

1 **Title:** Revisiting the dimensionality of biological diversity

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11 Abstract

12 Biodiversity can be represented by different dimensions. While many diversity
13 metrics try to capture the variation of these dimensions they also lead to a
14 ‘fragmentation’ of the concept of biodiversity itself. Developing a unified measure
15 that integrates all the dimensions of biodiversity is a theoretical solution for this
16 problem, however, it remains operationally impossible. Alternatively, understanding
17 which dimensions better represent the biodiversity of a set of communities can be a
18 reliable way to integrate the different diversity metrics. Therefore, to achieve a
19 holistic understand of biological diversity, we explore the concept of dimensionality.
20 We define dimensionality of diversity as the number of complementary components
21 of biodiversity, represented by diversity metrics, needed to describe biodiversity in an
22 unambiguously and effective way. We provide a solution that joins two components
23 of dimensionality — correlation and the variation — operationalized through two
24 metrics, respectively: Evenness of Eigenvalues (EE) and Importance Values (IV).
25 Through simulation we show that considering EE and IV together can provide
26 information that is neglected when only EE is considered. We demonstrate how to
27 apply this framework by investigating the dimensionality of South American small
28 mammal communities. Our example evidenced that, for some representations of
29 biological diversity, more attention is needed in the choice of diversity metrics
30 necessary to effectively characterize biodiversity. We conclude by highlighting that
31 this integrated framework provides a better understanding of dimensionality than
32 considering only the correlation component.

33 Keywords: biodiversity metrics, communities, biodiversity measurement, Importance
34 Values, Evenness of Eigenvalues.

35 **Introduction**

36 Biodiversity encompasses all variation present in life, from genetic material to
37 populations, communities and higher levels of biological organization like entire
38 ecosystems (Wilson 1997). In addition to its broadness in scale and complexity, the
39 central position of the concept of biodiversity in ecological studies justifies efforts to
40 develop measures that properly operationalize the concept. These efforts are reflected
41 in the immensurable number of diversity metrics that have appeared as attempts to
42 encompass all the variation in biodiversity. However, although these diversity metrics
43 allow the description of different dimensions, as the number of them increases the
44 concept of biodiversity becomes operationalized in disparate ways that convey no
45 precise information. This lack of consensus in operationalization of the concept of
46 biodiversity led Hurlbert (1971) to propose the idea of the non-concept of species
47 diversity, in which he advocated that the many metrics of biodiversity be summarized
48 in only a few relevant ones that can be used to express adequately and unambiguously
49 the concept of biodiversity.

50 Long since Hurlbert's seminal work, there has been a pronounced increase in
51 the number of metrics that quantify characteristics of biological diversity other than
52 the traditional taxonomic-based metrics, revealing that patterns of diversity for some
53 communities can be best described using other components of biological diversity,
54 such as functional and phylogenetic components (Graham and Fine 2008, Cisneros et
55 al. 2014). However, these findings are not consensual (e.g Lamb et al. 2009), since
56 some phylogenetic and functional metrics can be strongly correlated with traditional
57 metrics (Tucker and Cadotte 2013, Tucker et al. 2018), deepening the question of
58 which metrics represent the fundamental components of biological diversity
59 (Hurlbert, 1971). A theoretical approach to searching for fundamental variation in

60 biodiversity is to integrate the many sources of information in a unique framework.

61 This integration can be achieved by investigating the relationships among existing

62 metrics. A previous work that proposed this integration based it on quantifying a

63 characteristic of biodiversity known as dimensionality (Stevens and Tello 2014).

64 Dimensionality can be defined, at the community scale of biological

65 organization, as the amount of information needed to effectively characterize the

66 variation presented in a given biodiversity representation, by means of diversity

67 metrics. Communities with high dimensionality require more dimensions to be

68 effectively described than communities with low dimensionality (Stevens and Tello

69 2014). Quantifying the dimensionality of biodiversity currently involves searching for

70 the degree of complementarity in spatial or temporal variation among multiple metrics

71 of diversity, which is obtained mainly through a measure denominated Evenness of

72 Eigenvalues (hereafter EE) (Stevens and Tello 2014).

73 Stevens and Tello's EE metric is obtained by Principal Component Analysis

74 (PCA) of a matrix of diversity metrics (hereafter matrix \mathbf{M} , *sensu* Ricotta 2005) for a

75 set of communities, and calculating an evenness metric for the eigenvalues of the axes

76 that represent this fundamental biodiversity space. The logic behind EE is that, if the

77 diversity metrics used to characterize communities have low complementarity, almost

78 all of the fundamental variation in biodiversity will be centered in a few axes,

79 producing a low EE. On the other hand, if diversity metrics are completely

80 complementary with each other (variation in biodiversity will be equally distributed

81 among axes) the EE of the communities will be 1.

82 The EE metric represents, in a simple way, the degree of complementarity

83 among the dimensions of biodiversity represented by diversity metrics, which

84 comprises what we call here the correlation component of dimensionality (see also

85 Tucker and Cadotte 2013, Lamb et al. 2006 for uses of correlation component).
86 However, EE ignores another source of information in dimensionality — the amount
87 of variation, or importance, that each diversity metric presents in fundamental
88 biodiversity space. This comprises what we call here the variation component of
89 dimensionality.

90 Suppose a situation in which diversity metrics are highly correlated (Figure 1
91 A) and each metric accounts for a similar amount of variation in fundamental
92 biodiversity space (Figure 1B). This situation has low complementarity among
93 dimensions of biodiversity and high redundancy in the amount of variation that each
94 metric captures in fundamental biodiversity space (represented as the length of the
95 arrows in 1B). Consequently, we could rely on any of these diversity metrics to
96 effectively represent the variation in biodiversity of these communities. On the other
97 hand, communities with low complementarity may present a situation in which one of
98 the metrics captures almost all the variation in the fundamental biodiversity space
99 (Metric 2 in Figure 1C), indicating low redundancy of metrics. Following the current
100 approach to measuring dimensionality, EE would indicate similar patterns of
101 dimensionality for communities in 1B and 1C. However, the choice of metric in 1C is
102 of greater importance than in 1B, in which the metrics are highly redundant regarding
103 the information captured. Therefore, considering only the correlation component does
104 not provide enough evidence to support the decision of which diversity metrics to use
105 to effectively characterize biological diversity for two communities with similar EE,
106 because it disregards the variation component inherent to dimensionality.

107 Finding a measure that captures the variation component of dimensionality is
108 not an impediment for effectively characterizing dimensionality, since it can be
109 operationalized by the metric Importance Values (hereafter IV) proposed by Wilsey et

110 al. (2005). However, since the common way to quantify dimensionality (Stevens and
111 Tello 2014) is limited to capturing only the correlation component, the development
112 of a unified framework that combines both correlation and variation components
113 would provide a way to better represent the dimensionality of biodiversity.

