Quantifying GC-biased gene conversion in great ape genomes	1
using polymorphism-aware models	2
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Abstract

As multi-individual genome-wide population-scale data is becoming available, more-complex 10 modeling strategies are needed to quantify the patterns of nucleotide usage and associated 11 mechanisms of evolution. Recently, the multivariate neutral Moran model was proposed. 12 However, it was shown insufficient to explain the distribution of alleles in great apes. Here, we 13 proposed a new model that includes allelic selection. Our theoretical results constitute the 14 basis of a new Bayesian framework to estimate mutation rates and selection coefficients from 15 population data, which was employed to quantify the patterns of genome-wide GC-biased gene 16 conversion in great apes. Importantly, we showed that great apes have patterns of allelic 17 selection that vary in intensity, a feature that we correlated with the great apes' distinct 18 demographies. We also demonstrate that the AT/GC toggling effect decreases the probability 19 of a substitution, which promotes more polymorphisms in the base composition of great ape 20 genomes. We assessed the impact of CG-bias in molecular analysis and we found that mutation 21 rates and genetic distances are estimated under bias when gBGC is not properly accounted. 22 Our results stress the need for gBGC-aware models in population genetics and phylogenetics. 23 Keywords: Moran model, boundary mutations, allelic selection, great apes, GC-bias, gBGC 24

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1 Introduction

The field of molecular population genetics is currently been revolutionized by progress in data 26 acquisition. New challenges are emerging as new lines of inquiry posed by increasingly large 27 population-scale sequence data (Casillas and Barbadilla, 2017). Mathematical theory 28 describing population dynamics has been dormant since the early days of population genetics 29 (e.g. Fisher (1930); Wright (1931); Moran (1958); Kimura (1964)), but now that data is 30 available to perform statistical inference, many models have been revisited. Recently the 31 multivariate Moran model with boundary mutations was developed and applied to exome-wide 32 allele frequency data from great ape populations. However, drift and mutation are not fully 33 sufficient to explain the observed allele counts (Schrempf and Hobolth, 2017). It was 34 hypothesized that other forces, such as directional selection and GC-biased gene conversion 35 (gBGC), may also play a role in shaping the distribution of alleles in great apes. 36

Directional selection and gBGC have different causes but similar signatures: under directional 37 selection, the advantageous allele increases as a consequence of differences in survival and 38 reproduction among different phenotypes; under gBGC, the GC alleles are systematically 39 preferred. gBGC is a recombination-associated segregation bias that favors GC-alleles (or 40 strong alleles, hereafter) over AT-alleles (or weak alleles, hereafter) during the repair of 41 mismatches that occur within heteroduplex DNA during meiotic recombination (Marais, 42 2003). The process of gBGC was studied in several recent publications (e.g. Webster et al. 43 (2006); Escobar et al. (2011); Pessia et al. (2012); Serres-Giardi et al. (2012); Galtier et al. 44 (2018)), but its impact was mainly studied in mammalian genomes (Duret and Galtier, 2009; 45 Romiguier et al., 2010). Apart from some studies in human populations (Katzman et al., 2011; 46 Glémin et al., 2015), a population-level perspective of the intensity and diversity of patterns of 47 gBGC among closely related populations is still lacking. 48

Several questions remain open regarding the tempo and mode of gBGC evolution. The effect 49 of demography on gBGC is still controversial. While theory and some empirical studies 50 advocate a positive relationship between the effective population size and the intensity of 51 gBGC (Nagylaki, 1983; Glémin et al., 2015), Galtier et al. (2018) failed to detect such 52 relationship. Another aspect that is not completely understood is the impact of GC-bias on 53 the base composition of genomes (Phillips et al., 2004; Romiguier et al., 2013). In particular, 54 the individual and joint effect of gBGC and mutations shaping the substitution process 55 remains elusive. Here, we address these questions by revisiting the great ape data 56 (Prado-Martinez et al., 2013) with a Moran model that accounts for allelic selection, which in 57 principle may be able to capture both, episodes of directional selection and gBGC. 58

The Moran model (Moran, 1958) has a central position describing populations' evolution: it 59 models the dynamics of allele frequency changes in a finite haploid population. Recently, an 60 approximate solution for the multivariate Moran model with boundary mutations (i.e. low 61 mutation rates) was derived (Schrempf and Hobolth, 2017). In particular, the stationary 62 distribution was shown useful to infer population parameters from allele frequency data (De 63 Maio et al., 2015; Schrempf et al., 2016; Schrempf and Hobolth, 2017). Here, we present the 64 Moran model with boundary mutations and allelic selection, derive the stationary distribution, 65 and we build a Bayesian framework to estimate population parameters (base composition, 66 mutation rates, and selection coefficients) from population data. 67

Furthermore, our application to great apes shows that most great apes have patterns of GC-bias consistent with gBGC. Our results suggest further that demography has a major role in determining the intensity of gBGC among great apes, as the intensity of allelic selection among the great ape populations significantly correlates with their effective population size. We also show that not accounting for GC-bias may considerably distort the reconstructed evolutionary process, as mutation and substitution rates are estimated under bias.

