

Multi-model inference of mate choice effects from an information theoretic approach

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2 model inference, InfoMating

3

4

Abstract

5 Mate choice is an important biological phenomenon with a significant impact on the
6 evolution of organisms. Here, we relied in previous results on the description of mate
7 choice effects as a flow of information, for developing different classes of models
8 distinguished by their evolutionary effects: sexual selection and/or isolation.

9 We obtained formulas for the maximum likelihood estimates of each model and compared
10 three different information criteria (AICc, KICc, and BIC) for performing multimodel
11 inference. Simulation results showed a good performance of both model selection and
12 parameter estimation. We applied the modelling framework to real data, and estimated the
13 mating preference parameters in the Galician *Littorina saxatilis* ecotypes, confirming the
14 pattern of positive assortative mating in this species.

15 As far as we know, this is the first standardized methodology for model selection and
16 multimodel inference of mate choice effects. The full methodology was implemented in a
17 software called InfoMating (available at
18 <http://acraaj.webs.uvigo.es/InfoMating/Infomating.htm>).

19

20

21 *Introduction*

22 Mate choice is an active area of evolutionary research. It can be broadly described as the
23 effect of the expression of some traits leading to non-random mating. The biological
24 complexity lying behind mate choice has generated an extensive research producing
25 different theoretical descriptions and empirical tests (Edward 2015a, b; Hoquet 2015;
26 Hughes 2015; Roff 2015; Ah-King and Gowaty 2016).

27 There are several types of models than could be linked to the causes of mate choice. Mate
28 choice has been approached from the phenotypic, population genetics, and quantitative
29 genetics sides, being developed under either a probabilistic or a deterministic framework.
30 More realism can be still added by including ecological and behavioural aspects (Kokko et
31 al. 2006; Roff 2015).

32 However, from an evolutionary perspective, non-random mating has importance by its
33 consequences, as long as it implies a systematic change in phenotype and genotype
34 frequencies. Therefore, when studying mate choice, we can also focus on its evolutionary
35 consequences.

36 From an evolutionary point of view, mate choice is defined as the difference between the
37 observed and expected (by random) mating frequencies. So defined, the consequences of
38 mate choice can be partitioned into sexual selection (intrasexual selection) and sexual
39 isolation (behavioural isolation or intersexual selection).

40 The decomposition of mating behaviour into sexual selection and sexual isolation effects
41 has been made from multiple-choice experiments in *Drosophila* (Merrell 1950; Spieth and

42 Ringo 1983; Knoppien 1985); see also (Rolán-Alvarez and Caballero 2000) and references
43 therein.

44 Sexual selection refers to the observed change in gene or phenotype frequencies in mated
45 individuals with respect to population frequencies (Anderson et al. 1994; Hartl and Clark
46 1997). Because population frequencies are involved, the comparison includes non-mated
47 individuals.

48 Sexual isolation (intersexual selection) considers the deviation from random mating within
49 mated individuals. Because only frequencies within the sample of mating are considered,
50 the comparison does not involve non-mated individuals. Note that for simplicity, by sexual
51 isolation we are referring to intersexual selection that can be consequence of any of both,
52 positive or negative assortative mating (Merrell 1950; Lewontin et al. 1968; Spieth and
53 Ringo 1983).

54 The immediate causes of sexual selection and/or isolation can be a different preference or
55 discrimination between different mating types (e.g. A females prefer A males) and/or
56 different mating energy or vigour of a type from one sex for mating whatever couple (e.g. B
57 males systematically invest more energy on mating).

58 In a previous work (Carvajal-Rodríguez 2018), we showed that both types of cause can be
59 modelled jointly, by means of the parameters m_{ij} that represent the mutual mating
60 propensity between a female of type i and a male j .

61 Therefore, if A females prefer A males then this is modelled as a higher mutual mating
62 propensity between these types as compared with the mating propensity of the A females

63 with B males ($m_{AA} > m_{AB}$). On the other hand, if B males are more vigorous or prone for
64 mating whatever the female is, this is modelled by a higher average male propensity of such
65 males (marginal propensity, see below).

66 By modelling the mating as a differential mutual mating propensity among different types
67 of couples, it is possible to express the mean change in mating phenotypes as the
68 information gained due to mate choice (Carvajal-Rodríguez 2018). The expression of the
69 effects of mate choice in terms of information also permitted to identify the necessary and
70 sufficient conditions of random mating (see below).

71 Thus, a mate choice information based framework provides a formal approach for
72 developing a set of hypotheses. In addition, data-based evidence can be used for ranking
73 each hypothesis and perform multi-model-based inference (Link and Barker 2006;
74 Burnham et al. 2011; Aho et al. 2014).

75 We proceed as follows:

76 1.- First, we set the conditions for random mating obtained from the mutual mating
77 propensity formalism. Afterwards, we develop different classes of effects models and their
78 maximum likelihood estimates, namely: sexual selection, sexual isolation or double
79 (sexual selection and isolation) models with different number of independent mating
80 propensity parameters.

81 2.- We apply distinct information criteria for selecting the best candidate models and
82 estimating the values of the mutual mating propensity parameters based on the most
83 supported models.

84 3.- Finally, we demonstrate the methodology by analysing simulated and real data.

85

86 *Mutual Mating Propensity Models*

87 Consider a sample of n' matings. The total number of possible mating types is $K = k_1 \times k_2$

88 where k_1 is the number of female types and k_2 the number of male types. Let have n'_{ij}

89 matings of i -type females with j -type males from the sample. If the probability of the

90 mating $i \times j$ is q'_{ij} , then the logarithm of the likelihood function $\ln L$ of the sample is

91

$$92 \quad \ln L = C + \sum_{ij}^K n'_{ij} \ln (q'_{ij})$$

93

94 where C is the multinomial coefficient which is constant given the sample. As it is well-

95 known, the maximum likelihood estimator of the multinomial probability of the mating $i \times j$

96 is n'_{ij} / n' .

97

98 **SATURATED NON-RANDOM MATING MODEL M_{SAT}**

99 Consider a population with n_{1i} females of type i from a total of n_1 females and n_{2j} males of

100 type j from a total of n_2 males in the population. Therefore, the population frequency of

101 females of type i is $p_{1i} = n_{1i} / n_1$ and the population frequency of males of type j is $p_{2j} = n_{2j} /$

102 n_2 .

