Estimating parameters from multiple time series of population dynamics using Bayesian inference

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Abstract

1. Empirical time series of interacting entities, e.g. species abundances, are highly useful to study ecological mechanisms. Mathematical models are valuable tools to further elucidate those mechanisms and underlying processes. However, obtaining an agreement between model predictions and experimental observations remains a demanding task. As models always abstract from reality one parameter often summarizes several properties. Parameter measurements are performed in additional experiments independent of the ones delivering the time series. Transferring these parameter values to different settings may result in incorrect parametrizations. On top of that, the properties of organisms and thus the respective parameter values may vary considerably. These issues limit the use of a priori model parametrizations.

2. In this study, we present a method suited for a direct estimation of model parameters and their variability from experimental time series data. We combine numerical simulations of a dynamical population model with Bayesian inference, using a hierarchical framework that allows for variability of individual parameters. The method is applied to a comprehensive set of time series from a laboratory predator-prey system that features both steady states and cyclic population dynamics.

3. Our model predictions are able to reproduce both steady states and cyclic dynamics of the data. Additionally to the direct estimates of the parameter values, the Bayesian approach also provides their uncertainties. We found that fitting cyclic population dynamics, which contain more information on the process rates than steady states, yields more precise parameter estimates. We detected significant variability among parameters of different time series and identified the variation in the maximum growth rate of the prey as a source for the transition from steady

states to cyclic dynamics.

4. By lending more flexibility to the model, our approach facilitates parametrizations and shows more easily which patterns in time series can be explained also by simple models. Applying Bayesian inference and dynamical population models in conjunction may help to quantify the profound variability in organismal properties in nature.

Keywords

Bayesian inference, chemostat experiments, hierarchical model, ordinary differential equation, parameter estimation, population dynamics, predator prey, time series analysis, trait variability, Stan

Introduction

Trophic interactions provide the elementary link in all food webs. Long-term data sets of such interactions become increasingly available, both from field observations, and highly controlled large-scale and laboratory experiments (G. F. Fussmann et al. 2000; Tirok and Gaedke 2007; Becks et al. 2010; Magurran et al. 2010; Weigelt et al. 2010; Becks et al. 2012; Boit and Gaedke 2014). To mechanistically understand these trophic interactions, models are employed with the goal of reproducing the observed population dynamics, which can either settle to a steady state, or include more complex patterns like limit cycles or chaos (G. F. Fussmann et al. 2000; Boit et al. 2012; Becks and Arndt 2013; Barraquand et al. 2017). Obtaining an agreement of such non-static experimental observations and modelled time series remains a demanding task. Often, modellers are faced with situations where they can reproduce basic features of the dynamics, but not on the right time scales or not on biomass levels close to the data. One logical reaction would be to refine the model structure and include a higher level of biological detail. However,

another reason for such disagreements between model and data may be a generally valid model structure but an incorrect parametrization. In this study, we present a method to obtain the relevant model parameters directly from the experimental data by applying Bayesian inference to a comprehensive set of time series data for a laboratory predator-prey system that features both steady states and cyclic population dynamics.

Incorrect parametrizations can arise for different reasons. Firstly, a model always has to abstract from reality. This implies that individual model parameters summarize a multitude of different ecological processes. The impact of these processes on the model parameter likely changes over time. Due to these non-modelled processes a one-to-one relationship between empirically determined parameter values and model parameters is impossible. For example, typical predator-prey models consider a conversion efficiency by which prey biomass is converted into predator growth. Among others this efficiency depends on the variable prey abundance (S. Schälicke *in prep.*) and the relative importance of basal and activity respiration (Kath et al. 2018). Secondly, model parameters are often obtained experimentally under slightly different settings than the actually observed population dynamics. An example would be to measure the prey's population growth rate in batch experiments and then use this parameter in a chemostat model. By design, growth conditions in batch and chemostat cultures differ in some aspects, such as the dynamics of nutrient limitation or the selection pressure, e.g. for higher nutrient affinities versus maximum growth rates. Therefore, the parameters that were obtained for one experimental setting might be of limited value for a different one.

It is more and more recognized that the functional traits of organisms that determine trophic interactions comprise a considerable variability (Litchman and Klausmeier 2008; Bolnick et al. 2011; Violle et al. 2012; Bolius et al. 2017; Gaedke and Klauschies 2017). This trait variability can have far-reaching consequences both at the population level (Abrams 1999; Post and Palkovacs 2009; Becks et al. 2010; Ehrlich et al. 2017; Raatz et al. 2017; Cortez 2018) and at the community level (McGill et al. 2006; Hillebrand

and Matthiessen 2009). In such cases, employing just one parametrization for different time series of the same system may be insufficient and hamper the agreement of experimental and modelled population dynamics. Instead, the model will potentially support parts of the data or comprise certain general features, but will fail to reproduce its entire behaviour. Our Bayesian framework allows to retrieve information on such parameter-related uncertainties directly from the time series data, which might even become apparent as between-replicate differences, and provide individual parameter estimates for each data set.

Dynamical population models have traditionally been fitted to time series data by least squares or maximum likelihood methods (Bolker 2008; Cao et al. 2008; Aster et al. 2012), see e.g. DeLong et al. (2014), Rall and Latz (2016), and Curtsdotter et al. (2018) for recent applications. While they offer point estimates for unknown parameters, confidence intervals rely on a local approximation and normality assumptions. Also, their statistical flexibility is limited.

Bayesian methods, on the other hand, are highly flexible. They do not depend on normality assumptions and allow, e.g., utilizing prior information, defining hierachical levels among parameters, and recovering unobserved system states (Kindsvater et al. 2018). Using Markov chain Monte Carlo (MCMC) sampling, they globally explore the parameters' posterior probability distribution, thus quantify uncertainty exactly, and provide direct inference on sought parameters and derived quantities.

For discrete-time population dynamics, Bayesian methods have received growing attention over the last years (Almaraz and Oro 2011; Elderd and Miller 2015; Wittwer et al. 2015; Compagnoni et al. 2016; Robinson et al. 2017). In a discrete setup, state-space models (SSM) are feasible and allow, e.g., the separation of process and observation error (Hefley et al. 2013), recovering latent states (Hosack et al. 2012), incorporating age-structure (Taboadai and Anadón 2016), adding environmental covariates (Almaraz et al. 2012; Koons et al. 2015), or spatially explicit models (Iijima et al. 2013). These

advances were facilitated by the probabilistic programming environments BUGS (Lunn et al. 2009) and JAGS (Plummer 2003).

However, the implementation of continuous-time population dynamics (described by ordinary differential equations, ODEs) was not available in the mentioned software. Until recently, modellers had to combine numerical simulations of ODEs manually with MCMC routines (Gilioli et al. 2008; Toni et al. 2009; Johnson et al. 2013; Smith et al. 2015; Papanikolaou et al. 2016; Boersch-Supan et al. 2017). The probabilistic programming language Stan (Carpenter et al. 2017) offers an integrated solution. It comes with interfaces to R, Python, Matlab and more, a built-in numerical ODE solver and a Hamiltonian Monte Carlo (HMC) sampler (Monnahan et al. 2017). Thus, it supports fitting dynamical population models to time series data in a Bayesian framework, see K. E. Fussmann et al. (2017) and Carpenter (2018) for recent applications.

In this study, we will apply Bayesian inference in Stan to a set of time series of a predator-prey system in a chemostat, i.e. a continuous flow-through culture (Novick and Szilard 1950). The parameters of a well-established chemostat ODE model will be estimated yielding posterior distributions for the parameters, which allow also to quantify their uncertainties. By comparing the posteriors for the individual time series we can deduce a variability among them that manifests in different types of population dynamics and pin-point to specific parameters that seem to determine this variability.

Materials and Methods

Data collection

Chemostat experiments were performed to obtain predator-prey time series at a high temporal resolution in a highly controlled environment (Guntram Weithoff, Bernd Blasius, Gregor F. Fussmann, Ursula Gaedke, *et al.*, unpublished) resulting in a large collection of long-term time series with several different species. From these we selected

a subset of 13 experiments, which were replicates in the sense that the same species were used at the same inflow nutrient concentration and dilution rate, and daily counts of prey and predators are available. The experiments were conducted with the metazoan predator *Brachionus calyciflorus* (Rotifera) and its prey *Monoraphidium minutum*, a unicellular green alga. The algae grew on nitrogen-limited medium. However, daily nitrogen concentrations are not available. The experiments were performed within a time span of seven years, with some individual experiments lasting longer than one year. They yielded time series which differed with respect to the degree that they exhibited more or less regular cyclic dynamics, or more or less constant predator and prey densities. From these 13 replicates we selected shorter time series that showed either clear and pronounced predator-prey cycles or steady-state equilibria, and excluded initial transient phases. We chose a minimum sample length of 20 days which would typically allow for at least two predator-prey cycles in this system. This process resulted in a set of 18 samples, 10 of which featured a steady state and eight contained cyclic dynamics.

Dynamical population model

The population dynamics of a predator-prey system in a chemostat with nitrogen S, algae A and rotifers R are described by the equations:

$$\frac{dS}{dt} = (S^* - S)\,\delta - \frac{1}{c_A}\frac{f_A S}{h_A + S}A \tag{1}$$

$$\frac{dA}{dt} = \frac{f_A S}{h_A + S}A - \frac{1}{c_R}\frac{f_R A}{h_R + A}R - \delta A \tag{2}$$

$$\frac{dR}{dt} = \frac{f_R A}{h_R + A}R - \delta R \tag{3}$$

This formulation represents a slightly simplified version of the one originally presented by G. F. Fussmann et al. (2000) and neglects age structure of the predator. S^* is the concentration of nutrients in the inflow of rate δ. The factors c_A and c_R define the conversion of nutrients into algal biomass and algal into rotifer biomass, respectively.

The growth rate of algae is described by Monod kinetics $\frac{f_A S}{h_A+S}$. f_A is the maximum growth rate and h_A is the half-saturation constant (the resource density at which the growth rate equals half of the maximum growth rate). The same applies to the resource dependent growth rate of rotifers feeding on algae $\frac{f_R A}{h_R+A}$, which is described by a type II functional response (Real 1977). The maximum growth rates $f_A[\mathrm{d}^{-1}]$, $f_R[\mathrm{d}^{-1}]$ and the conversion factors $c_A[\mathrm{cells\,\mu mol^{-1}}]$, $c_R[\mathrm{ind\,cells^{-1}}]$ are free parameters, which are estimated as described in the next section. We used constant values for the following parameters from a similar system which instead used *Chlorella vulgaris* as the prey species: $S^* = 80\ \mu\mathrm{mol\,l^{-1}}$ and $\delta = 0.55\ \mathrm{d}^{-1}$, as they were carefully controlled in the experiments, and $h_A = 4.3\,\mu\mathrm{mol\,l^{-1}}$ and $h_R = 7.5 \cdot 10^8\,\mathrm{cells\,l^{-1}}$ (G. F. Fussmann et al. 2000). We chose to not use half-saturation constants h_A (or h_R) as free parameters, since the estimates can be highly correlated with maximum growth rates f_A (or f_R). Their combined effects on the resource-dependent growth rate $\frac{f_A S}{h_A+S}$ (or $\frac{f_R A}{h_R+A}$) can only be disentangled if the data cover a large range in the resource states S (or A) (Rosenbaum and Rall 2018), which is not the case for the present chemostat experiments.

Model fitting and inference

We combined numerical simulations of the dynamical population model (eqns. 1–3) with Bayesian parameter estimation by drawing samples from the posterior probability distribution $P(\theta|y)$ of the free parameters θ given the data y, based on the likelihood $P(y|\theta)$ and the prior distribution $P(\theta)$. We used Hamiltonian Monte Carlo sampling in Stan, accessed via the `RStan` R-package (Stan Development Team 2018). The Stan software comes with a built-in Runge-Kutta ODE solver with adaptive stepsize control for generating predictions $\widehat{y}(\theta)$.

The likelihood calculation $P(y|\theta)$ is carried out automatically by the software when provided with predictions $\widehat{y}(\theta)$ and the distribution of residuals $\widehat{y}(\theta) - y$. The predictions $\widehat{y}(\theta)$ are defined by the numerical solutions of the ODE $\widehat{A}(t_i)$ and $\widehat{R}(t_i)$, evaluated at

times t_i, for a given parameter combination θ. We chose a log-normal distribution of the residuals, i.e. $\ln(\widehat{A}(t_i)) \sim \mathcal{N}(\ln(A_i), \sigma)$ and $\ln(\widehat{R}(t_i)) \sim \mathcal{N}(\ln(R_i), \sigma)$, with a scale parameter σ. This trajectory matching method technically corresponds to treating the model deterministically and to assuming pure observation errors in the data without any process error (Bolker 2008, Chap. 11). Note that, even without data for the concentration nitrogen S, it is possible to fit the ODE model by including the initial densities of the predictions $\widehat{S}_0^{(j)}$, $\widehat{A}_0^{(j)}$ and $\widehat{R}_0^{(j)}$ as free parameters (Carpenter 2018). However, one-step-ahead fitting (i.e., assuming pure process error) would only be possible for this ODE model if data for all state variables S, A and R was available. We did not consider full state-space models accounting for both process and observation error.

