

1 **Self-determination motivation theory in R: The software package SDT**

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3
4 **Abstract**

5 This paper presents the technical details of the software package SDT in the R
6 computing and graphics environment, implementing a convex quadratic program that
7 was recently proposed in the literature on self-determination theory of human
8 motivation. Three main features are addressed, with their accompanying code for
9 computation in R: first, the application of the quadratic program and corresponding
10 code for the analysis of the extent of motivation internalization or externalization;
11 second, for exploring the simplex structure assumption of motivation; and third, for
12 adjusting the confounded scoring protocol, called the self-determination or relative
13 autonomy index, to account for the mixture of internal motivation and external
14 motivation. We describe the functions of the R package SDT. The computations are
15 demonstrated with example data accompanying the package, so researchers can run the
16 methodology on their own datasets.

17
18 **Keywords: self-determination theory, motivation, internalization, simplex**
19 **structure, self-determination or relative autonomy index, optimization, convexity,**
20 **quadratic program, R software package**

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26 1. INTRODUCTION

27 This paper discusses the software package SDT for self-determination theory (SDT)
28 measures in the R computing and graphics environment (The R Core Team, 2016). The
29 package is available at no cost from the Comprehensive R Archive Network (CRAN),
30 <http://CRAN.R-project.org/package=SDT>. SDT was introduced by Deci and Ryan (1985,
31 2000, 2002) and provides a theoretical framework for studying motivation. With SDT,
32 researchers can describe the motivational basis of human behavior. On the one hand,
33 there are the extrinsic forces acting on people (e.g., grades or evaluations), and on the
34 other, there are the intrinsic motives inherent in humans (e.g., interests or curiosity).
35 The general aim of SDT is to study the interplay between these extrinsic and intrinsic
36 factors. In particular, types of motivation were postulated in SDT (Figure 1). For the
37 introjected regulation and the identified regulation types of extrinsic motivation, their
38 internalizations were delineated as “*somewhat external*” and “*somewhat internal*”,
39 respectively. These notions remained undetermined in SDT (Ünlü & Dettweiler, 2015;
40 Ünlü, 2016). The basic concepts of SDT are explained in Section 2.

41 What makes this software package so exciting is threefold. First, the function
42 `internalization` of the package SDT implements the constrained regression
43 analysis approach proposed by Ünlü and Dettweiler (2015). Based on this approach, the
44 vaguely expressed intermediate motivations can be estimated from data. The approach
45 can be extended to simplex structure analysis, that is, for validating whether or not
46 motivation regulations theoretically closer to one another are more strongly
47 interrelated. Second, simplex structure analysis in R is realized with the function
48 `simplex` of the package SDT. Third, the function `sdi` of the package provides the
49 popular self-determination or relative autonomy index (SDI or RAI), in the common and
50 adjusted variants. This is a scoring protocol that aggregates the subscale scores to imply

51 an overall informative measure. The original SDI or RAI index is confounded. Generally,
52 it does not accommodate biasing effects on the index value that may result from mixed
53 internal and external motivation. Because of this, the function `sd_i` also implements an
54 adjusted scoring protocol variant of this measure, as discussed by Ünlü (2016). Thus,
55 this package gives the user the ability to calculate adjusted SDI scores for all
56 participants. Examples of this can be found in Section 5. In addition, the package SDT
57 provides `plot`, `print`, and `summary` methods for objects of specific classes for
58 conveniently graphing, printing, and outlining the results obtained from SDT analyses.

59 The paper has the following structure. In Section 2, the theory of self-
60 determination is reviewed. In Section 3, we recapitulate the issues of motivation
61 internalization, motivation simplex structure, and of adjusting the SDI or RAI measure
62 for mixed internal and external motivation. Thus, we introduce the theoretical
63 optimization framework, which the R package SDT is based on and that allows to
64 compute the solutions to the afore mentioned issues. Section 4 presents the package
65 SDT and we describe the functions of it. Section 5 demonstrates the package by
66 examples and we apply its functions to an accompanying dataset named
67 `learning_motivation`. In Section 6, we summarize and conclude with final remarks.

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69

2. SELF-DETERMINATION THEORY

70 SDT provides a framework for the study of human motivation (Deci & Ryan, 1985, 2000,
71 2002; Ryan & Deci, 2000a). Empirical data corroborate that there are three basic
72 psychological needs “essential in promoting life satisfaction and well-being”, the
73 “opportunities to experience autonomy, competence, and relatedness” (Levesque,
74 Zuehlke, Stanek, & Ryan, 2004, p. 68). Hereby, “autonomy refers to volition – the
75 organismic desire to self-organize experience and behavior and to have activity be

100 Applications of SDT are numerous. An extensive reference list, including
101 comprehensive materials on the theory and the questionnaires developed to assess the
102 different SDT constructs, is available at <http://www.selfdeterminationtheory.org>.

103 Figure 1 displays the self-determination continuum. Thus, according to SDT, the
104 behavior of a person can shift from extrinsic to intrinsically motivated. From left to right,
105 the behavior is more and more internalized through the regulation types that are
106 ordered along the continuum. Introjected regulation and identified regulation are
107 relevant to the discussion of this paper. Introjected regulation refers to a person that is
108 acting on the basis of external societal expectations only partially internalized and that
109 remain external to the self. Identified regulation means the person has identified with
110 the external values of his/her behavior and has internalized these more into her/his
111 value system. Details can be found in Deci and Ryan (1985, 2000, 2002).

112 The subscales of external regulation and intrinsic regulation are, by theory,
113 completely external motivation and internal motivation, respectively. For the
114 intermediate subscales of introjected regulation and identified regulation, on the other
115 hand, their internalizations are expressed as “*somewhat external*” and “*somewhat*
116 *internal*”, respectively. That is, these intermediate regulation types are mixtures of
117 external motivation and internal motivation and remain vaguely specified in SDT. The
118 constrained regression analysis approach to quantifying these notions, along with the
119 major implementation components, are theoretically presented in the following section.

120

121 3. CONVEX OPTIMIZATION AND MOTIVATION INTERNALIZATION

122 We start with a general introduction to convex optimization in the first paragraph of this
123 section. But in the second and following paragraphs, it will be clear why we should care
124 about convex optimization, meaning what problem convex optimization is going to

125 solve in SDT.

126 Optimization is omnipresent in many different scientific fields and especially
127 powerful if based on convexity (e.g., Boyd & Vandenberghe, 2009; Dattorro, 2009). The
128 general problem of convex optimization can be stated as:

129 $minimize g(\pi) \text{ subject to } \pi \in \mathcal{D},$

130 where $g: \mathbb{R}^k \rightarrow \mathbb{R}$ is a convex function mapping k arguments of interest

131 into a real-valued summary or target criterion,

132 and, determined by convex inequality (and affine equality) constraints,

133 $\mathcal{D} \subset \mathbb{R}^k$ is the convex set of all feasible values for the arguments.

134 The program is to minimize an objective function with respect to parameters of interest,
135 under given side constraints on the parameters. The convexity assumptions for the
136 objective function and the constraints ensure useful mathematical properties such as
137 the characterization of (global) optimality based on the important in optimization
138 Karush-Kuhn-Tucker conditions (Karush, 1939; Kuhn & Tucker, 1951; Kuhn, 1976;
139 Roberts & Varberg, 1973; Hiriart-Urruty & Lemaréchal, 2001; Boyd & Vandenberghe,
140 2009).

