1   **Multiobjective Strain Design: A Framework for Modular Cell Engineering**

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ABSTRACT

Diversity of cellular metabolism can be harnessed to produce a large space of molecules. However, development of optimal strains with high product titers, rates, and yields required for industrial production is laborious and expensive. To accelerate the strain engineering process, we have recently introduced a modular cell design concept that enables rapid generation of optimal production strains by systematically assembling a modular cell with an exchangeable production module(s) to produce target molecules efficiently. In this study, we formulated the modular cell design concept as a general multiobjective optimization problem with flexible design objectives derived from mass action. We developed algorithms and an associated software package, named ModCell2 to implement the design. We demonstrated that ModCell2 can systematically identify genetic modifications to design modular cells that can couple with a variety of production modules and exhibit a minimal tradeoff among modularity, performance, and robustness. Analysis of the modular cell designs revealed both intuitive and complex metabolic architectures enabling modular production of these molecules. We envision ModCell2 provides a powerful tool to guide modular cell engineering and sheds light on modular design principles of biological systems.

Keywords

Modular cell engineering; modular cell; production module; modular design; modularity; multiobjective optimization; multiobjective evolutionary algorithm.

INTRODUCTION

Engineering microbial cells to produce bulk and specialty chemicals from renewable and sustainable feedstocks is becoming a feasible alternative to traditional chemical methods that rely on petroleum feedstocks (1). However, only a handful of chemicals, out of the many possible molecules offered by nature, are industrially produced by microbial conversion, mainly because the current strain engineering process is laborious and expensive for profitable biochemical production (2). Thus, innovative technologies to enable rapid and economical strain engineering are needed to harness a large space of industrially-relevant molecules (3).

The modular organization of biological systems has been a source of inspiration for synthetic biology and metabolic engineering (4, 5). Modular pathway engineering breaks down target pathways into tractable pathway modules that can be finely tuned for optimal production of desirable chemicals (6, 7). Harnessing combinatorial pathways (e.g., fatty acid biosynthesis, reverse beta oxidation, polyketide or isoprenoid biosynthesis) is one excellent example of modular pathway engineering. These pathways contain metabolic similarity (or combinatorial characteristics) such as a group of common specific enzymes capable of catalyzing linear reaction steps (8) and/or elongation cycles (9-11) and hence are capable of producing a large library of unique molecules (12). Since these molecules are derived from a common precursor metabolite(s), the optimal production strains often share common genotypes and phenotypes, and hence, the costly strain optimization process is only performed once for these molecules. Remarkably, this advantageous strain optimization strategy can be applied even for production of molecules derived from different precursors, using the concept of modular cell (ModCell) design (2, 13, 14).

With the arrival of steady-state, constraint-based stoichiometric models of cellular metabolism, various computational algorithms have been developed to guide strain engineering

(15-17). These methods have featured the design of strains capable of growth-coupled product

synthesis (*GCP*), enabling adaptive laboratory evolution of these designed strains to enhance

product titers, rates, and yields (14, 18-20). Two approaches on growth-coupled production have

been formulated ─ one based on the coexistence of maximum growth and product synthesis rates

during the growth phase (21) and the other based on the obligate requirement of optimal product

synthesis in any growth phase (22). The distinction between these two types of growth coupling

are also referred to weak coupling (*wGCP*) and strong coupling (*sGCP*) (16, 23).

Development of most strain design algorithms has been focused on overproduction of only

one target molecule. The first algorithm proposed for modular cell design compatible for

overproduction of multiple target molecules is MODCELL (13), which guided several

experimental studies (14, 24-27). It works by generating *sGCP* strain designs for each target

product based on elementary mode analysis (28), and then comparing the design strategies of

different products to identify common genetic modifications among them. A similar approach was

adapted in a subsequent work (29). For MODCELL to find optimal solutions for multiple target

products, it requires: 1) enumerating all possible designs above a predefined minimum product

yield and with minimal reaction deletion sets for each production network, which might lead to a

large number of solutions for each network and hence make the problem computationally

intractable, and 2) the resulting designs for all products must be compared to identify common

interventions, which is a computationally-hard, set-covering problem. Thus, the current

enumerative approach of MODCELL might become intractable very quickly, especially for large-

scale metabolic networks and potentially generate non-optimal designs, i.e., requiring more knock-

outs than necessary or including fewer products than possible.

In this study, we generalized the concept of modular cell design and addressed the

computational limitation of implementing it. We developed a novel computational platform

(ModCell2), based on multiobjective optimization and analysis of mass action of cellular

metabolism, to guide the design of modular cells for large-scale metabolic networks. We

demonstrated that ModCell2 can systematically identify genetic modifications to design modular

cells that can couple with a variety of production modules and exhibit a minimal tradeoff among

modularity, performance, and robustness. By analyzing these designs, we further revealed both

intuitive and complex metabolic architectures enabling modularity in modular cell and production

modules required for efficient biosynthesis of target molecules.

