Learning the sequence of influenza A genome assembly during viral replication using point

process models and fluorescence in situ hybridization

Timothy D. Majarian,[1,2*] Robert F. Murphy,[1-4#] and Seema S. Lakdawala[5#]

[1]Computational Biology Department, [2]Department of Biological Sciences, [3]Department of

Biomedical Engineering, [4]Machine Learning Department, Carnegie Mellon University

[5]Department of Microbiology and Molecular Genetics , University of Pittsburgh School of

Medicine

*Present address: Broad Institute of MIT and Harvard

#Corresponding authors: murphy@andrew.cmu.edu, Lakdawala@pitt.edu

Abstract

Within influenza virus infected cells, viral genomic RNA are selectively packed into progeny

virions, which predominantly contain a single copy of 8 viral RNA segments.

Intersegmental RNA-RNA interactions are thought to mediate selective packaging of each

viral ribonucleoprotein complex (vRNP). Clear evidence of a specific interaction network

culminating in the full genomic set has yet to be identified. Using multi-color fluorescence

in situ hybridization to visualize four vRNP segments within a single cell, we developed

image-based models of vRNP-vRNP spatial dependence. These models were used to

construct likely sequences of vRNP associations resulting in the full genomic set. Our

results support the notion that selective packaging occurs during cytoplasmic transport

and identifies the formation of multiple distinct vRNP sub-complexes that likely form as

intermediate steps toward full genomic inclusion into a progeny virion. The methods

employed demonstrate a statistically driven, model based approach applicable to other

interaction and assembly problems.

Author Summary

Influenza virus consists of eight viral ribonucleoproteins (vRNPs) that are assembled by

infected cells to produce new virions. The process by which all eight vRNPs are assembled

is not yet understood. We therefore used images from a previous study in which up to four

vRNPs had been visualized in the same cell to construct models that measure how well the

subcellular distribution of one vRNP can be predicted from one or more other vRNPs. We

used the likelihood of these models as an estimate of the extent of association between

vRNPs and thereby constructed likely sequences of vRNP assembly that would produce full

virions. Our work identifies the formation of multiple distinct vRNP sub-complexes that

likely form as intermediate steps toward production of a virion. The results may be of use

in designing strategies to interfere with virus assembly. We also anticipate that the

approach may be useful for studying other assembly processes, especially for complexes

with modest affinities and more components than can be visualized simultaneously.

Introduction

Influenza A virus, part of the *orthomyxoviridae* family, has a segmented genome of eight

distinct viral RNA segments coding at least 11 major proteins and several auxiliary

peptides. Most notable of the 11 viral proteins are hemagglutinin (HA) and neurominidase

(NA), the canonical *H* and *N* in influenza strain designation. A segmented genome offers

potential evolutionary advantages during viral replication, in the form of reassortment. The

exchange of genomic material between two distinct viral strains in a co-infected cell often

causes a massive shift within the genome, greatly increasing viral genetic diversity. In only

the last century, influenza pandemics of 1957 (H2N2), 1968 (H3N2), and 2009 (H1N1)

have emerged through reassortment of at least two viruses (1). Segmented genomes also

come with an inherent mechanistic challenge: ensuring that progeny receive a complete

and unique set of genomic segments. Random packaging is possible but comes at the cost of

producing an overwhelming majority of progeny that are not viable. Evidence suggests that

there exists an active mechanism of selective packaging ensuring progeny viability through

efficient and accurate genomic packaging (2).

Within a virion, the viral genome is organized into individual viral ribonucleoprotein

(vRNP) complexes, composed of the viral RNA, virally encoded nucleoprotein (NP), and a

heterotrimeric polymerase complex made up of PB1, PB2 and PA. The vRNP structure is

known classically as a helical panhandle, where the RNA is wrapped around NP with RNA

bases exposed and the 3' and 5' ends associated with the polymerase complex (2). We have

recently demonstrated that the structure of vRNP is more complex than the classically

depicted "beads-on-a-string" schematic. The NP and viral RNA associate in a non-uniform

manner, where regions of the RNA are unbound by NP and capable of forming complex

structures (3). Macro-organization of vRNPs within the viral capsid show tight packaging,

organized in a '7+1' orientation, a single vRNP center shaft surrounded by 7 others (4-6).

Given the need to package one copy of all 8 vRNP segments, there is strong evidence for the

existence of a selective packaging mechanism prior to budding (2, 4-7). Intersegmental

RNA-RNA interactions have been proposed to mediate selective packaging of all eight

influenza vRNP segments. Specifically, the 5' and 3' non-coding regions of each segment are

highly conserved, essential for efficient packaging, and may be critical for the proposed

RNA-RNA interactions (2, 8-11). Previous studies using *in vitro* transcribed viral RNA have

successfully demonstrated RNA-RNA binding. These studies also show that a set of eight

reproducible interactions can suffice to form a full genome complex (12-14). However,

these studies were performed on viral RNA in the absence of NP which would alter the

regions of viral RNA accessible for the proposed RNA-RNA interactions. Recent *in vivo*

experiments have also pointed towards RNA-RNA interaction as the driving mechanism for

selective packaging (15). While there is consensus of its existence, a clear network of

segment interactions leading to a full genome complex has yet to be elucidated.