We fitted the maximum growth rates f_A and f_R and the conversion factors c_A and c_R on their logarithmic scale (see model code, Supporting Information). The dynamics and the statistical model are equivalent to fitting them on their original scale. But since they differ by several orders of magnitude, we found that log-transforming the parameter search space makes the iterative MCMC routine more robust.

We used a *hierarchical model* for the maximum growth rates f_A and f_R and for the conversion factors c_A and c_R by using time-series identity $j = 1, \ldots, m$ as a *random effect*. This means that every time series j is fitted with their individual set of parameters $\{f_A^{(j)}, f_R^{(j)}, c_A^{(j)}, c_R^{(j)}\}$, while each of them originates from a joint distribution across all m time series. In a Bayesian framework, this can be modeled via hierarchical dependencies in the prior distributions. Including logarithmic scaling, they read

$$\ln(\theta^{(j)}) \sim \mathcal{N}(\mu_{\ln(\theta)}, \sigma_{\ln(\theta)}),\ j = 1, \ldots, m,\ \theta = f_A, f_R, c_A, c_R, \tag{4}$$

where $\mu_{\ln(\theta)}$ are the overall means and $\sigma_{\ln(\theta)}$ the standard deviations across all m time series. $\mu_{\ln(\theta)}$ and $\sigma_{\ln(\theta)}$ are also free parameters with their own prior distribution (see Table 1 for a full description of the priors). We will test for variation between the

dynamics of the different time series as follows.

In linear models, classical ANOVA can be used within a hierarchical Bayesian framework to compare means of batches of data. Individual predictors' standard deviations are used to quantify the relative importance of different sources of variation in the data (Gelman and Hill 2007, Chap. 22). Here, this is not possible because of the highly nonlinear relationship between the parameters θ and the data y. But we can use the posterior distribution for an ANOVA-like analysis. Instead of checking for differences in means of samples of *data*, we directly uncover differences in *parameters* between two time series j and k by pairwise contrasts $\theta^{(j)} - \theta^{(k)}$. I.e., we compute posterior probabilities $P_{jk} = P(\theta^{(j)} > \theta^{(k)})$ that quantify these differences. These quantities P_{jk} are directly computed from the posterior distribution by dividing the number of samples featuring $\theta^{(j)} > \theta^{(k)}$ by the total number of samples, see also Kruschke (2015, Chap. 19).

To further investigate the importance of variation among the parameter estimates for different time series, we also fitted the ODE model (eqns. 1–3) using a single set of parameters $\{f_A, f_R, c_A, c_R\}$ (i.e. *fixed effects*) for all 18 time series as a null model. Only for the initial densities $\{\widehat{S}_0^{(j)}, \widehat{A}_0^{(j)}, \widehat{R}_0^{(j)}\}$ we allowed distinct values for each of the time series $j = 1, \ldots, 18$.

We briefly comment on numerical issues that can arise when combining numerical solutions of ODEs with MCMC sampling. When the MCMC sampler explores the parameter space, points can be sampled that make the computation of the likelihood by numerical simulation of the ODE infeasible (e.g. by requiring an immensely small integration step-size or simply by divergent state variables). Still, the sampling algorithm requires the computation of the likelihood to proceed with the iterations. To prevent the sampler from entering regions of the parameter space where, over a whole range of values, the likelihood is not available, we used two strategies. First, we implemented a numerical condition which prevents the numerical ODE solution from diverging or crossing the lower boundary of zero by setting the right-hand-side of the ODE to zero

if one of the state variables exceeds a reasonable range of $[10^{-6}, 10^{16}]$ (see model code in supporting information). Second, we used weakly informative priors on the overall mean parameters $\mu_{\ln(\theta)}$ (see Table 1) as follows. We initially fitted each of the 18 time series separately using priors based on the parametrization in G. F. Fussmann et al. (2000): $\ln(f_A^{(j)}) \sim \mathcal{N}(\ln(3.3), 1)$, $\ln(f_R^{(j)}) \sim \mathcal{N}(\ln(2.25), 1)$, $\ln(c_A^{(j)}) \sim \mathcal{N}(\ln(5.0 \cdot 10^7), 1)$, $\ln(c_R^{(j)}) \sim \mathcal{N}(\ln(2.5 \cdot 10^{-5}), 1)$. Then, we used the averages across the 18 estimated values as the priors' means for the joint hierarchical model (and also as priors for the fixed-effects model).

Results

Model convergence

We fitted the hierachical model by running 10 individual MCMC chains in parallel with an adaptation phase of 1,000 iterations and a sampling phase of 5,000 samples each, summing up to 50,000 samples of the posterior distribution. The runtime was approximately 10 days on a 2.2GHz Intel Xeon server architecture. Visual inspection of the trace plots and density plots showed a good mixture of the chains. Gelman-Rubin statistics of $\widehat{R} < 1.01$ and an adequate effective sample size n_{eff} (i.e. the estimated number of independent samples) verified convergence (Gelman and Hill 2007). See Supporting Information (Tables A1–A4) for a full list of parameter estimates and their statistics.

Identifiability of parameters

Some correlations between the free model parameters could be observed, especially between the logarithmic conversion factors $\ln(c_A)$ and $\ln(c_R)$ (see Supporting Information, Figs. A1–A5; plots were produced using the `BayesianTools` R-package (Hartig et al. 2017)). But the estimates' respective uncertainty (quantified by the marginal posteriors' standard deviations) do not substantially exceed the uncertainty in the uncorrelated esti-

mates of the logarithmic maximum growth rates $\ln(f_A)$ and $\ln(f_R)$ of prey and predators (Supporting Information, Tables A1–A4). This suggests that the chosen free parameters are truly informative on the systems behavior.

Posterior predictions

We generated posterior predictions by numerical simulations of the ODE system (eqns. 1–3) with all samples of the posterior distribution (Figs. 1,2). After a short transient phase, the median predicted trajectories feature either a steady-state equilibrium (time series 1–9), cyclic behavior (time series 13–18); or the posterior distribution includes parameter samples producing steady states as well as cyclic trajectories (time series 10–12). Correspondingly, we found multimodalities in the posterior distributions of time series 10–12 (see Supporting Information, Figs. A1–A6).

Interestingly, the relative uncertainty of the predictions (quantified by 95% confidence intervals) for all state variables is substantially reduced in time series 13–18 where the predictions feature cyclic behavior compared to other time series (Figs. 1,2). We measured the predictive accuracy in the univariate time series by normalized root mean-square-errors (cf. Fig. A6, Supporting Information). Here we see that the error distributions are shifted to smaller values and become more narrow for cyclic dynamics, also indicating a better fit. This may be explained by acknowledging that steady-state data, which covers a smaller range in the state space than cyclic data, contains less information about the process rates and hence the parameters.

As no data constrains the predictions for nitrogen the uncertainty is even higher here than in algae or rotifers in time series 1–9. Also we found that our ODE model is able to predict the full amplitude of cycles in algal states better than in rotifer states (Figs. 1,2; Fig. A6, Supporting Information). This is likely caused by a higher regularity in the algal data, which covers a larger amplitude decreasing the relative counting error. Also, algae feature a less complex life cycle than rotifers and their dynamics should thus be

less variable.

Variation among time series

For assessing the variation in the parameters across the time series ($j = 1, \ldots, 18$), we plotted the marginal (i.e. one-dimensional projections of the multivariate) posterior probability distributions of the logarithmic maximum growth rates $\ln(f_A^{(j)})$ and $\ln(f_R^{(j)})$ and the logarithmic conversion factors, $\ln(c_A^{(j)})$ and $\ln(c_R^{(j)})$ of prey and predators, respectively (Fig. 3). We also computed probabilities of pairwise contrasts $P_{jk} = P(\theta^{(j)} > \theta^{(k)})$ for a more detailed examination of the differences across time series ($\theta = f_A, f_R, c_A, c_R$, Tables 2–5). Values close to one or close to zero indicate significant pairwise differences. Note that the tables are symmetrical in the sense of $P_{jk} = 1 - P_{kj}$.

We found that time series with predicted steady states feature systematically higher values of f_A than time series with cyclic dynamics (Fig. 3a, Table 2, top right and bottom left blocks). While in cyclic time series 13–18 values of f_A differ among each other (bottom right block), no evidence was found for differences among steady-state time series 1–9 (top left block). Steady state time series also exhibit a high uncertainty in f_A estimates with confidence intervals spanning from $2.48\,\mathrm{d}^{-1}$ to $49.90\,\mathrm{d}^{-1}$ when transformed back to a linear scale (Fig. 3a, see also supporting information, Table A1). This uncertainty is substantially reduced in cyclic time series 13–18 with confidence intervals spanning from $2.27\,\mathrm{d}^{-1}$ to $5.26\,\mathrm{d}^{-1}$. Here, the predicted parameter values are close to the published value in G. F. Fussmann et al. (2000), which however was published for *Chlorella* instead of *Monoraphidium*, but should be in the same range. Similar to the uncertainty in predictions (see above) we can speculate that, in combination with the lack of nitrogen data, steady-state time series, which cover a smaller range in the state space, contain less information about the density dependent growth rates of algae on nitrogen $\frac{f_A S}{h_A + S}$ than cyclic time series.

For the rates f_R we found pairwise differences across all time series (Fig. 3b, Table 3).

No systematic effect of cyclic or steady-state time series was observed (i.e., f_R estimates for cyclic time series are not systematically smaller than estimates for steady-state time series or vice versa). Regarding the estimates' uncertainty we find that, in contrast to f_A, the data seem to provide enough information on the growth rates of rotifers on algae $\frac{f_R A}{h_R + A}$, even in steady-state time series: While f_A describes the interaction of algae A and nitrogen S with data only available for A, f_R describes the interaction of rotifers R and algae A with data available for both trophic levels.

The conversion factors c_A and c_R did not show significant pairwise differences for steady-state time series 1–9 (Fig. 3c,d; Tables 4,5, top left blocks; with few exceptions for c_R). Some pairwise differences among cyclic time series 13–18 (bottom right blocks) and to steady-state time series (top right and bottom left blocks) were observed, without being as systematic as in f_A. The uncertainty in parameter estimates is slightly larger in steady-state time series than in cycles time series, but the effect is not as prominent as in f_A estimates (see also full tables of estimates, Tabs. A1–A4, Supporting Information).

Comparison to fixed-effects model fitting

To further support the importance of variation among the parameter estimates, we also fitted a fixed-effects model using a single set of parameters $\{f_A, f_R, c_A, c_R\}$ for all 18 time series as a null model. We used the same priors and model fitting specifications as above. Although a formal model comparison via information criteria is generally available for Bayesian statistics (Vehtari et al. 2018), it is not applicable to our dynamical model, since predictions (and therefore residuals) are correlated along time. Hence, we compare the random and the fixed effects model qualitatively and quantitatively via their predictions. In contrast to the previous random effects model, the fixed effects model produces transient and steady-state predictions for all time series. Cyclic dynamics can not be reproduced (Supporting Information, Figs. A7&A8). Also, the predicted equilibrium fails to reproduce the correct level of algae and rotifer states in some cases. This

corresponds to a generally lower predictive accuracy when fitting algal and rotifers time series with the fixed effects model as compared to the random effects model (Supporting Information, Fig. A6). The effect is highest for cyclic time series 13–18, but also pronounced in time series 2, 3, 6 and 10, where the equilibrium states are not reproduced correctly.

Transition from cyclic dynamics to steady states

To support the observation that low growth rates f_A cause cyclic dynamics while high growth rates yield a steady-state equilibrium (Fig. 3), we performed a simulation study. We numerically simulated the chemostat model (eqns. 1–3) while systematically varying growth rates f_A between $1\,\mathrm{d}^{-1}$ and $7.389\,\mathrm{d}^{-1}$ (corresponding to parameter values $\ln(f_A)$ between 0 and 2). The maximum growth rate of the predator and the conversion factors were held constant at their estimated overall means $\exp(\mu_{\ln(\theta)})$ across the 18 time series ($f_R = 1.416\,\mathrm{d}^{-1}$, $c_A = 3.856 \cdot 10^7\,\mathrm{cells\,\mu mol^{-1}}$, $c_R = 1.063 \cdot 10^{-5}\,\mathrm{ind\,cells^{-1}}$). The bifurcation diagram (Fig. 4) shows system states of simulations over 200 days after discarding the first 100 days and clearly indicates cycles for low growth rates and steady states for high growth rates, with the Hopf bifurcation located in $f_A = 4.473\,\mathrm{d}^{-1}$.

