



## 7 Summary

8 A diverse immune repertoire is considered a hallmark of good health. However, other things  
9 being equal, current methods for measuring repertoire diversity do not distinguish between a  
10 repertoire that is composed of similar sequences, clonotypes, or clones and a repertoire that is  
11 composed of different ones, even though the latter is intuitively more diverse. Here we describe  
12 a framework for incorporating similarity into diversity measures, and illustrate using ... define  
13 diversity with binding similarity as functional diversity, and measure functional diversity on 391  
14 large-scale antibody and T-cell receptor (TCR) repertoires. We find that while repertoires often  
15 contain millions of unique sequences, functional diversity reveals a landscape defined by at  
16 most a few thousand unrelated CDR3 binding targets. Naïve/IgM repertoires have more unique  
17 sequences than memory/IgG, but memory/IgG repertoires are more functionally diverse.  
18 Functional diversity is sensitive to vaccination, infection, and aging, and unlike raw diversity is  
19 robust to sampling error. Finally, according to functional diversity, repertoires from different  
20 people overlap significantly, suggesting a definable ceiling for the functional diversity of  
21 humanity. Similarity redefines diversity in complex systems.

## 22 Introduction

23 Immune repertoires are famously diverse. Collectively, a person's  $\sim 10^{12}$  B and T cells express  
24 many millions of unique recombined antibody and TCR genes as part of millions of clonal  
25 lineages, more unique sequence than in the entire germline genome (Jiang et al., 2013). At the  
26 sequence level, the repertoires of any two people overlap by only a fraction of a percent,  
27 indicating still higher diversity in the population (Arnaout et al., 2011; Robins et al., 2010). Yet  
28 repertoires are formed from V, D, and J gene segments that almost all people share and that  
29 are expressed at similar frequencies across individuals targets (de Bourcy et al., 2017; DeWitt  
30 et al., 2016), and repertoires are shaped by similar antigenic exposures and a consequent need  
31 to recognize and bind similar. Squaring the diversity that is seen with the similarity that must  
32 exist is a major goal in immunology.

33 This goal has relevance for disease stratification and clinical management across a range of  
34 conditions. B- and T-cell diversity fall with age, as specific exposures expand a few select  
35 lineages at the expense of others (Messaoudi et al., 2004). Chronic infection appears to have a  
36 similar effect, impairing vaccination (Jiang et al., 2013). Low B-cell diversity is associated with  
37 physiological frailty, a syndrome seen alongside conditions that are traditionally considered to  
38 be unrelated to adaptive immunity (e.g., atherosclerotic cardiovascular disease), independent of  
39 chronological age (Gibson et al., 2009). In cancer, a rise in sequence-level T-cell diversity is  
40 thought to predict a successful response to immune-checkpoint inhibitors, drugs that make  
41 tumors more visible to the immune system (Hopkins et al., 2018).

42 Traditionally, diversity has been measured as a simple count of the number of different  
43 sequences, lineages, or clones in a sample, a measure known formally as species richness.  
44 However, species richness ignores a key feature of repertoire diversity, species frequency: the  
45 fact that some sequences are common and others rare. In an intuitive sense, a repertoire with a  
46 single dominant (e.g., leukemic) clone is less diverse than a repertoire that has the same  
47 number of clones but no dominant clone. To incorporate frequency into measurements of  
48 diversity, there exist a family of measures that includes Shannon entropy, the Simpson index  
49 (and related Gini coefficient), and the Berger-Parker index (Hill, 1973). These differ from each  
50 other in how much weight they place on frequency: i.e., how much more a large clone adds to  
51 the total diversity than a small one. Mathematically, weight can be represented as a parameter,  
52  $q$ , in the so-called Hill framework, a master equation for diversity in which species richness,  
53 Shannon entropy, the Simpson index, and the Berger-Parker index, among others, have been

54 shown to correspond to different values of  $q$  ( $q=0, 1, 2,$  and  $\infty,$  respectively). It is understood  
55 that no single diversity measure is best: the different measures provide complementary  
56 information about a given complex system (Morris et al., 2014). Robust methods exist for  
57 correcting sampling error for species richness and the frequency-weighted measures, and these  
58 methods are becoming standard for measuring immunological diversity (Greiff et al., 2015;  
59 Kaplinsky and Arnaout, 2016).

60 However, there is a second key feature of repertoire diversity that the frequency-weighted  
61 measures fail to capture: species similarity. A repertoire made up of all-different sequences is  
62 intuitively more diverse than a repertoire that has the same number of sequences, present at  
63 the same frequencies as in the first repertoire, but all drawn from the same lineage or clone. In  
64 the literature, this fact is sometimes addressed indirectly by grouping sequences together before  
65 measuring diversity, for example by clustering reads, collapsing clones, or binning by V(D)J  
66 segment usage (DeWitt et al., 2016; Jiang et al., 2013; Ju et al., 2018; Kaplinsky et al., 2014;  
67 Vollmers et al., 2013). However, grouping usually imposes a binary threshold—in or out—on  
68 what is by nature a continuous and overlapping relationship among sequences and their  
69 encoded proteins. Grouping also usually zeros out or ignores any diversity that might exist  
70 within groups. It is unclear what is lost by ignoring similarity, or what might be gained from a  
71 more complete synthesis of diversity with similarity. This is true not only for the immunome, but  
72 for other complex systems such as microbiomes, images, and tissues. Here we sought to  
73 develop and explore a continuous framework for measuring diversity-with-similarity on B- and T-  
74 cell repertoires.

## 75 **Results**

76 **Framework.** We measured diversity-with-similarity on high-throughput B- and T-cell repertoires  
77 using a robust mathematical framework initially proposed for studying diversity in ecology and  
78 environmental settings (Leinster and Cobbold, 2012). This framework provides “with-similarity”  
79 counterparts for species richness and the frequency-weighted diversity measures: species  
80 richness with similarity ( ${}^0D_s$ , which places a very small weight on frequency, and  ${}^\infty D_s$ , which like  
81  ${}^0D$  ignores frequency altogether), the exponential form of entropy with similarity ( ${}^1D_s$ , henceforth  
82 simply “entropy with similarity,” and likewise for other named indices), and so on. In  ${}^qD_s$   
83 notation,  $q$  is the frequency-weighting parameter,  $D_s$  denotes diversity-with-similarity, and  $D$   
84 without the subscript means diversity without similarity, which we refer to as “raw diversity.”

85 Mathematically, the key innovation of diversity-with-similarity relative to raw diversity is inclusion  
86 of a similarity matrix whose entries quantify how similar each pair of species (sequences,  
87 clonotypes, etc.) is. Constructing the similarity matrix necessitates a choice of similarity  
88 measure. (Note the difference between similarity measures and diversity measures: a similarity  
89 measure is used to build the similarity matrix, which then is used to calculate diversity  
90 measures.) The choice of similarity measure depends on the biological feature(s) of interest. For  
91 example to study somatic hypermutation, one might use the Hamming distance. Just as  
92 different weightings provide complementary information about rare vs. frequent species—for  
93 example, the number of new thymic emigrants (species richness;  ${}^0D$  or  ${}^{\infty}D_s$ ) vs. large leukemic  
94 clones (Berger-Parker index;  ${}^{\infty}D$ )—different similarity measures are expected to reveal different  
95 systems-level features of repertoires' sequence-level configuration. Also as with raw diversity  
96 measures, expressing results for the new diversity-with-similarity measures as effective  
97 numbers, also known as number equivalents, (Macarthur, 1965; Hill, 1973; Jost, 2007; Marion  
98 et al., 2015), as opposed to as bits or nats (for  ${}^1D_s$ ) or as various fractions (for  ${}^{>1}D_s$ ), makes it  
99 possible to compare them to each other, regardless of weighting or similarity measure, on a  
100 single intuitive scale (Box 1).

101 **Similarity measure.** We were interested in the single most fundamental mechanistic feature of  
102 antibodies and TCRs: binding to specific targets (Fig. 1). Therefore for our similarity measure,  
103 we used a proxy for binding affinity that follows from the empirically observed changes in  
104 dissociation constant ( $K_d$ ) associated with amino-acid substitution in antibody and TCR CDRs  
105 (Jankauskaite et al., 2018). We found that on average, a single amino-acid substitution at an  
106 antibody-antigen or TCR-peptide binding surface lowers affinity by 4-5 fold (geometric mean),  
107 with a long tail corresponding to rare orders-of-magnitude effects (Fig. 2a). We focused on  
108 CDR3, the third complementarity determining region, of IgH and TCR $\beta$ , since this is the single  
109 most important contributor to binding specificity (Xu and Davis, 2000); however, our approach  
110 can be applied to other regions. Because the relationship between sequence and specificity  
111 remains non-predictive and therefore complex, for any given sequence pair the similarity  
112 imputed from the observed distribution will be approximate; however, averaged over the many  
113 millions of pairs in each repertoire, it was expected to be a reasonably accurate first-pass  
114 repertoire-level view of immunological diversity with binding similarity.

115 Using this similarity measure, diversity-with-similarity is interpreted as the effective number of  
116 sequences in a repertoire if the sequences were equally common and had no binding overlap  
117 with each other (Box 1), or equivalently, the number of equally common non-overlapping binding

118 targets that a repertoire can recognize. We therefore refer to this version of diversity-with-  
119 similarity as "functional diversity" (Fig. 1). Functional diversity can be interpreted in the context  
120 of a "shape space" (Perelson and Oster, 1979) that contains all possible CDR3 binding targets,  
121 with nearby targets having similar three-dimensional shapes and conformations (Fig. 1a). Each  
122 CDR3 binds a (possibly overlapping) subset of targets; together, a repertoire's CDR3s cover  
123 some part of shape space (Fig. 1b). Functional diversity measures the size of this region,  
124 controlling for similarity and overlap in binding among different CDR3s (Fig. 1c).

125 **Validity.** We first established that our similarity measure behaved sensibly, with closely related  
126 sequences scoring high and unrelated sequences scoring low (Fig. 2b). We next established  
127 that it resulted in intuitive values for functional similarity by testing against expectations on  
128 simple *in silico* repertoires. In a representative test, we constructed four repertoires with 34  
129 unique sequences each and 752 sequences total (Fig. 2c-d). In each repertoire, a few  
130 sequences were common (larger circles) while most were rare (smaller circles), representing  
131 the long-tailed frequency distribution seen in real repertoires (Arnaout et al., 2011; Weinstein et  
132 al., 2009). Importantly, the species-frequency distribution for all four repertoires was identical,  
133 meaning that raw diversity was also identical across the repertoires, for all frequency  
134 weightings. The only difference between the repertoires was in the pairwise similarity among  
135 sequences.

