

When more is less: Dual phosphorylation protects

signaling off-state against overexpression

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ABSTRACT Kinases in signaling pathways are commonly activated by multisite phosphorylation. For example, the mitogen-activated protein kinase Erk is activated by its kinase Mek by two consecutive phosphorylations within its activation loop. In this article, we use kinetic models to study how the activation of Erk is coupled to its abundance. Intuitively, Erk activity should rise with increasing amounts of Erk protein. However, a mathematical model shows that the signaling off-state is robust to increasing amounts of Erk, and Erk activity may even decline with increasing amounts of Erk. This counter-intuitive, bell-shaped response of Erk activity to increasing amounts of Erk arises from the competition of the unmodified and single phosphorylated form of Erk for access to its kinase Mek. This shows that phosphorylation cycles can contain an intrinsic robustness mechanism that protects signaling from aberrant activation e.g. by gene expression noise or kinase overexpression following gene duplication events in diseases like cancer.

INTRODUCTION

The MAPK signaling pathway is one of the best studied signaling pathways due to its role in cell fate decisions like proliferation, migration and apoptosis and its critical role in development. Growth factors activate a receptor localised to the cell membrane, from where the signal is relayed by a cascade of kinases that activate each other by (reversible) phosphorylation on multiple sites. The terminal kinase, Erk, activates hundreds of cytoplasmic and nuclear targets (1). The activation of transcription factors induces a transcriptional response which ultimately manifests the cell fate decision.

An understanding of how such kinase cascades operate dynamically and quantitatively has been gained through a number of theoretical and experimental investigations. An early theoretical study showed that a single phosphorylation cycle can create a switch-like response (2). Later on, it was shown that the switch-like stimulus response profile of MAPK activity in *Xenopus* oocytes (3) can be explained by the *in vitro* distributive two-step activation mechanism of Erk (4). The mathematical description of phosphorylation cycles has its unique challenges as, opposed to metabolic networks, enzymes and substrates, all being kinases, mostly occur in similar concentrations. General concepts for modelling multisite-phosphorylation (5–7) and for the analysis of multistability of these systems have been provided (8–10). Many studies focused on the stimulus-response relationship of a kinase that is activated by multisite-phosphorylation. The profile can be graded, biphasic, switch-like or bistable depending on a multitude of factors like the order (11) and/or processivity (12, 13) of multisite phosphorylation, competition effects between modifying enzymes (5) or the sequestration of components within enzyme-substrate complexes (14–17). Some of the effects of competition and sequestration have

been shown experimentally *in vivo*. For instance, the activity of Erk depends on the expression level of its substrates, as deactivating phosphatases and Erk substrates compete for access to Erk in *Drosophila* (18).

Next to the ability to process all-or-none decisions, signaling pathways should provide their response in a robust fashion: the signaling off-state needs to be robust to fluctuating levels of signaling pathway components and to transient weak signals (19, 20). Negative feedbacks are common in MAPK signaling and can provide robustness to Erk activity at various expression levels of Erk (21). However, some robustness might emerge from the phosphorylation cycle motif alone, as e.g. the amount of modified substrate approaches a limit for increasing levels of the substrate in a single modification cycle in its basal state (22).

Here we present a new mechanism that leads to robust stationary Erk activity at Erk overexpression, which emerges from the distributive kinetics of Erk phosphorylation. We find that for low pathway activity and increasing levels of total Erk, the stationary amount of active dual phosphorylated Erk shows a bell-shaped response: With increasing amounts of Erk, Erk activity increases until it reaches a maximum after which active Erk starts to decrease and eventually approaches zero. This bell-shaped response is due to the gradual saturation of Mek with its substrate and the subsequent competition of unmodified and single phosphorylated Erk for access to Mek. This response can be seen regardless of the order of Erk (de)activation and the kind of phosphatases involved in dephosphorylation of threonine and tyrosine on Erk. We derive an analytical approximation of the maximum in the bell-shaped response which allows to estimate the biological relevance of the phenomenon based on the catalytic rate constants.

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param. [$s^{-1}\mu M^{-1}$]	value	comment
k_{on1}	0.18	measured in (26)
$k_{on2}, k_{onp1}, k_{onp2}$	0.18	as k_{on1}
k_{cat1}/K_{M1}	$3.9 \cdot 10^{-2}$	measured in (26)
k_{cat2}/K_{M2}	$2.1 \cdot 10^{-2}$	measured in (26)
param. [s^{-1}]	value	comment
d_1	$6.7 \cdot 10^{-3}$	$pYErk \rightarrow Erk$ (26)
d_2	$4.0 \cdot 10^{-3}$	$pTpYErk \rightarrow pYErk$ (26)
k_{off1}	0.27	measured in (26)
$k_{off2}, k_{offp1}, k_{offp2}$	0.27	as k_{off1}
k_{cat1}	$7.47 \cdot 10^{-2}$	*calculated from (26)
k_{cat2}	$3.57 \cdot 10^{-2}$	*calculated from (26)
k_{catp1}	$5.85 \cdot 10^{-2}$	*calculated from (26)
k_{catp2}	$3.15 \cdot 10^{-2}$	*calculated from (26)
param. [μM]	value	comment
Mek total	1.2	measured in (26)
Erk total	0.74	measured in (26)

Table 1: Table of parameters used in the basic model and in the model with two different phosphatases. Dephosphorylation rates $d_{1/2}$ are used in the simplified model where we assume mass-action kinetics for Erk deactivation. *Measured apparent rates $r = k_{cat}/K_M$ were used to derive the catalytic rates according to the equation $k_{cat} = \frac{r \cdot k_{off}}{k_{on} - r}$.

Overexpression of signaling proteins is a common consequence of the massive genomic alterations in cancer and it is generally believed that this alteration will increase pathway activity or may cause spontaneous pathway activation. However, our results show that a distributive two-step activation of Erk has the potential to suppress excessive Erk activity and thus protects the signaling off-state against Erk overexpression, which may explain why Erk overexpression is rarely seen in tumors (23–25).