Discussion

In this study we presented how a differential equations model can be fitted to observed time series data of species abundances, taking predator-prey dynamics as an example. Next to obtaining key model parameters in situ, this method allows to decipher variability in the outcomes among replicates and to point towards probable sources of this variability.

The comparison of choosing the time series identity as random or fixed effects shows that allowing for such a variability in the parameters between replicates can be crucial to

see whether time series data agree with a model. Especially if a certain parameter is close to a bifurcation, as it seems to be the case in our system for the maximum growth rate of the algae f_A, minor deviations in this parameter result in different predictions for the system dynamics. In such cases, in-situ parameter estimation allows to detect parameter sets for both steady states and cyclic dynamics, which can be separated by a multi-dimensional bifurcation boundary in models with a high number of parameters. Thereby, the chosen model structure may be accepted for all replicates, even if their dynamics differ, without increasing the model complexity. Instead, more than one parameter set might be needed to cover the whole diversity of possible and observable patterns in population dynamics.

Of course, this method requires that the model structure used for fitting the data applies sufficiently well to the mechanisms acting in the system delivering the data. Also, the data quality has to be high enough. We see that the dynamics of the prey are fitted better than those of the predator, which may be explained by different mechanisms: (i) The densities of the prey in the data are much larger than those of the predator. Assuming that the relative experimental error of counting decreases with larger individual numbers, this error should be smaller for the prey than for the predator. This increases the regularity of the dynamical patterns and simplifies the fitting of the prey time series. (ii) Rotifers, as metazoan animals, possess a more complex life-cycle than algae. Their dynamics may be affected by age-structure, variable resource co-limitation and other factors, which, for simplicity, were not included in the model. These non-modelled processes obviously decrease the goodness of fit. (iii) As no data was available for the resource of the prey, the parameters of the prey were confined only by the prey data, leaving more flexibility to improve the fitting in the prey's states. Contrarily, the parameters of the predator were more restricted, as both prey and predator data was available, leaving less flexibility for fitting the predator's states.

Interestingly, the type of population dynamics affects the quality of the inference as

well. Population cycles contain a higher degree of information on the system than steady states, as also rates of change are apparent, additional to the biomasses of the trophic levels. This manifests in more narrow parameter estimates (Fig. 3) and leads to more certain predictions (Figs. 1,2). Also, steady-state fits give rather (and partially unrealistically) high estimates for algal growth rates, whereas those from cyclic population dynamics are estimated close to published values.

Variability in key parameters suggests a heterogeneity in the traits that are encoded by these parameters. Heterogeneous traits imply intra-specific variability which may enable populations to escape perturbations and to persist in the presence of strong stressors (Reusch et al. 2005; Bell and Gonzalez 2009; Chevin et al. 2010). Bayesian parameter inference provides uncertainty estimates on key parameters and thereby allows to detect such variability in experiments. We propose that this technique may also help to quantify the presence of trait heterogeneity in nature.

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Authors' contributions

BR, UG and MR conceived the ideas, GW and GF contributed the data, BR developed the methods, BR and MR led the writing of the manuscript. All authors contributed critically to the drafts and gave final approval for publication.

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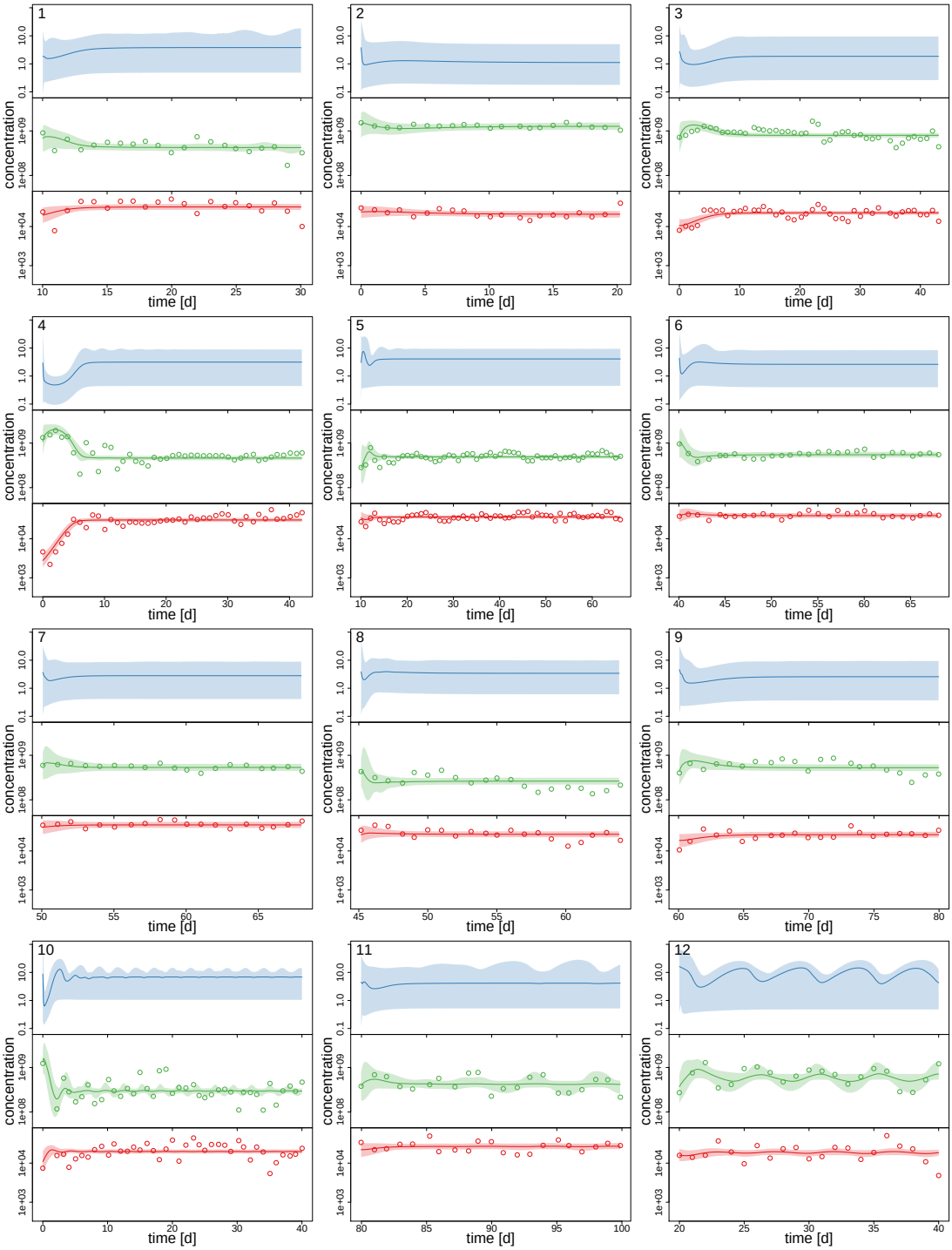

Figure 1: Time series 1–12 of 18, data (dots) and posterior predictions of fitted model for nitrogen (blue), algae (green) and rotifers (red). Solid lines represent median predictions, shaded areas depict 95% highest density intervals of the predictions. Predicted trajectories 1–9 featured a steady-state equilibrium. Time series 10–12 featured multimodalities in the posterior distribution and the predictions did not exhibit a clear tendency towards a steady state or cycles.

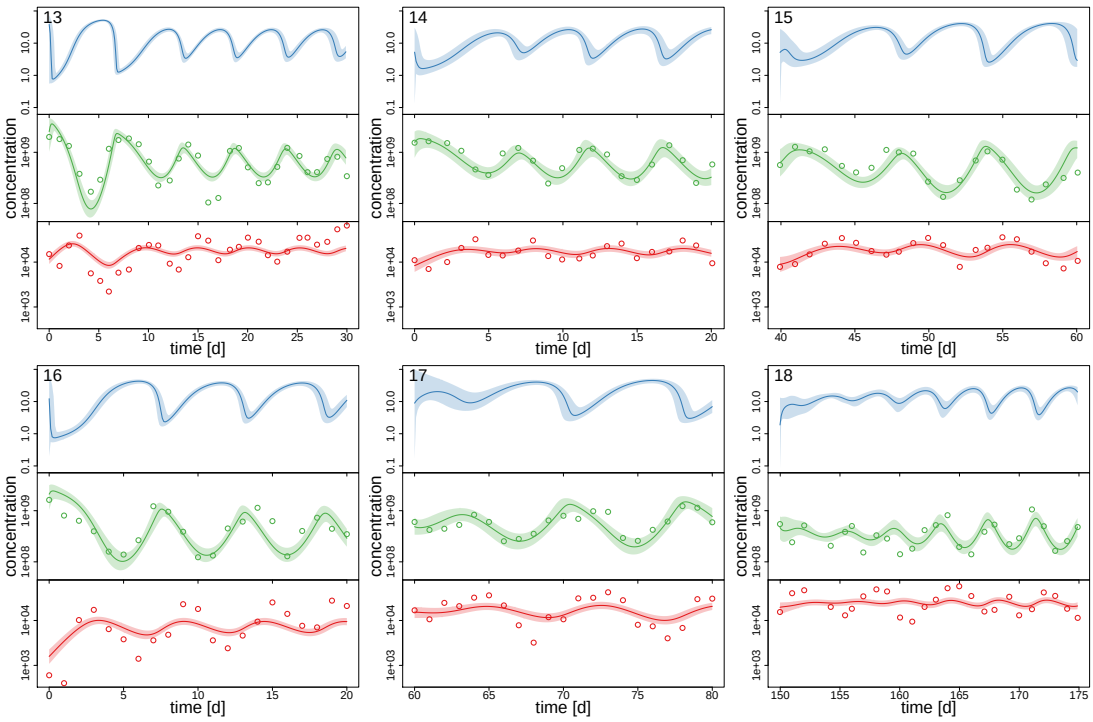

Figure 2: Time series 13–18 of 18, data (dots) and posterior predictions of fitted model for nitrogen (blue), algae (green) and rotifers (red). Solid lines represent median predictions, shaded areas depict 95% highest density intervals of the predictions. Shown predicted trajectories 13–18 featured cyclic behavior.

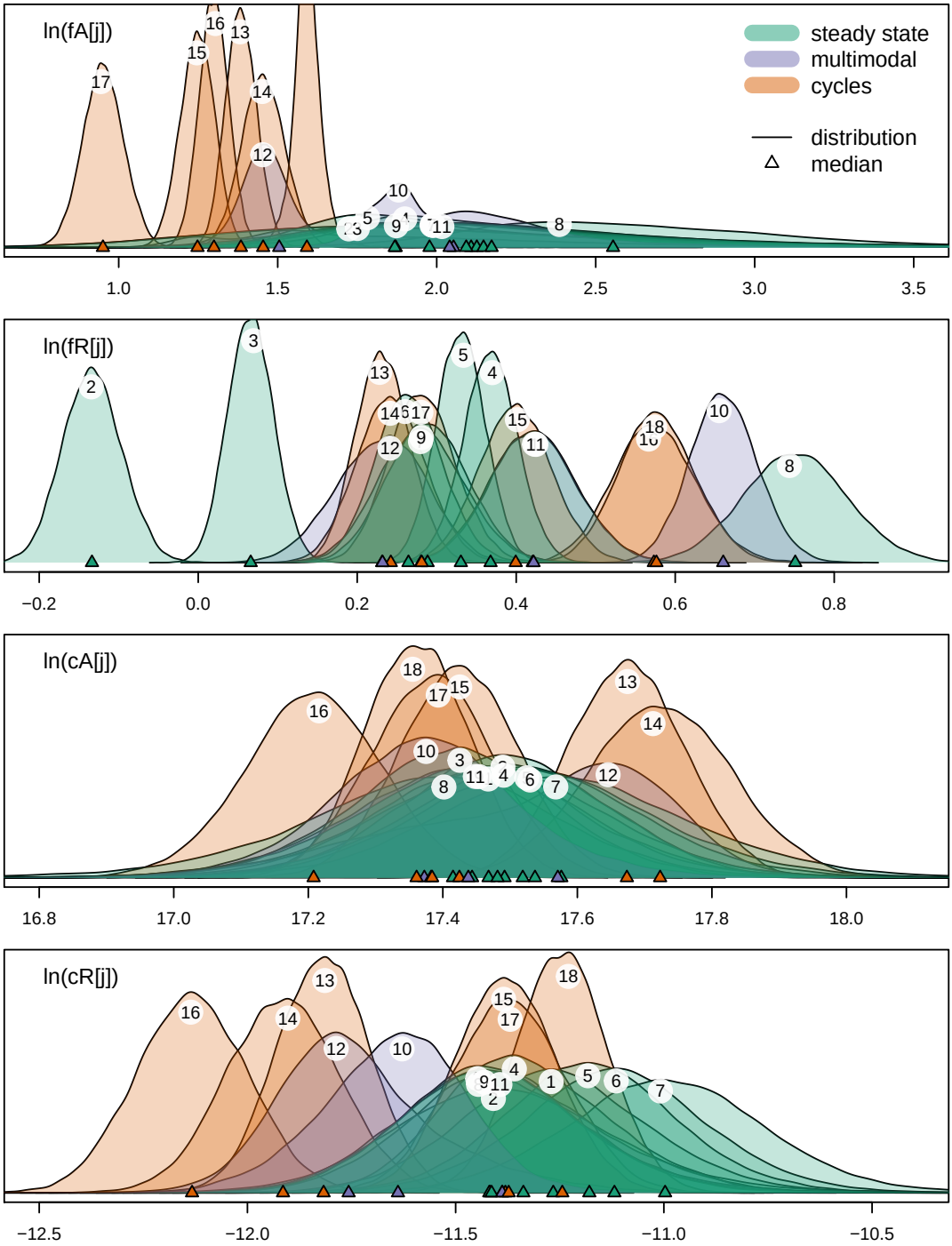

Figure 3: Marginal posterior distributions of maximum growth rates $\ln(f_A^{(j)})[\mathrm{d}^{-1}]$, $\ln(f_R^{(j)})[\mathrm{d}^{-1}]$, and conversion factors $\ln(c_A^{(j)})[\mathrm{cells\,\mu mol^{-1}}]$, $\ln(c_R^{(j)})[\mathrm{ind\,cells^{-1}}]$ $(j = 1, \ldots, 19)$. Predicted trajectories 1–9 featured a steady-state equilibrium (green), while predicted trajectories 13–18 featured cyclic behavior (orange). Time series 10–12 (purple) featured multimodalities in the posterior distribution and the predictions did not exhibit a clear tendency towards a steady state or cycles.