136 For the first repertoire (Fig. 2d, top row), we chose closely related sequences from a single real-  
137 world CDR3 clone. We expected that species richness with similarity—functional species  
138 richness—would be close to 1. (We used  ${}^0D_s$  here;  ${}^{\infty}D_s$  performed similarly.) We observed a  
139 value of 1.5; the extra 0.5 reflected sequence diversity within the clone. For the second  
140 repertoire (Fig. 2d, second row), we swapped out half the unique CDR3s with CDR3s from a  
141 different, unrelated real-world clone. As expected, we observed a rough doubling of functional  
142 diversity, to 2.4. For the third repertoire (Fig. 2d, third row), we replaced all the sequences with  
143 34 randomly chosen real-world CDR3s. We expected a functional diversity that was much  
144 higher than in the first two repertoires but less than 34 because of the inherent sequence  
145 similarities that make a CDR3 a CDR3, and, consistent with this expectation, observed a value  
146 of 22. For the final repertoire (Fig 2d, bottom row), we replaced the CDR3s with random amino-  
147 acid sequences (controlling for length), expecting a functional similarity of nearly 34, and this  
148 again was observed ( ${}^0D_s=32$ ). In contrast to these differences in functional diversity, raw  
149 diversity was indistinguishably 34 for all four repertoires. In every example, functional diversity fit  
150 an intuitive sense of what diversity should mean (Fig. 2c), while raw diversity failed to detect a

151 difference. These results support the validity of our functional-diversity framework for immune  
152 repertoires.

153 **Robustness.** Sampling error—the “missing-species” problem (Bunge and Fitzpatrick,  
154 1993(Bunge and Fitzpatrick, 1993))—is known to be a major potential confounder when  
155 measuring raw diversity, necessitating large (e.g.) blood volumes and/or post-hoc statistical  
156 correction for measurements on the sample to reflect repertoire diversity in the individual as a  
157 whole (Kaplinsky and Arnaout, 2016). A practical feature of functional similarity is that the  
158 values are smaller than those of raw diversity (reflecting clustering of similar sequences; Fig. 1-  
159 2). The effective coverage is therefore greater, meaning that less information about the  
160 functional diversity of an individual’s overall repertoire is lost upon sampling than is the case for  
161 raw diversity. This observation suggested that functional diversity is more robust to sampling  
162 error, possibly even making it accurate enough to use without statistical correction, and thus  
163 useful for the sample sizes typically available for sequencing (10,000-1 million cells).

164 To test this possibility, we systematically downsampled from a representative TCR $\beta$  repertoire  
165 and two representative IgH (IgG) repertoires, one prepared from mRNA and one from genomic  
166 DNA, each with  $\sim 10^6$  unique sequences, and compared raw vs. functional diversity on the  
167 subsamples to those of the full sample (Fig. 3). (We wished to consider possibility of lower  
168 diversity from mRNA than DNA, since transcriptionally less active cells may be less likely to be  
169 sampled.) For TCR $\beta$ , we found that functional species richness saturated at a sample size of  
170  $\sim 30,000$  sequences and functional entropy at  $\sim 10,000$  sequences (Fig. 3b, first and third  
171 columns). Functional diversity for higher frequency weightings ( $q$ ) saturated with even fewer  
172 sequences. For IgH from mRNA, functional species richness did not saturate but did plateau,  
173 with a final increase of  $\leq 2$  percent per order of magnitude. Assuming that each unique sequence  
174 corresponds to a cell and  $10^{10}$  B cells in the body, this final measured rate of increase means  
175 that the individual's total functional species richness is no more than 50 percent higher than the  
176 value measured on the sample (Fig. 3, middle row). This is the maximum expected sampling  
177 error. For IgH from DNA, functional species richness had begun to plateau at the full sample  
178 size, resulting in the value for the individual being no more than three times as much as in the  
179 sample (maximum three-fold error). Meanwhile, functional entropy saturated at 30,000 cells for  
180 IgH from mRNA and 300,000 cells for IgH from DNA. This behavior was in marked contrast to  
181 that of raw diversity, which did not saturate or plateau for species richness (Fig. 3, white  
182 symbols), consistent with previous reports and illustrating the need for statistical correction  
183 (Kaplinsky and Arnaout, 2016).

184 We then asked whether the robustness of functional diversity can be expected to generalize for  
185 any IgH or TCR repertoire. We reasoned that a “meta-repertoire” comprising sequences drawn  
186 uniformly (i.e., without regard to frequency) from many individuals will be more diverse, by any  
187 measure, than any single repertoire (which will have fewer sequences, and in which the same  
188 sequence may appear multiple times). Downsampling from a meta-repertoire therefore provides  
189 an upper bound or worst-case scenario for the sampling requirements for any single repertoire.  
190 To build meta-repertoires, we pooled and then uniqued CDR3s from 114 different IgH  
191 repertoires from 79 individuals including Americans of African, European, and Hispanic descent  
192 (Bild et al., 2002; DeWitt et al., 2016; Vollmers et al., 2013) to build an IgH meta-repertoire of  
193 roughly 36 million unique sequences—as many as or more than ever observed or currently  
194 estimated to be in a typical individual—and similarly for CDR3s from TCR $\beta$  repertoires from 69  
195 healthy individuals (of mostly European but some Asian descent) (Emerson et al., 2017) to build  
196 a TCR $\beta$  meta-repertoire of 10 million unique sequences, and downsampled from each of these  
197 meta-repertoires as above (Fig. 3, large circles). We found that functional diversity plateaued for  
198 all  $q$ , saturated for  $q \geq 1$  and reflected overall diversity to within a few percent from sample sizes  
199 of 50,000 for TCR $\beta$  and IgH RNA and 100,000 for IgH DNA for  $q=0$ , and 30,000 for TCR $\beta$  and  
200 IgH RNA and 300,000 for IgH DNA for  $q \geq 1$  (Fig. 3, large colored circles). Together, these results  
201 confirmed that functional diversity measured on samples is an accurate measure of overall  
202 functional diversity in the individual, at conventional sample sizes.

203 **Raw and functional diversity.** We measured raw and functional diversity on 141 healthy  
204 human subjects (Fig. 4). For IgH, we found a (geometric) mean functional species richness  
205 ( ${}^qD_s$ ) of 677 (range, 487-916) from mRNA and 2,205 (range, 2,042-2,485) from DNA,  
206 suggesting that on average, the human antibody repertoire is capable of recognizing the  
207 equivalent of no more than a few thousand unique non-overlapping heavy-chain CDR3 binding  
208 targets. (As above, lower diversity from mRNA was not unexpected, since inactive cells, which  
209 produce less IgH mRNA than active cells, may be underrepresented.) For TCR $\beta$  the mean  
210 functional diversity was 140 targets (range, 115-167). Functional diversity can be thought of as  
211 clustering similar sequences together, although functional clusters can overlap and sequences  
212 can belong to multiple clusters. An indication of the average size of these clusters can be  
213 obtained by taking the ratio of raw to functional diversity measures. For species richness, we  
214 found that IgH typically had hundreds of sequences per cluster, while TCR $\beta$  had thousands.  
215 Thus by both functional diversity and average functional-cluster size, IgH CDR3 repertoires are  
216 roughly 5-10 times as diverse as TCR $\beta$  (for small  $q$ ). Repertoires with higher raw diversity might



217 be expected to be more functionally diverse, but we found no consistent trend across all  
218 repertoire types. Thus, functional diversity generally complements raw diversity, adding  
219 information not captured by raw diversity alone.

220 *Naïve vs. memory B cells.* We next sought to investigate what this information might add to our  
221 understanding of adaptive immunity. We began with two widely studied B-cell subsets, naïve  
222 (IgM) and memory cells (predominantly IgG). Previous studies have shown that naïve  
223 repertoires have higher raw diversity than memory repertoires (DeWitt et al., 2016). This is at  
224 least superficially consistent with the fact that only a subset of naïve cells are selected to enter  
225 into the memory compartment. However, in a functional sense there is a case to be made that  
226 memory/IgG repertoires should be more diverse, since somatic hypermutation differentiates  
227 memory cells from naïve cells, and indeed from each other. Using well-characterized publicly  
228 available repertoires from DNA from three healthy human subjects, we confirmed that by raw  
229 species richness, naïve (CD27<sup>-</sup>IgM<sup>+</sup>) B-cell repertoires are ~10 times as diverse as memory  
230 (CD27<sup>+</sup>IgM<sup>-</sup>) repertoires (DeWitt et al., 2016) (Fig. 5a-b). Yet by functional species richness, we  
231 found that memory repertoires were at least as diverse as naïve (Figs. 5a-b). Comparing raw  
232 and functional diversity for 34 IgM and 32 IgG repertoires from mRNA (repertoires with less than  
233 100,000 total sequences were discarded) from 28 additional healthy individuals from a separate  
234 dataset showed a similar pattern as for the three DNA repertoires: in all but a few outliers, IgM  
235 had higher raw diversity but IgG had higher functional diversity (Fig. 5c-d). For raw diversity, the  
236 IgM:IgG ratio rose from ~3:1 at  $q=0$  to peak at 10:1 around  $q=1$ , due to a large fraction of rare  
237 IgG sequences (Fig. 5d). This effect was more pronounced for naïve:memory (Fig. 5b). For  
238 functional diversity, the absence of a peak in the IgM:IgG ratio suggests that these many rare  
239 sequences must nonetheless be similar to others in the repertoire, possibly because they are  
240 members of clones (Fig. 5b,d).

241 *Cytomegalovirus (CMV) exposure.* CMV is a herpesvirus to which half of the adult population  
242 has been exposed and results in life-threatening opportunistic infections in newborns, transplant  
243 recipients, and immunocompromised individuals (Emery, 2001). In most healthy individuals, it  
244 causes a chronic infection marked by clonal expansion of both B cells and T cells and a  
245 consequent fall in raw diversity, an effect also seen during aging (see below) (de Bourcy et al.,  
246 2017; Qi et al., 2014). We measured raw and functional TCR $\beta$  CDR3 diversity for 120  
247 individuals: 69 CMV-seronegative and 51 CMV-seropositive subjects aged 19-35 (Emerson et  
248 al., 2017), the narrow age range helping control for any age-related effects. There was a clear  
249 trend toward lower diversity in the CMV-seropositive group relative to the CMV-seronegative

250 group by both raw and functional diversity, for all weighting parameters (Fig. 6a). Combining raw  
251 with functional diversity facilitated identification of two subgroups among the subjects with  
252 known CMV status: subjects with a high raw Berger-Parker Index ( ${}^{\circ}D$ ) were almost always  
253 CMV-seronegative (Fig. 6b), whereas subjects with low functional Berger-Parker Index ( ${}^{\circ}D_s$ )  
254 were almost always CMV-seropositive (Fig. 6c). The reverse—low  ${}^{\circ}D$  or high  ${}^{\circ}D_s$ —did not  
255 distinguish between the groups. Using both measures gave a better indication of CMV status  
256 than did either one alone (Fig. 6d). The conclusion is that CMV is unlikely in the absence of  
257 large clones/expanded lineages, as has been reported, but is likely only if the large  
258 clones/expanded lineages that are present exhibit high similarity to other clones/lineages in the  
259 repertoire, or else are indeed very large (Fig. 6e). Again, the addition of functional diversity  
260 offers insight that raw diversity alone does not.