114 Therefore, our aim was to update the concept of dimensionality of biodiversity
115 and its operationalization by integrating the correlation and variation components
116 through EE and IV in a framework for quantification of dimensionality. To do this we
117 show, through simulation, how EE and IV together can distinguish situations with
118 different degrees of complementarity of dimensions of diversity and redundancy of
119 information that each metric captures. We then present an empirical example of the
120 investigation of dimensionality by applying the integrated framework to communities
121 of small mammals (cricetids and marsupials). Specifically, we evaluated the level of
122 complementarity and redundancy for different sets of diversity metrics used to
123 describe the biodiversity of cricetids and marsupials, highlighting how the proposed
124 dimensionality framework facilitates the first step of biological assessment — the
125 choice of metrics to be used for characterizing biodiversity.

126

127 **Material and Methods**

128 *Investigating the dimensionality of biodiversity: obtaining EE and IV*

129 Our framework for investigating the dimensionality of biodiversity comprises
130 three steps. The first step is to calculate matrix **M**, which, for the sake of simplicity,
131 will contain three metrics of diversity for the simulation analysis: a measure of
132 functional diversity (FD [Petchey & Gaston 2006]), a measure of phylogenetic
133 diversity (PD [Faith 1992]) and richness. We chose a simplistic approach with only
134 three metrics since our objective with the simulation analysis was to focus on showing

135 how IV can reveal patterns that are not detected by using only EE. We were more
136 interested in the patterns of correlation and variation of diversity metrics in
137 biodiversity space than the particularity of the metrics themselves. We present a more
138 realistic exploration of the integrated framework in the section *Assessing the*
139 *dimensionality of biodiversity in small mammal communities*.

140 The second step involves performing a PCA of matrix **M** using a standardized
141 correlation matrix. As will be shown next, the standardization method applied to
142 matrix **M** prior to the PCA must differ between the calculation of EE and IV.

143 The third step is to calculate the dimensionality metrics EE and IV. We
144 calculate EE using Camargo's evenness index in Equation 1, following the original
145 proposition of Stevens and Tello (2014):

146
$$EE = 1 - \left(\sum_{p=1}^{A(A-1)/2} |e_{ip} - e_{jp}| \right) / A$$
 Equation 1

147 Camargo's evenness index (Camargo 1993) is calculated using the axes (A) and their
148 respective eigenvalues (e_{ih} and e_{jh}) from a PCA of the standardized matrix **M**, in
149 which the metrics were scaled to have a mean of zero and equal variances. The higher
150 the value of EE, the higher the complementarity the communities have in relation to
151 the dimensions of biodiversity represented in matrix **M**. On the other hand, lower EE
152 values indicate lower complementarity in the dimensions used to characterize the
153 communities. IV is calculated according to the method proposed by Wilsey et al.
154 (2005), using a matrix (**M**) standardized by the maximum values of each diversity
155 metric. This standardization removes the effect that the different units of each
156 diversity metric have, without modifying their original variation. To obtain IV for
157 each diversity metric in matrix **M** we apply Equation 2, in which IV_i represents the IV
158 of diversity metric i , r^2_{ij} is the squared correlation of diversity metric i with PC_j , and

159 R_j^2 is the amount of variation that PC_j accounts for in ordination space (biodiversity
160 space).

161
$$IV_i = \sum r_{ij}^2 \times R_j^2 \quad \text{Equation 2}$$

162 PC varies from 1 to j and corresponds to the number of significant
163 eigenvectors in the PCA, evaluated by the Kaiser-Gutmann criterion. The greater the
164 IV the more variation the diversity metric accounts for in biodiversity space. IV
165 approaches 1 when the diversity metric accounts for almost all the variation and
166 approaches zero when the metric accounts for little variation. Sets of communities
167 with highly uneven IV values for diversity metrics possess low redundancy in metric
168 importance, while communities with highly even IV values possess high redundancy
169 regarding the amount of information captured by each metric.

170 *Testing the assessment of the dimensionality of diversity using EE and IV*

171 To assess the effectiveness of EE and IV in acquiring information regarding
172 correlation and variation of dimensionality in matrix **M**, the following conditions
173 must be met: (1) EE values must not differ for set of communities simulated in
174 scenarios with the same level of correlation among diversity metrics, and must differ
175 among communities that have different levels of correlation among diversity metrics;
176 (2) for scenarios with low and high correlation, IV must be similar among metrics that
177 have similar variation in biodiversity space (e.g. Figure 1B), and differ for scenarios
178 in which variation in biodiversity space is mainly due to a single metric (e.g. situation
179 represented Figure 1C, Metric 2 must have a higher IV than Metric 1). We evaluate
180 whether EE and IV can recover these patterns by simulating communities with
181 varying degrees of correlation and variation for each metric in biodiversity space
182 obtained from matrix **M**.

183 The simulations were based on a pattern-oriented procedure, producing
184 diversity metrics with patterns of correlation and variation that represent four
185 scenarios with the following characteristics: In the HiC/EqV (High Correlation and
186 Equal Variation) scenario the diversity metrics are highly correlated and have similar
187 variation in biodiversity space. The HiC/DifV (High Correlation and Different
188 Variation) scenario has diversity metrics that are highly correlated and vary in
189 importance of each metric in biodiversity space. The LoC/EqV (Low Correlation and
190 Equal Variation) scenario has diversity metrics with low correlation and similar
191 importance in biodiversity space. Finally, the LoC/DifV (Low Correlation and
192 Different Variation) scenario has diversity metrics with low correlation and dissimilar
193 importance in biodiversity space.

194 We generate scenarios HiC/EqV and HiC/DifV by starting with a phylogeny
195 that was simulated by a birth-death processes (function *sim.bdtree* from the package
196 *geiger* [Harmon, Weir, Brock, Glor, & Challenger, 2008]) where a species, chosen
197 randomly, initiates the procedure by colonizing a given community. Subsequent
198 addition of species to the community depends on the species that are already present
199 in that community. Communities at one extreme will only contain species that are
200 phylogenetically closely related to each other (top 10%), with the phylogenetic filter
201 becoming less restrictive until communities do not have any phylogenetic filter that
202 restricts coexistence of species (least restrictive condition). Since we simulated a
203 continuous trait that was conserved over the phylogenetic tree — evolved according
204 to a Brownian motion model, using the function *rTraitCont* (Paradis et al. 2004) with
205 the ρ [*rho*] parameter set to 3 — with the number of species in each community
206 gradually increasing (less phylogenetic filter, more species), the procedure created a
207 gradient of phylogenetic, functional and taxonomic diversity metrics. In order to

208 generate differences in variation of the diversity metrics, in scenario HiC/ DifV we
209 simulated a trait that evolves according to a regime of stabilizing selection (Ornstein-
210 Uhlebeck model with the strength of selection set by the parameter α at 0.8) that
211 restricts trait variation to within an optimal range (represented by a θ [theta] of 0).
212 This allowed us to generate a set of communities in which the diversity metrics were
213 highly correlated but variation of FD was much lower than that of richness and PD
214 since the traits that were used in the calculation of FD were restricted by the selection
215 process.