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2 Methods

2.1 The multivariate Moran model with allelic selection

We define the multivariate Moran model with boundary mutations and allelic selection following the terminology proposed by Vogl and Bergman (2015) and Schrempf and Hobolth (2017).

Consider a haploid population of N individuals and a single locus with K alleles: i and j are two possible alleles. The evolution of this population in the course of time is described by a continuous-time Markov chain with a discrete character-space defined by N and K, each of which represents a specific assortment of alleles. Two types of states can be defined: if all the individuals in a populations have the same allele, the population is monomorphic $\{i\}$, i.e. the N individuals have the allele i; differently, if two alleles are present in the population, the population is polymorphic $\{ni, (N - n)j\}$, meaning that n individuals have the allele i and (N - n) have the allele j.

The trajectory of alleles is defined based on the rate matrix Q. Time was accelerated by a factor of N, and therefore instead of describing the Moran dynamics in terms of Moran events (Moran, 1958), we developed a continuous version in which the time is measured in coalescent time.

Drift is defined according to the neutral Moran model: the transition rates of the allelic frequency shifts, only depend on the allele frequency and are therefore equal regardless the allele increases or decreases in the population (Durrett, 2008).

$$q^{\{ni,(N-n)j\} \to \{(n+1)i,(N-n-1)j\}} = q^{\{ni,(N-n)j\} \to \{(n-1)i,(N-n+1)j\}} = \frac{n(N-n)}{N}$$
(1)

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We accommodated mutation and selection in the framework of the neutral Moran model by assuming them to be decoupled (Baake and Bialowons, 2008; Etheridge et al., 2010). 95

Mutation is incorporated based on a boundary mutation model, in which mutations only occur 96 in the boundary states. The boundary mutations assumption is met if the mutation rates μ_{ij} 97 are small (and N is not too large); more specifically, Schrempf et al. (2016) established that 98 $N\mu_{ii}$ should be lower than 0.1, by comparing the expectations of the diffusion equation with 99 the polymorphic diversity under the Moran model. In fact, most eukaryotes fulfill this 100 condition (Lynch et al., 2016). Another assumption of our boundary mutation model is that 101 the polymorphic states can only be biallelic. However, this assumption is not a significant 102 constraint as tri-or-more allelic sites are rare for low mutation rates. 103

We employed the strategy used by Burden and Tang (2016) and separated our model into a 104 time-reversible and a flux part. We wrote the mutation rates as the entries of a specific 105 mutation model, the general time-reversible model (GTR) (Tavaré, 1986): $\mu_{ij} = \rho_{ij}\pi_j$, where ρ 106 represents the exchangeabilities between any two alleles and π the allele base composition 107 (rate matrix (2)). Here, we restricted ourselves to the GTR, as this model simplifies obtaining 108 formal results (Burden and Tang, 2016). Because π has K-1 free parameters and ρ includes 109 the exchangeabilities for all the possible pairwise combinations of K alleles, we ended up 110 having K(K+1)/2 - 1 free parameters in the GTR-based boundary mutation model. 111

We modeled allelic selection by defining K - 1 relative selection coefficients σ : an arbitrary selection coefficient is fixed to 0. Defining the fitness as the probability that an offspring of allele *i* is replaced with probability $1 + \sigma_i$ (Durrett, 2008), we can formulate the component of allelic selection alongside with drift, and thus among the polymorphic states (rate matrix (2)).

Altogether, the instantaneous rate matrix Q of the multivariate Moran model with boundary mutations and allelic selection can be defined as: 117

$$q^{\{ui,(N-u)j\} \to \{vi,(N-v)j\}} = \begin{cases} \mu_{ij} = \rho_{ij}\pi_j & u = N, \ v = N-1 \\ \mu_{ji} = \rho_{ij}\pi_i & u = 0, \ v = 1 \\ \frac{n}{N}(N-n)(1+\sigma_i) & u = n, \ v = n+1, \ 0 < n < N \\ \frac{n}{N}(N-n)(1+\sigma_j) & u = n, \ v = n-1, \ 0 < n < N \\ 0 & |u-v| > 1 \end{cases}$$
(2)

where u and v represent a frequency change in the allele counts (though N remains constant). ¹¹⁸ The diagonal elements are defined by the mathematical requirement such that the respective ¹¹⁹ row sum is 0. ¹²⁰

As the parameters of the population size, mutation rate and selection coefficients are confined, ¹²¹ it is possible to scale down them to a value small value N while keeping the overall dynamics ¹²² unchanged. The virtual population size N becomes a parameter describing the number of bins ¹²³ the allele frequencies can fall into. As a result, we can think of N either as a population size or ¹²⁴ a discretization scheme. ¹²⁵