MATERIALS AND METHODS

Ordinary differential equation models

Basic model of Erk (de)activation

We model the 2-step activation and deactivation of Erk by assuming that the kinase and phosphatase forms a complex with its substrate in a reversible fashion (association rate constants k_{onx} , dissociation rate constants k_{offx}). (De)phosphorylation and release of the phosphatase/kinase from their respective modified substrates is assumed to proceed as one irreversible step with rate constant k_{catx} . Within the index of kinetic rate constants $x \in \{1, 2\}$ indicates the phosphorylation reaction in phosphorylation cycle 1 or 2, $x \in \{p1, p2\}$ the dephospho-

rylation the reaction in cycle 1 or 2. We denote the total concentration of active kinase ppMek as K_T , the total concentration of phosphatase as P_T and the total concentration of Erk as Erk_T . Complexes of kinase/phosphatase with their substrates are named C_x/D_x where $x \in \{1, 2\}$ indicates the 1st and 2nd phosphorylation cycle, see also the pathway scheme in Fig. 1A. The following ODE system describes the kinetics of its components:

$$\frac{d}{dt} C_1 = k_{on1} \cdot Erk \cdot K - (k_{off1} + k_{cat1}) \cdot C_1 \quad (1)$$

$$\frac{d}{dt} C_2 = k_{on2} \cdot pErk \cdot K - (k_{off2} + k_{cat2}) \cdot C_2 \quad (2)$$

$$\frac{d}{dt} D_1 = k_{onp1} \cdot pErk \cdot P - (k_{offp1} + k_{catp1}) \cdot D_1 \quad (3)$$

$$\frac{d}{dt} D_2 = k_{onp2} \cdot ppErk \cdot P - (k_{offp2} + k_{catp2}) \cdot D_2 \quad (4)$$

$$\frac{d}{dt} pErk = k_{cat1} \cdot C_1 - k_{on2} \cdot pErk \cdot K + k_{off2} \cdot C_2 + k_{catp2} \cdot D_2 - k_{onp1} \cdot pErk \cdot P + k_{offp1} \cdot D_1$$

$$\frac{d}{dt} ppErk = k_{cat2} \cdot C_2 - k_{onp2} \cdot ppErk \cdot P + k_{offp2} \cdot D_2 \quad (6)$$

The concentrations of Erk, kinase K and phosphatase P can be calculated from the conservation relations:

$$K = K_T - C_1 - C_2 \quad (7)$$

$$P = P_T - D_1 - D_2 \quad (8)$$

$$Erk = Erk_T - pErk - ppErk - C_1 - C_2 - D_1 - D_2 \quad (9)$$

The kinetic parameters used for numerical simulation are shown in table 1. Several parameters of the model have been estimated *in vivo* in HeLa cells (26). All rate constants that describe the formation of an enzyme-substrate complex have been assumed to be identical, the same was assumed for the dissociation rates of these complexes.

Model with Erk deactivation by two different phosphatases

We describe the model in terms of modifications to the basic model. The conservation relations for the total kinase K_T and the total amount of Erk, Erk_T , remain unchanged, however, we have to replace equation (8) by two equations for the conservation relations for one phosphatase, P_1 and the 2nd phosphatase, P_2

$$P_1 = P_{1T} - D_1 \quad (10)$$

$$P_2 = P_{2T} - D_2 \quad (11)$$

Model equations (1) and (2) remain unchanged. In equations (3) and (5) variable P is replaced by P_1 , in equations (4) and

129 (6) variable P is replaced by P₂. Kinetic parameters remain
130 unchanged and can be found in table 1.

131 Ordered Model of Erk (de)activation

132 In this model we consider the two different forms of single
133 phosphorylated Erk, pYErk (phosphorylated on tyrosine)
134 and pTERk (phosphorylated on threonine). We model that
135 Erk is phosphorylated and dephosphorylated on tyrosine
136 first. Just like in the basic model we assume that binding of
137 enzyme and substrate is reversible with rates k_{onx}/k_{offx} . Here
138 $x \in \{c1, cy2, ct2, dy1, dt1, d2\}$ identifies the enzyme-substrate
139 complex involved, where C1/CY2/CT2 is the complex of
140 activating kinase with Erk/pYErk/pTERk and DY1/DT1/D2
141 the complex of phosphatase and pYErk/pTERk/pYpTERk. See
142 also the pathway scheme in Fig. 6A. For qualitative analysis
143 of this model we set the values of all kinetic parameters and
144 of the kinase/phosphatase concentration to 1, unless stated
145 otherwise. The following ODEs describe all components in
146 this model:

$$\frac{d}{dt} pYErk = k_{catc1} \cdot C1 + k_{offcy2} \cdot CY2 \quad (12)$$

$$+ k_{offdy1} \cdot DY1 - k_{oncy2} \cdot pYErk \cdot K$$

$$- k_{ondy1} \cdot P \cdot pYErk$$

$$\frac{d}{dt} pTERk = k_{offdt1} \cdot DT1 + k_{catd2} \cdot D2 \quad (13)$$

$$+ k_{offct2} \cdot CT2 - k_{ondt1} \cdot pTERk \cdot P$$

$$- k_{onct2} \cdot pTERk \cdot K$$

$$\frac{d}{dt} pYpTERk = k_{catcy2} \cdot CY2 + k_{catct2} \cdot CT2 \quad (14)$$

