Antagonistic coevolution between multiple quantitative traits:

Matching dynamics can arise from difference interactions

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ABSTRACT

Coevolution is one of the major drivers of complex dynamics in population ecology. Historically,

antagonistic coevolution in victim-exploiter systems has been a topic of special interest, and

involves traits with various genetic architectures (e.g., the number of genes involved) and effects

on interactions. For example, exploiters may need to have traits that "match" those of victims for

successful exploitation (i.e., a matching interaction), or traits that exceed those of victims (i.e., a

difference interaction). Different models exist which are appropriate for different types of traits,

including Mendelian (discrete) and quantitative (continuous) traits. For models with multiple

Mendelian traits, recent studies have shown that antagonistic coevolutionary patterns that appear

as matching interactions can arise due to multiple difference interactions with costs of having

large trait values. Here we generalize their findings to quantitative traits and show, analogously,

that the multidimensional difference interactions with costs sometimes behave qualitatively the

same as matching interactions. While previous studies in quantitative genetics have used the

dichotomy between matching and difference frameworks to explore coevolutionary dynamics, we

suggest that exploring multidimensional trait space is important to examine the generality of

results obtained from one-dimensional traits.

Keywords: bidirectional axis of vulnerability, fluctuating selection, gene-for-gene,

matching allele, unidirectional axis of vulnerability

INTRODUCTION

Theories developed from specific models are initially only as general as their underlying models,

until they are shown to hold in other modeling frameworks. Hence, it is critical to compare

results from a variety of modeling frameworks to test their generality. Antagonistic coevolution

in victim-exploiter systems (e.g., prey-predator or host-pathogen systems) has been intensively

studied theoretically (Abrams, 2000; Nuismer, 2017). It is thought to be a major driver for the

emergence and maintenance of biodiversity (Thompson, 2005) and feedbacks between interacting

species can create complex ecological (e.g., Cortez and Weitz, 2014) and evolutionary (e.g.,

Dercole et al., 2010) dynamics. Historically, various coevolution models with differing genetic

architectures (e.g., the number of genes involved) and interspecific interactions (e.g., a "matching"

vs. a "difference" interaction: see below) have been proposed and analyzed (Nuismer, 2017). To

account for genetic architecture, one approach is to consider coevolution with a few major genes

that interact and produce a discrete phenotype (Mendelian trait-based models, or hereafter

Mendelian models) (e.g., Mode, 1958; Seger, 1988). On the other hand, quantitative trait-based

models (hereafter quantitative models) assume the genetic architecture consists of many genes of

small effect and produce continuous phenotypes (e.g., Gavrilets, 1997; Saloniemi, 1993).

Here we highlight the connection between Mendelian and quantitative models of

victims and exploiters, and then clarify the relationships between existing modeling frameworks

for quantitative traits. We begin by reviewing the different modeling frameworks for Mendelian

traits. We then draw analogies between Mendelian and quantitative models (e.g., Boots et al.,

2014; Cortez and Weitz, 2014). The analogies suggest that the matching framework, in which

exploitation is more successful when traits "match" (i.e., a bidirectional axis of vulnerability

sensu Abrams 2000), can arise from the difference framework, in which exploitation is more

successful when traits differ (i.e., a unidirectional axis of vulnerability *sensu* Abrams 2000), in

multidimensional quantitative models with costs of having large trait values. Finally, as a proof

of concept, we give an example model of theoretical quantitative genetics where a

victim-exploiter coevolutionary dynamics arising from two pairs of difference traits can appear as

a matching interaction in a single pair of traits.

BACKGROUND

Historically, in Mendelian models, two major interactions were often assumed and

contrasted: the matching allele (MA) framework, in which exploitation is successful when traits

match (e.g., Grosberg and Hart, 2000) (Figure 1a), and the gene-for-gene (GFG) framework, in

which a generalist exploiter can exploit a range of victim genotypes, but pays a cost for

generalism (Flor, 1956) (Figure 1c). Researchers have investigated the relative importance and

relationships of the two interactions (e.g., Agrawal and Lively, 2002; Frank, 1996; Parker, 1994).

Recently, Ashby and Boots (2017) found that, in multidimensional Mendelian (multilocus)

models, the GFG framework can behave effectively as the MA framework (Figure 2a).

