 13: Two representative scans that demonstrate the theoretical bounds on ΔDFC. In the first column, we show the DFC estimates obtained between the PCC and IPS seeds. In these scans the DFC estimates show a large degree of correlation with the norm of the WM time course ($r = 0.66$) in the first row, and with the norm of the GS time course ($r = 0.7$) in the second row. Performing full or block regression does not reduce these correlations and the correlations are larger than 0.7 after each regression technique. In the second column, we show the ΔDFC values versus the orthogonal nuisance fraction for the WM and GS regressors. Each point in these plots corresponds to the orthogonal nuisance fraction and ΔDFC values in a specific window. We also superimpose the theoretical bounds. For the WM nuisance term, the effects of regression are limited by the tight bound imposed by the large orthogonal nuisance fractions. For the GS nuisance term, the bounds are more relaxed due to the smaller orthogonal nuisance fraction. However, the ΔDFC values are clustered around −0.5 and the difference between the Pre DFC and Post DFC values in the first column is also fairly constant. Thus, the Post DFC values remain correlated with the nuisance norms after GS regression.

to two representative scans. Consistent with the discussion in the previous sections, when the orthogonal nuisance fraction $\frac{|n_O|^2}{|n|^2}$ is close to 1.0, the theoretical bound (black dashed line) approaches 0.0. Consequently, the post regression correlation coefficients for the WM signal in the first column of Figure 13 are constrained to be close to the pre regression coefficients.

On the other hand, as the orthogonal nuisance fraction $\frac{|n_O|^2}{|n|^2}$ approaches zero, the theoretical bound relaxes and approaches ± 2. This corresponds to the case where the post regression correlation coefficient approaches either -1.0 or 1.0. Since the pre regression correlation coefficient is bounded between -1.0 and 1.0, the maximum absolute difference in coefficients is 2, consistent with the theoretical bound on $|\Delta\text{DFC}|$. The post regression correlation coefficients for the GS signal in the first column of Figure 13 have a noticeable negative offset when compared to the pre regression coefficients.

To demonstrate the general validity of the bound, in Figure 14, we plot ΔDFC versus the fraction $\frac{|n_O|^2}{|n|^2}$ for six different nuisance regressors using the data from all scans. For head motion (HM), the nuisance regressor was defined as the first principal component of the 6 motion regressors. All of the empirical DFC differences are found to lie within the theoretical bounds. For WM, CSF, HM, RVf, and HRf regressors, the mean of the orthogonal fractions range between 0.71 and 0.81, reflecting the fact that for most of the data windows there is a large orthogonal fraction and a fairly tight bound on ΔDFC such that mean difference between the pre and post regression values is small, ranging from -0.078 to -0.0005. In contrast, for the GS regressor, the mean orthogonal fraction is 0.35, and so the data for most of the windows lie in a range where the bounds on ΔDFC are not as tight and the mean difference between pre and post regression values is -0.42. The smaller orthogonal nuisance fraction observed for GS reflects the fact that it is derived as the mean of all the voxel time courses in the brain and therefore is expected to exhibit a greater similarity (and hence a greater in-plane component) with the seed voxel time courses, as compared to the other regressors.

5.3. Approximate Constant Offset observed in ΔDFC for GS regression

In both the previous section and Section 4.5, we noted that GS regression resulted in a relatively constant difference ΔDFC between the Pre and Post DFC values. In addition, we can see from the upper righthand panel in Figure 14 that the per-window ΔDFC values with GS regression are clustered close to the lower bound.

To further investigate this effect, we computed the mean and standard deviation of the ΔDFC values for each scan. We also computed the mean lower theoretical bound for each scan by computing the lower

bound for each window and then averaging over windows. Figure 15a plots the mean ΔDFC values versus the mean theoretical bounds, with the data for GS and non-GS regressors indicated by the circle and square markers, respectively. The line of unity is indicated by the dashed green line. Due to the additive property of inequality, the mean bound is strict so that all points must lie above this line. A linear fit to the GS data is shown with the magenta line ($R^2 = 0.91$, Slope = 0.86), which is fairly close to the line of unity. This indicates that the mean ΔDFC values after GS regression approximately follow the mean theoretical bound over a large range of ΔDFC values. In contrast, the mean ΔDFC values shown for the non-GS regressors are relatively small, consistent with the large orthogonal nuisance fractions observed

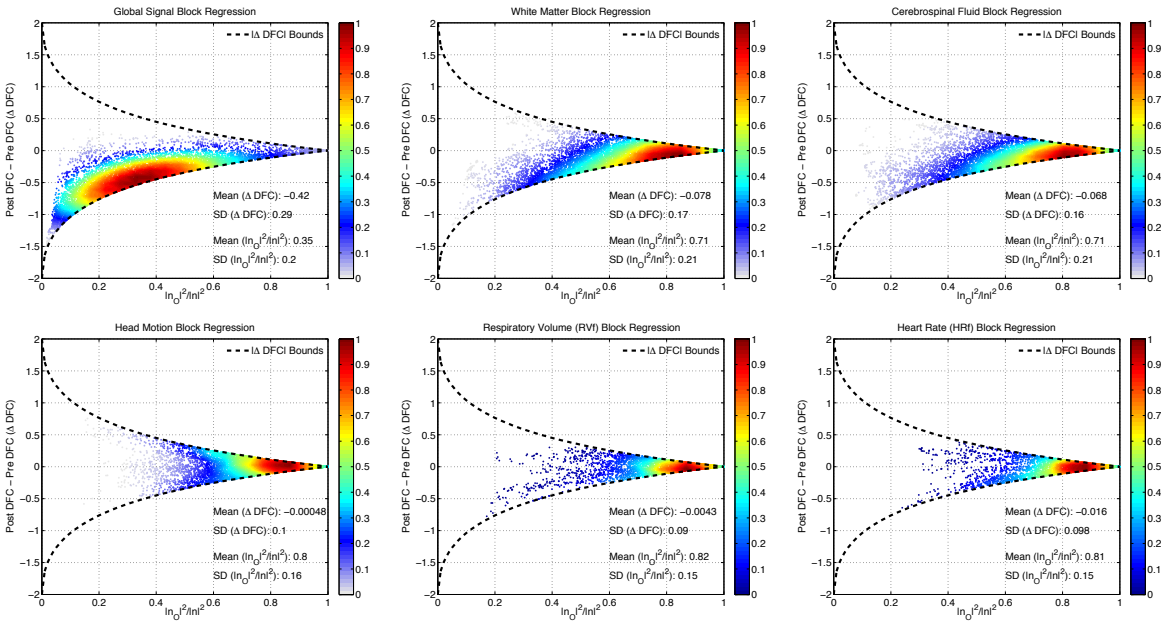

Figure 14: ΔDFC versus the fraction $\frac{|n_O|^2}{|n|^2}$ for six different nuisance regressors (from left to right and top to bottom: GS, WM, CSF, HM, RVf, and HRf) using the data from all scans. Each point in a given plot corresponds to a single window. The theoretical bounds for block regression are shown with the dashed black lines. The empirical ΔDFC values were found to lie within the theoretical bounds. For WM, CSF, HM, RVf and HRf regressors, the mean of the orthogonal fractions ranged between 0.71 and 0.81. In this region, the bounds are fairly narrow such that the mean differences between the pre and post regression values were small, ranging from −0.078 to −0.0005. For GS, the mean orthogonal fraction was 0.35 and the bounds on ΔDFC were not as tight. However, the the ΔDFC values were clustered about a mean value of −0.42 and closely followed the lower bound. Note that the relative density of data points (maximum density is normalized to 1.0) is indicated by the color map on the right-hand side of each plot.