$$+ k_{offd2} \cdot D2 - k_{ond2} \cdot P \cdot pYpTERk$$

$$\frac{d}{dt} C1 = k_{onc1} \cdot Erk \cdot K \quad (15)$$

$$- (k_{offc1} + k_{catc1}) \cdot C1$$

$$\frac{d}{dt} CY2 = k_{oncy2} \cdot pYErk \cdot K \quad (16)$$

$$- (k_{offcy2} + k_{catcy2}) \cdot CY2$$

$$\frac{d}{dt} CT2 = k_{onct2} \cdot pTERk \cdot K \quad (17)$$

$$- (k_{offct2} + k_{catct2}) \cdot CT2$$

$$\frac{d}{dt} DY1 = k_{ondy1} \cdot pYErk \cdot P \quad (18)$$

$$- (k_{offdy1} + k_{catdy1}) \cdot DY1$$

$$\frac{d}{dt} DT1 = k_{ondt1} \cdot pTERk \cdot P \quad (19)$$

$$- (k_{offdt1} + k_{catdt1}) \cdot DT1$$

$$\frac{d}{dt} D2 = k_{ond2} \cdot pYpTERk \cdot P \quad (20)$$

$$- (k_{offd2} + k_{catd2}) \cdot D2$$

147

148 where the concentrations of Erk, kinase K and phosphatase P

149 are given by the conservation relations

$$K = K_T - C1 - CY2 - CT2 \quad (21)$$

$$P = P_T - DY1 - DT1 - D2 \quad (22)$$

$$Erk = Erk_T - pYErk - pTERk - pYpTERk \quad (23)$$

$$- C1 - CY2 - CT2 - DY1 - DT1 - D2 .$$

150 Numerical simulations and calculations

151 All numerical simulations were carried out using MATLAB
152 R2013b. To determine the steady state phosphorylation levels,
153 the ODE system was solved by numerical integration (using the
154 solver ode23s) until a time point where the solution approaches
155 an equilibrium. Using the numerical root finding routine fsolve,
156 the steady state was confirmed. Uniqueness of the steady-state
157 was checked by starting from two opposing initial conditions,
158 where either no Erk was phosphorylated initially, or all Erk
159 dual phosphorylated. All analytical calculations have been
160 verified using Wolfram Mathematica 8.

161 RESULTS AND DISCUSSION

162 Mechanistic model predicts reduced Erk 163 activity at high Erk expression levels

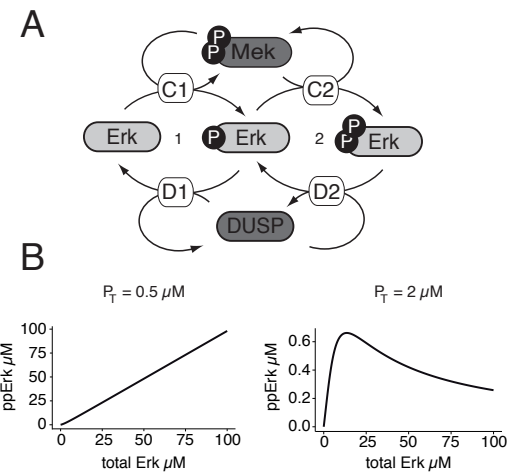


Figure 1: Bell-shaped response of active Erk as function of total Erk.

A, Distributive (basic) model of Erk (de)phosphorylation. Enzyme-substrate complexes C/D_{1/2} are formed in a reversible fashion. DUSP = dual-specificity phosphatase. B, Simulation of stationary ppErk versus level of total Erk using the basic model for high (left) and low (right) pathway activity. Total amount of active Mek equals $K_T = 1.2 \mu\text{M}$. The amount of phosphatase has been chosen arbitrarily and is indicated with P_T at the top of the respective panel. All other parameters set as shown in table 1.

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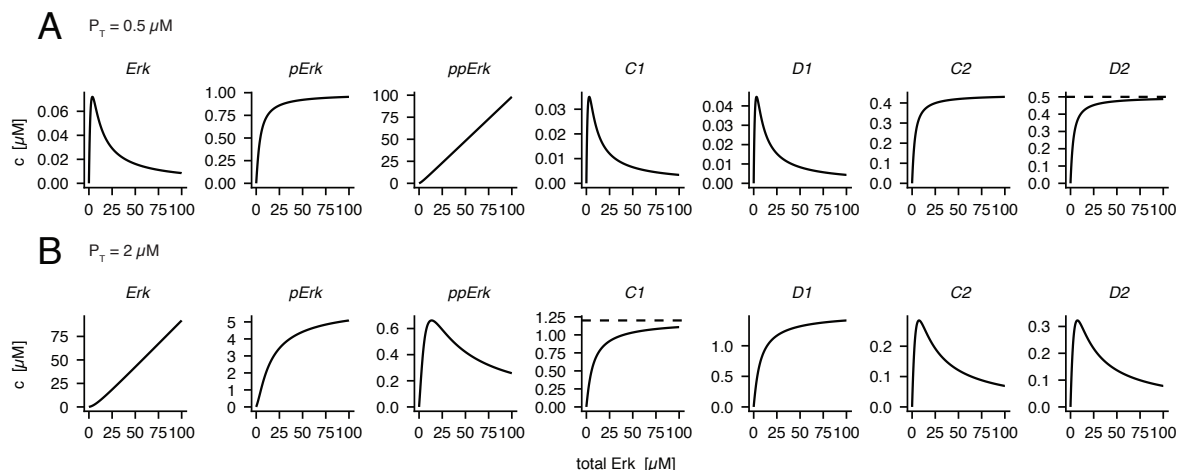


Figure 2: Steady state of the dual phosphorylation cycle when varying total amount of Erk. Simulation of the basic model with a total phosphatase concentration set to $P_T=0.5 \mu\text{M}$ in A and $P_T=2 \mu\text{M}$ in B. All other kinetic parameters set as listed in table 1. Dashed lines indicate the total concentration of the phosphatase in A and of the kinase in B.