METHODS

Design principles of modular cell engineering

In the conventional strain engineering approach, a parent strain is genetically modified to yield an

optimal production strain to make only a target product. To produce each new molecule, the

design-build-test cycles of strain engineering must be repeated, which is laborious and expensive

(Figure 1). To minimize the cycles, modular cell engineering is formulated by genetically

transforming a parent strain into a modular (chassis) cell that must be assembled with exchangeable

modules to create optimal production strains (13). A modular cell is designed to contain core

metabolic pathways shared across designed optimal production strains. Exchangeable modules are

production pathways designed to synthesize desirable chemicals. A combination of a modular cell

and a production module(s) is required to balance redox, energy, and precursor metabolites for

sustaining cellular metabolism during growth and/or stationary phases and exhibiting only

desirable phenotypes. Practically, modular cell engineering can be applied to monocultures and

polycultures, where a production module(s) can be embedded in a modular cell and activated by

intracellular and/or extracellular cues such as light and/or signaling molecules.

Multiobjective strain design framework for modular cell engineering

For modular cell engineering, we seek to design a chassis cell compatible with as many production

modules as possible to achieve only desirable production phenotypes while requiring minimal

genetic modifications. Since all production modules must leverage cellular resources of the

modular cell (e.g. precursor metabolites, cofactors, and energy), they form competing objectives.

Therefore, the framework of modular cell engineering can be formulated as a multiobjective

optimization problem, named ModCell2, as described below.

$\underset{over\ y_j,\ z_{jk}}{\text{maximize}}\ \boldsymbol{F} = \left(f_1, f_2, \dots, f_{|K|}\right)^T$ [1]

subject to

$f_k \in \arg max\{ \sum_{j \in J_k} c_{jk} v_{jk}$

subject to

112            $\sum_{j \in J_k} S_{ijk} v_{jk} = 0, for\ all\ i \in I_k$ [2]

113            $l_{jk} \leq v_{jk} \leq u_{jk}, for\ all\ j \in J_k$ [3]

114            $l_{jk} d_{jk} \leq v_{jk} \leq u_{jk} d_{jk}, for\ all\ j \in C\ and\ C \subseteq J_k$ [4]

115   *where* $d_{jk} = y_j\ V\ z_{jk}\}, for\ all\ k\ \epsilon\ K$

$z_{jk} \leq (1 - y_j), for\ all\ j \in C, k\ \in\ K$ [5]

$\sum_{j \in C}(1 - y_j) \leq \alpha$ [6]

$\sum_{j \in C}\ z_{jk} \leq \beta_k, for\ all\ k\ \epsilon\ K$ [7]

where i, j, and k are indices of metabolite i, reaction j, and production network k, respectively; f_k

is a design objective for network k; c_{jk} represents the cellular objective for reaction j in network k

associated with a design objective defined in eqns. 8-10; v_{jk} (mmol/g DCW/h) is metabolic flux of

reaction j bounded by l_{jk} and u_{jk} in network k, respectively; y_j and z_{jk} are binary design variables

for deletion reaction j and module reaction j in network k, respectively; α and β_k are design

parameters for deletion and module reactions, respectively; S_{ijk} is a stoichiometric coefficient of

metabolite in reaction j of network k; and C (eqn. 4) is the candidate reaction set (Supplementary

File S1). The goal of the optimization problem is to simultaneously maximize all design objectives

f_k.

Steady-state mass balance constraint of cellular metabolism. Quasi steady-state flux

balance of cellular metabolism (eqn. 2) is used as metabolic constraints for eqn. 1 (30, 31). A

model corresponding to each modular production strain (i.e. production network k) will be derived

from a parent strain (i.e. parent network) by adding necessary reactions (e.g., a production module)

to produce a target molecule. A feasible flux distribution for each production network is described

by mass balance (eqn. 2) and reaction flux bounds (eqns. 3 and 4). For a given production network,

the phenotypic space can be illustrated by the gray area that is projected onto the two-dimensional

space spanned by product synthesis and growth rates (Figure 2).

Design variables. In our formulation for modular cell engineering, we introduced two

design variables − binary reaction deletions (y_j) inherent to the modular cell and module-specific

reaction insertions (z_{jk}) (eqn. 4). These variables can be experimentally manipulated to constrain

the desirable phenotypes of production strains as shown in Figure 2. Specifically, $y_j = 0$ if reaction

j is deleted from the modular cell; otherwise, $y_j = 1$. Deleting metabolic reactions removes

undesired functional states of the network and leaves those with high design objectives. Likewise,

$z_{jk} = 1$ if reaction j is present in the production network k; otherwise, $z_{jk} = 0$. These module reactions

are endogenous reactions removed from the parent network (eqn. 5), but are added back to a

specific production module to enhance the compatibility of a modular cell. The maximum number

of reaction deletions (α) and module-specific reaction insertions (β_k) are user-defined parameters.

Design objectives. To generalize ModCell2 design, we allow three different types of

design objectives (f_k, eqn. 1) that determine production phenotypes for each production network.