We and others have previously used fluorescence *in situ* hybridization (FISH) studies to

visualize the intracellular localization of multiple vRNP segments during a productive viral

infection (16, 17). These studies have shown that distinct vRNP segments colocalize within

the cytoplasm. Based on our previous studies, we proposed a model whereby vRNP

segments are exported from the nucleus, the site of vRNP synthesis, as subcomplexes that

undergo further assembly en route to the plasma membrane through dynamic fusion or

colocalization events (17). We also performed a series of binary colocalization

comparisons, which would postulate a simple linear vRNP interaction network. However,

no such network was identified, suggesting the presence of a more complex vRNP

interaction network, including higher order subcomplexes as intermediate steps (17).

The lack of a method to image all eight vRNP segments within a single infected cell has

limited our ability to examine the precise spatial relationship between segments. In this

study, we developed a model based approach, rooted in point process theory, to quantify

vRNP spatial dependence as a metric for RNA-RNA interaction using multi-color FISH

images. A spatial point process model is a statistical model that captures the probability

that an event will happen at any position within a given geometry. The probability can

depend on the spatial distributions of other events or objects, often referred to as

covariates or factors. In this context, an event is the observance of fluorescent signal

(corresponding to one or more vRNPs) at a given location. We have previously used point

process modeling to construct generative models reflecting the spatial dependencies

between various punctate cell organelles and other cellular structures, such as the cell and

nuclear membranes, microtubules and the endoplasmic reticulum (18, 19). Related

analytical methods have been used to analyze clustering of molecules reflecting spatial

dependencies between copies of the same structure (20, 21) Here we use point process

models to present a statistically rigorous analysis of intracellular influenza A genome

assembly dynamics from a spatial perspective during a productive viral infection.

For an imaged cell, the locations of each vRNP segment can be represented as "realizations"

of an underlying probability density specific to that vRNP, calculated over the cytoplasm,

and dependent on the spatial proximity to various other structures within the cell. That is,

at any given position within the cell, there exists a probability of observing a single

segment, of a particular identity, that depends on the positions of cellular organelles and

other vRNP segments. By modeling the locational densities of each vRNP, we can therefore

learn a dependency network from which likely vRNP interactions are implied.

FISH imaging yields many distinct observations of individual cells, each with multiple vRNP

patterns observed. These point patterns, when viewed as realizations of point processes,

allow for statistical learning of spatial dependencies over many replicates for each vRNP

segment. By defining a set of covariates as the minimum distances between a given

segment and other segments observed within the same cell, we produce a set of models

that describe the spatial relationship between distinct segments. The extent to which two

(or more) vRNPs are dependent upon each other (e.g., likely to be found at nearby

positions) is reflected in the likelihood that images of those vRNPs would have been

produced by a learned model: high likelihoods signify that the positions of a given vRNP

can be predicted well from the others in that model. Model likelihoods can then be seen as

a metric of vRNP interaction, with higher likelihoods indicating more probable vRNP

association. These likelihoods can then be used to construct the most probable sequence of

interactions to form the full genome with methods borrowed from evolutionary tree

construction (22).

Results:

<u>Imaging influenza infected cells</u>

To investigate the spatial dependence between vRNP segments, we utilized images from a

previous study of infected cells stained for different combinations of vRNP segments using

four-color FISH (17). The data set included 14 different probe combinations that covered

all pairwise vRNP associations and 32 out of 56 possible triple vRNP combinations

(Supplemental Table 1 and 2). Prior to initiating pattern analysis, each image was

processsed to remove background, isolate individual cells, and find punctate vRNPs. Figure

1a and b presents representative raw and segmented images with detected points.

<u>Assessing randomness of the spatial distributions of vRNP</u>

We began by determining the extent to which the individual vRNPs could be considered

randomly distributed throughout the cell, since our search for spatial dependencies

between different vRNPs would be illogical if their positions were all random. We defined

the null hypothesis as complete spatial randomness, as would be exhibited by a

homogenous Poisson process, and estimated a p-value for this hypothesis using Monte

Carlo methods. Briefly, for a given vRNP in a particular cell, we started by segmenting the

cytoplasm and counting the number of points observed within. We then simulated

(generated) many random point configurations, of the same number of points, within the

segmented cytoplasm. We calculated a test statistic for each random configuration (see

Methods) and used these to assign a p-value to the test statistic for the observed pattern for

that cell; these were averaged for each vRNP. As shown in Table 1, every vRNP segment

showed significant deviation, with low variance, from a spatially random distribution. This

indicates the presence of a spatial trend and some dependency on cellular structures for

their location.

<u>Modeling cell and nuclear membrane dependence</u>

Given that the localizations of vRNPs are not random, we sought to determine if some

relationship to the cell and nuclear membranes could explain the spatial trend of vRNP

segments. Three covariates were defined for each position in the cell (minimum distance to

the cell membrane, minimum distance to the nuclear membrane, and the minimum ratio of

cell to nuclear distance for each point) and models were created for each vRNP using each

of the covariates separately. We estimated the accuracy of each model by cross validated

likelihood calculation (see Methods). It is important to note that, because the initial

pseudolikelihoods were used only to fit parameters, actual likelihoods can be compared

between models of different complexity.

Dependency on the cell membrane produced the lowest likelihood (worst) models relative

to all others (Table 2). Congregation of vRNP to the apical cell membrane is a potential

skewing factor. The many observed points within close proximity to the membrane

potentially caused low fitted probability of observing other points closer to the nucleus.

Cell shape differences combined with lower resolution in the z-dimension may have also

influenced these models.