261 *Flu vaccination.* Vaccination with a seasonal trivalent influenza vaccine (TIV) triggers clonal  
262 expansion in B cells. Previous work on five vaccinees showed likely flu-specific memory IgG  
263 lineages emerging by day 7 post-vaccination (Vollmers et al., 2013). We found that combining  
264 raw and functional diversity reveals a signature of clonal expansion and selection without the  
265 need for lineage tracking (Fig. 7). We measured raw and functional diversity for IgM and IgG at  
266 day 0 (pre-administration) and day 7 from all 14 vaccinees in Vollmers' dataset. We found that  
267 for most subjects, for IgG, raw species richness rose from day 0 to day 7 while functional  
268 species richness fell (Fig. 7a-b). This means that even as the number of sequences increased,  
269 many of the new sequences were similar to each other (or to existing sequences), and they  
270 tended to replace different-looking sequences. Meanwhile, there was no obvious pattern in IgM  
271 (Fig. 7c). Together, these results are what we would expect from clonal expansion and selection  
272 in a memory response, and thus represent a repertoire-level signature of these phenomena.  
273 Interestingly, in most cases, raw and functional entropy both fell (Fig. 7a-b, right panels). This  
274 suggests that most of the new sequences at day 7 were rare, while at the same time a subset of  
275 sequences and functional clusters grew. Thus overall, the addition of functional diversity reveals  
276 a key feature of clonal dynamics, which is not evident from raw diversity alone.

277 *Aging.* To explore the effect of age, we measured raw and functional diversity for TCR $\beta$  CDR3  
278 repertoires from 41 healthy individuals aged 6-90 years old (Britanova et al., 2014) (Fig. 8). We  
279 found that raw diversity falls with age regardless of weighting parameter; a fall in raw species  
280 richness had been reported previously (Britanova et al., 2014). Functional diversity also fell,  
281 regardless of weighting parameter. However, for species richness, four septuagenarians bucked  
282 the trend (Fig. 8, arrows): even as their raw species richness was unremarkable relative to that

283 of other individuals of similar age, their functional species richness was similar to that of  
284 children. Only one of these four had a high likelihood of being CMV-negative; the probability that  
285 all four were CMV-negative was low. We therefore consider CMV unlikely as an explanation for  
286 their high functional species richness. Unlike their peers, these four appear to have retained  
287 functional diversity among their rarest sequences. (The alternative hypothesis is that these four  
288 saw a rise in functional species richness from a lower level earlier in life, but we think this  
289 unlikely given the overall downward trend across individuals.) We considered but excluded  
290 PCR/sequencing artifacts as the cause, as we expected such artifacts would have led to larger  
291 raw species richness, which was not observed. Thus, functional diversity identified for further  
292 study individuals who were unremarkable by raw diversity alone.

## 293 **Discussion**

294 Diversity both affects function, and reflects it. In the adaptive immune system, the defining  
295 tradeoff is breadth vs. depth: a repertoire must be sufficiently diverse to contain sequences that  
296 can recognize a given target and lead to useful clones, but not so diverse that cells that express  
297 such sequences are too rare to find the target on biologically relevant timescales (Schober et  
298 al., 2018; Zarnitsyna et al., 2013). To monitor immunological diversity, either diagnostically or  
299 therapeutically, we must be able to measure it, and to measure it, we must define it. It is  
300 increasingly recognized that a reasonable definition of immunological diversity must account for  
301 differences in species frequency. Here we argue such a definition must also account for  
302 species' pairwise similarity, and show that binding similarity, which leads to what we call  
303 functional diversity, provides useful insight into repertoire function.

304 Pairwise similarity can be seen as governed by a tunable parameter that helps define the  
305 similarity matrix, analogous to how  $q$  is a tunable parameter that governs the effect of  
306 differences in frequency (Chao et al.). In our study, the similarity matrix is defined by the  
307 average single amino-acid change in  $K_d$ , an average based on over 1,300 independent  
308 measurements, and the assumption of multiplicative independence. This source data is not  
309 systematic, but to our knowledge is the best available. While our study is to our knowledge the  
310 first of its kind, it follows a long tradition of attempts to estimate the number of binding targets  
311 that can be recognized by the adaptive immune system immunization (Bachmann et al., 1994;  
312 Obar et al., 2008). In these past studies, typically a sample of B or T cells was diluted until  
313 binding to/protection from a given target was abolished, using whatever thresholds the  
314 investigators deemed appropriate. If the limiting frequency for binding/protection was found to

315 be, for example, 1:3,000, the conclusion was that the repertoire could recognize 3,000 different  
316 targets. This conclusion was based on the assumption that on average, all targets behave the  
317 same as the one under study. Such studies gave functional diversities of 100 to 100,000 targets  
318 for various T-cell populations and for B cells after antigen exposure, a wide range (Bachmann et  
319 al., 1994; Obar et al., 2008). The width of this range may reflect real differences in the  
320 frequencies of cells that are specific for different targets, or variability in stringency or  
321 experimental setup. Interestingly, this “how-many-can-fit” logic seems not to have been used  
322 when testing so-called natural antibodies, which bind many targets at low affinity (Frank, 2002;  
323 Holodick et al., 2017; Notkins, 2004). For example, when one in five natural antibodies were  
324 found to bind insulin (Chen et al., 1998), this was not taken to mean that the repertoire could  
325 recognize only five targets, because of presumed overlapping specificity of these antibodies for  
326 other targets. Meanwhile, theoretical studies have suggested a need for  $\leq 10,000$  binding  
327 targets, and fewer for T cells than B cells, because of major histocompatibility complex (MHC)  
328 restriction (Langman and Cohn, 1987; Zarnitsyna et al., 2013)

329 For raw diversity, we found that a typical repertoire contains on the order of 10 million unique  
330 CDR3s, well above the upper end of previous estimate for the number of binding targets. These  
331 findings are in line with other recent estimates that were likewise based on a combination of  
332 deep sequencing and statistical correction (Britanova et al., 2014; DeWitt et al., 2016; Kaplinsky  
333 and Arnaout, 2016). On average, these raw species richnesses mean that each of 100-100,000  
334 putative binding targets can be bound by 100-100,000 unique CDR3s ( $10^7/10^5$  to  $10^7/10^2$ ). From  
335 a medical perspective, such redundancy is good for treatment, because it supports the  
336 prevailing view that there are many ways to design an antibody- or TCR-based drug that will  
337 recognize a given target, but potentially a complicating factor for attempts to diagnose specific  
338 diseases based on repertoire sequence, because it suggests that signatures of exposure to a  
339 given target may be quite variable.

340 One of our key findings is that functional diversity is much lower than raw diversity: repertoires  
341 contain only a few hundred functional clusters for TCR $\beta$  CDR3s and at most a few thousand for  
342 IgH. The fact that functional diversity is based on  $K_d$ s suggests that functional diversity should  
343 correlate with the number of structurally unique, non-overlapping target clusters that CDR3s can  
344 recognize (Fig. 1). Yet our measurements of functional species richness lie at the low end of the  
345 range of prior estimates. We propose two explanations. First, our measurements are limited to  
346 CDR3s; variability in the rest of the antibody or TCR protein must add to the total number of  
347 potential binding targets. This possibility is testable by extending our method to more or indeed

348 all of the antibody or TCR sequence. Second, functional diversity may be providing a less  
349 detailed description of shape space than limiting-dilution studies: i.e., functional diversity may be  
350 a coarse graining of the target-binding landscape (Fairlie-Clarke et al., 2009; Smith et al., 1997)  
351 (Fig. 9). A pair of sequences may be similar enough to lie near each other in shape space, but  
352 only one may bind a given target above a threshold level of specificity in a binding study. In  
353 short, binding studies may be counting peaks while functional diversity is counting mountains. If  
354 true, our results suggest that the landscape of TCR $\beta$  CDR3 binding is more clustered than that  
355 of IgH, such that there are on average several times as many functional IgH clusters as TCR $\beta$ .  
356 This prediction is testable, at least in principle, through large-scale systematic binding assays to  
357 measure  $K_d$ , or by measuring binding as a binary outcome at multiple stringency thresholds.  
358 Both explanations may contribute. To our knowledge ours is the first attempt a quantitative  
359 summary of this landscape using data from large-scale binding studies and high-throughput  
360 repertoire sequencing.

361 Why is CDR3 functional diversity higher for IgH than for TCR $\beta$ ? We hypothesize that it is for the  
362 same three reasons that there is more sequence diversity for IgH than TCR $\beta$ . First, humans  
363 have 23  $D_H$  gene segments vs. only 2  $D_\beta$  segments, and D is the largest germline contributor to  
364 CDR3. V and J segments tend to directly contribute little more than the canonical starts and  
365 ends of CDR3s, and besides there are similar numbers of V-J combinations in IgH as TCR $\beta$   
366 ( $49 \times 6 = 294$  and  $48 \times 13 = 624$ , respectively). Second is somatic hypermutation, which diversifies  
367 IgH but not TCR $\beta$ . And third, IgH CDR3s are longer than TCR $\beta$  CDR3s, allowing for a larger  
368 number of possible sequences. Further analysis will be needed to test these hypotheses.

369 We have shown how functional diversity complements raw diversity to offer insight into the  
370 difference between naïve and memory repertoires, to aid in identification of disease states, and  
371 to illustrate clonal selection and other repertoire dynamics. We hope these examples will  
372 encourage others to use and/or expand our framework to investigate repertoire dynamics in  
373 other conditions, in other subsets, in the other chains (TCR $\alpha$  and IgL), and in other model  
374 systems such as zebrafish (Weinstein et al., 2009) and mouse (Arnaout et al., 2011; Kaplinsky  
375 et al., 2014). We draw attention to the fascinating difference between the number of unique  
376 sequences, which ran into the millions in most of the repertoires we investigated, and the much  
377 smaller numbers of what we call functional clusters (the effective numbers of functional  
378 diversity). The result is a “functional degeneracy” among sequences that are organized into  
379 functional clusters. Characterizing these clusters is an interesting topic for future work.