To illustrate, consider a simple GFG model with haploid victims and exploiters with

two loci with two alleles each, labelled 1 and 0. The alleles 1 and 0 indicate the presence and

absence, respectively, of an exploitation or defense trait (Figure 2a). If both of the allelic values

of the exploiter are greater than or equal to those of the victim, maximum exploitation will occur

(black squares in Figure 2a). If the victim has exactly one allelic value greater than that of the

exploiter, so that the other allelic value of the victim is less than or equal to the corresponding

value of the exploiter, then exploitation will be partial (gray squares in Figure 2a). Finally, in the

case where both allelic values of the victim are strictly greater than that of the exploiter (which

only occurs when the victim has genotype 11 and the exploiter has 00), then the victim is

defended against exploitation (a white square in Figure 2a).

When coevolutionary dynamics occurs between four genotypes with the alleles 1 and 0,

the type of interaction (indicated by a dashed white line in Figure 2a) effectively reduces to a MA

interaction: the exploiter genotype 01 (10) is more successful at exploiting its matching counter

victim genotype 01 (10) as in Figure 1a. Indeed, by investigating a multilocus GFG model,

Ashby and Boots (2017) found that, in the case where an intermediate number of alleles is

optimal, the victim/exploiter genotypes alternate between various subsets of defense/exploitation

alleles, but the total number of alleles stays constant. Hence, the multidimensional GFG

framework effectively behaves as the MA framework (Figure 2a).

As with Mendelian models, quantitative models have assumed an apparent dichotomy

between two frameworks (Abrams, 2000; Abrams and Matsuda, 1997; Gilman et al., 2012;

McPeek, 2017): the matching trait framework, in which exploitation is increasingly more

successful with the extent that trait values of exploiters match those of victims (Figure 1b), and

the difference trait framework, in which exploitation is increasingly more successful with the

extent that trait values of exploiters exceed those of victims (Figure 1d). They are also known as

the bidirectional and unidirectional trait cases, respectively (Abrams, 2000). The matching

framework is analogous to the MA model and is used, for example, when an exploiter can best

exploit a victim with body size or color within a certain range (Brown and Vincent, 1992;

Calcagno et al., 2010; Dercole et al., 2010; Dieckmann et al., 1995; Fleischer et al., 2018;

Gavrilets, 1997; Khibnik and Kondrashov, 1997; Kopp and Gavrilets, 2006; Marrow et al., 1992;

Mougi, 2012; Nuismer et al., 2005; Yamamichi and Ellner, 2016). This has been observed in the

coevolution between the African tawny-flanked prinia and its brood parasite, the cuckoo finch:

the parasite varies its egg color to match that of the victim (Spottiswoode and Stevens, 2012). On

the other hand, the difference framework is analogous to the GFG model and is appropriate when

exploitation success is determined by speed vs. speed, strength vs. armor, or toxicity vs.

resistance (Cortez and Weitz, 2014; Frank, 1994; Mougi and Iwasa, 2010; Mougi and Iwasa,

2011; Northfield and Ives, 2013; Nuismer et al., 2007; Saloniemi, 1993; Sasaki and Godfray,

1999; Tien and Ellner, 2012; van Velzen and Gaedke, 2017; Yamamichi and Miner, 2015). For

example, garter snakes evolve stronger resistance to rough-skinned newt toxicity in areas where

newts are more toxic (Brodie III and Brodie Jr, 1990).

In the matching framework, exploitation success is typically a unimodal function of the

trait difference, and is often modeled as a Gaussian: $\exp[-(x-y)^2]$, where x and y are traits of

victims and exploiters, respectively (Figure 1b) (e.g., Gavrilets, 1997; Khibnik and Kondrashov,

1997; Mougi, 2012; Nuismer et al., 2005). On the other hand, in the difference framework,

exploitation success is often modeled as a sigmoidal function of the trait difference: $1/[1 + \exp(x$

$- y)]$ (Figure 1d) (e.g., Mougi and Iwasa, 2010; Nuismer et al., 2007; van Velzen and Gaedke,

2017). In this framework, it is always advantageous for both species to increase the trait values

for better exploitation/defense (McPeek, 2017). Empirical evidence suggests that there is an

energetic trade-off to exploitation and defense and hence, most models of difference traits assume

there is a cost associated with each trait, which prevents runaway evolution. For example, garter

snakes resistant to newt toxin have costs associated with slower sprinting speeds (Brodie III and

Brodie Jr, 1999), milkweed bugs' production of enzymes to break down milkweed toxins has

metabolic costs (Dalla and Dobler, 2016), and numerous plants produce defense chemicals with a

cost to plant growth (reviewed in Herms and Mattson, 1992).