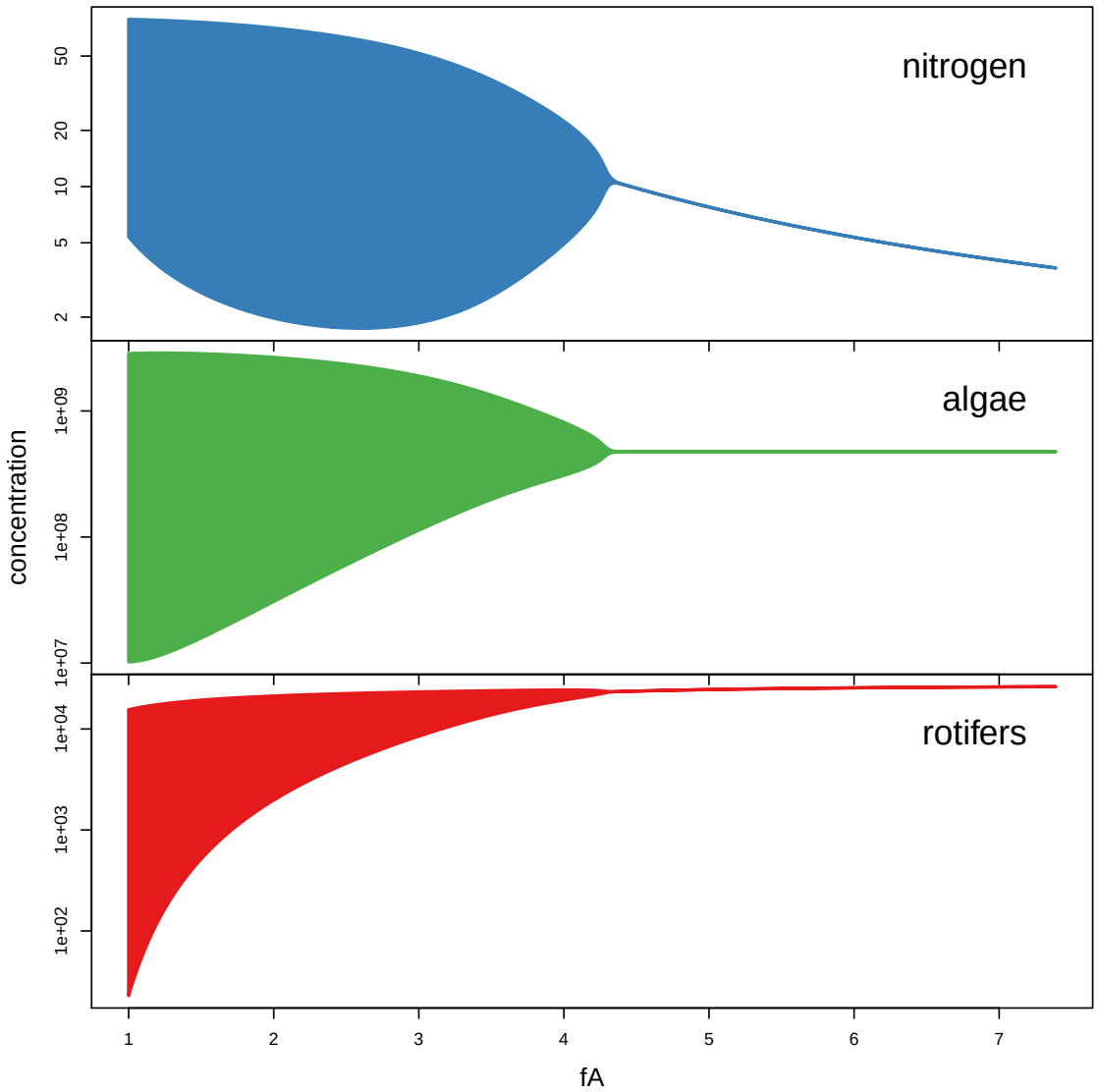

Figure 4: Bifurcation diagram for simulations with varying maximum growth rate f_A, while keeping the remaining parameters constant at the fitted overall means. A Hopf bifurcation happens at $f_A = 4.473$.

Table 1: Free parameters and their prior distributions. $\mathcal{N}(\cdot,\cdot)$ denotes a normal distribution, $\mathcal{N}(0,\cdot)^+$ denotes a positive half-normal distribution truncated at zero.

parameter		distribution	type
max. growth rates	$\ln(f_A^{(j)})$	$\mathcal{N}(\mu_{\ln(f_A)}, \sigma_{\ln(f_A)})$	hierarchical
max. growth rates	$\ln(f_R^{(j)})$	$\mathcal{N}(\mu_{\ln(f_R)}, \sigma_{\ln(f_R)})$	hierarchical
conversion factors	$\ln(c_A^{(j)})$	$\mathcal{N}(\mu_{\ln(c_A)}, \sigma_{\ln(c_A)})$	hierarchical
conversion factors	$\ln(c_R^{(j)})$	$\mathcal{N}(\mu_{\ln(c_R)}, \sigma_{\ln(c_R)})$	hierarchical
overall mean	$\mu_{\ln(f_A)}$	$\mathcal{N}(1.591, 1)$	weakly informative
overall mean	$\mu_{\ln(f_R)}$	$\mathcal{N}(0.356, 1)$	weakly informative
overall mean	$\mu_{\ln(c_A)}$	$\mathcal{N}(17.190, 1)$	weakly informative
overall mean	$\mu_{\ln(c_R)}$	$\mathcal{N}(-11.067, 1)$	weakly informative
overall standard deviation	$\sigma_{\ln(f_A)}$	$\mathcal{N}(0, 1)^+$	weakly informative
overall standard deviation	$\sigma_{\ln(f_R)}$	$\mathcal{N}(0, 1)^+$	weakly informative
overall standard deviation	$\sigma_{\ln(c_A)}$	$\mathcal{N}(0, 1)^+$	weakly informative
overall standard deviation	$\sigma_{\ln(c_R)}$	$\mathcal{N}(0, 1)^+$	weakly informative
initial densities	$\ln(\widehat{S}_0^{(j)})$	$\mathcal{N}(2, 2)$	weakly informative
initial densities	$\ln(\widehat{A}_0^{(j)})$	$\mathcal{N}(20, 2)$	weakly informative
initial densities	$\ln(\widehat{R}_0^{(j)})$	$\mathcal{N}(10, 2)$	weakly informative
residual standard deviation	σ	$\mathcal{N}(0, 2)^+$	weakly informative

Table 2: Pairwise contrasts $P_{jk} = P(f_A^{(j)} > f_A^{(k)})$ of maximum growth rates f_A. Bold numbers indicate significant differences, quantified by probabilities $P_{jk} > 0.90$ ($f_A^{(j)}$ significantly larger than $f_A^{(k)}$) and probabilities $P_{jk} < 0.10$ ($f_A^{(j)}$ significantly smaller than $f_A^{(k)}$). Block partitioning refers to time series 1–9 featuring a steady-state equilibrium, 10–12 having multimodal posterior distributions featuring both steady states and cycles, and 13–18 featuring cyclic dynamics.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		0.64	0.64	0.47	0.55	0.48	0.46	0.23	0.51	0.51	0.56	0.80	**0.97**	**0.95**	**0.99**	**0.99**	**1.00**	**0.90**
2	0.36		0.50	0.33	0.39	0.34	0.32	0.16	0.37	0.36	0.41	0.63	0.82	0.78	0.89	0.86	**0.97**	0.70
3	0.36	0.50		0.33	0.40	0.34	0.32	0.16	0.37	0.36	0.42	0.63	0.82	0.78	0.89	0.87	**0.97**	0.70
4	0.53	0.67	0.67		0.59	0.50	0.49	0.24	0.54	0.55	0.59	0.82	**0.99**	**0.98**	**1.00**	**0.99**	**1.00**	**0.95**
5	0.45	0.61	0.60	0.41		0.42	0.40	0.19	0.46	0.44	0.51	0.78	**0.98**	**0.96**	**1.00**	**0.99**	**1.00**	**0.90**
6	0.52	0.66	0.66	0.50	0.58		0.48	0.25	0.53	0.54	0.59	0.81	**0.98**	**0.96**	**0.99**	**0.99**	**1.00**	**0.92**
7	0.54	0.68	0.68	0.51	0.60	0.52		0.26	0.55	0.56	0.60	0.82	**0.98**	**0.97**	**1.00**	**0.99**	**1.00**	**0.93**
8	0.77	0.84	0.84	0.76	0.81	0.75	0.74		0.77	0.84	0.81	**0.92**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**
9	0.49	0.63	0.63	0.46	0.54	0.47	0.45	0.23		0.50	0.55	0.79	**0.97**	**0.94**	**0.99**	**0.98**	**1.00**	0.88
10	0.49	0.64	0.64	0.45	0.56	0.46	0.44	0.16	0.50		0.54	0.82	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
11	0.44	0.59	0.58	0.41	0.49	0.41	0.40	0.19	0.45	0.46		0.73	**0.92**	0.89	**0.96**	**0.94**	**1.00**	0.81
12	0.20	0.37	0.37	0.18	0.22	0.19	0.18	**0.08**	0.21	0.18	0.27		0.87	0.67	**0.98**	**0.97**	**1.00**	0.38
13	**0.03**	0.18	0.18	**0.01**	**0.02**	**0.02**	**0.02**	**0.00**	**0.03**	**0.00**	**0.08**	0.13		0.18	**0.98**	0.89	**1.00**	**0.00**
14	**0.05**	0.22	0.22	**0.02**	**0.04**	**0.04**	**0.03**	**0.00**	**0.06**	**0.00**	0.11	0.33	0.82		**0.99**	**0.97**	**1.00**	**0.04**
15	**0.01**	0.11	0.11	**0.00**	**0.00**	**0.01**	**0.00**	**0.00**	**0.01**	**0.00**	**0.04**	**0.02**	**0.02**	**0.01**		0.22	**1.00**	**0.00**
16	**0.01**	0.14	0.13	**0.01**	**0.01**	**0.01**	**0.01**	**0.00**	**0.02**	**0.00**	**0.06**	**0.03**	0.11	**0.03**	0.78		**1.00**	**0.00**
17	**0.00**	**0.03**	**0.03**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**		**0.00**
18	**0.10**	0.30	0.30	**0.05**	**0.10**	**0.08**	**0.07**	**0.01**	0.12	**0.00**	0.19	0.62	**1.00**	**0.96**	**1.00**	**1.00**	**1.00**	