For most vRNPs, models incorporating nuclear distance showed higher likelihoods than

those using cell membrane distance (Table 2). Some vRNP segments, such as PB2, PB1, PA

and NS, showed greater dependency on nuclear distance than the others. The ratio of cell

membrane to nuclear distance produced the best (highest likelihood) models in all cases

(Table 2). This ratio was a more accurate descriptor of location across varying cell shapes,

providing the most cogent model for dependence on host cell structures. Ratio models for

PB2, PB1, PA, and NS yielded higher likelihoods than other vRNP segments, showing a

greater dependence on both the nuclear and cell membrane location.

<u>Inter-segment dependence in pairs, triplets, and quadruplets</u>

Every pair of vRNP segments was observed in at least three cells, allowing for modeling of

vRNP-vRNP dependence among all segment pairs. For each pair of vRNP segments, two

models were trained, switching the dependent (primary) and independent (secondary)

patterns. Since the minimum inter-pattern distance measure is not commutative, the two

models need not be the same. More simply, a given vRNP location may depend on its

proximity to another vRNP, but the converse is not necessarily true.

In most cases, pairwise inter-pattern distance models (Table 3) had higher likelihoods than

those produced with the cell and nuclear membrane factors (Table 2). Some pairs showed

higher likelihoods than others, specifically PB1-NP, NP-PB1, PA-NP, HA-PB1, and PB1-M.

These pairings also showed relatively low variance in cross-validated likelihood estimates

(Supplemental Table 3). Many of the highest likelihood vRNP pairs are observed between

segments encoding the viral polymerase components, which is not surprising given the

dependence between these proteins both structurally and functionally (23-25). Figure 2

highlights the pairwise models by indiciating related vRNP pairs that are above -0.2 log

likelihood. Overall, vRNPs showed the greatest dependence on PA and PB1, with the most

connections between these two segments and the others, and the least dependence on NA,

which had the fewest connections.

Extending the pairwise models, the ratio of cell to nuclear distance was added as another

covariate. Observed point location in these models depended both on inter-pattern

distance and relative distance to the cell and nuclear membranes. Improvements in model

likelihood were seen in some models (Table 4), yet overall there was an increase in the

variance of our estimates (Supplemental Table 4). Overfitting to the cell and nuclear

distance may be the cause of this increased variance.

Thirty-two of 56 possible triplet vRNP segment complexes (referred to as triplets) were

present within the dataset analyzed and could therefore be modeled within the same cell.

This process allows for modeling of complex vRNP relations containing more than two

vRNP segments. Three models were trained for each observed triplet, one for each protein

depending on the other two. The highest likelihood model was chosen. Model likelihoods

tended to be higher than that of either pair model for a given vRNP (Table 5). Consistently,

we observed that the most likely triplet models for each vRNP included the five most likely

vRNP pairings (PB1-NP, NP-PB1, PA-NP, HA-PB1, and PB1-M) observed in pair likelihood

models (Table 3). Model likelihoods for all triplets observed can be found in Supplemental

Table 5.

A small subset of all possible four vRNP segment complexes (referred to as quadruplets)

were contained within the dataset and were included in our modeling of the complex

interactions. Again, considerable overlap is seen between the most likely quadruplet

models and both triplet and pair models (Table 6). The consistency in vRNP composition

between observed likely quadruplet, triples, and pairs further validates the spatial

dependence of vRNP segments and the presence of vRNP subcomplexes as influenza A

assembly intermediates. Model likelihoods for all quadruplets observed can be found in

Supplemental Table 6.

One of our major goals was to attempt to extrapolate higher order relationships from the

pair, triple and quadruplet models. To project how accurately this might be done, we

compared our observed triple and quadruple models with predicted models calculated

from only the pair models. Figure 3 illustrates that most of our observed triples and

quadruplets could be predicted accurately from the pair models, with a r^2 value of 0.266.

The exceptions are the triple and quadruple clusters containing the PB2-NA pair with

either NS, PB1 or both. These combinations were predicted to occur more often than was

observed in the experimental data sets (Figure 3). The pairwise likelihood of NA with PB2

is >-0.5, which is consistent with the observed likelihood and may be the driving force

behind the score compared to the other pairwise interactions. Without the triples and

quadruples containing PB2-NA, the r^2 value increases to 0.442. Overall, these data support

the notion that pairwise point process models can be used to predict higher order

complexes with reasonable accuracy.

<u>Interaction network construction</u>

Given these results, we sought to create a model of the set of interactions (complexes) that occur at all steps in the formation of a complete viral genome. To do this, we represented all possible interactions between vRNPs as a weighted, directed, acyclic graph with nodes labeled by each unique, unordered set of vRNPs. A path from all single nodes to the root represents a set of vRNP interactions resulting in a full genome complex (see Methods). For illustration, if all eight vRNPs directly formed a complete virion without forming any intermediate complexes (a highly unlikely scenario), this would be represented by weights of 1 for the connection between each vRNP and the complete virion and weights of zero for all other complexes. We then proceeded to estimate the weight for each possible vRNP complex containing at least 2 and up to eight segments based upon either the observed or predicted likelihoods (see Methods). Dynamic programming was then used to find the most probable path within the graph (Figure 4a). This tree produces two distinct clusters HA, M, NA, NS (hereafter referred to as C1) and PB1, PB2, NP (C2) that merge with PA as a final step towards forming the full genome.