141 An interesting and basic instance of this general convex optimization problem
142 appears in SDT, related to the problem of motivation internalization. We apply
143 optimization to gauge the internal and external motivation shares of the intermediate
144 regulation types, presupposing that the relevant subscales have been measured using
145 reliable and valid inventories. Each of the identified regulation and introjected
146 regulation is modeled as a convex combination of the fully internal and fully external
147 regulation types of intrinsic motivation and extrinsic motivation, respectively. The
148 optimization problem implied is: to find for the identified and introjected regulation
149 types those shares that minimize the discrepancy between the observed values based on

150 the inventory scores and the values predicted by the convex combinations, having the
151 unknown and to be estimated shares as their weights in the apparently extreme poles of
152 the theory. There are two inequality constraints to consider, namely that the two shares
153 in regards to intrinsic regulation and external regulation are nonnegative, and the
154 equality constraint is that these shares must add up to 1.

155 For this outlined SDT convex optimization problem, which is a basic one, the
156 question can be phrased as a quadratic program. This means that we can have a special
157 convex quadratic form for the objective function, with corresponding affine inequality
158 and equality constraints, which then makes possible the application of readily available
159 numerical algorithms for efficiently solving the program. For this purpose, we will use
160 the method by Goldfarb and Idnani (1982, 1983). The latter is a numerically stable dual
161 method for computing the solutions of quadratic programming problems of the type we
162 encounter in this paper.

163 Let $X_1 = InR$ and $X_2 = ExR$ be the intrinsic regulation (InR) and external
164 regulation (ExR) types, which are assumed to be internal and external, respectively. Let
165 Y stand for either identified, IdR , or introjected, IjR , regulation, for which we want to
166 compute the internalization or externalization shares. The basic model is

$$Y = \pi_{1,Y}X_1 + \pi_{2,Y}X_2,$$

167 where $\pi_{1,Y} \geq 0$, $\pi_{2,Y} \geq 0$, and $\pi_{1,Y} + \pi_{2,Y} = 1$, and Y, X_1 , and X_2 stand for the data and
168 are taken over all sample units (e.g., students). The parameters $\pi_{1,Y}$ and $\pi_{2,Y}$ are
169 unknown and estimated from the data. In other words, IdR and IjR can be modeled as a
170 convex combination of InR and ExR . That is, the degree of internalization is gauged by
171 the shares $\pi_{1,Y}$, as the internal extent of identified or introjected regulation, and $\pi_{2,Y}$, as
172 the external extent of identified or introjected regulation, relative to the extreme
173 internal and external poles of the self-determination continuum (cf. Figure 1).

174 The extension of this model to more than two components is straightforward:

175 $Y = \pi_{1,Y}X_1 + \pi_{2,Y}X_2 + \dots + \pi_{k,Y}X_k$, where $\pi_{i,Y} \geq 0$ for $1 \leq i \leq k$, and $\sum \pi_{i,Y} = 1$.

176 Subsequently, we consider the general formulation and omit the subscript Y , having in
177 mind that one of both IdR or IjR (or any other SDT target variable) is being considered.

178 The X_i 's for $1 \leq i \leq k$ form what we call the reference system of base elements,
179 according to which the convex decomposition of the target variable Y is made, with the
180 π_i 's for $1 \leq i \leq k$ interpreted as the corresponding shares in this system.

181 A special choice of the target variable and reference system can be made for the
182 analysis of the motivation simplex structure posited by SDT. (For example, the target
183 variable can be intrinsic regulation, and the reference system can consist of identified
184 regulation, introjected regulation, and external regulation. This choice is exemplified in
185 Section 5). Ünlü and Dettweiler (2015, p. 685): “The *simplex structure* of self-
186 determination theory means that motivation regulation types theoretically closer to one
187 another are more strongly interrelated, indicating that the self-determination theory
188 regulatory styles can be linearly ordered along the underlying continuum (Ryan &
189 Connell, 1989; Deci & Ryan, 2000).” In the SDT literature, “interrelated” is synonymous
190 with “correlated”, and Ünlü and Dettweiler (2015) have proposed assessing that
191 structure based on optimal shares instead. Thus, under a simplex structure assumption,
192 we expect in this new approach that the computed shares are larger for motivation
193 regulation types theoretically closer to one another.

194 A numerical solution to the optimization problem raised in SDT can be derived as
195 follows. Formulated in analogy to the general convex optimization problem, we

196 *minimize*

$$g(\pi = (\pi_1, \pi_2, \dots, \pi_k)^T) = \sum (Y - (\pi_1X_1 + \pi_2X_2 + \dots + \pi_kX_k))^2,$$

197 where “ \cdot^T ” stands for the transpose of a matrix and the sum is taken over
198 all sample units (e.g., students),

199 *subject to*

$$\pi \in \mathcal{D} = \{(\pi_1, \pi_2, \dots, \pi_k)^T : \pi_1 + \pi_2 + \dots + \pi_k = 1, \pi_i \geq 0 \text{ for } 1 \leq i \leq k\}.$$

200 There are k inequality constraints and one equality constraint. It can be
201 proven that the target function g is convex and that the feasible set \mathcal{D} is
202 convex (and even compact).

203 This problem can be viewed as a quadratic program. Obviously, an equivalent
204 formulation of the problem is:

205 *minimize*

$$\frac{1}{2} \sum (\pi_1 X_1 + \pi_2 X_2 + \dots + \pi_k X_k)^2 - \sum Y (\pi_1 X_1 + \pi_2 X_2 + \dots + \pi_k X_k)$$

206 *subject to*

$$\pi_1 \geq 0, \pi_2 \geq 0, \dots, \pi_k \geq 0, \text{ and}$$

$$\pi_1 + \pi_2 + \dots + \pi_k = 1.$$

207 This can be written in matrix notation yielding the required quadratic program
208 expression. More precisely, the first term is equal to

$$\frac{1}{2} \pi^T D \pi,$$

209 where $D = (X_1, X_2, \dots, X_k)^T (X_1, X_2, \dots, X_k)$ and the SDT variables X_1, X_2, \dots, X_k , and Y are
210 column vectors and observed data. The second term above equals

$$-d^T \pi,$$

211 where $d = (X_1, X_2, \dots, X_k)^T Y$ and the surveyed SDT variables are used as column vectors
212 in this notation. Moreover, the k inequality constraints can be written as

$$A^T \pi \geq b_1,$$

213 where $A = I_k$ is the $k \times k$ identity matrix, and $b_1 = 0_k$ is the column vector of length k
214 containing only 0's. The equality constraint is $a^T \pi = b_2$, where $a = 1_k$ is the column
215 vector of length k consisting of 1's only, and the scalar $b_2 = 1$.

216 In sum, this yields the required (convex) quadratic program that corresponds to
217 our initial SDT question:

218 *minimize*

$$\frac{1}{2} \pi^T D \pi - d^T \pi$$

219 *subject to*

$$A^T \pi \geq b_1, \text{ and}$$

$$a^T \pi = b_2.$$

220 Given the quadratic form above, we can use software to calculate a solution. This is
221 implemented in the R package SDT, described in Section 4.

222 The computable shares not only can be used for internalization or simplex
223 structure analyses, but can also provide an adjusted self-determination or relative
224 autonomy index, SDI or RAI. Scoring protocols such as the original SDI or RAI index are
225 summary statistics that aggregate test scores to give an overall informative measure
226 (see Grolnick & Ryan, 1989; Ryan & Connell, 1989; Vallerand, 2007; Wilson, Sabiston,
227 Mack, & Blanchard, 2012; Chemolli & Gagné, 2014).