2.2 The stationary distribution

The stationary distribution of a Markov process can be obtained by computing the vector p 127 satisfying the condition $\phi Q = 0$ (File S1). ϕ is the normalized stationary vector and has the 128 solution: 129

$$\phi_x = \begin{cases} \pi_i (1+\sigma_i)^{N-1} k^{-1} & \text{if } x = \{i\} \\ \pi_i \pi_j \rho_{ij} (1+\sigma_i)^{n-1} (1+\sigma_j)^{N-n-1} \frac{N}{n(N-n)} k^{-1} & \text{if } x = \{ni, (N-n)j\} \end{cases}$$
(3)

k is the normalization constant

$$k = \sum_{i \in \mathcal{A}} \pi_i (1 + \sigma_i)^{N-1} + \sum_{ij \in \mathcal{A}^C} \sum_{n=1}^{N-1} \pi_i \pi_j \rho_{ij} (1 + \sigma_i)^{n-1} (1 + \sigma_j)^{N-n-1} \frac{N}{n(N-n)},$$
(4)

where \mathcal{A} is the alphabet of the K alleles $\{a_1, \ldots, a_K\}$, representing the monomorphic states, and \mathcal{A}^C all the possible pairwise combinations of \mathcal{A} representing the K(K-1)/2 types of polymorphic states $a_1a_2, a_1a_3, \ldots, a_{K-1}a_K$.

2.3 Expected number of Moran events

From Q and ϕ , we can compute the expected number of Moran events (mutations, drift and selection). These are the expected state-changes per unit of time for the multivariate Moran model with selection (File S2)

$$d_S(t=1) = d_S = \frac{2}{k} \sum_{ij \in \mathcal{A}^C} \sum_{n=1}^N \pi_i \rho_{ij} \pi_j (1+\sigma_i)^{n-1} (1+\sigma_j)^{N-n}.$$
 (5)

The quantity (5) can also be interpreted as the overall rate of the model. The expected 138 number of Moran events for the neutral model can be easily calculated by letting $\sigma \to 0$. To 139 compare the Moran distance d_S with the standard models of evolution, we recalculated the 140 Moran distance to only account for substitutions events d_S^* : we corrected d_S by the probability 141 of a mutation and a subsequent fixation under the Moran model (File S3) 142

$$d_{S}^{*} = \frac{2}{k} \sum_{ij \in \mathcal{A}^{C}} \frac{\pi_{i} \pi_{j} \rho_{ij} (1 + \sigma_{i})^{N} (1 + \sigma_{j})^{N}}{\sum_{n=1}^{N} (1 + \sigma_{j})^{n} (1 + \sigma_{i})^{N-n+1}}.$$
(6)

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2.4 Bayesian inference with the stationary distribution

We can define a likelihood function for the stationary distribution for a set of S independent sites in N individuals by taking the product of p over the number of monomorphic and polymorphic sites: $\#\{i\}$ and $\#\{ni, (N-n)j\}$, respectively 146

$$p(\boldsymbol{x}|\boldsymbol{\pi}, \boldsymbol{\rho}, \boldsymbol{\sigma}) = \prod_{s=1}^{S} p(x_s) = k^{-S} \prod_{i \in \mathcal{A}} \left[\pi_i (1+\sigma_i)^{N-1} \right]^{\#\{i\}} \times \prod_{ij \in \mathcal{A}^C} \prod_{n=1}^{N-1} \left[\pi_i \pi_j \rho_{ij} (1+\sigma_i)^{n-1} (1+\sigma_j)^{N-n-1} \frac{N}{n(N-n)} \right]^{\#\{ni,(N-n)j\}}$$
(7)

We employed a Bayesian approach: we define the prior distributions independently, a Dirichlet 147 prior for π and an exponential prior for ρ and σ ; a Dirichlet and multiplier proposals were set 148 for the aforementioned parameters with tuning parameters guaranteeing a target acceptance 149 rate of 0.234. We employed the Metropolis-Hastings algorithm (Hastings, 1970) for each 150 conditional posterior in a Markov chain Monte Carlo sequence to obtain random samples from 151 the posterior. The algorithm was coded in the R statistical programing language (R Core 152 Team, 2015): the packages MCMCpack and expm were integrated in our code to obtain samples 153 from the Dirichlet density and to compute the matrix exponential, respectively (Martin et al., 154 2011; Goulet et al., 2017). The R script can be assessed in the GitHub branch 155 pomo-dev/pomo_selection. 156

2.5 Polymorphism-aware phylogenetic model

The multivariate Moran model can be also referred as a polymorphism-aware phylogenetic ¹⁵⁸ model (PoMo) if we set k = 4 alleles (De Maio et al., 2013, 2015; Schrempf et al., 2016), those ¹⁵⁹ representing the 4 nucleotide bases. We write \mathcal{A} as the alphabet of the 4 nucleotide bases ¹⁶⁰ $\{A, C, G, T\}$ and \mathcal{A}^C as all the possible pairwise combinations of the four nucleotide bases ¹⁶¹ $\{AC, AG, AT, CG, CT, GT\}$. For a population of size N we have 4 + 6(N - 1) possible states, ¹⁶² four of which are monomorphic (Figure 1). Applications and results presented in the following ¹⁶³ pages were obtained using the 4-variate model. ¹⁶⁴

2.6 Application: great ape population data

The stationary distribution of 4-multivariate model was employed to infer the distribution of 166 allele frequencies, selection coefficients and mutation rates from 4-fold degenerate sites of 167 exome-wide population data from great apes (Prado-Martinez et al., 2013). We used 11 168 populations with up to 23 individuals, totaling ~ 2.8 million sites per population (Table 1). 169