Depending on the application, a phenotype can be designed to be weak coupling (*wGCP*), strong

coupling (*sGCP*), and/or non-growth production (*NGP*) (Figure 2). The constrained phenotypic

spaces based on these design objectives are shown in color; any point within these spaces is a

feasible physiological state of the cell that can be represented by a metabolic flux distribution.

The *wGCP* design seeks to achieve a high product rate at maximum growth rate (Figure

2A). The *wGCP* design objective, f_k^{wGCP} ($\in [0, 1]$), is calculated as follows:

$$154 \qquad f_k^{wGCP} = \frac{v_{Pk}^{\mu}}{v_{Pmaxk}^{\mu}} \qquad\qquad [8]$$

where v_{Pk}^{μ} is the minimum synthesis rate of the target product P at the maximum growth rate for

production network k and v_{Pmaxk}^{μ} is the maximum synthesis rate of P (Supplementary File S1).

This *wGCP* design formulation is equivalent to RobustKnock (32) or OptKnock with a tilted

objective function (21, 33, 34). In eqn. 8, f_k^{wGCP} is scaled from 0 to 1 for proper comparison among

products. The *wGCP* design is appropriate for applications where growth rate is not limited by the

nutrients, and the product is formed during the growth phase.

The *sGCP* design seeks to achieve a high product rate not only at optimal growth rate but

also during non-growth phase (Figure 2B). The *sGCP* design objective, f_k^{sGCP} ($\in [0, 1]$), is

calculated as follows:

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$$f_k^{sGCP} = \frac{v_{Pk}^{\mu}}{v_{Pmaxk}^{\mu}} \cdot \frac{v_{Pk}^{\bar{\mu}}}{v_{Pmaxk}^{\bar{\mu}}} \qquad [9]$$

where $v_{Pk}^{\bar{\mu}}$ and $v_{Pmaxk}^{\bar{\mu}}$ are the minimum and maximum product formation rates for production

network k in the stationary phase, respectively (Supplementary File S1). The *sGCP* design

objective is comparable to the one implemented in MODCELL (13). Different from *wGCP*, *sGCP*

requires high product synthesis rate for any growth phase. However, the additional constraint of

optimal product synthesis during the stationary phase requires more genetic manipulations or

specific experimental conditions (e.g., anaerobic growth condition, supply of intermediate

metabolites). Both *wGCP* and *sGCP* designs enable fast growth selection to attain the optimum

product rates by adaptive laboratory evolution (35, 36).

The *NGP* design aims to maximize the minimum product rate during the non-growth phase

by eliminating carbon fluxes directed to biomass synthesis (Figure 2C). The *NGP* design objective,

f_k^{NGP} ($\in [0, 1]$), is calculated as follows:

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$$f_k^{NGP} = \frac{v_{Pk}^{\bar{\mu}}}{v_{Pmaxk}^{\bar{\mu}}} \qquad [10]$$

While the *NGP* design is not suitable for growth selection, it can be derived from a *wGCP* (or

sGCP) design by imposing additional genetic modifications. Practically, *NGP* design strains can

be activated during cell culturing using a regulatory genetic circuit to toggle switch between

production phases.

Design solutions. Optimal solutions for eqn. 1 are a Pareto set (***PS***) that correspond to

design variables, including reaction deletions (y_j) and module reaction insertions (z_{jk}). Each

solution constitutes a design of a modular cell:

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$$PS := \{\mathbf{x} \in \Omega : \nexists \mathbf{t} \in \Omega, \mathbf{F(t)} \prec \mathbf{F(x)}\} \qquad [11]$$

185   Here, $\mathbf{F(t)} \prec \mathbf{F(x)}$ means $\mathbf{F(t)}$ *dominates* $\mathbf{F(x)}$ if and only if $f_i(\mathbf{t}) \geq f_i(\mathbf{x})$ for all i, and $\mathbf{F(t)}$ differs

186   from $\mathbf{F(x)}$ in at least one entry. The feasible space of design variables, Ω, is defined by the problem

187   constraints (eqn. 2-7, also see Supplementary File S1). Phenotypes of modular cells will be the

188   image of the Pareto set in the objective space, known as the Pareto front (***PF***):

$$PF := \{\mathbf{F(x)} : \mathbf{x} \in PS\} \qquad\qquad [12]$$

For the multiobjective strain design framework, the input parameters include α (eqn. 6), β_k (eqn.

7), and the production networks as input metabolic models. Each model contains a production

module to produce one target chemical. The output is a Pareto set (genetic modifications) and its

respective Pareto front (desirable production phenotypes). For a special case with no trade-off

among the design objectives, an optimal solution, named a utopia point, exists where each

objective achieves its maximum value. The multiobjective strain design formulation presented is

general and can be applied to design modular cells for any organism.

Algorithm and implementation

ModCell2 algorithm. To solve the multiobjective optimization problem for modular cell

engineering, we used multiobjective evolutionary algorithms (MOEAs) (37). MOEAs were

selected because they can efficiently handle linear and non-linear problems and do not require

preferential specification of design objectives (38). MOEAs start by randomly generating a

population of individuals (a vector of design variables), each of which is mapped to a design

objective vector (i.e., a fitness vector). In ModCell2 (Supplementary File S1), the objective values

of an individual are calculated by solving the linear programming problems for each production

network. Next, individuals are shuffled to generate an offspring, from which the most fit

individuals are kept. This process was repeated until the termination criteria was reached, for

instance, either the solutions cannot be further improved or the simulation time limit is reached.