Table 3: Pairwise contrasts $P_{jk} = P(f_R^{(j)} > f_R^{(k)})$ of maximum growth rates f_R. Bold numbers and block partitioning as in Table 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		**1.00**	**1.00**	0.80	**0.93**	**0.99**	**0.97**	**0.00**	**0.96**	**0.00**	0.50	**0.99**	**1.00**	**1.00**	0.62	**0.02**	**0.98**	**0.02**
2	**0.00**		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
3	**0.00**	**1.00**		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
4	0.20	**1.00**	**1.00**		0.79	**0.97**	**0.91**	**0.00**	**0.90**	**0.00**	0.20	**0.98**	**1.00**	**0.99**	0.29	**0.00**	**0.94**	**0.00**
5	**0.07**	**1.00**	**1.00**	0.21		0.89	0.79	**0.00**	0.76	**0.00**	**0.07**	**0.95**	**0.98**	**0.95**	**0.10**	**0.00**	0.82	**0.00**
6	**0.01**	**1.00**	**1.00**	**0.03**	**0.11**		0.41	**0.00**	0.36	**0.00**	**0.01**	0.69	0.73	0.65	**0.02**	**0.00**	0.39	**0.00**
7	**0.03**	**1.00**	**1.00**	**0.09**	0.21	0.59		**0.00**	0.46	**0.00**	**0.03**	0.74	0.78	0.71	**0.05**	**0.00**	0.49	**0.00**
8	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		**1.00**	0.88	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**	**1.00**	**0.98**
9	**0.04**	**1.00**	**1.00**	**0.10**	0.24	0.64	0.54	**0.00**		**0.00**	**0.04**	0.78	0.83	0.76	**0.05**	**0.00**	0.54	**0.00**
10	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.12	**1.00**		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.91**	**1.00**	**0.91**
11	0.50	**1.00**	**1.00**	0.80	**0.93**	**0.99**	**0.97**	**0.00**	**0.96**	**0.00**		**0.99**	**1.00**	**1.00**	0.62	**0.02**	**0.98**	**0.02**
12	**0.01**	**1.00**	**1.00**	**0.02**	**0.05**	0.31	0.26	**0.00**	0.22	**0.00**	**0.01**		0.50	0.44	**0.01**	**0.00**	0.24	**0.00**
13	**0.00**	**1.00**	**1.00**	**0.00**	**0.02**	0.27	0.22	**0.00**	0.17	**0.00**	**0.00**	0.50		0.42	**0.00**	**0.00**	0.18	**0.00**
14	**0.00**	**1.00**	**1.00**	**0.01**	**0.05**	0.35	0.29	**0.00**	0.24	**0.00**	**0.00**	0.56	0.58		**0.01**	**0.00**	0.26	**0.00**
15	0.38	**1.00**	**1.00**	0.71	**0.90**	**0.98**	**0.95**	**0.00**	**0.95**	**0.00**	0.38	**0.99**	**1.00**	**0.99**		**0.00**	**0.97**	**0.00**
16	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.01**	**1.00**	**0.09**	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**		**1.00**	0.48
17	**0.02**	**1.00**	**1.00**	**0.06**	0.18	0.61	0.51	**0.00**	0.46	**0.00**	**0.02**	0.76	0.82	0.74	**0.03**	**0.00**		**0.00**
18	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.02**	**1.00**	**0.09**	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**	0.52	**1.00**	

Table 4: Pairwise contrasts $P_{jk} = P(c_A^{(j)} > c_A^{(k)})$ of conversion factors c_A. Bold numbers and block partitioning as in Table 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		0.45	0.60	0.48	0.41	0.37	0.31	0.66	0.55	0.68	0.55	0.38	0.13	**0.09**	0.59	**0.91**	0.67	0.72
2	0.55		0.66	0.53	0.45	0.41	0.34	0.70	0.59	0.73	0.61	0.42	0.14	**0.10**	0.65	**0.95**	0.74	0.79
3	0.40	0.34		0.38	0.30	0.27	0.22	0.57	0.45	0.59	0.45	0.29	**0.06**	**0.04**	0.47	0.89	0.57	0.63
4	0.52	0.47	0.62		0.42	0.38	0.32	0.67	0.56	0.70	0.58	0.39	0.13	**0.09**	0.62	0.92	0.70	0.75
5	0.59	0.55	0.70	0.58		0.46	0.39	0.74	0.64	0.76	0.65	0.46	0.20	0.14	0.70	0.94	0.76	0.81
6	0.63	0.59	0.73	0.62	0.54		0.43	0.77	0.67	0.79	0.68	0.50	0.24	0.17	0.73	**0.95**	0.79	0.84
7	0.69	0.66	0.78	0.68	0.61	0.57		0.81	0.73	0.83	0.74	0.57	0.32	0.24	0.78	**0.96**	0.84	0.87
8	0.34	0.30	0.43	0.33	0.26	0.23	0.19		0.39	0.51	0.40	0.27	**0.06**	**0.04**	0.41	0.80	0.49	0.54
9	0.45	0.41	0.55	0.44	0.36	0.33	0.27	0.61		0.63	0.51	0.35	**0.10**	**0.07**	0.54	0.89	0.63	0.68
10	0.32	0.27	0.41	0.30	0.24	0.21	0.17	0.49	0.37		0.37	0.24	**0.03**	**0.02**	0.37	0.85	0.47	0.53
11	0.45	0.39	0.55	0.42	0.35	0.32	0.26	0.60	0.49	0.63		0.33	**0.10**	**0.06**	0.52	0.89	0.61	0.67
12	0.62	0.58	0.71	0.61	0.54	0.50	0.43	0.73	0.65	0.76	0.67		0.26	0.17	0.71	**0.92**	0.76	0.79
13	0.87	0.86	**0.94**	0.87	0.80	0.76	0.68	**0.94**	**0.90**	**0.97**	**0.90**	0.74		0.35	**0.98**	**1.00**	**0.99**	**1.00**
14	**0.91**	**0.90**	**0.96**	**0.91**	0.86	0.83	0.76	**0.96**	**0.93**	**0.98**	**0.94**	0.83	0.65		**0.99**	**1.00**	**0.99**	**1.00**
15	0.41	0.35	0.53	0.38	0.30	0.27	0.22	0.59	0.46	0.63	0.48	0.29	**0.02**	**0.01**		**0.96**	0.63	0.71
16	**0.09**	**0.05**	0.11	**0.08**	**0.06**	**0.05**	**0.04**	0.20	0.11	0.15	0.11	**0.08**	**0.00**	**0.00**	**0.04**		**0.09**	**0.10**
17	0.33	0.26	0.43	0.30	0.24	0.21	0.16	0.51	0.37	0.53	0.39	0.24	**0.01**	**0.01**	**0.37**	**0.91**		0.58
18	0.28	0.21	0.37	0.25	0.19	0.16	0.13	0.46	0.32	0.47	0.33	0.21	**0.00**	**0.00**	0.29	**0.90**	0.42	

Table 5: Pairwise contrasts $P_{jk} = P(c_R^{(j)} > c_R^{(k)})$ of conversion factors c_R. Bold numbers and block partitioning as in Table 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		0.66	0.71	0.59	0.37	0.29	0.17	0.71	0.71	**0.93**	0.67	**0.95**	**0.99**	**1.00**	0.69	**1.00**	0.67	0.47
2	0.34		0.54	0.42	0.25	0.19	0.11	0.54	0.54	0.83	0.50	0.88	**0.95**	**0.98**	0.49	**1.00**	0.48	0.30
3	0.29	0.46		0.37	0.19	0.14	**0.08**	0.50	0.49	0.82	0.46	0.88	**0.96**	**0.98**	0.44	**1.00**	0.43	0.23
4	0.41	0.58	0.63		0.28	0.21	0.12	0.63	0.63	**0.91**	0.59	**0.94**	**0.99**	**1.00**	0.59	**1.00**	0.58	0.35
5	0.63	0.75	0.81	0.72		0.41	0.25	0.81	0.81	**0.97**	0.78	**0.97**	**1.00**	**1.00**	0.81	**1.00**	0.79	0.62
6	0.71	0.81	0.86	0.79	0.59		0.32	0.85	0.85	**0.98**	0.83	**0.98**	**1.00**	**1.00**	0.86	**1.00**	0.85	0.71
7	0.83	0.89	**0.92**	0.88	0.75	0.68		**0.92**	**0.92**	**0.99**	**0.91**	**0.99**	**1.00**	**1.00**	**0.93**	**1.00**	**0.92**	0.84
8	0.29	0.46	0.50	0.37	0.19	0.15	**0.08**		0.49	0.83	0.45	0.88	**0.96**	**0.98**	0.44	**1.00**	0.43	0.24
9	0.29	0.46	0.51	0.37	0.19	0.15	**0.08**	0.51		0.83	0.46	0.88	**0.96**	**0.98**	0.45	**1.00**	0.44	0.24
10	**0.07**	0.17	0.18	**0.09**	**0.03**	**0.02**	**0.01**	0.17	0.17		0.15	0.65	0.80	0.89	**0.09**	**0.99**	**0.10**	**0.02**
11	0.33	0.50	0.54	0.41	0.22	0.17	**0.09**	0.55	0.54	0.85		**0.90**	**0.97**	**0.99**	0.49	**1.00**	0.48	0.28
12	**0.05**	0.12	0.12	**0.06**	**0.03**	**0.02**	**0.01**	0.12	0.12	0.35	**0.10**		0.64	0.79	**0.07**	**0.97**	**0.07**	**0.03**
13	**0.01**	**0.05**	**0.04**	**0.01**	**0.00**	**0.00**	**0.00**	**0.04**	**0.04**	0.20	**0.03**	0.36		0.72	**0.00**	**0.96**	**0.00**	**0.00**
14	**0.00**	**0.02**	**0.02**	**0.00**	**0.00**	**0.00**	**0.00**	**0.02**	**0.02**	0.11	**0.01**	0.21	0.28		**0.00**	0.87	**0.00**	**0.00**
15	0.31	0.51	0.56	0.41	0.19	0.14	**0.07**	0.56	0.55	**0.91**	0.51	**0.93**	**1.00**	**1.00**		**1.00**	0.48	0.20
16	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.03**	**0.04**	0.13	**0.00**		**0.00**	**0.00**
17	0.33	0.52	0.57	0.42	0.21	0.15	**0.08**	0.57	0.56	**0.90**	0.52	**0.93**	**1.00**	**1.00**	0.52	**1.00**		0.23
18	0.53	0.70	0.77	0.65	0.38	0.29	0.16	0.76	0.76	**0.98**	0.72	**0.97**	**1.00**	**1.00**	0.80	**1.00**	0.77	

Supplement

Stan model code

```
functions{
  // ODE right hand side
  real[] frmodel(real t, real[] B, real[] p, real[] x_r, int[] x_i){
    // p[1] S0, inflow concentration
    // p[2] delta, inflow rate
    // p[3] fA, max. growth rate
    // p[4] hA, half-saturation density
    // p[5] 1/cA, conversion factor
    // p[6] fR,  max. growth rate
    // p[7] hR, half-saturation density
    // p[8] 1/cR, conversion factor
    real dBdt[3];
    real feeding_S;
    real feeding_A;
    if( B[1]<1e-6 || B[1]>1e16 || B[2]<1e-6 || B[2]>1e16 || B[3]<1e-6 || B[3]>1e16 ){
      dBdt[1] = 0;
      dBdt[2] = 0;
      dBdt[3] = 0;
    }
    else{
      feeding_S = (p[3]/(p[4]+B[1]))*B[1]*B[2];
      feeding_A = (p[6]/(p[7]+B[2]))*B[2]*B[3];
      dBdt[1] = (p[1]-B[1])*p[2] - p[5]*feeding_S;
      dBdt[2] = feeding_S        - p[8]*feeding_A - p[2]*B[2];
      dBdt[3] = feeding_A                         - p[2]*B[3];
    }
    return dBdt;
  }
}

data{
  int m;              // number of time series
  int n[m];           // lengths of the m time series
  int nmax;           // max(n)
  real time0[m];      // initial timepoints of the m time series
  real logA0[m];      // initial algal densities (log)
  real logR0[m];      // initial rotifer densities (log)
  real time[m,nmax]; // each row i contains times of observations and is filled with
placeholder 0s for n[i]<j<=nmax
  real logA[m,nmax]; // algal observations, placeholders as in time[,]
  real logR[m,nmax]; // rotifer observations, placeholders as in time[,]
}

transformed data { // just empty placeholders for function call
  real x_r[0];
  int x_i[0];
}

parameters {
```

```
  real mu_fA;
  real<lower=0> sigma_fA;
  real fA[m]; // maximum growth rates, A <- S

  real mu_fR;
  real<lower=0> sigma_fR;
  real fR[m]; // maximum growth rates, R <- A

  real mu_cA;
  real<lower=0> sigma_cA;
  real cA[m]; // conversion factors, A <- S

  real mu_cR;
  real<lower=0> sigma_cR;
  real cR[m]; // conversion factors, R <- A

  real<lower=0> sdev;  // standard deviation of residuals

  real logB0[m,3]; // initial values for ODE
}

model {
  // intermediate parameters
  real p[8];      // hand over to ODE
  real Bsim[nmax,3]; // simulated values, matrix. dim1 = time, dim2 = dim_ODE = 3
  real B0[3]; // initial values for ODE

  // priors (informative)
  mu_fA ~ normal(1.591, 1.0);
  sigma_fA ~ normal(0, 1.0);

  mu_fR ~ normal(0.356, 1.0);
  sigma_fR ~ normal(0, 1.0);

  mu_cA ~ normal(17.190, 1.0);
  sigma_cA ~ normal(0, 1.0);

  mu_cR ~ normal(-11.067, 1.0);
  sigma_cR ~ normal(0, 1.0);

  for(j in 1:m){
    fA[j] ~ normal(mu_fA, sigma_fA);
    fR[j] ~ normal(mu_fR, sigma_fR);
    cA[j] ~ normal(mu_cA, sigma_cA);
    cR[j] ~ normal(mu_cR, sigma_cR);
    logB0[j, 1] ~ normal( 2,2);
    logB0[j, 2] ~ normal(20,2);
    logB0[j, 3] ~ normal(10,2);
  }
  sdev ~ normal(0, 2);

  // likelihood
  for (j in 1:m){ // loop over time series
```

```
    p[1] = 80.0;          // S0, inflow concentration
    p[2] = 0.55;          // delta, inflow rate
    p[3] = exp(fA[j]);    // max. growth rate
    p[4] = 4.3;           // half-saturation density
    p[5] = 1/exp(cA[j]); // conversion factor
    p[6] = exp(fR[j]);    // max. growth rate
    p[7] = 7.5e8;         // half-saturation density
    p[8] = 1/exp(cR[j]); // conversion factor

    // ODE simulation
    for (i in 1:3) {
      B0[i] = exp(logB0[j,i]);
    }
    Bsim[1:n[j], 1:3] = integrate_ode_rk45(frmodel,B0,time0[j],time[j,1:n[j]],p,x_r,x_i);

    // residuals
    logA0[j] ~ normal(logB0[j,2], sdev);
    logR0[j] ~ normal(logB0[j,3], sdev);

    for (i in 1:n[j]){
      logA[j,i] ~ normal(log(Bsim[i,2]), sdev);
      logR[j,i] ~ normal(log(Bsim[i,3]), sdev);
    }

  } // end for (j in 1:m)
}
```