103 The mating probability between types i and j can be expressed as $q'_{ij} = m_{ij}q_{ij}$ (Carvajal-
104 Rodríguez 2018) where q_{ij} is the product of the population female and male frequencies of
105 each type ($q_{ij} = p_{1i} \times p_{2j}$) and m_{ij} is the normalized mutual mating propensity that refers to
106 the number of matings after an encounter between females of type i and males of type j .
107 The values m_{ij} are normalized so that $\sum q'_{ij} = 1$.

108 Under this multinomial model, the log-likelihood of the sample is

$$109 \quad \ln L_{sat} = C + \sum_{ij}^K n'_{ij} \ln(m_{ij}q_{ij}) \quad (1)$$

110 We note this model as saturated (M_{sat}) because there are as many parameters as independent
111 mating propensities, $P_{sat} = K - 1$. The last propensity is not independent and can be
112 calculated from the others as

$$113 \quad m_{k1k2} = (1 - \sum m_{ij}q_{ij}) / q_{k1k2}$$

114 where the summation is over the $K - 1$ remaining categories.

115 The female and male population frequencies, p_1 and p_2 , are considered to be known or,
116 alternatively, they need to be estimated in all the models. Therefore, in terms of model
117 comparison we can ignore the population frequencies when counting the number of
118 parameters involved in each model.

119 The maximum likelihood estimate (MLE) of m_{ij} is $(n'_{ij} / n) / q_{ij} = \text{PTI}_{ij}$ where PTI_{ij} is the
120 pair total index i.e. the frequency of the observed pair types divided by the expected
121 frequency under random mating (Rolán-Alvarez and Caballero 2000).

122 The model M_{sat} is the most complex model that can be fitted to the available data. The
123 principle of parsimony suggests that we may consider reduced special cases of this
124 saturated model.

125 Following, we computed the ML estimates of different classes of reduced general models
126 that require less fixed parameters. All of them can be expressed as particular cases of the
127 saturated model.

128

129 **RANDOM MATING MODEL M_0**

130 The random model M_0 corresponds to the simplest, most reduced model, and it is nested
131 with all the other (is a particular case of any other model) while we cannot derive any
132 simplified version from it. This model assumes that the within-population mating
133 probability is at random.

134 When mating is at random, the mating probability between types i and j is $q'_{ij} = q_{ij} = p_{1i} \times$
135 p_{2j} . This model is a particular case of the saturated one when the mating propensity is equal
136 for every mating type. The number of mating parameters is $P_0 = 0$.

137 The log-likelihood of the sample of mating is

$$138 \quad \ln L_0 = C + \sum_{ij}^K n'_{ij} \ln(q_{ij}) \quad (2)$$

139 Now, let's define the marginal propensity m_{Fem_i} for a female of type i as

$$140 \quad m_{Fem_i} = \sum_j^{males} p_{2j} \frac{m_{ij}}{M} = \sum_j^{males} p_{2j} m_{ij} \quad (3)$$

141 Similarly for a male of type j

$$m_{Male_j} = \sum_i^{females} p_{1i} \frac{m'_{ij}}{M} = \sum_i^{females} p_{1i} m_{ij}$$

142 Where we have noted the absolute (unnormalized) propensity as m'_{ij} and the mean
143 propensity M

$$M = \sum_{i,j} q_{ij} m'_{ij}$$

144 Interestingly, the M_0 model corresponds to M_{sat} subjected to the following restrictions:

145 i) $m_{ij} = m_{Fem_i} \times m_{Male_j} \forall i, j$ (4)

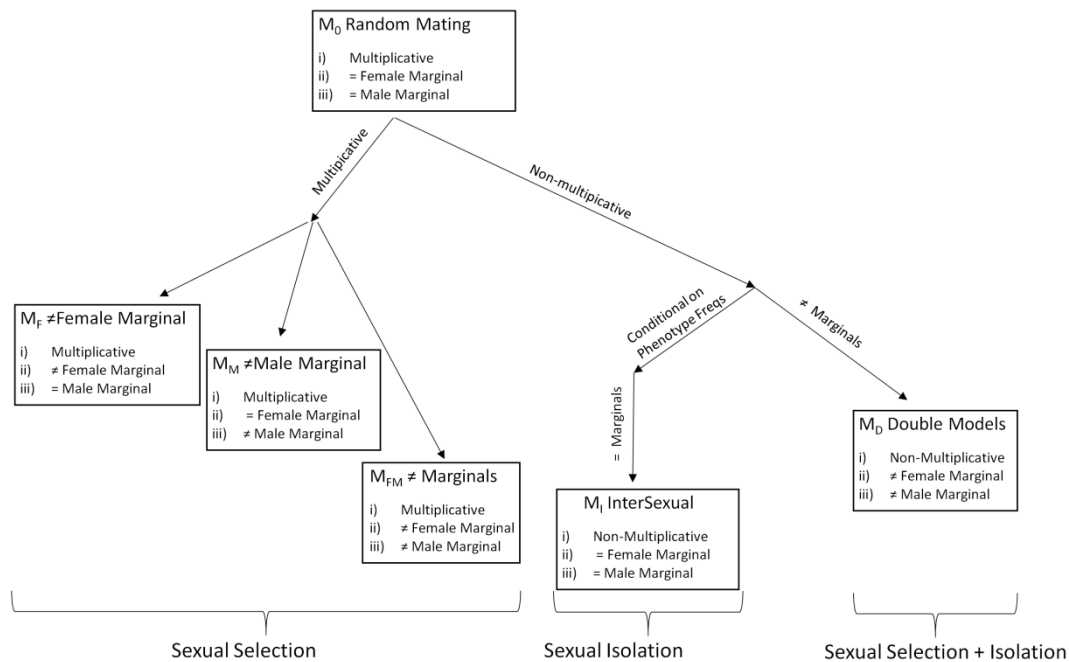
146 ii) $m_{Fem_i} = m_{Fem_j} \forall i, j$

147 iii) $m_{Male_i} = m_{Male_j} \forall i, j$

148

149 Thus, given the model M_{sat} , we may define M_0 as a particular case that has three additional
150 restrictions. These restrictions provoke that all the propensities are the same, so mating just
151 depends on the frequencies of each type. It is useful to express M_0 in terms of the three
152 restrictions because by relaxing some of them, we can generate different classes of models.