216 We generated the scenario LoC/DifV by following the same procedures
217 described above for scenario HiC/EqV, however, the trait was simulated to have low
218 phylogenetic signal and the phylogenetic tree used to calculate PD was modified to
219 simulate a process of evolution in which most speciation occurs near the root (a star-
220 like phylogeny). This procedure resulted in low correlation between PD and FD, since
221 the relationship between phylogeny and traits was disrupted. Additionally, low
222 variability for PD and richness metrics was obtained since we set the simulations to
223 produce communities with the same number of species but with the phylogenetic
224 filtering acting in community assembly. Consequently, most of the variation in this
225 scenario is due to the FD metric. Finally, to generate scenario LoC/EqV we simulated
226 communities in which all species in the phylogenetic tree had an equal probability of
227 occurring in any community (no phylogenetic filtering acting on the assembly), and
228 set the richness to be very similar for all communities. This procedure generated
229 metacommunities with low correlation and similar amounts of variation for all
230 diversity metrics.

231 We generated 999 sets of communities for each scenario described above,
232 with the metacommunities of all scenarios being composed of 50 communities with a

233 minimum of 20 and a maximum of 200 species. The phylogenetic filter was set to act
234 gradually on the communities, increasing by the order of 10% (start by selecting the
235 top 10% most phylogenetically similar species, followed by the top 20% and so on
236 until 90% of the species have been selected from the pool). Details and an illustration
237 of the simulation procedures and scenarios are presented in the supplementary
238 material Appendix S1, along with a link to an interactive module that we produced to
239 illustrate the simulation procedure used in this work.

240 Finally, we tested whether the values of EE and IV met our theoretical
241 expectations. We checked if EE values differed between scenarios with low
242 correlation and scenarios with high correlation (scenarios HiC/DifV and HiC/EqV
243 versus scenarios LoC/EqV and LoC/DifV). To effectively capture the correlation
244 component of dimensionality EE must be higher in scenarios with low correlation
245 among diversity metrics than in scenarios with high correlation. To test for
246 differences among IV values of each metric in the scenarios we used a graphical tool
247 called profile of importance (Wilsey et al. 2005) and quantified differences in IV of
248 each metric by calculating F values obtained from a linear model (Equation 3). F
249 values allow the IV values of the three dimensions (PD, FD and richness) to be
250 compared and to determine if the IV values of the DifV scenarios (scenarios
251 HiC/DifV and LoC/DifV) differed more from each than did the IV values calculated
252 for the EqV scenarios (scenarios HiC/EqV and LoC/EqV). The simulation scenarios
253 and the theoretical expectations regarding EE and IV follow the schematic
254 representation present in Figure 2.

255 *Assessing the dimensionality of biodiversity in small mammal communities*

256 We illustrate the application of the dimensionality framework with a database
257 of small mammal communities (marsupial and cricetid mammals) distributed

258 throughout the South American continent. We constructed matrix **M** for these
259 communities by calculating eight diversity metrics that represent different dimensions
260 of taxonomic, functional and phylogenetic components of biological diversity. The
261 choice of metrics was based on the works of Tucker et al. (2017) and Scheiner (2019),
262 which together represent the most complete compilation and classification of metrics
263 of taxonomic, functional (Scheiner, 2019) and phylogenetic diversity (Tucker et al.
264 2017). We chose at least one metric for each of the richness, divergence and
265 regularity dimensions of the three components of biodiversity considered here. The
266 taxonomic component was represented by richness; the functional component by FD
267 (richness dimension, Petchey and Gaston 2006b), FEve (regularity dimension) and
268 FDiv (divergence dimension, Villéger et al. 2008); and the phylogenetic component
269 by PD (richness dimension, Faith 1992), MNTD (divergence dimension, Webb et al.,
270 2002), PSV (divergence dimension, Helmus et al., 2007) and PE_{ve} (regularity
271 dimension, Villéger et al. 2014).

272 Traits used to calculate functional metrics comprised life-history attributes —
273 weight, head-body length, diet and form of locomotion. Species were categorized
274 according to their diet as insectivores, herbivores, granivores, omnivores, frugivores,
275 piscivores, seed predators and leaf predators, and according to their modes of
276 locomotion as terrestrial, semifossorial, semiaquatic, arboreal and scansorial. Some
277 species were allocated to more than one diet and locomotion category. All calculated
278 diversity metrics require a distance matrix or a functional dendrogram obtained from
279 a distance matrix. Therefore, to obtain the functional distance matrix we used Gower
280 distance (Pavoine et al. 2009) for traits that have different statistical characteristics
281 (numerical and categorical).

282 The phylogenetic hypothesis used to calculate phylogenetic indices was
283 obtained from the mammalian phylogenies of Bininda-Emonds et al. (2007) and
284 Fabre et al. (2012), the latter of which was used to improve the phylogenetic
285 resolution to species level. Seven species present in our data were not included in the
286 phylogeny Fabre et al. (2012), so we included these species as polytomies within their
287 respective genera. Divergence times for our phylogeny were estimated in millions of
288 years by equally distributing the ages of undated nodes, based on the know ages
289 present in Bininda-Emonds et al. (2007) and Fabre et al. (2012), using the BLADJ
290 algorithm of Phylocom software (Webb et al. 2008). The phylogenetic hypothesis and
291 the original references compiled to assemble the community data used in this work
292 are provided in Figure S2 and Table S1 of Appendix 2 of the supplementary material.

293 The metrics EE and IV were calculated as previously described, with the
294 number of axes used in IV calculation being determined by the Kaiser-Gutmann stop
295 criterion. We also compared the observed values of EE with a null distribution of 999
296 EE values generated by a null model that randomizes a species incidence matrix while
297 preserving differences in richness among sites and mixing species frequency
298 (performed with the *sim3* function from the *EcoSimR* package [Gotelli and Ellison
299 2013a]). Using this null model we tested the null hypothesis that observed EE values
300 do not differ from expected EE values according to variation in richness. We
301 implemented a function called *dimensionality* to calculate EE values from matrix **M**.
302 The function allows the user to choose the evenness method that will be used in the
303 calculation. It can be accessed at
304 https://github.com/GabrielNakamura/dimensionality_function.