To assess the contribution of the observed quadruplets and triplets on this particular network, we performed the network construction both without observed quadruplets (Figure 4b) and without observed quadruplets and triplets (Figure 4c). Interestingly, exclusion of all observed quadruplets did not drastically alter the network, with both C1

and C2 still being present. However, networks built only with observed pairs resulted in a

quite different network lacking larger subcomplexes (Figure 4c).

Given that the dynamic programming approach yields only a single most likely tree, we

sought to assess the robustness of that inferred tree to potential inaccuracy in the

estimated dependencies. We therefore assessed the stability of the tree by adding various

amounts of random noise to the estimated likelihoods (see Methods, Figure 4d).

Surprisingly, the addition of noise of up to a third of the mean log-likelihood had little effect

on the network found.

Since the vRNP sets constituting C1 and C2 had been directly measured (reactions L and B,

respectively), we also considered the possibility that the presence of measured (rather

than inferred) likelihoods for them drove network construction and biased our results only

towards those sets of vRNAs that we observed. If this is not the case, removal of all

observations of these subcomplexes would ideally not significantly change the final

network; C1 and C2 should be recaptured from other interaction models. Removal of data

derived from images of the sets of vRNPs in C1 and C2 (Figure 4e) yielded a network that

largely overlapped with Figure 4a. Here we see HA, M, NS, *PB1* in place of C1 and NP, *PA*,

PB2 in place of C2. This same tree was generated with the removal of only C1.

Removal of only C2 again yielded a network with substantial overlap to Figure 4a. C1 was

replaced by HA, M, NA, NS with NP, PA, PB1, PB2 replacing C2 (Figure 4f). Together, these

constructed networks demonstrate both the stability of the primary interaction network

and the importance of triplet and quadruplet observations in predicting higher order vRNA

complexes.

Discussion:

Influenza A vRNP segments selectively assemble within an infected cell to produce progeny

virions containing one copy of all eight segments. The mechanism driving selective

assembly is still largely unknown, but RNA-RNA interaction between the segments has

been proposed (12-14). In this study, we have modeled the *in vivo* spatial dependencies of

influenza A vRNP segments using multi-color fluorescent *in situ* images to generate

possible vRNP-vRNP interaction networks and propose a new perspective on genome

packaging during viral replication. By utilizing a rigorous statistical framework, we have

extended the efforts of previous groups and present a novel method for construction of

vRNP interaction networks based on their precise spatial information within actively

infected cells. Using spatial proximity as a proxy for physical interaction, point process

modeling in this study suggests clear spatial dependenencies between certain vRNP

segments, and confirms our previous observations of subcomplex formation during

cytoplasmic transport (18).

Previous studies using *in vitro* transcribed RNA suggest multiple interactions between

vRNP segments are expected (2, 4, 12, 14, 26). Similarly, the modeled likelihoods show

potentially multiple interactions for each vRNP segment. Electron tomography studies have

revealed a conserved '7+1' supramolecular structure within the viral interior, where a

center shaft is surrounded by seven remaining segments (2, 4, 26), demonstrating an

ordered process in genome assembly.

Our results suggest two candidates for the center shaft or 'master segment': PA and PB1.

From the 6 constructed networks, PA is the most variable, switching between

subcomplexes. This would be expected if a segment was evenly dependent on most others,

a possible signature of a core segment. PB1 also behaves similarly. Both PB1 and PA show

the highest average pairwise model likelihood over all other segments.

A few vRNP pairs were seen often in the most likely pairwise, triplet, and quadruplet

models and in the final constructed network. PB1-PB2, HA-M, and HA-PA show

commutative relationships, appearing in the top most likely triplet and quadruplet models

for each vRNP within the pair. Some non-commutative dependencies were also seen, most

notably in NP depending on PB2 in both triplet and quadruplet models without PB2-NP

dependence.

By using dynamic programming, we were able to generate the most likely interaction

network based on all models. Subtle variations in the vRNP interaction network were

observed by the addition of noise and exclusion of higher order datasets, suggesting that

there may exist multiple interaction pathways all acting at once in a single cell. Some of the

variation in our results may also be due to the presence of some lower specificity vRNA-

vRNA interactions also occurring during genome assembly. This would explain some of the

highly likely pairwise interactions observed that were not included in the final interaction

network, such as PB1 and HA. A system with multiple possible interaction networks could

be potentially advantageous in both ensuring packaging of a full genome in viral particles

and in future reassortment events. For example, reassortment of one vRNA segment during

a coinfection may decrease the binding efficacy necessary for a single interaction network

but a second network may serve to rescue viral viability while also increasing genetic

diversity.

With this, we also point out the consistent presence of the PB2, PB1 and NP cluster. Since

these three proteins function together, with PA, to form the viral polymerase, which

promote vRNA replication and transcription, this cluster is potentially important. Viruses

capable of packaging complementary polymerase segments into progeny virions will have

a fitness adavantage compared to viruses with polymerase mismatches. This phenotype

has been observed in experimental reassortment experiments between 2009 H1N1

pandemic and seasonal H3N2 viruses(23, 25).

More generally, the computational pipeline presented may function as a useful tool in

elucidating spatial dependency over a wide range of biological phenomena observable

through microscopy. As we have demonstrated, point process models can capture key

aspects of point distributions while retaining a basis in probability. Network construction

synthesizes the results derived through modeling into a cogent, most-likely set of spatial

interactions or dependencies.