153 Therefore, we produced three general classes of models defined by their consequences after
154 relaxing some of the conditions in (4), namely, sexual selection, sexual isolation, and
155 double effect models (Fig. 1).



156

157 **Fig 1.** Mating models as defined by their effect after relaxing some of the conditions imposed to the
 158 random mating model M_0 .

159

160 **SEXUAL SELECTION EFFECTS MODELS**

161 This class of model corresponds to relaxing the second and/or third conditions in M_0 while
 162 maintaining the condition of multiplicativity (4-i). The maintenance of the first condition
 163 implies that the propensity of a mating pair (i,j) is the product of the marginal female (m_{Fem})
 164 and male (m_{Male}) propensities. And because 4-i) is maintained, sexual isolation effects
 165 cannot occur (Carvajal-Rodríguez 2018). We distinguished models with sexual selection in
 166 just one sex or in both sexes.

167

168 **Female Sexual Selection**

169 For generating models having sexual selection only in females, we relaxed condition (4-ii).

170 This implies that at least one female marginal propensity, say female of type A , is different

171 from the rest of female types i.e. $m_{Fem_A} \neq m_{Fem_j}$ with j being any other type than A .

172 Therefore, we set a different absolute (unnormalized) propensity a_h for each female type h

173 as follows

$$174 \quad m'_{11} = m'_{12} = \dots = m'_{1k_2} = a_1$$

$$175 \quad m'_{21} = m'_{22} = \dots = m'_{2k_2} = a_2$$

176 .

177 .

178 .

$$179 \quad m'_{(k_1-1)1} = m'_{k_12} = \dots = m'_{k_1k_2} = a_{k_1-1}$$

$$180 \quad m'_{k_11} = m'_{k_12} = \dots = m'_{k_1k_2} = 1 \quad (5)$$

181 with $a_h > 0 \forall h$.

182 Note that the relationships among the propensities will not be altered if we divide by a_{k_1} so

183 that we have fixed $a_{k_1} = 1$. Thus, under female sexual selection models we can consider H_1

184 $\in [1, k_1 - 1]$ different parameters.

185 The normalization factor is the mean propensity $M = \sum m'_{ij}q_{ij}$. Now, if we compute the

186 marginal female and male propensities (3) we see that

$$187 \quad m_{Fem_1} = a_1/M; m_{Fem_2} = a_2/M \dots; m_{Fem_k_1} = 1/M$$

$$188 \quad m_{Male_1} = m_{Male_2} = m_{Male_3} = \dots = m_{Male_k_2} = M/M = 1$$

189 For any parameter a_h , $1 \leq h < k_1$, the MLE under the female sexual selection models was

$$190 \quad \hat{a}_h = \frac{\lambda(a_h)}{\lambda(1)} \quad (6)$$

191 where $\lambda(\theta)$ was defined in general for any set A of mating pair types having the same value
192 of propensity θ as

$$193 \quad \lambda(\theta) = \frac{\sum_t^A x_t}{\sum_{t_1 t_2}^A p_{1t_1} p_{2t_2}} \quad (7)$$

194 So, $\lambda(a_h)$ expresses the sum of the observed matings with propensity a_h divided by the sum
195 of the product of the population frequencies from each partner type (see section S1 in
196 Supporting Information for detailed explanation). Similarly, $\lambda(1)$ corresponds to the sum of
197 the observed matings having expected propensity 1 divided by the sum of the
198 corresponding products of population frequencies.

199

200 **Male Sexual Selection**

201 Sexual selection in only males is generated in a similar way, by relaxing (4-iii),
202 interchanging rows with columns in (5). If we note the parameters as b_h instead of a_h , the
203 maximum likelihood estimate was

$$204 \quad \hat{b}_h = \frac{\lambda(b_h)}{\lambda(1)} \quad (8)$$

205 with $1 \leq h < k_2$.

206

207 **Sexual Selection In Both Sexes**

208 For generating models with sexual selection in both sexes, we only needed to maintain the
209 condition (4-i). Therefore, for $K = k_1 \times k_2$ mating types we may introduce as much as $(k_1-$

210 $1) \times (k_2 - 1)$ parameters for the most parameterized model, and a minimum of two (female and
 211 male effects) for the less parameterized model, in order to get a multiplicative model (no
 212 sexual isolation) with sexual selection in both sexes. By notational convenience, we fix the
 213 category k_1 in females and k_2 in males as having unitary propensity. Therefore

$$214 \quad m'_{ij} = a_i b_j, \quad i < k_1, j < k_2$$

$$215 \quad m'_{ik_2} = a_i, \quad i < k_1$$

$$216 \quad m'_{k_1j} = b_j, \quad j < k_2,$$

$$217 \quad m'_{k_1k_2} = 1.$$

218 with $a_i > 0, b_j > 0 \forall i, j$.

219 The maximum likelihood estimates were

$$220 \quad \hat{a}_i = \left(\frac{p_{1k_1}}{p_{1i}} \right) \frac{\sum_{j=1}^{k_2} x_{ij}}{\sum_{j=1}^{k_2} x_{k_1j}} = \frac{\lambda(a_i + \sum_j a_i b_j)}{\lambda_{fem}(1)} \quad (9)$$

$$221 \quad \hat{b}_j = \left(\frac{p_{2k_2}}{p_{2j}} \right) \frac{\sum_{i=1}^{k_1} x_{ij}}{\sum_{i=1}^{k_1} x_{ik_2}} = \frac{\lambda(b_j + \sum_i a_i b_j)}{\lambda_{male}(1)}$$

222 Where $\lambda(a+b)$ stands for the sum of observed mating types having propensity a or b divided
 223 by the sum of their expected random mating frequencies (e.g. the sum of the expected
 224 frequencies of matings with propensity a_1 or $a_1 b_j$ for every male type, is p_{1i}), and $\lambda_{sex}(1)$
 225 refers to the sum of cases that contribute with 1 to the propensity by the side of such sex
 226 divided by the corresponding sum of expected frequencies (e.g. in a multiplicative model as
 227 the above, every mating $k_1 \times j$ has contribution 1 by the female side and the sum of
 228 frequencies is p_{1k_1}).