164 To investigate the effect of changing concentrations of the
 165 target in a covalent modification cycle, we chose to model the
 166 activation of Erk. Erk needs to be phosphorylated on threonine
 167 and tyrosine within the TEY motif to be fully active (27). The
 168 only enzyme that catalyzes these two phosphorylation steps is
 169 Mek1/2. *In vitro* it has been shown that Mek cannot catalyze
 170 these two phosphorylations in one reaction (as processive
 171 enzymes do), but Mek preferentially phosphorylates Erk on
 172 tyrosine first (4, 28), and then the enzyme substrate complex
 173 dissociates and reforms for the second phosphorylation step
 174 (distributive mechanism) (29).

175 Erk is dephosphorylated and thereby inactivated by dif-
 176 ferent types of phosphatases. Ubiquitous phosphotyrosine
 177 phosphatases like PTP remove the phosphorylation on ty-
 178 rosine. DUSPs remove phosphates on both threonine and
 179 tyrosine (30). Another special characteristic of DUSPs is their
 180 specific localisation either to the nucleus or cytoplasm and
 181 their regulation by MAPKs themselves. Dephosphorylation
 182 by DUSPs is believed to follow a distributive scheme as well
 183 (31).

184 The direct proof for distributive kinetics has been provided
 185 by *in vitro* studies (28, 29). But a distributive mechanism has
 186 the potential to be converted to a quasi-processive one *in*
 187 *vivo*. Either molecular crowding (26, 32) or the anchoring
 188 to molecular scaffolds could increase the stability of the
 189 Mek-pErk complex and/or enable rapid rebinding of the
 190 latter. However, it has been shown that in mouse embryonic
 191 fibroblasts only the scaffold KSR and Mek1/2 form rather
 192 stable complexes in the cytoplasm, whereas the interaction
 193 of the scaffold with Raf and Erk is highly dynamic (33).
 194 Up to now the experimental evidence for distributive Erk
 195 phosphorylation *in vivo* outweighs the evidence for a quasi-

196 processive mechanism (34–37).

197 We therefore developed a kinetic model which accounts
 198 for the (reversible) binding of Mek to Erk, its phosphorylation,
 199 and the (reversible) binding of DUSPs to Erk with subsequent
 200 dephosphorylation (see scheme in Fig. 1A). Phosphorylation
 201 and dephosphorylation were assumed to follow a distributive
 202 scheme. The ordinary differential equations (ODEs) and
 203 kinetic parameters that describe the kinetics associated with
 204 the presented reaction scheme can be found in Materials and
 205 Methods.

206 We then performed numerical simulations of the model,
 207 where we varied the total concentration of Erk. We noticed
 208 that the change of ppErk (dual phosphorylated Erk) upon
 209 increase of total Erk is qualitatively different for different
 210 activity ratios of the modifying kinase and phosphatase. For
 211 low concentration of the phosphatase, such as at $P_T=0.5$
 212 μM (see Fig. 1B), when the maximal turnover rate of the
 213 kinase $v_{\text{max},K} = k_{\text{cat},K} \cdot K_T$ exceeds the maximal turnover
 214 rate of the phosphatase, ppErk rises linearly with total Erk.
 215 However, when the phosphatase dominates with $P_T=2 \mu\text{M}$,
 216 ppErk shows a nonlinear, bell-shaped dependence on total Erk
 217 (Fig. 1B). While ppErk increases first, it reaches a maximum
 218 and subsequently decreases for higher levels of Erk.

219 Puzzled by this non-intuitive behavior, we inspected how
 220 the different forms of Erk and its complexes with kinases or
 221 phosphatases change when the total amount of Erk is increas-
 222 ing. The single phosphorylated Erk increases monotonically
 223 with the Erk expression level, however, it approaches a limit
 224 (Fig. 2B). The ppMek-Erk enzyme-substrate complex C_1
 225 shows a similar behavior as it approaches the concentration of
 226 total ppMek, here called K_T (Fig. 2B). This shows that ppMek
 227 becomes saturated with unphosphorylated Erk at increasing

228 levels of the latter. This is reminiscent of a mechanism de-
 229 scribed previously as kinetic tumor suppression for a single
 230 modification cycle. (22), and this mechanism will be key to
 231 understand the bell-shaped response of dual phosphorylated
 232 Erk, as shown below.

233 Limited activation in a single phosphorylation 234 cycle

235 For now, let us assume that Erk is activated by a single phos-
 236 phorylation that is provided by a kinase and removed by a
 237 phosphatase. Then, at low pathway activity, the amount of
 238 activated Erk has an upper limit (22). As the steady state
 239 of a single phosphorylation cycle has an analytical solution
 240 (2), this upper limit can be derived by calculating the math-
 241 ematical limit of pErk as total Erk approaches infinity (22).
 242 However, there is an easier approach. As we consider a sce-
 243 nario involving large amounts of total Erk, we can assume
 244 Michaelis-Menten kinetics for the modifying enzymes, so the
 245 velocity of kinase/phosphatase is determined by its affinity to
 246 the substrate, $K_{M,K/P}$, and its maximum turnover rate $v_{max,K/P}$
 247 (see Fig. 3). At low pathway activity $v_{max,P}$ is larger than
 248 $v_{max,K}$. As we consider a phosphorylation cycle, the velocities
 249 of kinase and phosphatase have to be identical in steady state
 250 (indicated by the black horizontal lines in Fig. 3). In conse-
 251 quence, the amount of pErk will be significantly smaller than
 252 the amount of unmodified Erk, as shown in Fig. 3A. If the
 253 level of total Erk is increased further, both enzymes are pushed
 254 to higher velocities, but the smaller $v_{max,K}$ sets an upper limit
 255 to this steady state velocity (see Fig. 3B). In consequence,
 256 unmodified Erk accumulates while pErk approaches an upper
 257 limit. This limit can be derived from the steady state condition
 258 when the kinase operates at saturation:

$$v_p = \frac{v_{max,P} \cdot pErk_{max}}{K_{M,P} + pErk_{max}} = v_{max,K} \leftrightarrow$$

$$pErk_{max} = \frac{K_{M,P}}{\frac{v_{max,P}}{v_{max,K}} - 1}. \quad (24)$$

259 We see from equation (24) that the activity ratio of kinase
 260 and phosphatase directly influences the stationary level of
 261 phosphorylated Erk. When the maximal turnover rate of the
 262 phosphatase is twice the maximal turnover rate of the kinase,
 263 the maximal amount of phosphorylated Erk complies to the
 264 Michaelis-Menten constant of the phosphatase. The role of
 265 the phosphatases' K_M is intuitive, as a weaker affinity of the
 266 phosphatase helps to pile up more of the activated species
 267 pErk.