ModCell2 implementation.   To streamline the modular cell design, we developed the

ModCell2 software package based on three core classes (Figure S1 in Supplementary File S2).

The Prodnet class parses and pre-processes production network models, and computes production

phenotypes. The MCdesign class serves as an interface between the MOEA optimization method

and metabolic models. Finally, the ResAnalysis class loads the Pareto set computed by MCdesign

and identifies the most promising modular cell designs.

The code was written in MATLAB 2017b (The Mathworks Inc.) using the function

gamultiobj() from the MATLAB Optimization Toolbox that implements the NSGA-II algorithm

(39) to solve the multiobjective optimization problem. The solution and analysis methods were

parallelized using the MATLAB Parallel Computing Toolbox. The linear programs to calculate

metabolic fluxes were solved using the GNU Linear Programming Kit (GLPK). The COBRA

toolbox (40, 41) and F2C2 0.95b (42) were also used for COBRA model preprocessing and

manipulation.

Metabolic models. In our study, we used three parent models including i) a small metabolic

network to illustrate the modular cell design concept (13), ii) a core metabolic network of

Escherichia coli to compare the performance of ModCell2 with respect to the conventional single-

product strain design strategy and the first-generation modular cell design platform MODCELL

(13), and iii) a genome-scale metabolic network of *E. coli* (i.e., iML1515 (33)) for biosynthesis of

a library of endogenous and heterologous metabolites, including 4 organic acids, 6 alcohols, and

10 esters (Figures S2 in Supplementary File S2) (8, 22, 25, 43-49).

Simulation protocols. Anaerobic conditions were imposed by setting oxygen exchange

fluxes to be 0, and the glucose uptake rate was constrained to be at most 10 mmol/gCDW/h, as

experimentally observed for *E. coli*. When using the genome-scale model iML1515 to simulate

wGCP designs, the commonly observed fermentative products (acetate, CO_2, ethanol, formate,

lactate, succinate) were allowed for secretion as described elsewhere (50). For simulation of *sGCP*

and *NGP* designs, the glucose uptake rate was fixed (i.e., -10 mmol/gCDW/h); otherwise, the flux

is not active during the no-growth phases, resulting in the product synthesis rate of 0 regardless of

genetic manipulations. To compare ModCell2 with Optknock, we applied the OptKnock algorithm

with a titlted objective function (51) to generate *wGCP* designs for each production network, using

the open-source algebraic modeling language Pyomo (52). The MILP problems were solved using

CPLEX 12.8.0 with a time limit of 10,000 seconds set for each product. ModCell2 is provided as

an open-source software package and is freely available for academic research. The software

package and documentation can be download via either web.utk.edu/~ctrinh or Github

https://github.com/TrinhLab.

Analysis methods for design solutions

Compatibility. The compatibility, C ($\in Z^+$), of a design is defined as the number of products

that are coupled with a modular cell and has objective values above a specified cutoff value θ. As

a default, we set $\theta = 0.6$ for the *wGCP* and *NGP* design objectives and $\theta = 0.36$ (0.6^2) for the *sGCP*

design objective. For example, a *wGCP* design for 3 products that has the design objective values

of 0.4, 0.9, and 0.6 has a compatibility of 2, given a cutoff value of $\theta \geq 0.6$.

Compatibility difference and loss. Robustness is the ability of a system to maintain its

function against perturbations, and hence is very important of designing biological and engineered

systems (53). To evaluate the robustness of modular cell designs, we defined two metrics, the

compatibility difference (*CD*) and compatibility loss (CL ϵ [0,1]) as follows:

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$$CD = C_{initial} - C_{final}$$
[13]

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$$CL = \frac{C_{initial} - C_{final}}{C_{initial}}$$
[13]

where $C_{initial}$ and C_{final} are the compatibilities of a modular cell design before and after a single

reaction deletion, respectively. The value $CD > 0$ (or $CL > 0$) means the modular gains fitness

while $CD < 0$ (or $CL < 0$) means that it loses its fitness. In the analysis, we did not consider essential

and blocked reactions for our single-deletion analysis; for instance, there are only 1139 potential

reaction deletions in the iML1515 model.

Metabolic switch design. A metabolic switch design is a modular cell that can possess

multiple production phenotypes (i.e., *wGCP*, *sGCP*, and *NPG*), activated by an environmental

stimulus (e.g. metabolites, lights). The metabolic switch design is enforced to have a set of reaction

(gene) deletions in one production phenotype to be a subset of the other, for instance, $\{\mathbf{y}_{wGCP}\} \subseteq$

$\{\mathbf{y}_{NPG}\}$, $\{\mathbf{y}_{wWCP}\} \subseteq \{\mathbf{y}_{sGCP}\}$. The metabolic switch design is beneficial for multiphase fermentation

configurations that enable flexible genetic modification and implementation. Specifically, the

metabolic switch design can exhibit the *wGCP* phenotype during the growth phase and the *NPG*

(or *sGCP*) phenotype during the stationary phase. The metabolic switches can be implemented

using the genetic switchboard (54).

