

1 **Title:** Expected differences in diversity and rarity between communities containing sexually  
2 versus asexually reproducing taxa

3 **Running Title:** Reproductive method alters diversity, rarity

4 **Authors:** Cristina M. Herren<sup>1, 2, 3</sup>

5 **Author affiliations:**

6 <sup>1</sup>Harvard Data Science Initiative, Harvard University, Cambridge, Massachusetts, USA

7 <sup>2</sup>Department of Biostatistics, Harvard T. H. Chan School of Public Health, Boston,  
8 Massachusetts, USA

9 <sup>3</sup>Department of Biomedical Informatics, Harvard Medical School, Boston, Massachusetts, USA

10 Email: [cristina\\_herren@hms.harvard.edu](mailto:cristina_herren@hms.harvard.edu)

11 310 Countway Medical Library, 10 Shattuck St, Boston MA 02115

12

## 12 **Abstract**

13           Recent studies evaluating the community structures of microorganisms and macro-  
14 organisms have found greater diversity and rarity within micro-scale communities, compared to  
15 macro-scale communities. However, reproductive method has been a confounding factor in these  
16 comparisons; the microbes considered generally reproduce asexually, while the macro-organisms  
17 considered generally reproduce sexually. Sexual reproduction imposes the constraint of mate  
18 finding, which can have significant demographic consequences by depressing birth rates at low  
19 population sizes. Here, I examine theoretically how the effects of mate finding in sexual  
20 populations translate to the emergent community properties of diversity, rarity, and dominance.  
21 Using continuous-time Markov chain models, I compare communities with and without  
22 constraints of mate limitation. In mate-limited sexual populations, the decreased growth rates at  
23 low population densities translates to a much higher extinction rate. In communities consisting of  
24 sexually reproducing taxa, the increased extinction rate due to mate limitation decreases  
25 expected diversity. Furthermore, mate limitation has a disproportionately strong effect on taxa  
26 with low population density, leading to fewer rare taxa. These shifts in community structure  
27 mirror recent empirical studies of micro versus macro communities, indicating that reproductive  
28 method may contribute to observed differences in emergent properties between communities at  
29 these two scales.

## 30 **Introduction**

31 Ecologists have historically been fascinated by the diversity of microbial communities  
32 (Hutchinson 1961), and several recent studies have indeed demonstrated differences in  
33 community structure between microbes and larger “macro” organisms (Nemergut et al. 2013,  
34 Hansen and Carey 2015, Locey and Lennon 2016, Shoemaker et al. 2017, Meyer et al. 2018).  
35 Generally, microbial communities have higher diversity that results in part from the large  
36 number of rare taxa (Neufeld and Lynch 2015). But, other properties, such as abundance of the  
37 most dominant taxon, are indistinguishable between communities at the two different scales  
38 (Locey and Lennon 2016). Despite increasing data on which to base these comparisons, the  
39 mechanisms generating these patterns of population distributions within and between  
40 communities are poorly understood (Shade 2017). One prominent additional difference between  
41 many of the microbial populations and macro populations in prior comparative studies is  
42 reproductive method; the microbial populations considered (bacteria, archaea, and most  
43 phytoplankton) reproduce asexually, while most macro populations considered have sexual  
44 reproduction. Here, I examine theoretically whether reproductive method can contribute to  
45 observed differences in community structure between asexually reproducing microorganisms and  
46 sexually reproducing macro organisms.

47 Individuals in sexually reproducing populations must encounter a mate before  
48 reproducing, whereas asexual individuals do not have this constraint. Mate finding and its  
49 consequences on population dynamics have been extensively studied in the theoretical literature  
50 (beginning with Volterra 1938), in part because it is one mechanism that causes Allee effects  
51 (reviewed in Gascoigne et al. 2009). An Allee effect is defined as positive density dependence  
52 within a population, meaning that individual-level growth rates increase as population density

53 increases (Odum and Allee 1954). When an Allee effect is present, the benefit of encountering  
54 another individual from the population outweighs negative interactions, such as competition, and  
55 individuals become more reproductively successful as density increases (Courchamp et al. 1999).  
56 In populations with sexual reproduction, sparse populations are slow growing due to the inability  
57 to find a mate. Mate encounters become more frequent as the population grows, such that per-  
58 capita fitness increases as density increases. The effects of mate finding on population growth  
59 are prominent when population sizes are small, but decrease when population size is large and  
60 mates are no longer limiting (Dennis 1989).

61         Many previous theoretical models have considered mate finding and Allee effects using  
62 differential or difference equations describing the population growth rate (Odum and Allee 1954,  
63 Dennis 1989, Boukal and Berec 2002). Strong reductions in birth rates due to mate limitation can  
64 cause population declines at low abundance, effectively setting a “critical density” below which  
65 the population becomes extinct (Gerritsen 1980). When the population size is greater than the  
66 critical density, the population continues to grow until reaching a stable equilibrium at its  
67 carrying capacity (Stephens et al. 1999). However, a major drawback of deterministic models is  
68 the inability to consider time to extinction for populations with a positive stable equilibrium;  
69 with deterministic equations, any population with a positive stable state will persist indefinitely.  
70 This result conflicts with the empirical observation that smaller populations are more vulnerable  
71 to extinction (Purvis et al. 2000).

72         Stochastic models are promising for studying demographic consequences of mate  
73 finding, because they allow for extinction in populations that would otherwise reach a positive  
74 carrying capacity (Lande 1993). Whereas the persistence of populations in deterministic  
75 equations is governed by local population growth rates around an equilibrium, persistence in

76 stochastic models depends on the growth rates at every density (Assaf and Meerson 2010). In  
77 other words, the chance of extinction in real populations is related to population growth rates  
78 near zero, which are important in stochastic models but rarely considered in deterministic  
79 models. Several forms of stochastic populations models have been used to study populations  
80 with Allee effects, often with discrete time models (Stephan and Wissel 1994, Allen et al. 2005,  
81 Sun 2016). These studies have concluded that diminished growth rates at low population  
82 densities can substantially decrease expected time to extinction (Stephan and Wissel 1994,  
83 Dennis 2002). However, it is computationally difficult to model multiple interdependent  
84 populations or populations with overlapping generations in discrete time models (Allen and  
85 Allen 2003), which is often a prohibitive barrier to such studies.

