Antagonistic coevolution between multiple quantitative traits:

matching dynamics can arise from difference interactions

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ABSTRACT

Coevolution is one of the major drivers of complex dynamics in population ecology. Historically,

antagonistic coevolution in victim-exploiter systems has been a topic of special interest, and

involves traits with various genetic architectures (e.g., the number of genes involved) and effects

on interactions. For example, exploiters may need to have traits that "match" those of victims for

successful exploitation (i.e., a matching interaction), or traits that exceed those of victims (i.e., a

difference interaction). Different models exist which are appropriate for different types of traits,

including Mendelian (discrete) and quantitative (continuous) traits. For models with multiple

Mendelian traits, recent studies have shown that antagonistic coevolutionary patterns that appear

as matching interactions can arise due to multiple difference interactions. Here we generalize

their findings to quantitative traits and show, analogously, that the multidimensional difference

interactions sometimes behave qualitatively the same as matching interactions. While previous

studies in quantitative genetics have used the dichotomy between matching and difference

frameworks to explore coevolutionary dynamics, we suggest that exploring multidimensional

trait space is important to examine the generality of results obtained from one-dimensional traits.

Keywords: bidirectional axis of vulnerability, fluctuating selection, gene-for-gene,

matching allele, unidirectional axis of vulnerability

INTRODUCTION

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Theories developed from specific models are initially only as general as their underlying models,

until they are shown to hold in other modeling frameworks. Hence, it is critical to compare

results from a variety of modeling frameworks to test their generality. Antagonistic coevolution in

victim-exploiter systems (e.g., prey-predator or host-pathogen systems) has been intensively

studied theoretically (Abrams 2000, Nuismer 2017). It is thought to be a major driver for the

emergence and maintenance of biodiversity (Thompson 2005) and feedbacks between interacting

species can create complex ecological (e.g., Cortez and Weitz 2014) and evolutionary (e.g.,

Dercole et al. 2010) dynamics. Historically, various coevolution models with differing genetic

architectures (e.g., the number of genes involved) and interspecific interactions (e.g., a "matching"

vs. a "difference" interaction: see below) have been proposed and analyzed (Nuismer 2017). To

account for genetic architecture, one approach is to consider coevolution with a few major genes

that interact and produce a discrete phenotype (Mendelian trait-based models, or hereafter

Mendelian models) (e.g., Mode 1958, Seger 1988). On the other hand, quantitative trait-based

models (hereafter quantitative models) assume the genetic architecture consists of many genes of

small effect and produce continuous phenotypes (e.g., Saloniemi 1993, Gavrilets 1997).

Here we highlight the connection between Mendelian and quantitative models of

victims and exploiters and then clarify the relationships between existing modeling frameworks

for quantitative traits. We begin by reviewing the different modeling frameworks for Mendelian

traits. We then draw analogies between Mendelian and quantitative models (e.g., Boots et al.

2014, Cortez and Weitz 2014). The analogies suggest that the matching framework, in which

exploitation is more successful when traits "match" (i.e., a bidirectional axis of vulnerability

sensu Abrams 2000), can arise from the difference framework, in which exploitation is more

successful when traits differ (i.e., a unidirectional axis of vulnerability *sensu* Abrams 2000), in

multidimensional quantitative models. Finally, as a proof of concept, we give an example model

of theoretical quantitative genetics where a victim-exploiter coevolutionary dynamics arising

from two pairs of difference traits can appear as a matching interaction in a single pair of traits.

BACKGROUND

Historically, in Mendelian models, two major interactions were often assumed and

contrasted: the matching allele (MA) framework, in which exploitation is successful when traits

match (e.g., Grosberg and Hart 2000) (Fig. 1A), and the gene-for-gene (GFG) framework, in

which a generalist exploiter can exploit a range of victim genotypes, but pays a cost for

generalism (Flor 1956) (Fig. 1C). Researchers have investigated the relative importance and

relationships of the two interactions (e.g., Parker 1994, Frank 1996, Agrawal and Lively 2002).