228 The original SDI or RAI index is defined as

$$\text{RAI} \equiv \text{SDI} = (2InR + IdR) - (2ExR + IjR).$$

229 This scoring protocol does not take into consideration the fact that the identified and
230 introjected regulation types are mixtures of internal and external motivation. The
231 resulting overall measure may be confounded and therefore may lack interpretability,
232 because in weighting the subscale scores the same weights are used for the two shares
233 of internal and external motivation of a regulation type.

234 The adjusted SDI or RAI (see Ünlü, 2016), which is weighted according to the
235 extent to which these regulation types are internal and external, is given by

236 $RAI_{adj} \equiv SDI_{adj} = \text{mean internal motivation} - \text{mean external motivation}.$

237 Mean internal motivation, \overline{IM} , and mean external motivation, \overline{EM} , are quantified using
238 the π -weights obtained from the quadratic program described above (with identified
239 regulation or introjected regulation as the target variable, and with intrinsic regulation
240 and external regulation as the reference system):

$$\overline{IM} = \frac{(InR - 1) + \pi_{1,IdR}(IdR - 1) + \pi_{1,IjR}(IjR - 1)}{3}$$

241 and

$$\overline{EM} = \frac{(ExR - 1) + \pi_{2,IdR}(IdR - 1) + \pi_{2,IjR}(IjR - 1)}{3}.$$

242 Translation with -1 and averaging guarantee that all of the instrument variables
243 $InR - 1, IdR - 1, IjR - 1, ExR - 1$, the components \overline{IM} and \overline{EM} , and the new scoring
244 protocol $RAI_{adj} \equiv SDI_{adj}$ range in the same interval. This is not true for the original
245 index. In the R package SDT, both indices are implemented, described in the following
246 section.

247

248 4. R PACKAGE SDT

249 We briefly describe the functions and relevant parts of the package. How to actually use
250 the software is demonstrated by examples in Section 5. The description of the package
251 will be short, and detailed information can be found in the comprehensive
252 documentation files and commented source code for the package in R (for an overview,
253 type `package?SDT`).

254 The package SDT uses the S3 system and consists of the following main
255 functions: `internalization` (motivation internalization analysis), `sdi` (original and

256 adjusted SDI or RAI index), and `simplex` (motivation simplex structure analysis). It
257 contains further functions, which are `plot`, `print`, and `summary` methods: `plot.sdi`,
258 `print.sdi`, and `summary.sdi`, and `plot.share` and `print.share`. There is the
259 accompanying dataset `learning_motivation` (described and analyzed in Section 5),
260 based on which the features of the package SDT are illustrated.

261 One of the main functions of the package is `internalization`:
262 `internalization(intermediate_regulation, intrinsic_regulation,`
263 `external_regulation)`

264 This function provides the motivation internalization or externalization computation
265 (see Section 3). It takes an intermediate regulation type, either identified or introjected,
266 as the target variable, and returns its shares, a numeric vector containing two named
267 components `internal_share` and `external_share`, with respect to the poles of
268 intrinsic regulation and external regulation as the reference system. The arguments
269 `intermediate_regulation`, `intrinsic_regulation`,
270 `external_regulation` are numeric vectors of respective subscale motivation
271 scores, where no infinite, undefined, or missing values are allowed.

272 The original and adjusted SDI or RAI indices can be computed using the function
273 `sdi`:

274 `sdi(intrinsic_regulation, identified_regulation,`
275 `introjected_regulation, external_regulation,`
276 `compute.adjusted = TRUE, minscore = 1)`

277 This function takes as input the four regulation types, which are numeric vectors of
278 intrinsic, identified, introjected, and external regulation subscale motivation scores,
279 respectively, where no infinite, undefined, or missing values are allowed. The argument
280 `compute.adjusted = TRUE` indicates adjusted index computation, whereas
281 specifying `FALSE` for it allows to compute the original index. The argument `minscore`

282 gives the minimum score of the scale procedure (typically 1) and only needs to be
283 specified for the adjusted index (for `compute.adjusted = FALSE`, this argument is
284 irrelevant and ignored). As mentioned in Section 3, for the adjusted variant, we translate
285 with “`– minscore`” and `average` to warrant that all variables and component and index
286 values range in the same interval.

287 The function `sdi` returns a named list. The returned list contains three
288 components, in both the cases of original or adjusted index computation. For the original
289 index computation, the components `confounded_internal_locus`,
290 `confounded_external_locus`, and `sdi_original` are numeric vectors of the
291 confounded internal locus values ($2InR + IdR$), confounded external locus values
292 ($2ExR + IjR$), and the overall original index values, for all students or rows of the
293 dataset. For the adjusted index computation, the components
294 `adjusted_internal_locus`, `adjusted_external_locus`, and `sdi_adjusted`
295 are numeric vectors of the adjusted internal locus values (\overline{IM}), adjusted external locus
296 values (\overline{EM}), and the overall adjusted index values, for all students or rows of the
297 dataset.

298 For simplex structure analysis, the function `simplex` can be used:

```
299 simplex(target_regulation, base_regulation_1,  
300         base_regulation_2, base_regulation_3)
```

301 The simplex structure shares are calculated of a target regulation type, either intrinsic,
302 identified, introjected, or external regulation subscale motivation scores, in the
303 reference system of the remaining base regulation types. In these numeric vectors, no
304 infinite, undefined, or missing values are allowed. The function `simplex` returns a
305 numeric vector consisting of three components: `base_regulation_1 share`,
306 `base_regulation_2 share`, and `base_regulation_3 share`; these are the

307 respective shares of the target regulation relative to the remaining base regulation types
308 of the theory.

309 The interdependencies among the functions are as follows. In the functions
310 `internalization` and `simplex`, the function `solve.QP` of the R package
311 `quadprog` (S original by Berwin A. Turlach R port by Andreas Weingessel, 2013) is
312 used, to solve the SDT related convex quadratic program (see Section 3). `solve.QP`
313 implements the quadratic program minimizer by Goldfarb and Idnani (1982, 1983). For
314 calculation of the adjusted index, the function `sdi` calls the function
315 `internalization`.

316 The functions `internalization` and `simplex` return objects (denoted `x`) of
317 the class “share”. For these, `S3 plot` and `print` methods are implemented. The
318 `plot` method
319 `plot(x, target = NULL, reference = NULL, ...)`
320 graphs the results of SDT share analyses by means of stacked bar plots of the
321 `internalization`, `externalization`, or `simplex` structure shares of a target regulation
322 relative to a reference system. Generic or user-specified labeling of the plot axes are
323 possible. The default values `target = NULL` and `reference = NULL` correspond to
324 generic labeling. If in user-specified labeling character strings for the arguments
325 `target` or `reference` are specified, these are used to label the *x*-axis and *y*-axis of the
326 bar plot, respectively. What this means is also shown by examples in Section 5, and
327 detailed information on the labeling can also be found in the comprehensive
328 documentation files in R. The `print` method
329 `print(x, ...)`
330 outputs on the console the shares of `internalization`, `externalization`, or `simplex`
331 structure, stripped off the attributes.