Here, by drawing analogies between the Mendelian trait and quantitative trait

frameworks (Figure 1), we further extend the generalization of Ashby and Boots (2017) and

demonstrate that the difference framework can result in similar dynamics to the matching

framework in multidimensional quantitative models. The matching model is a continuous trait

analog to the MA model and the difference model is a continuous trait analog to the GFG model

(Figure 1). In a multilocus GFG model, under certain conditions the victim/exploiter genotypes

alternate between various subsets of defense/exploitation alleles, but the total number of alleles

stays constant, which is effectively what occurs under the MA framework (Ashby & Boots 2017).

This is analogous to a bivariate difference trait model, in which the victim/exploiter genotypes

alternate between having more investment in one defense/exploitation trait vs. another, which is

effectively what occurs under the matching framework. Instead of switching between the

extremes of a single matching trait, the species are switching between two difference-trait-based

strategies. We consider a victim-exploiter model in which each has two difference traits that

influence exploitation success. An empirical example that motivates our study is a stepwise

infection process (Hall et al., 2017), where host species prevent exploitation of parasites by

several difference traits. In the case where intermediate trait values are optimal due to the costs of

defense/offense, the victim/exploiter traits may switch between combinations where a trait is

large while the other trait is small, but the total costs stay constant. The resultant dynamics may

appear as the matching interaction, and this is the quantitative-model analog to the findings in

Ashby and Boots (2017) in Mendelian models.

 AN EXAMPLE

We develop a two-dimensional quantitative model with difference interactions (Figure

2b) to demonstrate that matching dynamics can emerge from a multidimensional difference

framework (see Appendix S1 and Supplementary Mathematica file, Supporting Information, for

details). We consider a quantitative genetic model (Iwasa et al., 1991; Lande, 1976) in continuous

time (i.e., Malthusian fitness) (Abrams et al., 1993). We assume that victims and exploiters each

have two quantitative traits. We let x_i be the value of victim trait i ($= 1, 2$) and assume greater

values allow for defense against trait i of the exploiter. We let y_i be the value of exploiter trait i

and assume greater values allow for more exploitation. Each trait is associated with a cost that

decreases individual fitness with greater trait values. The difference between the exploiter trait

and the corresponding victim trait, $y_i - x_i$, affects exploitation success (Figure 2b), which is

expressed by a sigmoid function (Figure 1d). We assume that fitness of victims and exploiters are,

respectively,

$$m_x = r\left(x_1, x_2\right) - a_0 \prod_{i=1}^{2} \frac{1}{1 + \exp\left[-\theta_i\left(y_i - x_i\right)\right]},$$

$$m_y = c\left(y_1, y_2\right) a_0 \prod_{i=1}^{2} \frac{1}{1 + \exp\left[-\theta_i\left(y_i - x_i\right)\right]},$$

(1)

where a_0 is the effects of successful exploitation on fitness (e.g., maximum attacking rate), and θ_i

($i = 1, 2$) determine the sensitivity of exploitation success to the trait difference. In our model, the

product $\prod_{i=1}^{2} 1 / \left\{ 1 + \exp\left[-\theta_i \left(y_i - x_i \right) \right] \right\}$ means that victims need to overcome their exploiters at

only one trait for escaping from exploitation, whereas exploiters must overcome the two defense

traits for successful exploitation (Gilman et al. 2012). This sequential interaction is also found in

many host-parasite systems (Hall et al. 2017).