RESULTS AND DISCUSSION

Illustrating ModCell2 for modular cell design of a simplified network

An example parent network, adapted from (13), was used to illustrate ModCell2 (Figure 3A).

Inputs for the multiobjective optimization problem include i) three production networks (Figure

3B), comprising of one endogenous production module (module 1) and two heterologous production modules (modules 2 and 3) and ii) design parameters (Figure 3C), containing design objective type, maximum number of deletion reactions (α), and maximum number of module reactions (β_k). The output of ModCell2 generated the Pareto set and the corresponding Pareto front for modular cell designs (Figure 3D). The 2-D plots of product yields versus growth rates presented the feasible phenotypic spaces of the wildtype (gray area) and the designed strain (blue area).

Using various α and β_k values, ModCell2 simulation generated three $wGCP$-α-β_k-d, four $sGCP$-α-β_k-d designs, and three NGP-α-β_k-d designs, where d is the design solution index (Figure 3D). For instance, by setting $\alpha = 3$ and $\beta_k = 0$, we found three $sGCP$ designs including $sGCP$-3-0-1, $sGCP$-3-0-2, and $sGCP$-3-0-3. The first design $sGCP$-3-0-1 has a compatibility of 2 (given $\theta > 0$) with the design objective values of 0.42 and 0.97 for the products P2 and P3, respectively. In contrast, the $sGCP$-3-0-2 and $sGCP$-3-0-3 designs have compatibilities of only 1 with the design objectives of 0.63 for P2 and 0.45 for P1, respectively.

Based on all designs, we can clearly see the trade-offs for optimization of different products for $\beta_k = 0$. However, setting $\beta_k \geq 1$ helps increase the compatibility of a modular cell with different production modules. In addition, we found that the Pareto front collapses into a utopia point as seen in the $wGCP$-1-1-1, $sGCP$-3-1-1, and NGP-3-1-1 designs. For instance, the modular cell, $sGCP$-3-1-1, is compatible with all three products. The three corresponding optimal production strains can couple growth and product formation during the growth phase. During the stationary phase, these strains produce the products at maximum theoretical yields. In theory, a universal modular cell always exists, provided that enough reaction deletions and module reactions are used. It might be more tractable to construct such a modular cell from a synthetic minimal cell using the bottom-up approach. However, construction of a universal modular cell from a host

298   organism (e.g., *E. coli*, *S. cerevisiae*) using the top-down approach will require a significantly large

299   number of genetic modifications, that might be challenging.

301   **Comparing ModCell2 designs with first-generation MODCELL and single product designs**

302   *ModCell2 can generate more and better designs than the first-generation modular cell*

303   *design platform*. To evaluate the algorithms and performance of ModCell2, we directly compared

304   it with MODCELL (13) in two case studies, using the same core model of *E. coli* for production

305   of five alcohols (ethanol, propanol, isopropanol, butanol, and isobutanol) and 5 derived butyrate

306   esters (ethyl butyrate, propyl butyrate, isopropyl butyrate, butyl butyrate, and isobutyl butyrate)

307   from glucose (Figure 4A).

308   In the first case study, we fixed the reaction module, i.e. $\beta_k = 2$ for ethanol dehydrogenase

(FEM5) and ethanol export reaction (TRA1), in ModCell2 to emulate the same input as

MODCELL (Supplementary File S3). The results showed that ModCell2 generated all the designs

with the same *sGCP* objective values like MODCELL (Figure 4B, 4C, 4D) together with other

alternative solutions (Supplementary File S3). Interestingly, ModCell2 only required 5 and 6

reaction deletions as opposed to 7 and 7 for the *sGCP-5-0-5* and *sGCP-5-0-6* designs, respectively.

By setting the maximum reaction deletions to $\alpha \geq 6$, ModCell2 could find better design solutions

with fewer deletion reaction requirement and higher objective values (Supplementary File S3).

316   In the second case study, we used the same model configuration but did not enforce the

module reactions. By setting $\alpha = 5$ and $\beta_k = 1$, we found the *sGCP-5-1-8* design that is

compatible with all products and achieves the same objective values for products found in *sGCP-*

5-0-5, *sGCP-5-0-6*, and *sGCP-5-0-2* (Figure 4E). The desirable phenotypic spaces can be further

constrained for many products if α is increased from 5 to 6 (Figure 4F). Remarkably, by setting

$\alpha = 8$ and $\beta_k = 2$, we found a utopia point design, *sGCP-8-2-9*, without any trade-off among

design objectives (Figure 4G). This utopia point design could not be achieved with $\alpha < 8$

regardless of any β_k value.

Overall, the results demonstrate that ModCell2 can efficiently compute the Pareto front of

modular cell designs. It can find better designs with fewer reaction deletion and module reaction

requirements, improve design objective values, and enhance compatibility.