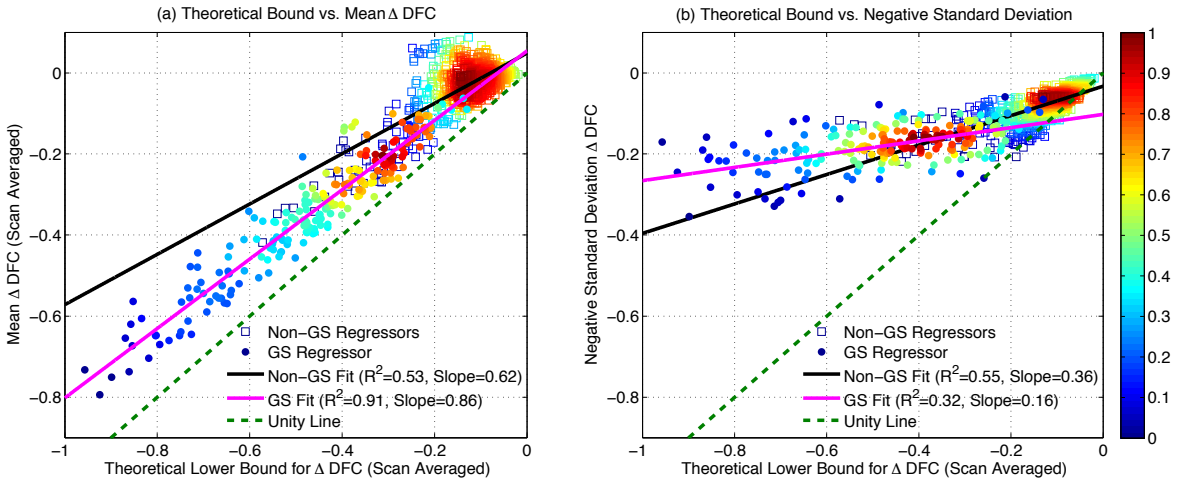

Figure 15: (a) Mean ΔDFC versus scan-averaged theoretical bound. Data for GS and non-GS regressors are indicated by the circle and square markers, respectively. For GS regression, the linear fit (magenta line, $R^2 = 0.91$, Slope = 0.86) is close to the line of unity (dashed green line). For non-GS regressors, the linear fit is indicated by the black line and most values occur at small ΔDFC values. (b) Negative Standard Deviation (NSD) of ΔDFC (over each scan) versus the mean theoretical bound. For GS regression, both the slope of the best fit line (magenta line, $R^2 = 0.32$, Slope = 0.16) and the magnitude of the NSD values are considerably lower than the slope and magnitudes observed for the mean ΔDFC values. The relative density of data points (maximum density is normalized to 1.0) is computed for GS and non-GS data separately and indicated by the color map on the right-hand side.

for these regressors.

In Figure 15b, we plot the negative standard deviation (NSD) of ΔDFC for each scan versus the average bound, where the standard deviation is negated for display purposes. Note that in contrast to the mean ΔDFC values, there is no requirement that these points lie above the line of unity (dashed green line). The slope of the linear fit (magenta line, $R^2 = 0.32$, Slope = 0.16) is much smaller than the slope of 0.86 observed for mean ΔDFC. In addition, the magnitudes of the SD values are significantly smaller than the magnitudes of the mean ΔDFC values ($p < 10^{-6}$, paired t-test). Thus, the primary effect of GS regression is to induce a negative offset in the mean of the DFC estimates accompanied by a relatively smaller change in the DFC fluctuations about the mean. As a result, the Post DFC estimates after GS regression can be approximated to first order as a shifted version of the Pre DFC estimates and will therefore largely retain the correlation with the nuisance norm.

5.4. DFC estimates after full regression

In the Results section we have shown that effects of block regression and full regression on the DFC estimates were very similar. Here, we present the expression for the DFC estimate after full regression as a modified version of the expression obtained for block regression. The main difference between the two approaches is that the regression fit coefficients for full regression are computed from the entire time series data, whereas the fit coefficients for block regression are computed using only the data in the window of interest. However, it important to note that both approaches subtract out a scaled version of the nuisance term in the window of interest.

As derived in Appendix C, the correlation coefficient after full regression for the kth window can be written as:

$$\frac{\hat{x}_{1,k}^T \hat{x}_{2,k}}{|\hat{x}_{1,k}||\hat{x}_{2,k}|} = \frac{x_{1,k}^T x_{2,k} - [1-\gamma_{1,k}\gamma_{2,k}]\frac{|n_{I,k}|^2}{|n_k|^2} x_{1,k}^T P_{n_{I,k}} x_{2,k}}{|x_{1,k}||x_{2,k}|\sqrt{1-[1-\gamma_{1,k}^2]\frac{|n_{I,k}|^2}{|n_k|^2}\frac{x_{1,k}^T P_{n_{I,k}} x_{1,k}}{|x_{1,k}|^2}}\sqrt{1-[1-\gamma_{2,k}^2]\frac{|n_{I,k}|^2}{|n_k|^2}\frac{x_{2,k}^T P_{n_{I,k}} x_{2,k}}{|x_{2,k}|^2}}} \tag{6}$$

where the hat notation is used to refer to time courses after full regression, and $\gamma_{1,k}$ and $\gamma_{2,k}$ are scalar correction terms that account for the difference between full and block regression. When these correction terms are zero, the expressions for block and full regression are identical.

The correction terms $\gamma_{1,k}$ and $\gamma_{2,k}$ are empirically determined and vary with the specific features of the signal and nuisance time series. Thus, in contrast to the block regression, it is challenging to derive theoretical bounds on ΔDFC for the full regression case. However, as shown in Figure 16, the empirical values of ΔDFC for full regression are largely within the theoretical bounds obtained for block regression. This is consistent with the empirical similarity of the DFC estimates obtained after block and full regression.