229 The formulae in (9) is similar to (7) and (8). Note that (9) becomes (7) by considering every
230 b as 1 while becomes (8) by considering every a as 1.

231 The percentage of sexual selection information corresponding to each sex, J_{S1} and J_{S2} in
232 (Carvajal-Rodríguez 2018), would depend on the within sex population frequencies and on
233 the propensity estimates.

234

235 **SEXUAL ISOLATION EFFECT MODELS**

236 Isolation (intersexual selection) corresponds to the class of non-multiplicative models, i.e.
237 they can be obtained by relaxing the condition (4-i). Recall that we refer as sexual isolation
238 to the effect of either positive or negative assortative mating.

239 If there is no sexual selection neither in females nor males, we may have only sexual
240 isolation. However, this cannot be guaranteed in general as sexual selection is frequency
241 dependent under non-multiplicative models (see below).

242 Let consider a model where unnormalized propensities are

$$243 \quad m'_{ii} = a_i > 0 \quad \forall i \in \min\{k_1, k_2\} \text{ and } m'_{ij} = 1 \text{ for } i \neq j.$$

244 Thus, homotype mating ($i \times i$) propensities are parameterized while heterotype are not. This
245 model is non-multiplicative in general, because the contribution of the type i to the
246 propensity is distinct in m_{ii} than in m_{ij} or in m_{ji} (although with an even number of types we
247 could define a multiplicative model by setting $m_{ii} = 1 / m_{ij}$).

248 By recalling the definition of marginal propensities in (3) we see that the condition for
249 equal female marginal $m_{Fem_i} = m_{Fem_j}$ is

$$250 \quad p_{2i}(a_i - 1) = p_{2j}(a_j - 1) \quad (10)$$

251 and in males

$$252 \quad p_{1i}(a_i - 1) = p_{1j}(a_j - 1)$$

253 If there is only one parameter, the different homotypes have the same propensity, thus $m'_{ii} =$
254 a , whatever the type i . Then the condition (10) becomes $p_{2i} = p_{2j}$ and $p_{1i} = p_{1j}$ which for k_1
255 female and k_2 male types imply $p_{2i} = p_{2j} = 1 / k_2$ and $p_{1i} = p_{1j} = 1 / k_1$. Which means that
256 under this symmetric model the occurrence of sexual isolation without sexual selection
257 requires uniform frequencies.

258 In general, depending on the conditions in (10), the sexual isolation models are double
259 effect models i.e. with sexual selection in at least one sex.

260 The maximum likelihood estimate was

$$261 \quad \hat{a}_h = \frac{\lambda(a_h)}{\lambda(1)} \quad (11)$$

262 with $1 \leq h \leq \min\{k_1, k_2\}$.

263 Therefore, (11) provides the maximum likelihood estimate for the parameters of any sexual
264 isolation effects model in which the mating considered as heterotype has an absolute
265 propensity of 1 while the homotype mating has a different value. Note that this may involve
266 models in which the homotype mating has higher propensity (positive assortative mating,

267 $a_h > 1$) or vice versa, the homotype mating has lower propensity than the heterotype one
268 (negative assortative, $a_h < 1$).

269 The number of parameters ranges from 1 to the maximum possible number of mating types
270 considered as distinct homotypes, which is the minimum of $\{k_1, k_2\}$.

271 We could further define other sexual isolation models in which the homotype mating has
272 absolute propensity of 1 while the different heterotypes have absolute value of a_{ij} (see S1
273 section in Supporting Information).

274

275 **DOUBLE EFFECT MODELS**

276 We have seen that the sexual isolation models may have double effect (sexual isolation +
277 sexual selection) depending on the within sex frequencies. While it is not possible to assure
278 that the isolation model have not sexual selection, we can however, develop models that are
279 double in their effects still under frequencies satisfying (10).

280

281 **Double Models Under Uniform Frequencies**

282 The details of these models are given in the Supporting Information S1 section. Some of
283 them include an additive parameter of the type $m'_{jj} = 1 + c$ restricted to $|c| < 1$ instead of m'_{jj}
284 $= a$ restricted to $a > 0$. These additive parameters are required for the model to produce
285 double effects under uniform frequencies.

286 The ML estimates for some of these double effect models were obtained using the Nelder-
287 Mead simplex numerical algorithm (see below).

288

289 **General Double Effect Models**

290 We also generated any particular model with propensities $m_{ij} = \theta_{ij}$ with the restriction that
291 at least one propensity had value of 1. The MLE of the parameters was

$$292 \quad \hat{\theta}_{ij} = \frac{\lambda(\theta_{ij})}{\lambda(1)} \quad (12)$$

293 where $\lambda(\theta_{ij})$ is defined as in (7).

294 The most parameterized model has $K-1$ parameters and coincides with the saturated model
295 so that, only in this case, the estimates in (12) are the corresponding pair total indices (PTI).

296 All the above derived MLE formulae have also been checked by numerical approximation
297 using the bounded Nelder-Mead simplex algorithm (Press 2002; Singer and Singer 2004;
298 Gao and Han 2012). The set of described models jointly with their effects are summarized
299 in Table 1.

300

301 **Table 1.** Mutual mating propensity models as defined by different parameters and their
302 effects in a case with two different types at each sex ($k_1 = k_2 = 2$). When the value of m'_{ij} is
303 not explicitly given is assumed to be 1.

Name (abbreviation)	Model	MLE	Effect
---------------------	-------	-----	--------

Random (M_0)	$m'_{ij} = 1 \forall i,j$		Random mating
Multiplicative Sexual Selection Models			
Female sexual selection ($S_{\text{Fem}}-1P$)	$m'_{11} = m'_{12} = a$	$a = \lambda(a) / \lambda(1)$	Fem sexual selection
Male sexual selection ($S_{\text{Male}}-1P$)	$m'_{11} = m'_{21} = b$	$b = \lambda(b) / \lambda(1)$	Male sexual selection
Two sex sexual selection ($S2-2P$)	$m'_{11} = ab$ else		2-sex sexual selection
	$m'_{12} = a$ else	$a = \lambda_{\text{fem}}(a) / \lambda_{\text{fem}}(1)$	
	$m'_{21} = b$	$b = \lambda_{\text{male}}(b) / \lambda_{\text{male}}(1)$	
Sexual Isolation Models			
	non-multiplicative		Isolation freq-dep
Symmetric sexual isolation ($I-1p$)	$m'_{11} = m'_{22} = a$	$a = \lambda(a) / \lambda(1)$	Isolation UF
Full sexual isolation ($I-2p$)	$m'_{11} = a_1, m'_{22} = a_2$	$a_i = \lambda(a_i) / \lambda(1)$	Isolation freq-dep
General Double Models ($D-gp$)			
Saturated (M_{sat})	$m'_{ij} = a_{ij}, m'_{k_1k_2} = 1$	$a_i = \lambda(a_i) / \lambda(1)$	Isolation+Sex Sel

304 k_1 : number of female categories; k_2 : number of male categories; UF: uniform frequencies; $K = k_1 \times k_2$.