Future work in spatial modeling of influenza A packaging should incorporate a temporal

dimension through live cell imaging. Point process models have been adapted for

spatiotemporal modeling (27). Incorporation of intracellular markers important for

influenza A vRNP transport may increase the accuracy of models. Influenza A vRNP

transport from the nucleus, the site of vRNP synthesis, to the plasma membrane is a

complex process utilizing a variety of host proteins (28). Rab11A-containing vesicles are

thought to be the primary mode of transport although there is evidence that a Rab11A-

independent mechanism exists (29-31). A recent study has implicated the ER to mediating

transport of vRNP segments through anchored Rab11A proteins(32). Novel methods in

fluorescent multiplexing (33) present an exciting opportunity for observing many cellular

structures which will provide a holistic image the spatial dependence of vRNP segments

upon subcellular structures and to each other.

Methods:

Multi-color Fluorescent In Situ Hybridization Images

This study used previously published multi-color FISH images from (17) that were

generated at the National Institutes of Health. Briefly, MDCK cells were infected with

recombinant WSN/1933 H1N1 for 8 hours and then fixed and stained with FISH probes,

obtained from Biosearch technologies, against four distinct vRNP segments. DAPI was

included as well to label DNA. Multi-color FISH samples were imaged on an Leica SP5

white light laser to ensure spectral separation of the five colors (Dapi, Alexa 488, Quasar

570, Cal Fluor Red 590, Quasar 670). The specificity of the probes and spectral separation

between fluorophores was previously confirmed (17).

Image preprocessing, cell segmentation and point detection

All image preprocessing was performed in MATLAB, v. 2015a. Prior to point detection and

segmentation, image noise was removed by convolution with a Gaussian mask. Each image

channel was denoised individually. As described in (17), each image was captured with

0.17 um z-step size spanning the entire cell volume, defined for each cell using both the

nuclear and FISH staining to define the apical cell membrane. Each image was captured

with a pixel size of ~50x50x168 nm.

Individual vRNP segments were identified by finding connected areas of signal within the

denoised image. The center of each object was taken as the maximum intensity pixel

within, yielding a set of point coordinates.

Since there was no fluorescent tag for cell membrane components, we estimated the cell

boundary using the vRNP images. We assumed that fluorescent signal would be denser

within the cell cytoplasm than in the surrounding area with cell-cell junctions subtly

defined by 'valleys', curves of low signal density whose normal vectors point towards

increasing density. The gradient of point density over the image was then used to segment

individual cells from both the background and each other by the mean-shift algorithm (34).

Hand segmentation was also used to ensure accurate cell membrane segmentation. In most

cases, the convex hull of all points within an identified cell region then defined the cell

membrane. The nucleus was segmented through simple thresholding and smoothing.

Point processes and patterns

Within each cell, the vRNP segments form a point pattern, $\mathbf{x}$ = {x_1, x_2, ..., x_n}, where n is the

number of observed points and each x_i is the 3-dimensional coordinate vector for point i.

The point pattern is defined over a bounded region, W, the segmented cell cytoplasm. $\mathbf{x}$ is

then viewed as a realization of some random point process $\mathbf{X}$, the generating distribution

for all patterns of the particular vRNP identity. The process $\mathbf{X}$ then represents the

culmination of biological factors that determines vRNP segment location, eg. nuclear

export, directed transport over the microtubule network, inhibition by other organelles,

etc.

To model $\mathbf{X}$, first define its locational density, f, a function over the cytoplasm, where $f(\mathbf{u})$ is

the probability of observing a point at position $\mathbf{u}$. The simplest model for this density is the

Homogenous Poisson, characterized by complete randomness over space, or uniform

probability:

$$f(\mathbf{x}|n) = \frac{1}{Z}\lambda^n \qquad\qquad\qquad (1)$$

where λ determines point density, and Z a normalizing constant. In the point process

literature, λ is referred to as *intensity*, but we refer to it as point density to avoid confusion

with fluorescence intensity.

Characterizing spatial randomness

Hypothesis testing for spatial randomness, comparing an observed pattern to an

expectation under a Poisson assumption, was used as an initial motivation for further

modeling. Ripley's K-function describes the number of neighboring points within a given

distance (r) of each observed point in a pattern, a measure of clustering or inhibition of

points within space. Under a Poisson assumption,

$$K_{poi}(r) = \pi r^2 \qquad\qquad (2)$$

443   The expected K-function value can then be used to measure the difference between a given

444   pattern and that of a Homogenous Poisson with an equal number of points. To assess this

445   for observed point patterns, we used the test statistic

$$T = \int_0^{r_{max}} \left(\widehat{K}(r) - K_{poi}(r) \right)^2 dr \qquad\qquad (3)$$

449   where $\widehat{K}(r)$ is the estimated K-function for a given pattern and r a radius defining the point

450   neighborhood. To provide a background distribution for this statistic for a given cell

451   geometry, we generated 100 samples from a Homogenous Poisson process defined over its

452   cell cytoplasm and calculated the test statistic. We then obtained a p-value for the

453   hypothesis that an observed distribution was drawn from a homogenous Poisson process:

455
$$p = \frac{1 + \sum_{i=1}^{m} I(t_{obs} \geq t_i)}{m+1} \tag{4}$$

where m is the number of samples drawn. The p-values were averaged over all cells for a

given vRNP.