Table A1: Summary statistics for maximum growth rates $f_A[\mathrm{d}^{-1}]$. Gelman-Rubin statistics $\widehat{R} < 1.01$ for all parameters verify convergence, n_{eff} is the number of independent samples. Block partitioning refers to time series 1–9 featuring a steady-state equilibrium, 10–12 having multimodal posterior distributions featuring both steady states and cycles, and 13–18 featuring cyclic dynamics.

	median	sdev	2.5%	97.5%	n_{eff}
$\mu_{\ln(f_A)}$	1.83	0.26	1.48	2.47	4731
$\sigma_{\ln(f_A)}$	0.58	0.25	0.29	1.24	4000
$\ln(f_A^{(1)})$	2.11	0.59	1.35	3.69	5155
$\ln(f_A^{(2)})$	1.87	0.67	0.91	3.52	13204
$\ln(f_A^{(3)})$	1.87	0.67	0.95	3.57	10406
$\ln(f_A^{(4)})$	2.13	0.56	1.48	3.68	7883
$\ln(f_A^{(5)})$	1.98	0.59	1.44	3.66	5382
$\ln(f_A^{(6)})$	2.15	0.59	1.42	3.71	7145
$\ln(f_A^{(7)})$	2.17	0.59	1.44	3.73	7665
$\ln(f_A^{(8)})$	2.55	0.54	1.80	3.91	5529
$\ln(f_A^{(9)})$	2.09	0.60	1.35	3.69	7660
$\ln(f_A^{(10)})$	2.05	0.42	1.72	3.28	5052
$\ln(f_A^{(11)})$	2.04	0.61	1.22	3.60	3924
$\ln(f_A^{(12)})$	1.50	0.52	1.31	3.20	2166
$\ln(f_A^{(13)})$	1.38	0.05	1.29	1.48	14744
$\ln(f_A^{(14)})$	1.45	0.07	1.32	1.59	16137
$\ln(f_A^{(15)})$	1.25	0.05	1.14	1.35	22348
$\ln(f_A^{(16)})$	1.30	0.05	1.21	1.39	16600
$\ln(f_A^{(17)})$	0.95	0.06	0.82	1.08	17680
$\ln(f_A^{(18)})$	1.59	0.05	1.50	1.68	3796

Table A2: Summary statistics for maximum growth rates $f_R[\mathrm{d}^{-1}]$. Gelman-Rubin statistics $\widehat{R} < 1.01$ for all parameters verify convergence, n_{eff} is the number of independent samples. Block partitioning as in Table A1.

	median	sdev	2.5%	97.5%	n_{eff}
$\mu_{\ln(f_R)}$	0.35	0.06	0.24	0.46	50000
$\sigma_{\ln(f_R)}$	0.23	0.05	0.16	0.34	34632
$\ln(f_R^{(1)})$	0.42	0.05	0.32	0.53	50000
$\ln(f_R^{(2)})$	-0.13	0.04	-0.20	-0.06	50000
$\ln(f_R^{(3)})$	0.07	0.03	0.01	0.12	50000
$\ln(f_R^{(4)})$	0.37	0.03	0.30	0.43	50000
$\ln(f_R^{(5)})$	0.33	0.03	0.27	0.39	50000
$\ln(f_R^{(6)})$	0.26	0.04	0.18	0.35	50000
$\ln(f_R^{(7)})$	0.28	0.05	0.18	0.39	50000
$\ln(f_R^{(8)})$	0.75	0.07	0.62	0.88	50000
$\ln(f_R^{(9)})$	0.29	0.05	0.19	0.39	50000
$\ln(f_R^{(10)})$	0.66	0.04	0.58	0.75	50000
$\ln(f_R^{(11)})$	0.42	0.05	0.32	0.53	50000
$\ln(f_R^{(12)})$	0.23	0.06	0.12	0.33	3524
$\ln(f_R^{(13)})$	0.23	0.03	0.17	0.30	14531
$\ln(f_R^{(14)})$	0.24	0.04	0.16	0.33	17246
$\ln(f_R^{(15)})$	0.40	0.05	0.31	0.49	16198
$\ln(f_R^{(16)})$	0.57	0.05	0.48	0.68	14219
$\ln(f_R^{(17)})$	0.28	0.04	0.20	0.37	17048
$\ln(f_R^{(18)})$	0.58	0.05	0.48	0.67	19616

Table A3: Summary statistics for conversion factors c_A[cells µmol^{-1}]. Gelman-Rubin statistics $\widehat{R} < 1.01$ for all parameters verify convergence, n_{eff} is the number of independent samples. Block partitioning as in Table A1.

	median	sdev	2.5%	97.5%	n_{eff}
$\mu_{\ln(c_A)}$	17.47	0.08	17.30	17.62	5732
$\sigma_{\ln(c_A)}$	0.19	0.06	0.10	0.33	10185
$\ln(c_A^{(1)})$	17.47	0.18	17.10	17.81	12071
$\ln(c_A^{(2)})$	17.49	0.16	17.18	17.81	15177
$\ln(c_A^{(3)})$	17.42	0.15	17.11	17.71	13572
$\ln(c_A^{(4)})$	17.48	0.17	17.13	17.78	11188
$\ln(c_A^{(5)})$	17.52	0.18	17.17	17.87	9840
$\ln(c_A^{(6)})$	17.54	0.18	17.19	17.91	11120
$\ln(c_A^{(7)})$	17.58	0.19	17.21	17.97	9908
$\ln(c_A^{(8)})$	17.38	0.20	16.95	17.74	9982
$\ln(c_A^{(9)})$	17.44	0.18	17.08	17.79	13187
$\ln(c_A^{(10)})$	17.37	0.15	17.08	17.67	11517
$\ln(c_A^{(11)})$	17.44	0.17	17.09	17.78	10586
$\ln(c_A^{(12)})$	17.57	0.20	17.10	17.83	2033
$\ln(c_A^{(13)})$	17.67	0.09	17.51	17.84	12284
$\ln(c_A^{(14)})$	17.72	0.11	17.51	17.93	11447
$\ln(c_A^{(15)})$	17.42	0.09	17.25	17.60	13512
$\ln(c_A^{(16)})$	17.21	0.10	17.00	17.40	12653
$\ln(c_A^{(17)})$	17.38	0.09	17.20	17.56	13303
$\ln(c_A^{(18)})$	17.36	0.08	17.21	17.52	13427

Table A4: Summary statistics for conversion factors $c_R[\text{ind cells}^{-1}]$. Gelman-Rubin statistics $\widehat{R} < 1.01$ for all parameters verify convergence, n_{eff} is the number of independent samples. Block partitioning as in Table A1.

	median	sdev	2.5%	97.5%	n_{eff}
$\mu_{\ln(c_R)}$	-11.45	0.12	-11.68	-11.19	7217
$\sigma_{\ln(c_R)}$	0.35	0.09	0.22	0.57	11319
$\ln(c_R^{(1)})$	-11.27	0.23	-11.70	-10.79	9467
$\ln(c_R^{(2)})$	-11.39	0.27	-11.87	-10.82	12979
$\ln(c_R^{(3)})$	-11.42	0.22	-11.82	-10.96	12983
$\ln(c_R^{(4)})$	-11.34	0.20	-11.68	-10.89	10493
$\ln(c_R^{(5)})$	-11.18	0.22	-11.60	-10.74	8791
$\ln(c_R^{(6)})$	-11.12	0.23	-11.56	-10.66	9880
$\ln(c_R^{(7)})$	-11.00	0.24	-11.47	-10.51	9396
$\ln(c_R^{(8)})$	-11.42	0.24	-11.84	-10.90	8524
$\ln(c_R^{(9)})$	-11.41	0.23	-11.84	-10.94	12085
$\ln(c_R^{(10)})$	-11.64	0.18	-12.01	-11.31	8873
$\ln(c_R^{(11)})$	-11.39	0.23	-11.81	-10.90	5625
$\ln(c_R^{(12)})$	-11.76	0.20	-12.06	-11.25	4368
$\ln(c_R^{(13)})$	-11.82	0.11	-12.04	-11.60	15393
$\ln(c_R^{(14)})$	-11.91	0.14	-12.18	-11.65	14288
$\ln(c_R^{(15)})$	-11.38	0.12	-11.62	-11.14	18293
$\ln(c_R^{(16)})$	-12.13	0.13	-12.39	-11.87	14837
$\ln(c_R^{(17)})$	-11.37	0.14	-11.64	-11.10	16239
$\ln(c_R^{(18)})$	-11.24	0.11	-11.46	-11.02	14780

Table A5: Summary statistics for fixed effects model. Gelman-Rubin statistics $\widehat{R} < 1.01$ for all parameters verify convergence, n_{eff} is the number of independent samples.

	median	sdev	2.5%	97.5%	n_{eff}
$\ln(f_A)$	1.72	0.86	0.30	3.58	50000
$\ln(f_R)$	0.34	0.01	0.31	0.37	50000
$\ln(c_A)$	15.77	0.05	15.68	15.86	34768
$\ln(c_R)$	-7.74	0.34	-8.35	-7.02	23816

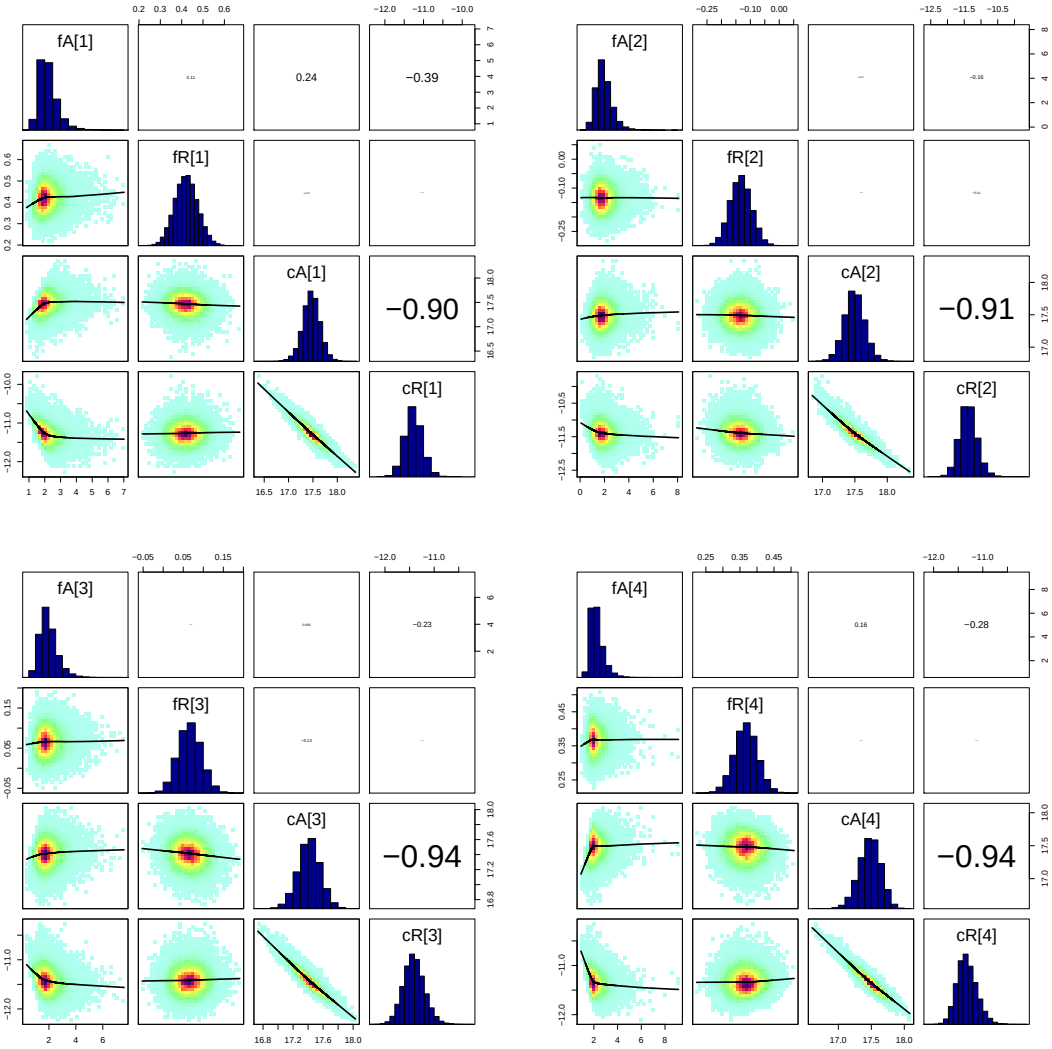

Figure A1: Pairs plots of time series 1–4 parameters (on ln-scale). Marginal posterior densities (diagonal panels), pairwise posterior densities (lower left panels, red indicates high density, black line represents nonlinear fit of the correlation) and correlation coefficients (upper right panels, fontsize indicates high correlation).