305 We calculated IV for the small-mammal metacommunities according to
306 Equation 2, applying *ImportanceVal* — the R code for the IV function (the function

307 can be accessed at https://github.com/GabrielNakamura/IV_function). We used the
308 Kaiser-Gutmann stop criterion and a bootstrap procedure that re-sampled matrix **M**
309 999 times and recalculated IV for each metric so that we generated confidence
310 intervals for the IV value of each diversity metric. We performed all calculations with
311 a standardized matrix **M** (scaled to a mean of zero and unit variance for the
312 calculation of EE values and standardized by the maximum values of each metric for
313 the calculation of IV values). Bootstrapped IV values were submitted to an Ordinary
314 Least Square (OLS) linear model to test for differences in the importance of the
315 components of diversity that assemble matrix **M**:

$$316 \quad IV_i = \alpha + \beta * dimension_i + \varepsilon_i \quad , \quad \text{Equation 3}$$

317 Equation 3 represents the effects parametrization model in which IV_i represents the
318 predicted value of IV for the diversity metric i , β the effect of a given dimension over
319 another and ε_i the error term associated with the residuals, which follow a Gaussian
320 distribution. Each value of IV was classified as belonging to the phylogenetic (PD,
321 PEve, PSV and mntd), the functional (FD, FDis and FDiv) or the taxonomic
322 (richness) dimension. Through this model we aimed to determine if any of the
323 components of diversity (functional, phylogenetic or taxonomic) captures a greater
324 amount of information from biodiversity space. Additionally, we performed another
325 linear OLS model using the same set of data but considering each metric as the
326 explanatory variable, in order to assess differences in importance among diversity
327 metrics. For both models we performed a Tukey test to assess pairwise differences in
328 importance among dimensions and metrics.

329 The dimensionality framework was applied to four different configurations of
330 matrix **M**: all metrics; a combination of phylogenetic metrics and richness; a
331 combination of functional metrics and richness; and a combination of functional and

332 phylogenetic metrics. We performed these analyses to show how dimensionality can
333 change depending on the components of diversity used in matrix **M**, and what the
334 implications of different values of EE and different similarities among metrics IV
335 (represented as Camargo's evenness of IV metrics) are on the choice of diversity
336 metrics to be used to represent the biodiversity. For these analysis we also computed
337 EE as the mean value calculated from a bootstrap procedure equivalent to that used
338 for the IV metric, in order to generate confidence intervals.

339 **Results**

340 *Simulated data*

341 Our simulation revealed that EE and IV, when used together, acquire information
342 regarding two aspects of dimensionality: correlation among metrics and the variation
343 that each metric accounts for in biodiversity space. This complementary information
344 that IV brings to the analysis of dimensionality is evidenced in Figure 3. Thus,
345 different patterns of redundancy in information captured by the metrics can be
346 obtained for a given level of correlation, with greater differences among IV values in
347 scenarios HiC/DifV and LoC/DifV (right side of Figure 3) than in HiC/EqV and
348 LoC/EqV (left side of Figure 3).

349 The differences in EE between scenarios of high and low correlation (Figure
350 1, comparison between EE of upper and lower graphics), but not between scenarios of
351 different and equal variation (Figure 1, comparison between EE bars in the same row)
352 support our argument that this metric captures only the correlation component of
353 dimensionality.

354 The ability of IV to capture the degree of redundancy in biodiversity
355 information of the metrics was clear mainly for the HiC/DifV scenario, in which the
356 attribute used to generate communities exhibited low variation (OU model) and,

357 consequently, the FD metric presented lower IV than richness and PD metrics. It is
358 worth noting that differences among the IV of metrics was greater in scenario
359 LoC/EqV than in scenario HiC/EqV (Figure 1, lower right graphic), since it is not
360 possible to obtain high redundancy in metric information (indicated by similar IV
361 values among metrics) along with high values of complementarity (indicated by high
362 EE). High redundancy in the importance of metrics is only possible for communities
363 with low EE (low complementarity of dimensions), as demonstrated by scenario
364 HiC/EqV. The magnitude of the differences in IV among metrics for each scenario is
365 shown in Figure S3 of Appendix S3 of the supplementary material.

366 *Small mammal communities*

367 We obtained a moderate value for complementarity for the small mammal
368 communities, as indicated by an EE of 0.49 for matrix **M** calculated with all eight
369 diversity metrics. The correlation component of dimensionality, at least for the three
370 analyzed components of diversity (functional, phylogenetic and taxonomic), may be a
371 consequence of spatial gradients of species richness, as evidenced by comparing
372 observed EE with that expected by the null model distribution of EE (Figure S4 in
373 Appendix 3 of the supplementary material).

374 Only two axes of the PCA were significant according Kaiser-Guttman criterion
375 (representing 70% of all the variation in matrix **M**), and composed the fundamental
376 biodiversity space in which IV was calculated. Observed IV values for the eight
377 diversity metrics ranged from 0.19 for PSV (27% of all the variation in biodiversity
378 space) to 0.003 to FDiv (0.3% of all the variation in biodiversity space). Bootstrap
379 means and confidence intervals for IV for all metrics are illustrated in Figure 4
380 through the IV profile (*sensu* Willig and Hollander 1995), evidencing PSV as the
381 metric capturing most of the variation in biodiversity space, followed by richness.

382 The linear OLS model showed significant variation in IV among diversity metrics
383 (F-value= 3.428; $p < 0.05$), while the Tukey test revealed that the greatest difference in
384 importance was between taxonomic and functional components of biodiversity
385 followed by the difference between phylogenetic and functional components
386 (difference between observed means of 0.092 and 0.064, respectively; Figure S5 of
387 Appendix 3). This finding highlights the importance of considering the taxonomic and
388 phylogenetic dimensions in characterizing the biodiversity of communities of
389 cricetids and marsupials.

390 Analysis of dimensionality for matrix **M** containing functional metrics and
391 richness had the highest complementarity (highest EE) and lowest redundancy in
392 metric importance (biodiversity representation with similar values of IV, as indicated
393 by a lower evenness of IV than obtained for other sets of metrics) (Figure 5). PSV
394 was the metric that captured the most information in matrix **M** containing
395 phylogenetic metrics and richness (30% of all the variation in biodiversity space) and
396 phylogenetic and functional metrics (31% of all the variation in biodiversity space), as
397 well as for matrix **M** containing all metrics (24% and of all the variation in
398 biodiversity space). For matrix **M** that considered only functional metrics and
399 richness, richness captured most of variation (47% of all the variation in biodiversity
400 space). Despite the high variability, as indicated by the confidence intervals of IV and
401 EE evenness, it is worth noting that IV evenness remains constant for different mean
402 values of EE, with the greatest IV evenness being for the set of metrics that had the
403 lowest EE value (matrix **M** with phylogenetic metrics and richness).