Recently, Ashby and Boots (2017) found that, in multidimensional Mendelian (multilocus)

models, the GFG framework can behave effectively as the MA framework (Fig. 2A).

65   To illustrate, consider a simple GFG model with haploid victims and exploiters with

two loci with two alleles each, labelled 1 and 0. The alleles 1 and 0 indicate the presence and

absence, respectively, of an exploitation or defense trait (Fig 2A). If both of the allelic values of

the exploiter are greater than or equal to those of the victim, maximum exploitation will occur

(black squares in Fig. 2A). If the victim has exactly one allelic value greater than that of the

exploiter, so that the other allelic value of the victim is less than or equal to the corresponding

value of the exploiter, then exploitation will be partial (gray squares in Fig. 2A). Finally, in the

case where both allelic values of the victim are strictly greater than that of the exploiter (which

only occurs when the victim has genotype 11 and the exploiter has 00), then the victim is

defended against exploitation (a white square in Fig. 2A).

75   When coevolutionary dynamics occurs between four genotypes with the alleles 1 and 0,

the type of interaction (indicated by a dashed white line in Fig. 2A) effectively reduces to a MA

interaction: the exploiter genotype 01 (10) is more successful at exploiting its matching counter

victim genotype 01 (10) as in Fig 1A. Indeed, by investigating a multilocus GFG model, Ashby

and Boots (2017) found that, in the case where an intermediate number of alleles is optimal, the

victim/exploiter genotypes alternate between various subsets of defense/exploitation alleles, but

the total number of alleles stays constant. Hence, the multidimensional GFG framework

effectively behaves as the MA framework (Fig. 2A).

As with Mendelian models, quantitative models have assumed an apparent dichotomy

between two frameworks (Abrams and Matsuda 1997, Abrams 2000, Gilman et al. 2012, McPeek

2017): the matching trait framework, in which exploitation is increasingly more successful with

the extent that trait values of exploiters match those of victims (Fig. 1B), and the difference trait

framework, in which exploitation is increasingly more successful with the extent that trait values

of exploiters exceed those of victims (Fig. 1D). They are also known as the bidirectional and

unidirectional trait cases, respectively (Abrams 2000). The matching framework is analogous to

the MA model and is used, for example, when an exploiter can best exploit a victim with body

size or color within a certain range (Brown and Vincent 1992, Marrow et al. 1992, Dieckmann et

al. 1995, Gavrilets 1997, Khibnik and Kondrashov 1997, Nuismer et al. 2005, Kopp and

Gavrilets 2006, Calcagno et al. 2010, Dercole et al. 2010, Mougi 2012, Yamamichi and Ellner

2016, Fleischer et al. 2018). This has been observed in the coevolution between the African

tawny-flanked prinia and its brood parasite, the cuckoo finch: the parasite varies its egg color to

match that of the victim (Spottiswoode and Stevens 2012). On the other hand, the difference

framework is analogous to the GFG model and is appropriate when exploitation success is

determined by speed vs. speed, strength vs. armor, or toxicity vs. resistance (Saloniemi 1993,

Frank 1994, Sasaki and Godfray 1999, Nuismer et al. 2007, Mougi and Iwasa 2010, Mougi and

Iwasa 2011, Tien and Ellner 2012, Northfield and Ives 2013, Cortez and Weitz 2014, Yamamichi

and Miner 2015, van Velzen and Gaedke 2017). For example, garter snakes evolve stronger

resistance to rough-skinned newt toxicity in areas where newts are more toxic (Brodie III and

Brodie Jr 1990).