268 It is now clear that when Erk is overexpressed the formation
 269 of active Erk is limited, because the kinase saturates and the
 270 phosphatase does not. The only parametric prerequisite for
 271 this effect is a lower v_{max} of the kinase compared to the
 272 phosphatase.

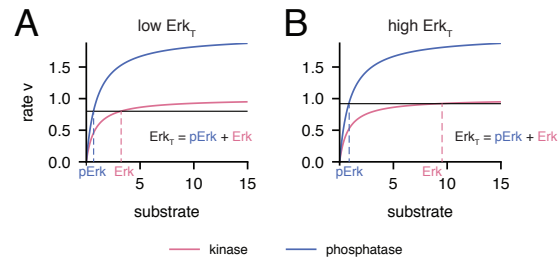


Figure 3: Overexpression insensitivity in a single phosphorylation cycle.

The velocity of the kinase (pink) and of the phosphatase (blue) are shown as a function of substrate level according to Michaelis-Menten, where $v_{max,K} < v_{max,P}$. In steady state, the velocity of the kinase equals the velocity of the phosphatase, which is indicated by the black horizontal line. The amount of substrates (Erk and pErk) follows as indicated by the dashed lines. A, for low amounts of total Erk, B, for high amounts of total Erk.

273 The signal is attenuated further in a dual 274 phosphorylation cycle

275 Also in the dual phosphorylation cycle the stationary level
 276 of single phosphorylated Erk rises with the total amount of
 277 Erk and finally approaches a limit, given that $v_{max,K} < v_{max,P}$
 278 (Fig. 2). This is due to progressing saturation of ppMek
 279 - however, now ppMek can either be bound in a complex
 280 with Erk (C_1) or pErk (C_2). As C_2 approaches 0 and C_1
 281 approaches K_T , (see Fig. 2B) active Mek apparently becomes
 282 sequestered within the first phosphorylation cycle. That means,
 283 two mechanisms shape the basal steady state amount of ppErk
 284 at Erk overexpression: saturation of ppMek and sequestration
 285 of ppMek in the first phosphorylation step. Consequently, the
 286 phosphatase is also drawn into the first phosphorylation cycle
 287 - complexes D_2 and C_2 decrease for rising levels of total Erk
 288 (Fig. 2B).

289 When the condition is reversed, so when $v_{max,K} > v_{max,P}$,
 290 all intermediate species of the dual phosphorylation cycle
 291 behave in a mirror-inverted fashion, e.g. unphosphorylated
 292 Erk exchanges its concentration profile with the profile of
 293 dual phosphorylated Erk. The phosphatase saturates in the
 294 2nd phosphorylation cycle and draws most of the kinase into
 295 the 2nd cycle (Fig. 2A).

296 As either the kinase (phosphatase) is sequestered in the first
 297 (second) phosphorylation cycle, the limit of single phos-
 298 phorylated Erk in a dual phosphorylation cycle can be calculated
 299 like in a single phosphorylation cycle:

$$pErk_{max} = \begin{cases} \frac{K_{M,P1}}{\frac{v_{max,P1}}{v_{max,K1}} - 1} & \text{when } v_{max,K} < v_{max,P} \\ \frac{K_{M,K2}}{\frac{v_{max,K2}}{v_{max,P2}} - 1} & \text{when } v_{max,K} > v_{max,P} \end{cases} \quad (25)$$

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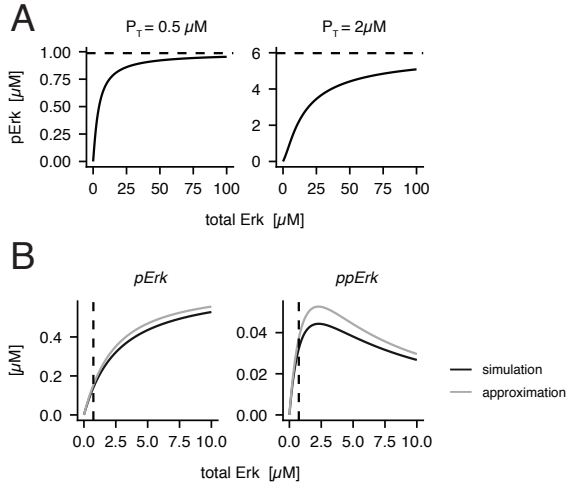


Figure 4: Quantification of Erk activation limit.

A, Numerical simulation of the amount of pErk in a dual phosphorylation cycle according to the basic model (black line) for high (left) and low (right) pathway activity. The analytical limit of pErk (see eq. (25)) is indicated by the dashed line. All parameters chosen as listed in table 1. B, The steady state level of pErk and ppErk at varying levels of Erk_T was simulated with the simplified model that features distributive dual phosphorylation of Erk by Mek and mass action rates of dephosphorylation. The conservation relation for K_T and Erk_T is either exact (simulation, black line) or approximated according to eq. (28) and (29) (analytical approximation, gray line). The dashed line indicates the concentration of Erk in HeLa cells (26).

$K_{M,P1}$ refers to the affinity of the phosphatase in cycle 1, which is its affinity to pErk. Likewise $K_{M,K2}$ refers to the affinity of the kinase in cycle 2 – the affinity of the kinase to pErk. Maximum turnover rates v_{\max} are labelled accordingly. Figure 4A shows the amount of single phosphorylated Erk in a dual phosphorylation cycle for increasing amounts of Erk and the calculated limits using the equation (25).