404 **Discussion**

405 Our results with simulated data evidence the need for a dimensionality framework
406 that integrates both EE and IV in order to effectively characterize dimensionality by
407 considering its two components —correlation and variation in biodiversity space.

408 Operationalizing these two components through EE and IV reveals their
409 complementarity (by means of EE) and, given some level of complementarity, the
410 degree of redundancy in information captured by the metrics used to express these
411 dimensions (through IV). Therefore, our proposed dimensionality framework
412 represents a step beyond the current approach to operationalizing dimensionality, as
413 proposed by Stevens and Tello (2014) by distinguishing the degree of redundancy in
414 information that each diversity metric captures.

415 Our integrated dimensionality framework joins other propositions in helping
416 to choose metrics for the biological characterization of communities. We are aware
417 that the main guide for choosing diversity metrics must be the objectives of the work.
418 However, regardless of the objective, it is desirable to use diversity metrics that
419 encompass complementary components of biological diversity and account for a
420 satisfactory amount of the information present in the biodiversity component being
421 investigated (Ricotta 2005b). In this respect, Saito et al. (2015) showed that
422 phylogenetic, functional and traditional taxonomic indices present complementary
423 information and should be used to adequately characterize and monitor biodiversity of
424 stream macroinvertebrate communities. Ouchi-Melo et al. (2018) performed an
425 integrated assessment to identify areas of conservation interest in the Cerrado biome,
426 and evidenced the importance of considering traditional together with functional and
427 phylogenetic metrics. Although both of these works considered the complementarity
428 component by accounting for correlation among metrics, they did not account for
429 redundancy in the amount of variation that each metric captures in biodiversity space,
430 thus facing the same problem presented by using the EE metric alone. The
431 dimensionality framework presented here, therefore, represents the most general and
432 complete framework to date for guiding researchers in their choice of metrics to be

433 used for biological assessment by considering both complementarity among
434 biological dimensions and the amount of information that metrics can capture.

435 It is worth pointing out that the dimensionality of diversity can be investigated
436 at any spatial and temporal scale, and using any configuration of matrix **M**. Even for
437 works that focus on only one component of biodiversity, the investigation of
438 dimensionality can be important for knowing which aspects of biodiversity are worthy
439 of being included in biological assessment. Tucker et al. (2017) identified three
440 complementary components of the phylogenetic component: richness, divergence and
441 regularity. Thus, research focused on phylogenetic diversity can address whether
442 these three components are complementary dimensions in the analyzed communities
443 and which metrics are the most important to measure in order to best represent
444 variation in these dimensions. As we showed in our empirical example with small
445 mammal communities, dimensionality will depend on the representation of biological
446 diversity used in matrix **M**, which influences practical decisions regarding which
447 metrics are the most important for characterizing biodiversity.

448 At least for the cricetid and marsupial communities analyzed here,
449 characterizing diversity through functional and taxonomic components requires great
450 care in the choice of diversity metrics to be used. This is because this situation has the
451 highest complementarity regarding diversity dimensions (highest EE value),
452 indicating the need to rely on different components of diversity to effectively describe
453 biodiversity, and a moderate level of redundancy in metrics, indicating that some
454 metrics account for disproportionately more information than others. In this example,
455 richness accounted for more information than the other metrics, but consideration of
456 other components that represent functional information is also important for
457 effectively characterizing biological diversity. This functional component can be

458 represented by FDiv or FEve, which are very redundant in information. On the other
459 hand, if the characterization of small mammal communities was focused on
460 phylogenetic and taxonomic components, the choice of metrics to be used would
461 require less caution since complementarity among dimensions is lower and
462 redundancy of information is greater, indicating that all the metrics capture similar
463 amounts of information of biodiversity space.

464 When considering matrix **M** with all eight diversity metrics, applying the
465 dimensionality framework to small mammal communities revealed that cricetids and
466 marsupials possess intermediate to low levels of complementarity (mean EE of 0.51
467 ± 0.025). Together with low complementarity, low levels of redundancy among the
468 metrics was found when considering the three components of biodiversity together
469 (mean IV evenness of 0.63 ± 0.082). Consequently, we suggest that the choice of
470 diversity metrics to effectively represent these communities must encompass the three
471 components of diversity — choosing the PSV metric, which accounts for the highest
472 IV, and two other complementary metrics to represent taxonomic (richness) and
473 functional components (FD that has the highest IV among functional metrics, as
474 shown in Figure 6).

475 The patterns of IV values for small mammal communities contrasted with the
476 findings of Wilsey et al. (2005) and Lyashevskaya and Farnsworth (2012), who
477 concluded that richness was the least important diversity metric for representing
478 variation in community structure (grassland and marine benthic communities,
479 respectively). Although we did not consider abundance-based metrics, as these
480 studies did, we point out that patterns of complementarity and redundancy can differ
481 depending on the taxonomic group being investigated and the metrics being used (as
482 already emphasized by our empirical application of the IV framework with different

483 configurations of matrix \mathbf{M}). This finding highlights the need to understand
484 contingencies in the correlation and variation components of the dimensionality of
485 different communities.

486 We only used metrics that capture three sources of information from
487 biodiversity (phylogenetic, functional and taxonomic), since they are the main
488 assessed components of diversity and represent important metrics for capturing
489 different dimensions of these components (Tucker et al. 2017). Despite the limited
490 number of metrics presented in this work, the dimensionality framework used here is
491 highly flexible in the sense that it can be applied to a matrix \mathbf{M} that contains many
492 more dimensions (Ricotta 2005). Therefore, we could represent diversity in a much
493 more complete manner, with metrics that capture other quantifiable components such
494 as genomic (*e.g.* Nei 1978), proteomic (*e.g.* Gotelli et al. 2013b) or any other
495 dimension that can be quantified.

496 *Conclusion and future directions*

497 This work represents an upgrade of the operationalization of the concept of
498 dimensionality presented by previous works. We demonstrate that including the
499 correlation component of dimensionality with the variation component, through the
500 use of EE and IV, in the same framework more effectively characterizes the
501 dimensionality of biodiversity.

502 Besides conceptual and operational advances, the dimensionality framework
503 proposed here provides evidence regarding practical situations in which the choice of
504 diversity metrics is more critical for effectively characterizing biodiversity. The use of
505 this dimensionality framework can help identify these different situations and assist in
506 choosing metrics.