86         Here, I compare population and community dynamics between communities that must  
87 find mates before reproducing and communities where populations have no mate limitation (a  
88 case equivalent to asexual reproduction). I use stochastic models to evaluate demographic  
89 consequences of mate finding. First, I use continuous-time Markov chain models (CTMCs) to  
90 study how mate limitation alters time to extinction for single populations. These models use a  
91 computationally efficient simulation algorithm, which allows for simulation of multiple  
92 coexisting populations. Such models have been extensively used for simulating chemical  
93 reaction networks (Gillespie 2007), but can also be used for modeling population dynamics  
94 (Dobramysl et al. 2018). After obtaining the mean times to extinction obtained from these  
95 models, I use the island biogeography framework to evaluate how varying extinction times  
96 translate to changes in community diversity. The island biogeography framework posits that the  
97 expected long-term community diversity can be calculated by identifying the number of taxa  
98 where immigration and extinction rates are equal (MacArthur and Wilson 1967). In these

99 models, I assume identical immigration rates between the various communities, but extinction  
100 rates are a function of mate limitation. Finally, I simulate communities consisting of populations  
101 with differing growth rates to evaluate how consequences of mate limitation scale to  
102 heterogeneous communities. I show that the constraint of mate search decreases diversity,  
103 primarily by excluding rare taxa, whereas dominance of the most abundant population is  
104 unaffected.

105

## 106 **Methods**

### 107 *Single population dynamics*

108 First, I studied the effects of mate limitation on the time to extinction for single  
109 populations. I used CTMCs to evaluate time to extinction, implemented with the Gillespie  
110 algorithm (Gillespie 1977). Briefly, these models record births and deaths in a population as  
111 events that occur with varying frequency, depending on population size. Births are marked by the  
112 addition of a single individual to the population, whereas deaths remove a single individual. The  
113 overall rate at which any event (birth or death) occurs is the sum of the birth and death rates. The  
114 time until the next event is exponentially distributed with a parameter equal to the summed event  
115 rates. Therefore, as event rates increase, waiting time until the next event decreases. After  
116 drawing a random value from the exponential distribution for the time increment, the magnitudes  
117 of the instantaneous birth and death rates indicate whether a birth (add one individual) or death  
118 (remove one individual) is more likely to occur. Another random number is generated to  
119 determine whether a birth or death event transpires. After an individual is added or removed  
120 from the population, birth and death rates are updated based on the new population size, and the  
121 steps repeat. Extinction occurs at the first time point where the population equaled zero.

122 Throughout this study, I consider populations that are self limiting. In deterministic  
123 models, self-limiting populations experiencing logistic growth reach a stable carrying capacity  
124 determined by the intrinsic birth rate ( $b$ ) and the density-dependent death rate ( $d$ ) (Eq. 1).

125

$$\frac{dN}{dt} = bN - dN^2 \quad \text{Eq. 1}$$

126

127  
128 In the stochastic model formulation, births and deaths are modeled as discrete events,  
129 also referred to as “reactions” (Anderson and Kurtz 2011). The equivalent birth event rate for  
130 logistic growth model is equal to  $bN$  (Eq. 2):

131

$$\text{Event : } N \rightarrow N + 1 \quad \text{Rate : } bN \quad \text{Eq. 2}$$

132

133  
134 To study effects of mate limitation, I modified the birth event rate to include mate search.  
135 Previous work has yielded an equation governing the encounter rate between one individual and  
136 other individuals when searching in three-dimensional environments (Gerritsen and Strickler  
137 1977). Results for the two-dimensional case yield qualitatively equivalent results and are shown  
138 in the supplementary material. The mate encounter rate is dependent upon the speed at which  
139 individuals move ( $V$ ) and the radius at which they can detect a mate ( $R$ ). Here, I assume that  
140 males and females move at the same speed and that there is a 1:1 male to female ratio.

141 Multiplying the intrinsic birth rate ( $b$ ) by the probability that at least 1 mate will be encountered  
142 yields the following birth event rate for mate-limited populations (Eq. 3):

143

144 
$$Event : N \rightarrow N + 1 \quad Rate : bN \cdot \left(1 - e^{-2\pi R^2 VN/3}\right) \quad Eq. 3$$

145

146 The death rate functions used for the two cases were identical (Eq. 4):

147

148 
$$Event : N \rightarrow N - 1 \quad Rate : dN^2 \quad Eq. 4$$

149

150 As an illustration of the difference between deterministic and stochastic models, I  
151 investigated cases where long-term population dynamics of the various populations would be  
152 equivalent in the deterministic case; both mate-limited and non-limited populations would have  
153 nearly identical carrying capacities if these dynamics were translated to deterministic models. I  
154 simulated population trajectories and evaluated time to extinction using CTMCs. I used the two  
155 birth rates expressions above (Eq. 2 and 3) for the scenarios with and without mate limitation,  
156 and Eq. 4 for the death rate in all models. I recorded the mean time to extinction (MTE) for 1000  
157 simulated chains (populations).

158

### 159 *Evaluating diversity with island biogeography theory*

160 The theory of island biogeography formalized the concept that long-term community  
161 diversity is governed by the rate at which taxa enter the community (i.e. immigration) and the  
162 rate that taxa leave the community (i.e. extinction). In island biogeography models, the  
163 immigration rate and extinction rate of taxa within a community change as a function of the  
164 number of taxa currently present in a community (MacArthur and Wilson 1967). Thus, the  
165 expected diversity (defined here as the number of coexisting taxa) of the community is identified  
166 by finding the number of taxa where the immigration and extinction rates are equal. To evaluate



167 the accuracy of this analytical approximation for these simulations, I calculated the expected  
168 long-term diversity for a community consisting of populations with identical birth and death  
169 rates (and, therefore, identical MTEs).

170 I compared results of the analytical estimates of diversity to simulations of diversity in a  
171 stochastic reaction network model (coupled, simultaneous CTMCs) explicitly tracking each  
172 population. In the stochastic reaction network, the community-level immigration event rate was a  
173 function of current diversity. Immigration events were modeled as a population increasing from  
174 0 to a small population size, in this case of 2 individuals (Eq. 5):

175

$$176 \quad \text{Event : } 0 \rightarrow 2 \quad \text{Rate : } (100 - \text{Diversity}) \cdot i \quad \text{Eq. 5}$$

177

178 The immigration rate decreased linearly as a function of current community diversity, and  
179 reached zero when 100 taxa were present. I used an immigration constant of  $i = 0.001$ , and  
180 conducted these simulations across different parameters governing mate finding.