305

306 *Model Selection And Multimodel Inference*

307 Information-based model selection and multi-model inference can be applied for describing

308 uncertainty in a set of models and performing inference on the parameters of interest

309 (Burnham et al. 2011; Grueber et al. 2011; Barker and Link 2015; Claeskens 2016).

310 There are several information criteria at hand, although trusting on a single form of

311 information criterion is unlikely to be universally successful (Brewer et al. 2016; Aho et al.

312 2017). In consequence, we considered two Kullback-Leiberg divergence-based measures

313 plus the well-known Bayesian information criterion.

314 INFORMATION CRITERIA

315 The Akaike information criterion (AIC) provides the link between the Kullback-Leiberg
316 divergence and the maximized log-likelihood of a given model (Akaike 1973). The sample-
317 corrected version AICc is asymptotically equivalent and may work better for small sample
318 size so from herein we use the AICc version computed as

$$319 \quad AIC_c = -2\ln(L) + 2P_m + (2P_m(P_m+1))/(n' - P_m - 1)$$

320 where L is the maximum likelihood of the model, P_m the total number of mating parameters
321 estimated in the model and n' is the number of matings.

322 There is also a version for the symmetric K-L divergence (Jeffrey's) called the KICc
323 criterion (Cavanaugh 2004; Keerativibool 2014). Because mate choice models can be
324 described by the informational flow in the mating phenotypes, in the form of the Jeffrey's
325 divergence (Carvajal-Rodríguez 2018), it seems adequate considering KICc criterion
326 besides the AICc one.

$$327 \quad KIC_c = -2\ln(L) + n'\ln(n' / (n' - P_m)) + P_2$$

$$328 \quad \text{with } P_2 = n'[(n' - P_m)(2P_m + 3) - 2] / [(n' - P_m - 2)(n' - P_m)]$$

329 Finally, the Bayesian information criterion (BIC Schwarz 1978) permits (via the difference
330 between BIC values) an asymptotic approximation to the Bayes factor applied for model
331 comparison (Wagenmakers 2007)

$$332 \quad BIC = -2\ln(L) + P_m\ln(n')$$

333 Overdispersion

334 In the context of model selection, data overdispersion, i.e. greater observed variance than
335 expected, could generate the selection of overly complex models.

336 The simplest approach to estimate overdispersion is by computing a single variance
337 inflation factor (ν). This inflation factor is the observed variation divided by the expected
338 under the most complex model (M_c) among the proposed ones (Richards 2008; Symonds
339 and Moussalli 2011). It can be asymptotically approximated by the deviance i.e. twice the
340 difference between the log-likelihood of the saturated (M_{sat}) and the M_c model, divided by
341 the difference in the number of parameters ($P_{M_{\text{sat}}} - P_{M_c}$) between both models

$$342 \quad \nu = 2[\ln(L_{M_{\text{sat}}}) - \ln(L_{M_c})] / df$$

343 where $df = P_{M_{\text{sat}}} - P_{M_c}$.

344 If $1 \leq \nu \leq 4$ this indicate overdispersion while if higher than 4-6 this may indicate poor
345 model structure and the construction of the set of models should be reconsidered (Burnham
346 and Anderson 2002). For ν values around 1 to 4, quasi-likelihood theory provides a way to
347 analyse overdispersed data (Anderson et al. 1994; Richards 2008). The quasi-likelihood is
348 the likelihood divided by an estimate of ν , and so we can consider a quasi-likelihood
349 version of the various information criteria, namely QAICc, QKICc (Kim et al. 2014) and
350 QBIC simply by replacing the likelihood with the quasi-likelihood in the corresponding
351 formula. In such cases the number of parameters is increased by one and the model
352 variance is multiplied by ν (see below). When the quasi-likelihood version is used, it must
353 be done for all models and criteria.

354

355 **Model Weights**

356 Hereafter, we denote by IC when referring generically to any information criteria. For a
357 particular criterion and for any set of R models there is a minimum criterion value e.g.
358 AIC_{\min} , BIC_{\min} , etc. Thus, the models can be ranked regarding the difference with that
359 minimum

$$360 \quad \Delta_i = IC_i - IC_{\min}, \text{ for } i = 1, 2, \dots, R.$$

361 where IC_i refers to any specific information criterion for the model i .

362 Models can also be ranked by their weights from higher to lower. The weight w_i refers to
363 the strength of evidence for that model (Burnham et al. 2011; Claeskens 2016)

$$364 \quad w_i = l_i / \sum l_j \text{ for } j = 1, 2, \dots, R$$

365 where $l_i = \exp(-0.5\Delta_i)$ is the relative likelihood of each model given the data.

366

367 **MULTI MODEL INFERENCE**

368 If we are interested in estimating a mating parameter, we can obtain the value from the
369 best-fit model. Alternatively, we can make the inference based on a group of the most
370 credible models. The latter strategy is called multi-model-based inference (Burnham and
371 Anderson 2002; Burnham et al. 2011; Symonds and Moussalli 2011).

372 Therefore, we can perform a model averaged prediction for the parameters that are
373 variables in the best model.

374 Before performing the average, we should translate the different models to the same scale
375 of propensity. For example, a model like $m'_{11} = 2, m'_{12} = m'_{21} = m'_{22} = 1$, is not in the same
376 scale that $m'_{11} = 2, m'_{12} = m'_{21} = m'_{22} = 0.5$. Without loss of generality, the latter can be
377 transformed into a equivalent model $m'_{11} = 4, m'_{12} = m'_{21} = m'_{22} = 1$, which is now in the
378 same scale that the first model.