507 Since the evidence presented in the literature regarding characterization of
508 dimensionality is limited (Lyashevskaya and Farnsworth 2012, Stevens and Tello 2014,
509 2018, Stevens and Gavilanez 2015), and based only on specific groups of organisms,
510 some questions still need to be addressed to provide a more complete understanding
511 and generalization of the role that some factors play in the dimensionality of
512 ecological communities. For instance, one might wonder if some dimensions of
513 diversity are consistently more informative than others when describing diversity
514 patterns among different taxa, or if distinct factors (historical, evolutionary and/or
515 ecological) generate predictably higher or lower levels of dimensionality across
516 communities.

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607 **Figure legends**

608 Figure 1: A) A set of communities described by two diversity metrics (Metric 1 and
609 Metric 2) that are highly correlated. This pattern of correlation can be related to two
610 diversity metrics that account for similar amounts of variation in the reduced
611 biodiversity space (B, high redundancy), or be a situation in which one metric has
612 disproportional importance for capturing variation in biodiversity space (C, low
613 redundancy).

614 Figure 2: Schematic representation of simulated scenarios and expected outcomes for
615 EE and IV. The abscissa represents the variation component of dimensionality.
616 Metacommunities were simulated to show similar values of variation among metrics
617 (lower left quadrant) or different values of variation among metrics (lower right
618 quadrant), so that, respectively, similar and different IV values among diversity
619 metrics are expected. The ordinate represents the correlation component of
620 dimensionality. Metacommunities were simulated that had metrics with high (upper
621 right panel) and low correlation, so that, respectively, low and high EE values are
622 expected.

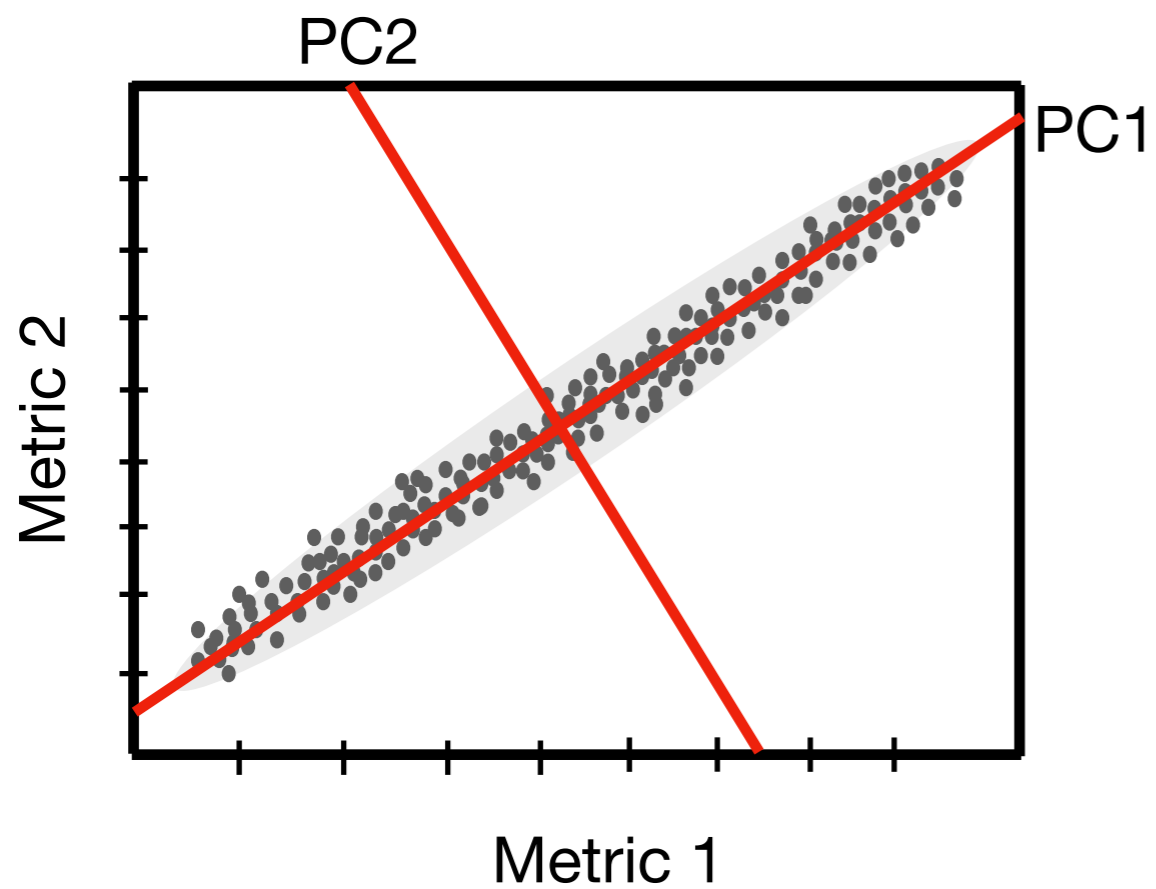
623 Figure 3: Bar plots showing IV and EE calculated for metacommunities simulated
624 according different scenarios (HiC/EqV, HiC/DifV, LoC/EqV and LoC/DifV) using
625 PD, FD and richness metrics in matrix **M**. For each of these scenarios situations were
626 presented in which the metrics contribute similarly or unequally in biodiversity space
627 (variation in ordinate axis) and are highly or lowly correlated (variation in abscissa
628 axis).

629 Figure 4: IV profile for marsupial and cricetid mammal communities from South
630 America calculated using matrix **M** containing eight diversity metrics. Bar height

