

1 Regulation of evidence accumulation by pupil-linked
2 arousal processes

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Abstract

Integrating evidence over time is crucial for effective decision making. For simple perceptual decisions, a large body of work suggests that humans and animals are capable of integrating evidence over time fairly well, but that their performance is far from optimal. This suboptimality is thought to arise from a number of different sources including: (1) noise in sensory and motor systems, (2) unequal weighting of evidence over time, (3) order effects from previous trials and (4) irrational side biases for one choice over another. In this work we investigated these different sources of suboptimality and how they are related to pupil dilation, a putative correlate of norepinephrine tone. In particular, we measured pupil response in humans making a series of decisions based on rapidly-presented auditory information in an evidence accumulation task. We found that people exhibited all four types of suboptimality, and that some of these suboptimalities covaried with each other across participants. Pupillometry showed that only noise and the uneven weighting of evidence over time, the ‘integration kernel’, were related to the change in pupil response during the stimulus. Moreover, these two different suboptimalities were related to different aspects of the pupil signal, with the individual differences in pupil response associated with individual differences in integration kernel, while trial-by-trial fluctuations in pupil response were associated with trial-by-trial fluctuations in noise. These results suggest that different sources of suboptimality in human perceptual decision making are related to distinct pupil-linked processes possibly related to tonic and phasic norepinephrine activity.

1 Introduction

The ability to integrate evidence over time is a crucial component of perceptual decision making. This is true whether we are integrating visual information from saccade to saccade as we scan a scene, or integrating auditory information from word to word as we listen to someone talk. In recent years much work has been devoted to understanding how humans and animals perform evidence integration over short time scales (on the order of one second) in simple perceptual tasks [1, 2, 3, 4]. In a classic paradigm from

33 this literature, known as the Random Dot Motion Task, participants are presented with
34 a movie of randomly moving dots that have a weak tendency to drift in a particular
35 direction (e.g. left or right) and they must decide which way the dots are drifting [5].
36 The optimal strategy in this task is to count, i.e. integrate, the number of dots moving
37 to the left and right over the time course of the stimulus and choose the side that had
38 the most dots moving in that direction. Amazingly, this optimal strategy can account for
39 many of the qualitative properties of human and animal behaviour and neural correlates
40 of integrated evidence can be found in several areas of the brain [2, 6, 3, 7].

41 Despite the ability of the optimal model to qualitatively account for a number of
42 experimental findings, the quantitative performance of even highly trained humans and
43 animals is suboptimal [1, 8]. This suboptimality is thought to arise from at least four
44 different sources: (1) neuronal noise, (2) unequal weighting of evidence over time, (3)
45 order effects from previous trials and (4) side biases.

46 The first source of suboptimality is neuronal noise. While the exact cause of neuronal
47 noise is subject to debate [9, 10, 11, 8], it is thought that variability in neural firing impacts
48 perceptual decision making in one of two ways. First, noisy sensory information reduces
49 the quality of the evidence going into the accumulator in the first place [12, 1, 13, 14].
50 Second, noisy action selection causes mistakes to be made even after the integration
51 process is complete [15, 16, 17].

52 The second source of suboptimality comes from the unequal weighting of evidence over
53 time, which we call here the ‘integration kernel’. In particular, while the optimal kernel
54 in most perceptual decision making tasks is flat — i.e. all information is weighed equally
55 over time — a number of studies have shown that humans and animals can have quite
56 suboptimal kernels. For example, in the Random Dot Motion Task, monkeys exhibit
57 a ‘primacy’ kernel, putting more weight on the early parts of the stimulus relative to
58 the later parts of the stimulus [4]. Conversely, in a slightly different integration task,
59 humans exhibit the opposite ‘recency’ kernel, weighing later information more than early
60 information [18, 19]. Finally, in some experiments this second source of suboptimality
61 appears to be absent, with a ‘flat’ integration kernel being found in both rats and highly

62 trained humans [1].

63 The third source of suboptimality reflects the tendency to let previous decisions and
64 outcomes interfere with the present choice. Thus, when making multiple perceptual
65 decisions, the current decision is influenced by the choice we just made, for example by
66 repeating an action when it is rewarded and choosing something else when it is not, a
67 reinforcement learning effect [15, 16, 8, 20], or simply repeating a choice regardless of the
68 outcome associated with it, a choice kernel effect [8, 20, 21]. Such sequential dependence
69 can be advantageous when there are temporal correlations between trials, as is the case in
70 many reinforcement learning tasks [15, 16], but is suboptimal in most perceptual decision
71 making tasks when each trial is independent of the past [22, 23, 8, 20].

72 Finally, the fourth suboptimality is an overall side bias where both humans and animals
73 develop a preference for one option (e.g. left) even though that leads to more errors overall
74 [20].

75 Evidence from a number of studies suggests that pupil-linked arousal processes, puta-
76 tively driven by the locus coeruleus norepinephrine system [24, 25, 26, 27, 28], are well
77 placed to modulate all four of these different sources of suboptimality. With regard to
78 noise, increased pupil response has been associated with a number of different cognitive
79 processes such as effort, arousal, mood, attention and memory, all of which might influ-
80 ence noise [26, 29]. In the specific case of perceptual decisions, previous work suggests
81 a role for pupil-linked arousal systems to modulate the overall neuronal noise, i.e. the
82 signal-to-noise ratio of sensory cues, in the evidence accumulation process [30, 31]. With
83 regard to kernel and side bias, pupil response has been associated with a change the ‘gain’
84 of other neural systems, which in turn is thought to modulate the strength of internal
85 and external cognitive biases on decision making [26, 28, 32, 33, 34, 35]. In addition,
86 recent empirical and theoretical work has also suggested that norepinephrine, a putative
87 driver of pupil dilation, modulates the urgency of decision making in a sequential sam-
88 pling task such that the higher the norepinephrine level, the more urgently a decision is
89 made [36, 37, 38]. Taken together these studies point to the possibility of pupil-linked
90 norepinephrine systems to modulate the integration kernel and side bias by changing the

91 strength of pre-existing biases during the integration process. Finally, with regard to
92 sequential effects, pupil changes have been related to how humans integrate relevant in-
93 formation from previous trials to infer uncertainty and expectation [21, 39, 40], suggesting
94 a role for pupil-linked arousal systems in modulating sequential effects.

95 In this work we investigated all four sources of suboptimal perceptual decision making
96 and their relationship to between pupil-linked arousal processes in a single task. By
97 quantifying all four sources of suboptimality in the same task we were able to assess the
98 relationships between the suboptimalities and determine the extent to which pupil-linked
99 arousal processes were related to each.

100 **2 Results**

101 To study the effects of pupil response on evidence accumulation, we designed an auditory
102 discrimination task based on the Poisson Clicks Task [1]. In this task, participants listened
103 to two trains of clicks in the left and right ears, and were instructed to indicate which side
104 they thought had more clicks (Figure 1a). Clicks in our task were generated according
105 to a Bernoulli process, such that there was always a click every 50ms that was either on
106 the left, with probability p_{left} , or otherwise on the right. This process meant that the
107 total number of clicks was always fixed at 20 clicks and the clicks occurred at a fixed
108 frequency of 20 Hz. This generative process for the clicks represented a slight departure
109 from [1], in which clicks were generated by a Poisson process with a refractory period
110 of 20ms. The main reason for using a Bernoulli process was to simplify the subsequent
111 logistic regression analysis for quantifying the different sources of suboptimality, without
112 imposing too much a priori assumptions. To indicate this difference we refer to our task
113 as the Bernoulli Clicks Task.

114 **2.1 Psychometric and chronometric functions**

115 108 participants each performed between 666 and 938 trials (mean 760.7) of the Bernoulli
116 Clicks Task. Basic behaviour was consistent with behaviours in similar pulsed-accumulation

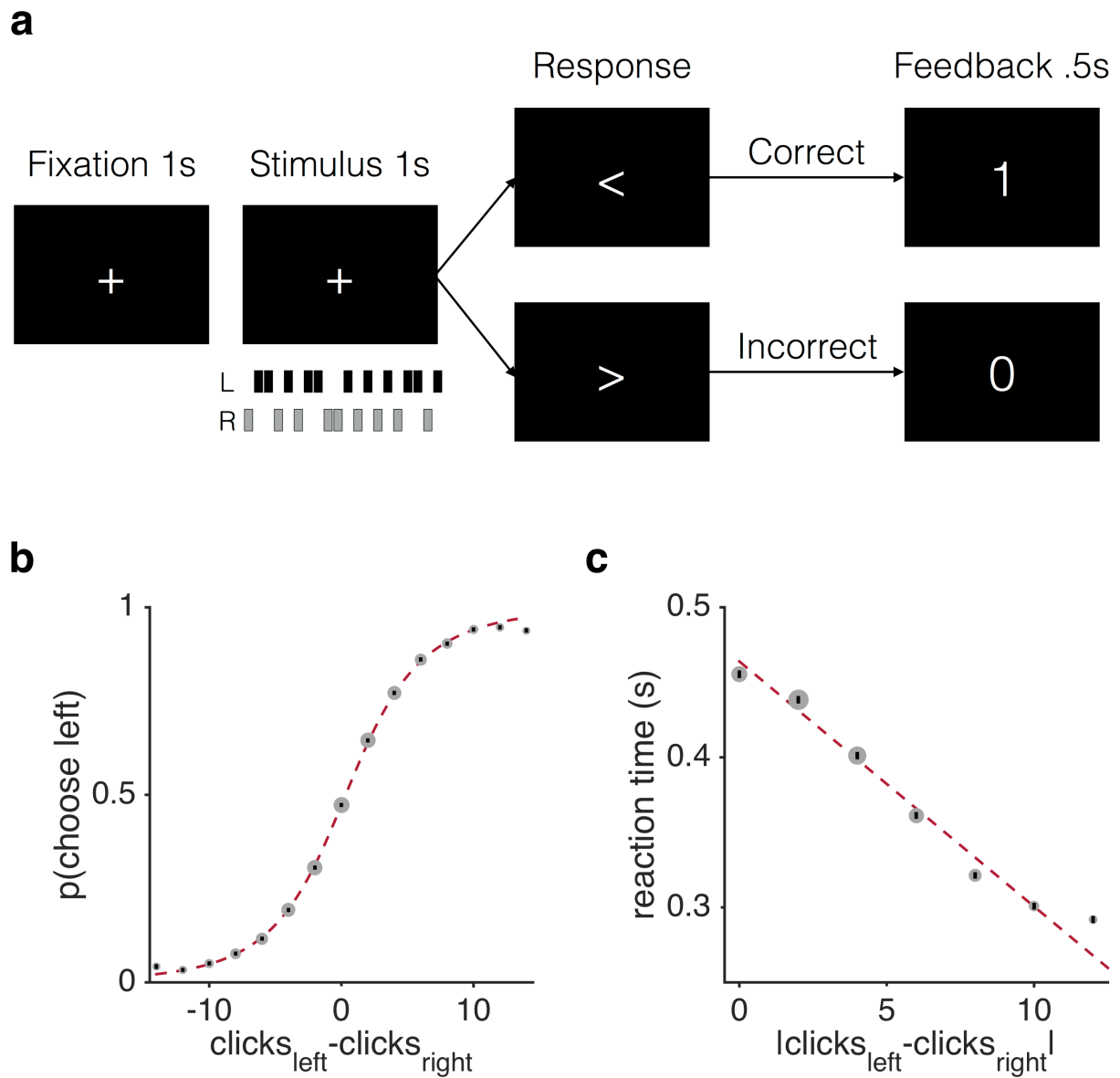


Figure 1: Basic behaviour of 108 participants. (a) Participants listened to a train of twenty clicks coming in either the left (L, black bars) or right (R, grey bars) ear for one second, and decided which side had more clicks. (b) Choice probability (probability of choosing left) showed sigmoidal relationship with difficulty (the difference in number of clicks between left and right). (c) Reaction times were higher on more difficult trials. Size of grey dots scaled by number of trials. All error bars (black bars) indicate s.e.m. across participants. Dotted red lines are fits with sigmoidal function and linear function respectively.

117 tasks [1, 4]. Choice exhibited a sigmoidal dependence on the net difference in evidence
 118 strength, i.e. the difference in number of clicks between the right and left, Δclick (Figure
 119 1b). A simple logistic regression of the form:

$$\text{logit}(p_{\text{left}} \text{ at trial } t) = \beta_0 + \beta_{\Delta\text{click}}\Delta\text{click} \quad (1)$$

120 revealed a significant effect of Δclick ($\beta_{\Delta\text{click}} = 0.3500$, $p = 0.0001$). Reaction times were
 121 also modulated by net evidence strength (Figure 1c) and linear regression of the form:

$$\text{RT at trial } t \text{ in seconds} = \beta_0 + \beta_{\Delta\text{click}}|\Delta\text{click}| \quad (2)$$

122 found a significant effect of the absolute value of Δclick on RT ($\beta_{\Delta\text{click}} = -0.017$, $p =$
 123 3.7×10^{-13}). These results indicated that participants were faster and more accurate
 124 when the difference of number of clicks was large (easy trials), and less accurate and
 125 slower when that difference was small (hard trials).