379 The averaged parameter estimates were computed as a weighted mean where the weights
380 are the strength of evidence for each model as obtained under a given information criterion.
381 The parameters were averaged only over the models for which they appear as a variable.
382 Because the weights need to sum up to 1, it was necessary renormalize them by dividing by
383 the accumulated weight in the confidence subset.

384 Therefore, for a parameter included in the confidence subset R_s we have

$$385 \quad \hat{m} = \frac{\sum_i^{R_s} w_i \hat{m}_i}{\sum_i^{R_s} w_i}$$

386 This way of performing the model averaged prediction is called natural averaging
387 (Symonds and Moussalli 2011).

388 Finally, the reliability of each parameter estimate was measured as the unconditional
389 standard error

$$390 \quad Se(\hat{m}) = \sum_i^{R_s} w_i \sqrt{vV(\hat{m}_i) + (\hat{m}_i - \hat{m})^2}$$

391 where $V(\hat{m}_i) = V(m_i | \text{model } i) = V(q') = q'(1-q') / n'$ is the model standard error squared and
392 v is the variance inflation factor.

393 The use of the sum of weights to estimate variable importance in regression models has
394 been criticized because multicollinearity among the predictor variables and the imprecision
395 of the weight measures (Galipaud et al. 2014; Cade 2015; Galipaud et al. 2017).

396 However, the mutual propensity parameters do not belong to a regression model and their
397 average is performed in the same scale and with comparable units. Therefore, under the
398 mutual mating propensity setting, the multimodel inference would work well as we
399 confirmed by simulation (next section).

400

401 *Simulations*

402 To test how well the exposed methodology is able to distinguish the different classes of
403 effects models and estimate the mating parameters, we used the sampling with replacement
404 algorithm in the program MateSim (Carvajal-Rodriguez 2018b) for generating mating
405 tables by Monte Carlo simulation (see section S2 in Supporting Information for detailed
406 explanation).

407 We have simulated mating tables corresponding to sexual isolation and sexual selection
408 mate choice effects. These effects correspond to an a priori mating pattern defined by the
409 mating propensity distribution (Table S1 and Fig. S1). The resulting mating tables were
410 consequence of the mating pattern and the sampling process.

411 The mating tables consisted in two types of information (Fig. S2). First, the population
412 frequencies (pre-mating individuals) which were generated randomly for each simulation

413 run. Note that the minimum phenotype frequency (MPF) allowed was 0.05. Second, we
414 counted 500 mating pairs ($n' = 500$) for a hypothetical trait with two classes at each sex.
415 Because we simulated a species with high population size ($n = 10,000$) the mating process
416 was represented as a sampling with replacement, and the population frequencies were
417 constant over the mating season.

418 Concerning the mating pairs, the number of occurrences for each mating type $i \times j$ was
419 obtained as

$$420 \quad Q(i, j) = n' \times p_{1i} \times p_{2j} \times m_{ij}$$

421 where n' is the sample size and m_{ij} is the normalized propensity of the mating pair.

422 The propensities assayed corresponded to random mating, $m'_{11}=m'_{22} = m'_{12}=m'_{21}=1$ (M_0 in
423 Table 2); positive assortative mating, $m'_{11}=m'_{22} = 2$, $m'_{12}=m'_{21}=1$ (Isol in Table 2); and
424 female sexual selection, $m'_{11}=m'_{12} = 2$, $m'_{21}=m'_{22}=1$ (SS_{Fem} in Table 2). The parameter
425 estimates were averaged over 1000 runs for each case.

426 In Table 2, we present the results of the multimodel inference from the simulated tables.

427 We may appreciate that random mating was perfectly estimated by the three IC methods.

428 The mate choice effects models were also similarly estimated by the three criteria. The

429 sexual selection model estimates were slightly less accurate which is caused by the

430 variation in the population frequencies that may generate scenarios with double effect

431 models (sexual selection + sexual isolation) and/or sexual selection in both sexes.

432

433 **Table 2. Average (standard error) parameter estimates for different models under**
 434 **sample size 500.**

Model		m'_{11}	m'_{12}	m_{21}	m_{22}
M₀	Expected	1	1	1	1
	AICc	1.0±0.0000	1.0±0.0000	1.0±0.0000	1.0±0.0000
	KICc	1.0±0.0000	1.0±0.0000	1.0±0.0000	1.0±0.0000
	BIC	1.0±0.0000	1.0±0.0000	1.0±0.0000	1.0±0.0000
Isol	Expected	2	1	1	2
	AICc	2.0±0.004	1.0±0.013	1.0±0.001	2.0±0.004
	KICc	2.0±0.005	1.0±0.009	1.0±0.001	2.0±0.005
	BIC	2.0±0.006	1.0±0.003	1.0±0.001	2.1±0.006
SS_{Fem}	Expected	2	2	1	1
	AICc	2.1±0.183	1.9±0.013	1.0±0.007	1.0±0.006
	KICc	2.1±0.163	1.9±0.010	1.0±0.007	1.0±0.006
	BIC	1.8±0.010	1.8±0.013	1.0±0.009	1.0±0.006

435 M₀: Random mating model. Isol: isolation model; SS_{Fem}: Female sexual selection model.

436 The results were similar with lower sample size ($n'=100$, Table S2) but female sexual
 437 selection were less accurately estimated specially under the BIC criteria. The loss of
 438 accuracy is caused by some few cases having frequency values close to the minimum in

439 which, after sampling, there were no power to distinguish from the random mating pattern.
440 However, if the frequencies were uniformly distributed, i.e. $p_{1i}=1/2$ and $p_{2j} = 1/2$, then even
441 under sample size of 100, the estimation of female sexual selection was accurate resulting
442 in the correct parameter estimates ($m'_{11}=2$, $m'_{12}=2$, $m'_{21}=1$, $m'_{22}=1$) for the three information
443 criteria (see SS_{FEMU} case in Table S2).