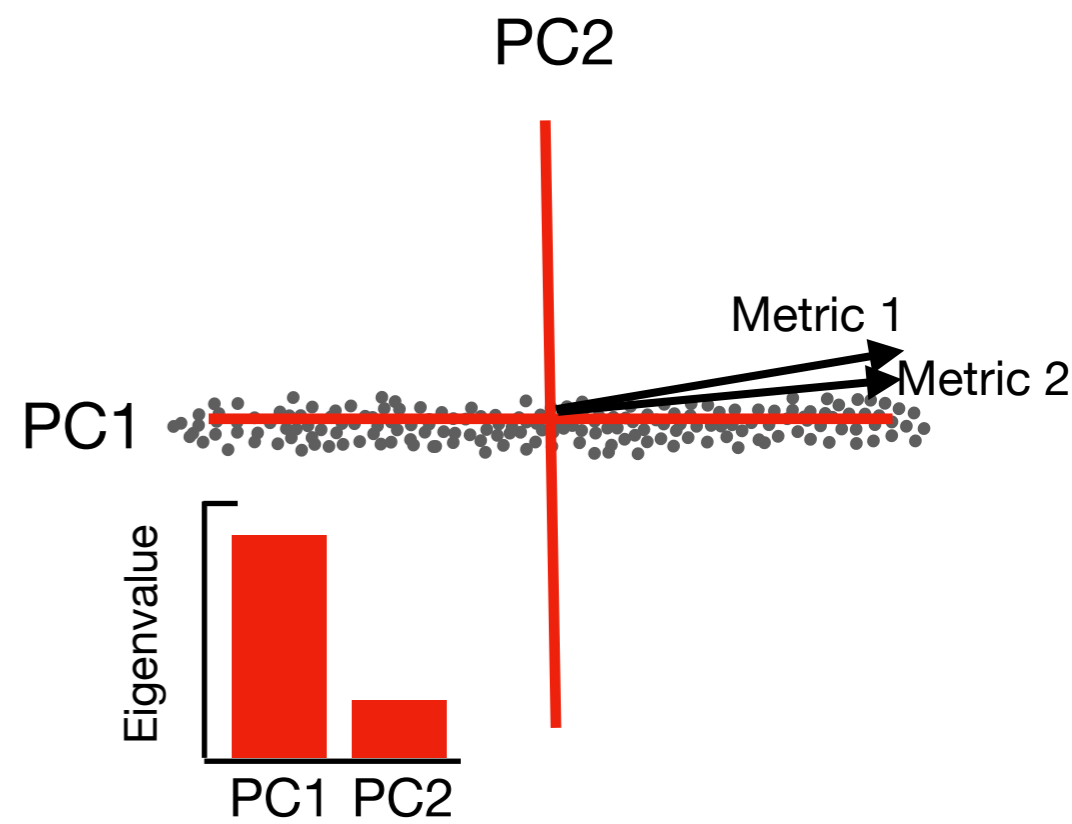
631 corresponds to the mean IV for each diversity metric while lines represent 95%
632 confidence intervals, both calculated via a bootstrap procedure.

633 Figure 5: Values of EE and evenness of IV calculated for four different
634 configurations of matrix **M**. Symbols represent mean values for each matrix
635 configuration while lines represent confidence intervals. Bar graphics represent
636 IV profiles calculated for matrix **M** with all metrics of diversity; functional
637 metrics and richness; phylogenetic metrics and richness; and functional and
638 phylogenetic metrics. Bars represent means while lines represent confidence
639 intervals obtained via a bootstrap procedure applied to each matrix **M**.

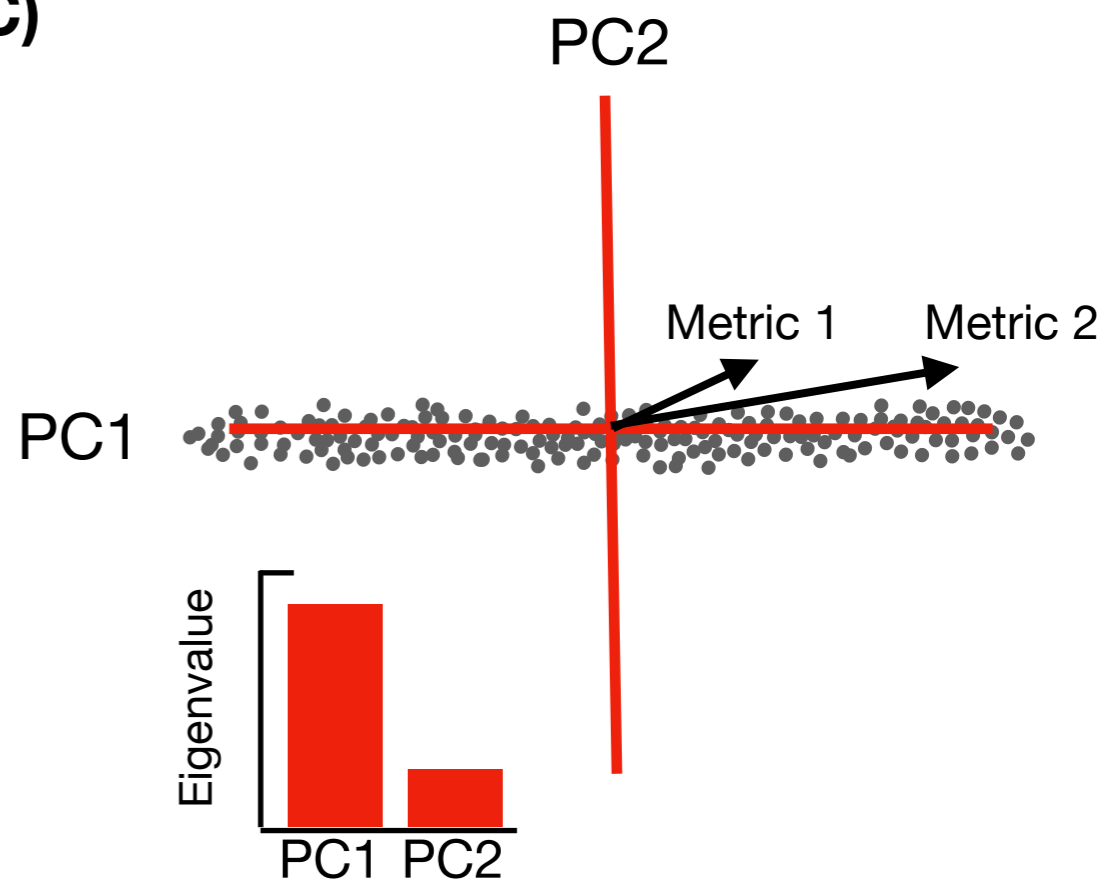
A)

