

21 **Abstract**

22 1. In order to study colour signals as animals perceive them, visual ecologists usually
23 rely on models of colour vision that do not consider patterns—the spatial arrangement of
24 features within a signal.

25 2. HMAX describes a family of models that are used to study pattern perception in
26 human vision research, and which have inspired many artificial intelligence algorithms.
27 In this article, we highlight that the sensory and brain mechanisms modelled in HMAX
28 are widespread, occurring in most if not all vertebrates, thus offering HMAX models a
29 wide range of applications in visual ecology.

30 3. We begin with a short description of the neural mechanisms of pattern perception in
31 vertebrates, emphasizing similarities in processes across species. Then, we provide a
32 detailed description of HMAX, highlighting how the model is linked to biological vision.
33 We further present *sparse*-HMAX, an extension of HMAX that includes a sparse coding
34 scheme, in order to make the model even more biologically realistic and to provide a
35 tool for estimating efficiency in information processing. In an illustrative analysis, we
36 then show that HMAX performs better than two other reference methods (manually-
37 positioned landmarks and the SURF algorithm) for estimating similarities between faces
38 in a nonhuman primate species.

39 4. This manuscript is accompanied with MATLAB codes of an efficient implementation of
40 HMAX and *sparse*-HMAX that can be further flexibly parameterized to model non-human
41 colour vision, with the goal to encourage visual ecologists to adopt tools from computer
42 vision and computational neuroscience.

43

44 **Key words:** colour patterns, animal colour vision, computer vision, visual
45 communication signals, visual ecology, artificial neural networks, face similarity.

46 **Introduction**

47 Understanding the evolution and the ecological significance of communicative traits
48 requires studying these traits in the eyes of beholders (Endler *et al.* 2005). In visual
49 communication, colour spaces—which model perceived differences between colours—
50 have thus become very popular among visual ecologists who study socio-sexual
51 communication, camouflage, mimicry and plant-animal interactions (Renoult, Kelber &
52 Schaefer 2017). In colour spaces, the design of visual stimuli is usually studied as a
53 collection of isolated plain colours, without considering the influence of their spatial
54 arrangement on perception. However, the effectiveness of a communication system
55 strongly depends on how colour patches, lines or dots are arranged spatially to form
56 colour patterns. For example, the diurnal hawkmoth *Macroglossum stellatarum* innately
57 prefers radial blue and white patterns to ring patterns with the same colours (Kelber
58 2002). Modelling the perception of colour patterns is thus a necessary step toward a
59 better understanding of natural communication systems.

60 In this article, we highlight the benefits of the HMAX family of models for
61 analysing patterned colour stimuli, as vertebrates perceive them. HMAX was originally
62 developed by computational neuroscientists to model information processing in the
63 ventral stream of the visual pathway, that is, the brain area involved in shape and colour
64 perception in humans (Serre & Riesenhuber 2004). Yet, due to the generality of the
65 hierarchical mechanisms involved in this family of models, the sensory and brain
66 processes modelled in HMAX are certainly widespread, occurring in most if not all
67 vertebrate taxa, thus offering HMAX a wide range of applications in visual ecology.

68 We begin with a short description of the neural mechanisms of pattern
69 perception in vertebrates, emphasizing similarities in processes across species. Then,
70 we provide a detailed description of HMAX, highlighting how it is connected to biological

71 vision. We further present *sparse-HMAX*, an extension of HMAX that includes a sparse
72 coding scheme. Sparse coding describes the strategy of neural systems to minimize the
73 number of neurons activated simultaneously (Olshausen & Field 2004). By adding
74 sparse coding to HMAX, we aim both to develop an even more biologically realistic
75 model of perception of colour patterns, and to provide a framework for estimating
76 efficiency in information processing (Renoult & Mendelson 2019). A fast and easily
77 customizable version of HMAX that can be used to model the perception of colour
78 patterns in most vertebrates and our *sparse-HMAX* algorithm are available at:
79 <https://github.com/EEVCOM-Montpellier/HMAX>. In the last part of this article, we
80 apply HMAX to estimate similarities between faces in a nonhuman primate species as an
81 example application.

82

83 **Perception of colour patterns in vertebrates**

84 Despite structural differences in how vertebrates perceive colour patterns, a number of
85 general principles governing the processing of visual information are shared across
86 species. In this section, we review four of these principles: the hierarchical processing of
87 information, the tuning of neurons to stimulus features, the sparse encoding of
88 information, and the opponent processing of colour information.