163   Here r and c represent costs of defense and exploitation for victims and exploiters,

respectively (e.g., prey growth and predator conversion efficiency are decreasing functions of

defense and offense, respectively: see Appendix S1, Supporting Information), and they are

linearly decreasing functions of the sum of the two traits values (i.e., lower values of r and c

represent higher costs). The costs can prevent escalation toward positive infinite values in the two

traits. However, the two-dimensional difference model with costs behaves like a one-dimensional

matching model, in which runaway evolution occurs toward a positive infinite value in one trait

and a negative infinite value in the other trait (Appendices S2, S3, Figures S1, S2, Supporting

Information). To prevent this, we add stabilizing selection to the victim cost function as in

Gavrilets (1997). This can occur in quantitative models but not in Mendelian models because it is

customary to assume unbounded trait values in quantitative models, which permits the existence

of runaway escalation, whereas Mendelian traits are typically modeled as taking values in finite

sets. We employ the following functions:

$$r = r_0 \left(1 - \sum_{i=1}^{2} \rho_{xi} x_i \right) \exp\left[-\sum_{i=1}^{2} s_i \left(x_i - x_{i0} \right)^2 \right],$$

177   (2)

$$c = c_0 \left(1 - \sum_{i=1}^{2} \rho_{yi} y_i \right),$$

where r_0 and c_0 are basic parameter values (e.g., for prey growth rate and predator conversion

efficiency) and ρ_{ij} ($i = x, y, j = 1, 2$) determine the slope of the functions. Stabilizing selection

drives victim evolution toward $(x_1, x_2) = (x_{10}, x_{20})$, where s_i ($i = 1, 2$) determines the strength of

stabilizing selection (Figure S3, Supporting Information). Note that adding stabilizing selection

to the exploiter function instead of the victim function results in victim's escape from

exploitation. We can add stabilizing selection to the exploiter function in addition to the victim

function, but here we keep the model as simple as possible.

Then coevolutionary dynamics are described by four ordinary differential equations,

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} v_{x11} & v_{x12} \\ v_{x12} & v_{x22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial m_x}{\partial x_1} \\ \dfrac{\partial m_x}{\partial x_2} \end{pmatrix},$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} v_{y11} & v_{y12} \\ v_{y12} & v_{y22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial m_y}{\partial y_1} \\ \dfrac{\partial m_y}{\partial y_2} \end{pmatrix},$$

(3)

where v_{ijj} ($i = x, y, j = 1, 2$) represents additive genetic variance, v_{i12} ($i = x, y$) represents

covariance, and partial derivatives represent fitness gradients (Abrams et al., 1993). For

simplicity, we assume that the traits evolve independently (genetic covariance, v_{i12}, is zero). For

example, this is likely in hosts that defend against parasites by a cellular immune response and a

behavioral response, but further work is needed to explore the effects of trait correlations. Also,

our trait symmetry assumptions (i.e., the additive genetic variances are the same, $v_{i11} = v_{i22}$, and

the two traits affect exploitation success and fitness costs in the same way, $\theta_1 = \theta_2$, $\rho_{x1} = \rho_{x2}$, $\rho_{y1} =$

ρ_{y2}, and $s_1 = s_2$) should be carefully examined in future studies as this assumption may not hold

for most traits (Figure S4, Supporting Information), and it may result in, for example, time-scale

separation in evolutionary dynamics of the two traits.

By numerically solving the model, we find that the two-dimensional difference model

can behave as a one-dimensional matching model. First, when there is no stabilizing selection in

the victim cost ($s_i = 0$) and the exploiter genetic variance is small enough ($v_{xii} \gg v_{yii}$, $i = 1, 2$),

coevolutionary dynamics shows runaway escalation toward extreme trait values, and the

coevolutionary outcome depends on initial conditions (Figure S2, Supporting Information).