444

445 *Example of application*

446 *Littorina saxatilis* is a marine gastropod mollusc adapted to different shore habitats in
447 Galician rocky shores. There are two different ecotypes, an exposed-to-wave (smooth un-
448 banded, SU), and a non-exposed (rough banded, RB) ecotype. Several experimental studies
449 have shown that these ecotypes have evolved local adaptation at small spatial scale. For
450 example, stronger waves on the lower shore may provoke that the SU ecotype becomes
451 sexually mature at smaller size than the upper-shore (RB) ecotype. In addition, in some
452 areas of the mid-shore habitat, the two ecotypes occasionally mate, producing apparently
453 fertile intermediate morphological forms that are called hybrids (HY) (Rolan-Alvarez et al.
454 2015).

455 Sexual isolation between RB and SU morphs was observed in wild mating pairs in the mid-
456 shore zone, likewise size-assortative mating in all shore levels (Rolán-Alvarez et al. 1999;
457 Cruz et al. 2001). It is assumed that the size is the key trait causing the increase of sexual
458 isolation in this model system, being the male the choosy sex in this species (Rolan-Alvarez
459 2007).

460 Here, we analysed two *L. saxatilis* data sets (Table 3) corresponding to two sampling years
461 (Rolán-Alvarez et al. 1999; Cruz et al. 2001) for estimating the mutual mating propensity
462 parameters between the RB, SU and HY morphs.

463

464 **Table 3.** The population frequencies by year and sex, from Rolán-Alvarez et al. (1999) and
465 Cruz et al. (2001) autumn data.

	Year	Sample size	RB	HY	SU
Female	1999	1222	0.36	0.12	0.52
	2001	598	0.29	0.13	0.58
Male	1999	1140	0.35	0.12	0.53
	2001	476	0.36	0.15	0.49

466

467 We first computed the information partition (Carvajal-Rodríguez 2018) and saw that there
468 was significant sexual isolation in both data sets (J_{PSI} p value < 0.0001) while no significant
469 sexual selection was detected.

470 Then, we considered 27 models from the different classes (Fig. 1 and Table S1), including
471 M_0 (random mating, 0 mating parameters), various isolation, sexual selection and double
472 effect models with up to 7 parameters and the M_{sat} (8 independent mating parameters)
473 model. Each data set was analysed separately, computing the maximum likelihood
474 estimates and the corresponding information criteria.

475 The data from 1999 were more dispersed and the uncertainty in the models higher than in
476 the data from 2001. However, in both years and for the three information criteria, the

477 models with at least 5% of weight were always double effect models (sexual isolation +
478 sexual selection) with only 1 or 2 parameters. These models implied a pattern of positive
479 assortative mating.

480 To better compare the mating pattern for the different data we evaluated the mating
481 parameter values obtained by multimodel inference. The major difference among the
482 different criteria and years corresponded to the mating involving hybrids. However, in
483 every case, the most favourable matings were RB×RB and SU×SU and the less favourable
484 mating was SU female by RB male (SU×RB). The three criteria produced similar relative
485 estimates. Therefore, we present only the AICc multimodel estimates averaged over the two
486 years (Table 4). For clarity, the estimates within each year were divided by the estimate of
487 the HY×SU mating so that the value of m_{HYSU} is always 1.

488 The scenery depicted showed that the within-ecotype mating was preferred. RB males
489 chose about twice as much the mating with RB females (m_{RBRB}) than with HY females
490 (m_{HYRB}), while the mating with SU females (m_{SURB}) was ten times less frequent.

491 HY males had a quite variable mating pattern depending on the year, but it seems they
492 preferred RB and HY females more than SU ones. The SU males showed the opposite
493 pattern, they preferred SU over HY and RB females. Thus, as RB, the SU males also
494 preferred the within ecotype mating although with not so strong effect.

495

496 **Table 4.** *Littorina saxatilis* data. Mutual-propensity estimates from multimodel inference
497 under the AICc. Values are the average over the two year data plus/minus the standard error

		Males		
		RB	HY	SU
	RB	1.9±0.09	1.5±0.38	1.0±0.0
Females	HY	1.0±0.0	1.5±0.34	1.0±0
	SU	0.16±0.01	1.0±0.0	1.7±0.23

498

499 *Discussion*

500

501 **SIMULATIONS**

502 We have simulated mating tables corresponding to random mating, sexual isolation and
503 sexual selection mate choice effects. The random mating pattern was perfectly assessed
504 independently of the sample size. Under the higher sample size ($n' = 500$), the multimodel
505 inference accurately estimated the true effects both for sexual isolation and sexual
506 selection. With low sample size ($n' = 100$) the accuracy was good for the isolation pattern
507 but not so for the sexual selection pattern. This occurs because the sexual selection pattern
508 underlying in the propensities matrix was not so well represented in the mating table in
509 some runs, so the accuracy was lower in these cases because the pattern was not reflected in
510 the sampling.

511 Not surprisingly, BIC was slightly more conservative, while AICc presented slightly more
512 accurate estimates in some cases. The KICc criterion performed similar to the best AICc
513 and BIC cases.

514 In general, the estimation was accurate and even in the worst cases (lowest sample size and
515 extreme phenotypic frequencies), the mean estimates were closer to the real value than to
516 random mating.

517

518 **EXAMPLE**

519 Previous studies in the Galician *L. saxatilis* hybrid zone showed that mate choice favours
520 within-morph pairs, whereas pairs including at least one hybrid morph seems to mate
521 randomly. The latter is not so clear in our analysis, may be due to the low number of hybrid
522 mating in the data (Fig. S3).

523 The estimates obtained by multimodel inference support the positive assortative mating for
524 the ecotype. In addition, another result emerged from the analysis: The pair RB male with
525 SU female, has less mutual preference than SU male with RB female ($m_{\text{SURB}} < m_{\text{RBSU}}$).

526 This pattern could be related with a physical difficulty for the mating involving bigger RB
527 males with the smaller SU females, as mating pairs with female bigger than the male (a
528 typical trend in gastropods) are somehow more frequent (E. Rolán-Alvarez personal
529 communication).

530 In addition to the mating pattern depicted by the multi-model approach, we also got the
531 relative estimates of the mating parameters. Testing the reliability of these estimates is,
532 however, out of the scope of the present manuscript, and it was left for future work.