A simplified model explains limited activation in a dual phosphorylation cycle

To improve our understanding of how the various rate constants shape the maximum of Erk activation in a dual phosphorylation cycle we sought to simplify our basic ODE model (1)-(6) in a way that will allow us to calculate a closed form of the steady state. Limited activation of Erk is seen when ppMek is shared between two cycles and eventually saturates and sequesters in one of the cycles. The phosphatases keep working far from saturation, so that we can model their catalysis with mass-action kinetics instead. Thus the model equations (1) and (2) remain unchanged but the equations (3) and (4) that describe the temporal development of the phosphatase

in complex with its two different substrates can be dropped. Assuming that dephosphorylation of single phosphorylated Erk proceeds with rate d_1 and dephosphorylation of dual phosphorylated Erk with rate d_2 , equations (5) and (6) are rewritten to

$$\frac{d}{dt} \text{pErk} = k_{\text{cat}1} \cdot C_1 - k_{\text{on}2} \cdot \text{pErk} \cdot K + k_{\text{off}2} \cdot C_2 - d_1 \cdot \text{pErk} + d_2 \cdot \text{ppErk} \quad (26)$$

$$\frac{d}{dt} \text{ppErk} = k_{\text{cat}2} \cdot C_2 - d_2 \cdot \text{ppErk}. \quad (27)$$

Even with this modification, the explicit description of all components in steady state is impossible, which is generally true when the various enzyme-substrate complexes are appreciable compared to the concentration of free substrate and product (38). However, we can approximate

$$K \approx K_T - C_1 \quad (28)$$

$$\text{Erk} \approx \text{Erk}_T - C_1 - \text{pErk} \quad (29)$$

because the concentration of the complex formed by ppMek and monophosphorylated Erk, C_2 , is significantly smaller than C_1 and ppErk has the smallest contribution to the total level of Erk.

In equation (28) and (29) Erk_T and K_T denote the respective total enzyme concentrations of Erk and ppMek. The steady state of this simplified system has a closed form and reads:

$$C_1 = \alpha - \sqrt{\alpha^2 - \beta} \quad \text{with} \quad (30)$$

$$\alpha = \frac{d_1(K_{M1} + \text{Erk}_T) + K_T(d_1 + k_{\text{cat}1})}{2(d_1 + k_{\text{cat}1})} \quad \text{and}$$

$$\beta = \frac{d_1 \text{Erk}_T K_T}{d_1 + k_{\text{cat}1}}$$

$$\text{pErk} = \frac{k_{\text{cat}1}}{d_1} \cdot C_1 \quad (31)$$

$$\text{Erk} = \frac{K_{M1} \cdot C_1}{K_T - C_1} \quad (32)$$

$$\text{ppErk} = \frac{k_{\text{cat}1} k_{\text{cat}2} \cdot C_1 (K_T - C_1)}{d_1 d_2 K_{M2}} \quad (33)$$

$$C_2 = \frac{k_{\text{cat}1} \cdot C_1 (K_T - C_1)}{d_1 K_{M2}} \quad (34)$$

$$K = K_T - C_1. \quad (35)$$

Here, $K_{M1/2}$ refers to the Michaelis-Menten constant of the kinase in the first/second phosphorylation cycle. The approximation of the steady state captures the correlation of phosphorylated Erk and total Erk qualitatively as well as the order of magnitude in phosphorylation, as can be seen in a direct comparison of the numerical solution of the system with mass-action kinetics for dephosphorylation with the analytical approximation (Fig. 4B) where the conservation relations of

Erk and ppMek have been truncated as shown in equation (28) and (29).

Using the analytical solution from equation (33) we can now derive the concentration of Erk at which ppErk is maximal. The derivative of ppErk by the level of total Erk

$$\frac{d \text{ppErk}(\text{Erk}_T)}{d \text{Erk}_T} = \gamma \frac{dC_1(\text{Erk}_T)}{d \text{Erk}_T} [\text{K}_T - 2C_1(\text{Erk}_T)] \stackrel{!}{=} 0 \quad (36)$$

equals zero at the maximum with

$$\gamma = \frac{k_{\text{cat}1} k_{\text{cat}2}}{d_1 d_2 K_{M2}}. \quad (37)$$

Condition (36) is only fulfilled when

$$C_1(\text{Erk}_T) = \frac{\text{K}_T}{2}, \quad (38)$$

as C_1 grows with the amount of Erk_T until saturation of the kinase with Erk, the first factor, $\frac{dC_1}{d \text{Erk}_T}$, is never zero. The level of total Erk in the cell leading to maximal activation is the one where half of the total available kinase ppMek is sequestered in a complex with unphosphorylated Erk. Condition (38) allows for the exact calculation of the maximum coordinate to

$$(\text{Erk}_T, \text{ppErk})_{\text{max}} = \left(K_{M1} + \left[1 + \frac{k_{\text{cat}1}}{d_1} \right] \frac{\text{K}_T}{2}, \frac{k_{\text{cat}1} k_{\text{cat}2}}{d_1 d_2 K_{M2}} \cdot \frac{\text{K}_T^2}{4} \right). \quad (39)$$

Note that this model will always create a bell-shaped response as it is built on the assumption that the phosphatases cannot saturate - changing any parameter in this model will only alter the position and/or height of the peak of activation. The maximal ppErk level is proportional to the square of the kinase concentration which reflects the two step nature of the activation process. A higher affinity of the kinase to un-phosphorylated Erk (smaller K_{M1}) enforces sequestration and thus shifts the position of the peak to smaller levels of Erk. A higher affinity in catalysis of the 2nd phosphorylation (smaller K_{M2}) increases the activation level. Only the catalytic rates of the 1st modification cycle (d_1 and $k_{\text{cat}1}$) influence the peak position, which suggests that the activity ratio of kinase and phosphatase in the cycle converting between Erk and pErk creates the prerequisite for limited activation.