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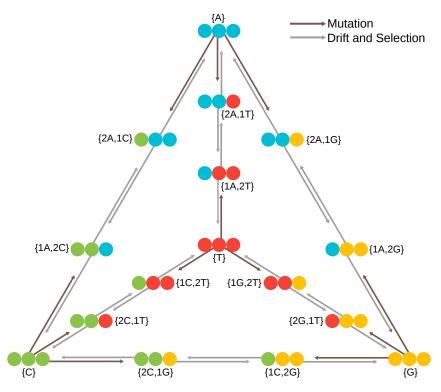


Figure 1: **PoMo state-space using** N = 3. The 4 alleles represent the four nucleotide bases. Brow and grey arrows indicate mutations, and genetic drift and selection, respectively. Monomorphic or boundary states $\{i\}$ are represented in the tetrahedron's vertices, while the polymorphic states $\{ni, (N - n)j\}$ are represented in its edges. Monomorphic states interact with polymorphic states via mutation, but a polymorphic can only reach a monomorphic state via drift or selection. Between polymorphic states only drift and selection events occur.

Data preparation follows the pipeline described in De Maio et al. (2015). The allelic counts of 170 all 11 primate subspecies are available in the GitHub branch pomo-dev/pomo_selection. 171 Estimates of the Watterson's θ genetic diversity is below 0.003 for all the studied populations 172 (Schrempf et al., 2016), validating the boundary mutations assumption of 0.1. 173

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3 Results

3.1 Simulations and algorithm validation

To validate the analytical solution for the stationary distribution of the multivariate Moran 176 model, we compare it to the numerical solution obtained by calculating the probability matrix 177 of Qt for large enough t. We confirmed that the numerical solution converges to the analytical 178 solution (Figure S1). 179

We validated the Bayesian algorithm for estimating population parameters from the stationary ¹⁸⁰ distribution by performing simulations (Table S1 and Figures S2-S5). Our algorithms ¹⁸¹ efficiently recover the true population parameters from simulated allele counts. We tested the ¹⁸² algorithms for different number of sites $(10^3, 10^6 \text{ and } 10^9)$ and state-spaces $(N = 5, 10 \text{ and} 10^8)$

50). While the number of sites does not increase the computation time substantially and is not 184 being a limiting factor for genome-wide analysis, the size of the state-space influences the 185 computational time. For larger state-spaces N, more iterations are needed to obtain 186 converged, independent and mixed MCMC chains during the posterior estimation. 187

3.2 Patterns of allelic selection in great apes

To test the role of allelic selection defining the distribution of alleles in the great apes, we 189 compared the neutral multivariate Moran model (M_M) and the model with allelic selection 190 (M_S). Using the predictive stationary distribution and the observed allele counts, we computed 191 the Bayes' factors favoring the more complex model M_S (i.e. log BF > 0 favors the model with 192 allelic selection) for all populations. It is clear that M_S fits the data considerably better for 193 most of the studied great apes (log BF > 100, Table 1). The only exception is the Eastern 194 gorillas population, for each a lower log BF was obtained (log BF = 5.497, Table 1).

Table 1: Evidence of allelic selection among the great ape populations. The number of individuals and the number 4-fold degenerate sites per population are indicated by I and S, respectively. The log Bayes' factors (log BF) were calculated as the sum over the product of the allele counts and the posterior predictive probabilities under the Moran model with boundary mutations (M_M) and allelic selection (M_S). BF favor the model with allelic selection when higher than 1.

Population	Ι	S	$\log p(\boldsymbol{x} \mathbf{M}_{\mathrm{M}})$	$\log p(\boldsymbol{x} \mathbf{M}_{\mathrm{S}})$	log BF
African humans	6	2827135	-3941390.98	-3940993.95	397
Non-African humans	12	2826956	-3940071.64	-3939858.12	213
Eastern gorillas	6	2823830	-3917375.00	-3917370.00	5
Western gorillas	54	2813092	-3955462.98	-3954663.09	799
Western chimpanzees	10	2823911	-3935188.83	-3934928.50	260
Nigeria-Cameroon chimpanzees	20	2825739	-3980386.43	-3979429.05	957
Eastern chimpanzees	12	2822976	-3961202.57	-3960561.15	641
Central chimpanzees	8	2822685	-3958674.29	-3957704.55	969
Bonobos	26	2824240	-3948520.55	-3947835.54	685
Bornean orangutans	10	2824768	-3952527.89	-3952358.67	169
Sumatran orangutans	10	2824618	-3973247.40	-3972725.44	521

We have also corroborated our Bayes' factors by inspecting the fit of the predictive distribution of alleles of M_M and M_S with the allele counts (Figure S6A-K). The allele counts for the polymorphic states are not symmetric, generally one allele if preferred and so are the polymorphic states that have it in higher proportions. As expected, we observed that M_S better reproduces the skewed distribution of allele counts among great apes.