89

90 *Hierarchical processing of visual information*

91 The perception of colour pattern is one step of the whole vision process that ultimately
92 leads to recognition. In mammals, vision starts with the stimulation of retinal
93 photoreceptors that convert the light arising from a stimulus into electro-chemical
94 signals. These signals are then conducted through retinal ganglion cells to reach the
95 lateral geniculate nucleus (LGN), a relay centre that connects the retina to the primary

96 visual cortex (V1). Signals continue to flow bottom-up through V2 and V4, and then
97 through the inferior temporal cortex (IT), which feeds the prefrontal cortex that connects
98 perception to memory and action (Felleman & Van Essen 1991). Areas V3 and V5 are
99 involved mainly in motion vision (Zeki *et al.* 1991).

100 As signals flow from photoreceptors up to IT, the information extracted becomes
101 increasingly complex (Mély & Serre 2017). At the receptor level, light contrasts are
102 recorded locally without any information about their spatial organization. In V1,
103 neurons become sensitive to short and oriented line segments (Tootell *et al.* 1988).
104 Basic shapes such as curved lines (i.e. combinations of oriented line segments) are
105 mainly processed in V4. More complex shapes representing entire objects (i.e.
106 combinations of curved lines; e.g., a lion, a house or a face) are processed in IT and in
107 subsequent specialised areas (e.g., the fusiform face area for faces). In addition,
108 throughout the visual pathway neurons are increasingly invariant to orientation, scale,
109 position and lighting conditions. Neurons in IT thus fire in response to specific items yet
110 they are insensitive to how tilted, distant, centred in the field of view and shaded these
111 items are (Mély & Serre 2017). How the visual system achieves the dual increase in
112 sensitivity and invariance has been a central question of vision science and is still one of
113 the most active research topics in computer vision (e.g., Anselmi *et al.* 2016).

114 In their seminal article, Hubel and Wiesel (1962) proposed a physiological model
115 of V1 that copes with the complexity-invariance problem. The model assumes a
116 feedforward, hierarchical flow of information within V1 that involves two different types
117 of neurons: the simple cells and the complex cells. Simple cells pool afferents (LGN cells
118 with circular receptive fields; RFs hereafter) sampled along oriented line segments.
119 Complex cells pool inputs from several spatially contiguous simple cells with different
120 orientations, phases, positions or scales. Consequently, one complex cell keeps the

121 complex selectivity of its afferent simple cells but it is tolerant to local variation in
122 stimulus orientation, phase, position or scaling. This hierarchical model has been
123 extended to V2, V4 and IT, where neurons equivalent to simple and complex cells have
124 been discovered (Soto & Wasserman 2012). Information thus flows through the visual
125 pathway alternating between simple cells, with increasingly complex selectivity, and
126 complex cells, with increasingly large tolerance to geometrical transformations.
127 Furthermore, because both types of cells pool information from multiple afferents, the
128 neurons' RFs become increasingly larger as signals flow up (Smith *et al.* 2001).

129 The hierarchical processing of visual information is a general principle found in
130 other, non-mammalian vertebrates. Most neurons in the tecto-isthmic system—the
131 functionally analogous structure to the visual cortex in non-mammals—are selective to
132 orientation (e.g., in fish see Ben-Tov *et al.* 2013). In birds, it was found that the elongated
133 RF of isthmic cells is generated by pooling afferents from aligned tectal cells with
134 circular RFs, and that this elongated RF underlies the orientation selectivity of these
135 cells (Li, Xiao & Wang 2006). In fishes, the laminar organisation of orientation-selective
136 inputs coming from the retina suggests that different layers in the tectum may be
137 dedicated to processing specific visual features (Abbas & Meyer 2014). RF size has been
138 shown to increase along the bird visual pathway too (Engelage & Bischof 1996). Overall,
139 the available behavioural and neurophysiological data indicate that, although they use
140 ontogenetically different structures, a similar hierarchical and feedforward processing
141 occurs in the tecto-isthmic system and in the visual cortex (Soto & Wasserman 2012).

142 Besides feedforward projections, the visual pathway of vertebrate vision also
143 involves horizontal and feedback neural projections (Treue 2003). Yet, because of the
144 short response latency of IT's neurons to visual stimuli (~100 ms), and the ability of
145 primates to recognize objects in a very short time (~150 ms; Thorpe, Fize & Marlot

146 1996), it is generally recognized that core object representation, and *a fortiori* pattern
147 perception, are essentially feedforward mechanisms (VanRullen & Koch 2003; DiCarlo,
148 Zoccolan & Rust 2012). This is why the simple hierarchical model of Hubel and Wiesel
149 has been consistently efficient in explaining and predicting empirical results in
150 neuroscience over the years (Ferster & Miller 2000; Reid & Usrey 2004).