ModCell2 can identify designs with more compatibility than the conventional single-

product designs. To evaluate if the conventional, single-product design strategy is suitable for

modular cell engineering, we first used OptKnock to generate *wGCP* designs for the same 10 target

molecules independently with various allowable reaction deletions ($\alpha = 2, 3, ..., 7$). Likewise, we

employed ModCell2 to produce *wGCP* designs using the same α and various β. To directly

compare OptKnock and ModCell2 solutions, we calculated the *wGCP* design objective values for

all products based on each OptKnock solution (Supplementary File S3). As expected, our result

showed that ModCell2 and OptKnock designs have the same highest objective values for each

product (Figure 5A). However, several OptKnock solutions were always dominated by ModCell2

solutions in all parameter configurations (Figure 5B). With $\alpha \geq 4$, ModCell2 could identify

wGCP-α-1 designs with the maximum compatibility of 10, while the best OptKnock designs only

achieved the highest compatibility of 5 (Figure 5C, 5D).

Overall, ModCell2 can generate modular cells compatible with the maximum number of

modules and achieve high objective values. Single-product designs might not be compatible with

a large number of products, and the solutions might be far from Pareto optimality.

Exploring emergent features of modular cell design using an *E. coli* genome-scale network

Modcell2 can design modular cells using a large-scale metabolic network. To

demonstrate that ModCell2 can be applied for a genome scale metabolic network, we tested it to

generate *wGCP* designs for 20 target molecules with $\alpha = 4$ and various β_k (Supplementary File

S4). The α value was chosen because with 4 deletions, OptKnock could identify single product

designs with objectives above 60% of the theoretical maximum (Supplementary File S5). With

$\beta_k = 0$, ModCell2 could identify modular cell designs with compatibility of 17, for example, the

wGCP-4-0-50 design featuring deletion of ACALD (acetaldehyde dehydrogenase, *adhE*),

ACKr/PTAr (acetate kinase, *ack;* phosphotransacetylase, *pta*), GLYAT (glycine C-

acetyltransferase, *kbl*), and LDH_D (lactate dehydrogenase, *ldhA*) (Figure 6A, 6D, Supplementary

File S4). By analyzing all *wGCP-4-β_k-d* designs (257 total for β_k = 0, 1, 2, and 3), we found that

the ethanol and D-lactate production modules are most compatible with all modular cell designs

(Figure 6A, 6C, Supplementary File S4). Among reaction deletions, *LdhA* (86% of designs), *Pta*

(38%), and *AdhE* (25%) are the most frequent deletion reactions (Figure 6B). This finding is

consistent with a comprehensive survey of metabolic engineering publications (55) showing that

these deleted reactions appeared in most of *E. coli* engineered strains for production of fuels and

chemicals. The result supports the potential use of modular cell engineering to systematically build

modular platform strains.

ModCell2 designs can capture combinatorial characteristics of production modules. To

evaluate whether ModCell2 could capture the combinatorial properties among production

modules, we analyzed the Pareto front of *wGCP-4-0-d* that have a total of 58 designs. Hierarchical

clustering of this Pareto front revealed certain products with similar objective values across

solutions, such as ethyl esters and butyrate esters (Figure 6A). These products together were

compatible with different modular cells and exhibited metabolic similarity in their production

modules. Thus, ModCell2 could generate designs that capture the combinatorial properties useful

for modular cell engineering.

ModCell2 can identify highly compatible modular cells. Analysis of compatibility shows

that certain modular cells can couple with production modules that may not exhibit the

combinatorial properties (Figure 6D). However, there exists a tradeoff between the number of

feasible designs and degree of compatibility. Some modular cell designs are compatible with up

to 17 out of 20 products, for instance, the most compatible design, *wGCP-4-0-48*, featuring

deletions of ACALD (*adhE*), ACKr/PTAr (*ack*, *pta*), GND (phosphogluconate dehydrogenase,

gnd) and LDH_D (*ldhA*) (Supplementary File S4). An alternative design *wGCP-4-0-48-alternative*

also exists where deletion of G6PDH2r (gluose-6-phosphate dehydrogenase, *zwf*) is replaced by

that of GND, the first step in the oxidative pentose phosphate pathway. The gene deletions in the

design *wGCP-4-0-48-alternative* are a subset of the modular *E. coli* strain TCS095, whose modular

properties have recently been validated experimentally (14).

To determine if modular cell design is a viable alternative to single-product design, we also

analyzed a potential tradeoff between design performance and modularity by comparing the

maximum value of each objective across all solutions in the Pareto front and the single-product

design optima. If production modules exhibit competing phenotypes, a modular cell will not

achieve the same performance in all modules as a single-product design strain. Analysis of the

most compatible design *wGCP-4-0-48-alternative* showed that it could achieve objectives within

4% of the single-product optima in 14 products and within 10% in 3 products (Figure 6E). This

result indicates that it is feasible to identify highly compatible modular cell designs without a

significant tradeoff between performance and modularity.