The correction term for a time course x_1 is given in Appendix C as $\gamma_{1,k} = \frac{\beta_{1,F} - \beta_{1,k}}{\beta_{1,k}}$, where $\beta_{1,F}$ is the regression fit coefficient using full regression and $\beta_{1,k}$ is the per-window fit coefficient. As defined, the correction term $\gamma_{1,k}$ blows up for very small $\beta_{1,k}$ values. However, in practice this term is actually multiplied by $\beta_{1,k}$ (see Equation (C.5)), so that it is sufficient to consider the difference between the fit coefficients $m_{1,k} = \beta_{1,F} - \beta_{1,k}$. Similarly, for the second time series x_2, we may consider the difference term $m_{2,k}$. Note that as $m_{1,k}$, $m_{2,k}$, and the product term $m_{1,k}m_{2,k}$ go to zero, then $\gamma_{1,k}$, $\gamma_{2,k}$, and the product $\gamma_{1,k}\gamma_{2,k}$ all approach zero, and full regression and block regression become identical operations. To provide examples of the behavior of these differences, Supplementary Figure 2 shows that $m_{1,k}$ and

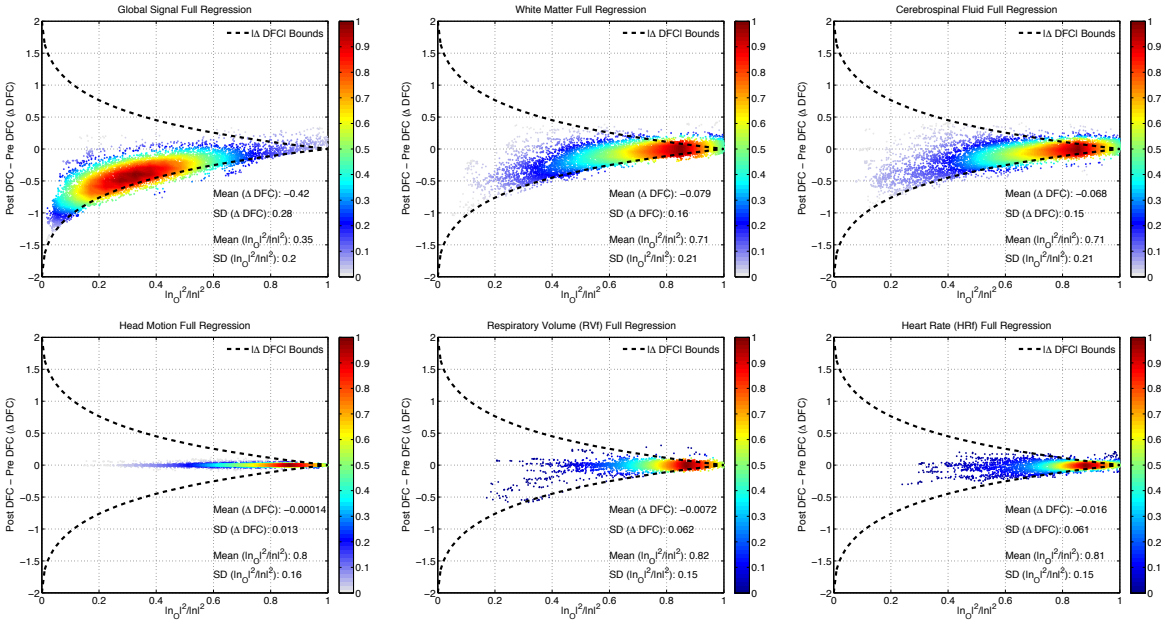

Figure 16: The empirical ΔDFC versus the $\frac{|n_O|^2}{|n|^2}$ fraction for the same six nuisance regressors used in Figure 14. This time full regression was performed to obtain the empirical values but the same bounds are taken from the block regression derivation. The empirical values of ΔDFC for full regression are largely within the theoretical bounds obtained for block regression, consistent with the empirical similarity of the DFC estimates obtained after block and full regression. Note that the relative density of data points (maximum density is normalized to 1.0) is indicated by the color map on the right-hand side of each plot.

$m_{1,k}$ as well as the product $m_{1,k}m_{2,k}$ are centered around zero for the GS and WM regressors.

6. Discussion

6.1. Summary

We have shown that sliding window correlation DFC estimates can be significantly correlated with the sliding window norms of various nuisance measurements. This relationship between the DFC estimates and the nuisance norms can exist even when the correlations between the underlying nuisance and seed time courses are relatively weak. Moreover, we found that significant correlations between the DFC estimates and nuisance norms can persist even after performing nuisance regression. We derived mathematical expressions to describe the effects and limitations of nuisance regression on DFC estimates and demonstrated that the empirical results lie within the theoretically predicted bounds.

Based on our empirical and theoretical findings, we identified two main mechanisms for the inefficacy of nuisance regression. First, as shown in Figure 5, the DFC estimates can be significantly correlated with the nuisance norms even when the underlying correlation between the nuisance terms and the seed signals is relatively low. As a result, for most cases a large fraction of the nuisance term is orthogonal to the subspace spanned by the seed signals. This greatly reduces the efficacy of nuisance regression, such that the difference between the Pre and Post DFC values is relatively small and the relation between the DFC estimates and nuisance norms is largely unaffected. We observed this major limitation of nuisance regression particularly for WM, CSF, RVf, HRf, and HM regressors as demonstrated in Figure 14.

The second mechanism applies primarily to the GS nuisance term which has a smaller orthogonal nuisance fraction. The reduced orthogonal nuisance fraction reflects the fact that the GS is computed as the average of the BOLD time courses across the brain and will therefore tend to have a higher correlation with the seed time courses. As shown in Figure 15, the mean ΔDFC values observed for GS regression closely follow the mean theoretical bound, which approaches -1 as the orthogonal fraction approaches zero. In contrast, the standard deviations of the ΔDFC values exhibit a much weaker dependence on the orthogonal fraction. As a result, the Post DFC estimates after GS regression can be approximated to first order as a shifted version of the Pre DFC estimates, as illustrated in the second row of Figure 13 for a representative subject. Because the Post DFC estimates largely retain the fluctuations in the Pre DFC estimates, they will also retain the correlation with the nuisance norm. Although this mechanism is most often observed for the GS nuisance term, it can sometimes be observed for WM and CSF nuisance terms (e.g. upper righthand panel in Figure 7) in which partial volume effects with gray matter can make the WM and CSF nuisance terms to behave more like the GS.