151

152 *The tuning of neuronal selectivity*

153 For visual ecologists, an important question is whether the selectivity of cortical/tecto-
154 isthmic neurons adapts to the environment, and whether this adaptation is determined
155 developmentally or evolutionarily. In contrast to many studies supporting a spectral
156 tuning of photoreceptors to lighting conditions (listed in Cummings & Endler 2018),
157 analyses comparing the shape and orientation selectivity of neurons between species
158 inhabiting contrasting visual environments are still lacking. Nevertheless, a few studies
159 with model species reveal interesting relationships between orientation selectivity and
160 environmental stimuli. One study in kittens showed that, at eye opening, a proportion of
161 cells in V1 show the orientation selectivity typical of a mature visual cortex (Sengpiel &
162 Kind 2002). Moreover, the visual cortex of kittens reared in a striped environment
163 responded to all orientations, even those never seen by the animal, even though twice as
164 much cortical area was devoted to the experienced orientation (Sengpiel, Stawinski &
165 Bonhoeffer 1999). For more complex shapes, and thus in higher levels of information
166 processing, there is also evidence that some stimuli are innately categorized (e.g., faces
167 in primates: Johnson *et al.* 1991); nevertheless it is generally accepted that complex
168 shape selectivity is mostly tuned by learning (e.g., Freedman *et al.* 2005). Overall, it
169 appears that shape selectivity is innate, but that it can be retuned to environmental

170 stimuli through learning in some neurons; moreover, the proportion of neurons that can
171 be retuned seems to increase in higher levels of the visual pathway.

172

173 *Sparse coding*

174 At a given level of information processing, signals from individual neurons are combined
175 into a neural code, i.e. a neural representation of a visual stimulus at this particular level.

176 Yet the high metabolic cost of neuronal activation (in the human visual system, it
177 accounts for 2.5 to 3.5 % of a resting body's overall energy requirements; Attwell &
178 Laughlin 2001) imposes constraints onto the neural code (Graham & Field 2006).

179 Olshausen and his colleagues analysed the importance of this constraint by training
180 artificial neurons to encode images of natural scenes as efficiently and precisely as
181 possible (Olshausen & Field 1996; Olshausen & Field 1997). To match the selectivity of

182 these artificial neurons to that of real neurons measured in mammalian V1, the authors
183 had to implement a sparseness criterion in the neural code. In this context, sparseness

184 can describe both the fact that only a small fraction of neurons are active at any time
185 (population sparseness), and that individual neurons activate shortly and rarely

186 (temporal sparseness). Sparse coding is metabolically effective because frequently firing
187 a few generalist neurons is far more costly than maintaining a large population of highly

188 selective yet sparsely activated neurons (Lennie 2003; Olshausen & Field 2004).
189 Experimentally, following its discovery in V1, sparse coding has been demonstrated at

190 all levels of perception, from the retina (Pitkow & Meister 2012) to V4 (Carlson *et al.*
191 2011) and IT (Brincat & Connor 2004). Sparse coding is also ubiquitous in other sensory

192 modalities and has been found in all organisms investigated so far, including
193 invertebrates (e.g.; Hromádka, DeWeese & Zador 2008).

194

195 *Opponent colour coding*

196 The studies described previously have investigated the selectivity of neurons to
197 luminance (i.e. along a greyscale) contrasts; yet most vertebrates additionally use colour
198 as a source of information. It is well established that opponent coding is crucial for
199 modelling colour perception: compared to analysing raw excitation values, analysing
200 *differences* in photoreceptor excitations dramatically improves the fit between predicted
201 and actual perceived differences between colours (Renoult, Kelber & Schaefer 2017). In
202 Old World primates, colour opponency is achieved by combining outputs of the three S,
203 M and L photoreceptors (standing for short, medium and long wavelength, respectively)
204 into two opponent channels: the red-green and the blue-yellow channels computed as L-
205 M and S-(M+L), respectively. However, the predictive power of these two opponent
206 channels strongly depends on the size and shape of the stimulus (Derrington, Krauskopf
207 & Lennie 1984). Although it has been traditionally assumed that luminance and colour
208 are processed separately in the visual system of vertebrates, and that shape perception
209 is mediated by the luminance channel, evidence has accumulated that colour and shape
210 are inextricably linked (for a review, see Shapley & Hawken 2011).