380 Functional similarity also offers a new perspective on similarities and differences between  
381 people. To show that functional diversity is robust to sampling, we generated “meta-repertoires”  
382 by pooling sequences from scores and in some cases over a hundred people, including people  
383 from different ethnic backgrounds. Surprisingly, and somewhat unexpectedly given the low  
384 sequence overlap between pairs of individuals, the functional diversity of these meta-repertoires  
385 never exceeded the functional diversity of any given repertoire by more than a few fold;  
386 moreover, the functional diversity trended toward saturating in samples of just a million  
387 sequences (Fig. 3). Together, these findings predict that any two individuals share a majority of  
388 their functional clusters, in stark contrast to the vanishingly small fraction of sequences they  
389 share. Further, these findings suggest that the functional diversity of the entire population is only  
390 a few hundred clusters for TCR $\beta$  CDR3s and a few thousand for IgH, and imply that these  
391 clusters can be sampled exhaustively by sequencing fewer than 20 individuals. It will be  
392 fascinating to test this finding with additional ethnically and geographically diverse populations,  
393 to further examine our prediction that, contrary to conventional wisdom, the functional limits of  
394 the adaptive immune system are in a practical sense both finite and within reach.

395 The focus of this study was binding similarity, but we expect that the utility of the diversity-with-  
396 similarity framework will extend to other facets of immunology (e.g., somatic hypermutation) and  
397 to other fields, most readily metagenomics, sociology, oncology, and cellular cartography  
398 (Almendro et al., 2014; Heindl et al., 2016; Koopmans and Schaeffer, 2013; Li et al., 2012;  
399 Taraska, 2015). We hope this study will serve as a template for incorporating similarity into the  
400 study of other complex systems.

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410 staff, and participants of the MESA study for their valuable contributions (a full list of  
411 participating MESA investigators and institutions can be found at <http://www.mesa-nhlbi.org>), as

412 well as the Research Computing Group at the High Performance Computing Cluster at Harvard  
413 Medical School.

#### 414 **Online methods**

415 **High-throughput repertoires.** We obtained 391 quantitative high-throughput IgH and TCR $\beta$   
416 repertoires from 202 human subjects. These included IgH from naïve and memory B cells from  
417 DNA ( $n=3$  individuals) (DeWitt et al., 2016); TCR $\beta$  chains from DNA from healthy subjects  
418 known to be serologically negative for cytomegalovirus (CMV) ( $n=69$  individuals) (Emerson et  
419 al., 2017) and from healthy subjects whose CMV serostatus was unknown ( $n=41$  individuals)  
420 (Britanova et al., 2014); pooled barcoded IgG and IgM heavy chains from mRNA from healthy  
421 subjects before and seven days after administration of one of two influenza vaccines ( $n=28$   
422 individuals) (Vollmers et al., 2013); quantitative pooled TCR $\beta$  chains from DNA for subjects who  
423 were otherwise healthy but serologically CMV positive ( $n=51$  individuals) (Emerson et al., 2017);  
424 and IgH chains from DNA for subjects enrolled in the Multi-Ethnic study of Atherosclerosis  
425 (MESA;  $n=41$  individuals) (Bild et al., 2002). CDR3 annotation was performed using our in-  
426 house pipeline as previously reported (Kaplinsky et al., 2014) and standard tools (e.g. IMGT).  
427 Details for obtaining these datasets are available from the primary publications referenced  
428 above.

429 **Similarity measures.** A functional measure of similarity between polypeptides is how well they  
430 bind the same target. We were interested in similarity as a function of the number of amino acid  
431 substitutions (i.e., as a function of edit distance). The effect of substitutions on binding is  
432 complex and depends on the position and identity of the specific amino acids involved; many  
433 substitutions may have little or no effect, while a few may abolish binding entirely (Lunzer et al.,  
434 2010). When comprehensive data are available, detailed statistical models can offer reasonable  
435 predictions of the effect of specific amino-acid substitutions (Hopf et al., 2017; Salinas and  
436 Ranganathan, 2018; Lee et al., 2008). However, this type of data does not yet exist across  
437 entire antibody and TCR repertoires, and so simpler models are required. These models are not  
438 expected to precisely predict the effects of specific substitutions, but should accurately reflect  
439 the effects of substitutions when averaged over many pairs of proteins, such as the millions of  
440 pairs in megacell-scale repertoires.

441 To develop a model for our similarity measure,  $s$ , we downloaded SKEMPI 2.0, which is to our  
442 knowledge the largest and best-curated database of experimentally measured effects of amino-

443 acid substitution on protein-protein binding (Jankauskaite et al., 2018). Each entry includes a  
444 Protein Data Bank (PDB) identifier (Berman et al., 2000), the type of structural region (Levy,  
445 2010) that contains the substitution(s), one or more PDB coordinates, and (in nearly all cases)  
446 the dissociation constant ( $K_d$ ) of each member of the pair (referred to in the database and Fig.  
447 2a as “wild type” and “mutant”). We extracted entries for all single amino-acid substitutions for  
448 which  $K_d$  for both wild type and mutant were recorded, and considered only entries that involved  
449 binding between antibody and antigen ( $n=797$ ) or TCR and peptide/MHC ( $n=531$ ; total  
450  $n=1,328$ ). Although amino-acid substitutions anywhere in a protein may affect binding,  
451 substitutions at the core of the binding interface are more likely to affect binding than  
452 substitutions elsewhere (Levy, 2010). Therefore we split the data into core ( $n=584$ ) and non-  
453 core ( $n=744$ ) groups and analyzed the effect of substitution binding, measured as  
454  $|\log_{10}(K_{dmut}/K_{dwt})|$ , separately for each group.

455 As expected, the probability distributions for the two groups differed substantially from each  
456 other (Mann-Whitney U  $p$ -value  $2.0 \times 10^{-33}$ ). Substitution of a core residue had a 13-fold  
457 (geometric) mean effect on binding, consistent with prior reports (Whittaker et al., 2001), while  
458 substitution of a non-core residue had a 4-fold effect. Both probability distributions were long-  
459 tailed, and were reasonably well described by exponential probability-density functions (i.e., of  
460 the form  $ke^{-kx}$ ). We confirmed that the distributions for antibody-antigen core residues ( $n=352$ )  
461 and TCR-peptide/MHC core residues ( $n=232$ ) were similar to each other, that the distributions  
462 for antibody-antigen non-core residues ( $n=445$ ) and TCR-peptide/MHC non-core residues  
463 ( $n=299$ ) were also similar to each other, that within each of the antibody-antigen and TCR-  
464 peptide/MHC subgroups the distributions for core and non-core residues were different, and that  
465 these results held separately for human and non-human (nearly all of which were mouse)  
466 sequences (using the Structural Antibody Database (Dunbar et al., 2014) and the Structural  
467 TCR Database (Leem et al., 2018) to assign species), all using Mann-Whitney U and visualized  
468 as histograms. Using PyMol v2.2.0 (The PyMOL Molecular Graphics System, Version 2.0  
469 Schrödinger, LLC), we next manually reviewed nine structures containing substitutions in  
470 human IgH or TCR $\beta$  CDR3s (1BD2, 1OGA, 3BN9, 3QDJ, 3SE8, 3SE9, 4I77, 5C6T, 5E9D) and  
471 found that to a good approximation, a constant fraction  $0.15 \pm 0.05$  of CDR3 amino acids consist  
472 of core residues, with no obvious difference between chain types. To estimate the effect of a  
473 single amino-acid substitution in a CDR3 in our datasets, we therefore combined core and non-  
474 core distributions with a weighting of 0.15:0.85.



475 The resulting distribution was again long-tailed, with most substitutions having small effects and  
476 a few having effects of many orders of magnitude (Fig. 2a). There were small spikes in the tail  
477 for substitutions with  $\geq 60$ -fold effects, i.e.  $|\log_{10}(K_{dmult}/K_{dwt})| \geq 1.8$ . A review of sources cited by  
478 SKEMPI suggested that these spikes likely reflect ascertainment bias: selective experimentation  
479 on amino acids with unusually strong effects (e.g. Pons et al., 1999; Taylor et al., 1998). To  
480 counteract such bias, we therefore built a high-confidence dataset using 1.8 as the cutoff.  
481 Ascertainment bias in two- and three-amino-acid substitutions is expected to follow the square  
482 and cube of the bias in single-amino-acid substitutions, respectively, precluding rigorous  
483 conclusions from being drawn from independence testing. However, comparison with those  
484 groups was broadly consistent with either multiplicative ( $s=c^m$ , where  $s$ =similarity;  $c$ =the cost of  
485 binding, i.e.,  $1/(\text{fold effect})$ ;  $m$ =edit distance) or additive ( $s=c/m$  for  $m \geq 1$ ) independence.  
486 Because additive effects result in higher pairwise similarities and therefore smaller repertoire  
487 diversities than multiplicative effects, the multiplicative-independence model is more  
488 conservative for studying the effects of similarity on diversity. We therefore chose the  
489 multiplicative model for further analysis.

490 To determine the similarity between two CDR3s with edit distance  $m$ , we sampled  
491 independently from the high-confidence dataset  $m$  times, and multiplied the costs together. We  
492 confirmed that on average, the results of this stochastic sampling were the same as  
493 deterministic calculation of  $s=c^m$  with  $c \approx 0.55$ . We performed sensitivity analysis based on lower-  
494 confidence cutoffs (down to  $c=0.48$ ) and alternative assumptions (up to  $c=0.60$ ). This resulted in  
495 somewhat higher or lower diversity values, but qualitative patterns were robust to these  
496 perturbations.

497 **Diversity measures.** We calculated  ${}^qD$  as previously described (Hill, 1973; Kaplinsky and  
498 Arnaout, 2016) and  ${}^qD_s$  according to Leinster and Cobbold (Leinster and Cobbold, 2012). We  
499 corrected  ${}^qD$  for sampling error using Recon (default settings) as previously described  
500 (Kaplinsky and Arnaout, 2016). We note Hill's framework (Hill, 1973) has inspired several  
501 methods for incorporating similarity into diversity measurements, each of which retains useful  
502 features of Hill's framework (Chao et al., 2018; Chiu and Chao, 2014; Leinster and Cobbold,  
503 2012; Scheiner, 2012). Two of the new frameworks were introduced with explicit discussion of  
504 how to decompose population-level diversity into within- and between-group components  
505 (Leinster and Cobbold, 2012; Chiu and Chao, 2014). Each of these has advantages and  
506 disadvantages over the other (discussed in Botta-Dukát, 2018) . We chose Leinster and

507 Cobbold's framework here because we found it easier to apply and interpret. For readability, we  
508 made a minor change to the notation, from  ${}^qD^Z$  to  ${}^qD_s$ .