181

### 182 *Simulation of community structure with heterogeneous taxa*

183 To study how demographic consequences of mate finding scale to communities with  
184 heterogeneous populations, I simulated communities where populations had varying intrinsic  
185 growth rates ( $b$ ). I evaluated how changes in mate searching parameters affected the diversity,  
186 rarity, and dominance of taxa within the simulated communities. Taxa within each community  
187 experienced the same degree of mate limitation, which was determined by changing the values of  
188  $R$  and  $V$  over the range of 0.5 to 1.2. Across these combinations of search radius and search

189 speed, mate finding is limiting to population growth in small populations, but not limiting in  
190 large populations (those with 20 or more individuals).

191 For each combination of search radius and search speed for the mate limited populations,  
192 I simulated a stochastic reaction network where intrinsic birth parameters ( $b$ ) were randomly  
193 drawn from a lognormal distribution with a mean of the underlying normal distribution was 0  
194 and the standard deviation was 0.25. The death parameter ( $d$ ) was fixed at 0.1 for all populations.  
195 I used a lognormal distribution of growth rates, because a lognormal distribution provides a good  
196 fit to the observed abundance distributions of microbial taxa (Shoemaker et al. 2017).  
197 Additionally, populations defined by these growth rates routinely become extinct within  
198 computationally tractable time scales. However, I verified that simulation results were  
199 qualitatively similar when using a normal distribution of birth rates. Again, immigration was a  
200 linearly decreasing function of current diversity, where an immigration event was modeled as a  
201 change in population size from 0 to 2 (Eq. 5). Immigrant populations were assigned a new birth  
202 parameter from the lognormal distribution. After a burn in period of 5 000 000 events, I recorded  
203 instantaneous measurements of diversity, dominance (largest population size), and mean  
204 population size every 200 000 events. I compared results of simulations where mates were  
205 limiting to results of simulations where mates were not limiting.

206

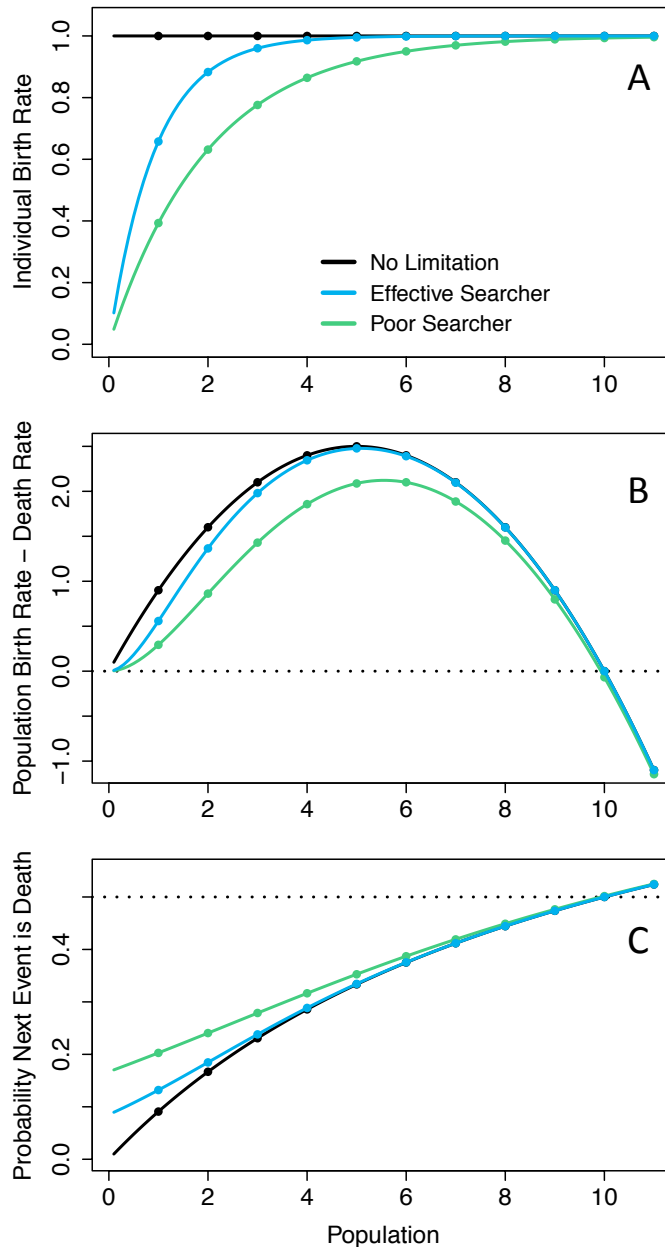
## 207 **Results**

### 208 *Single population dynamics*

209 First, I compared time to extinction for non-limited populations (i.e. asexual reproducers)  
210 and for sexual populations where mates must be encountered. For sexual populations, I evaluated  
211 two combinations of search radius and search speed. One scenario indicates a poor searcher with

212 low search radius and speed ( $R$  and  $V = 0.62$ ), and the other was a more effective searcher with  
213 higher search radius and speed ( $R$  and  $V = 0.8$ ). I chose parameter values that would yield  
214 equivalent long-term population dynamics if these populations were modeled deterministically;  
215 all three scenarios have nearly identical population densities where the birth rate equals the death  
216 rate, indicating equal carrying capacities in the absence of stochasticity (Fig. 1b). The per-capita  
217 birth rate is much higher in small populations for the asexually reproducing populations than for  
218 sexually reproducing populations (Fig. 1A). However, the birth rate in the sexual populations  
219 increased as individuals became more effective at finding mates. Multiplying individual birth  
220 rates by population size yields population-level birth minus death rates (Fig. 1B). Effects of mate  
221 limitation are prominent at small populations, but negligible as population size increases. With  
222 CTMC models, it is also possible to calculate the probability that the next event in the model will  
223 be a birth or a death. In models with asexual reproduction, it is highly unlikely that a death will  
224 occur in a small population. This probability of population decline at low population sizes is  
225 increased when mate limitation is present (Fig. 1C).

Fig. 1



226

227 **Figure 1:** Mate limitation decreases the individual level birth rate at low population density

228 (panel A), which influences both the population-level growth rate (panel B) and the probability

229 that the next event in the model will be a death (panel C). Effects of mate limitation on

230 population growth become negligible as population sizes increase, as visible by the convergence

231 of the three scenarios at larger populations. The effect of mate limitation is the difference  
232 between the asexually reproducing populations (black line) and the sexually reproducing  
233 populations (blue and green lines). Population growth rates are suppressed more strongly in poor  
234 mate searchers (green line) than effective mate searchers (blue line). The dashed line in C  
235 indicates a probability of 0.5, where a birth and death are equally likely.