533

534 **GENERAL**

535 The advantages of model selection and multimodel inference in evolutionary ecology has
536 already been widely discussed, jointly with the pros and cons of applying any information
537 criteria (Link and Barker 2006; Burnham et al. 2011; Aho et al. 2014; Barker and Link
538 2015; Aho et al. 2017) or the reliability of the obtained estimates (Galipaud et al. 2014;
539 Cade 2015; Giam and Olden 2016; Galipaud et al. 2017).

540 Multimodel inference has been however, rarely utilized in the context of sexual selection
541 and isolation effects of mate choice. Here, by proposing general models causing different
542 type of effects, jointly with their maximum likelihood estimates, we are providing the first
543 standardized methodology for model selection and multimodel inference of the mating
544 parameters involved in sexual selection and isolation effects.

545 The set of *a priori* models (including user-defined ones) permits to perform an *a posteriori*
546 quantification of the data-based evidence and provide confidence sets on plausible non-
547 trivial models while letting at the same time, multi-model inference on the parameters.

548 The approach was implemented by allowing three different information criteria. Under the
549 scenarios assayed, they performed similarly for simulated and real data.

550 The proposed methodology should ideally work under scenarios for which the availability
551 of individuals is not affected by the matings that have already occurred, as expected for
552 polygamous species, or even for monogamous species, when the number of available
553 individuals is higher than the mating pairs. In the case of monogamous species with low
554 population size, the population frequencies may be altered during the reproductive season
555 and so, the distribution of propensities could not define the same sexual selection pattern
556 over the season. This could happen because the sexual selection effects are frequency
557 dependent. On the contrary, the sexual isolation effects are not frequency dependent, and
558 should not be affected by the variation of the population frequencies (Carvajal-Rodriguez
559 2018a).

560 The developed methodology has been fully implemented in a program called InfoMating
561 available at <http://acraaj.webs.uvigo.es/InfoMating/Infomating.htm> or upon request to the
562 author.

563

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571 **REFERENCES**

- 572 Akaike, H. 1973. Information theory and an extension of the maximum likelihood
573 principle. Pp. 267-281 in B. N. Petrov, and F. Csaki, eds. Second International
574 Symposium on Information Theory, Budapest: Akademiai Kiado.
- 575 Anderson, D. R., K. P. Burnham, and G. C. White. 1994. AIC Model Selection in
576 Overdispersed Capture-Recapture Data. *Ecology* 75:1780-1793.
- 577 Burnham, K. P. and D. R. Anderson. 2002. Model selection and multimodel inference: a
578 practical information-theoretic approach. Springer-Verlag, New York, NY.
- 579 Burnham, K. P., D. R. Anderson, and K. P. Huyvaert. 2011. AIC model selection and
580 multimodel inference in behavioral ecology: some background, observations, and
581 comparisons. *Behavioral Ecology and Sociobiology* 65:23-35.
- 582 Carvajal-Rodríguez, A. 2018a. A generalization of the informational view of non-random
583 mating: Models with variable population frequencies. *bioRxiv*:279075.
- 584 Carvajal-Rodríguez, A. 2018b. MateSim: Monte Carlo simulation for the generation of
585 mating tables. *bioRxiv*:239178.
- 586 Carvajal-Rodríguez, A. 2018. Non-random mating and information theory. *Theoretical*
587 *Population Biology* 120:103-113.
- 588 Cavanaugh, J. E. 2004. Criteria for linear model selection based on Kullback's symmetric
589 divergence. *Australian & New Zealand Journal of Statistics* 46:257-274.
- 590 Claeskens, G. 2016. Statistical Model Choice. *Annual Review of Statistics and Its*
591 *Application* 3:233-256.
- 592 Cruz, R., E. Rolán-Álvarez, and C. García. 2001. Sexual selection on phenotypic traits in a
593 hybrid zone of *Littorina saxatilis* (Olivi). *Journal of Evolutionary Biology* 14:773-
594 785.
- 595 Hartl, D. L. and A. G. Clark. 1997. *Principles of Population Genetics*. Sinauer Associates,
596 Inc., Sunderland, MA.
- 597 Keerativibool, W. 2014. Unifying the Derivations of Kullback Information Criterion and
598 Corrected Versions. *Thailand Statistician* 12:37-53.
- 599 Kim, H.-J., J. E. Cavanaugh, T. A. Dallas, and S. A. Foré. 2014. Model selection criteria for
600 overdispersed data and their application to the characterization of a host-parasite
601 relationship. *Environmental and ecological statistics* 21:329-350.
- 602 Rolan-Alvarez, E. 2007. Sympatric speciation as a by-product of ecological adaptation in
603 the Galician *Littorina saxatilis* hybrid zone. *Journal of Molluscan Studies* 73:1-10.
- 604 Rolan-Alvarez, E., C. Austin, and E. G. Boulding. 2015. The contribution of the genus
605 *Littorina* to the field of evolutionary ecology. *Oceanography and Marine Biology:*
606 *an Annual Review* 53:157-214.
- 607 Rolán-Alvarez, E. and A. Caballero. 2000. Estimating sexual selection and sexual isolation
608 effects from mating frequencies. *Evolution* 54:30-36.
- 609 Rolán-Alvarez, E., J. Erlandsson, K. Johannesson, and R. Cruz. 1999. Mechanisms of
610 incomplete prezygotic reproductive isolation in an intertidal snail: testing
611 behavioural models in wild populations. *Journal of Evolutionary Biology* 12:879-
612 890.
- 613 Schwarz, G. 1978. Estimating the dimension of a model. *The Annals of Statistics* 6:461-
614 464.

615 Symonds, M. R. E. and A. Moussalli. 2011. A brief guide to model selection, multimodel
616 inference and model averaging in behavioural ecology using Akaike's information
617 criterion. Behavioral Ecology and Sociobiology 65:13-21.

618 Wagenmakers, E.-J. 2007. A practical solution to the pervasive problems of p values.
619 Psychonomic bulletin & review 14:779-804.

620

621

622

623 **SUPPORTING INFORMATION**

624

625 The Supporting Information file includes:

626 S-1) Mutual Mating Propensity Models

627 S-2) Monte Carlo simulation of mating tables

628 S-3) Example data input file.

629

630 **FIGURE LEGENDS**

631 **Fig 1.** Mating models as defined by their effect after relaxing some of the conditions imposed to the
632 random mating model M_0 .

633