126 **2.2 Humans exhibited all four suboptimalities in the Bernoulli** 127 **Clicks Task**

128 We used a logistic regression model to characterize the four different types of subopti-
 129 malities in human decision making in our task. This model quantified the impact of each
 130 click, the reinforcement learning and choice kernel effects from the five previous trials and
 131 the side bias on participants' choices. In particular, we assumed that the probability of
 132 choosing left on trial t was given by

$$\text{logit}(p_{\text{left}} \text{ at trial } t) = \underbrace{\sum_{i=1}^{20} \beta_i^{\text{click}} c_i}_{\text{intergration kernel}} + \underbrace{\sum_{j=1}^5 \beta_j^{\text{RL}} a_{t-j} r_{t-j}}_{\text{reinforcement learning}} + \underbrace{\sum_{j=1}^5 \beta_j^{\text{CK}} a_{t-j}}_{\text{choice kernel}} + \underbrace{\beta^{\text{side}}}_{\text{side bias}} \quad (3)$$

133 where c_i was the i th click (+1 for a left click and -1 for right), a_{t-j} was the choice made
 134 on the $t - j$ th trial (+1 for a left choice and -1 for right), and r_{t-j} was the 'reward' on the

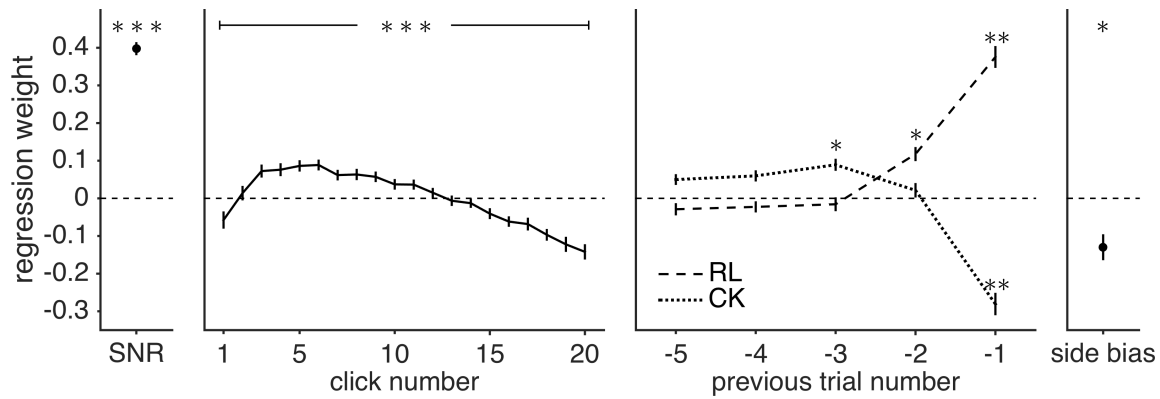


Figure 2: Regression model. (Leftmost) Mean of click weights is significantly above zero. T-test against zero, two-tailed, ***: FDR corrected for multiple comparisons $p < 0.00001$. (Second from left) Deviation of click weights from the mean has an uneven shape. repeated measures ANOVA, ***: $p < 0.00001$. (Second from right) Effect of previous trials: RL (the correct side in previous trial) positively predicts choice, indicating a reinforcement learning effect, while as CK (the choice made in previous trial) negatively predicts choice, indicating a alternating choice kernel. T-test against zero, two-tailed, **: FDR corrected for multiple comparisons $p < 0.00001$, Cohen's $d > 1$; *: FDR corrected for multiple comparisons $p < 0.00001$, Cohen's $d > 0.5$. (Rightmost) Side bias. T-test against zero, two-tailed, *: FDR corrected for multiple comparisons $p = 0.0001$. All error bars (black bars) indicate s.e.m. across participants.

135 $t - j$ th trial (+1 for correct and -1 for incorrect). Therefore, $a_{t-j}r_{t-j}$ indicated the correct
 136 side on the $t - j$ th trial (+1 when left was correct and -1 when right was correct). The
 137 relative effect of each of these terms on the decision was determined by the regression
 138 weights: β_i^{click} (the effect of each click), β_j^{RL} (the reinforcement learning (RL) effect, i.e.
 139 effect of previous correct side), β_j^{CK} (the choice kernel (CK) effect, i.e. effect of previous
 140 choice) and β^{side} (an overall side bias).

141 Each of the four suboptimality could be quantified using different parameters from
 142 this model (Figure 2). First, the signal-to-noise ratio (SNR), corresponding to subopti-
 143 mality arising from neuronal noise, was quantified as the average weight given to all clicks
 144 ($\frac{1}{20} \sum_{i=1}^{20} \beta_i^{\text{click}}$). The higher the average click weight, the higher the SNR or equivalently,
 145 the lower the relative level of the noise. This average was significantly different from
 146 zero ($T(107) = 27.65$, two-tailed, FDR corrected for multiple comparisons $p < 0.00001$,
 147 Cohen's $d = 2.66$) (Figure 2 leftmost panel), indicating that participants based their
 148 decision on (at least some of) the clicks and that each click increased the log odds of
 149 ultimately choosing that direction by about 0.4.

150 The second suboptimality, i.e. deviations from a flat integration kernel, was quantified
151 as the deviation of the click weights from the average, i.e. $\beta_i^{\text{click}} - \frac{1}{20} \sum_{j=1}^{20} \beta_j^{\text{click}}$ (Figure
152 2 second from left panel). Here we found that participants did not weigh all the clicks
153 equally (repeated measures ANOVA, $F(19, 2033) = 28.21$, $p < 0.00001$, partial $\eta^2 =$
154 0.21). This was not consistent with previous reports with a similar task where all clicks
155 received equal weighting on average [1].

156 Sequential effects, the third suboptimality, were captured by the effects from previous
157 trials. Specifically, the terms β_j^{RL} and β_j^{CK} quantified the reinforcement learning (RL)
158 effect (effect of past correct side) and choice kernel (CK) effect (effect of past choice) for
159 the past five trials on the current choice. In line with earlier work [20], we found that
160 previous trials had both significant RL and CK effects on participants' choices (Figure
161 2 second from right panel). Notably, the positive RL regression weight demonstrated a
162 positive reinforcement learning effect, in that participants tended to choose whichever
163 side that was shown to be correct on the previous trial ($T(107) = 14.40$, two-tailed, FDR
164 corrected for multiple comparisons $p < 0.00001$, Cohen's $d = 1.39$). The negative CK
165 regression weight indicated an alternating choice kernel — participants tended to choose
166 the opposite of what they had chosen on the previous trial ($T(107) = -10.45$, two-tailed,
167 FDR corrected for multiple comparisons $p < 0.00001$, Cohen's $d = -1.01$).

168 Finally, the side bias was quantified by the intercept term β^{side} in the model (Figure 2
169 rightmost panel). This term quantified the extent to which a participant chose the left
170 side on all trials regardless of which side was the correct side. Here we saw a significant
171 right bias indicated by a significantly negative regression weight ($T(107) = -4.12$, two-
172 tailed, FDR corrected for multiple comparisons $p = 0.0001$, Cohen's $d = -0.40$).

173 **2.3 Sequential effects and signal-to-noise ratio covary across** 174 **participants**

175 We then inspected how these suboptimalities correlated with each other across partici-
176 pants. We used a three way mixed ANOVA to inspect the effect of previous trials on both
177 SNR and kernel shape. The three factors we investigated were: RL regression weights,

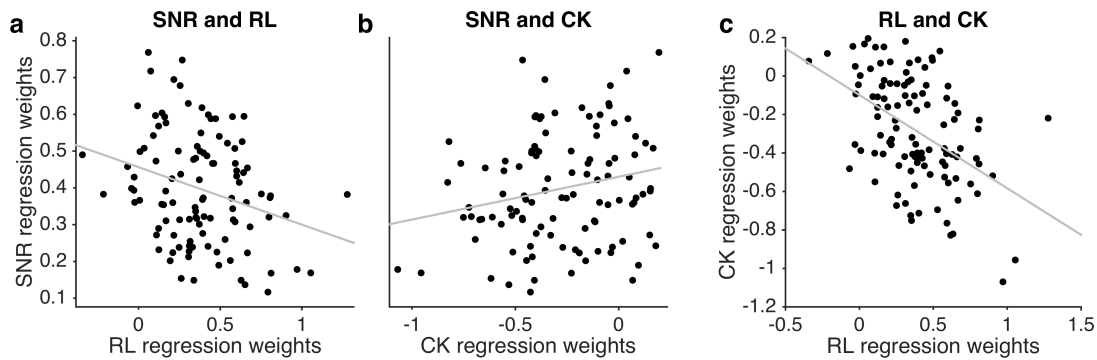


Figure 3: Interaction between suboptimalities across participants. (a) RL is significantly negatively correlated with SNR ($r = -0.28$, $p = 0.003$). (b) CK is significantly positively correlated with SNR ($r = 0.22$, $p = 0.03$). (c) RL is significantly negatively correlated with CK ($r = -0.46$, $p = 5 \times 10^{-7}$).

178 CK regression weights, and time. The ANOVA was set up to investigate the effect of
179 these three factors on the regression weights of clicks. In this ANOVA, the main effects of
180 either RL or CK on kernel weights told us whether RL or CK correlated with the overall
181 SNR. The interaction effect between either RL or CK and time on kernel weights told us
182 whether RL or CK correlated with the kernel shape.

183 We found that both RL and CK had significant main effects on SNR (RL: $F(1, 2080) =$
184 50.66 , $p = 1.5 \times 10^{-12}$, CK: $F(1, 2080) = 56.82$, $p = 7.1 \times 10^{-14}$), but not kernel shape
185 (RL \times time: $F(19, 2080) = 0.80$, $p = 0.71$, CK \times time: $F(19, 2080) = 0.19$, $p = 0.99$).
186 Specifically RL is negatively correlated with SNR ($r = -0.28$, $p = 0.003$) (Figure 3a),
187 while CK is positively correlated with SNR ($r = 0.22$, $p = 0.03$) (Figure 3b).

188 Importantly, since the RL effect was positive (Figure 2), i.e. participants tended to
189 choose, on the current trial, whichever side was correct in the previous trial), a negative
190 correlation indicated that participants who relied more on feedback from the previous
191 trial tended to rely less on information on the current trial. Conversely, since CK effect
192 was negative (i.e. participants tended to alternate their choices of sides between trials)
193 (Figure 2), a positive correlation indicated that the more participants alternated their
194 choices (i.e. relied on past choice history), again the less they relied on evidence from the
195 current trial.

196 Together these results suggested a ‘subtractive’ effect between choice history and signal-
197 to-noise ratio on the current trial - participants who rely more on history (RL and CK)

198 tend to rely less on evidence from the current trial. This result could also be interpreted as
199 that participants who were worse at making decisions based on evidence from the current
200 trial tended to rely more on previous history. Interestingly, we also found a similar small
201 but significant relationship between sequential effects and SNR at the within participant
202 level (Supplementary Materials Section S.1).

203 In addition, we also saw a negative correlation between RL and CK across participants
204 ($r = -0.46$, $p = 5 \times 10^{-7}$) (Figure 3c), which indicated that participants who relied more
205 on past feedback also relied more on past choice (stronger alternating effect).

206 **2.4 Individual differences in pupil change correlate with indi-** 207 **vidual differences integration kernel**

208 To examine the interaction between individual differences in pupil response and integra-
209 tion behaviour, we first computed the pupil diameter change during the presentation of
210 clicks stimulus. We time-locked the pupillary response to the onset of the clicks stimulus,
211 and averaged the pupil diameter within each participant. We then took the difference
212 between the peak and the trough of the pupil diameter within the clicks stimulus, which
213 we called the the ‘pupil change’ for each participant (Figure 4a). As shown by a me-
214 dian split in Figure 4b, there were considerable individual differences in the pupil change
215 with some participants showing almost no change while others changed a lot during the
216 stimulus.