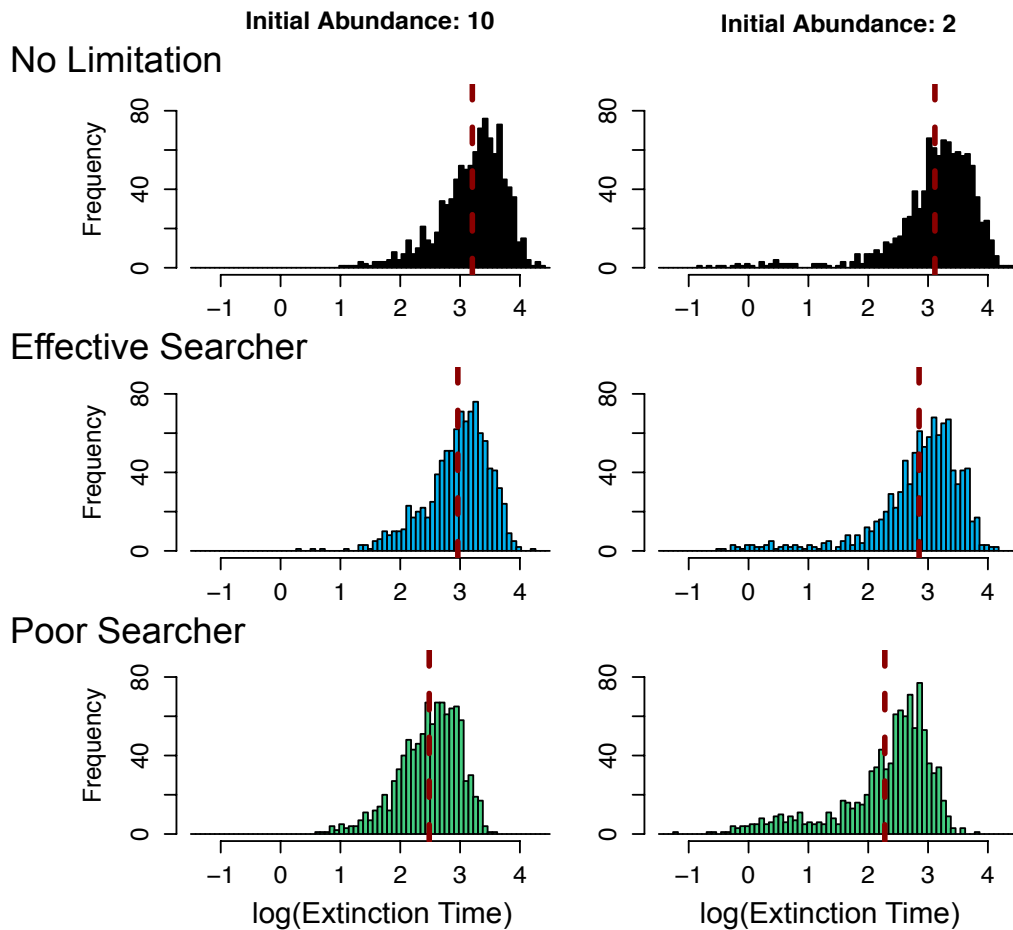
236

237 I recorded the time to extinction for 1000 simulated populations parameterized with the  
238 three birth rate scenarios shown in Fig 1. All populations had equivalent death rates. To evaluate  
239 the effect of initial conditions, I used initial population sizes of 10 and 2. Asexual populations  
240 persisted longest of the three scenarios, with a MTE of 2792 for an initial population size of 10  
241 and 2664 for an initial population size of 2. In mate-limited populations, MTEs for effective  
242 searchers were 1579 and 1557 (for initial size of 10 or 2 individuals), while for poor searchers  
243 MTEs were 515 and 477.

244 Across all populations, rapid extinction (very short MTE) was more common than when  
245 the initial population size was 2, rather than 10. The decrease in MTEs between populations with  
246 an initial population of 10 and an initial population of 2 is partially due to this higher frequency  
247 of very short times to extinction (Fig. 2).

248

Fig. 2



249

250 **Figure 2:** Times to extinction from 1000 simulated populations for communities with no mate  
251 limitation (top), mate limitation with effective searchers (middle), and mate limitation with poor  
252 searchers (bottom). Mean time to extinction (dashed red lines) decreases as the probability of  
253 encountering a mate decreases, and is therefore lowest for poor mate searchers. Time to extinction  
254 is also affected by initial population size, and decreases when the population is initiated with 2  
255 individuals (right column) as compared to 10 individuals (left column).

256

257 *Evaluating diversity with island biogeography theory*

258            Assuming that a community consisted of populations with identical birth and death rates,  
259 I calculated the estimated long-term diversity for the three birth rate scenarios from the  
260 associated extinction rates (shown in Fig. 2). The extinction rate for a single population is  
261  $1/\text{MTE}$ , meaning the extinction rate for a community of  $m$  taxa is  $m/\text{MTE}$ . I used the same rate of  
262 immigration in each scenario. The immigration rate was a linearly decreasing function of current  
263 diversity, and reached 0 when 100 taxa were present (Fig. 3). Thus, no more than 100 taxa could  
264 exist in the community. An approximation of long-term diversity under these assumptions can be  
265 found using the formula (Eq. 6). For the expected diversity calculations and associated  
266 simulations, I used an immigration constant  $i = 0.001$ , which determines the slope and intercept  
267 of the immigration function. However, the stochastic nature of the simulations means that these  
268 calculations will be inexact, because the populations never reach equilibrium.