6.2. Nuisance effects in DFC studies

It has been previously noted that nuisance effects might account for a significant portion of the fluctuations in DFC estimates (Preti et al., 2017; Hutchison et al., 2013). The effects may be especially pronounced for sliding window DFC estimates since transient nuisance effects can greatly alter the correlation estimates within a short temporal window (Hutchison et al., 2013). Chang and Glover (2010) looked at the inter-subject correlation between measures of motion and DFC variability, but did not find a significant relation. They also found that intra-subject correlations between sequences of motion-based and DFCs-based deviations were not significant. However, they did not consider the intra-subject correlation between DFC estimates and motion nuisance norms, as was done in this study.

Nikolaou et al. (2016) found that dynamic fluctuations in network degree were significantly correlated with the spectral power of end-tidal CO_2 and heart rate measurements. Since the network degree calculated in that study was based on the summation of DFC magnitudes, these findings are roughly consistent with our observation of a significant correlation between the DFC estimates and the norms of the RVf and HRf regressors.

Glomb et al. (2017) reported that measures of instantaneous BOLD variance (averaged over the entire brain) were significantly correlated with instantaneous average correlation (over all pairwise correlations). These results are consistent with our findings of significant correlations between DFC estimates and the GS norm. A distinction is that the GS norm is the magnitude of an average signal whereas the instantaneous BOLD variance was calculated as the average across variances from different regions. However, prior work has shown that these two measures are highly related (He and Liu, 2012; Wong et al., 2012).

While the GS is widely used to capture global nuisance effects such as head motion and respiration (Power et al., 2017; Liu et al., 2017), there is also growing evidence that the GS reflects neural and vigilance related fluctuations (Schölvinck et al., 2010; Chang et al., 2016; Falahpour et al., 2018). For instance, Falahpour et al. (2018) reported the existence of significant temporal correlations between the GS and EEG measures of vigilance. The findings in this paper suggest that DFC estimates may reflect vigilance changes both before and after GSR. However, further work is needed to better understand the connection between DFC estimates and vigilance.

A key finding of this paper is that DFC estimates can be significantly correlated with the nuisance norms even when the underlying nuisance time courses are largely orthogonal to the observed seed time courses. As discussed in the Interpretation section, one plausible scenario that can give rise to this effect is the presence of an additive nuisance term consisting of time-varying in-plane and orthogonal terms, where the magnitudes of the two terms are roughly in sync and the magnitude of the orthogonal term is much larger than that of the in-plane term. Under this scenario, the time-varying in-plane component can cause DFC fluctuations in the observed vectors (since it affects the inner-angle between the observed vectors) even when the correlation of the putative underlying signal vectors (which are not observed) remains constant. Because of the assumed relation between the in-plane and orthogonal terms, the resulting DFC fluctuations are correlated with the overall nuisance norm. In addition, because the in-plane term is much smaller than the orthogonal term, the overall nuisance term exhibits a weak correlation with the observed vectors. It is important to note that while this plausible scenario provides some insight into the empirical observations and helps to motivate the theoretical findings, it is by no means intended to serve

as a "model" of the data. The modeling of nuisance effects in fMRI is still an area of active investigation (Liu, 2016; Murphy et al., 2013) and future work will be needed to more fully characterize the impact of nuisance terms on DFC estimates. Such efforts are likely to require a consideration of the non-linear and non-stationary aspects of the underlying signals (Sugihara et al., 2012; Hutchison et al., 2013).

6.3. Efficacy of Nuisance regression in DFC studies

Although nuisance regression is widely performed in DFC studies (Hutchison et al., 2013; Preti et al., 2017), its effects on DFC estimates have received relatively little attention. Nikolaou et al. (2016) reported that regression reduced but did not eliminate the correlation between dynamic measures of network degree and measures of heart rate and respiratory spectral power. These findings are aligned with our results showing that regression has a limited effect on the correlation between DFC estimates and HRf and RVf nuisance norms.

In recent work, Xu et al. (2018) found that global signal regression (GSR) had a maximal impact on DFC estimates in temporal windows where the GS mean absolute magnitude was large and a lesser impact in windows where the magnitude was small. They interpreted their findings using the framework introduced in (Nalci et al., 2017b), where it was shown that GSR can be approximated as a temporal down-weighting process. The authors noted that the main effect of GSR was a spatially heterogenous negative shift in the sliding window correlation values, an observation consistent with the predominantly negative ΔDFC values found in this study (see the upper righthand panel in Figure 14). However, in contrast to our study, the authors did not consider the relationship between the DFC estimates and the GS norm.

There is related work for static FC studies regarding the inefficacy of nuisance regression (Power et al., 2017; Chang et al., 2009; Power et al., 2012, 2014). For instance, Power et al. (2017) observed that frame-wise displacement (FD), a summary measure for head motion, remained highly related with resting-state fMRI time courses even after HM regression. As we have discussed in detail, the efficacy of nuisance regression increases as the orthogonal nuisance fraction decreases. Due to the inverse relationship between the orthogonal fraction and the percent variance explained (see Figure 10), this means that the efficacy of regression will decrease as the amount of variance that can be explained by the nuisance regressors decreases. Chang et al. (2009) found that RVf and HRf regressors together explained only 15.8% of the average total variance in voxels which showed a significant correlation between the BOLD time series and nuisance regressors, and the RVf regressor explained only 11.7% of the average total variance when used

on its own. This is roughly in-line with the results shown in Figure 5 where the percent variance values observed for non-GS regressors were largely below 20%, with 51% of the significant correlations between the DFC estimates and nuisance norms occurring when the percent variance was less than 6.25%.

In this work, we used the sliding window correlation approach, which is widely used in DFC studies (Preti et al., 2017; Hutchison et al., 2013; Calhoun et al., 2014) both as a primary analysis approach and as an intermediate analysis step (e.g. used to generate DFC estimates that are then further analyzed with k-means clustering or principal components analysis). Other approaches include time-frequency methods such as wavelet transform coherence (Chang and Glover, 2010; Yaesoubi et al., 2015) and probabilistic methods such as hidden Markov modeling (Vidaurre et al., 2017). Regardless of the approach, nuisance regression is a standard preprocessing step using nuisance regressors based on either independent measures (e.g. physiological measurements) or data-driven measures (e.g. GS, WM+CSF, or independent component analysis (ICA) components). Our findings regarding the limited efficacy of nuisance regression for sliding-window DFC estimates suggest that caution must also be exercised when interpreting DFC estimates obtained with other approaches. Nevertheless, future work to assess the impact of nuisance terms and the efficacy of nuisance regression when applied to additional DFC approaches would be of great interest.

6.4. Nuisance Norm Regression

Given the inefficacy of nuisance regression in reducing the correlations between the DFC estimates and the nuisance norms, it is reasonable to consider alternative approaches. A potential solution is to compute the sliding window norm for each regressor and then project these out from the DFC estimates using linear regression. This procedure is described in Appendix D and referred to as *nuisance norm regression* (NNR). This approach differs from traditional nuisance regression techniques which regress out nuisance measures directly from the fMRI time courses. Instead, NNR acts on the correlation coefficients. Further work is needed to characterize the potential advantages and disadvantages of this approach.