332 The function `sdi` returns an object (`x` or `object`) of the class “`sdi`”. There are
333 corresponding `S3 plot`, `print`, and `summary` methods. The `plot` method
334 `plot(x, minscore = 1, maxscore = 5, ...)`
335 visualizes the results of the original or adjusted SDI or RAI index computation. A
336 scatterplot is drawn of the confounded or adjusted external locus values on the y -axis,
337 and the confounded or adjusted internal locus values on the x -axis. The line $y = x$ is
338 shown as the red full line, for graphical comparison of the two value types. The
339 admissible range for the original or adjusted component values is displayed in gray
340 dashed lines. Points with larger overall index values are portrayed in darker gray tone.
341 The `minscore` and `maxscore` arguments are used to define the admissible range.
342 `minscore` is the minimum score of the scale procedure (typically 1). `maxscore` is the
343 scale procedure maximum score (typically 4, 5, or 7). For the adjusted index, the
344 admissible range is $[0, \text{maxscore} - \text{minscore}]$, with $[0, 4]$ in the default values. The
345 admissible range for the original index is
346 $[(2 \cdot \text{minscore}) + \text{minscore}, (2 \cdot \text{maxscore}) + \text{maxscore}]$, which is $[3, 15]$ for the
347 default values.

348 The `print` method
349 `print(x, ...)`
350 prints the original or adjusted overall SDI scores, for all students or rows of the dataset.
351 The `summary` method
352 `summary(object, ...)`
353 outputs simple summary statistics for the confounded or adjusted internal locus
354 component values, confounded or adjusted external locus component values, and for the
355 values of the original or adjusted SDI overall index. The summary statistics printed are
356 the minimum, first quartile, median, mean, third quartile, and the maximum.

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5. EXAMPLES

The goal of this section is to illustrate by examples how the functions of the package can be run technically. As such, following the here described use cases, the functions can be analogously applied in any other empirical data set. This section simply shows how. In particular, the goal of this paper cannot be to systematically investigate and research, centered around real-world applications, the scope and limitations of the techniques of SDI adjustment, simplex structure analysis, and motivation internalization, from a substantial point of view. This is more out of the scope rather than a limitation of this software paper. The present paper provides the software basis for such substantial future research work.

5.1. Dataset

The package SDT contains a real dataset, on learning motivation from Austrian school classes in mathematics, information sciences, and natural sciences (Müller et al., 2007): `learning_motivation`. (I would like to thank Professor Dr. Florian Müller and his colleagues from University of Klagenfurt, Austria for providing the author with this dataset.) We use this dataset to illustrate the package's functions. The `learning_motivation` data frame consists of 1,150 rows/students and 6 columns/variables. The students comprise 578 girls and 572 boys (mean age 14.1, with standard deviation 1.9). The variables are sex (integer vector, female = 1, and male = 2), age (integer vector, years), and the learning motivation scores for the subscales of intrinsic regulation, identified regulation, introjected regulation, and external regulation. The motivation variables of the data frame are numeric vectors, which contain aggregate subscale scores, that is, the means taken over all test items that form a respective subscale.

408 Identified regulation is composed of approximately 57% internal share and 43%
409 external share, which is more internal motivation than external motivation, as expected
410 by theory. For introjected regulation, which according to theory ought to be more
411 external motivation than internal motivation, we have the internal and external shares
412 of approximately 33% and 67%, respectively.

413 We can access the attribute value and class of the object `idr`, or print all
414 attributes of the object `ijr`:

```
415 R> attr(idr, "analysis")  
416 [1] "internalization"  
417 R> class(idr)  
418 [1] "share"  
419 R> attributes(ijr)  
420 $names  
421 [1] "internal share" "external share"  
422 $analysis  
423 [1] "internalization"  
424 $class  
425 [1] "share"
```

426 Objects such as `ijr` of the class “share” can be plotted:

```
427 plot(ijr)  
428 gives the stacked bar plot with generic labels of the axes shown in Figure 2.
```

429 [Figure 2 about here]

430 We can have a similar plot for the object `idr` with user-specified labels

```
431 plot(idr, target = "identified regulation", reference =  
432 c("intrinsic regulation", "external regulation"))
```

433 which is shown in Figure 3.

434 [Figure 3 about here]

435

436 5.3. Simplex Structure Analysis

437 We can perform a simplex structure analysis with intrinsic regulation as the target
438 variable, and with identified regulation, introjected regulation, and external regulation
439 as the reference system:

```
440 R> (simstr <- simplex(intrinsic, identified, introjected, external))  
441 base_regulation_1 share base_regulation_2 share base_regulation_3 share  
442           0.6999234           0.3000766           0.0000000
```

443 We can see that the posited simplex structure assumption is fulfilled for this choice of
444 variables. The computed shares are plausible with theory. Intrinsic regulation, which is
445 completely internal, is more interrelated with identified regulation with a share of
446 approximately 70%, followed by introjected regulation with a share of approximately
447 30%, and has a 0% share in regard to external regulation, which is completely external.

448 The object `simstr` is a numeric vector with an attribute value and class:

```
449 R> mode(simstr)
```

```
450 [1] "numeric"
```

```
451 R> attr(simstr, "analysis")
```

```
452 [1] "simplex"
```

```
453 R> class(simstr)
```

```
454 [1] "share"
```

455 and can be plotted with user-specified labels

```
456 R> plot(simstr, target = "intrinsic regulation", reference =  
457       c("identified regulation", "introjected regulation",  
458       "external regulation"))
```

459 shown in Figure 4, where the external regulation share of the computed value zero is
460 omitted.

461 [Figure 4 about here]

462 A similar plot can be produced with external regulation as the target variable, and
463 with intrinsic regulation, identified regulation, and introjected regulation as the
464 reference system, with generic labels:

```
465 R> plot(simplex(target_regulation = external,  
466   base_regulation_1 = intrinsic,  
467   base_regulation_2 = identified,  
468   base_regulation_3 = introjected))
```

469 This is shown in Figure 5, where the intrinsic regulation share of value zero is omitted.

470 [Figure 5 about here]

471 The respective shares in this case are:

```
472 R> simplex(external, intrinsic, identified, introjected)  
473 base_regulation_1 share base_regulation_2 share base_regulation_3 share  
474           0.0000000           0.3388902           0.6611098
```

475 Again, the shares are in accordance with theory, and the simplex structure assumption is
476 satisfied with this set of variables. External regulation is most interrelated with
477 introjected regulation (66%), followed by identified regulation (34%), and with a zero
478 share regarding intrinsic regulation.

479

480 5.4. Original and Adjusted Indices

481 We can compute, for each student or row pattern of the dataset, the corresponding
482 values of the original and adjusted SDI or RAI indices, using the function `sdi`. Adjusted
483 index computation can be performed by

```
484 adj <- sdi(intrinsic, identified, introjected, external)
```

```
485 with the attributes
486 R> attributes(adj)
487 $names
488 [1] "adjusted_internal_locus" "adjusted_external_locus"
489 "sdi_adjusted"
490 $variant
491 [1] "adjusted"
492 $class
493 [1] "sdi"
494     We can inspect the first six elements of each list component vector:
495 R> lapply(adj, head)
496 $adjusted_internal_locus
497 [1] 1.9666013 0.6611796 1.2663977 1.4860787 1.3610451 0.8580979
498 $adjusted_external_locus
499 [1] 2.033399 1.105487 1.150269 1.547255 1.222288 1.225235
500 $sdi_adjusted
501 [1] -0.06679746 -0.44430747 0.11612865 -0.06117589 0.13875686
502 -0.36713743
```

```
503     The original index computation can be performed by
504 orig <- sdi(intrinsic, identified, introjected, external,
505     compute.adjusted = FALSE)
506 and summarized using the corresponding method
507 R> summary(orig)
508 summary of confounded internal locus values:
509     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
510     3.000   7.650   9.700   9.547  11.600  15.000
511 summary of confounded external locus values:
```

```
512      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
513      3.000   7.000   8.750   8.622  10.500  15.000
```

514 summary of original SDI scores:

```
515      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
516     -8.7500 -1.2500   0.6500   0.9257   3.1000  11.2500
```

517 In contrast, the summary for the adjusted measure yields

```
518 R> summary(adj)
```

519 summary of adjusted internal locus values:

```
520      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
521      0.0000   0.9664   1.3660   1.3360   1.7100   2.5330
```

522 summary of adjusted external locus values:

```
523      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
524      0.000   1.002   1.369   1.365   1.747   2.801
```

525 summary of adjusted SDI scores:

```
526      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
527     -1.47900 -0.37830 -0.07886 -0.02832   0.30720   1.48000
```

528 It is interesting to plot objects of the class “sdi”. Plotting the objects `adj` and
529 `orig` (minimum and maximum scores of the scale procedure were the default values 1
530 and 5, respectively)

```
531 plot(adj)
```

```
532 plot(orig)
```

533 produce the scatterplots shown in Figures 6 and 7, respectively.