217 To examine the relationship of pupil change with overall signal-to-noise ratio and inte-
218 gration kernel, we used a two way mixed ANOVA to compare the effects of pupil change
219 (coded as a continuous variable) and time on participants’ regression weights from equa-
220 tion (β_i^{click} s from equation (3)). If pupil change had an effect on the overall signal-to-noise
221 ratio, we should see a main effect of pupil change on the regression weights. Conversely,
222 if pupil change had an effect on the integration kernel, we should see a significant interac-
223 tion effect between pupil and time on regression weights. Only the interaction effect was
224 significant (interaction $F(19, 2014) = 2.225$, $p = 0.0018$, partial $\eta^2 = 0.02$; main effect
225 $F(1, 106) = 2.761$, $p > 0.05$). Moreover, these results were robust to a number of different

226 assumptions in the analysis such as the size and location of the window size for comput-
 227 ing pupil change (Supplementary Materials Section S.4), and whether we performed the
 228 analysis on the raw regression weights or on the top two principal components (Supple-
 229 mentary Materials Section S.3). Taken together these findings suggested that individual
 230 differences in pupil change affected the shape of the integration kernel but not the overall
 231 signal-to-noise ratio (illustrated using a median split in Figure 4c left two panels).

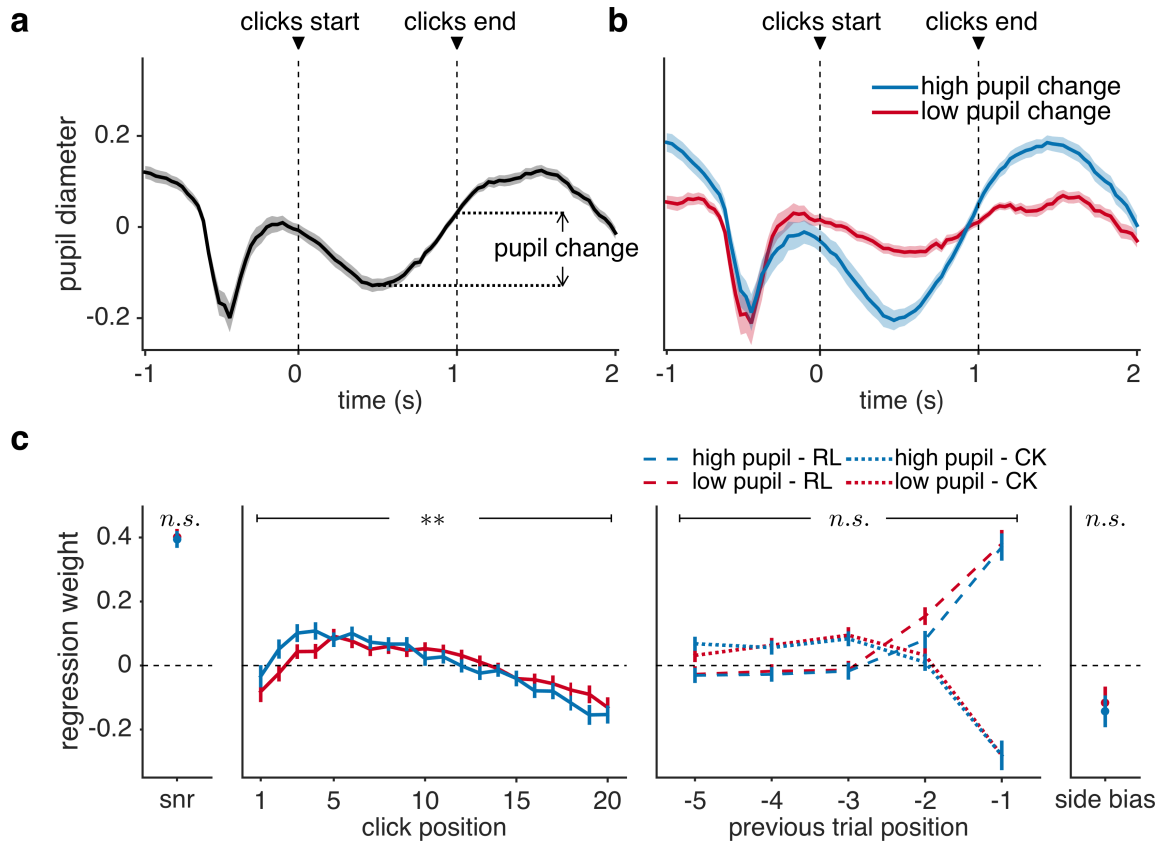


Figure 4: Interaction between pupil change and integration behaviour across participants. (a) Pupil diameter time-locked to the onset of clicks, averaged within participants across trials and then across participants. All shaded areas indicate s.e.m. across participants. (b) Averaged pupil response across participants split into two groups — high (blue) vs. low (red) change in pupil response. All shaded areas indicate s.e.m. across participants. (c) Regression weights averaged across participants split into high vs. low pupil change groups for visualization. (Leftmost) Mean regression weight showed no change across groups. (Second from left) Pupil had a significant interaction effect with time on regression weights. Two way mixed ANOVA, **: $p = 0.0018$. (Second from right) Effect of previous trials showed no differences across groups. (Rightmost) Side bias showed no change across groups. All error bars indicate s.e.m. across participants.

232 To understand which click weights were driving this interaction effect we performed
 233 a correlation analysis between individual differences in the regression weights for each

234 click and individual differences in the pupil change. These post hoc tests suggested that
 235 the main change occurred in the second and third clicks, whose weights were increased in
 236 participants with high pupil change. Specifically, pupil change was significantly correlated
 237 with 2nd ($r = 0.29$, FDR corrected for multiple comparisons $p = 0.02$) and 3rd ($r = 0.30$,
 238 FDR corrected for multiple comparisons $p = 0.02$) kernel weights (Figure S2).

239 To examine the relationship between pupil change and sequential effects and side bias,
 240 we looked at the correlation between pupil change and regression weights for the previous
 241 trials (RL and CK) and side bias. We found no significant relationship between pupil
 242 change and either sequential effects (absolute correlation $r < 0.16$, FDR corrected for
 243 multiple comparisons $p > 0.05$) or side bias (correlation $r = -0.01$, FDR corrected for
 244 multiple comparisons $p > 0.05$) (illustrated using a median split in Figure 4c right two
 245 panels)

246 These results combined suggest that individual differences in pupil change were asso-
 247 ciated with individual differences in only one of the four suboptimalities, the kernel of
 248 integration such that participants with larger pupil change had more uneven integration
 249 kernels.

250 **2.5 Trial-by-trial variability in pupil change correlates with trial-** 251 **by-trial variability in signal-to-noise ratio**

252 To quantify how trial-by-trial pupil change relates to the four suboptimalities in evidence
 253 accumulation, we modified the regression model (equation 3) to include interaction terms
 254 between clicks, previous trials, and trial-by-trial fluctuations in pupil:

$$\begin{aligned}
 \text{logit}(p_{\text{left}} \text{ at trial } t) = & \sum_{i=1}^{20} \beta_i^{\text{click}} c_i + \beta_1^{\text{RL}} a_{t-1} r_{t-1} + \beta_1^{\text{CK}} a_{t-1} + \beta^{\text{side}} \\
 & + \underbrace{\beta^{\Delta \text{click} \times \Phi_t} \Delta c \Phi_t}_{\text{SNR} \times \text{pupil}} + \underbrace{\beta_1^{\text{RL} \times \Phi_t} a_{t-1} r_{t-1} \Phi_t}_{\text{reinforcement learning} \times \text{pupil}} + \underbrace{\beta_1^{\text{CK} \times \Phi_t} a_{t-1} \Phi_t}_{\text{choice kernel} \times \text{pupil}} + \underbrace{\beta^{\text{side} \times \Phi_t} \Phi_t}_{\text{side bias} \times \text{pupil}}
 \end{aligned}
 \tag{4}$$

255 where Δc was number of clicks on the left minus number of clicks on the right, correspond-
256 ing to the mean click regression weight in Figure 2, indicating the average signal-to-noise
257 ratio. (It is also worth noting that here the interaction between side bias and pupil is
258 equivalent to a main effect of pupil change - since the regressor indicating a side bias is
259 all 1s.)

260 We found that trial-by-trial pupil change interacted significantly with Δclick after
261 correction for multiple comparisons ($T_{107} = -3.27$, two-tailed, FDR corrected for multiple
262 comparisons $p = 0.0016$, Cohen's $d = -0.31$), but not with side bias, RL (previous
263 correct) or CK (previous choice) (Figure 5).

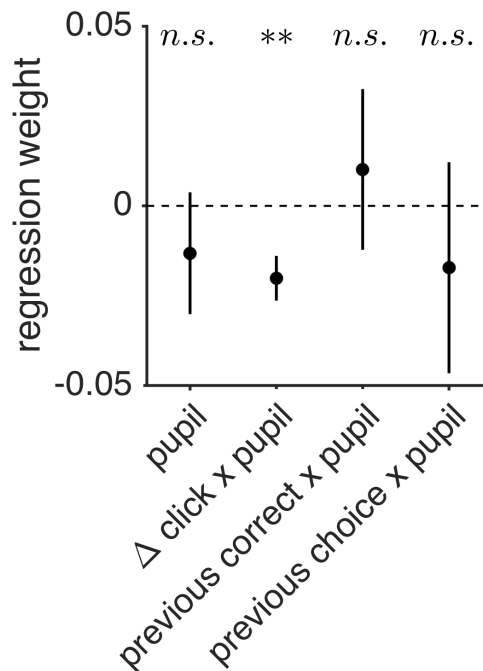


Figure 5: Trial-by-trial interaction between pupil change and integration behaviour. T-test against zero, two-tailed, **: FDR corrected for multiple comparisons $p = 0.0016$. All error bars (black bars) indicate s.e.m. across participants.

264 We then tested whether there was an interaction between pupil change and integration
265 kernel shape with a slightly modified version of equation (4):

$$\begin{aligned}
 \text{logit}(p_{\text{left}} \text{ at trial } t) = & \sum_{i=1}^{20} \beta_i^{\text{click}} c_i + \beta_1^{\text{RL}} a_{t-1} r_{t-1} + \beta_1^{\text{CK}} a_{t-1} + \beta^{\text{side}} \\
 & + \underbrace{\sum_{i=1}^{20} \beta_i^{\text{click} \times \Phi_t} c_i \Phi_t}_{\text{kernel} \times \text{pupil}} + \underbrace{\beta_1^{\text{RL} \times \Phi_t} a_{t-1} r_{t-1} \Phi_t}_{\text{reinforcement learning} \times \text{pupil}} + \underbrace{\beta_1^{\text{CK} \times \Phi_t} a_{t-1} \Phi_t}_{\text{choice kernel} \times \text{pupil}} + \underbrace{\beta^{\text{side} \times \Phi_t} \Phi_t}_{\text{side bias} \times \text{pupil}}
 \end{aligned}
 \tag{5}$$

266 where Φ_t was the pupil change measure at trial t . The first four terms in this model
 267 were the same as equation (3), and the last four terms were the respective interaction
 268 terms of clicks (integration kernel), previous correct side (RL), previous choice (CK),
 269 and side bias with pupil change. With repeated measures ANOVA, we did not find a
 270 significant effect of time on $\beta^{\text{click}_i \times \Phi_t}$ ($F(19, 2033) = 0.72$, $p = 0.81$), suggesting that
 271 pupil change did not modulate the integration kernel on a trial-by-trial level. We did
 272 not find a significant interaction effect between pupil and RL ($T(107) = 1.78$, two-tailed,
 273 FDR corrected $p = 0.14$), CK ($T(107) = -0.78$, two-tailed, FDR corrected $p = 0.62$),
 274 or side bias ($T(107) = -1.63$, two-tailed, FDR corrected $p = 0.19$) either. These results
 275 combined suggested that pupil change on a trial-by-trial level specifically modulated the
 276 overall signal-to-noise ratio, and not integration kernel or sequential effects.

277 3 Discussion

278 In this paper we investigated four sources of suboptimality in human evidence integra-
 279 tion: neuronal noise (as reflected in the signal-to-noise ratio), uneven integration kernel,
 280 sequential effects, and side bias, and their relationship with pupil diameter at the across-
 281 participants and within-participants level. We showed that all four types of suboptimality
 282 were at play in our perceptual decision making task. These included variance that could
 283 not be explained by another source, i.e. ‘noise,’ a predominantly ‘bump’ integration ker-
 284 nel, sequential effects in the form of a positive reinforcement learning (RL) effect (choosing
 285 the previous correct answer) and an alternation choice kernel (CK) effect (choosing the

286 opposite of previous choice), and an overall side bias (Figure 2). In addition, across the
287 population, participants with stronger sequential effects (RL and CK) tended to rely less
288 on evidence from the current trial (smaller signal-to-noise ratio), and participants with
289 one kind of sequential effects (RL) also tended to have the other kind (CK) (Figure 3).
290 At the physiological level, two of the four suboptimalities were associated with pupil di-
291 lation, at the trial-by-trial and individual difference levels respectively. At the individual
292 difference level, only the integration kernel was associated with pupil change, with a more
293 uneven profile of integration being associated with larger pupil change during stimulus
294 presentation (Figure 4). Conversely, at the trial-by-trial level only noise was associated
295 with pupil change, with a smaller signal-to-noise ratio being associated with larger pupil
296 change on that trial (Figure 5). Our work adds to a growing literature on the subopti-
297 malities in evidence accumulation and perceptual decision making and their relationship
298 with pupil dilation. In the following we discuss the implications of our behavioral and
299 pupillometric findings.