509 Use of this framework raised two issues that we addressed. First, its  $q=0$  measure,  ${}^0D_s$ ,  
510 depends on frequency albeit to a very small extent, unlike the Hill framework's  $q=0$  measure,  ${}^0D$ ,  
511 which is species richness (and is independent of frequency). Therefore, as a more direct  
512 comparison to species richness, we calculated  ${}^0D_s$  both with frequency information and without  
513 it (i.e., setting the frequencies of each of the  $n$  species to  $1/n$ ). We refer to the latter as  ${}^\varnothing D_s$  ("D-  
514 null"). Second, it has been shown that this framework can result in unreasonably low diversity  
515 values when most of the off-diagonal entries of the similarity matrix are far from zero, resulting  
516 in an insensitivity to  $q$  (Botta-Dukát, 2018; Chiu and Chao, 2014). We expected most of our off  
517 diagonals to be close to zero, since our similarity measures directly or indirectly involve  
518 exponential decay, which generates small values, but confirmed that most of the off diagonals in  
519 our similarity matrices were indeed close to zero by plotting histograms. Consequently, our  
520 measures were sensitive to  $q$ , as desired and expected.

521 **Robustness analyses.** For robustness analyses, IgH and TCR $\beta$  were analyzed separately. The  
522 upper-bound/worst-case scenario for IgH was evaluated by constructing a "meta-repertoire" by  
523 combining IgG sequences of subjects before vaccination ( $n=28$  individuals; Vollmers et al.  
524 2013), sequences from memory cells from healthy subjects from public database ( $n=3$ ; DeWitt  
525 et al. 2016), and sequences from subjects enrolled in MESA study ( $n=41$ ; Bild et al. 2002), and  
526 sampling from this meta-repertoire without regard to the frequency of sequences. We chose  
527 IgG/memory sequences where possible because those sets exhibited higher functional diversity  
528 than naïve sets, and we were interested in maximizing diversity. We ignored the frequency of  
529 sequences for the same reason: uniform frequency maximizes diversity, other things equal. For  
530 TCR $\beta$ , we constructed a meta-repertoire by combining sequences from CMV seronegative  
531 individuals ( $n=69$ ; Emerson et al. 2017) and again sampling at uniform frequency. We chose  
532 CMV seronegative individuals for the same reason as we chose memory/IgG sequences above:  
533 seronegative individuals exhibited higher diversity. For both IgH and TCR $\beta$ , including all  
534 sequences lowered diversities slightly. The representative samples were from subject D3 for  
535 IgH (from DNA), subject SRR960344 for IgH (from mRNA), and subject Keck0070 for TCR $\beta$   
536 (CMV seronegative). CDR3 sequences were sampled proportional to their frequency in the  
537 repertoire.

## 538 **Figure titles and legends**

539 **Figure 1: Functional diversity.** (a) Each dot represents a binding target (e.g. an epitope) with  
540 a different shape. Nearby targets have similar shapes (inset). Targets form clusters of similarity.  
541 (b) Each colored region represents the targets that can be bound by one of six unique CDR3  
542 sequences in a representative repertoire; this repertoire has a raw species richness of 6.  
543 Together, the colored regions cover the part of shape space that can be bound by the  
544 repertoire. (The unbound region might include, e.g., self-antigens.) Note the substantial overlap  
545 in binding targets for the orange, yellow, green, and blue CDR3s. This overlap reflects binding  
546 similarity among these CDR3s. (c) Because of this similarity, the repertoire covers only the  
547 region denoted by the four identical non-overlapping squares. The functional species richness of  
548 this repertoire is therefore 4: this repertoire has the same species richness as a repertoire  
549 comprising four CDR3s that have zero overlap in binding specificity.

550 **Figure 2: Validity.** (a) Single amino-acid mutations in antibody and TCR molecules have a  
551 range of effects on affinity, as measured by change in dissociation constant,  $K_d$  (gray). This was  
552 well fit by a simple exponential (black line), providing parameterization for the similarity metric.  
553 (b) CDR3s with high sequence identity have high similarity, while different CDR3s have low  
554 similarity. Shown are two clones, represented by red and white subnetworks, each composed of  
555 17 unique CDR3 sequences drawn from clonotypes of two real IgH repertoires. Node size  
556 corresponds to the frequency of each sequence; edges connect pairs of sequences that differ at  
557 a single amino-acid position. (c)-(d) Functional similarity agrees with an intuitive sense of  
558 repertoire diversity. Each of the four repertoires in (c) has the same number of unique  
559 sequences, present at the same frequencies; as a result, they all have identical raw diversity  
560 (for every value of  $q$ ) despite their obvious quantitative and qualitative differences. In contrast,  
561 functional diversity increases with the number of, and increasing difference between,  
562 repertoires' constituent sequences. Node size denotes sequence frequency. Shades denote  
563 different clonotypes. Comparing the third and fourth rows, note that even when two repertoires  
564 have the same number and frequencies of unique sequences, the repertoire whose sequences  
565 are more different from each other (random peptides) has the higher functional diversity.

566 **Figure 3: Robustness.** Results for raw and functional species richness ( $q=0$ ;  ${}^0D$  and  ${}^0D_s$ ) and  
567 raw and functional entropy ( $q=1$ ;  ${}^1D$  and  ${}^1D_s$ ). Raw, white shapes; functional, colored shapes.  
568 Large symbols give an upper bound/worst-case scenario based on sampling meta-repertoires;  
569 small symbols give results for a representative sample from DNA (circles) and mRNA

570 (triangles). First row: sample diversity is plotted as the effective number of sequences. For  $q=0$ ,  
571 functional diversity plateaus for TCR $\beta$  and IgH RNA and trends toward a plateau for IgH DNA at  
572 the tested sample sizes; all three plateau for  $q=1$ ; raw diversity does not plateau for either  $q=0$   
573 or  $q=1$ . Second row: discovery rate is the probability that the next sampled sequence will add to  
574 the diversity. For example, for the IgH DNA representative sample, for  $q=0$  raw diversity, at a  
575 sample size of 1 million sequences there is still a probability of  $\sim 0.5$  (a 50 percent chance) that  
576 the next sequence to be sampled will be one that has not yet been seen and will therefore add  
577 to the diversity. Third row: maximum error is the maximum fraction by which the diversity in the  
578 sample can underestimate the diversity in the individual from whom the sample was taken.  
579 Horizontal dashed lines indicate the threshold for two-fold error. For example, for the worst-case  
580 scenario for TCR $\beta$ ,  $q=0$  functional diversity measured on a sample of 10,000 sequences will be  
581 no more than a two-fold underestimate of diversity in the individual as a whole; in other words,  
582 the sample value will be at least 50 percent of the overall value.

583 **Figure 4: Diversity in individuals.** Raw (black lines; left vertical axis) and functional (colored  
584 bars; right vertical axis) species richness ( $q=0$ ) for 179 CDR3 repertoires from healthy  
585 individuals representing (a) IgH from mRNA (all isotypes: IgA, IgG, IgM, IgD, and IgE), (b) IgM  
586 and (c) IgG from mRNA from the subjects in (a), (d) IgH from DNA (all isotypes), (e) naïve IgH  
587 from DNA, (f) memory IgH from DNA, and (g) TCR $\beta$  from DNA. See Methods for references.  
588 Matched pairs of symbols below the horizontal axis denote replicates. Note the difference in the  
589 scale for functional diversity between IgH and TCR $\beta$ . Note also a general lack of correlation  
590 between raw and functional species richness, except in (c).

591 **Figure 5: Naïve vs. memory.** (a) Diversity profiles for naïve (red) and memory (black)  
592 compartments from three deeply sequenced individuals. A diversity profile is a way to show  
593 diversity across a range frequency-weighting parameter values at once. By raw diversity (left),  
594 the naïve compartment is more diverse across the range of weightings. By functional diversity  
595 (right), this distinction disappears. In (b), this disappearance is highlighted by plotting the ratio of  
596 naïve:memory diversity for raw diversity (red) and functional diversity (black). According to  
597 functional diversity, the naïve compartment is no more diverse, and indeed sometimes  
598 somewhat less diverse, than the memory compartment. This reversal is even more prominent in  
599 comparisons of repertoires from an additional 28 healthy subjects (c,d).

600 **Figure 6: Infection.** (a) Diversity profiles showing effective number of species as a function of  
601 weighting parameter  $q$  for diversity without similarity (left) and diversity with similarity (right)

602 showing a trend toward lower diversity in CMV-seropositive individuals (red) relative to CMV-  
603 seronegative individuals (black), especially for large  $q$ . **(b)** Raw Berger-Parker Index ( $q=\infty$ ),  
604 which measures the largest clones, showing that high diversity—an absence of large clones—is  
605 rare in CMV-seropositive individuals. **(c)** Functional Berger-Parker Index, showing that low  
606 diversity—the presence of large clones with similarity to other clones in the repertoire—is rare in  
607 CMV-seronegative individuals. **(d)** Combining raw and functional Berger-Parker Indices (first  
608 principal component of PCA, which explains 72 percent of variance) illustrates both of the trends  
609 in (c): for the third of subjects beyond the cutoffs indicated by the horizontal dashed lines CMV  
610 serological status is assigned with an accuracy of 95 percent. **(e)** Schematic representation of  
611 the three classes revealed by combining diversity with and without similarity. Each circle is a  
612 clone; each collection of clones is a representative repertoire. Top: subjects without large clones  
613 are almost always CMV seronegative. Bottom: subjects with large clones that are similar to  
614 other clones in the sample (shown in red) are almost always CMV seropositive. Middle:  
615 repertoires with large clones that are not similar to other clones in the repertoire may be either  
616 CMV seropositive or CMV seronegative. Receiver-operator characteristic (ROC) analysis gave  
617 an area under the curve (AUC) of 0.79.

618 **Figure 7: Vaccination.** Raw and functional diversity together reveal clonal expansion and  
619 selection without needing lineage analysis. **(a)** In the IgG compartment, raw species richness  
620 rises while functional species richness falls in most vaccinees (left). Meanwhile raw and  
621 functional entropy both fall (right). The difference vs. species richness suggests most new  
622 sequences at day 7 are rare. **(b)** Meanwhile, the IgM compartment changes less by these  
623 measures.

624 **Figure 8: Aging.** Raw and functional species richness ( $q=\emptyset$ ) for TCR $\beta$  CDR3 repertoires from  
625 41 healthy individuals. Arrows denote four septuagenarians who bucked the trend of lower  
626 functional species richness with age. Note that for each individual, the raw species richness is  
627 ~10-fold higher than previously reported (Britanova 2014), likely because the method we used  
628 to correct for missing species (Recon) is more sensitive than the method used in the previous  
629 report (Fisher).