6.5. Conclusion

We have provided a detailed examination of nuisance effects and regression in DFC measures. Our findings both confirm and significantly extend the limited prior work in this area. In particular, we have shown that DFC estimates can be significantly correlated with nuisance norms even when there is only a weak correlation between the nuisance and seed signals. We demonstrated that nuisance regression

is largely ineffective for the removal of this relationship. Furthermore, we provided a mathematical framework to describe the effects of nuisance regression and showed that the experimental findings are in agreement with the theoretical predictions.

This work highlights a potential confound in the interpretation of DFC studies, which typically make the implicit assumption that nuisance effects are largely minimized in the pre-processing stage. Our findings suggest that the interpretation of DFC measures should be expanded to consider potential correlations with the nuisance norms. Because nuisance effects such as subject motion, respiration, and cardiac activity originate in brain networks that control these functions (Liu, 2016; Iacovella and Hasson, 2011), a significant correlation between DFC estimates and the norms of these nuisance effects does not necessarily invalidate the use of the DFC estimates. Indeed, it may lead to a more complete interpretation that takes into account the role of functional networks responsible for physiological activity. Future DFC studies would benefit from the development of analysis methods that more effectively take into account the origins of the nuisance effects.

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Appendix A. Block regression on windowed time series

For the kth window, let $x_{1,k}$ and $x_{2,k}$ be a pair of windowed fMRI time series, with $\tilde{x}_{1,k}$ and $\tilde{x}_{2,k}$ corresponding to the time courses after block regression using n_k as the nuisance regressor. Since each window is treated independently in block regression, we can simplify the derivations by dropping the window index k for now. We also assume without loss of generality that the time series have zero mean.

For x_1 we can write the time course after regression as:

$$\tilde{x}_1 = x_1 - n(n^T n)^{-1} n^T x_1 = x_1 - P_n x_1, \tag{A.1}$$

where $P_n = n(n^T n)^{-1} n^T$ is the projection matrix onto n, and a similar expression holds for x_2. For the derivations that follow, we decompose the nuisance regressor into in-plane and orthogonal components such that $n = n_I + n_O$ where n_I is the component of n that lies in the subspace spanned by x_1 and x_2, and n_O is the orthogonal complement.

The squared norm for the time series after regression is:

$$\tilde{x}_1^T \tilde{x}_1 = (x_1 - P_n x_1)^T (x_1 - P_n x_1) \tag{A.2}$$

$$= x_1^T x_1 - x_1^T P_n x_1 \tag{A.3}$$

$$= x_1^T x_1 - \frac{x_1^T (n_I + n_O)(n_I + n_O)^T x_1}{|n|^2} \tag{A.4}$$

$$= x_1^T x_1 - \frac{x_1^T n_I n_I^T x_1}{|n|^2} \tag{A.5}$$

$$= x_1^T x_1 - \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_1, \tag{A.6}$$

where $|n|^2 = n_I^T n_I + n_O^T n_O$, $P_{n_I} = n_I (n_I^T n_I)^{-1} n_I^T$, and we have made use of the relation $n_O^T x_1 = 0$ and the symmetry of the projection matrices. The corresponding norm can then be written as:

$$|\tilde{x}_1| = \sqrt{x_1^T x_1 - \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_1} \tag{A.7}$$

$$= |x_1| \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_1^T P_{n_I} x_1}{|x_1|^2}}. \tag{A.8}$$

A similar derivation holds for $|\tilde{x}_2|$.

To compute the correlation coefficient after regression, we start with the dot product between $\tilde{x}_1$ and $\tilde{x}_2$ and use the orthogonal decomposition to write:

$$\tilde{x}_1^T \tilde{x}_2 = (x_1 - P_n x_1)^T (x_2 - P_n x_2) \tag{A.9}$$

$$= x_1^T x_2 - \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_2, \tag{A.10}$$

where the omitted steps in the derivation are similar to those shown above for the derivation of the norm. Normalizing the dot product by the appropriate norms yields the correlation coefficient after regression as:

$$\tilde{r} = \frac{\tilde{x}_1^T \tilde{x}_2}{|\tilde{x}_1||\tilde{x}_2|} = \frac{x_1^T x_2 - \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_2}{|x_1||x_2| \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_1^T P_{n_I} x_1}{|x_1|^2}} \sqrt{1 - \frac{|n_I|^2}{|n|^2} \frac{x_2^T P_{n_I} x_2}{|x_2|^2}}}. \tag{A.11}$$

This can be rewritten in the following form:

$$\tilde{r} = \lambda r + O, \tag{A.12}$$

where $r = \dfrac{x_1^T x_2}{|x_1||x_2|}$ is the correlation coefficient prior to regression, and the scaling λ and offset O terms are defined as:

$$\lambda = \frac{1}{\sqrt{1 - \frac{|n_I|^2}{|n|^2}\frac{x_1^T P_{n_I} x_1}{|x_1|^2}}\sqrt{1 - \frac{|n_I|^2}{|n|^2}\frac{x_2^T P_{n_I} x_2}{|x_2|^2}}} \tag{A.13}$$

$$O = \frac{-\lambda \frac{|n_I|^2}{|n|^2} x_1^T P_{n_I} x_2}{|x_1||x_2|}. \tag{A.14}$$

Note that if the orthogonal nuisance component n_O is large compared to the in-plane component n_I, the ratio $\dfrac{|n_I|^2}{|n|^2} = 1 - \dfrac{|n_O|^2}{|n_I|^2 + |n_O|^2}$ approaches 0, such that the scaling term λ approaches 1, and the offset term O goes to 0. As a result, the correlation coefficients before and after regression will be approximately equal $\tilde{r} \approx r$.

To gain further insight, it is useful to rewrite Equation (A.11) using trigonometric functions. First, note that the correlation coefficient prior to regression can be also written as $r = \cos\theta$, where θ is the angle between x_1 and x_2. Then without loss of generality, we can write $\theta = \theta_1 + \theta_2$ where θ_1 is the angle between n_I and x_1 and θ_2 is the angle between n_I and x_2, where both θ_1 and θ_2 are assumed to be non-negative. This corresponds to the case where the in-plane term n_I "lies" between the vectors x_1 and x_2. Note it also possible for the in-plane term to lie outside of the vectors, such that $\theta = \pm(\theta_1 - \theta_2)$. We refer to this second case as the complementary case.