In this model C_1 approaches the level of K_T for increasing concentrations of Erk. It follows from equation (31) that the limit of single phosphorylated Erk amounts to

$$\text{pErk}_{\text{max}} = \frac{k_{\text{cat}1}}{d_1} \cdot \text{K}_T. \quad (40)$$

Quantification of the activation limit

Using the equations (39) and (40) with the kinetic parameters measured in HeLa cells we can now estimate whether the kinetic suppression of excessive amounts of active Erk might

play a role *in vivo*. Assuming that only 5% of the cellular Mek is activated, maximal levels of active Erk can be found at 2.3 μM which is about 3 fold more than the average Erk expression level measured in HeLa cells (Fig. 4B). Also, only 2% of Erk is activated at the peak, which means that 5% Mek activity is attenuated to only 2% of Erk activation at the peak. For the physiological concentration of Erk, at 0.74 μM , indicated with the dashed vertical line in Fig. 4B, the relative Erk activation is at 4.5%. Single phosphorylated Erk approaches a limit, which accords to 0.67 μM .

With the help of the analytic equations derived here, the maximal activation level of a target can be estimated for any single or dual phosphorylation cycle, given that the catalytic rates are known. In case of Erk activation in HeLa cells, the mechanism which limits Erk activation is effective already at 3x overexpression, which can be considered mild in comparison to the observation that Erk concentrations vary about 3 fold between clonal cells (39).

Different phosphatases can be involved in Erk deactivation

So far we have assumed that one enzyme is responsible for (de)phosphorylation of threonine and tyrosine on Erk. But dual-specificity phosphatases are a class of phosphatases whose expression is highly regulated in concentration and location (40). Under some circumstances they might not even be the main phosphatases responsible for deactivation of Erk. In the scenario where dephosphorylation of threonine and tyrosine is carried out by two different phosphatases, the activity ratio of kinase and phosphatase may differ in the two cycles. To test the prerequisite for the bell-shaped response under these circumstances we have adapted the basic model to include two different phosphatases as shown in the scheme in Fig. 5A.

When the two phosphatases outcompete the kinase in both cycles, ppErk shows the same non-linear profile as was seen before (Fig. 5B). The bell-shaped response of ppErk is also found when phosphatase 1 has a larger turnover rate than the kinase, but not phosphatase 2 (Fig. 5C). However, if only the phosphatase 2 has a higher maximum turnover rate than the kinase, the formation of single phosphorylated Erk is proportional to the amount of available Erk and ppErk approaches a limit like in a single modification cycle (Fig. 5D).

It can be concluded that as long as the phosphatase dominates the activity of the kinase in at least one cycle, activation of Erk is limited even at higher expression levels. However, the model with two phosphatases clearly shows that a dominant activity of the phosphatase within the first phosphorylation cycle is sufficient for the bell-shaped profile of dual phosphorylated Erk.

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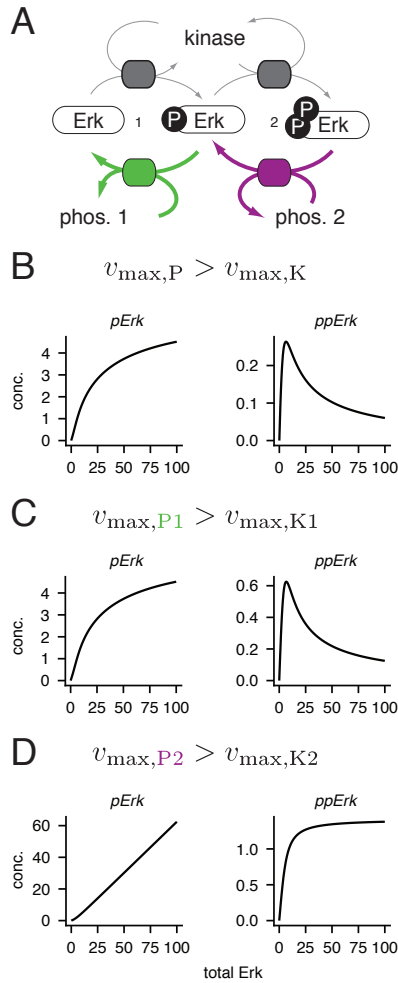


Figure 5: Limited activation in dual phosphorylation cycles where different phosphatases catalyse the first and second dephosphorylation.

A, The basic model was modified to a scheme in which two different phosphatases deactivate Erk (see Material and Methods section). We show the steady state amounts of pErk and ppErk for different levels of total Erk when v_{\max} of the phosphatase exceeds the level of v_{\max} of the kinase in both cycles (B) and when the phosphatase has a higher maximum turnover rate than the kinase in only one out of the two cycles as indicated at the top of the panels C and D.

432 Prediction for the ordered model of Erk 433 modification

434 Experimental evidence supports the hypothesis that the acti-
435 vating and deactivating modification of Erk proceeds in an
436 ordered fashion: Erk is phosphorylated and dephosphorylated
437 on tyrosine first. We have built a model to account for this
438 by explicitly considering the 3 different states of phospho-
439 rylated Erk, pYpTerk, pTerk and pYpTerk. We assume that
440 the conversion from Erk to pTerk as well as the conversion

441 from pYpTerk to pYerk do not occur (see model equations in
442 Material and Methods and a model scheme in Fig. 6A). From
443 the results above we concluded that the bell-shaped response
444 of active pYpTerk occurs only if the maximum turnover rate of the
445 phosphatase exceeds the maximum turnover rate of the
446 kinase within the first phosphorylation cycle. To test whether
447 this condition still holds, we simulate the stationary amount
448 of active Erk while varying the total amount of Erk with a
449 parameter set in which the concentration of the kinase and the
450 phosphatase equal $1 \mu\text{M}$ and all other kinetic parameters are
451 set to 1. Now the first phosphorylation cycle in the ordered
452 scheme constitutes the cycle between Erk and pYerk. If we
453 set the rate constant $k_{\text{catdy}1}$ to 2 (while keeping all other pa-
454 rameters at 1), the condition for the bell-shaped pYpTerk is
455 fulfilled. And indeed we find the previous saturation of pYerk
456 to a limit value and a bell-shaped profile of pYpTerk (see Fig.
457 6B). In contrast, as pTerk is only created from pYpTerk in
458 this ordered scheme, this species also shows a bell-shaped
459 response curve. Here, the kinase is saturated in complex C1
460 and the phosphatase operates far from saturation, as described
461 previously.