211 The neurons that compute the L-M and S-(M+L) signals represent only one of two
212 categories of opponent cells found in the vertebrate visual system, which have been
213 named single-opponent (SO) cells (Shapley & Hawken 2011). SO cells compute an
214 averaged difference in photoreceptor excitation locally, and thus they are mostly
215 activated by full-field stimuli. SO cells are useful for processing surface information and
216 are almost not selective to orientation. Another category of colour-selective cells
217 receives inputs from SO cells. These so-called double-opponent (DO) cells compute an
218 averaged difference in colour (L-M or S-(M+L)) signals between different regions of the
219 visual field (Shapley & Hawken 2011). DO cells located in the retina and LGN have a

220 circular RF and are thus unselective to orientation. In V1, however, many DO cells have
221 an oriented RF (Johnson, Hawken & Shapley 2001). Oriented DO cells are useful for
222 processing coloured line segments and thus are very important for the perception of
223 colour patterns. Furthermore, DO cells are selective to both colour and luminance
224 contrasts, which argues against the idea of a strong segregation of colour and luminance
225 processing beyond the LGN (Gegenfurtner 2003). Besides primates, SO and DO cells
226 have been found in other vertebrates including fishes (Daw 1968) and birds (Frost,
227 Scilley & Wong 1981).

228 In summary, the perception of colour patterns in vertebrates relies on general
229 principles that are likely shared among most species: a hierarchical processing of
230 signals, alternating shape-selective and spatial-pooling neurons, the adaptation of
231 shape-selective neurons, sparse coding and a simple/double opponent colour coding
232 scheme.

233

234 **HMAX and *sparse-HMAX***

235 For more than five decades, researchers from cognitive sciences, computer vision and
236 artificial intelligence have proposed mathematical models of colour pattern perception
237 that account for the aforementioned principles (reviewed in Poggio & Serre 2013; Serre
238 2013). HMAX (Riesenhuber & Poggio 1999; Serre *et al.* 2005) is one of the most popular
239 families of models of visual processing in humans. The original HMAX has been
240 improved in multiple ways; the version presented here is generalizable to non-human
241 vertebrates and can further account for sparse coding.

242

243

244

245 HMAX

246 HMAX is a hierarchical neural network starting with a scale image I as an input layer and
 247 alternating S layers that perform feature mapping and C layers that perform feature
 248 pooling. S and C stand for “Simple” and “Complex”, in reference to the simple and
 249 complex cells described previously. Feature mapping is achieved by convoluting
 250 different shape-selective filters (or artificial neurons) with I in the first layer, or with
 251 maps produced by the preceding layer. Feature pooling increases the invariance of
 252 feature maps to geometrical transformations. Classical versions of HMAX include four
 253 layers in addition to the input layer: $S1$, $C1$, $S2$ and $C2$ (Figure 1a). The output of $C2$ is a
 254 compact code describing the input image, typically a vector of a thousand values.