269

$$\text{Expected Diversity} = \frac{100 i}{\frac{1}{\text{MTE}} + i} \quad \text{Eq. 6}$$

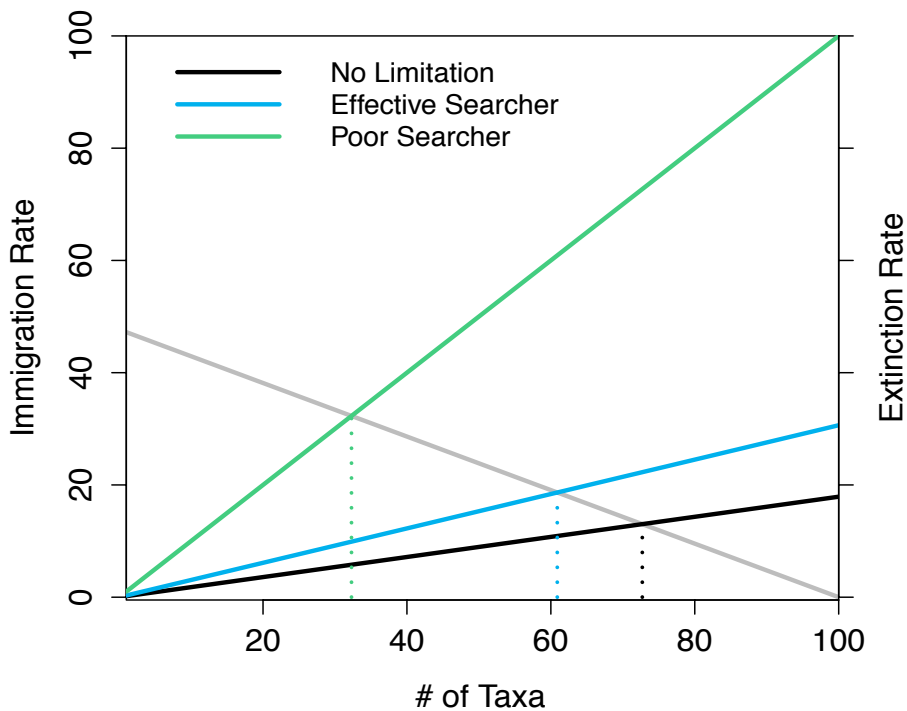
270

271

272 Eq. 6 shows that the long-term diversity is a function of mean time to extinction. As MTE  
273 approaches infinity, the expected diversity approaches the diversity level where immigration is  
274 zero (in this case, 100). Conversely, as MTE approaches zero, expected diversity also approaches  
275 zero. I evaluated the accuracy of this approximation using explicit simulations of simultaneously  
276 coexisting populations using the same parameters. The two estimates of diversity were within  
277 one unit (taxon). Approximations using Eq. 6 yielded expected long-term diversities of 72.6,  
278 60.9, and 32.3 for the scenarios of no limitation, effective searching, and poor searching, while  
279 simulations yielded long-term average diversities of 73.1, 61.8, and 32.9.

280

Fig. 3



281  
282 **Figure 3:** When using the same immigration function (grey line, left axis), mate limitation  
283 affects expected diversity by changing time to population extinction. A decreased time to  
284 extinction results in a greater slope in the community extinction rate (black, blue, and green lines  
285 on right axis). Expected diversity can be found by calculating the number of taxa where the  
286 immigration rate and extinction rate intersect (indicated by dashed lines). Communities are most  
287 diverse when there is no mate limitation (black lines). When populations are mate-limited, but  
288 individuals are effective at finding mates, there is a small decrease in expected diversity (blue  
289 lines). When individuals are poor searchers, there is a dramatic decline in diversity due to more  
290 rapid extinction (green lines). The time scale at which the immigration and extinction rates are  
291 shown here is the MTE of the shortest-lived populations (the poor searcher populations). Using a  
292 different time scale alters the y-axes, but does not change where lines intersect.  
293

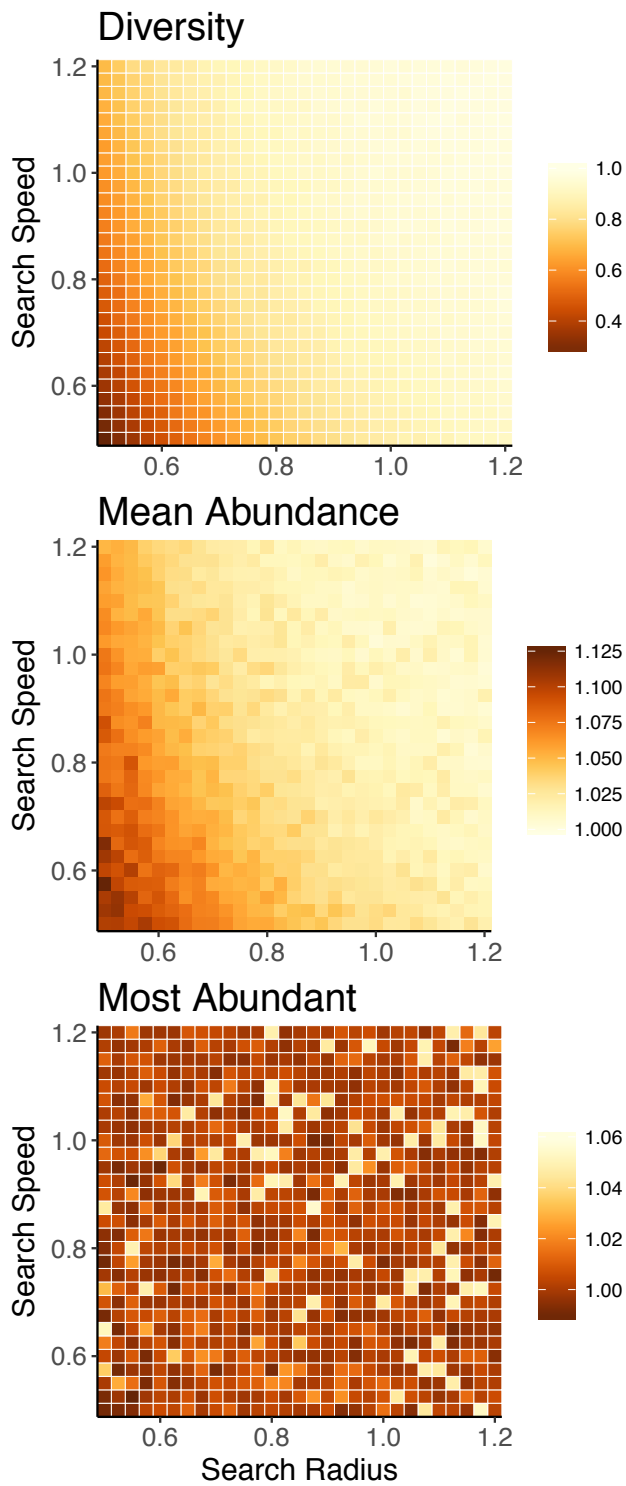


294 *Simulation of community structure with heterogeneous taxa*

295           Next, I simulated communities in which taxa had varied growth rates. I compared  
296 diversity, mean population size, and dominance (population size of most abundant taxon) of  
297 communities containing mate-limited taxa and those with non-limited taxa. When sexually  
298 reproducing taxa were highly effective searchers due to high search radius and/or speed, mate  
299 limitation had little effect on the effective birth rates of those taxa. Then, diversity and  
300 population size converged to results from communities containing asexual populations (Fig. 4).  
301 However, the abundance of the most dominant population was not affected by search efficacy or  
302 reproductive method (Fig. 4). Cells in the figure panels are scaled by the values from the no-  
303 limitation simulations (a value of 1 indicates equivalent results).