534 [Figure 6 about here]

535 [Figure 7 about here]

536 Points with larger overall index values are depicted in darker gray tone. The admissible
537 ranges for the original and adjusted indices are [3, 15] and [0, 4], respectively.

538 From Figures 6 and 7, we can see that the adjusted scores are more concentrated
539 around the diagonal line shown in red, whereas the confounded scores do scatter
540 messily over the broad range of admissible values. The adjusted scores in Figure 6
541 indicate that the external and internal motivation extents are distributed primarily in
542 the lower to middle regions, between 1 to 2 scale points. This renders possible to see, if
543 and where on the common scale from 0–4, there may be tolerance or scope for possible
544 interventions, to improve on pupils' learning motivation. This is not possible for the
545 original index, which messily scatters.

546 Moreover, according to the adjusted RAI index, the girls are slightly extrinsically
547 oriented:

```
548 R> mean(adj$sdi_adjusted[sex == 1])  
549 [1] -0.1062873
```

550 In contrast, with regard to the confounded original index, the girls can be deemed
551 clearly intrinsically motivated:

```
552 R> mean(orig$sdi_original[sex == 1])  
553 [1] 0.3730969
```

554 The former observation based on the adjusted index is more plausible. For,
555 mathematics, informatics, and natural sciences school classes are studied, and there is
556 empirical evidence that girls in these subject areas may typically behave extrinsically
557 motivated.

558 The `print` method lists the original and adjusted SDI overall index values, for all
559 students or rows of the dataset (the R output is omitted, for typographic reasons):

```
560 R> adj  
561 adjusted SDI scores:  
562 [1] -6.679746e-02 -4.443075e-01 1.161286e-01 -6.117589e-02  
563 1.387569e-01 -3.671374e-01 -2.008070e-01 4.658596e-01
```

```
564      [9]  0.000000e+00  1.062343e-01 -3.686244e-01  4.553563e-01
565 -3.712720e-01  1.220229e-01  8.130066e-01  7.724937e-01
566 ...
567 R> orig
568 original SDI scores:
569      [1]  1.25 -1.90  1.50  0.70  2.00 -1.50  0.00  4.00  0.00  2.05
570 -2.20  4.00 -0.25  2.00  6.90  5.20  1.15  5.50  4.85
571     [20] -0.80  0.75  1.50 -2.15  2.90  2.60  1.05  1.05  1.90 -4.35
572  2.65  6.30  1.75 -3.00 -0.90 -1.15  6.15  2.70 -1.75
573 ...
```

574

575 6. CONCLUSION

576 We have introduced the package SDT for computing self-determination theory (SDT)
577 measures in the R language and environment. The package contains functions for
578 computing the measures of motivation internalization, motivation simplex structure,
579 and the original and adjusted self-determination or relative autonomy indices (SDI or
580 RAI). The functions of the package SDT were described, and we demonstrated the
581 functions' usage on an accompanying example dataset.

582 With the package SDT in R we hope to have established a basis for computational
583 work in SDT. We plan to extend this package to incorporate such dimensionality
584 reduction approaches as principal component analysis and factor analysis, for SDT
585 questionnaire validation in R. Interactive visualization techniques for the exploration of
586 raw-data motivation variables could also be implemented and utilized in R, for
587 exploratory SDT analyses. The realization of SDT, for the first time in R, can also be
588 valuable in applying current or computational statistical methods to SDT. For instance,
589 the determination of confidence intervals and hypothesis testing in SDT for the
590 computed optimal shares and the original and adjusted SDI or RAI indices may likely be

591 realized using resampling methods. Future work of this sort would involve extensive
592 computer simulation, which could be ideally achieved with R.

593

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692 **Captions**

693

694 *Figure 1.* “The self-determination continuum, showing the motivational, self-regulatory,
695 and perceived locus of causality bases of behaviors that vary in the degree to which they
696 are self-determined” (Deci & Ryan, 2000, p. 237).

697

698 *Figure 2.* Internalization analysis plot for introjected regulation as the target variable.
699 Stacked bar plot is shown with generic labels.

700

701 *Figure 3.* Stacked bar plot of the internal and external shares of identified regulation.
702 User-specified labels are provided.

703

704 *Figure 4.* Simplex structure analysis plot, with user-specified labels, for intrinsic
705 regulation as the target variable, and with identified regulation, introjected regulation,
706 and external regulation as the reference system.

707

708 *Figure 5.* Stacked bar plot of the simplex structure shares of external regulation with
709 respect to intrinsic regulation, identified regulation, and introjected regulation,
710 displayed with generic labels.

711

712 *Figure 6.* Scatterplot for the adjusted self-determination index (SDI). Adjusted external
713 locus values versus adjusted internal locus values are plotted. The points in the
714 scatterplot are shown at different gray levels, determined by their adjusted SDI overall
715 index values. The red line, to assist visualization, is $y = x$, and the admissible range $[0, 4]$
716 is graphed in gray dashed lines.

717

718 *Figure 7.* Scatterplot for the original self-determination index (SDI). Confounded

719 external locus values versus confounded internal locus values are plotted. Points are

720 drawn at different gray levels, depending on the original SDI overall index values. The

721 red line, for comparison, is $y = x$, and the admissible range [3, 15] is shown in gray

722 dashed lines.