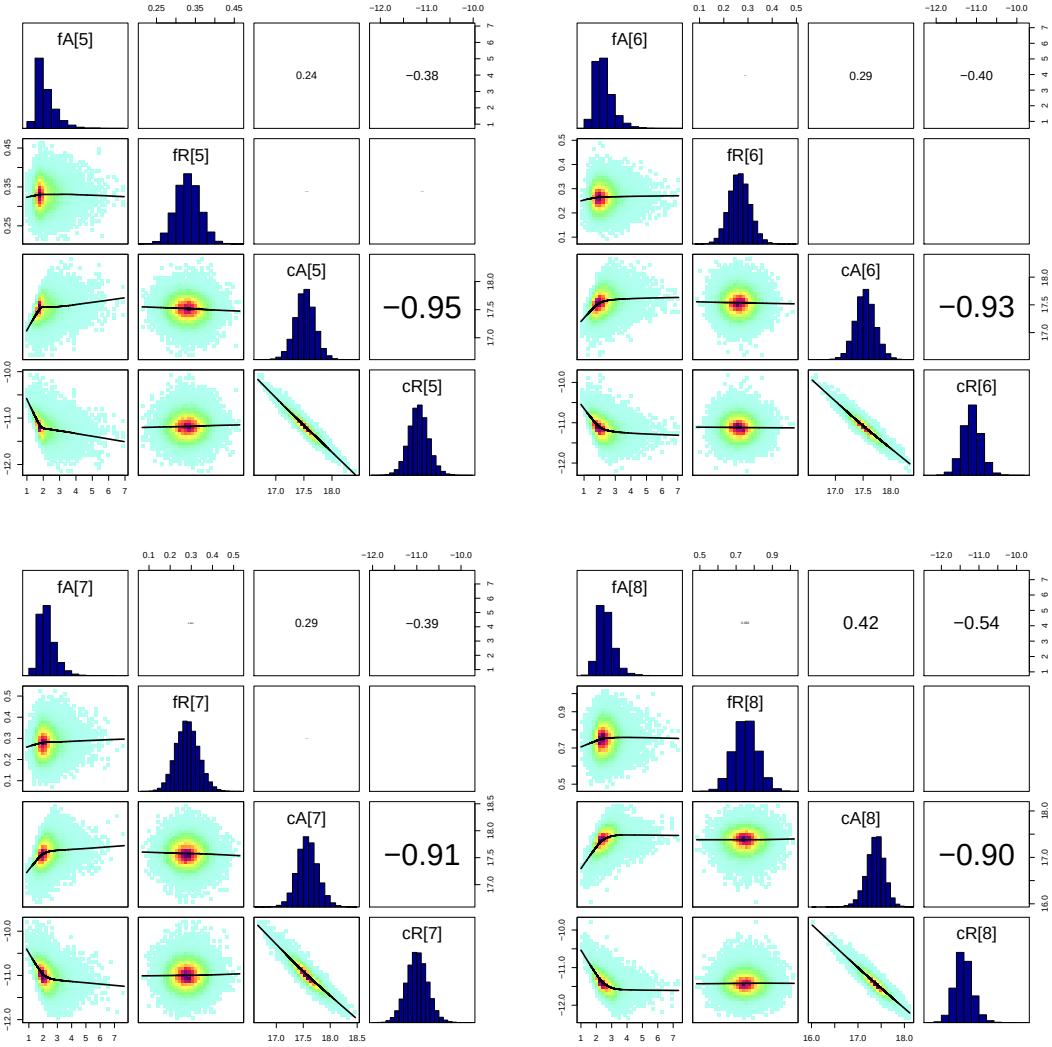

Figure A2: Pairs plots of time series 5–8 parameters. Panels as in Fig. A1.

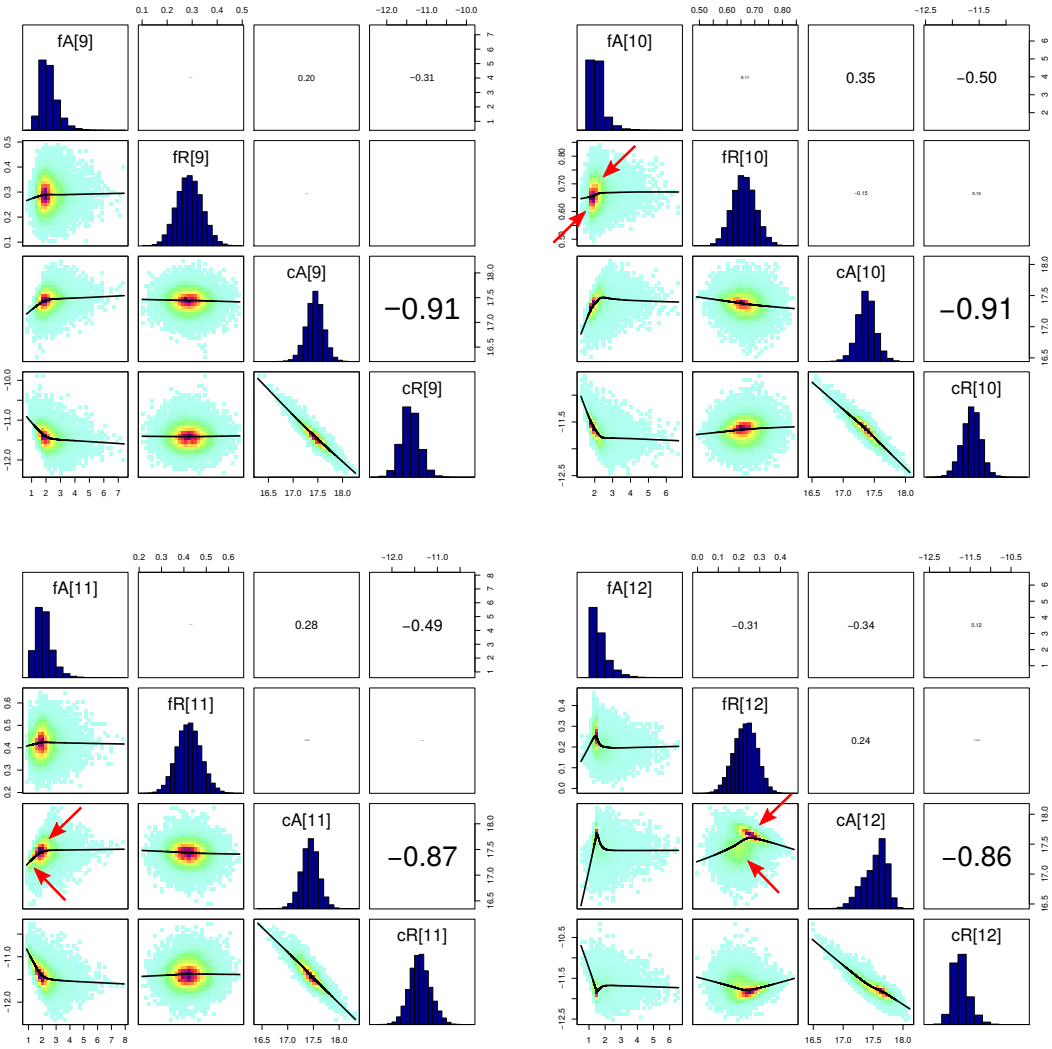

Figure A3: Pairs plots of time series 9–12 parameters. Panels as in Fig. A1. Red arrows indicate multimodalities in time series 10–12.

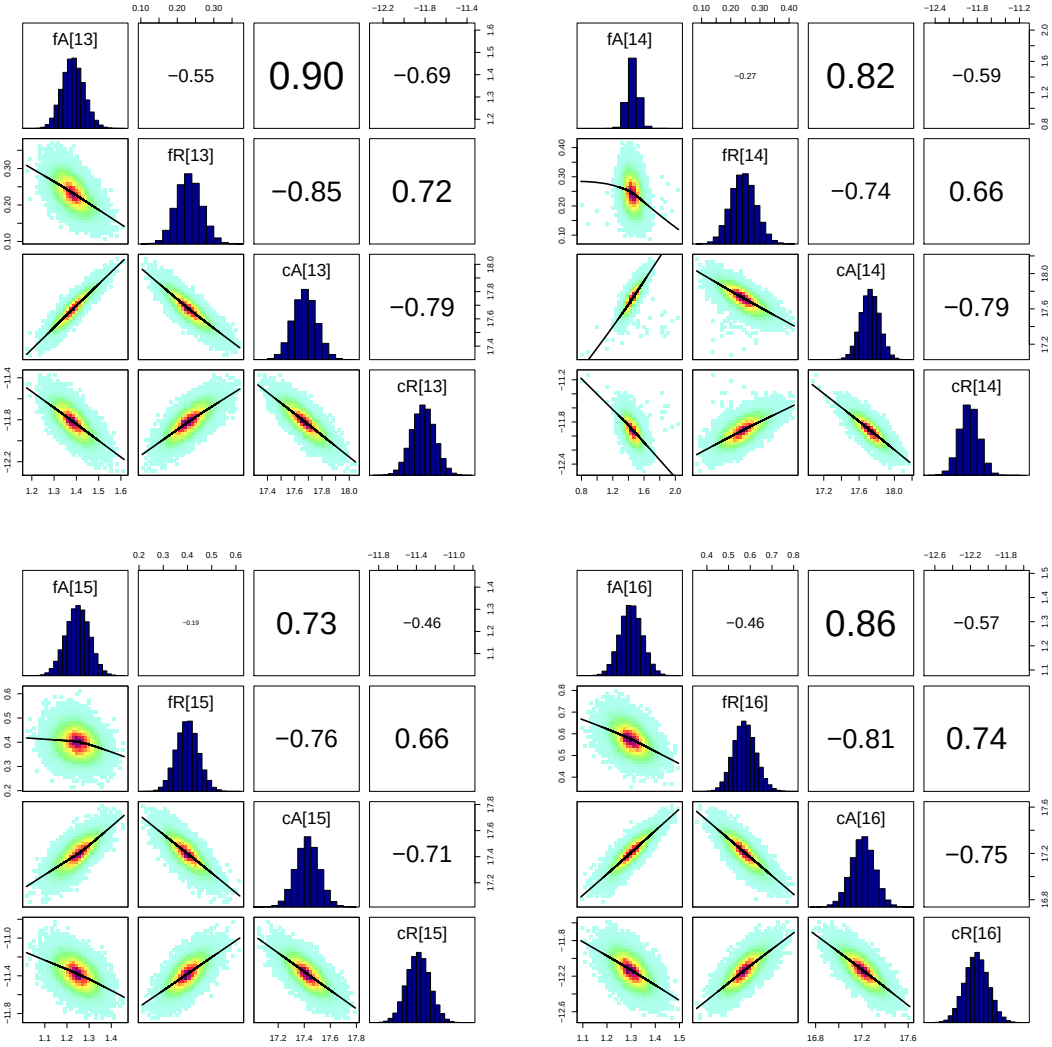

Figure A4: Pairs plots of time series 13–16 parameters. Panels as in Fig. A1.