In the matching framework, exploitation success is typically a unimodal function of the

trait difference, and is often modeled as a Gaussian: $\exp[-(x - y)^2]$, where x and y are traits of

victims and exploiters, respectively (Fig. 1B) (e.g., Gavrilets 1997, Khibnik and Kondrashov

1997, Nuismer et al. 2005, Mougi 2012). On the other hand, in the difference framework,

108 exploitation success is often modeled as a sigmoidal function of the trait difference: $1/[1 + \exp(x$

109 $- y)]$ (Fig. 1D) (e.g., Nuismer et al. 2007, Mougi and Iwasa 2010, van Velzen and Gaedke 2017).

110 In this framework, it is always advantageous for both species to increase the trait values for better

111 exploitation/defense (McPeek 2017). Empirical evidence suggests that there is an energetic

112 trade-off to exploitation and defense and hence, most models of difference traits assume there is a

113 cost associated with each trait, which prevents runaway evolution. For example, garter snakes

114 resistant to newt toxin have costs associated with slower sprinting speeds (Brodie III and Brodie

115 Jr 1999), milkweed bugs' production of enzymes to break down milkweed toxins has metabolic

116 costs (Dalla and Dobler 2016), and numerous plants produce defense chemicals with a cost to

117 plant growth (reviewed in Herms and Mattson 1992).

118  Here, by drawing analogies between the Mendelian trait and quantitative trait

119 frameworks (Fig. 1), we further extend the generalization of Ashby and Boots (2017) and

120 demonstrate that the difference framework can result in similar dynamics to the matching

121 framework in multidimensional quantitative models. The matching model is a continuous trait

122 analog to the MA model and the difference model is a continuous trait analog to the GFG model

123 (Fig. 1). In a multilocus GFG model, under certain conditions the victim/exploiter genotypes

alternate between various subsets of defense/exploitation alleles, but the total number of alleles

stays constant, which is effectively what occurs under the MA framework (Ashby & Boots 2017).

This is analogous to a bivariate difference trait model, in which the victim/exploiter genotypes

alternate between having more investment in one defense/exploitation trait vs. another, which is

effectively what occurs under the matching framework. Instead of switching between the

extremes of a single matching trait, the species are switching between two difference-trait-based

strategies. Here, we consider a victim-exploiter model in which each has two difference traits that

influence exploitation success. An empirical example that motivates our study is a stepwise

infection process (Hall et al. 2017), where host species prevent exploitation of parasites by

several difference traits. In the case where intermediate trait values are optimal due to the costs of

defense/offense, the victim/exploiter traits may switch between combinations where a trait is

large while the other trait is small, but the total costs stay constant. The resultant dynamics may

appear as the matching interaction, and this is the quantitative-model analog to the findings in

Ashby and Boots (2017) in Mendelian models.

 AN EXAMPLE

We develop a two-dimensional quantitative model with difference interactions (Fig.

2B) to demonstrate that matching dynamics can emerge from a multidimensional difference

framework (see Appendix and Supplementary Mathematica file for details). We consider a

quantitative genetic model (Lande 1976, Iwasa et al. 1991) in continuous time (i.e., Malthusian

fitness) (Abrams et al. 1993). We assume that victims and exploiters each have two quantitative

traits. We let x_i be the value of victim trait i (= 1, 2) and assume greater values allow for defense

against trait i of the exploiter. We let y_i be the value of exploiter trait i and assume greater values

allow for more exploitation. Each trait is associated with a cost that decreases individual fitness

with greater trait values. The difference between the exploiter trait and the corresponding victim

trait, $y_i - x_i$, affects exploitation success (Fig. 2B), which is expressed by a sigmoid function (Fig.

1D). We assume that fitness of victims and exploiters are, respectively,

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$$W_x = r(x_1, x_2) - a_0 \prod_{i=1}^{2} \frac{1}{1 + \exp\left[-\theta_i(y_i - x_i)\right]},$$

$$W_y = c(y_1, y_2) a_0 \prod_{i=1}^{2} \frac{1}{1 + \exp\left[-\theta_i(y_i - x_i)\right]},$$
(1)

where a_0 is the effects of successful exploitation on fitness (e.g., maximum attacking rate), and θ_i

($i = 1, 2$) determine the sensitivity of exploitation success to the trait difference. We assume that

victims need to overcome their exploiters at only one trait for escaping from exploitation,

whereas exploiters must overcome the two defense traits for successful exploitation (Gilman et al.