With the stated assumptions, we rewrite the post regression DFC in Equation (A.11) as:

$$\tilde{r} = \frac{\tilde{x}_1^T \tilde{x}_2}{|\tilde{x}_1||\tilde{x}_2|} = \frac{\dfrac{x_1^T x_2}{|x_1||x_2|} - \dfrac{|n_I|^2}{|n|^2}\dfrac{x_1^T P_{n_I} x_2}{|x_1||x_2|}}{\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\dfrac{x_1^T P_{n_I} x_1}{|x_1|^2}}\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\dfrac{x_2^T P_{n_I} x_2}{|x_2|^2}}} \tag{A.15}$$

$$= \frac{\cos\theta - \dfrac{|n_I|^2}{|n|^2}\cos\theta_1\cos\theta_2}{\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_1}\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_2}} \tag{A.16}$$

$$= \frac{\cos\theta - \dfrac{|n_I|^2}{|n|^2}(\cos\theta \pm \sin\theta_1\sin\theta_2)}{\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_1}\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_2}} \tag{A.17}$$

$$= \frac{\left(1 - \dfrac{|n_I|^2}{|n|^2}\right)\cos\theta \mp \dfrac{|n_I|^2}{|n|^2}\sin\theta_1\sin\theta_2}{\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_1}\sqrt{1 - \dfrac{|n_I|^2}{|n|^2}\cos^2\theta_2}}, \tag{A.18}$$

where we have used the identities:

$$x_1^T P_{n_I} x_2 = \frac{x_1^T n_I n_I^T x_2}{|n_I|^2} = |x_1||x_2|\cos\theta_1\cos\theta_2, \tag{A.19}$$

$$x_1^T P_{n_I} x_1 = |x_1|^2\cos^2\theta_1, \tag{A.20}$$

$$x_2^T P_{n_I} x_2 = |x_2|^2\cos^2\theta_2. \tag{A.21}$$

Also note that in Equation (A.17) we have used the trigonometric identity $\cos\theta_1\cos\theta_2 = \cos\theta \pm \sin\theta_1\sin\theta_2$. If $\theta = \theta_1 + \theta_2$, the sign of $\mp$ in Equation (A.18) is a minus ($-$), and in the complementary case for $\theta = \pm(\theta_1 - \theta_2)$ (i.e. when n_I lies outside of x_1 and x_2) the sign is a $+$.

Finally, by making the substitution $\dfrac{|n_I|^2}{|n|^2} = 1 - \dfrac{|n_O|^2}{|n|^2}$, the correlation coefficient after regression is:

$$\tilde{r} = \frac{\dfrac{|n_O|^2}{|n|^2}\cos\theta \mp \left(1 - \dfrac{|n_O|^2}{|n|^2}\right)\sin\theta_1\sin\theta_2}{\sqrt{\sin^2\theta_1 + \dfrac{|n_O|^2}{|n|^2}\cos^2\theta_1}\sqrt{\sin^2\theta_2 + \dfrac{|n_O|^2}{|n|^2}\cos^2\theta_2}}. \tag{A.22}$$

If the orthogonal component becomes small and the orthogonal fraction $\dfrac{|n_O|^2}{|n|^2} \to 0$, then $\tilde{r}$ reduces

to:

$$\tilde{r} = \pm \frac{\sin\theta_1 \sin\theta_2}{|\sin\theta_1||\sin\theta_2|} = \pm 1, \tag{A.23}$$

where the minus sign applies to the case where the nuisance vector lies between the observed vectors and the plus sign applies to the complementary case where the nuisance vector lies outside the vectors.

Appendix B. Derivation of limits on difference in DFC estimates

Here we derive the analytical bound on the difference $\Delta\text{DFC} = \tilde{r} - r$, between the pre and post regression DFC estimates. First, we provide derivations for the case when $\theta = \theta_1 + \theta_2$ (we will consider the complementary case $\theta = \pm(\theta_1 - \theta_2)$ later). In this case, the ΔDFC is:

$$\Delta\text{DFC} = \frac{\frac{|n_O|^2}{|n|^2}\cos(\theta_1+\theta_2) - \left(1 - \frac{|n_O|^2}{|n|^2}\right)\sin\theta_1\sin\theta_2}{\sqrt{\sin^2\theta_1 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_1}\sqrt{\sin^2\theta_2 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_2}} - \cos(\theta_1+\theta_2). \tag{B.1}$$

Note that since n_I lies between the observed vectors x_1 and x_2, nuisance regression will increase the angle (and decrease the correlation value) between the vectors x_1 and x_2, and we will have $\tilde{r} < r$. As a result, we are looking for a lower bound on ΔDFC as a function of $\frac{|n_O|^2}{|n|^2}$.

It can be shown empirically that the expression for ΔDFC is minimized when $\theta_1 = \theta_2$. Using this,

we simplify the original expression for ΔDFC in Equation B.1 as:

$$\Delta\text{DFC} = \frac{\frac{|n_O|^2}{|n|^2}\cos 2\theta_1 - \left(1 - \frac{|n_O|^2}{|n|^2}\right)\sin^2\theta_1}{\sin^2\theta_1 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_1} - \cos 2\theta_1 \tag{B.2}$$

$$= \frac{\left(-\sin^2\theta_1 - \frac{|n_O|^2}{|n|^2}\cos^2\theta_1 + \frac{|n_O|^2}{|n|^2}\right)\cos 2\theta_1 + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2\theta_1}{\sin^2\theta_1 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_1} \tag{B.3}$$

$$= \frac{\left(\frac{|n_O|^2}{|n|^2}\sin^2\theta_1 - \sin^2\theta_1\right)\cos 2\theta_1 + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2\theta_1}{\sin^2\theta_1 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_1} \tag{B.4}$$

$$= \frac{\left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2\theta_1(2\cos^2\theta_1 - 1) + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2\theta_1}{\sin^2\theta_1 + \frac{|n_O|^2}{|n|^2}\cos^2\theta_1} \tag{B.5}$$

$$= \frac{\left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2 2\theta_1}{(1 - \cos 2\theta_1) + \frac{|n_O|^2}{|n|^2}(1 + \cos 2\theta_1)} \tag{B.6}$$

$$= \frac{\left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2 2\theta_1}{1 + \frac{|n_O|^2}{|n|^2} + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\cos 2\theta_1}, \tag{B.7}$$

where in the numerator of Equation (B.6), we used the trigonometric identity: $2\sin^2\theta_1\cos^2\theta_1 = \sin^2 2\theta_1/2$, and in the denominator of the same equation, we used: $\sin^2\theta_1 = (1 - \cos 2\theta_1)/2$ and $\cos^2\theta_1 = (1 + \cos 2\theta_1)/2$. We then take the partial derivative with respect to θ_1 as:

$$\frac{\partial}{\partial\theta_1}[\Delta\text{DFC}] = \frac{\partial}{\partial\theta_1}\left[\frac{\left(\frac{|n_O|^2}{|n|^2} - 1\right)\sin^2 2\theta_1}{1 + \frac{|n_O|^2}{|n|^2} + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\cos 2\theta_1}\right] \tag{B.8}$$

$$= 2\sin 2\theta_1\left(1 - 4\frac{\frac{|n_O|^2}{|n|^2}}{\left(1 + \frac{|n_O|^2}{|n|^2} + \left(\frac{|n_O|^2}{|n|^2} - 1\right)\cos 2\theta_1\right)^2}\right). \tag{B.9}$$