304

Fig. 4



305

306 **Figure 4:** Heatmaps show average diversity (top), abundance (middle), and dominant population

307 size (bottom) from simulations of mate-limited communities where populations have varying

308 search radius and search speed. Cells within each heatmap show the results for communities  
309 consisting of populations with the given search radius and search speed. Cell values are scaled by  
310 results from communities without mate limitation. Thus, a value of 1 indicates results equivalent  
311 to those of non-limited communities. The diversity (top) and mean abundance size (middle)  
312 within communities containing mate-limited populations change in response to mate limitation,  
313 as stronger mate limitation corresponds to decreased diversity and larger average population size.  
314 When the search radius and search speed are large, the probability of finding a mate approaches  
315 1, and results from mate-limited and non-limited communities converge. However, the size of  
316 the most abundant population (bottom) is not affected by mate limitation.

317

318 Communities containing the poorest mate searchers experienced the greatest declines in  
319 diversity, in comparison to the communities with asexual populations. The communities with the  
320 strongest mate search limitation ( $R = 0.5$  and  $V = 0.5$ ) had mean diversity of 17.7 taxa;  
321 conversely, communities where mates were not limiting had mean diversity of 68.5 taxa.  
322 Similarly, mean population size was 11.2 for the most mate-limited communities, but 10.2 in  
323 non-limited communities. Another measurement of rareness, the skewness of population  
324 abundances, showed a similar result (see supplementary material). Higher skewness indicates  
325 more rare taxa. Average skewness in the distribution of population sizes was 0.18 in  
326 communities with greatest mate limitation, and 0.57 in non-limited communities. However, the  
327 abundance of the largest population was not consistently related to mate limitation. The  
328 abundance of the dominant population in communities with mate-limited populations could be  
329 higher or lower than the dominant population in the non-limited community.

330 Multiple regression analyses showed that mate search speed and search radius explained  
331 approximately 90% of variation in diversity and mean abundance in the communities with mate-  
332 limited sexual populations (see supplementary material). In contrast, search radius and search  
333 speed explain only 1% of variation in dominance (maximum population size). In mate-searching  
334 populations, the same degree of limitation could be generated with different combinations of  
335 search radius and search speed. Any combination of  $R$  and  $V$  that produces a constant value of  
336  $VR^2$  yields an equivalent probability of encountering a mate (Eq. 3).

337

## 338 **Discussion**

339 This study shows that the constraint of mate finding influences emergent community  
340 properties, including diversity and average population size. Mate limitation strongly suppresses  
341 birth rates when populations are small (Fig. 1), leading to a higher probability that sexually  
342 reproducing populations will decline when rare. These discrepancies in birth rate lead to shorter  
343 times to extinction in taxa that must find a mate, versus those that reproduce asexually (Fig. 2).  
344 This effect is particularly strong when populations are introduced at low density, which is a  
345 plausible scenario when considering newly established populations. In stochastic simulations,  
346 communities consisting of asexual taxa maintained greater diversity due to a longer expected  
347 persistence time of each population (Fig. 3). In the case where immigration is a linear function of  
348 current diversity, expected diversity increases as MTE increases (Eq. 6). When these simulations  
349 were extended to communities with heterogeneous taxa, differences in diversity were amplified,  
350 because mate limitation had especially strong negative effects on taxa with already-low growth  
351 rates (Fig. 4). Thus, mate limitation decreased the number of coexisting taxa, primarily by  
352 excluding low abundance taxa. However, mate limitation has minimal consequences in larger

353 populations, and therefore the population size of the most abundant taxon was not related to  
354 reproductive method or mate search efficacy.

355         The degree of mate limitation is a function of search ability, which is determined by  
356 search radius and search speed. As either search variable (radius or speed) increases, the  
357 probability of finding a mate approaches 1, indicating no limitation to the population birth rate.  
358 In this case, simulation results of sexual populations with mate finding converge to those of  
359 populations without limitation. This is also evident when looking at per capita and population  
360 growth curves (Fig. 1). Birth rates asymptotically reach the no-limitation case as mate search  
361 becomes more effective. Simulation results are qualitatively similar if instead considering a two-  
362 dimensional search (see supplemental material). As in the three-dimensional scenario, mate  
363 limitation led to lower diversity and rarity, but had no effect on maximum population size. When  
364 searching for mates in three dimensions, the search radius has greater consequence of finding a  
365 mate than the search speed. This observation suggests that a trait affecting search radius (such as  
366 eyesight) might have a greater fitness effect in a 3D environment than a trait affecting search  
367 speed (such as swimming velocity); an incremental increase in search radius would lead to a  
368 greater effective birth rate than an increase in search speed. Finally, this study highlights the  
369 utility of stochastic models for studying community structure. Deterministic models show  
370 equivalent long-term dynamics for populations with the same carrying capacity, whereas these  
371 stochastic models show pronounced differences. Thus, this study is concordant with prior models  
372 showing that Allee effects increase extinction rates (Brassil 2001, Leibhold and Bascompte 2003,  
373 Dennis et al. 2016), and further demonstrates that these population-level effects alter emergent  
374 community properties.