2012). This sequential interaction is also found in many host-parasite systems (Hall et al. 2017).

Here r and c represent costs of defense and exploitation for victims and exploiters,

respectively (e.g., prey growth and predator conversion efficiency are decreasing functions of

defense and offense, respectively: see Appendix S1), and they are linearly decreasing functions of

the sum of the two traits values (i.e., lower values of r and c represent higher costs). In addition,

we add stabilizing selection to the victim cost function as in Gavrilets (1997) to prevent runaway

evolution to infinite trait values. This can occur in quantitative models but not in Mendelian

models because it is customary to assume unbounded trait values in quantitative models, which

permits the existence of runaway escalation, whereas Mendelian traits are typically modeled as

taking values in finite sets. We employ the following functions:

$$r = r_0 \left(1 - \sum_{i=1}^{2} \rho_{xi} x_i \right) \exp\left[-\sum_{i=1}^{2} s_i \left(x_i - x_{i0} \right)^2 \right],$$

$$c = c_0 \left(1 - \sum_{i=1}^{2} \rho_{yi} y_i \right),$$

(2)

171 where r_0 and c_0 are basic parameter values (e.g., for prey growth rate and predator conversion

172 efficiency) and ρ_{ij} ($i = x, y, j = 1, 2$) determine the slope of the functions. Stabilizing selection

173 drives victim evolution toward $(x_1, x_2) = (x_{10}, x_{20})$, where s_i ($i = 1, 2$) determines the strength of

174 stabilizing selection.

175   Then coevolutionary dynamics are described by four ordinary differential equations,

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} v_{x11} & v_{x12} \\ v_{x12} & v_{x22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial W_x}{\partial x_1} \\ \dfrac{\partial W_x}{\partial x_2} \end{pmatrix},$$

(3)

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} v_{y11} & v_{y12} \\ v_{y12} & v_{y22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial W_y}{\partial y_1} \\ \dfrac{\partial W_y}{\partial y_2} \end{pmatrix},$$

where v_{ijj} ($i = x, y, j = 1, 2$) represents additive genetic variance, v_{i12} ($i = x, y$) represents

covariance, and partial derivatives represent fitness gradients (Abrams et al. 1993). For simplicity,

we assume that the traits evolve independently (genetic covariance, v_{j12}, is zero). This is likely in,

for example, hosts that defend against parasites by a cellular immune response and a behavioral

response, but further work is needed to explore the effects of trait correlations. We also assume

trait symmetry, but this should be carefully examined in future studies as the symmetry will be

broken with probability one in the real world (Fig. S4).

By numerically solving the model, we find that the two-dimensional difference model

can behave as a one-dimensional matching model. First, when there is no stabilizing selection in

the victim cost ($s_i = 0$) and the exploiter genetic variance is small enough ($v_{xii} \gg v_{yii}$, $i = 1, 2$),

coevolutionary dynamics shows runaway escalation toward extreme trait values, and the

coevolutionary outcome depends on initial conditions (Fig. S2). Second, when there is stabilizing

selection ($s_i > 0$) and the exploiter genetic variance is large enough (e.g., $v_{xii} < v_{yii}$), the system is

attracted to a stable equilibrium (the blue and red points in Figs. 3A, 3B), and there is disruptive

selection for victim traits (i.e., fitness minimum: vectors representing fitness gradients in Figs. 3A

are distracted from the equilibrium) and stabilizing selection for exploiter traits (i.e., fitness

maximum: vectors in Figs. 3B are attracted to the equilibrium), which is consistent with the

matching interaction (Appendix S2) (Abrams and Matsuda 1997, Gavrilets 1997). Third, when

there is stabilizing selection ($s_i > 0$) and the exploiter genetic variance is small enough ($v_{xii} \gg$

v_{yii}), the equilibrium is not stable (Appendix S3, Fig. S3) and the trait dynamics fluctuate (Fig.