We further explored the parameter estimates under M_S to know how strong and variable are the patterns of allelic selection among great apes. We analyzed the posterior distribution of the relative selection coefficients of C, G, and T (σ_A was set to 0). A general pattern of allelic 201 202

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selection is observed in great apes. The selection coefficients of C and G are similar (meaning 204 that their posterior distributions largely overlap), but different from the selection coefficient of 205 T, which in turn overlaps 0 (approximately equal to the selection coefficient of A) (Figure 2). 206 The only exception is the Eastern gorillas, for which the selection coefficients are all only 207 slightly higher than 0 and rather similar (Figure 2). This result corroborates the relatively low 208 Bayes' factor found for evidence of allelic selection in the Eastern gorilla population. 209

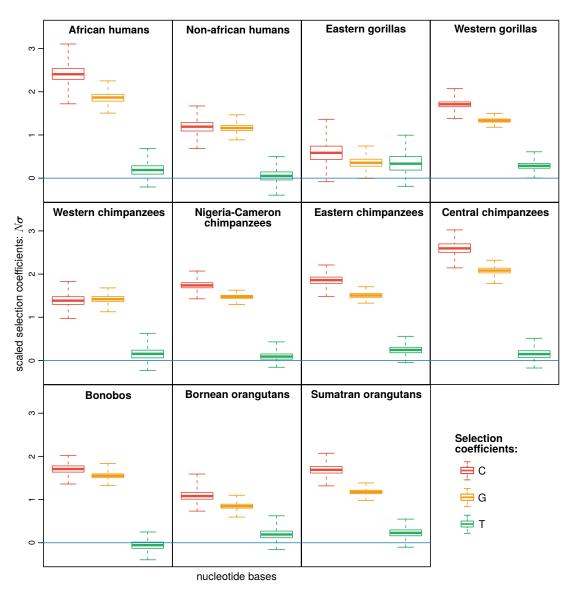


Figure 2: Scaled allelic selection coefficients for the great apes 4-fold degenerate synonymous sites. The boxplots represent the posterior distribution of the C, G and T scaled selection coefficients (σ_A was set to 0); the estimates were obtained using the 4-variate Moran model. The line in blue represents $\sigma_A = 0$. Table S2 summarizes the average scaled selection coefficients for each great ape population.

We further explored this result in order to check if the patterns of GC-bias found among great ²¹⁰ apes can be associated with gBGC. We correlate the GC-bias per chromosome ($\sigma_C + \sigma_G$) with ²¹¹ the chromosome size and recombination rate in the non-African human population (Figure ²¹² S7), for which this data is particularly well characterized (Jensen-Seaman, 2004). We found a ²¹³

significant positive correlation between the GC-bias and recombination rate (Spearman's $\rho = 214$ 0.57, *p*-value = 0.006), but a negative correlation with the chromosome length (Spearman's $\rho = -0.52$, *p*-value = 0.014).

Although the patterns of selection among great apes are similar qualitatively, they differ quantitatively. For example, the Central chimpanzees have patterns of GC-bias around 2.08/2.60 (σ_C/σ_G , Table S2 and Figure 2), while the closely related population of Western chimpanzees shows less strong patterns (around 1.38/1.42). Likewise, the GC-bias content in African and non-African human populations contrasts: 2.41/1.86 and 1.19/1.16, respectively. These results show that the patterns of allelic selection greatly vary among great apes, even among closely related populations. 218

It has been hypothesized that GC-bias is a compensation mechanism for the mutational bias 224 that exists in favor of the weak alleles, A and T (Duret and Galtier, 2009; Philippe et al., 225 2011): the AT/GC toggling effect. Congruently with this expectations, we observed that 226 mutation rates from strong to weak alleles are higher (by a factor of 3.05 in average), but 227 rather similar between alleles of the same type (around 1.02 in average; supplementary table 228 S2), while the selection coefficients, as shown, have a clear pattern of GC-bias in most of the 229 great ape populations. 230

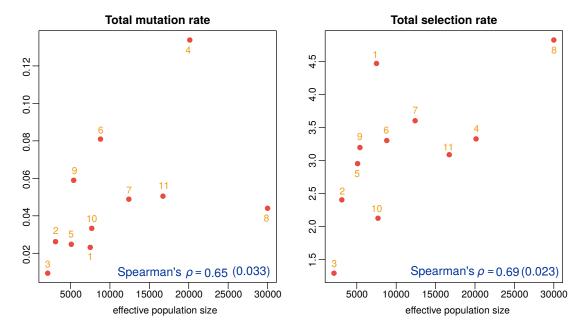


Figure 3: Correlating N_e and the total rate of mutation and selection in great apes. Great ape populations are numbered: 1. African human, 2. Non-African human, 3. Eastern gorilla, 4. Western gorilla, 5. Western chimpanzee, 6. Nigeria-Cameroon chimpanzee, 7. Eastern chimpanzee, 8. Central chimpanzee, 9. Bonobo, 10. Bornean orangutan and 11. Sumatran orangutan. Estimates of N_e were taken from Prado-Martinez et al. (2013) and Tenesa et al. (2007).