300 **3.1 Behavioral findings**

301 At the behavioral level, our findings are consistent with a number of previous results
302 showing the presence of noise in the integration process [1, 8], uneven weighting of in-
303 formation over time [4, 18, 19], and the presence of order effects [23, 21, 20] and side
304 biases [1]. In addition, by running our task in a large number of participants (something
305 not traditionally done in the animal literature), we were able to expose the relationships
306 *between* the suboptimalities at the individual difference level. Intriguingly this analysis
307 suggests an antagonistic relationship between the use of information from the past trial
308 (i.e. RL and CK effects) and processing of the current stimulus, as reflected in SNR. Such
309 an effect may reflect a kind of compensatory process in low performers. That is, people
310 who are less able to process the stimulus correctly (low SNR) may rely more on sequential
311 effects to (either explicitly or implicitly) try to compensate. While such a strategy is not
312 adaptive for this task, this approach would pay off if there was autocorrelation in the
313 task.

314 In addition to the correlations between suboptimalities, one unexpected behavioral
315 finding was the *shape* of the uneven integration kernel. Specifically the ‘bump’ kernel
316 where clicks in the middle are weighed more than those at the beginning or the end. This
317 contrasts with previous work on perceptual decision making from Brunton and colleagues
318 [1], who found that the integration kernel of rats and well-trained humans was flat, Yates
319 and colleagues [4], who showed a purely primacy driven integration kernel in monkeys, and
320 several studies that showed humans have a recency kernel [18, 19]. Given this difference
321 in results, one obvious question is whether the bump kernel is a genuine feature of the
322 integration process or some artifact of either the analysis pipeline or the task?

323 With respect to the analysis, one possibility is that the bump may result from a mixture
324 of subjects with primacy and recency kernels which average together to form the bump.
325 To test this we categorized the integration kernel for each participant into one of the
326 following four shapes: bump, primacy, recency, and flat (for categorization method, see
327 Supplementary Materials Section S.8). All 108 of these are plotted in Supplementary
328 Figure S14. From here it is easy to see that a large number of subjects (49%) exhibit
329 the bump kernel. This suggests that at least on the level of individual participants, the
330 bump kernel is a feature of the integration process, and not just a artifact of averaging.
331 Of course, the possibility remains that this bump kernel is a result of mixing a primacy
332 and recency kernels *within subject* (e.g. some trials have primacy kernels, and some have
333 recency). More detailed modeling work will be needed to tease these interpretations
334 apart.

335 With respect to the task, another possible cause for the bump kernel comes from the
336 number of clicks in each stimulus being fixed. This fixed number of clicks in each stimulus
337 means that an ideal observer, who is aware that there are only 20 clicks in each stimulus,
338 could safely stop integrating clicks if the excess number of clicks favoring on one side
339 exceeds the number of remaining clicks. That is, by fixing the number of clicks, we may
340 be implicitly favoring a bounded integration process (with a collapsing bound). Such
341 bound crossing would cause the later clicks to be down-weighted on average as we see
342 in the later part of the bump profile. Bound crossing would not, of course, account

343 for the initial rise in weights for the bump profile, which would need some additional
344 mechanism (perhaps a recency effect combined with a bound) to explain. Incidentally,
345 this account would fit with the recency bias found in perceptual categorization in previous
346 studies [18, 19]. An important direction for future research would be to test whether this
347 account fully explains the bump profile, both with more detailed modeling in addition to
348 more experiments in which the total number of clicks in each stimulus is not fixed.

349 **3.2 Physiological findings**

350 At the physiological level, our results add to a rapidly growing literature on the rela-
351 tionship between pupil dilation and decision making. In particular, this literature has
352 reported associations between pupil dilation and a number of suboptimalities including:
353 noise [21, 18], reinforcement learning effects [39], choice kernel effects [21] and pre-existing
354 biases [28, 33]. Ours is the first to examine the relationship between pupil dilation and
355 all of these suboptimalities in a single task, as well as being the first to look at the rela-
356 tionship between pupil dilation and the shape of integration kernel. Below we situate our
357 results with respect to this previous literature considering each of the suboptimalities in
358 turn.

359 **3.2.1 SNR and pupil**

360 With regard to SNR, we found that increased pupil change is associated with lower SNR
361 on each trial. This finding is consistent with much of the previous literature. For example,
362 in the Dot Motion paradigm, Murphy and colleagues showed that trial-by-trial variability
363 in the evidence accumulation process was associated with increased pupil dilation [31].
364 Likewise in other perceptual decision making tasks several authors have observed an asso-
365 ciation with increased pupil dilation and noise in behavior [21, 18]. Outside of perceptual
366 decision making, Jepma and colleagues observed the same relationship between pupil and
367 decision noise in a reinforcement-learning based explore-exploit task [41].

368 Of course, while the finding that trial-to-trial pupil dilation is associated with trial-
369 to-trial behavioral variability is robust across studies, exactly what this finding means is

370 open to interpretation. In this paper we have related it to signal-to-noise ratio, with the
371 interpretation that changes in pupil reflect change in SNR which causes poor performance.
372 If one takes pupil as an index of activity in the locus coeruleus, our interpretation is
373 consistent with the Adaptive Gain Theory of norepinephrine function, such that increased
374 LC activity causes more variability in behavior via changes in neural gain [?, 42].

375 An alternate interpretation, put forth by Urai and colleagues [21], is that pupil reflects
376 subjective uncertainty and that participants are more uncertain on trials in which they
377 perform poorly. In this interpretation the direction of causality is reversed: it is poor per-
378 formance that leads to changes in pupil, via its effect on uncertainty (which, incidentally,
379 may also be related to LC [43]). Distinguishing between these accounts, which predict
380 almost identical relationships between pupil and behavioral variability, will be difficult
381 with correlational experiments such as ours, and future work using pharmacological and
382 other causal interventions will be necessary to determine the direction of the relationship
383 between pupil (putatively LC) and noise.

384 **3.2.2 Sequential effects and pupil**

385 In contrast to our result showing no relationship between pupil and the reinforcement
386 learning and choice kernel, a number of studies have found relationships between pupil
387 and sequential effects. For example, Nassar and colleagues showed that both baseline
388 pupil and pupil change modulates how information from previous trials affect current
389 choice [39], a result which was recently replicated in a different version of the task [40].
390 Similarly, in a perceptual decision making task, Urai and colleagues [21] showed that
391 pupil dilation on the previous trial modulated the extent to which that trial influenced
392 the current choice.

393 One possible cause of the difference between our results and this previous work is the
394 overall magnitude of the sequential effects in the respective tasks. Specifically, in our
395 task the sequential effects were small, with the combined effect of reinforcement learning
396 and choice kernel equating to about 2 clicks, or 10% of the variance in the response.
397 Conversely, in [21] the previous trial effects account for almost 100% of the variance when

398 evidence on the current trial is weak. Likewise in [39] and [40] successful performance of
399 the tasks *required* the use of sequential effects and so the sequential effects observed were
400 huge. This difference in overall magnitude of the sequential effects could simply have
401 made modulation of these sequential effects by pupil too small to observe in our task.

402 Another possible cause of the difference in results is the timing of the pupil signal that
403 we focused on. Specifically, our task was optimized to look at pupil during presentation
404 of the click stimuli and not at pupil at other points in the task, such as baseline pupil
405 or pupil following the choice and feedback, which can have very different behavioral and
406 computational correlates [44]. This difference in timing is especially important for the
407 Urai et al. results [21] where the pupil signal modulating sequential effects was computed
408 250 ms before feedback, which is at least 2800 ms after stimulus onset. Such a time
409 lag would be well into the inter-trial interval and possibly even the next trial in our
410 task, making the corresponding pupil signal hard for us to compute. Indeed, when we
411 looked at the signal at these later times, there was no association between pupil and
412 sequential effects (Supplementary Materials Section S.5). Clearly, future experiments
413 with additional delays will be necessary to determine whether pupil does or does not
414 modulate sequential effects in our task.

415 **3.2.3 Other biases and pupil**

416 A number of other authors have related individual differences in pupil dilation to a number
417 of other biases including risk aversion [45], learning styles [28], and the framing effect [33].
418 While these biases are not directly related to integrating evidence over time, the more
419 general point that individuals with large pupil change have more bias across a range of
420 tasks is consistent with our result that individual differences in pupil change modulate
421 the integration kernel. In particular, we find that people with greater pupil change show
422 more deviation (that is more bias) from the ideal, flat, integration kernel. Taken together
423 these results suggests that, at least some, deviations from optimality are modulated by
424 pupil, possibly via its association with LC.

425 3.2.4 Difference between between- and within-participant results

426 More generally, the difference between the between- and within-participant pupil results
427 is intriguing. On the one hand, individual differences in pupil change correlate with kernel
428 shape, while on the other trial-by-trial fluctuations in pupil change correlate with SNR.
429 Why exactly would the individual differences and trial-by-trial correlates of the *same*
430 signal be so different?

431 One possibility, originally raised in [28], is that these slightly different measures of pupil
432 diameter — individual differences vs trial-by-trial fluctuation — may represent different
433 neural measures, with the average of pupil dilations representing baseline or tonic locus
434 coeruleus (LC) signals, and the trial-to-trial fluctuations of pupil reflecting transient, or
435 phasic, LC firing. At the individual difference level, Eldar and colleagues [28, 33] have
436 suggested that mean pupil response within a participant is a measure of tonic LC activity.
437 In contrast, at the trial-by-trial level, work in monkeys and in mice has suggested that
438 moment-to-moment pupil diameter changes track phasic LC firing [24, 25]. Applying
439 these interpretations to the present findings suggests that tonic LC activity changes the
440 kernel of integration while phasic LC decreases the signal-to-noise ratio.

441 The interpretation that tonic LC modulates the integration kernel between participants
442 is consistent with previous work showing that individual differences in pupil change cor-
443 relate with individual differences in susceptibility to a variety of cognitive and decision
444 biases [28, 33]. Importantly, theoretical work has shown with biophysically based neural
445 network model that high tonic LC activity acts to amplify attractor dynamics, essentially
446 causing the storage of impulsive decisions [38]. This can serve as a partial explanation
447 for why our results revealed a positive correlation between individual pupil (proxy for
448 tonic LC activity) and early kernel weights. Furthermore, empirical work has shown that
449 pupil-linked arousal (associated with LC-NE activity and neural gain) is related to time-
450 varying changes in the decision bound [36]. Specifically, higher pupil was found to reflect
451 a stronger urgency signal (lower decision bound). While this relationship between pupil
452 and urgency was only found for the case in which participants faced a decision deadline,
453 an effect on decision bound could potentially offer an explanation for the relationship

454 between pupil and the shape of the integration kernel. Clearly, more detailed modeling
455 and experimental work will be needed to test this hypothesis.

456 The interpretation that phasic LC, as indexed by trial-by-trial pupil change, modu-
457 lates signal-to-noise ratio is consistent with a number of pupil findings as outlined above
458 [31, 18, 21]. However, it is at odds with a number of findings from direct LC recordings
459 in monkeys, where enhanced phasic LC is associated with *better* task performance [46].
460 Understanding these results in more detail, with experiments in animals and neuroimag-
461 ing in humans, will be important if we are to fully understand that LC plays in these
462 decisions.

463 **4 Methods**

464 **4.1 Participants**

465 188 healthy participants (University of Arizona undergraduate students) took part in
466 the experiment for course credit. We excluded 55 participants due to poor performance
467 (accuracy lower than 60%), and then another 25 participants due to poor eye tracking
468 data (see Eye tracking section below). All participants provided informed written consent
469 prior to participating in the study. All procedures conformed to the human subject ethical
470 regulations. All study procedures and consent were approved by the University of Arizona
471 Institutional Review Board.

472 **4.2 The Bernoulli Clicks Task**

473 Participants made a series of auditory perceptual decisions. On each trial they listened
474 to a series of 20 auditory ‘clicks’ presented over the course of 1 second. Clicks could
475 be either ‘Left’ or ‘Right’ clicks, presented in the left or right ear. Participants decided
476 which ear received the most clicks. In contrast to the Poisson Clicks Task [1], in which
477 the click timing was random, clicks in our task were presented every 50 ms with a fixed
478 probability ($p = 0.55$) of occurring in the ‘correct’ ear. The correct side was determined
479 with a fixed 50% probability. Feedback appeared 500 ms after response, followed by a 1

480 s fixation delay before the next trial.