630 **Figure 9: Binding landscape.** **(a)** Schematic of the target-binding landscape. The gray  
631 distribution represents CDR3 sequences that bind a given target. Sequences are ordered by  
632 their similarity to each other. (In reality, similarity is a multidimensional property that makes it  
633 impossible to order sequences in a single dimension as shown here; this is done for illustrative

634 purposes only.) The height at each sequence denotes the affinity with which it binds the target  
635 (vertical axis; measured e.g. by  $K_d$ ). Many more sequences bind the target at low affinity than at  
636 high affinity, resulting in a “mountain-and-peak” appearance. This schematic is useful for  
637 interpreting functional diversity as described in this study, and the raw diversity estimates based  
638 on previous binding studies, as described in the Discussion. (Note that in this schematic, the  
639 raw diversity as measured simply corresponds to the total number of sequences along, i.e. the  
640 width of, the horizontal axis.) At high affinity, very few sequences bind a given target. At medium  
641 affinity, more sequences bind, and can be binned into two small clusters, represented by the  
642 small circles. At low affinity, many sequences bind, and can be binned into a single large  
643 cluster, represented by the large circle. In **(b)-(d)**, many targets are shown. Each color  
644 corresponds to a different target; nearby targets are structurally similar. As in **(a)**, each colored  
645 area denotes the sequences that bind a given target, as a function of binding affinity (vertical  
646 axis). Experiments usually detect the highest-affinity sequences: the peaks of the landscape  
647 (above the horizontal dotted line). The narrower the peak when it crosses the experimental  
648 threshold, the rarer specific sequences are, and the larger the number of targets that the  
649 repertoire will be estimated to bind. (For example, if 100,000 sequences are shown across the  
650 horizontal axis in each plot, and only one crosses the experimental threshold for a given target,  
651 the frequency of sequences specific for that target is 1:100,000, and the conclusion will be that  
652 there must be 100,000 such targets that the repertoire can bind. If 100 cross the experimental  
653 threshold, the conclusion will be that the repertoire can bind only 1,000 targets.) Functional  
654 diversity measures the overall contours of the landscape. Conceptually, this can be thought of  
655 as measuring the number and size of the “mountains” at a lower affinity threshold (horizontal  
656 solid lines). The differences in functional diversity between **(b)** memory IgH, **(c)** TCR $\beta$ , and **(d)**  
657 naïve IgH correspond to different landscapes. The raw species richness of memory IgH and  
658 TCR $\beta$  are comparable, represented here by the same width of all the plots. In addition, a similar  
659 number of sequences per target cross the experimental threshold, so estimates of the total  
660 number of targets that the repertoires can bind will also be comparable. However, less low-  
661 affinity overlap between the targets of the IgH sequences in **(b)** gives it higher functional species  
662 richness than the TCR $\beta$  repertoire in **(c)**: here, six functional clusters (white circles) vs. three.  
663 (The sizes of the clusters are related to frequency-weighted functional diversity measures, i.e.  
664 larger  $q$ .) The sequences in the naïve IgH repertoire in **(d)** have only low affinity for the six  
665 colored targets, and many recognize more than one target (overlapping colored areas). Note the  
666 lower experimental threshold (horizontal dotted line), consistent with the ~10% or more of  
667 antibodies that recognize a target and the high degree of cross-reactivity in studies of natural

668 antibodies (Frank, 2002; Holodick et al., 2017). The functional diversity threshold controlled by  
669 the average effect on  $K_d$  of a single-amino-acid change in CDR3. If the effect were larger—or if  
670 it were amplified by e.g. raising it to a power when building the similarity matrix—the threshold  
671 would be higher, and vice versa.

## 672 Tables and Figures

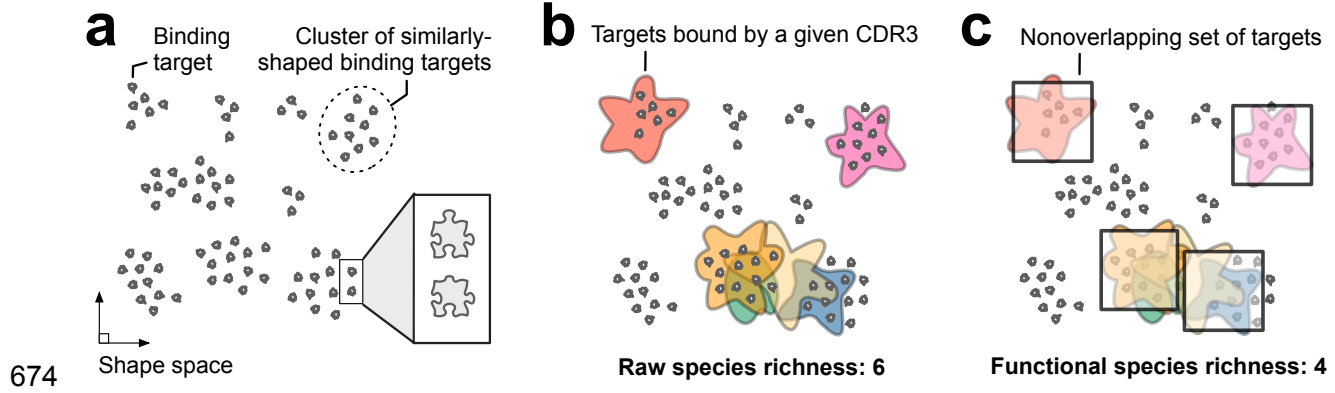
### **Box 1: Interpreting effective numbers**

Consider two repertoires of 100 clones each. In the first repertoire, one clone is large and accounts for 91 percent of all cells (e.g. a leukemic clone); the other 99 clones are small and account for the remaining 9 percent. In the second repertoire, all 100 clones are equally common, each accounting for 1 percent of cells. The Shannon entropies of the two repertoires are 1.0 bit and 6.6 bits. Entropy is converted to an effective number— ${}^1D$ —by exponentiation: the effective number of clones in the first repertoire is  $2^{1.0}=2$ , while in the second repertoire it is  $2^{6.6}=100$ . Thus per entropy, the first repertoire can be thought of as “effectively” consisting of just two clones: the 99 rare clones collectively count the same as the one large clone. In other words, the first repertoire has the same effective diversity as a repertoire that consists of just two clones that are equally common. The second repertoire already consists of clones that are equally common, so the effective number of clones in this repertoire,  $2^{6.6}=100$ , is the same as its species richness. Diversity with similarity is interpreted in a similar fashion: a repertoire with a  ${}^qD_s$  of  $n$  species has the same effective diversity as a repertoire with  $n$  species that are equally common (as above), with the additional constraint that these species are now also completely unrelated to/dissimilar from each other.



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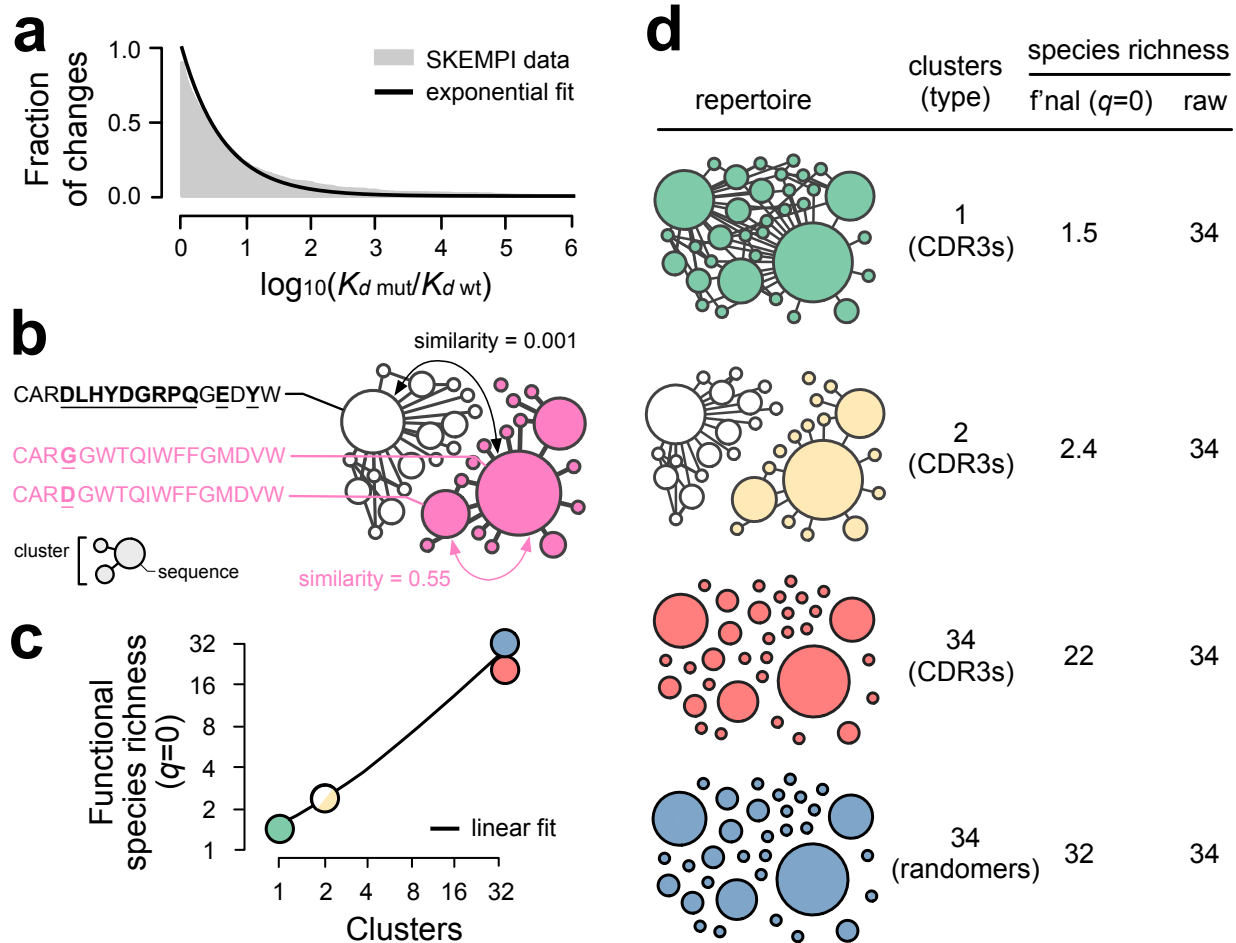
Figure 1