Second, when there is stabilizing selection ($s_i > 0$) and the exploiter genetic variance is large

enough (e.g., $v_{xii} < v_{yii}$), the system is attracted to a stable equilibrium (the blue and red points in

Figures 3a, 3b), and there is disruptive selection for victim traits (i.e., fitness minimum: vectors

representing fitness gradients in Figure 3a are distracted from the equilibrium) and stabilizing

selection for exploiter traits (i.e., fitness maximum: vectors in Figure 3b are attracted to the

equilibrium), which is consistent with the matching interaction (Appendix S2, Supporting

Information) (Abrams and Matsuda, 1997; Gavrilets, 1997). Third, when there is stabilizing

selection ($s_i > 0$) and the exploiter genetic variance is small enough ($v_{xii} \gg v_{yii}$), the equilibrium

is not stable (Appendix S3, Figure S3, Supporting Information) and the trait dynamics fluctuate

(Figure 3c) but are constrained to regions in which the total investments are nearly constant

(Figure 3d) and the two traits for each species show antiphase cycles (Figures 3c, 3e, 3f). These

simulation results demonstrate that the matching dynamics can arise from the difference

interaction, generalizing Ashby and Boots (2017)'s idea from Mendelian models to quantitative

models.

DISCUSSION

Victim-exploiter interactions are among the most fundamental type of ecological

interactions. In addition to their importance in ecological communities, these interactions are

widely recognized for playing an important role in ecological dynamics (e.g., extinction and

predator-prey cycles: Cortez and Weitz (2014); Northfield and Ives (2013)) and evolutionary

dynamics (e.g., sex, recombination, and epistasis: Otto and Lenormand (2002)), because each

species needs to continue to adapt to new selection pressures by the other species. Due to its

importance, many modeling frameworks have been proposed to investigate the dynamics of

coevolution between a victim and its exploiter, capturing various aspects of the traits

involved (Nuismer, 2017).

In this work, we highlight the relationship between two modeling frameworks of

quantitative traits in victim-exploiter coevolution. In particular, a multidimensional difference

trait framework, in which each trait confers an advantage in the victim-exploiter interaction but is

associated with some cost (Figures 1d, 2b) can generate dynamics like a matching trait

framework, in which traits between a victim and exploiter must "match" in some sense for

successful interactions (Figure 1b). Our investigation extends our knowledge of the

coevolutionary process by demonstrating how effectively matching dynamics can be generated

from difference traits with costs in a special case of multivariate coevolution as a proof of

concept (rather than a thorough investigation of the dynamics: Figure 3).

239   For some traits, the distinction between matching traits and difference traits is clear.

For both victims and exploiters, being stronger and faster are always more advantageous (but

may have an associated cost or energetic limitation) and hence, are difference traits. However, in

other cases, it is not so obvious. For example, body size of both victims and exploiters can be a

difference trait, in which bigger is better for exploitation/defense (e.g., gape-limited predation), or

a matching trait, in which exploiters must be within a certain size range depending on victims'

size for successful exploitation. Besides classifying difference vs. matching traits, a second

empirical difficulty arises when not all traits are simultaneously measured. Our simulations

suggest that, even when the body size is a difference trait, it may behave as a matching trait

because of another potentially unmeasured difference trait such as toxicity/resistance. This

correlated coevolution between the two traits can occur even without genetic covariance, as long

as the body size and toxicity/resistance affect exploitation success and the costs of being big

affect fitness together with the costs of the toxicity/resistance.

Previous work has suggested that matching traits are more likely to lead to

coevolutionary cycling of traits than difference interactions (Abrams, 2000; McPeek, 2017). In

comparing quantitative models with a one-dimensional trait, for example, McPeek (2017) found

cycles are observed in a smaller area of parameter space with difference traits compared to

matching traits. Furthermore, the underlying mechanism of cycles differs in the two frameworks.

In the matching framework, cycles consist of the exploiter tracking victim traits and the victim

escaping from exploiter traits (i.e., fluctuating selection between equally specific

defense/exploitation traits). In the difference framework, cycles consist of the victim and

exploiter investing more energy in their traits for better defense/exploitation, and then

abandoning their defense/exploitation because at some point they are too costly to maintain (i.e.,

fluctuating selection between strong and weak defense/exploitation traits). Even when empirical

researchers are focusing on a single trait with the difference interaction, the system may have

another difference trait that can affect exploitation success. In such a case, coevolutionary

dynamics can be driven by the matching-like interaction, which may cause cyclic dynamics more

easily (McPeek, 2017).