Analysis of potential tradeoff between robustness and modularity can identify conserved

metabolic features. To evaluate the robustness of modular cells, we analyzed the compatibility

change (*CD*) of *wGCP-4-0* designs with compatibilities of 4 or greater (Figure S3 in

Supplementary File 2). Remarkably, the result shows that only 7.5% of potential reaction deletions

were detrimental to the robustness of modular cells while the large remaining portion did not affect

CD values. Out of the 85 reactions whose deletion affected compatibility, only a few appeared

consistently across the designs. For instance, deletion of TPI (triose-phosphate isomerase, *tpi*) led

to an average compatibility loss of 95%, inactivating most modular cell designs. Based on flux

variability analysis, TPI must operate in the forward direction by converting glycerone phosphate

(dhap) to glyceraldehyde-3-phosphate (g3p) to drive sufficient flux through glycolysis and hence

preventing synthesis of undesired byproducts (D-lactate or 1,2-propanediol) from dhap. Likewise,

deletion of carbon dioxide and water transport and exchange reactions caused compatibility loss

across all designs. Pyruvate carboxylase (PPC) is an important reaction to channel carbon flux

through the Krebs cycle (56), and hence, deletion of PPC reduces compatibility in most modular

cell designs with an average *CL* of 43%.

While some reaction deletions are critical for modular cell robustness, others are associated

with specific products. For example, deletion of PDH (pyruvate dehydrogenase complex,

lpd/aceEF) removes compatibility in all butanol-derived designs, indicating PFL (pyruvate

formate lyase, *pfl*) is not an appropriate route. To make heterologous butanol-derived molecules

under anaerobic conditions, FDH (NADH-dependent formate dehydrogenase, *fdh*) is required in

butanol-derived modules where enzymatic reaction pairs of PFL and FDH could substitute PDH

known to be anaerobically inhibited.

Overall, analysis of tradeoff between modularity and robustness can identify not only the

conserved metabolic features of modular cells but also potential bottlenecks in specific production

modules.

Enabling metabolic switch among different design objectives using ModCell2

The ability to dynamically control growth and production phases can potentially enhance

product titers, rates, and yields. For instance, two-phase fermentation can be employed where

growth phase is optimized for biomass synthesis and stationary phase for chemical production

(57). Using ModCell2, we investigated the feasibility to design optimal strains to toggle switch

desirable production phenotypes.

To design a *wGCP→NPG* metabolic switch, we first used our reference *wGCP* design as

a parent strain (Figure 7A) and then employed ModCell2 to identify the most compatible

wGCP→NPG designs. With 5 additional deletions, we could find *wGCP→*NPG designs that

encompass both *wGCP* and *NPG* phenotypes, for instance, the *sup-NGP-5-0-23* design featuring

deletion of PGI (glucose-6-phosphate isomerase, *pgi*), MDH (malate dehydrogenase, *mdh*), ASPT

(L-aspartase, *aspT*), Tkt2 (transketolase, *tktB*), and ATPS4rpp (ATP synthase, *atp*) (Figure 7B).

The deletion reactions in the *wGCP→*NPG designs appear in both catabolic (PGI, ATPS4tpp) and

anabolic (ASP, TKT2) processes, responsible for growth disruption and direction of carbon flow

to the biosynthesis of target products.

Likewise, we used ModCell2 to design a *wGCP→sGCP* metabolic switch. We identified

the most compatible *wGCP→sGCP* designs with 6 additional deletions, for instance, the *sup-*

sGCP-6-0-39 design featuring the deletion of MGSA (Methylglyoxal synthase, *mgsA*), ALCD2x

(alcohol dehydrogenase, *adhE*), PFL, MDH, FADRx (FAD reductase, *fadI*), and GLUDy

(NADP$^+$dependent glutamate dehydrogenase, *gdhA*) (Figure 7C). Different from the *wGCP→NGP*

metabolic switch, all deletions in the *wGCP→sGCP* designs are involved the elimination of

biosynthesis pathways of undesirable byproducts.

While it is feasible to metabolically switch among different production phenotypes, it not

only requires more reaction deletions but also reduces the product compatibility. For instance, the

*wGCP→*sGCP and *wGCP→*NGP designs are only compatible with 5 products while the *wGCP*

parent design have a compatibility of 17 out of 20 products with 4 deletions. The main reason is

that both *wGCP→*sGCP and *wGCP→*NGP designs must eliminate all possible redundant

pathways that result in biosynthesis of undesirable byproducts.

CONCLUSION

In this study, we developed a multiobjective strain design platform for modular cell

engineering. With a new developed algorithm and computational platform, ModCell2 enables

flexible design of modular cells that can couple with production modules to exhibit desirable

production phenotypes. In comparison to the first-generation strain design platform, ModCell2 can

handle large-scale metabolic networks and identify better solutions that require fewer genetic

modifications and exhibit more product compatibility. Different from the conventional single-

product strain design, ModCell2 can find solutions that are Pareto optimal with negligible tradeoffs

among modularity, performance, and robustness. We envision ModCell2 is a useful tool to

implement modular cell engineering and fundamentally study modular designs in natural and

synthetic biological systems.