3C) but are constrained to regions in which the total investments are nearly constant (Fig. 3D)

and the two traits for each species show antiphase cycles (Figs. 3C, E, F). These simulation

results demonstrates that the matching dynamics can arise from the difference interaction,

generalizing Ashby and Boots (2017)'s idea from Mendelian models to quantitative models.

DISCUSSION

205   Victim-exploiter interactions are among the most fundamental type of ecological

interactions. In addition to their importance in ecological communities, these interactions are

widely recognized for playing an important role in ecological dynamics (e.g., extinction and

predator-prey cycles: Northfield and Ives (2013), Cortez and Weitz (2014)) and evolutionary

dynamics (e.g., sex, recombination, and epistasis: Otto and Lenormand (2002)), because each

species needs to continue to adapt to new selection pressures by the other species. Due to its

importance, many modeling frameworks have been proposed to investigate the dynamics of

coevolution between a victim and its exploiter, capturing various aspects of the traits

involved (Nuismer 2017).

In this work, we highlight the relationship between two modeling frameworks of

quantitative traits in victim-exploiter coevolution. In particular, a multidimensional difference

trait framework, in which each trait confers an advantage in the victim-exploiter interaction but is

associated with some cost (Figs. 1D, 2B) can generate dynamics like a matching trait framework,

in which traits between a victim and exploiter must "match" in some sense for successful

interactions (Fig. 1B). Our investigation extends our knowledge of the coevolutionary process by

demonstrating how effectively matching dynamics can be generated from difference traits in a

special case of multivariate coevolution as a proof of concept (rather than a thorough

investigation of the dynamics: Fig. 3).

For some traits, the distinction between matching traits and difference traits is clear. For

both victims and exploiters, being stronger and faster are always more advantageous (but may

have an associated cost or energetic limitation) and hence, are difference traits. However, in other

cases, it is not so obvious. For example, body size of both victims and exploiters can be a

difference trait, in which bigger is better for exploitation/defense (e.g., gape-limited predation), or

a matching trait, in which exploiters must be within a certain size range depending on victims'

size for successful exploitation. Our simulations suggest that, even when the body size is a

difference trait, it may behave as a matching trait because of another difference trait. This

correlated coevolution between the two traits can occur even without genetic covariance, as long

as the costs of being big affect fitness together with the cost of the other trait.

Previous work has suggested that matching traits are more likely to lead to

coevolutionary cycling of traits than difference interactions (Abrams 2000, McPeek 2017). In

comparing quantitative models with a one-dimensional trait, for example, McPeek (2017) found

cycles are observed in a smaller area of parameter space with difference traits compared to

matching traits. Furthermore, the underlying mechanism of cycles differs in the two frameworks.

In the matching framework, cycles consist of the exploiter tracking victim traits and the victim

escaping from exploiter traits (i.e., fluctuating selection between equally specific

defense/exploitation traits). In the difference framework, cycles consist of the victim and

exploiter investing more energy in their traits for better defense/exploitation, and then

abandoning their defense/exploitation because at some point they are too costly to maintain (i.e.,

fluctuating selection between strong and weak defense/exploitation traits). Even when empirical

researchers are focusing on a single trait with the difference interaction, the system may have

another difference trait that can affect exploitation success. In such a case, coevolutionary

dynamics can be driven by the matching-like interaction, which may cause cyclic dynamics more

easily (McPeek 2017).

In spite of recent interest in coevolution in multidimensional trait space (e.g., Gilman et

al. 2012, Débarre et al. 2014, Doebeli and Ispolatov 2017), the relation of the matching and

difference frameworks has not been well recognized. For example, in their study on evolutionary

escape, Gilman et al. (2012) contrasted coevolutionary dynamics with matching and difference

interactions in multidimensional trait space but did not find a significant difference between them

(see Fig. 1 of Gilman et al. (2012)). Our generalization offers a potential explanation for this

previous work, as coevolutionary dynamics with matching interactions can arise from difference

interactions in multidimensional quantitative models.