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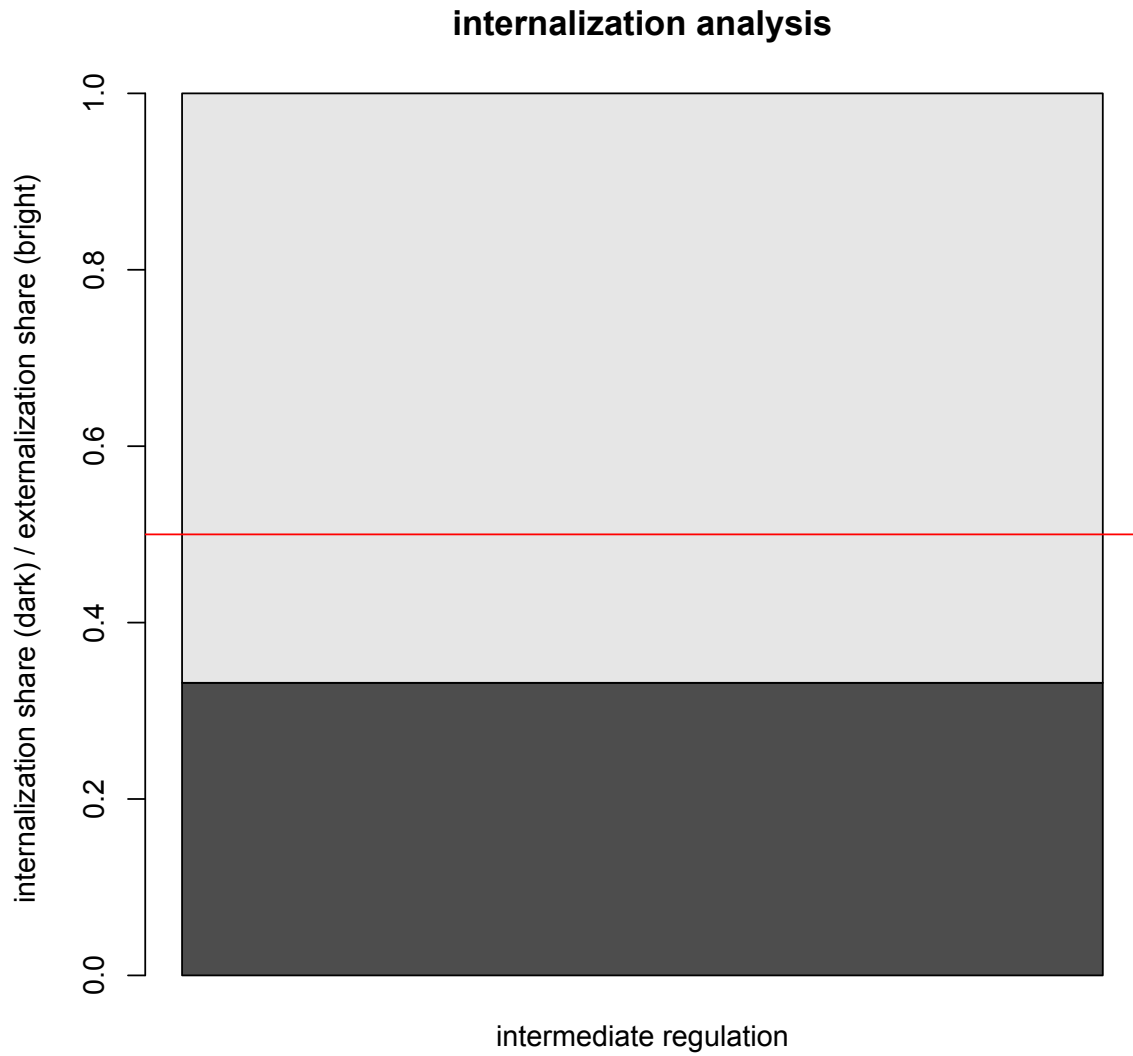
Figures

Figure 1

Behavior	Nonself-determined					Self-determined
Type of Motivation	Amotivation	Extrinsic Motivation				Intrinsic Motivation
Type of Regulation	Non-regulation	External Regulation	Introjected Regulation	Identified Regulation	Integrated Regulation	Intrinsic Regulation
Locus of Causality	Impersonal	External	Somewhat External	Somewhat Internal	Internal	Internal

763 *Figure 2*

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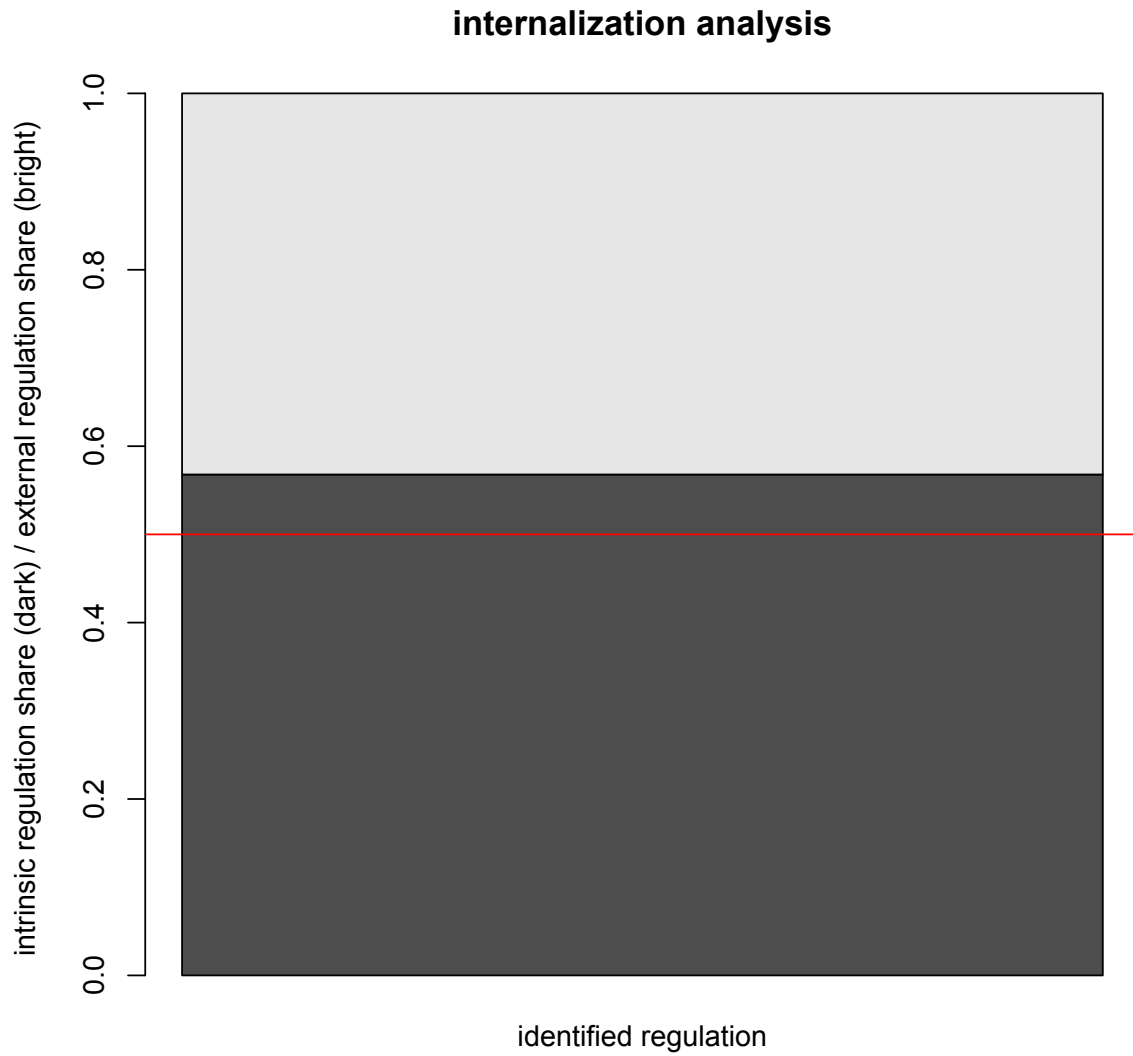
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773 *Figure 3*