Now solving for the non-trivial roots of the derivative yields $\theta_1^* = \pm tan^{-1}\left(\sqrt{\frac{|n_O|}{|n|}}\right)$. Finally, by using

this value in the ΔDFC expression we find the lower bound as:

$$\Delta\text{DFC}_{LB} = -\left(\frac{4}{1+|n_O|/|n|} - 2\right) = -2\left(\frac{|n|-|n_O|}{|n|+|n_O|}\right). \tag{B.10}$$

For the complementary case: $\theta = \pm(\theta_1 - \theta_2)$, in which n_I is positioned outside of x_1 and x_2, linear regression will decrease the angle between the vectors x_1 and x_2 and the correlation value after regression will increase yielding $\Delta\text{DFC} \geq 0$. Therefore, we are seeking an upper bound. It can be shown empirically that the expression for ΔDFC is maximized when $\theta_1 + \theta_2 = \pi$. Following a derivation similar to the one used for the lower bound, the upper bound is obtained as:

$$\Delta\text{DFC}_{UB} = +2\left(\frac{|n|-|n_O|}{|n|+|n_O|}\right). \tag{B.11}$$

Appendix C. Derivation of DFC estimate after full regression

In this section, we show that the correlation coefficients obtained after full regression can be expressed as a modified version of the correlation coefficients obtained with block regression. First, we note that for the kth window the residual after block regression can be written as:

$$\tilde{x}_{1,k} = x_{1,k} - P_{n_k} x_{1,k} \tag{C.1}$$

$$= x_{1,k} - \beta_{1,k} n_k, \tag{C.2}$$

where $P_{n_k} = n_k(n_k^T n_k)^{-1} n_k$ is the projection matrix onto the windowed nuisance time series n_k, and $\beta_{1,k} = (n_k^T n_k)^{-1} n_k^T x_{1,k}$ is the corresponding scalar fit coefficient computed for the windowed time series $x_{1,k}$.

For full regression the residual can be written as:

$$\hat{x}_{1,k} = x_{1,k} - \beta_{1,F} n_k, \tag{C.3}$$

where $\beta_{1,F} = (n^T n)^{-1} n^T x_1$ is the scalar fit coefficient computed using the entire time series x_1 and the full nuisance time series n.

Noting that the expressions in Equation (C.2) and Equation (C.3) differ only in the scalar fit coefficients $\beta_{1,F}$ and $\beta_{1,k}$, we can rewrite Equation (C.3) as:

$$\hat{x}_{1,k} = x_{1,k} - (\beta_{1,k} + m_{1,k})\, n_k \tag{C.4}$$

$$= x_{1,k} - \left(1 + \frac{m_{1,k}}{\beta_{1,k}}\right) \beta_{1,k} n_k \tag{C.5}$$

$$= x_{1,k} - (1 + \gamma_{1,k}) P_{n_k} x_{1,k}, \tag{C.6}$$

where $m_{1,k} = \beta_{1,F} - \beta_{1,k}$ is the difference in fit coefficients and $\gamma_{1,k} = \frac{m_{1,k}}{\beta_{1,k}}$ is a correction term that captures the difference between full and block regression. Note that if $\gamma_{1,k} = 0$ then full regression and block regression have identical effects for the kth window. The expression for $\hat{x}_{2,k}$ has the same form as Equation (C.6).

To compute the correlation coefficient between $\hat{x}_{1,k}$ and $\hat{x}_{2,k}$, we first note that the dot product can be written as:

$$\hat{x}_{1,k}^T \hat{x}_{2,k} = (x_{1,k} - (1+\gamma_{1,k})P_{n_k} x_{1,k})^T (x_{2,k} - (1+\gamma_{2,k})P_{n_k} x_{2,k}) \tag{C.7}$$

$$= x_{1,k}^T x_{2,k} - [1 - \gamma_{1,k}\gamma_{2,k}] x_{1,k}^T P_{n_k} x_{2,k} \tag{C.8}$$

$$= x_{1,k}^T x_{2,k} - [1 - \gamma_{1,k}\gamma_{2,k}] \frac{|n_{I,k}|^2}{|n_k|^2} x_{1,k}^T P_{n_{I,k}} x_{2,k}, \tag{C.9}$$

which is similar in form to the dot product expression previously derived in Equation (A.10) for block regression with the addition of a correction term $[1 - \gamma_{1,k}\gamma_{2,k}]$ and the window index k. The norms $|\hat{x}_{1,k}|$ and $|\hat{x}_{1,k}|$ can be derived using similar modifications.

Incorporating all the correction terms, we can write the correlation coefficient after full regression as:

$$\frac{\hat{x}_{1,k}^T \hat{x}_{2,k}}{|\hat{x}_{1,k}||\hat{x}_{2,k}|} = \frac{x_{1,k}^T x_{2,k} - [1 - \gamma_{1,k}\gamma_{2,k}] \frac{|n_{I,k}|^2}{|n_k|^2} x_{1,k}^T P_{n_{I,k}} x_{2,k}}{|x_{1,k}||x_{2,k}| \sqrt{1 - [1 - \gamma_{1,k}^2] \frac{|n_{I,k}|^2}{|n_k|^2} \frac{x_{1,k}^T P_{n_{I,k}} x_{1,k}}{|x_{1,k}|^2}} \sqrt{1 - [1 - \gamma_{2,k}^2] \frac{|n_{I,k}|^2}{|n_k|^2} \frac{x_{2,k}^T P_{n_{I,k}} x_{2,k}}{|x_{2,k}|^2}}}. \tag{C.10}$$

Note that if $\gamma_{1,k} = \gamma_{2,k} = 0$ and we drop the window index k, then this expression is identical to the expression for block regression previously shown in Equation (A.11).