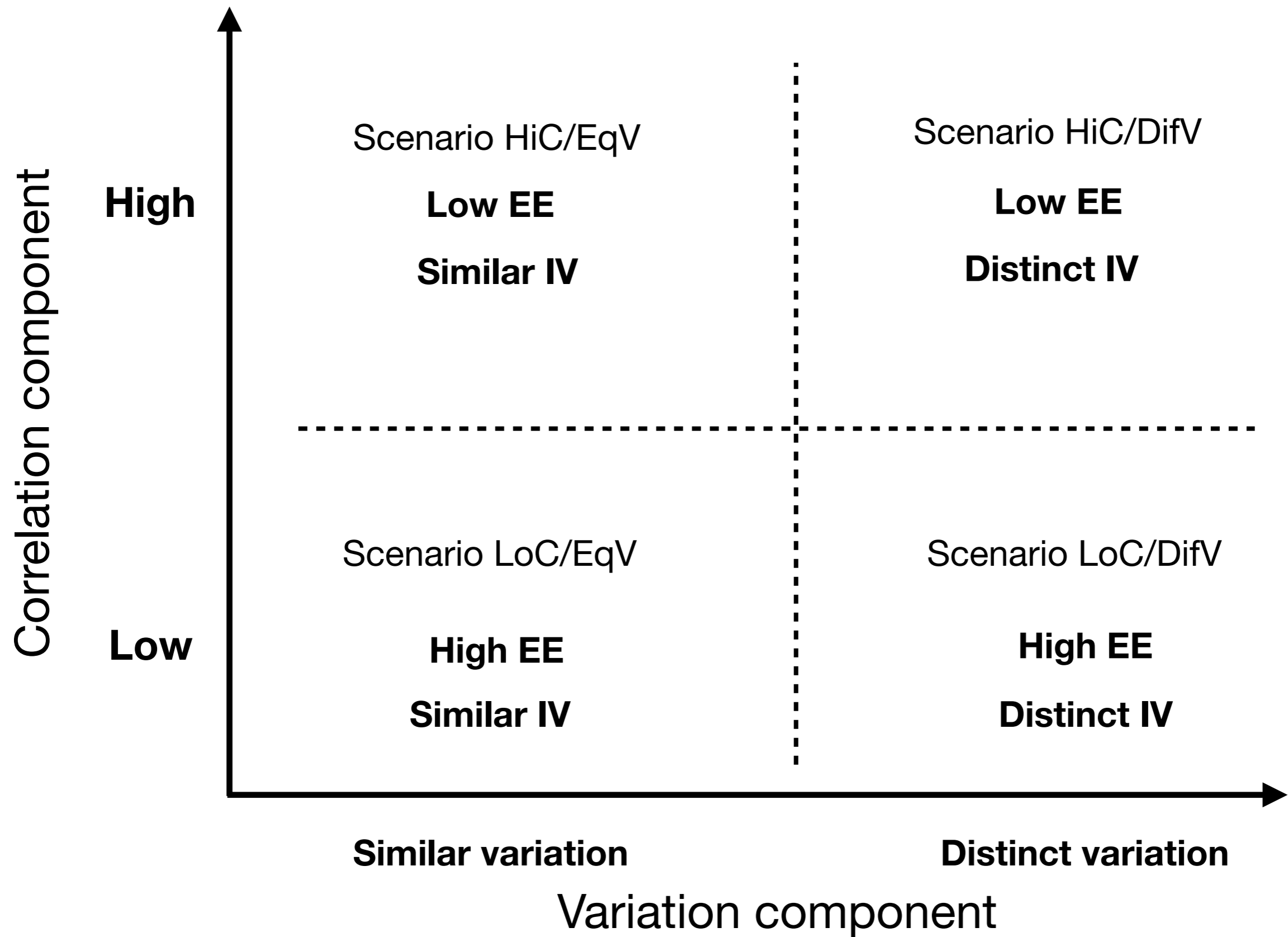


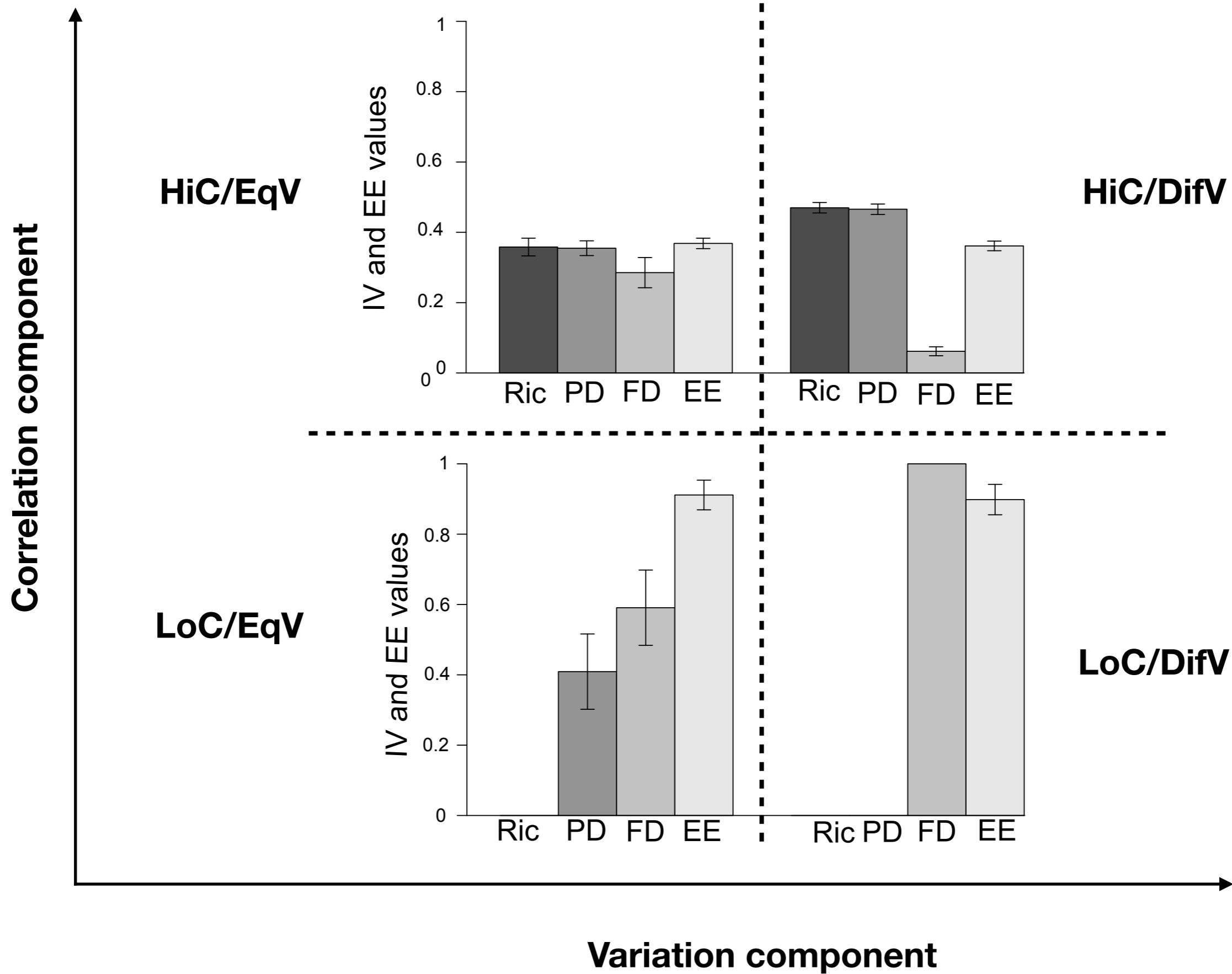
B)



C)







IV