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 Figure legends

Figure 1: Comparison between the conventional single-product strain design and modular cell

engineering. In the conventional approach, each target product requires to go through the iterative

optimization cycle. The modular cell engineering approach exploits common phenotypes

associated with high product titers, rates, and yields; and hence, the strain optimization cycle only

needs to be performed once for multiple products, which helps reduce the cost and time of strain

development.

Figure 2: Graphical representation of phenotypic spaces for different strain design objectives

including **(A)** weak growth coupling (*wGCP*), **(B)** strong growth coupling (*sGCP*), and **(C)** no-

growth production (*NPG*). v_{Pk}^{μ} is the minimum product formation rate at the maximum growth

rate for production network k, and $v_{P_{max}k}^{\mu}$ is the maximum product secretion rate attainable. $v_{Pk}^{\bar{\mu}}$

and $v_{P_{max}k}^{\bar{\mu}}$ are the minimum and maximum product formation rates for production network k

during the stationary phase, respectively.

Figure 3: Illustration of ModCell2 workflow and analysis including **(A)** parent model, **(B)**

production modules, **(C)** design parameters, and **(D)** simulation output for Pareto set and Pareto

front based on design input.

Figure 4: The 2-D metabolic phenotypic spaces of different *sGCP* designs using the core

metabolic model. **(A)** Metabolic map, **(B)** *sGCP-5-0-5* design, **(C)** *sGCP-5-0-6* design, **(D)** *sGCP-*

5-0-2 design, **(E)** *sGCP-5-1-8* design, **(F)** *sGCP-6-1-10* design, and **(G)** *sGCP-8-2-9* design. For

each panel, the gray and blue areas correspond to the phenotypic spaces of the wildtype and the

optimal production strain, respectively.

Figure 5: Comparison of strain design by OptKnock and Modcell2. (**A**) A correlation between the

maximum objective values for each product generated by OptKnock and the equivalent values

attained by ModCell2. Each point is colored based on the number of reaction deletions, with

warmer colors corresponding to more reaction deletions. (**B**) A comparison between the Optknock

objective vectors with at most 7 reaction deletions and the representative ModCell2 objective

vector, *wGCP-7-1* which dominates them. Each color circle represents a pair of dominating *wGCP*

design and dominated OptKnock solution (Supplementary File S3). (**C**) Maximum compatibility

of OptKnock designs (blue), *wGCP* designs ($\beta_k = 0$, orange), wGCP designs ($\beta_k = 1$, yellow). (**D**)

Compatibility distribution of Optknock ($\alpha = 7$, orange) and *wGCP-7-1* (blue).

Figure 6: (A) Pareto front of *wGCP-4-0-d*. The columns correspond to different designs labeled

by their design index, *d*, where the rows correspond to different products. (**B**) Frequency of the top

deletion reactions. (**C**) Product compatibility distribution across designs. (**D**) Design compatibility.

(**E**) Tradeoff between modularity and performance. The bars correspond to the maximum objective

values attainable for each product whereas the blue line represent the objective values of the

wGCP-4-0-48alternative design.

Figure 7: Production phenotypes of (**A**) the wild type (gray) and the representative, highly-

compatible design *wGCP-4-0-48-alternative* (blue), (**B**) the *wGCP→NPG* design, *sup-NGP-5-0-*

23, and (**C**) the *wGCP→sGCP* design, *sup-sGCP-6-0-39*.

Figure 1

Features	Conventional strain engineering	Modular cell engineering
Parent strain		
Modular cell	Absent	Optimized common production phenotypes
Exchangeable modules	1	Multiple
Optimal production strains		
Design-build-test cycle	Repeated for every new product	One time for many products

A

B

C

648 **Figure 3**

A — Parent model

B — Production modules

C — Design parameters

Design objective (f_k)
Maximum reaction deletions (α)
Maximum module reactions (β_k)

D

Objective	α	β_k	d	Reaction deletions	Module reactions P1	P2	P3	f_k P1	P2	P3	Prod. yield vs growth P1	P2	P3
wGCP	1	0	1	R10				0.45	0.00	0.00			
wGCP	1	0	2	R9				0.00	0.63	0.97			
wGCP	1	1	1	R9	R9			0.45	0.63	0.97			
sGCP	3	0	1	R8, R9, R11				0.00	0.42	0.97			
sGCP	3	0	2	R6, R9, R10				0.00	0.63	0.00			
sGCP	3	0	3	R6, R8, R11				0.45	0.00	0.00			
sGCP	3	1	1	R6, R9, R11	R9		R6	0.45	0.63	0.97			
NGP	3	0	1	R6, R11				1.00	0.00	0.00			
NGP	3	0	2	R4, R9, R11				0.00	1.00	1.00			
NGP	3	1	1	R4, R9, R11	R9			1.00	1.00	1.00			

Design parameters — *Pareto set* — *Pareto front*

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Figure 4

Figure 5

Figure 6

Figure 7