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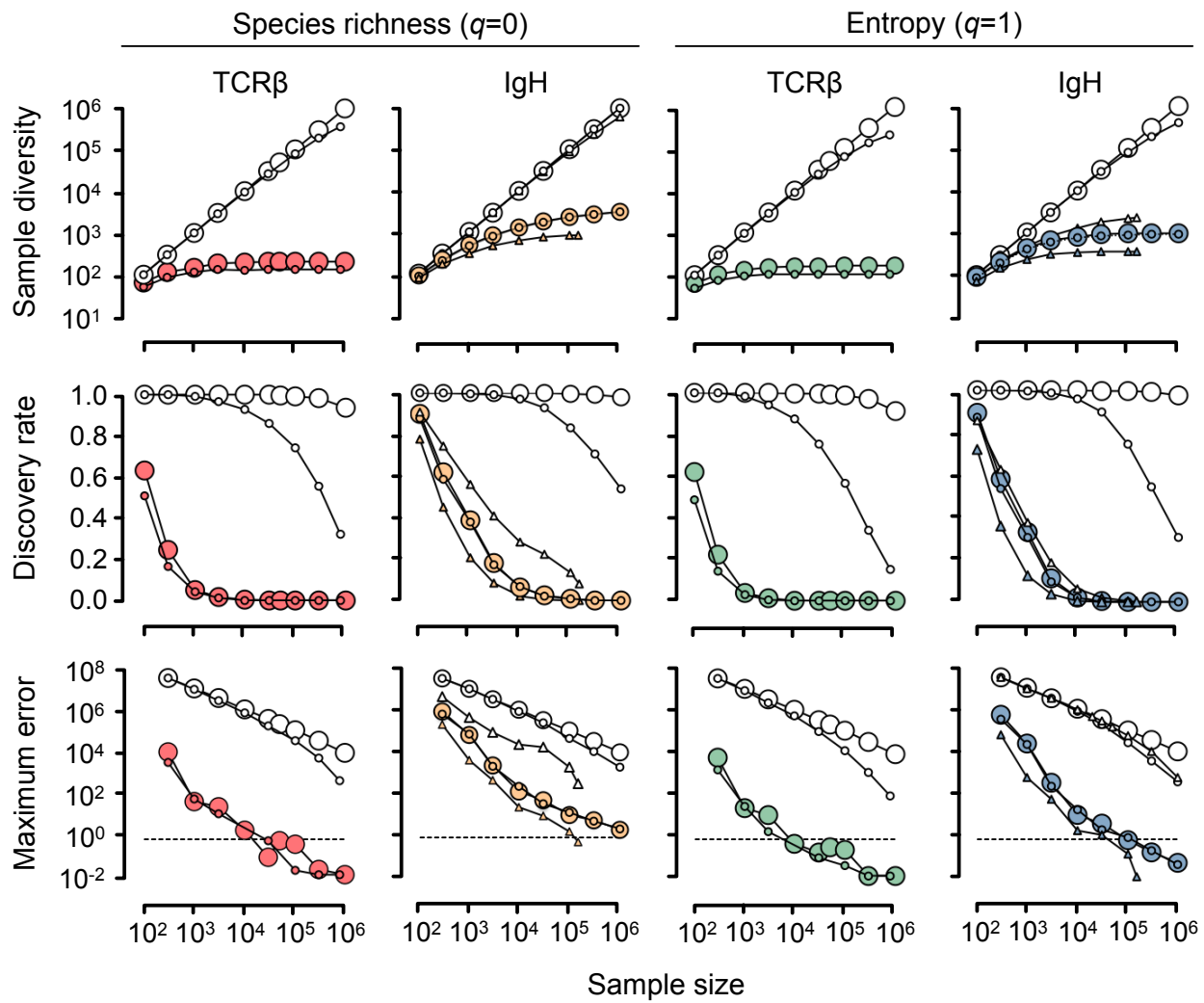
Figure 2



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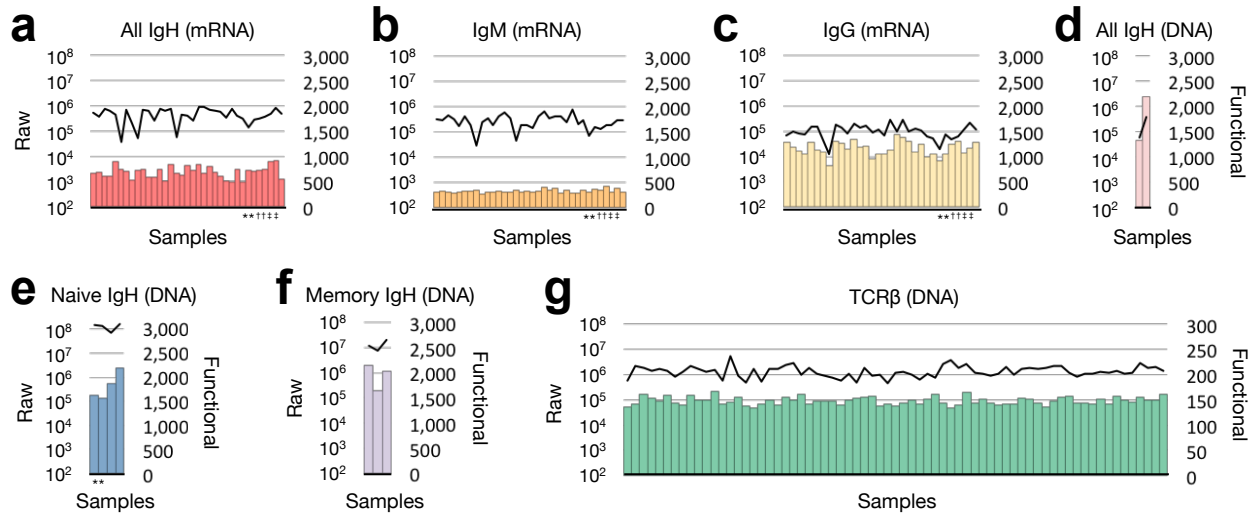
Figure 3



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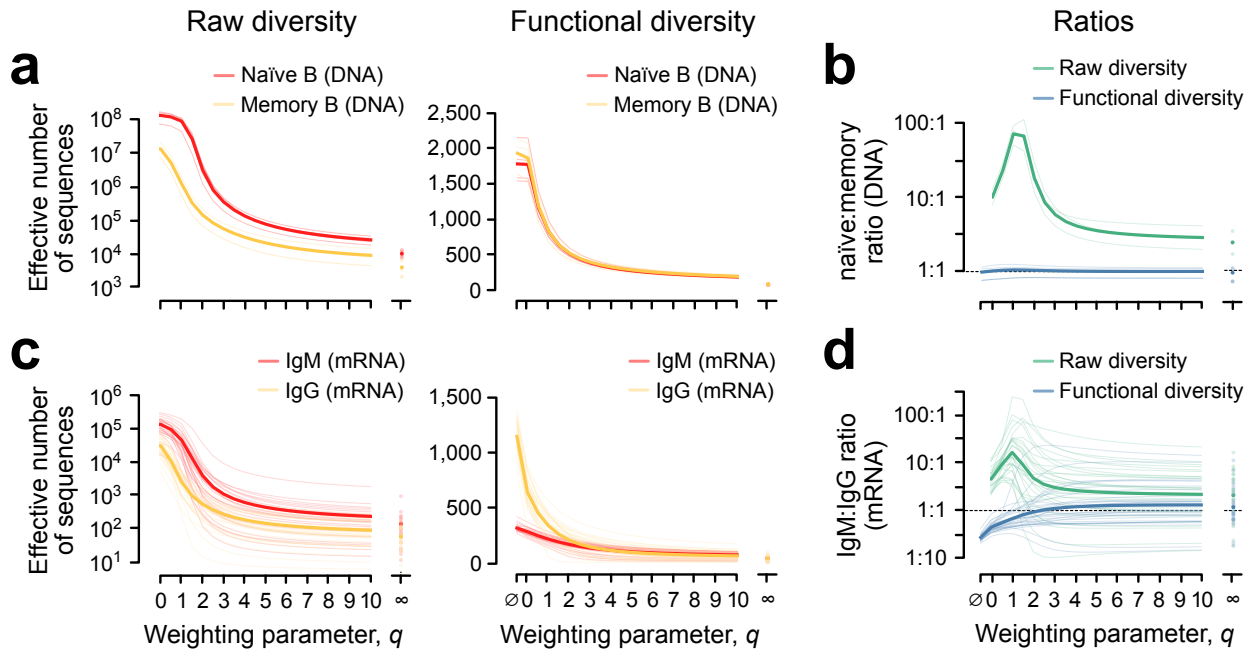
Figure 4



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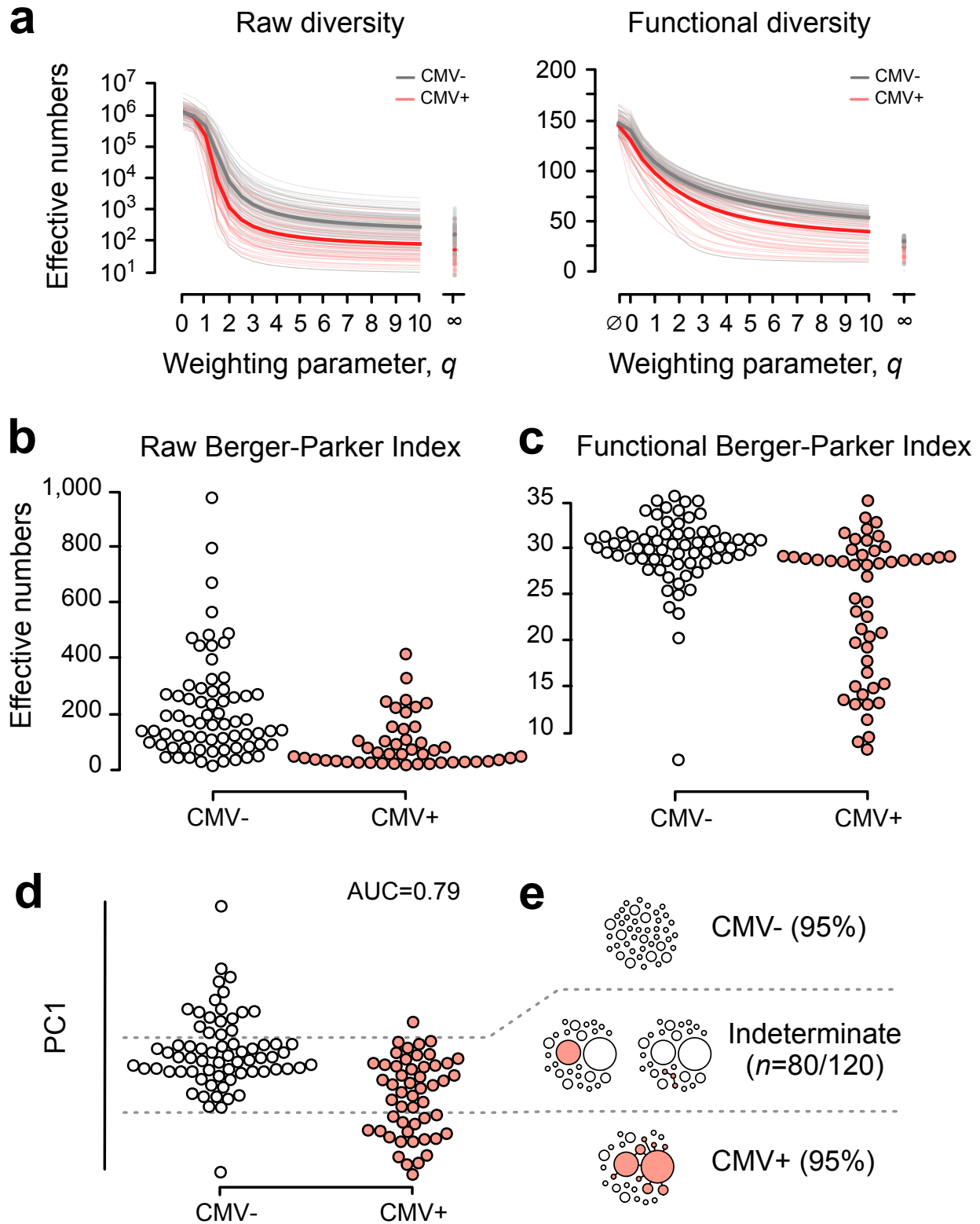
Figure 5