In spite of recent interest in coevolution in multidimensional trait space (e.g., Débarre

et al., 2014; Doebeli and Ispolatov, 2017; Gilman et al., 2012), the relation of the matching and

difference frameworks has not been well recognized. For example, in their study on evolutionary

escape, Gilman et al. (2012) contrasted coevolutionary dynamics with matching and difference

interactions in multidimensional trait space but did not find a significant difference between them

(see Figure 1 of Gilman et al. (2012)). Our generalization offers a potential explanation for this

previous work, as coevolutionary dynamics with matching interactions can arise from difference

interactions in multidimensional quantitative models.

This work was inspired by the recent work of Ashby and Boots (2017), who showed an

analogous relationship for two Mendelian trait models of coevolution. Here, we draw out the

parallels between two Mendelian (single-locus) trait frameworks and two quantitative trait

frameworks. Namely, we highlight that the matching allele (MA) framework is a Mendelian

analog of the matching framework, and the gene-for-gene (GFG) framework is a Mendelian

analog of the difference framework (Figure 1). While these frameworks are valuable, many traits

important to victim-exploiter interactions may actually lie somewhere in the middle of these two

extremes; they may be determined by many (but not infinite number of) loci. It is still not well

understood how coevolutionary dynamics differ between when traits are governed by a single

locus, multi-locus (with linkage disequilibrium and epistasis), and effectively infinite loci (as

assumed in classical quantitative genetics). While there have been a handful of papers to

investigate this question in specific scenarios, further investigation is need for a general

understanding.

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FIGURE LEGENDS

FIGURE 1. Interaction frameworks and genetic architectures in coevolution models. (a)

Matching allele (MA) is a matching interaction in Mendelian traits. Victims and exploiters are

haploid and their genotypes are represented by 1 and 0 at a particular locus. Dark shading

indicates greater exploitation success. (b) The matching interaction is a MA interaction in

quantitative traits. Exploitation success is maximized when the trait difference is zero. (c)

Gene-for-gene (GFG) is a difference interaction in Mendelian traits. Here 1 and 0 indicate the

presence and absence, respectively, of defense/exploitation alleles. (d) The difference interaction

is a GFG interaction in quantitative traits. Exploitation success is increased by larger exploiter

trait values and by smaller victim trait values.

FIGURE 2. (a) An example of exploitation matrix in a Mendelian GFG model with two loci

(Ashby and Boots, 2017). The region surrounded by a dashed white line shows that the MA

interaction arises from the GFG framework. Victims and exploiters are haploid and their

genotypes are represented by 1 and 0 at a locus, and they indicate the presence and absence,

respectively, of defense/exploitation alleles. Darker shading indicates greater exploitation success

and arrows indicate greater costs. (b) Exploitation success as a function of both trait differences

between exploiters and victims ($y_i - x_i$) in a quantitative difference model with two traits

(Equation 1).

FIGURE 3. Coevolutionary dynamics of two quantitative traits with difference interactions. (a,

b) Fitness landscapes of victims (a) and exploiters (b) when the exploiter genetic variance is large

enough and the system converges to a stable equilibrium. (a) A contour plot of fitness (m_x) and a

vector field of fitness gradients for the victim traits when the exploiter traits are at the equilibrium

trait value (the red point in b). (b) A contour plot of fitness (m_y) and a vector field of fitness

gradients for the exploiter traits when the victim traits are at the equilibrium trait value (the blue

point in a). Parameter values are $\rho_{xi} = 0.07$, $\rho_{yi} = 2$, $x_{10} = x_{20} = 0.2$, $a_0 = r_0 = c_0 = \theta_i = v_{jii} = 1$, $s_i =$

0.01, and $v_{j12} = 0$ ($i = 1, 2, j = x, y$). (c-f) Coevolutionary cycles when the victim genetic variance

is much larger than exploiter genetic variance ($v_{xii} \gg v_{yii}$). (c) Numerical solutions of the four

traits. (d) Numerical solutions of the functions (Equation 2) representing costs of

defense/exploitation (note that lower values indicate higher costs). (e, f) Coevolutionary cycles of

victims (e) and exploiters (f). Parameter values are the same as (a) and (b) except for $v_{xii} = 1$ and

$v_{yii} = 0.01$ ($i = 1, 2$). Note that c is negative in gray regions in (b) and (f).

Figure 1.

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457　**Figure 2.**

459   **Figure 3.**