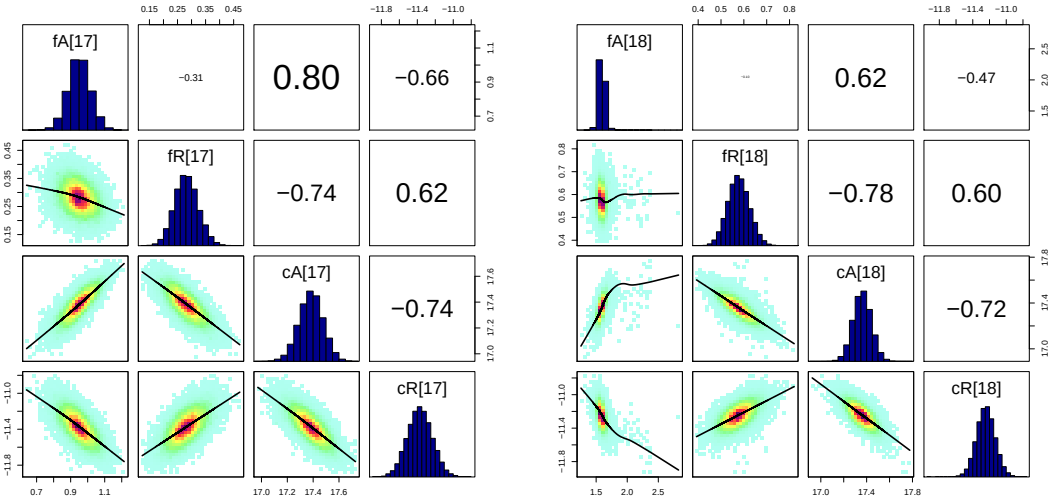

Figure A5: Pairs plots of time series 17, 18 parameters. Panels as in Fig. A1.

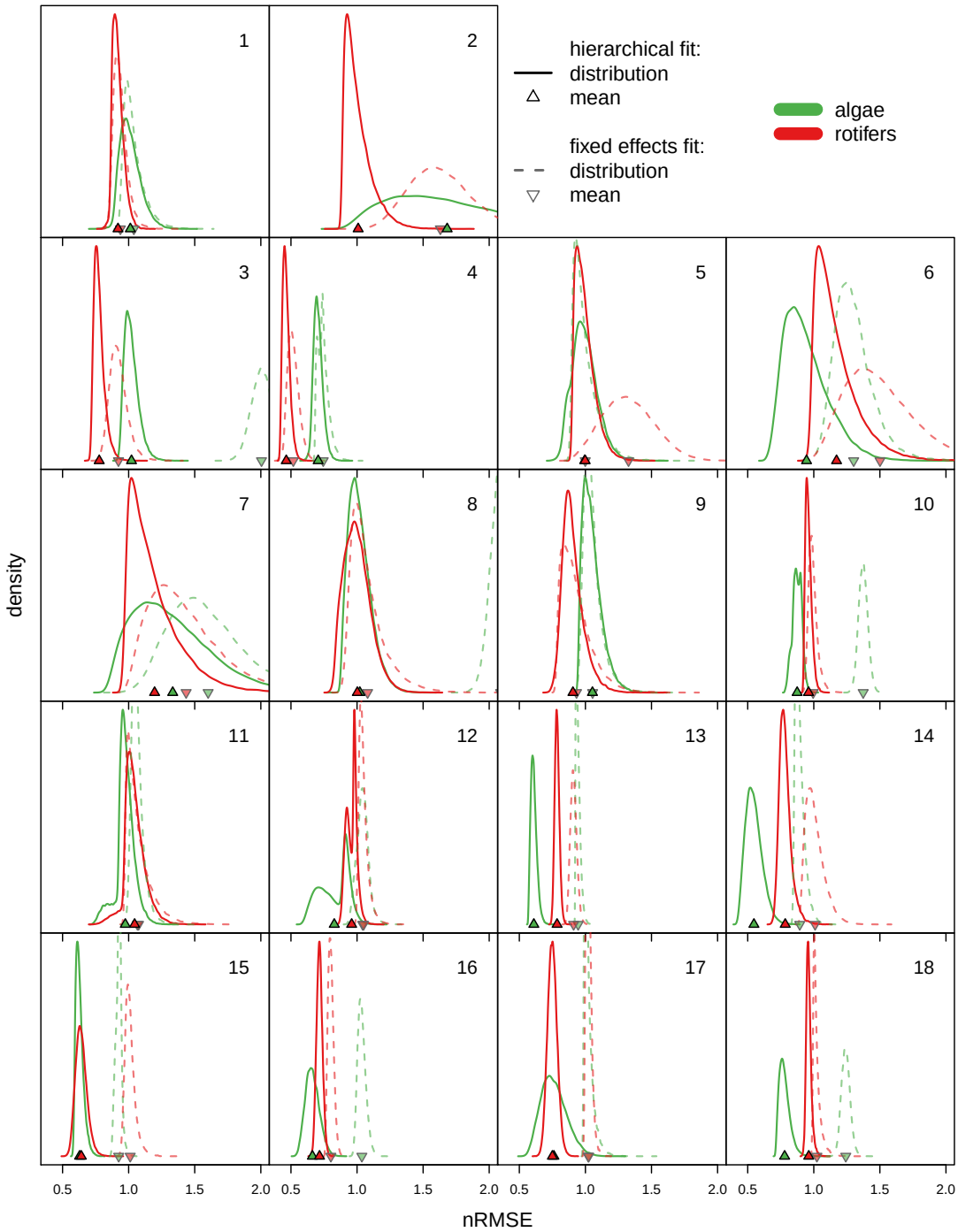

Figure A6: Predictive accuracy assessed by posterior distributions of the normalized root-mean-square error (nRMSE) $\frac{1}{\sigma(\ln(y))}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\ln(y_i)-\ln(\widehat{y}_i)\right)^2}$ with log-scaled data $\ln(y_i)$ and predictions $\ln(\widehat{y}_i)$, normalized by each time series' empirical standard deviation $\sigma(\ln(y))$, $y = A^{(j)}, R^{(j)}$, $j = 1, \ldots, 18$. Algae (green) and rotifer (red) nRMSE for random effects model using distinct parameters for each time series (solid lines) and fixed effects model using the same parameters across all time series (dashed lines).

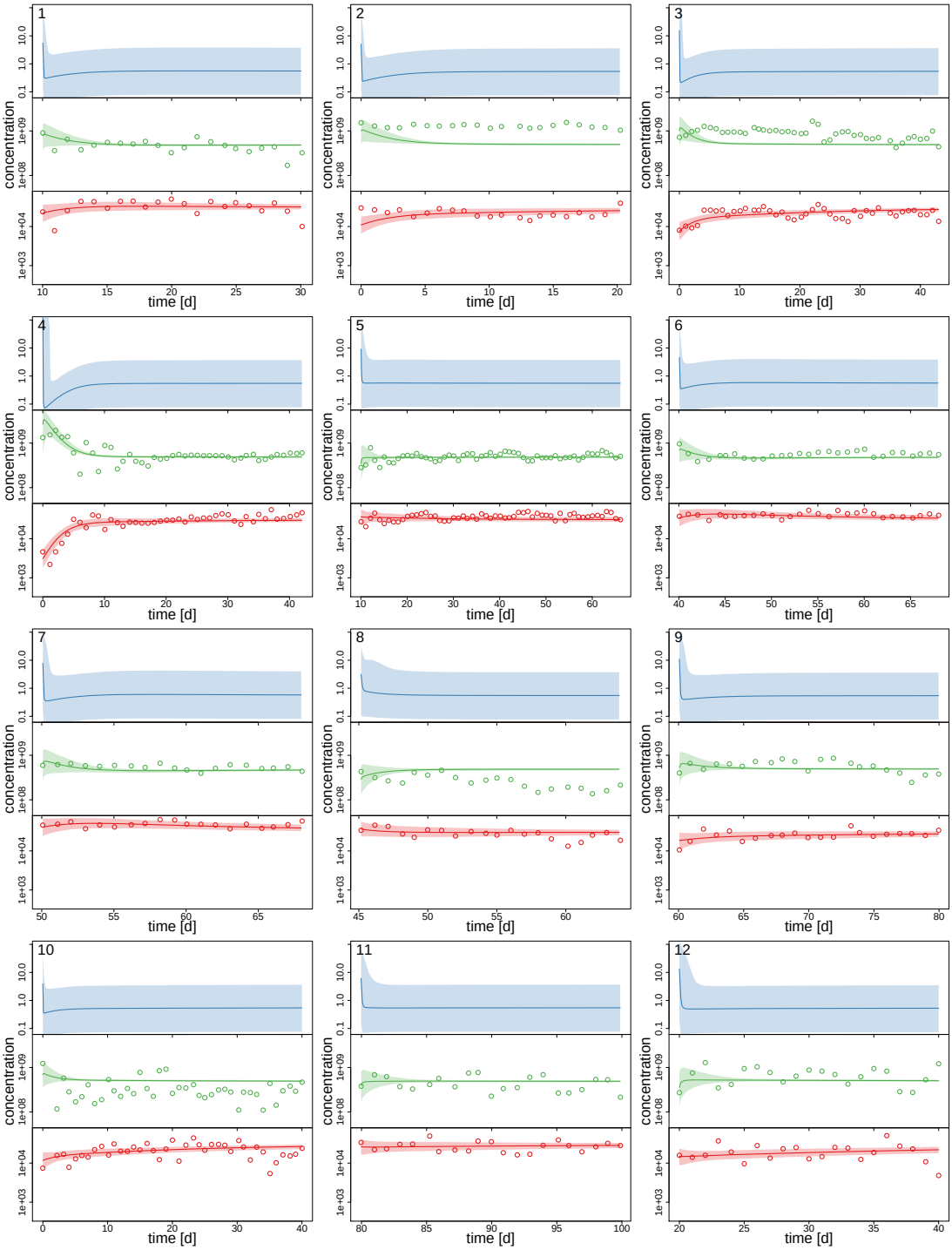

Figure A7: Time series 1–12 of 18, data (dots) and posterior predictions of fitted fixed effects model for nitrogen (blue), algae (green) and rotifers (red). Solid lines represent median predictions, shaded areas depict 95% highest density intervals of the predictions.

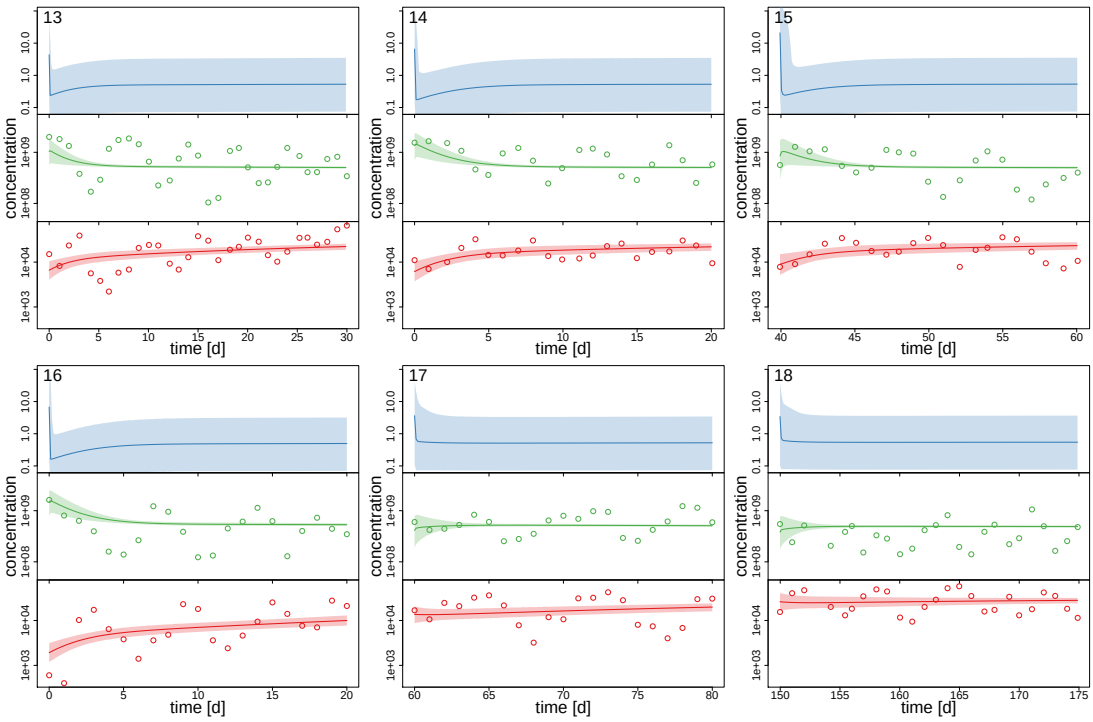

Figure A8: Time series 13–18 of 18, data (dots) and posterior predictions of fitted fixed effects model for nitrogen (blue), algae (green) and rotifers (red). Solid lines represent median predictions, shaded areas depict 95% highest density intervals of the predictions.