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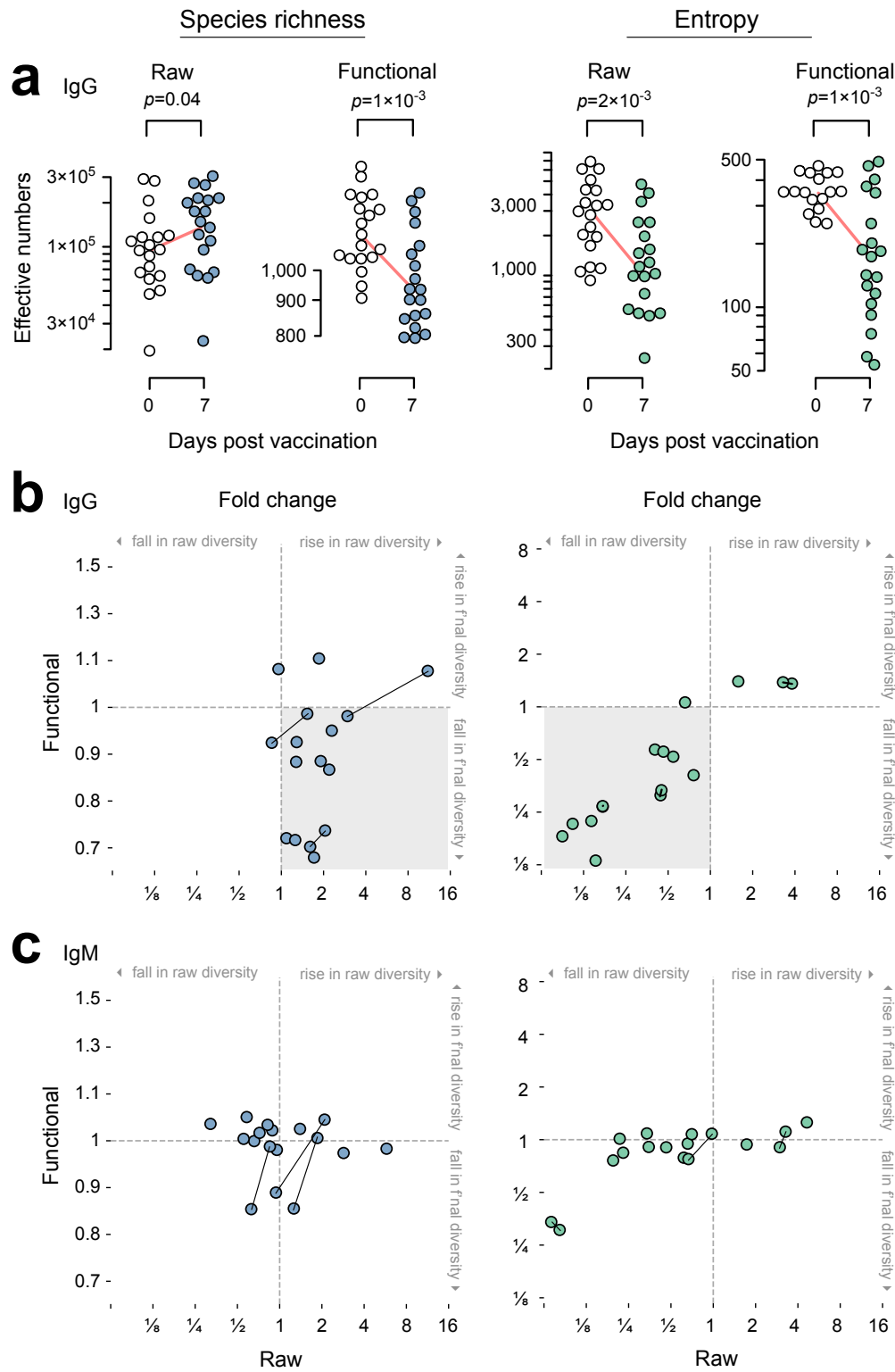
Figure 6



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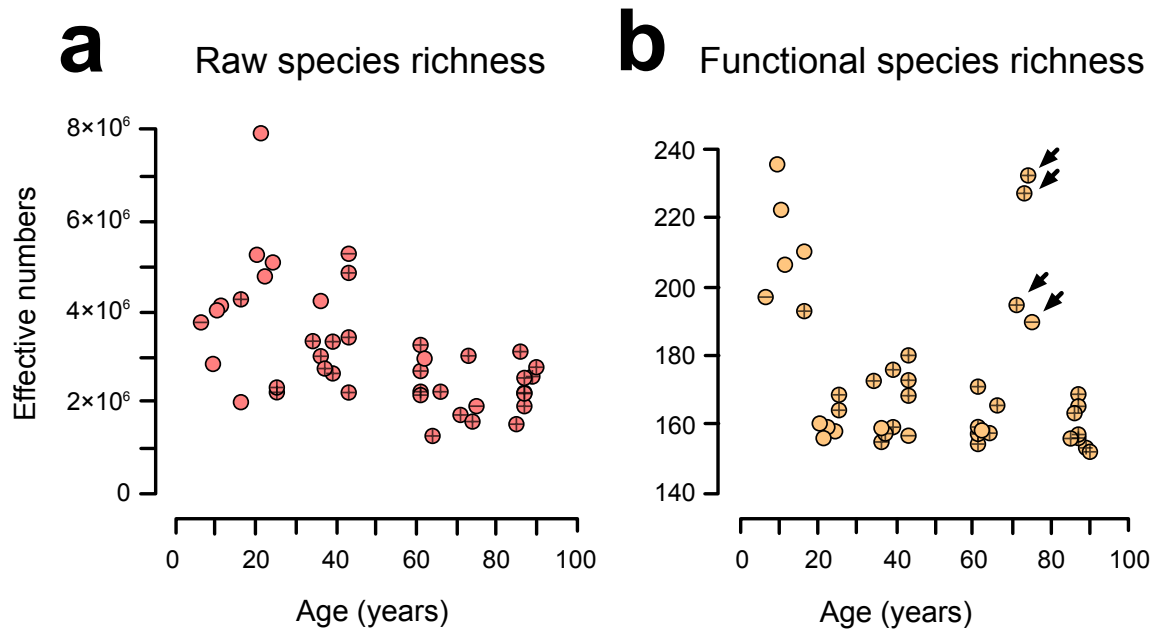
Figure 7



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Figure 8

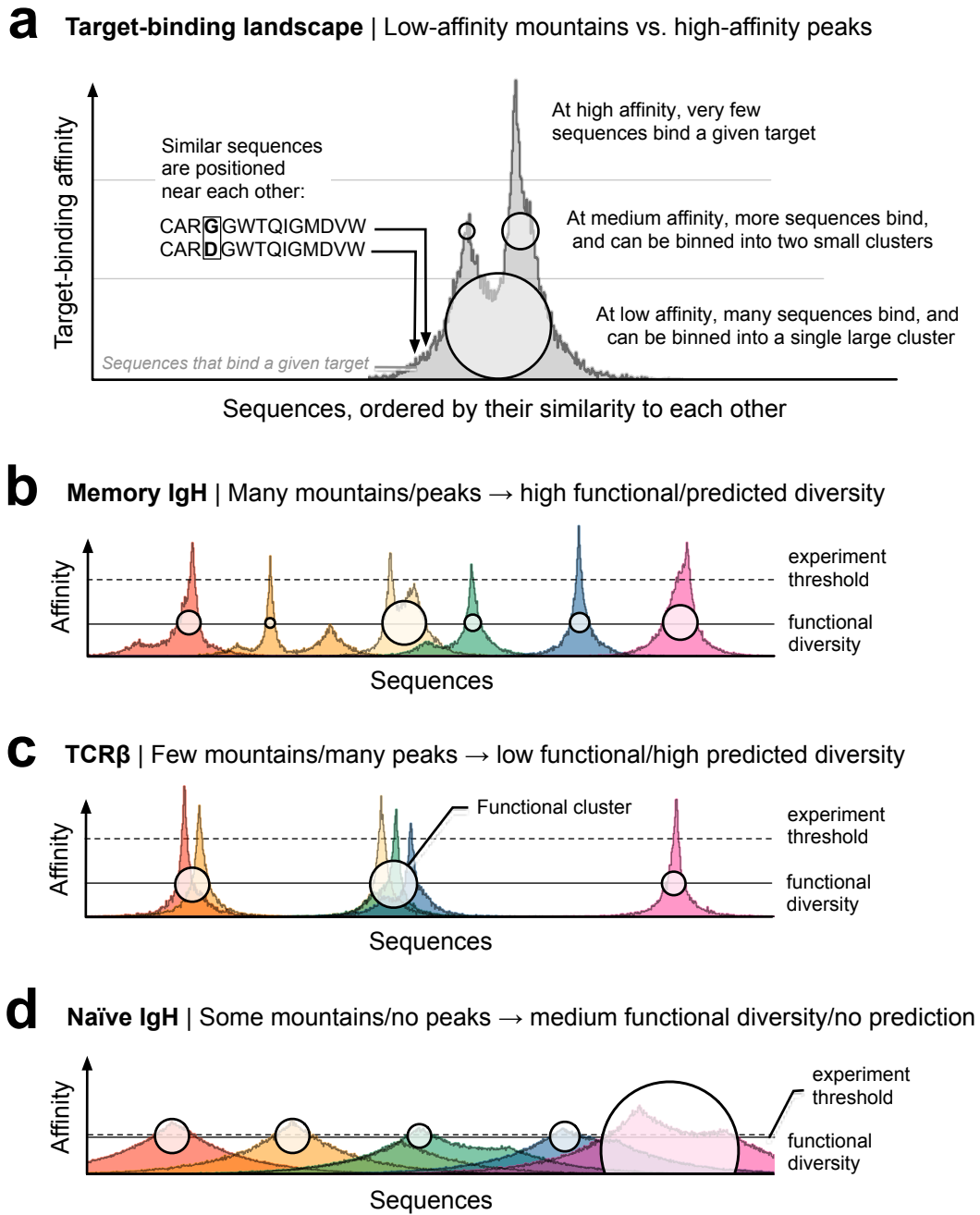


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Figure 9



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