This work was inspired by the recent work of Ashby and Boots (2017), who showed an

analogous relationship for two Mendelian trait models of coevolution. Here, we draw out the

parallels between two Mendelian (single-locus) trait frameworks and two quantitative trait

frameworks. Namely, we highlight that the matching allele (MA) framework is a Mendelian

analog of the matching framework, and the gene-for-gene (GFG) framework is a Mendelian

analog of the difference framework (Fig. 1). While these frameworks are valuable, many traits

important to victim-exploiter interactions may actually lie somewhere in the middle of these two

extremes; they may be determined by many (but not infinite number of) loci. It is still not well

understood how coevolutionary dynamics differ between when traits are governed by a single

locus, multi-locus (with linkage disequilibrium and epistasis), and effectively infinite loci (as

assumed in classical quantitative genetics). While there have been a handful of papers to

investigate this question in specific scenarios, further investigation is need for a general

understanding.

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Figure 1. Interaction frameworks and genetic architectures in coevolution models. A: Matching

allele (MA) is a matching interaction in Mendelian traits. Here victims and exploiters are haploid

and their genotypes are represented by 1 and 0 at a particular locus, and they indicate the

presence and absence, respectively, of defense/exploitation alleles. Dark shading indicates greater

exploitation success. B: The matching interaction is a MA interaction in quantitative traits.

Exploitation success is maximized when the trait difference is zero. C: Gene-for-gene (GFG) is a

difference interaction in Mendelian traits. D: The difference interaction is a GFG interaction in

quantitative traits. Exploitation success is increased by larger exploiter trait values and by smaller

victim trait values.

Figure 2. A: An example of exploitation matrix in a Mendelian GFG model with two loci (Ashby

and Boots 2017). The region surrounded by a dashed white line shows that the MA interaction

arises from the GFG framework. Victims and exploiters are haploid and their genotypes are

represented by 1 and 0 at a locus, and they indicate the presence and absence, respectively, of

defense/exploitation alleles. Darker shading indicates greater exploitation success and arrows

indicate greater costs. B: Exploitation success as a function of both trait differences between

exploiters and victims ($y_i - x_i$) in a quantitative difference model with two traits (Eq. 1).

Figure 3. Coevolutionary dynamics of two quantitative traits with difference interactions. A, B:

Fitness landscapes of victims (A) and exploiters (B) when the exploiter genetic variance is large

enough and the system converges to a stable equilibrium. A: A contour plot of fitness (W_x) and a

vector field of fitness gradients for the victim traits when the exploiter traits are at the equilibrium

trait value (the red point in B). B: A contour plot of fitness (W_y) and a vector field of fitness

gradients for the exploiter traits (B) when the victim traits are at the equilibrium trait value (the

blue point in A). Parameter values are $\rho_{xi} = 0.07$, $\rho_{yi} = 2$, $x_{10} = x_{20} = 0.2$, $a_0 = r_0 = c_0 = \theta_i = v_{jii} = 1$,

$s_i = 0.01$, and $v_{j12} = 0$ ($i = 1, 2, j = x, y$). C-F: Coevolutionary cycles when the victim genetic

variance is much larger than exploiter genetic variance ($v_{xii} \gg v_{yii}$). C: Numerical solutions of the

four traits. D: Numerical solutions of the functions (Eq. 2) representing costs of

defense/exploitation (note that lower values indicate higher costs). E, F: Coevolutionary cycles of

victims (E) and exploiters (F). Parameter values are the same as A and B except for $v_{xii} = 1$ and

$v_{yii} = 0.01$ ($i = 1, 2$). Note that c is negative in gray regions in B and F.

Figure 1.

Figure 2.

Figure 3.