<u>Modeling spatial dependence on cellular structures</u>

An Inhomogenous Poisson Process, in its simplest form, is characterized by a spatially

dependent locational density. Per point factors (eg: distance to cell membrane) determine

the probability of a point occurring at a given location. As above, let **X** be some point

process with realization **x**, and define $s(\mathbf{x})$, an n x k matrix where each $s_{i,j}$ is some factor j

that can be calculated for each point x_i. The Inhomogenous Poisson point density function

is:

$$\lambda(x_i; \theta) = \left(exp(\theta^T s(x_i))\right)$$

470   where θ is the k-dimensional parameter vector. The likelihood of the model given point

471   data is then defined by:

$$l(\mathbf{x}; \theta) = \left(\prod_{i=1}^{n} \lambda(x_i)\right) \exp\left(-\int_W \lambda(u)du\right)$$

475 To quantify vRNP dependence on the cell and nuclear membranes, we define the first factor

476 (f_1) as the ratio of nuclear to cell distance calculated per point:

$$f_1(\boldsymbol{x}) = {d_n(\boldsymbol{x})}\big/{d_c(\boldsymbol{x})}$$

479 **Modeling spatial dependence between vRNPs**

481 The second factor, minimum inter-pattern point distance, was used to explore whether

482 there is vRNP-vRNP interaction between vRNPs of different types. For each observed vRNP

483 segment of type l, and some other vRNP point pattern of type j, the minimum inter-pattern

484 point distance is:

$$f_2\big(x_i^l|l \neq t\big) = \min_j d\big(x_i^l, x_j^t\big)$$

For each vRNP type, we use a 'one depends on all' schema. That is, each model with the

same set of vRNPs is not equivalent but exclusively represents a single vRNP pattern (the

primary pattern) depending on others (the secondary patterns). Models were constructed

for every unique pair, triplet, and quadruplet of vRNP types.

For each model, all instances of its unique components were gathered, all images with tags

for every vRNP in the model. Parameters of the model were then fit for each instance using

the maximum pseudolikelihood (35). Hold one out cross validation was used to assess

quality of fit over all instances yielding cross-validated pseudo-likelihood estimated

parameters and variances. These fitted parameters were then used to calculate a true

model likelihood as in Li et al (19).

Interaction network assembly

We formulate the task of finding the most probable set of vRNA interactions yielding a full

genome as a graph problem. First, let the set of nodes in the graph be every possible

combination of vRNAs ranging in size from a single vRNA to all 8 segments. Directed edges

in the graph connect subset to super set nodes. We weight each edge of the graph by the

likelihood of the most likely model that results in the superset and contains the subset as a

primary or secondary segment (Figure 1). Edges that represent models that have not been

observed remain unweighted.

From this graph, a set of interactions yielding the full genome is a tree with leaves as the

vRNA singletons and root as the full 8-mer of vRNA segments. In these trees, we require

each node to have at least two incoming edges, barring the leaf nodes. The set of incoming

edges to a given node must emanate from nodes whose labels, when combined, exactly

equal the given node. A set of incoming edges essentially 'combines' nodes, directly

corresponding to the physical event of vRNP segments binding. The most likely set of

interactions is also the most highly weighted tree. The problem of finding the most

probable interaction network then reduces to finding the highest weighted tree in the

graph. This problem can easily be solved through dynamic programming within a paradigm

often used in evolutionary tree construction (22).

The likelihoods of models involving multiple vRNP segments can be taken as a measure of

spatial dependence of primary segments on secondary segments. vRNP segments that truly

interact are expected to be highly dependent either unidirectionally or bidirectionally. The

cross validated model likelihoods can be used to weight groups of edges in the graph. Since

the graph also has edges that were not observed in actual images (eg: any complex of more

than 4 vRNAs), composite or inferred likelihoods were generated through recursion in a

dynamic programming scheme. For each unobserved complex, we took the most likely set

of observed models that could generate the complex.

For a given unobserved complex $N = \{N_1, N_2, N_3, \ldots\}$,

531           $L(N) = \max(L(N_1 | N_2, N_3, \ldots), L(N_2 | N_1, N_3, \ldots), \ldots)$

533   Then for some likelihood in the above, $L(N_1 | N_2)$ where $N_1 = (n^1{}_1, \ldots n^1{}_k)$ and $N_2 = (n^2{}_1, \ldots$

534     $n^2{}_l)$, $|N_1| = k$, $|N_2| = l$,

$$L\big((N_1 | N_2)\big) = \sum_{i=1}^{k} \operatorname*{average}_{N_j^2 \in all\ unique\ sets\ of\ N_2} (L(n_i^1 | N_j^2))$$

Interaction network stability analysis

To assess the stability of the generated interaction network, two procedures of

perturbation were performed: introduction of noise and removing certain high likelihood

models. We simulated noise as a normal random distribution centered at 0 with varying

standard deviations, termed the noise levels. Under increasing noise levels, each observed

model likelihood was amended with a noise value drawn from this distribution prior to

interaction network construction. For each noise level, we simulated 100 trials and tallied

the proportion of output trees that contained each possible edge. Trees that align with the

non-noisy assembly signal stability while highly variable trees signal instability.

Reproducible Research Archive

All images, derived data and source code will be available upon publication at

http://murphylab.cbd.cmu.edu/software.