3.3 N_e and the total rate of mutation and selection in great apes

It is widely known that the intensity of mutation and selection reflect population demography. ²³² To check whether the estimated mutation and selection coefficients among great ape ²³³ populations may be explained by demography, we tested the correlation between the total rate ²³⁴ of mutation and selection and N_e (obtained from Tenesa et al. (2007); Prado-Martinez et al. ²³⁵ (2013)). Positive and significant correlations between the total mutation and selection rates ²³⁶ and the effective population size were obtained (Figure 3): Spearman's correlation coefficient ²³⁷ of 0.65 (*p*-value = 0.033) and 0.69 (*p*-value = 0.023), respectively. ²³⁸

This result shows that N_e plays an important role in determining the intensity of mutations 239 and selection. In particular, it becomes clear that the different patterns of GC-bias found 240 among great apes are, in part, due to different demographies. For example, Central 241 chimpanzees have the highest GC-bias among the studied great apes, and they are indeed the 242 population that was estimated with the largest N_e (30 000, Prado-Martinez et al. (2013)). 243 Eastern gorillas showed the opposite pattern: this population had no evidence of GC-bias 244 (with very homogeneous selection coefficients) and congruently Prado-Martinez et al. (2013) 245 estimated its N_e as only 2000, the lowest of the studied populations. 246

Table 2: Expected number of substitutions per unit of time. The expected number of substitutions for the multivariate Moran model with boundary mutations d_M^* and allelic selection d_S^* were calculated based on the posterior distributions of the model parameters and equation (6). The relative difference between these distances was calculated as the ratio between the average number of events between the two models (d_S^*/d_M^*) and was used to assess how dissimilar these distances are.

Population	$d_M^* \times 10^3$	$d_S^* \times 10^3$	d_{S}^{*}/d_{M}^{*}
African humans	0.123	0.120	0.978
Non-African humans	0.041	0.039	0.954
Eastern gorillas	0.061	0.064	1.045
Western gorillas	0.011	0.009	0.845
Western chimpanzees	0.054	0.052	0.956
Nigeria-Cameroon chimpanzees	0.045	0.038	0.858
Eastern chimpanzees	0.073	0.066	0.910
Central chimpanzees	0.130	0.114	0.873
Bonobos	0.019	0.016	0.821
Bornean orangutans	0.077	0.077	0.998
Sumatran orangutans	0.111	0.106	0.959

3.4 Comparing the expected number of substitutions in great apes

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We calculated the expected number of substitutions under M_M and M_S to evaluate the impact ²⁴⁸ of allelic selection (in particular, GC-bias) in the evolutionary process. With formula (6), we ²⁴⁹ calculated d_M^* and d_S^* using the posterior estimates of the respective model parameters. We ²⁵⁰

observe that for most of the great ape populations, the expected number of substitutions is 251 lower when allelic selection is accounted (Table 2). Eastern gorillas are an exception, and the 252 opposite pattern was observed. We also calculated the relative difference between the expected 253 number of substitutions in both models (i.e. d_S/d_M) and we obtained minor (-0.26% in 254 Bornean orangutans) to major (-17.8% in bonobos) relative differences; the average difference 255 is -7.3% (Table 2). These results suggest that not accounting for GC-bias may distort the 256 reconstructed evolutionary process by overestimating the expected number of substitutions. 257

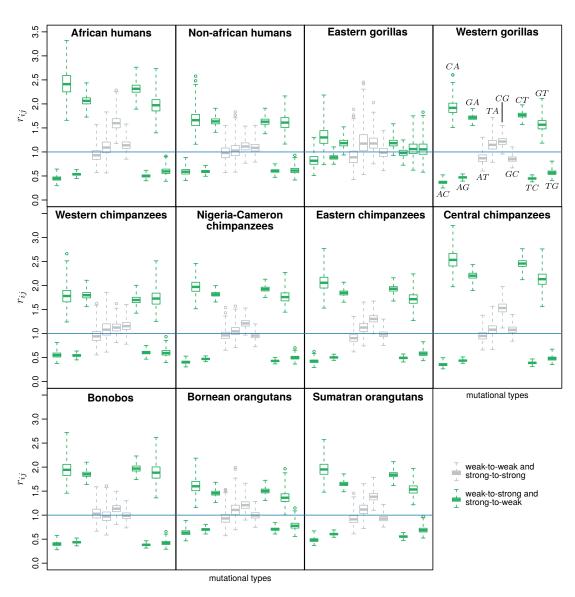


Figure 4: Relative difference in the mutation rates estimated under the neutral and nonneutral Moran model. r_{ij} represents the ratio between the mutation from allele *i* to allele *j* in the model with allelic selection and the model with boundary mutations: $r_{ij} = \mu_{ij}^S / \mu_{ij}^M$. The 12 mutational types are indicated in the western gorillas plot: all the plots follow this arrangement.

We complement this result by comparing the posterior distribution of the mutations rates in M_M and M_S . Because we wanted to identify the mutational types that may be differently estimated between these models, we calculated the relative difference between the mutation 260

rate from allele *i* to allele *j* under the models, respectively: $r_{ij} = \mu_{ij}^S / \mu_{ij}^M$. If $r_{ij} > 1$ for a certain mutation rate *ij*, then this mutation rate is being underestimated in M_M when compared to M_S (and *vice versa* if $r_{ij} < 1$); if $r_{ij} \approx 1$ the mutation rates are equally estimated in both models.