375           Results from these models mirror empirical findings that microbial populations (with  
376 asexual reproduction) tend to be high in diversity and rarity, although not distinct from other  
377 communities in the dominance of abundant taxa (Locey and Lennon 2016). In these simulated  
378 communities, eliminating the constraint of mate finding translated to greater diversity with a  
379 higher frequency of low-abundance taxa, while the population size of the most abundant taxon  
380 was unaffected (Fig. 4). Thus, allowing for mate finding generates a parsimonious explanation  
381 for the community-level patterns observed in comparisons of micro and macro ecological  
382 communities. Using empirical data to probe the hypothesis that mate limitation constrains  
383 diversity and rarity illustrates the plausibility of this explanation. For example, one study used  
384 very deep 16S amplicon sequencing to evaluate whether marine bacterial populations that  
385 appeared to only be present seasonally were, instead, consistently present at abundances below  
386 the usual detection limit (Caporaso et al. 2012). At a depth of approximately 11 million  
387 sequences, 48% of sequences appeared only once (Caporaso et al. 2012). Similarly, another  
388 study used deep sequencing of human gut samples to generate rarefaction curves illustrating how  
389 many taxa were observed in response to sequencing depth. New taxa continued to be identified  
390 after one million sequences were recovered (Turnbaugh et al. 2010). Even if many of the  
391 observed rare taxa are the product of sequencing errors, these findings suggest persistence of  
392 extremely low abundance taxa (fewer than 1 individual per million). For sexually reproducing  
393 populations, these relative abundances could be prohibitively low for individuals to find mates  
394 within a lifetime. Finally, asexual reproduction in microbes makes it possible for single  
395 individuals to establish populations in new environments. Given the demonstrated plausibility of  
396 immigration from microbial seed banks (Lennon and Jones 2011, Caporaso et al. 2012), the  
397 growth rate of small populations is especially relevant for the persistence of microbial taxa.

398           Diversity is a common outcome variable in ecological studies, though there is ongoing  
399   debate about how diversity is related to community function (Shade 2017). In the models studied  
400   here, diversity is a byproduct of population demographics, including birth rate and mate search  
401   ability. More generally, these models show that diversity can be affected by neutral and  
402   stochastic processes. Subsequent empirical studies using diversity as an outcome variable might  
403   also collect information about immigration and extinction rates to determine whether diversity  
404   reflects these processes. For example, surveys of human-associated microbial communities have  
405   found variation in diversity across body sites (Caporaso et al. 2011, Huttenhower et al. 2012).  
406   Gut and oral bacterial communities are especially diverse (Huttenhower et al. 2012), but these  
407   habitats could conceivably have higher immigration rates than other body sites due to daily  
408   introduction of bacteria within food (David et al. 2014). Similarly, a recent study found little  
409   evidence that fungi can persist within the healthy human gut, but still identified hundreds of  
410   fungal taxa in stool samples (Auchtung et al. 2018). The high diversity of fungi in the human gut,  
411   in spite of their inability to colonize this habitat, was attributed to persistent immigration of fungi  
412   on ingested foods (Auchtung et al. 2018). These studies, coupled with the modeling results  
413   presented here, demonstrate how diversity could change independently of community function.

414           Mathematical models serve as an unbiased line of inference for linking mechanisms to  
415   emergent community properties. In the context of comparing community structures of microbial  
416   communities and macro-organisms, the differing empirical methods used to study communities  
417   at these two scales can generate spurious patterns. Thus, it is hard to discern which findings are  
418   true distinctions between micro and macro communities, and which are artifacts of methodology.  
419   For instance, DNA sequence similarity is often used to define microbial taxa, whereas macro  
420   organisms are generally identified using direct observations. The differences in error rate and

421 detection limits between these two methods could also explain the higher diversity and rarity in  
422 microbial communities. Several steps in the workflow of generating 16S amplicon data,  
423 including variation in sample processing and sequencing errors, can generate observations of  
424 artifactual rare taxa (Fouhy et al. 2016). Furthermore, macro-organisms are often identified using  
425 morphological characteristics, but many more taxa can be differentiated if instead using DNA  
426 sequencing methods (Fontaneto et al. 2009). Thus, if methodology is a confounding factor when  
427 comparing communities, there is uncertainty about whether observed differences are spurious  
428 due to sampling bias. Theoretical studies can therefore reinforce empirical findings by  
429 determining, with unbiased methodology, whether an identified mechanism can reproduce  
430 observed community structure. These modeling results indicate that there are expected  
431 differences in diversity and average population size when comparing communities consisting of  
432 taxa that reproduce sexually versus asexually.

433

#### 434 **Acknowledgements**

435 This work was funded by a postdoctoral fellowship from the Harvard Data Science Initiative. I  
436 received valuable feedback from Michael Baym, Anurag Limdi, and members of the Baym lab.  
437 The research computing staff at Harvard Medical School provided technical support for model  
438 simulations.



439 **References**

- 440 Allen LJS, Allen EJ. A comparison of three different stochastic population models with regard to  
441 persistence time. *Theoretical Population Biology* 2003; **64**: 439–449.
- 442 Allen LJS, Fagan JF, Högnäs G, Fagerholm H. Population extinction in discrete-time stochastic  
443 population models with an Allee effect. *Journal of Difference Equations and*  
444 *Applications* 2005, **11**, 273-293.
- 445 Anderson DF, Kurtz TG. Continuous Time Markov Chain Models for Chemical Reaction  
446 Networks. *Design and Analysis of Biomolecular Circuits*. 2011. Springer, New York,  
447 NY, pp 3–42.
- 448 Assaf M, Meerson B. Extinction of metastable stochastic populations. *Physical Reviews E* 2010;  
449 **81**: 021116.
- 450 Auchtung TA, Fofanova TY, Stewart CJ, Nash AK, Wong MC, Gesell JR, et al. Investigating  
451 Colonization of the Healthy Adult Gastrointestinal Tract by Fungi. *mSphere* 2018; **3**:  
452 e00092-18.
- 453 Boukal DS, Berec L. Single-species Models of the Allee Effect: Extinction Boundaries, Sex  
454 Ratios and Mate Encounters. *Journal of Theoretical Biology* 2002; **218**: 375–394.
- 455 Brassil CE. Mean time to extinction of a metapopulation with an Allee effect. *Ecological*  
456 *Modelling* 2001; **143**: 9–16.
- 457 Caporaso JG, Lauber CL, Costello EK, Berg-Lyons D, Gonzalez A, Stombaugh J, et al. Moving  
458 pictures of the human microbiome. *Genome Biology* 2011; **12**: R50.
- 459 Caporaso JG, Paszkiewicz K, Field D, Knight R, Gilbert JA. The Western English Channel  
460 contains a persistent microbial seed bank. *The ISME Journal* 2012; **6**: 1089–1093.