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Figure 1: Illustration of the analysis pipeline. (A) Unprocessed fluorescence channels for cells FISH tagged for four vRNA segments and stained with DAPI. (B) Preprocessed image channels after filtering, nuclear and cell membrane segmentation, and point identification. The 3D images in (A) and (B) are displayed as maximum value projections. (C) Factors are calculated for each channel in each image, model parameters are fit, and a likelihood generated. (D) Dynamic programming is used to generate a most likely set of interactions yielding a full genome. Edges are possible interactions, solid lines indicate edges that are part of the most likely set of interactions. In this toy example, the most likely set of interactions is A and B binding to form AB, and then C binding to AB.

699 Figure 2: Pairwise interactions learning from MDCK cell images. Each vRNA segment is
700 arranged around the perimeter. Bands connect vRNA segments that interact with high
701 likelihood. The width of the base of each band is proportional to the likelihood of the
702 interaction model, larger bases correspond to more likely pairings. Only interactions whose
703 log likelihoods are above -0.2 are shown.
704

Figure 3: Comparison of composite and observed model likelihoods for triplet and quadruplet models. To ensure that the recursion in our dynamic programming scheme accurately predicts model likelihoods, we compared triplet and quadruplet observed likelihoods to those generated from only pairwise models. Ideally, all points would fall on the red line, matching observed and predicted values perfectly. The cluster in the lower left corner represent interactions that are predicted to occur with high likelihood but in reality, do not. All points within this cluster contain the same four vRNA segments (PB1, PB2, NA, NS)

Figure 4: Constructed interaction networks. Using dynamic programming, we constructed the single most likely set of interactions yeilding a full genome. First, we used all model likelihoods (a) to build a master tree. Hotter edge colors correspond to more likely interactions. Since the images contain only a subset of the possible triplets and quadruplet, we assessed how much this affected the master tree by removing all quadruplets (b) or both quadruplets and triplets (c). To test the stability of the master tree, trees were constructed with random noise added to each observed model likelihood before performing the dynamic programing. Multiple noise levels were tested up to 1/3 of the mean model likelihood. In all cases, two possible trees were generated: (a) or (d). Given the two observed clusters in the master tree, we next asked whether the composite likelihood function could recapture their interactions without actually observing them. Removing all models involving HA, M, NS, NA (C1) and NP, PB1, PB2 (C2) yielded (e). Removing only C2 yielded (f).

731 Table 1: Monte Carlo p-values and variances for the hypothesis that the spatial distribution
732 of each vRNA segment is distributed by a homogeneous Poisson process.

	Mean p-value	Variance
HA	0.00671	0.00015
PB1	0.00524	0.00034
PB2	0.0034	0.00011
NA	0.00249	0.00003
M	0.00005	0
PA	0.00467	0.0002
NP	0.01185	0.00036
NS	0.00306	0.00007

733
734
735 Table 2: Cross validated negative log-likelihoods for models dependent on distance to
736 different reference locations.
737

	Cell Model	Nuclear Model	Cell+Nuclear Model
PB1	2.398	0.408	2.073
PB2	2.362	0.410	0.746
PA	2.414	0.470	1.252
HA	2.412	2.447	5.210
NP	2.506	2.501	5.236
NA	2.493	2.350	1.748
M	2.554	2.585	3.490
NS	2.397	0.666	1.931

738
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740
741 Table 3: Cross validated negative log-likelihoods for pairwise interaction models.
742

	PB1	PB2	PA	HA	NP	NA	M	NS
PB1		0.241	0.203	0.205	0.125	0.403	0.171	0.291
PB2	0.387		0.268	0.284	0.315	0.561	0.208	0.398
PA	0.221	0.220		0.229	0.155	0.322	0.281	0.232
HA	0.128	0.333	0.184		0.272	0.233	0.258	0.288
NP	0.180	0.318	0.194	0.290		0.252	0.275	0.341
NA	0.325	0.320	0.309	0.300	0.274		0.285	0.300
M	0.225	0.329	0.284	0.289	0.297	0.293		0.291
NS	0.277	0.343	0.222	0.288	0.325	0.344	0.289	

743
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745

746 Table 4: Cross validated negative log-likelihoods for pairwise interaction models that also
747 include cell-to-nuclear distance ratio.
748

	PB1	PB2	PA	HA	NP	NA	M	NS
PB1		0.59	0.40	3.98	0.14	2.25	1.67	1.72
PB2	0.45		0.28	0.41	0.36	0.58	0.22	0.51
PA	0.37	0.31		1.77	0.56	1.08	0.91	1.17
HA	2.73	0.47	1.08		4.11	3.84	3.50	0.82
NP	0.19	0.56	0.49	2.54		5.36	6.98	0.72
NA	1.42	2.50	0.73	3.36	11.05		3.50	1.01
M	0.71	0.80	0.85	3.09	7.02	3.34		0.92
NS	2.15	0.41	0.99	0.88	0.53	0.72	1.12	

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Table 5: Cross validated log-likelihoods for the top two triple models with each vRNA segment as the primary pattern. Pairwise model likelihoods for the members of each model are shown in the right columns.