We observed a systematic bias among great apes. While weak-to-weak and strong-to-strong 265 mutation rates are generally non-differentially estimated in both models (most of their r266 overlap 1, Figure 4) the strong-to-weak and weak-to-strong mutation rates are generally biased 267 in M_M. In particular, we obtained that weak-to-strong mutation rates are augmented, while 268 mutations rates from strong-to-weak alleles are deprecated (Figure 4), which suggests that not 269 accounting for GC-bias may bias the estimation of population parameters. Eastern gorillas 270 behave differently by not showing significant differences between the estimated mutations rates 271 (all r_{ij} overlap 1, Figure 4). 272

4 Discussion

In this work, we built on the multivariate Moran model with boundary mutations and allelic 274 selection to explain the population processes shaping the observed distribution of alleles. We 275 obtained new formulae to characterize this model: in particular, we derived the stationary 276 distribution and the rate of the process. In addition, we built a Bayesian framework to 277 estimate population parameters (base composition, mutation rates, and selection coefficients) 278 from population data. This work accomplishes tasks set by Schrempf and Hobolth (2017) who 279 observed derivations from neutrality without having a model in place to enlighten the causes. 280

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4.1 Variable patterns gBGC among great apes

A genome-wide application in the great apes provides important insight into the strength and magnitude of GC-bias patterns and also the impact of gBGC in the evolutionary process. To our knowledge, this is the first work giving a population perspective of the patterns of GC-bias in non-human populations.

Here, we focus on GC-bias because it is a genome-wide effect. Mathematically speaking, it is difficult to disentangle gBGC from directional selection: they may have different biological explanations, but represent the exact same process modeling-wise (i.e. one allele is preferred over the others). Therefore, existing signatures of directional selection are most likely canceling out, when several site-histories (around 2.8 million sites in our case) are summarized to perform inferences.

The patterns of GC-bias we have found in great apes are in concordance with the well-known 292 process of gBGC. As expected, we observed that the larger the recombination rate or the lower 293 the chromosome length, the higher the GC-effect. Evidently, recombination promotes gBGC; 204 however, a negative association between gBGC and chromosome size is expected (because at 295 least one crossover per chromosome is necessary for proper segregation during meiosis, the 296 crossover rate (in cM per Mb) will be higher for small than for large chromosomes (Farré 297 et al., 2013)). We have performed these analyses in non-African Humans, for which this data is 298 available; however, we are confident that the patterns of GC-bias found in great apes are due 299 to gBGC. 300

In agreement with previous studies in mammals and humans (Spencer et al., 2006; Lartillot, 301 2013; Capra et al., 2013; Lachance and Tishkoff, 2014; Glémin et al., 2015), we found that 302 gBGC is weak on average. Indeed, among great apes, the effect of GC-bias ranges between 303 1.49 ± 0.53 , consistent with the nearly-neutral scenario (Ohta and Gillespie, 1996; Vogl and 304 Bergman, 2015). These estimates are in congruence with other estimates of the scaled 305 conversion coefficient in coding regions: Lynch (2010) estimated $4N_es$ as 0.82 in humans and 306 Lartillot (2013) adopted a phylogenetic approach to predict scaled conversion coefficients lower 307 than 1 in all apes. Notice that the latter works employed the Wright-Fisher model. As the rate 308 of genetic drift is twice as fast in the Moran model, we expect to estimate twice as hight 309 selection coefficients with our approach. 310

It has been hypothesized that GC-bias is a compensation mechanism for the mutational bias 311 that exists in favor of the weak alleles, A and T (Galtier et al., 2009; Duret and Galtier, 2009; 312 Philippe et al., 2011). Congruently with this expectations, we observed that mutation rates 313 from strong to weak alleles are higher but rather similar between alleles of the same type. 314 Interestingly, this symmetric manner by which mutations and selection are acting in great apes 315 leads, as we have demonstrated, the number of substitutions to decrease in average, which 316 suggests that the AT/GC toggling may actually increase the population variability by 317 promoting more polymorphic sites. 318

Glémin et al. (2015) hypothesized that differences in GC-bias intensity among human ³¹⁹ populations were due to effects of demography. We also advance that demography regulates ³²⁰ the intensity of gBGC in great apes. We obtained a positive correlation between the total rate ³²¹ of selection and N_e in great apes. An important conclusion of our study is that the patterns of ³²² gBGC can rapidly change due to demography, even among closely related populations. In fact, ³²³ most of the studied populations are known to have diverged less than 0.5 million years ago ³²⁴ (Prado-Martinez et al., 2013). ³²⁵ Here, we showed that GC-bias determines the genome-wide base composition of genomes in a 326 factor proportional to $(1 + \sigma_{C/G})^{N-1}$ (or $(1+s)^{N_e-1}$ in the true dynamic). Therefore, by 327 either changing N_e or s, we are able to change the AT/GC composition of genomes. Because 328 we were able to correlate N_e with the intensity of allelic selection, we are convinced that 329 demography has a major role determining the base composition of great apes genomes. 330 Intriguingly, Galtier et al. (2018) have not found this correlation at the species level in 331 animals. This is most likely happening because genome-wide recombination rate, length of 332 gene conversion tracts and repair biases should significantly vary across species, but not so 333 much across related populations, which explains why the correlation between the intensity of 334 gBGC and N_e was found in great app populations, but not more generally in animals. 335