Appendix D. Nuisance Norm Regression (NNR)

Nuisance norm regression aims to remove the nuisance norm-related variance from the DFC estimates. In this technique, nuisance removal can be performed in the relevant DFC metric space in addition to traditional nuisance removal on raw time series. For sliding window correlations obtained between pairs of ROIs, we define a sliding window correlation vector $R = [r_1, r_2, \cdots, r_k, \cdots, r_M]^T$, where k is the window index, M is the total number of windows, and r_k is the pairwise correlation value for the kth window. Similarly, we define a nuisance norm vector (or matrix for multiple nuisance terms)

$N = [|n_1|, |n_2|, \cdots, |n_k|, \cdots, |n_M|]^T$, where $|n_k|$ is the norm of the nuisance regressor for the kth window. Then the effects of a nuisance norm can be removed through linear regression to obtain a "clean" DFC estimate $\tilde{R} = R - N(N^T N)^{-1} N^T R$. In the case of multiple nuisance terms, the nuisance norm matrix can be expanded to include additional norm vectors. For example, let N_{WM}, N_{CSF}, N_{RVf}, N_{HRf}, and N_{HM} be the sliding window norm vectors for the WM, CSF, RVf, HRf, and HM (total norm) regressors, respectively. Then the corresponding nuisance norm matrix is $N = [N_{WM}, N_{CSF}, N_{RVf}, N_{HRf}, N_{HM}]$. Potential variations include expansion of the nuisance norm matrix to include additional terms, such as the squared norms or temporal derivatives of the sliding window norms.

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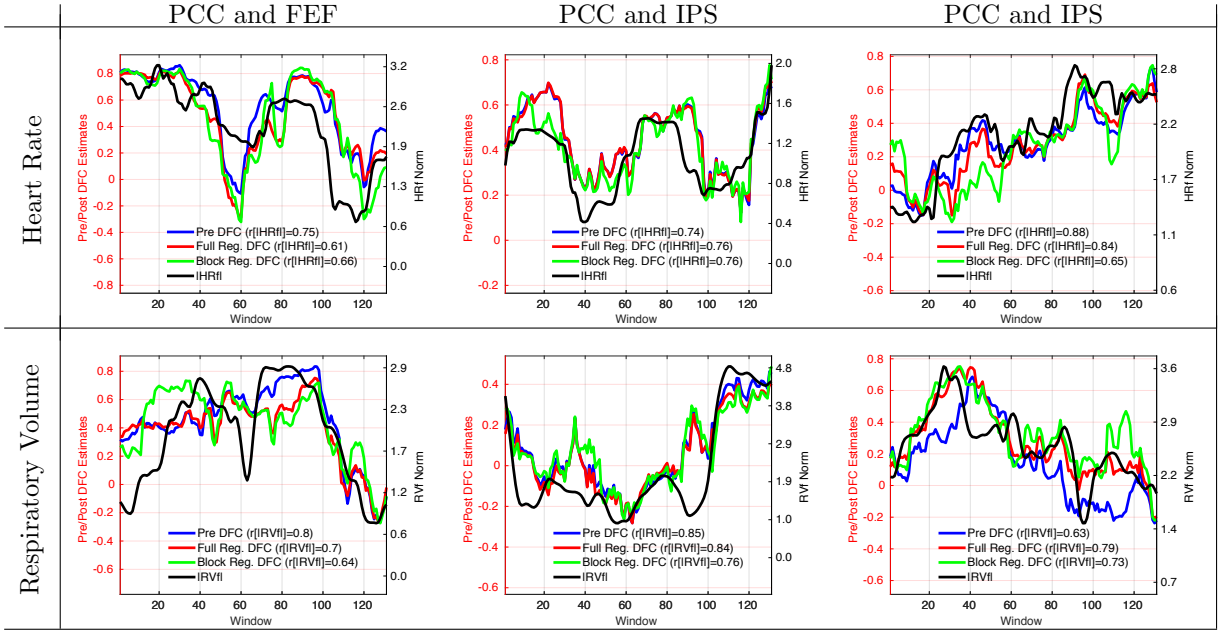
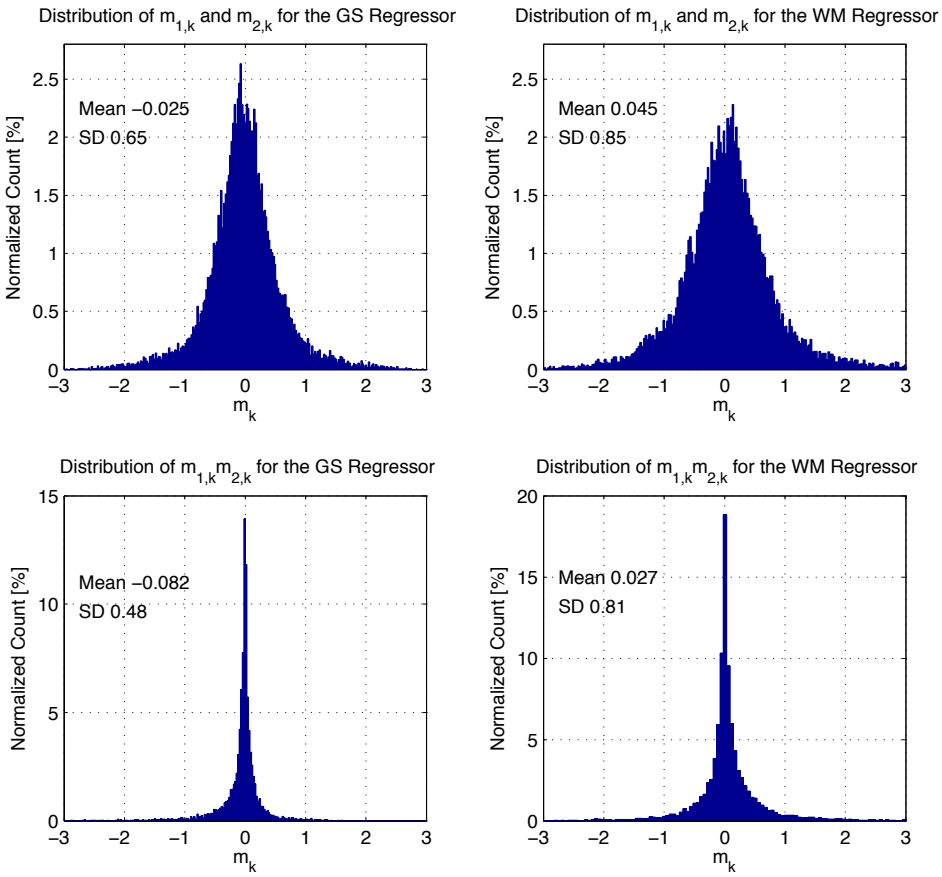
Appendix E. Supplementary Figures



Supplementary Figure 1: 6 representative scans showing that both before and after nuisance regression the DFC estimates (e.g. between PCC and IPS, PCC and MOT and PCC and FEF) are strongly correlated with the respective nuisance norms.



Supplementary Figure 2: Top row: Histograms of the difference terms $m_{1,k} = \beta_{1,F} - \beta_{1,k}$ and $m_{2,k} = \beta_{2,F} - \beta_{2,k}$ for GS and WM regressors. The plots combine the instances of the two terms. Bottom row: Histograms of the product term $m_{1,k}m_{2,k}$.