RNA	CV Likelihood	PB1	PB2	PA	HA	NP	NA	M	NS
PB1									
PB1 M PB2	0.168		0.241					0.171	
PB1 M PA	0.176			0.203				0.171	
PB2									
PB2 M PB1	0.164	0.387						0.208	
PB2 PA M	0.175			0.268				0.208	
PA									
PA M PB2	0.273		0.220					0.281	
PA PB1 M	0.336	0.221						0.281	
HA									
HA NS PA	0.313			0.184					0.288
HA NP PB2	0.326		0.333			0.272			
NP									
NP PB1 PB2	0.180	0.180	0.318						
NP PA PB1	0.302	0.180		0.194					
NA									
NA HA NP	0.255				0.300	0.274			
NA M HA	0.333				0.300			0.285	
M									
M PB2 PA	0.236		0.329	0.284					
M PB1 PA	0.257	0.225		0.284					
NS									
NS M PB2	0.300		0.343					0.289	
NS HA PA	0.356			0.222	0.288				

757 Table 6: Cross validated negative log-likelihoods for the top two quadruple models with
758 each vRNA segment as the primary pattern. Pairwise model negative log-likelihoods for the
759 members of each model are shown in the right columns.
760

RNA	CV Likelihood	PB1	PB2	PA	HA	NP	NA	M	NS
PB1									
PB1 M PA PB2	0.154		0.241	0.203				0.171	
PB1 PB2 NA NS	0.313		0.241				0.403		0.291
PB2									
PB2 PA PB1 M	0.159	0.387		0.268				0.208	
PB2 NS HA M	0.256				0.284			0.208	0.398
PA									
PA PB2 PB1 NP	0.202	0.221	0.220			0.155			
PA HA NS M	0.241				0.228			0.281	0.232
HA									
HA PA NS M	0.171			0.184				0.258	0.288
HA M NA NP	0.188					0.272	0.233	0.258	
NP									
NP PA PB2 PB1	0.234	0.180	0.318	0.194					
NP HA NA M	1.289				0.290		0.252	0.275	
NA									
NA HA NP M	0.242				0.300	0.274		0.285	
NA M PA NS	0.365			0.309				0.285	0.300
M									
M PB1 PB2 PA	0.245	0.225	0.329	0.284					
M NS HA PA	0.246			0.284	0.289				0.291
NS									
NS M PA HA	0.229			0.222	0.288			0.289	
NS PB2 M HA	0.340		0.343		0.288			0.289	

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764 Supp. Table 1: FISH imaging probe sets. The leftmost column is the experiment label with
765 the right 4 columns showing which vRNA segments were tagged. Each experiment also
766 used DAPI staining.

Reaction	Probe 1	Probe 2	Probe 3	Probe 4
A	M	HA	PB2	NS
B	PB1	PB2	PA	NP
C	PB1	M	HA	NS
D	M	PA	HA	NS
E	M	HA	NA	NP
F	PB1	PB2	NA	NS
G		PA	NA	
H			NP	NS
I	PB1	M	PB2	PA
J		M	NA	NP
K		PB2	HA	NP
L	M	HA	NA	NS
M	M	NA	HA	PA
N	M	PB2	NP	
Rxn 1	M	PA	NA	NS

767
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770 Supp. Table 2: Observed triplets within the reaction set. 32 of 56 possible triplets were
771 observed.
772

Observed triplets	Reaction
PB2, PB1, PA	B, I
PB2, PB1, NP	B
PB2, PB1, NA	F
PB2, PB1, M	I
Pb2, PB1, NS	F
PB2, PA, NP	B
PB2, PA, M	I
PB2, HA, NP	K
PB2, HA, M	A
PB2, HA, NS	A
PB2, NP, NS	N
PB2, NA, NS	F
PB2, M, NS	A
PB1, PA, NP	B
PB1, PA, M	I
PB1, HA, M	C
Pb1, HA, NS	C
PB1, NA, NS	F
PB1, M, NS	C
PA, HA, NA	M
PA, HA, M	D, M
PA, HA, NS	D
PA, NA, M	M, Rxn 1
PA, NA, NS	Rxn 1
PA, M, NS	Rxn 1
HA, NP, NA	E
HA, NP, M	E
HA, NA, M	E, L, M
HA, NA, NS	L
HA, M, NS	L
NP, NA, M	E, J
NA, M, NS	L, Rxn 1

773
774
775

776 Supp. Table 3: Cross validated log-variances for pairwise interaction models with cell-to-
777 nuclear distance ratio.
778
779 (large)
780
781 Supp. Table 4: Variance in the cross validated negative log-likelihoods for pairwise
782 interaction models with cell-to-nuclear distance ratio shown in Table 4.
783

	PB1	PB2	PA	HA	NP	NA	M	NS
PB1		0.0172	0.0025	0.0012	0.0010	0.0002	0.0012	0.0067
PB2	0.0552		0.0072	0.0310	0.0086	0.0028	0.0025	0.0919
PA	0.0055	0.0091		0.0054	0.0035	0.0061	0.0055	0.0103
HA	0.0018	0.0175	0.0011		0.0052	0.0064	0.0119	0.0146
NP	0.0006	0.0489	0.0015	0.0029		0.0060	0.0079	0.0003
NA	0.0002	0.0001	0.0023	0.0561	0.0143		0.0066	0.0025
M	0.0022	0.0146	0.0064	0.0146	0.0104	0.0037		0.0068
NS	0.0068	0.0033	0.0083	0.0038	0.0004	0.0011	0.0042	

784
785
786 Supp. Table 5: Cross validated log-likelihoods for all observed triplets.
787
788 (large)
789
790 Supp. Table 6: Cross validated log-likelihoods for all observed quadruplets.
791
792 (large)
793
794