481 Participants performed the task on a desktop computer, while wearing headphones,
482 and were positioned in chin rests to facilitate eye-tracking and pupillometry. They were
483 instructed to fixate on a symbol displayed in the center of the screen, where response and
484 outcome feedback was also displayed during trials, and made responses using a standard
485 keyboard. Participants played until they made 500 correct responses or 50 minutes of
486 total experiment time was reached.

487 **4.3 Behavioural analyses**

488 We modeled the choice with logistic regression using equation (3). In particular we
489 assumed that the probability of choosing left on trial t , is a sigmoidal function of the
490 impact from each click, the impact from five previous trial correct sides, the impact from
491 five previous trial choices, and an overall side bias. In this model, by giving the i th click
492 its own weight, we could account for the overall integration kernel.

493 **4.4 Eye tracking**

494 A desk-mounted EyeTribe eye-tracker was used to measure participants' pupil diame-
495 ter from both eyes at a rate of 30 samples per second while they were performing the
496 behavioural task with their head fixed on a chin rest. Pupil diameter data were pre-
497 processed to detect and remove blinks and other artifacts. Pupil diameter was z-scored
498 across entire experiment before analysis. For each trial, pupil response was extracted
499 time-locked to the start of the trial (Figure 4a). Change in pupil response was computed
500 as the difference between the peak diameter and the minimum diameter during the 1s
501 following trial onset. Pupil response measurements in which more than one-third of the
502 samples contained artifacts were considered invalid and excluded from the analysis. Only
503 participants with at least 200 valid trials were included in analysis ($n = 108$).

504 **4.5 Across participants pupil analysis**

505 For each participant, we took the mean pupil response across trials and computed the
506 change in pupil diameter as described in previous section. We then compared this pupil
507 change measurement with regression weights from equation (3). Specifically, we per-
508 formed a two way mixed ANOVA in which pupil change is a between subject variable,
509 time is a within subject variable, and regression weight is the dependent variable. We
510 inspected the main effect of pupil change, which informed whether the average regression
511 weight changed with pupil change across participants. We also inspected the interaction
512 effect between pupil change and time, which informed whether pupil change modulates
513 the effect of time on regression weights (i.e. the integration kernel).

514 **4.6 Trial-by-trial within participants pupil analysis**

515 For each trial, we took the pupil response and computed the change in pupil diameter.
516 We then modeled participants' choices with the logistic model in equations (4) and (5)
517 to parse out trial-by-trial effects of pupil on integration. The first three terms in both
518 equations were similar to equation (3). But in addition, we assumed that choice was
519 also a function of the interaction between trial-by-trial pupil change and clicks, previous
520 correct side, and previous choice.

521 **4.7 Statistics**

522 All data analyses and statistics were done in MATLAB and R. Repeated measures
523 ANOVA, two way mixed ANOVA, and the corresponding post hoc tests done in R. All
524 other analyses and statistical tests done in MATLAB.

525 **Author contributions**

526 W. Keung did the data analyses. T. Hagen collected and preprocessed the data. T. Hagen
527 and R. Wilson designed the experiment. W. Keung and R. Wilson wrote the manuscript.
528 All three authors contributed to interpretation of the results and critical discussion.

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537 **Competing Interests**

538 The authors declare no competing financial or non-financial interests as defined by Nature
539 Research.

540 **Data availability**

541 The data sets generated and analysed during the current study are available from the
542 corresponding author upon reasonable request.

543 **Code availability**

544 Experiment code was created with Psychtoolbox-3 and custom MATLAB code. All anal-
545 yses were created with custom MATLAB and R code. All code will be uploaded to
546 <https://github.com/janekeung129/clicks-pupil>.

547 **References**

548 [1] Brunton, B. W., Botvinick, M. M. & Brody, C. D. Rats and humans can optimally
549 accumulate evidence for decision-making. *Science* **340**, 95–98 (2013).

- 550 [2] Erlich, J. C., Brunton, B. W., Duan, C. A., Hanks, T. D. & Brody, C. D. Distinct
551 effects of prefrontal and parietal cortex inactivations on an accumulation of evidence
552 task in the rat. *Elife* **4** (2015).
- 553 [3] Katz, L. N., Yates, J. L., Pillow, J. W. & Huk, A. C. Dissociated functional sig-
554 nificance of decision-related activity in the primate dorsal stream. *Nature* **535**, 285
555 (2016).
- 556 [4] Yates, J. L., Park, I. M., Katz, L. N., Pillow, J. W. & Huk, A. C. Functional dissec-
557 tion of signal and noise in mt and lip during decision-making. *Nature neuroscience*
558 **20**, 1285 (2017).
- 559 [5] Newsome, W. T. & Pare, E. B. A selective impairment of motion perception following
560 lesions of the middle temporal visual area (mt). *Journal of Neuroscience* **8**, 2201–
561 2211 (1988).
- 562 [6] Hanks, T. D. *et al.* Distinct relationships of parietal and prefrontal cortices to
563 evidence accumulation. *Nature* **520**, 220 (2015).
- 564 [7] Gold, J. I. & Shadlen, M. N. Neural computations that underlie decisions about
565 sensory stimuli. *Trends in cognitive sciences* **5**, 10–16 (2001).
- 566 [8] Drugowitsch, J., Wyart, V., Devauchelle, A.-D. & Koechlin, E. Computational
567 precision of mental inference as critical source of human choice suboptimality. *Neuron*
568 **92**, 1398–1411 (2016).
- 569 [9] Faisal, A. A., Selen, L. P. & Wolpert, D. M. Noise in the nervous system. *Nature*
570 *reviews neuroscience* **9**, 292 (2008).
- 571 [10] Ma, W. J., Beck, J. M., Latham, P. E. & Pouget, A. Bayesian inference with
572 probabilistic population codes. *Nature neuroscience* **9**, 1432 (2006).
- 573 [11] Beck, J. M., Ma, W. J., Pitkow, X., Latham, P. E. & Pouget, A. Not noisy, just
574 wrong: the role of suboptimal inference in behavioral variability. *Neuron* **74**, 30–39
575 (2012).
- 576 [12] Smith, P. L. & Ratcliff, R. Psychology and neurobiology of simple decisions. *Trends*
577 *in neurosciences* **27**, 161–168 (2004).
- 578 [13] Osborne, L. C., Lisberger, S. G. & Bialek, W. A sensory source for motor variation.

- 579 *Nature* **437**, 412 (2005).
- 580 [14] Kaufman, M. T. & Churchland, A. K. Cognitive neuroscience: sensory noise drives
581 bad decisions. *Nature* **496**, 172 (2013).
- 582 [15] Sutton, R. S. & Barto, A. G. *Introduction to reinforcement learning*, vol. 135 (MIT
583 press Cambridge, 1998).
- 584 [16] Daw, N. D., O’doherly, J. P., Dayan, P., Seymour, B. & Dolan, R. J. Cortical
585 substrates for exploratory decisions in humans. *Nature* **441**, 876 (2006).
- 586 [17] Griffiths, T. L. & Tenenbaum, J. B. Optimal predictions in everyday cognition.
587 *Psychological science* **17**, 767–773 (2006).
- 588 [18] Cheadle, S. *et al.* Adaptive gain control during human perceptual choice. *Neuron*
589 **81**, 1429–1441 (2014).
- 590 [19] Wyart, V., Myers, N. E. & Summerfield, C. Neural mechanisms of human perceptual
591 choice under focused and divided attention. *Journal of neuroscience* **35**, 3485–3498
592 (2015).
- 593 [20] Abrahamyan, A., Silva, L. L., Dakin, S. C., Carandini, M. & Gardner, J. L. Adapt-
594 able history biases in human perceptual decisions. *Proceedings of the National*
595 *Academy of Sciences* **113**, E3548–E3557 (2016).
- 596 [21] Urai, A. E., Braun, A. & Donner, T. H. Pupil-linked arousal is driven by decision
597 uncertainty and alters serial choice bias. *Nature communications* **8**, 14637 (2017).
- 598 [22] Barraclough, D. J., Conroy, M. L. & Lee, D. Prefrontal cortex and decision making
599 in a mixed-strategy game. *Nature neuroscience* **7**, 404 (2004).
- 600 [23] Akrami, A., Kopec, C. D., Diamond, M. E. & Brody, C. D. Posterior parietal cortex
601 represents sensory history and mediates its effects on behaviour. *Nature* (2018).
- 602 [24] Joshi, S., Li, Y., Kalwani, R. M. & Gold, J. I. Relationships between pupil diameter
603 and neuronal activity in the locus coeruleus, colliculi, and cingulate cortex. *Neuron*
604 **89**, 221–234 (2016).
- 605 [25] Reimer, J. *et al.* Pupil fluctuations track rapid changes in adrenergic and cholinergic
606 activity in cortex. *Nature communications* **7**, 13289 (2016).
- 607 [26] Aston-Jones, G. & Cohen, J. D. An integrative theory of locus coeruleus-

- 608 norepinephrine function: adaptive gain and optimal performance. *Annu. Rev. Neu-*
609 *rosci.* **28**, 403–450 (2005).
- 610 [27] Rajkowski, J. Correlations between locus coeruleus (lc) neural activity, pupil diame-
611 ter and behavior in monkey support a role of lc in attention. *Soc. Neurosc., Abstract,*
612 *Washington, DC, 1993* (1993).
- 613 [28] Eldar, E., Cohen, J. D. & Niv, Y. The effects of neural gain on attention and
614 learning. *Nature neuroscience* **16**, 1146 (2013).
- 615 [29] Sara, S. J. The locus coeruleus and noradrenergic modulation of cognition. *Nature*
616 *reviews neuroscience* **10**, 211 (2009).
- 617 [30] Cavanagh, J. F., Wiecki, T. V., Kochar, A. & Frank, M. J. Eye tracking and
618 pupillometry are indicators of dissociable latent decision processes. *Journal of Ex-*
619 *perimental Psychology: General* **143**, 1476 (2014).
- 620 [31] Murphy, P. R., Vandekerckhove, J. & Nieuwenhuis, S. Pupil-linked arousal deter-
621 mines variability in perceptual decision making. *PLoS computational biology* **10**,
622 e1003854 (2014).
- 623 [32] Mather, M., Clewett, D., Sakaki, M. & Harley, C. W. Norepinephrine ignites local
624 hotspots of neuronal excitation: How arousal amplifies selectivity in perception and
625 memory. *Behavioral and Brain Sciences* **39** (2016).
- 626 [33] Eldar, E., Felson, V., Cohen, J. D. & Niv, Y. A pupillary index of susceptibility to
627 decision biases. *bioRxiv* 247890 (2018).
- 628 [34] de Gee, J. W., Knapen, T. & Donner, T. H. Decision-related pupil dilation re-
629 flects upcoming choice and individual bias. *Proceedings of the National Academy of*
630 *Sciences* **111**, E618–E625 (2014).
- 631 [35] de Gee, J. W. *et al.* Dynamic modulation of decision biases by brainstem arousal
632 systems. *Elife* **6** (2017).
- 633 [36] Murphy, P. R., Boonstra, E. & Nieuwenhuis, S. Global gain modulation generates
634 time-dependent urgency during perceptual choice in humans. *Nature communica-*
635 *tions* **7**, 13526 (2016).
- 636 [37] Hauser, T. U., Moutoussis, M., Purg, N., Dayan, P. & Dolan, R. J. Noradrenaline

- 637 modulates decision urgency during sequential information gathering. *bioRxiv* 252932
638 (2018).
- 639 [38] Eckhoff, P., Wong-Lin, K. & Holmes, P. Optimality and robustness of a biophysical
640 decision-making model under norepinephrine modulation. *Journal of Neuroscience*
641 **29**, 4301–4311 (2009).
- 642 [39] Nassar, M. R. *et al.* Rational regulation of learning dynamics by pupil-linked arousal
643 systems. *Nature neuroscience* **15**, 1040 (2012).
- 644 [40] Krishnamurthy, K., Nassar, M. R., Sarode, S. & Gold, J. I. Arousal-related adjust-
645 ments of perceptual biases optimize perception in dynamic environments. *Nature*
646 *human behaviour* **1**, 0107 (2017).
- 647 [41] Jepma, M. & Nieuwenhuis, S. Pupil diameter predicts changes in the exploration–
648 exploitation trade-off: Evidence for the adaptive gain theory. *Journal of cognitive*
649 *neuroscience* **23**, 1587–1596 (2011).
- 650 [42] Servan-Schreiber, D., Printz, H. & Cohen, J. D. A network model of catecholamine
651 effects: gain, signal-to-noise ratio, and behavior. *Science* **249**, 892–895 (1990).
- 652 [43] Angela, J. Y. & Dayan, P. Uncertainty, neuromodulation, and attention. *Neuron*
653 **46**, 681–692 (2005).
- 654 [44] O’Reilly, J. X. *et al.* Dissociable effects of surprise and model update in parietal
655 and anterior cingulate cortex. *Proceedings of the National Academy of Sciences*
656 201305373 (2013).
- 657 [45] Yechiam, E. & Telpaz, A. To take risk is to face loss: a tonic pupillometry study.
658 *Frontiers in psychology* **2**, 344 (2011).
- 659 [46] Aston-Jones, G., Rajkowski, J., Kubiak, P. & Alexinsky, T. Locus coeruleus neurons
660 in monkey are selectively activated by attended cues in a vigilance task. *Journal of*
661 *Neuroscience* **14**, 4467–4480 (1994).
- 662 [47] Menard, S. *Applied logistic regression analysis*, vol. 106 (Sage, 2002).
- 663 [48] Neter, J., Kutner, M. H., Nachtsheim, C. J. & Wasserman, W. *Applied linear*
664 *statistical models*, vol. 4 (Irwin Chicago, 1996).