- 461 Consortium THMP, Huttenhower C, Gevers D, Knight R, Abubucker S, Badger JH, et al.  
462 Structure, function and diversity of the healthy human microbiome. *Nature* 2012; **486**:  
463 207–214.
- 464 Courchamp F, Clutton-Brock T, Grenfell B. Inverse density dependence and the Allee effect.  
465 *Trends in Ecology & Evolution* 1999; **14**: 405–410.
- 466 David LA, Maurice CF, Carmody RN, Gootenberg DB, Button JE, Wolfe BE, et al. Diet rapidly  
467 and reproducibly alters the human gut microbiome. *Nature* 2014; **505**: 559–563.
- 468 Dennis B. Allee effects: population growth, critical density, and the chance of extinction.  
469 *Natural Resource Modeling* 1989; **3**: 481-538.
- 470 Dennis B. Allee Effects in Stochastic Populations. *Oikos* 2002; **96**: 389–401.
- 471 Dennis B, Assas L, Elaydi S, Kwessi E, Livadiotis G. Allee effects and resilience in stochastic  
472 populations. *Theoretical Ecol* 2016; **9**: 323–335.
- 473 Dobramysl U, Mobilia M, Pleimling M, Täuber UC. Stochastic population dynamics in spatially  
474 extended predator–prey systems. *Journal of Physics A: Mathematical and Theoretical*  
475 2018; **51**: 063001.
- 476 Fontaneto D, Kaya M, Herniou EA, Barraclough TG. Extreme levels of hidden diversity in  
477 microscopic animals (Rotifera) revealed by DNA taxonomy. *Molecular Phylogenetics*  
478 *and Evolution* 2009; **53**: 182–189.
- 479 Fouhy F, Clooney AG, Stanton C, Claesson MJ, Cotter PD. 16S rRNA gene sequencing of mock  
480 microbial populations- impact of DNA extraction method, primer choice and sequencing  
481 platform. *BMC Microbiology* 2016; **16**: 123.
- 482 Gascoigne J, Berec L, Gregory S, Courchamp F. Dangerously few liaisons: a review of mate-  
483 finding Allee effects. *Population Ecology* 2009; **51**: 355–372.

- 484 Gerritsen J. Sex and Parthenogenesis in Sparse Populations. *The American Naturalist* 1980; **115**:  
485 718–742.
- 486 Gerritsen J, Strickler JR. Encounter Probabilities and Community Structure in Zooplankton: a  
487 Mathematical Model. *Journal of the Fisheries Research Board of Canada* 1977; **34**: 73–  
488 82.
- 489 Gillespie DT. Exact stochastic simulation of coupled chemical reactions. *The journal of physical*  
490 *chemistry*, 1977; **81**: 2340-2361.
- 491 Gillespie DT. Stochastic Simulation of Chemical Kinetics. *Annual Review of Physical Chemistry*  
492 2007; **58**: 35–55.
- 493 Hansen GJA, Carey CC. Fish and Phytoplankton Exhibit Contrasting Temporal Species  
494 Abundance Patterns in a Dynamic North Temperate Lake. *PLOS ONE* 2015; **10**:  
495 e0115414.
- 496 Hutchinson GE. The paradox of the plankton. *The American Naturalist* 1961; **882**: 137-145.
- 497 Lande R. Risks of Population Extinction from Demographic and Environmental Stochasticity  
498 and Random Catastrophes. *The American Naturalist* 1993; **142**: 911–927.
- 499 Lennon JT, Jones SE. Microbial seed banks: the ecological and evolutionary implications of  
500 dormancy. *Nature Reviews Microbiology* 2011; **9**: 119–130.
- 501 Ley RE, Peterson DA, Gordon JI. Ecological and Evolutionary Forces Shaping Microbial  
502 Diversity in the Human Intestine. *Cell* 2006; **124**: 837–848.
- 503 Liebhold Andrew, Bascompte Jordi. The Allee effect, stochastic dynamics and the eradication of  
504 alien species. *Ecology Letters* 2003; **6**: 133–140.
- 505 Locey KJ, Lennon JT. Scaling laws predict global microbial diversity. *PNAS* 2016; **113**: 5970–  
506 5975.

- 507 Lynch MDJ, Neufeld JD. Ecology and exploration of the rare biosphere. *Nature Reviews*  
508 *Microbiology* 2015; **13**: 217–229.
- 509 MacArthur RH, Wilson EO. The Theory of Island Biogeography. 1967. Princeton University  
510 Press.
- 511 Meyer KM, Memiaghe H, Korte L, Kenfack F, Alonso A, Bohannan BJM. Why do microbes  
512 exhibit weak biogeographic patterns? *The ISME Journal* 2018.
- 513 Nemergut DR, Schmidt SK, Fukami T, O’Neill SP, Bilinski TM, Stanish LF, et al. Patterns and  
514 Processes of Microbial Community Assembly. *Microbiology and Molecular Biology*  
515 *Reviews* 2013; **77**: 342–356.
- 516 Odum HT, Allee WC. A Note on the Stable Point of Populations Showing Both Intraspecific  
517 Cooperation and Disoperation. *Ecology* 1954; **35**: 95–97.
- 518 Purvis A, Gittleman JL, Cowlshaw G, Mace GM. Predicting extinction risk in declining species.  
519 *Proceedings of the Royal Society of London B: Biological Sciences* 2000; **267**: 1947–  
520 1952.
- 521 Shoemaker WR, Locey KJ, Lennon JT. A macroecological theory of microbial biodiversity.  
522 *Nature Ecology & Evolution* 2017; **1**: 0107.
- 523 Shade A. Diversity is the question, not the answer. *The ISME Journal* 2017; **11**: 1–6.
- 524 Stephens PA, Sutherland WJ, Freckleton RP. What Is the Allee Effect? *Oikos* 1999; **87**: 185–  
525 190.
- 526 Stephan T, Wissel C. Stochastic extinction models discrete in time. *Ecological Modelling* 1994;  
527 **75–76**: 183–192.
- 528 Sun G-Q. Mathematical modeling of population dynamics with Allee effect. *Nonlinear*  
529 *Dynamics* 2016; **85**: 1–12.

- 530 Turnbaugh PJ, Quince C, Faith JJ, McHardy AC, Yatsunencko T, Niazi F, et al. Organismal,  
531 genetic, and transcriptional variation in the deeply sequenced gut microbiomes of  
532 identical twins. *PNAS* 2010; **107**: 7503–7508.
- 533 Volterra V. Population growth, equilibria, and extinction under specified breeding conditions: a  
534 development and extension of the theory of the logistic curve. *Human Biology* 1938; **10**:  
535 1–11.
- 536 Xu Z, Knight R. Dietary effects on human gut microbiome diversity. *British Journal of Nutrition*  
537 2015; **113**: S1–S5.