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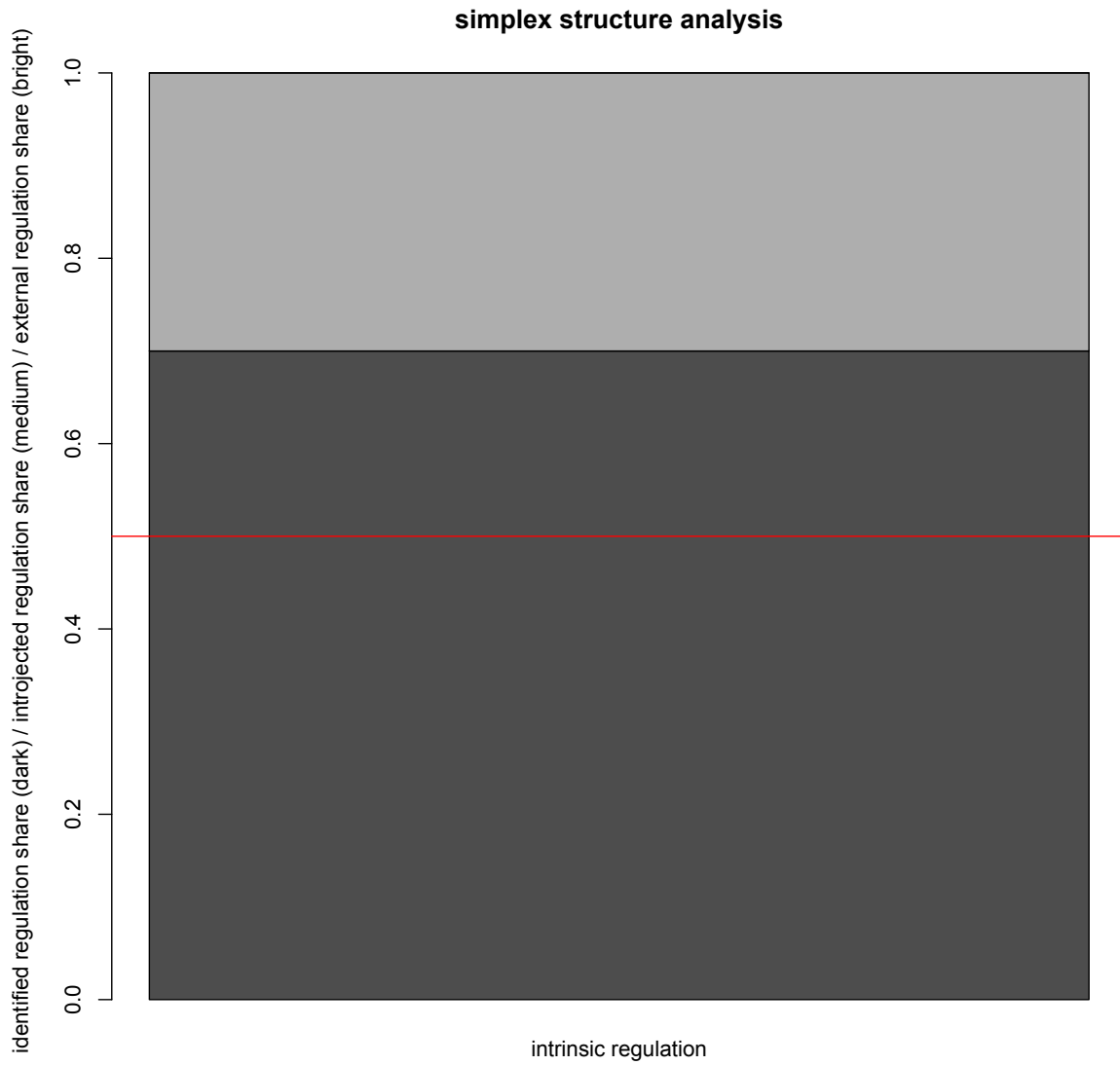
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783 *Figure 4*

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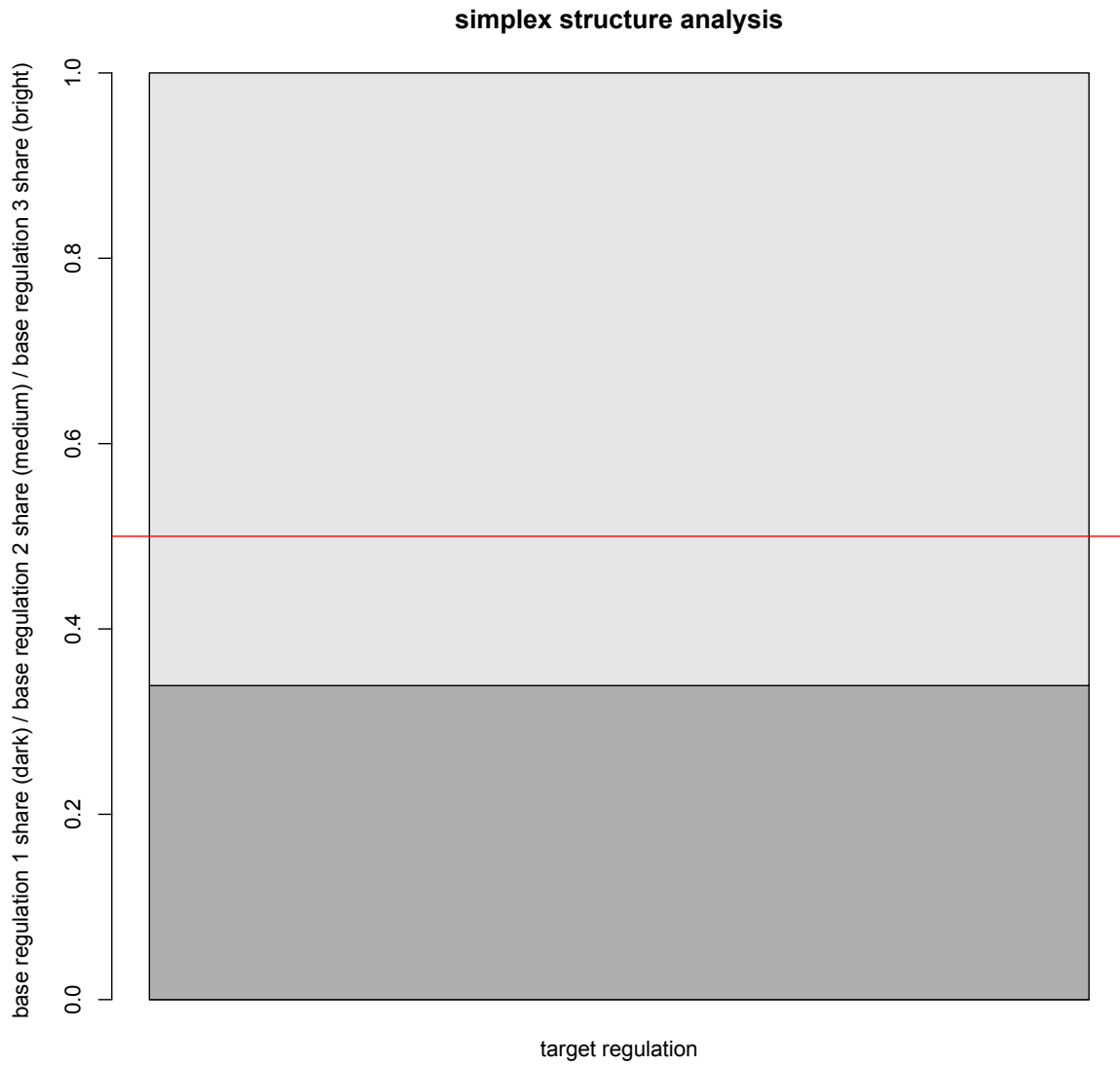
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793 *Figure 5*