665 Supplementary Material

666 S.1 Within participant analysis of interaction between sequen- 667 tial effects and signal-to-noise ratio (corresponding to Main 668 Text Section 2.3)

669 Another way to investigate whether choice history enhances or diminishes the effect of
670 current evidence is to inspect this interaction at a trial-by-trial level within participants.
671 We adapted a technique for measuring consistency used by Cheadle and colleagues [18]
672 – we measured the consistency θ between previous history (previous correct side and
673 previous choice, representing reinforcement learning effect (RL) and choice kernel effect
674 (CK) respectively) and the net difference in clicks on the current trial as the product
675 between the two:

$$\theta_{\text{RL}} = a_{t-1}r_{t-1}\Delta c \quad (6)$$

676

$$\theta_{\text{CK}} = a_{t-1}\Delta c \quad (7)$$

677 where Δc was the net difference in clicks (each click coded as +1 for left or -1 for right),
678 a_{t-1} was the previous choice (+1 for left or -1 for right), and r_{t-1} was the previous reward
679 (+1 for correct and -1 for wrong), and $a_{t-1}r_{t-1}$ indicated which side (left or right) was
680 correct in the previous trial (hereby referred to as previous correct side). If the previous
681 correct side or previous choice and Δc agreed in side (left or right), the consistency would
682 be positive, and if not, the consistency would be negative. Because previous correct
683 side and previous choice were always either +1 or -1, the magnitude of the consistency
684 depended solely on Δc , i.e. if the evidence on the current trial was strong and agreed
685 with the previous trial, the consistency was highly positive, whereas if the evidence on
686 the current trial was strong and disagreed with the previous trial, the consistency term
687 would be highly negative.

688 We used this consistency measure as a “weight” by multiplying the term with the
689 net difference in clicks, so that the weight of the current evidence is modulated by the

690 consistency term. For example, in the case of θ_{CK} , the regressor is computed as the
691 following:

$$\text{Consistency weighted } \Delta c = \Delta c \theta_{CK} / 14 = \Delta c \times a_{t-1} \Delta c / 14 \quad (8)$$

692 This regressor means that the impact of Δc is enhanced (quadratically) when it agrees
693 with previous choice, but is enhanced in the opposite direction if it does not agree with
694 previous choice. We also normalized Δc (by dividing it by the maximum absolute dif-
695 ference in clicks, which is 14) so that the Beta we estimate from this regressor will have
696 the same unit with that of Δc , for the purpose of comparing effect size. We added this
697 interaction term in to the regression model in addition to the pure net difference in clicks
698 term.

$$\text{logit}(p_{\text{left}}) = \beta \Delta c + \beta' a_{t-1} \Delta c / 14 \times \Delta c \quad (9)$$

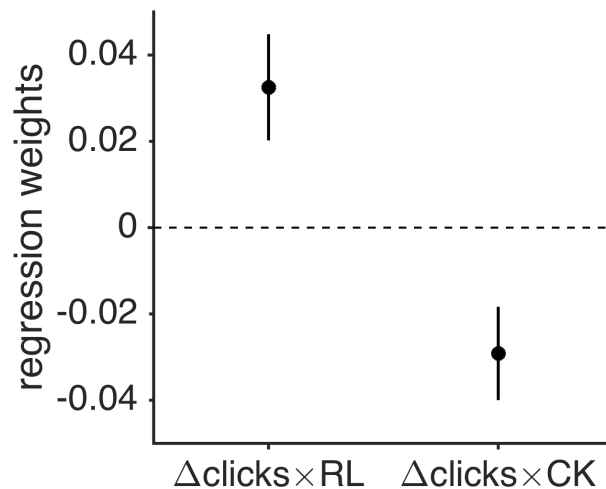


Figure S1: Trial-by-trial interaction between SNR and previous trial effects.

699 This consistency modulated Δc came out significantly negative for CK ($b = -0.029$,
700 FDR corrected for multiple comparisons $p = 0.0075$) and positive for RL ($b = 0.032$,
701 FDR corrected $p = 0.0087$) (Figure S1). Since the previous history biases are positive
702 for RL (reinforcement learning bias) and negative for CK (alternation bias), these results
703 suggest that if Δc is consistent with the reinforcement learning bias (previous correct
704 side) it's more highly weighted, and if Δc is consistent with the alternating bias (i.e. not

705 consistent with previous choice) it is again more highly weighted. This agrees with the
706 individual differences analysis above, and suggests a multiplicative effect of previous trial
707 bias on current trial on a trial-by-trial level. We do note that the effect sizes for both
708 consistency-weighted Δc are quite small, effectively amounting to about one-tenth of the
709 effect of an average click.

710 **S.2 Scatter plots of correlation between early kernel weights**
711 **and pupil (corresponding to Main Text Section 2.4)**

712 We plot the scatter plots between pupil change and early (second and third) kernel weights
713 below (Figure S2). Pupil change was significantly correlated with second ($r = 0.29$, FDR
714 corrected for multiple comparisons $p = 0.02$) and third ($r = 0.30$, FDR corrected for
715 multiple comparisons $p = 0.02$) kernel weights.

716 In addition, we removed a participant who may be an outlier (highlighted in red) in
717 both correlations. The correlation results still held after removing the outlier: across par-
718 ticipants, pupil change was significantly correlated with second ($r = 0.29$, FDR corrected
719 for multiple comparisons $p = 0.03$) and third ($r = 0.33$, FDR corrected for multiple
720 comparisons $p = 0.01$) kernel weights.

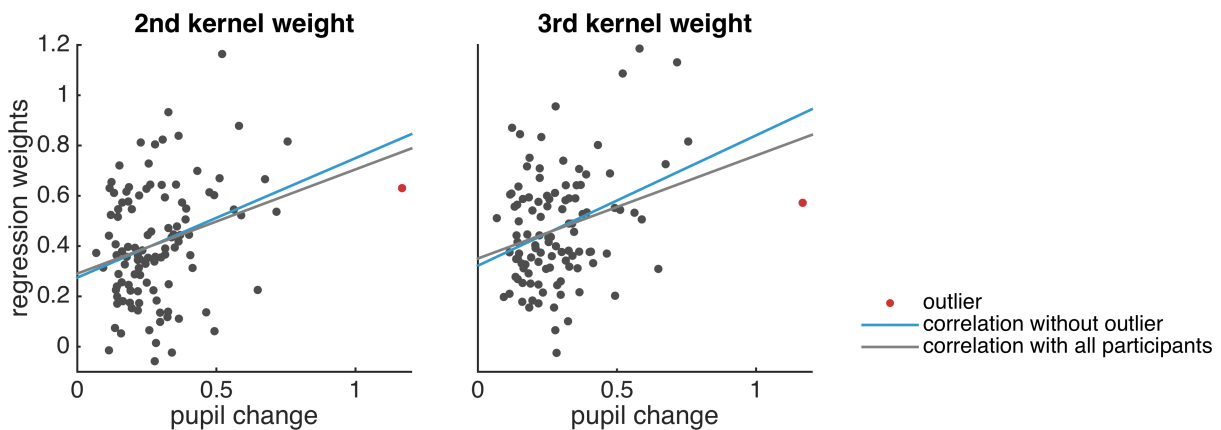


Figure S2: Scatter plots of second and third kernel weights and pupil change across participants. Grey line is least-squares line for all participants. Red dot is a potential outlier participant. Blue line is least-squares line for all participants except the outlier.

721 S.3 PCA on kernel weights (corresponding to Main Text Sec- 722 tion 2.4)

723 We performed PCA on the kernel weights for dimensionality reduction. We showed the
724 plot of the cumulative sum of the fraction of the total variance retained as the number of
725 components increases (Figure S3a). We computed this fraction by dividing cumulative
726 sum of the principal component variances (i.e. eigenvalues) by the sum of the variances.

727 We plotted the first two components out of the twenty components (Figure S3b), the
728 first one showed a smooth bump kernel and the second one showed a smooth primacy
729 kernel. Together they count for 70% of the total variance explained, which indicates that
730 the main factors contributing to kernel shape are not the high frequency oscillations but
731 these low frequency kernel shapes.

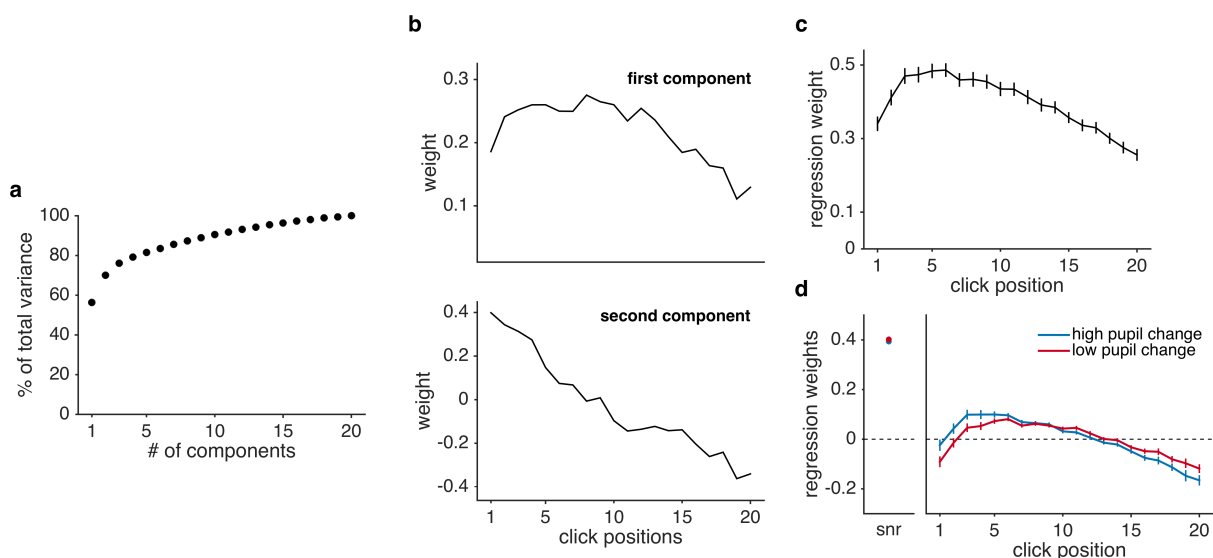


Figure S3: (a) Cumulative sum of fraction of total variance retained as the number of components increased. (b) First two principal components. (c) Integration kernel created using only the first two principal components. (d) Regression weights (recreated using only the first two principal components) averaged across participants split into high vs. low pupil change groups for visualization.

732 We performed dimensionality reduction on the kernel weights by picking these first
733 two components to recreate the kernel weights (Figure S3c), and tested for the effect
734 of interaction between pupil change and time on kernel weights. With two way mixed
735 ANOVA, we saw a significant effect of interaction between pupil change and time on
736 kernel weights ($F(19,2014) = 4.096$, $p = 6.9e-9$), which concur with what was reported in

737 the original manuscript. This analysis was done with pupil change a continuous variable,
738 but again for visualization purpose, we plot the kernel weights of participants split by
739 high vs low pupil change (Figure S3d). These results combined suggest that the pupil
740 effect on kernel shape is not due to the high frequency components of the kernel, but the
741 low frequency components (the bump and the primacy kernels).

742 **S.4 Relationship between kernel shape and pupil measured us-** 743 **ing different window sizes (corresponding to Main Text Sec-** 744 **tion 2.4)**

745 One potential concern is that the window for measuring pupil change to the stimulus
746 window (1 second after stimulus onset) is also a relatively early window for inspecting
747 pupil change, and that this relatively early window can bias the result to detecting re-
748 lationships between pupil and processes that happen earlier in time. We addressed this
749 concern in two ways:

- 750 1. An expanding window analysis, in which we time-locked the pupil response to stim-
751 ulus onset and expanded the window of analysis from between 0s and 1s to between
752 0s and 2s, and computed the pupil change as the difference between the maximum
753 and the minimum of pupil response for each window size.
- 754 2. A sliding window analysis, in which we computed the pupil change as the difference
755 between every point on the pupil response time course and baseline (.25 sec pre
756 stimulus).