While correlating the strength of selection with N_e , we obtained a correlation coefficient (0.69) 336 that suggests that other processes may be determining the strength of allelic selection: we can 337 refer two likely reasons. First, the effect of recent demographic effects. We have considered a 338 fixed population size and stationarity, which are good assumptions to recover long-standing 339 population processes, but may not capture the more-recent demographic events and therefore, 340 their impact on GC-bias. Second, variations in s due to species-specific recombination 341 landscapes may also contribute to different GC-bias. Indeed, variation in the karvotypes 342 (number and length of chromosomes) and the short-life and self-destructive nature of 343 recombination hotspots are known to contribute to generating different patterns of GC-bias 344 among species (Duret and Galtier, 2009; Lesecque et al., 2014). For the particular case of great 345 apes, changes in the karyotype should not be a major aspect, as it is very conserved among 346 primates: humans have 46 diploid chromosomes whereas the other great apes (orangutans, 347 gorillas, and chimps) have 48. However, it is known that humans and chimpanzees, and even 348 human populations share few recombination hotspots (Auton et al., 2012; Lesecque et al., 349 2014), which may explicate differences in the great apes' recombination landscapes and, 350 ultimately, why the intensity of allelic selection cannot be completely explained by the effects 351 of demography. 352

Knowing to what extent variations in N_e or s determine the base composition of genomes will require further studies. In particular, determining s experimentally, ideally in different populations, would help to assess the real impact of gBGC and how variable it is among species/populations. If as for the mutation rate, we could assume that s vary slightly among closely related populations/species, then we might attribute different intensities of GC-bias almost solely to demographic effects, which simplifies the task of accommodating gBGC in population models.

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4.2 gBGC calls for caution in molecular and phylogenetic analyses

The effects of gBGC in the molecular analysis have been extensively described in the literature 361 (reviewed in Romiguier and Roux (2017)), we complement these results by showing how 362 GC-bias affects the base composition of genomes, and how the mutation rates and genetic 363 distances may be biased if GC-bias is not properly accounted. In particular, we observed that 364 mutations rates from weak-to-strong and strong-to-weak alleles are systematically over and 365 underestimated, respectively. Biased estimators are not necessarily worthless (particularly 366 when the bias is known) for parameter estimation. Being able to describe the distribution of 367 alleles with fewer parameters is in principle a good aspect modeling-wise. However, we have 368 strong evidence that the model with allelic selection much better fits the observed allele counts 369 for all the studied great ape populations. 370

The idea that gBGC may distort the reconstructed evolutionary process comes mainly from 371 phylogenetic studies. For example, it is hypothesized that gBGC may promote substitution 372 saturation (Romiguier and Roux, 2017). We have shown that the number of substitutions may 373 be significantly overestimated if we do not account for GC-bias, meaning that gBGC may 374 indeed promote branch saturation. Based on this and other gBGC-related complications (e.g. 375 GC-bias promotes incomplete lineage sorting (Hobolth et al., 2011)), some authors advocate 376 that only GC-poor markers should be used for phylogenetic analysis (McCormack et al., 2012; 377 Romiguier et al., 2013). Contradicting this approach, our results show that we may gain more 378 inferential power if GC-bias is accounted for when estimating evolutionary distances. 379

Recently, a nucleotide substitution process that accounts for gBGC was proposed by Lartillot (2013). In this model, the scaled conversion coefficient is used to correct the substitution rates main a similar fashion as we have done to calculate the expected number of substitutions for the Moran distance (i.e. assessing the relative fixation probabilities under GC-bias, File S3). Therefore, we expect to obtain similar results with this nucleotide substitution model and our model: the only differences being that our model accounts for polymorphic sites and is based on the Moran model (while in Lartillot (2013) populations follow the Wright-Fisher model).

5 Conclusion

Despite the widespread evidence of gBGC in several taxa, several questions remain open regarding the role of gBGC determining the base composition of genomes. In this work, we quantify the patterns of gBGC in great apes while contributing to the discussion of the tempo and mode of gBGC evolution in vertebrate genomes.

Our Moran model adds a significant contribution to the endeavor of estimating population ³⁹² parameters from multi-individual, genome-wide population-scale data. Our model was used to ³⁹³ estimate genome-wide signature of gBGC, but it can also be more generally employed to ³⁹⁴ estimate patterns of nucleotide usage and associated mechanisms of evolution. Importantly, ³⁹⁵ our analysis showed that gBGC may significantly distort estimates of population parameters ³⁹⁶ and genetic distances, stressing that gBGC-aware models should be used when employing ³⁹⁷ molecular phylogenetics and population genetics analyses. ³⁹⁸

Here, we have not performed phylogenetic inference, but previous applications of the Moran³⁹⁹ model to phylogenetic problems (De Maio et al., 2015; Schrempf et al., 2016) show that it can⁴⁰⁰ be done. Therefore, a necessary future work would be testing the effect of gBGC in phylogeny⁴⁰¹ reconstruction, in particular, determining how much of its signal can be accounted for⁴⁰² increasing the accuracy of tree estimation both on the topology and branch lengths.⁴⁰³

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