462 Alternatively, one can ask what happens when we assume
463 that the dephosphorylation rate from pTerk exceeds the rate of
464 phosphorylation from Erk to pYerk, by setting all parameters
465 to 1 but the rate $k_{\text{catdt}1} = 2$ (Fig. 6C). Here, significant amounts
466 of pYerk can be formed which serve as substrate to the second
467 step of phosphorylation. In consequence, we see a plain limit
468 to the amount of active pYpTerk, as would be the behaviour in
469 a single modification cycle for high levels of substrate. Again,
470 pTerk has the same concentration profile as pYpTerk, because
471 it is only being created from it. Interestingly, both the kinase
472 and the phosphatase are drawn into the first phosphorylation
473 cycle here, i.e. the kinase is sequestered in complex C1 and
474 the phosphatase in complex DY1.

475 We can conclude that we still find a bell-shaped pYpTerk
476 response profile when the dephosphorylation rate of the ty-
477 rosine residue of Erk's activation loop exceeds the phospho-
478 rylation rate of this residue. However, also when dephospho-
479 rylation of the threonine residue dominates the activating
480 phosphorylation, we find robustness of the signaling off-state
481 to increasing amounts of total Erk, as pYpTerk does not rise
482 in a linear fashion, but approaches a limit.

483 CONCLUSION

484 When the activity of a signaling protein is modified by the
485 addition of one phospho-group, the signaling off-state is robust
486 to increasing amounts of the protein itself, as the modifying
487 kinase saturates eventually. If the activity of a protein is
488 regulated by two consecutive phosphorylation events, the
489 formation of dual phosphorylated active protein at increasing
490 levels of protein is suppressed even further as the modifying
491 kinase gets saturated with its substrate and additionally gets
492 sequestered within the first phosphorylation step, which makes
493 it less available for catalysis of the second phosphorylation step.

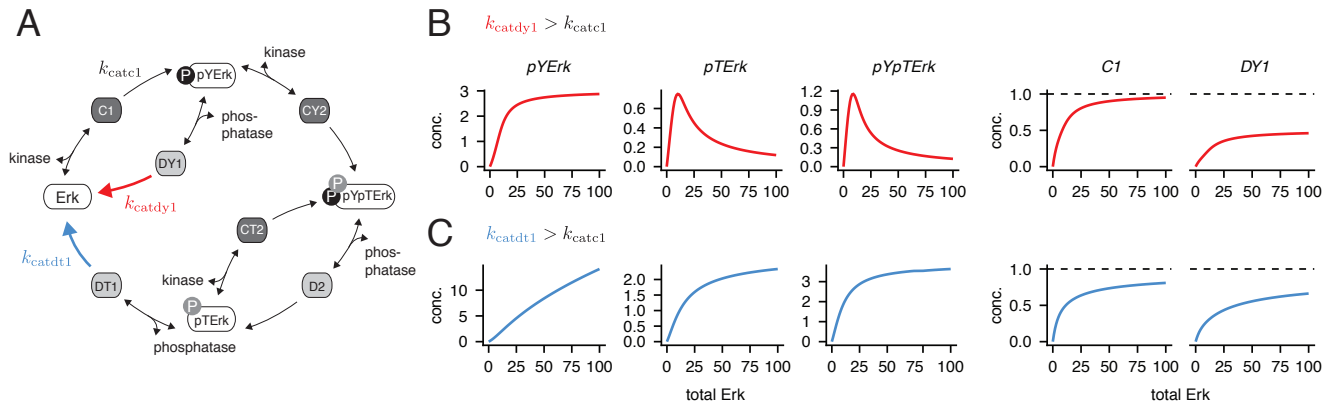


Figure 6: Limits to active Erk in the ordered model of Erk (de)activation.

A, According to the ordered model Erk is phosphorylated and dephosphorylated on tyrosine first. B, Simulation of steady state of the different modification states of Erk and of the enzyme-substrate complexes C1 and DY1 when varying the amount of total Erk. All kinetic parameters and concentrations of modifying kinase and phosphatase have been set to 1, except for $k_{catdy1} = 2$. As a consequence v_{max} of pYErk dephosphorylation exceeds v_{max} of Erk phosphorylation to pYErk. C, Like in B, but now k_{catdt1} is the only parameter set to 2, which makes v_{max} of pTErk dephosphorylation larger than v_{max} of Erk phosphorylation to pYErk.

494 The prerequisite for this phenomenon is the distributive nature
 495 of two-step activation. As of now there is no clear consensus
 496 as to whether Erk is activated in a distributive fashion *in*
 497 *vivo*. However if so, the kinetic suppression of excessive
 498 amounts of active Erk described here in combination with the
 499 multitude of negative feedbacks present in MAPK signaling
 500 might explain why increasing expression of Erk alone would
 501 not confer a growth advantage to cells and why overexpression
 502 of Erk is rarely found in cancer in contrast to e.g. the frequent
 503 overexpression of receptors of the HER family.

504 AUTHOR CONTRIBUTIONS

505 FW carried out all simulations and analytical calculations.
 506 NB and FW designed the research and wrote the article.

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