757 **S.4.1 Expanding window analysis**

758 We repeated the analysis with a two-way mixed ANOVA to quantify the effect of pupil
759 on signal-to-noise ratio and on kernel shape. As in the original reported results, we did
760 not see a significant influence of pupil on signal-to-noise ratio, but we did see a significant
761 effect of pupil on integration shape at every window size (FDR corrected p-values for all
762 points ≤ 0.011). We reported the corrected p-values of the interaction effect between
763 pupil and time on kernel weights in Figure S4a.

764 To inspect whether the direction of this result was consistent with the reported result,
765 and to inspect whether it was the same set of clicks (i.e. the clicks occurring early on in
766 the stimulus) that contribute to the change in kernel shape, we looked at the correlation
767 coefficients between pupil change (for every window size) and kernel weights. We found
768 that, as with the original reported results, only the 2nd and 3rd kernel weights correlate

769 significantly positively with pupil change across individuals (Figure S4b). This suggested
770 that later pupil change did not reveal relationship with integration that happened later
771 in time.

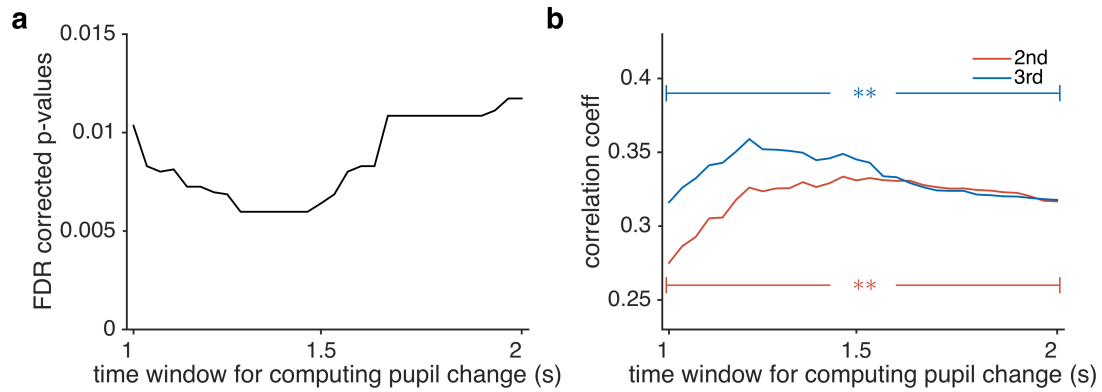


Figure S4: (a) FDR corrected p-values of effect of interaction between pupil and time on kernel weights. (b) All correlation coefficients are significantly positive (FDR corrected p-values for all coefficients ≤ 0.005)

772 S.4.2 Sliding window analysis

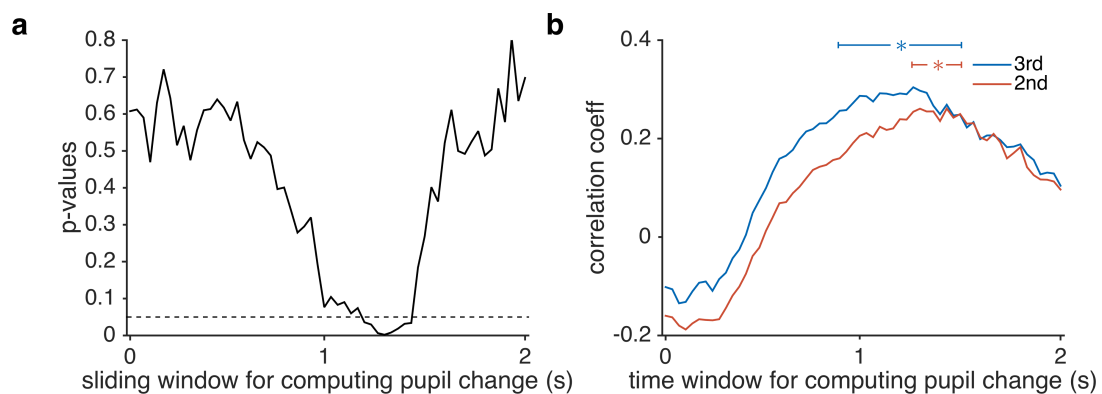


Figure S5: (a) Uncorrected p-values of effect of interaction between pupil and time on kernel weights. (b) Correlation coefficients between pupil and second/third kernel weights ($p \leq 0.01$ uncorrected)

773 We repeated the above analysis with a sliding window analysis, and saw the similar
774 results. Specifically, we used each point in time of the pupil response between 0 and
775 2 seconds after the clicks onset. Again a two way mixed ANOVA revealed that pupil
776 change modulated kernel shape, although the largest effect was observed about 250ms
777 after the end of the clicks (Figure S5a). In addition, we again saw that only the 2nd
778 and the 3rd kernel weights correlated with pupil change (Figure S5b). Combined, these

779 analyses support that the relationship between kernel shape and pupil is not a result of
780 early window of pupil change biasing the detection of modulation in early kernel weights.

781 **S.5 Relationship between sequential effects and pupil measured**
782 **using different window sizes (corresponding to Main Text**
783 **Section 2.5)**

784 In contrast to our results, Urai et al 2017 found an association between pupil response and
785 the magnitude of the history effects. The question here, then, is whether such a pupil-
786 history effect relationship exists in our data. One key difference between our analysis
787 and Urai's is that we only looked at pupil on the present trial, whereas Urai looked at
788 pupil on the previous trial. To address this question comprehensively, we repeated the
789 above sliding window analysis to inspect correlation between pupil on the previous trial
790 and history effects at the within participant level.

791 **S.5.1 Expanding window analysis for pupil on previous trial**

792 Here we used a similar expanding window analysis as described in the previous section,
793 and repeated the regression analysis described in Main Text Section 2.5 (equation (4)).
794 The main difference was that the pupil signal comes from the previous trial, not the
795 current trial. Repeating the regression analysis with these two measures we did not see
796 a significant interaction effect between previous trial effect and pupil change (Figure S6)
797 suggesting that the relationship between past pupil and history effects is not present in
798 our data.

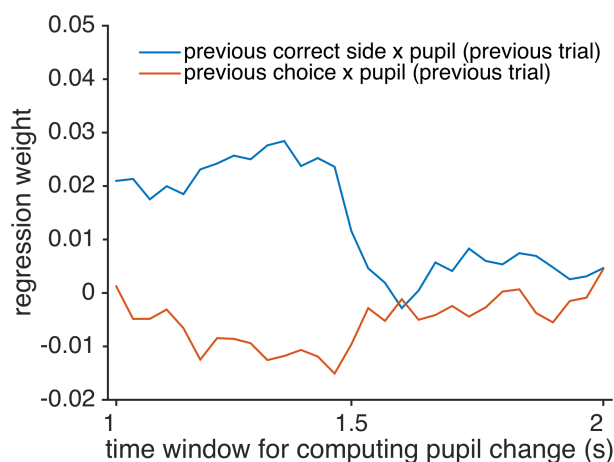


Figure S6: Expanding window analysis of interaction between previous trial effects and pupil change.

799 S.5.2 Sliding window analysis for pupil on previous trial

800 We also used a sliding window approach to investigate the interaction effect between
801 previous trial and pupil change on the last trial. Again, we did not observe any significant
802 effect (Figure S7), further confirming the lack of a relationship between pupil and history
803 effects in our data.

804 Clearly, these null results are quite different from the findings of Urai et al. and the
805 obvious question is why this difference arises. As discussed in the Discussion, we believe
806 there are two possible issues at play here. First, the history effects we observed in this
807 task are relatively small (having the same effect on choice as approximately 1 click). This
808 small effect size for past trials means that the modulation of this small effect by pupil
809 will be difficult to detect. The second reason we may not have observed the effect is down
810 to the timing of our trials. To maximize the number of trials in the task, the inter-trial
811 interval was short. This short ITI makes it hard to examine the late pupil components,
812 which is exactly the component that Urai et al. found correlating with history effects.

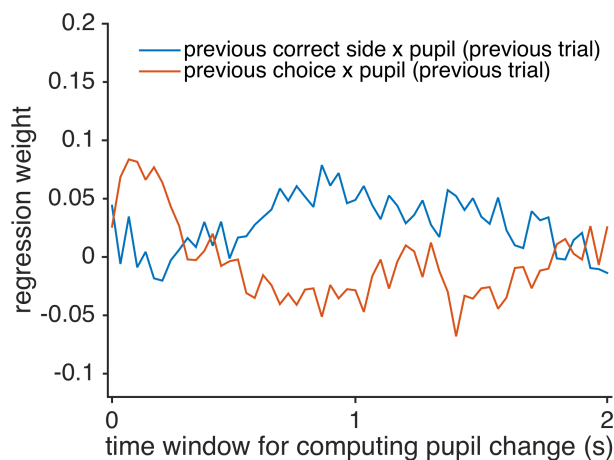


Figure S7: Sliding window analysis of interaction between previous trial effects and pupil change.

813 S.6 Variance inflation factor analyses for regression models

814 Most of the reported analyses require fitting a considerable number of regressors, which
 815 raises the concern that the regressors may be collinear with each other. Here we computed
 816 variance inflation factors (VIFs) for our regression models to examine if the regressors in
 817 our regression models are collinear with each other.

818 For equation (3), we note that VIFs are all around 1 (the minimum value for vari-
 819 ance inflation factors) for all participants (Figure S8), and thus it does not run into
 820 multicollinearity issues.

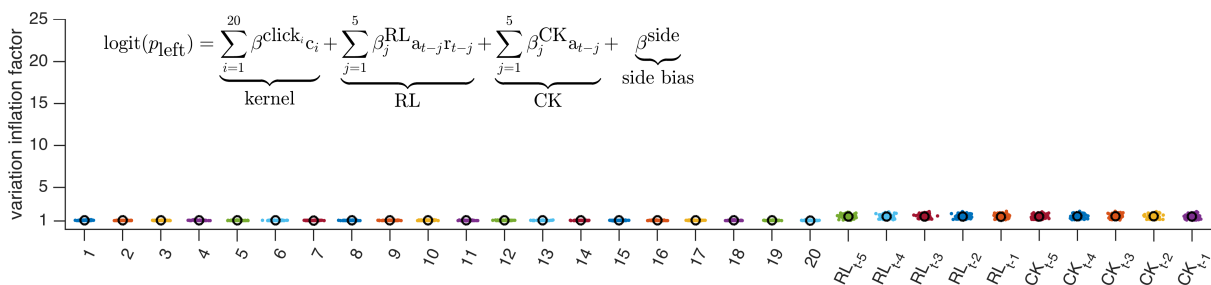


Figure S8: VIFs for equation (3). Equation replicated in figure. Each participant's VIF is shown as a colored dot, the mean across participants is shown as a black circle.

821 Beginning with equation (4) (which had fewer regressors than equation (5), namely
 822 trial-by-trial analysis showing interaction effect between pupil and signal-to-noise ratio),
 823 we find that VIFs for individual clicks (1 to 20) and side \times pupil regressors are low
 824 (Figure S9). Mean VIFs for all the other regressors are ≤ 10 , which is the standard
 825 cutoff for diagnosing multicollinearity in regression [47, 48].

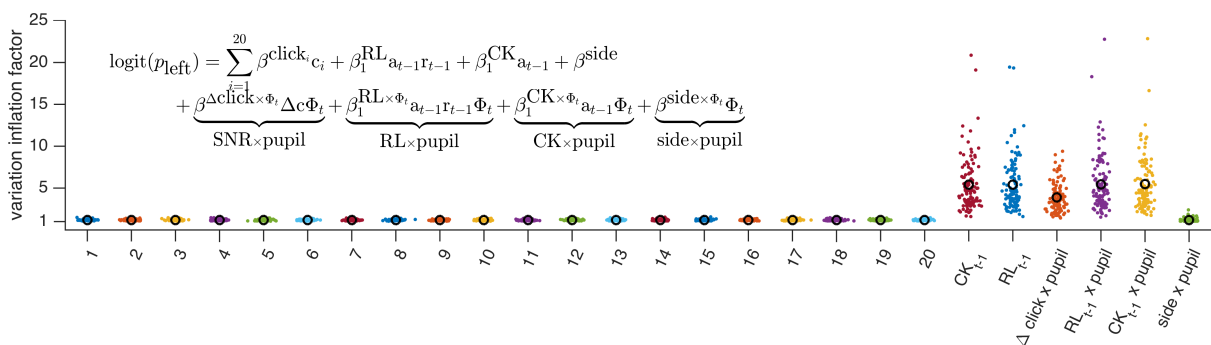


Figure S9: VIFs for equation (4).