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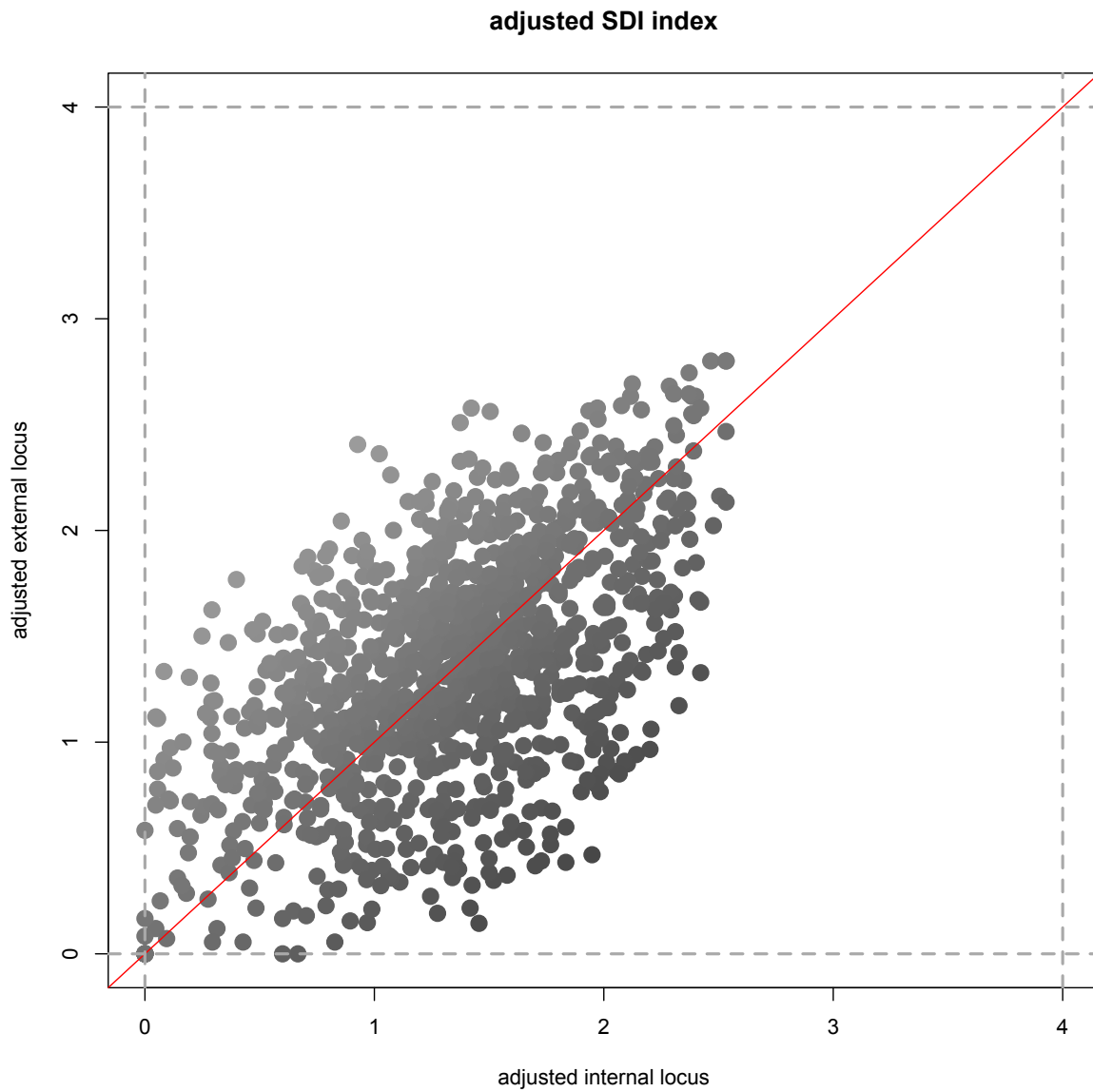
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803 *Figure 6*

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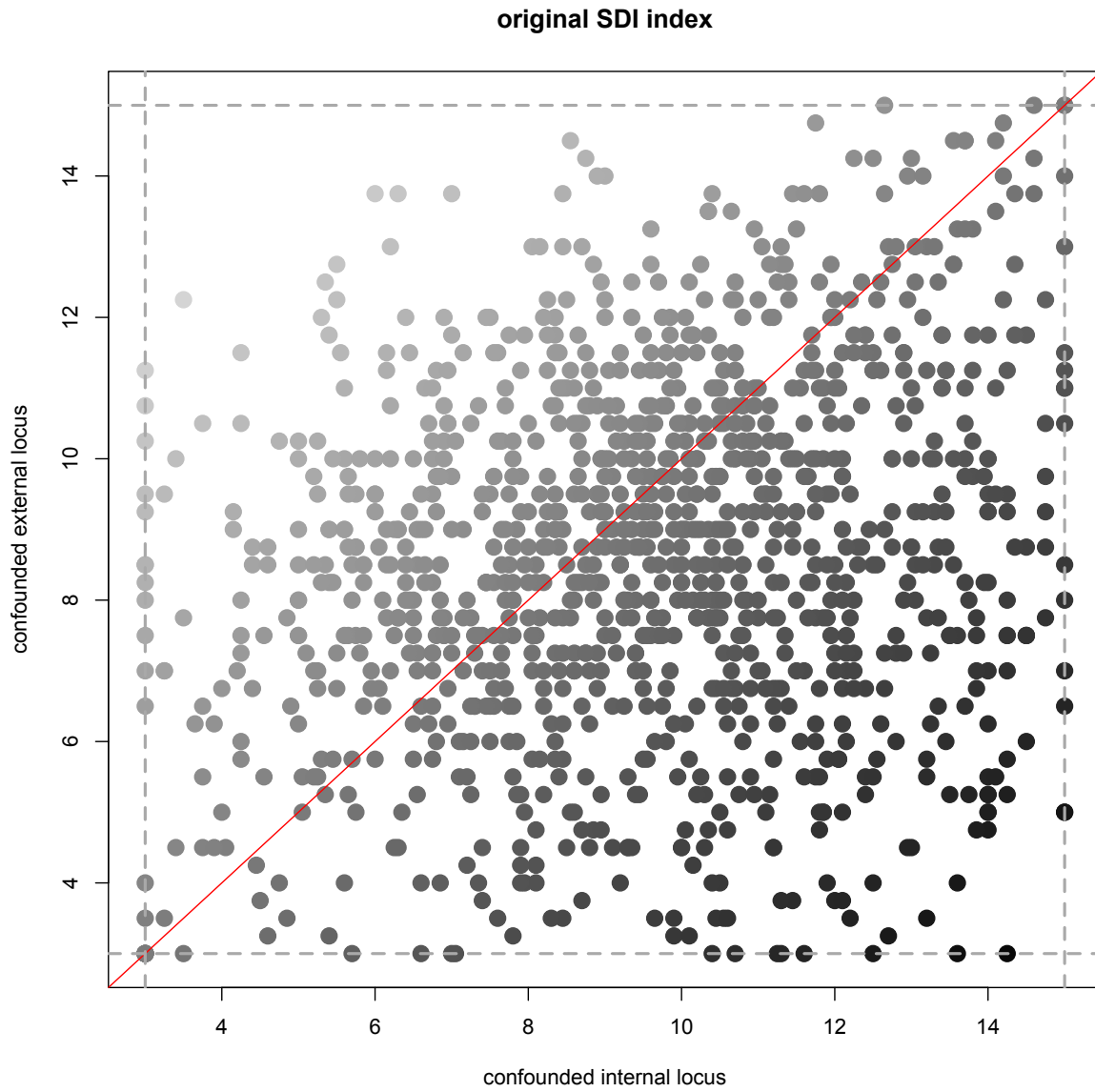
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813 *Figure 7*

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