826 However, we see some participants with VIFs much higher than 5. To assuage the
 827 validity of this regression analysis, we excluded these participants and picked only par-

828 ticipants for whom all the regressors have a VIF smaller than or equal to 5, and ran the
 829 regression analyses on only these participants ($n = 54$). We plot the VIFs from the new
 830 group of participants (Figure 8). In the new group of participants, we see the same results
 831 as reported in Main Text Figure 5): trial-by-trial analysis of interaction effect between
 832 pupil and signal-to-noise ratio is significantly negative (Beta = -0.0083, FDR corrected
 833 for multiple comparisons $p = 0.01$) (Figure 9).

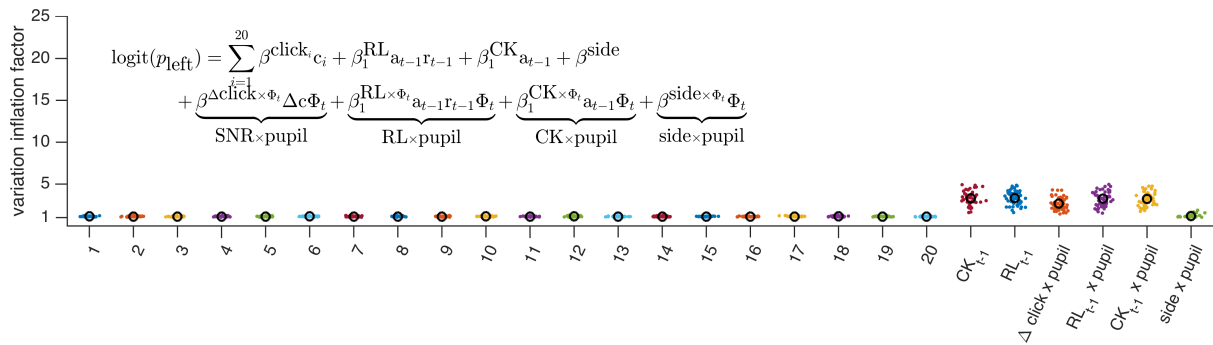


Figure S10: VIFs of only participants for whom all the VIFs are smaller than or equal to 5 ($n = 54$) for equation (4)

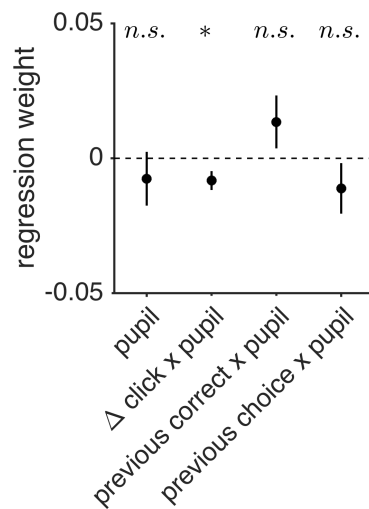


Figure S11: Trial by trial analysis of interaction effects between pupil and behavior with only participants for whom all the VIFs are smaller than or equal to 5 ($n = 54$).

834 For equation (5), due to the large number of regressors, the VIFs for all regressors
 835 are relatively high (Figure S12). We acknowledge that for this specific regression model
 836 (inspecting pupil interaction with kernel shape on a trial by trial basis) we are limited
 837 in what we can do with the amount of data we have. However, the mean VIFs for all
 838 regressors are still smaller than 10, which is not a definitive diagnosis that the model has

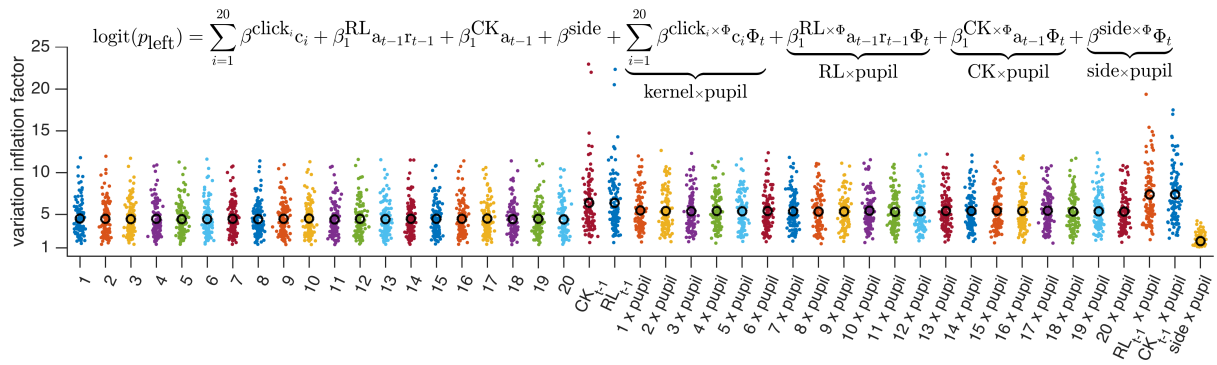


Figure S12: VIFs for equation (5)

839 serious multicollinearity issues [47].

840 S.7 Pupil analysis using pupil change residualized from previ- 841 ous trial pupil change

842 The ITI is relatively short for pupil signal, and one potential concern is a bleed over effect
843 of pupil signal from the previous trial into the current trial. To alleviate this concern, we
844 repeated our analyses with residualized pupil change. We took the regression of the pupil
845 change in the previous trial on pupil change in the current trial, and used the residual
846 as our new measure of pupil change. Repeating the trial-by-trial analyses, we see that
847 our results concur with the original results. Specifically we find that $\Delta click \times pupil$ has
848 a significant interaction effect on choice (Beta = -0.0184, FDR corrected p = 0.0025)
849 (Figure S13).

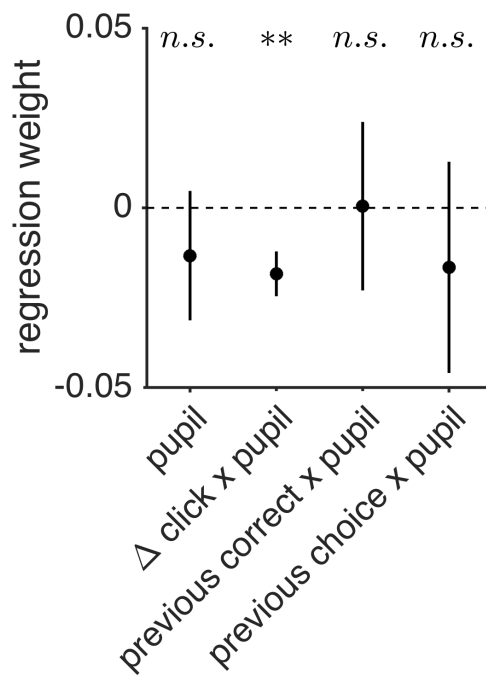


Figure S13:

850 S.8 Categorizing kernel shapes

851 To categorize the kernel for each participant into one of the four shapes, we fit polynomial
852 functions with different degrees to participants' choices, and selected the best fitting
853 model with model comparison using the Akaike Information Criterion (AIC). Specifically,
854 we assume that the probability of choosing left at trial t is (the logit of) the weighted sum
855 of clicks, where the weights are from a polynomial function, as shown in the following
856 equation:

$$\text{logit}(p_{\text{left}} \text{ at trial } t) = \sum_{i=1}^{20} \beta_i^{\text{poly}} c_i, \text{ where } \beta_i^{\text{poly}} = \sum_{n=0}^N \alpha_n i^n \quad (10)$$

857 We fitted three different polynomial functions by changing N from 0 to 2: constant,
858 linear, and quadratic. We then selected the best fitting function for each participant by
859 comparing the fits from different polynomials with AIC. We categorized each participant's
860 integration kernel into one of the four shapes using the following criteria: (1) flat: kernel
861 was best fit with the constant function; (2) primacy: kernel was best fit with linear
862 function with a negative slope (α_1), or with quadratic function with a minimum ($\alpha_2 > 0$)
863 and the minimum is located later than the 10th click, or with quadratic function with a
864 maximum ($\alpha_2 < 0$) and the maximum is located earlier than the 2nd click; (3) recency:
865 kernel was best fit with linear function with a positive slope, or with quadratic function
866 with a minimum ($\alpha_2 > 0$) and the minimum is located earlier than the 10th click, or with
867 quadratic function with a maximum ($\alpha_2 < 0$) and the maximum is located later than the
868 18th click; (4) bump: kernel that did not meet the previous three criteria (i.e. kernel was
869 best fit with quadratic function and was neither primacy nor recency).

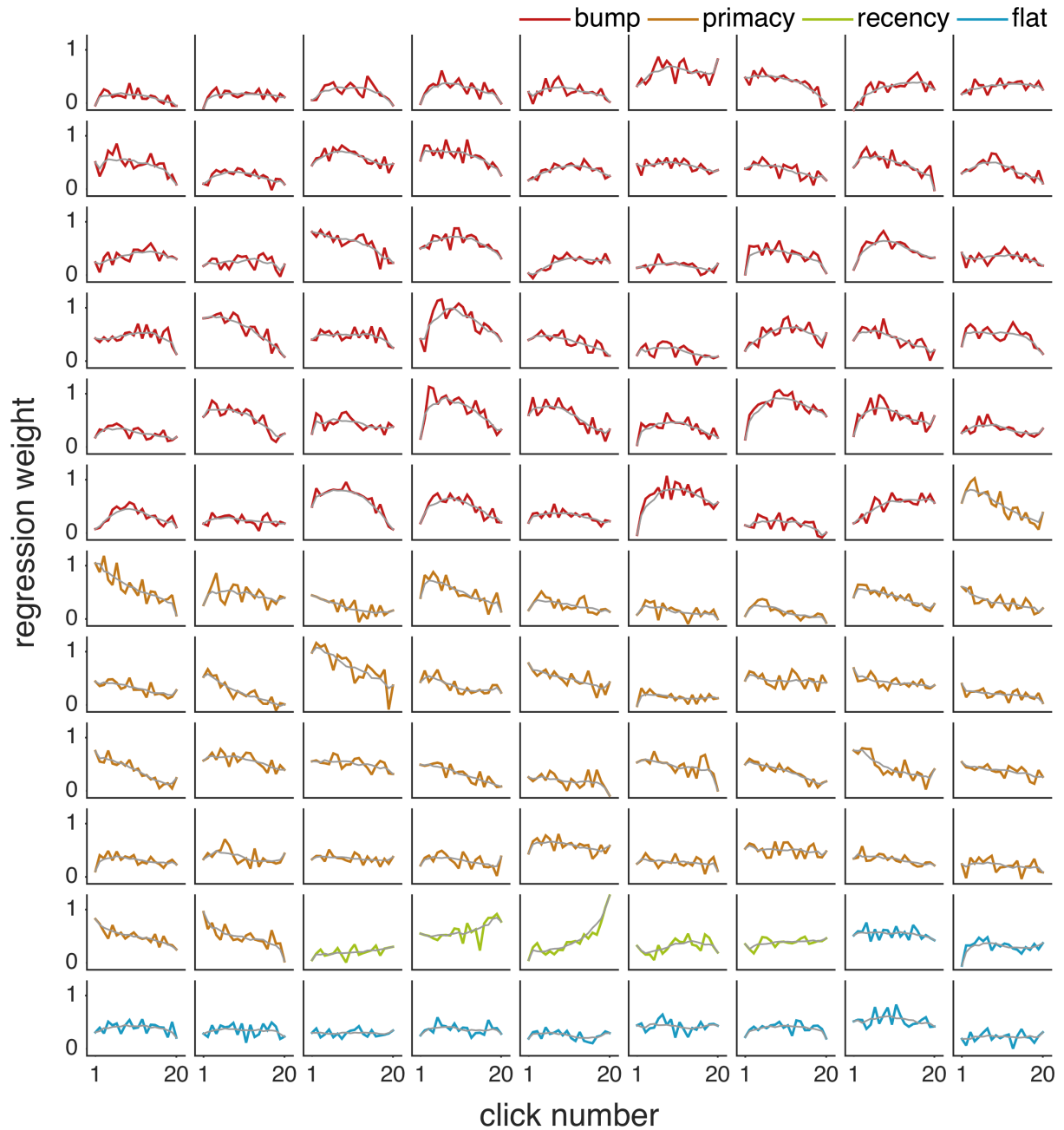


Figure S14: Individual integration kernel plots. Colored lines are regression weights of clicks from equation (3). Plots are sorted and color coded by kernel shape. Light grey line shows smoothed integration kernel.