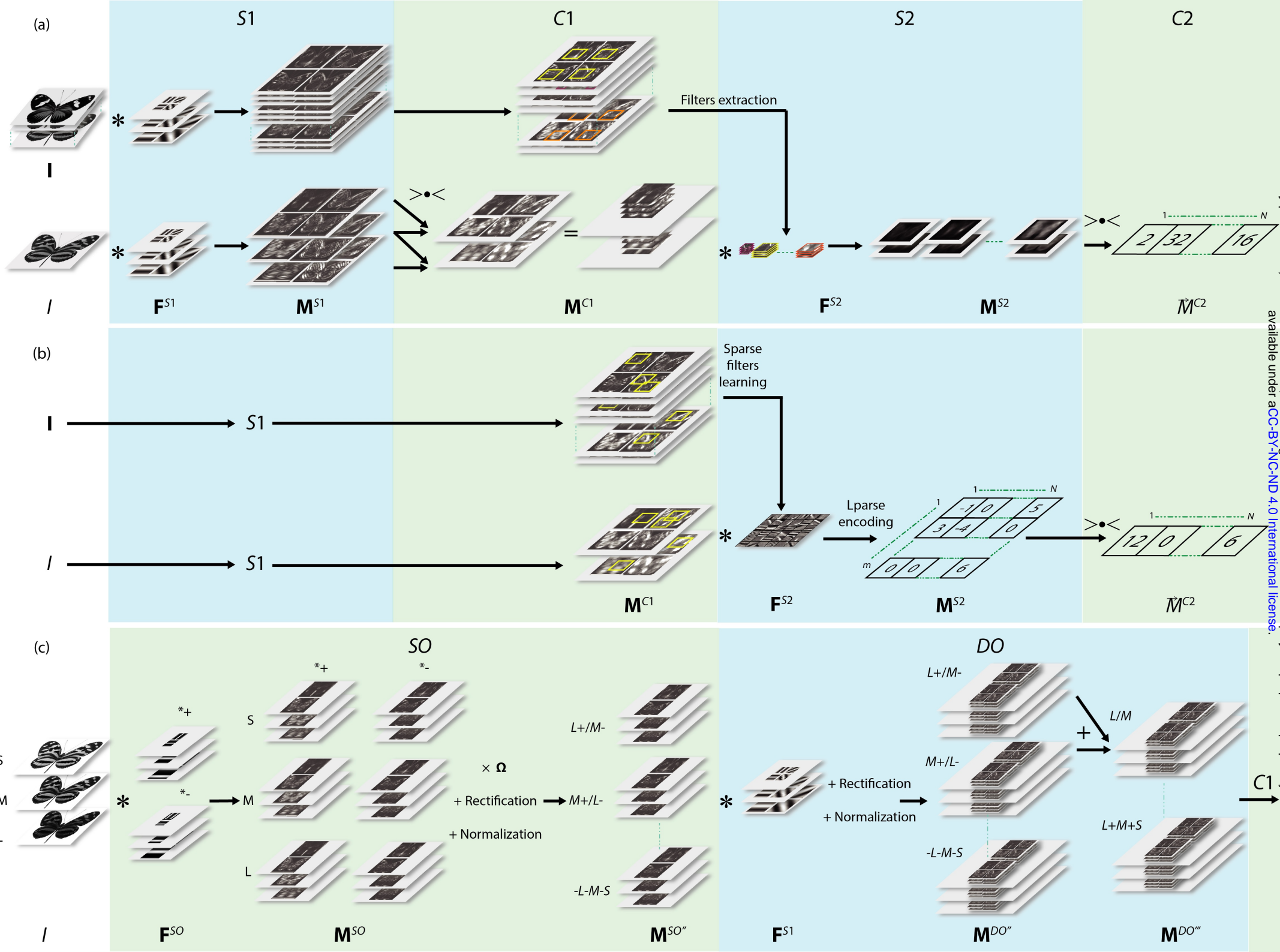
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256 **Figure 1** [See next page]. Overview of HMAX models. (a) Classical HMAX for scale images
 257 with filter learning (upper row) and stimulus encoding (lower row). (b) *sparse*-HMAX
 258 for scale images with filter learning (upper row) and stimulus encoding (lower row). (c)
 259 Simple opponent (SO) and double-opponent (DO) processes in HMAX for colour images
 260 (here, describing photoreceptor excitation maps of a trichromat species).

261

262 *Layer S1.*– In $S1$, feature-selective neurons are represented by a set \mathbf{F}^{S1} of Gabor filters
 263 $F_{\theta,\sigma}^{S1} \in \mathbb{R}^{q \times q}$ defined by orientation $\theta \in \mathbb{R}^t$ (t different values), scale $\sigma \in \mathbb{R}^{sc}$ and filter
 264 size $q \in \mathbb{Z}^u$. Gabor filters have been previously shown to accurately model the
 265 selectivity both of simple cells of V1 in mammals (Jones & Palmer 1987), and also (most
 266 likely) of tecto-isthmic neurons with an elongated RF in other vertebrates. Defining
 267 $x_g=1,\dots,q$ and $y_g=1,\dots,q$, a Gabor filter $F_{\theta,\sigma}^{S1}$ is described as

$$F_{\theta,\sigma}^{S1}(x_g, y_g) = \exp\left(-\frac{(x_g \cos\theta + y_g \sin\theta)^2 + \gamma(y_g \cos\theta - x_g \sin\theta)^2}{2\sigma^2}\right) \times \cos\left(2\pi \frac{1}{\lambda}(x_g \cos\theta + y_g \sin\theta)\right).$$



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268 with γ the aspect ratio of the filter and λ the wavelength parameter. Serre and
269 Riesenhuber (2004) found that λ had limited effect on tuning filter selectivity and thus
270 kept it constant. These authors further proposed to approximate σ and λ from q such
271 that

$$\begin{cases} \sigma = 0.0036 * q^2 + 0.35 * q + 0.18 \\ \lambda = \frac{\sigma}{0.8} \end{cases} .$$

272 The number of different scales (sc) is thus taken as identical to the number of different
273 filter sizes (u). \mathbf{F}^{S1} is determined by the set of SC/u and t values for σ/q and θ
274 respectively. Those should vary between species and, as discussed previously with the
275 example of kittens reared in a striped environment, with experience (for parameter
276 values that best fit to neurophysiological data in mammals, see Serre & Riesenhuber
277 2004). Having built the filter set \mathbf{F}^{S1} , following Theriault, Thome & Cord (2013), each
278 filter is then convolved with I to generate $t \times sc$ feature maps $M_{\theta,\sigma}^{S1}$ such that

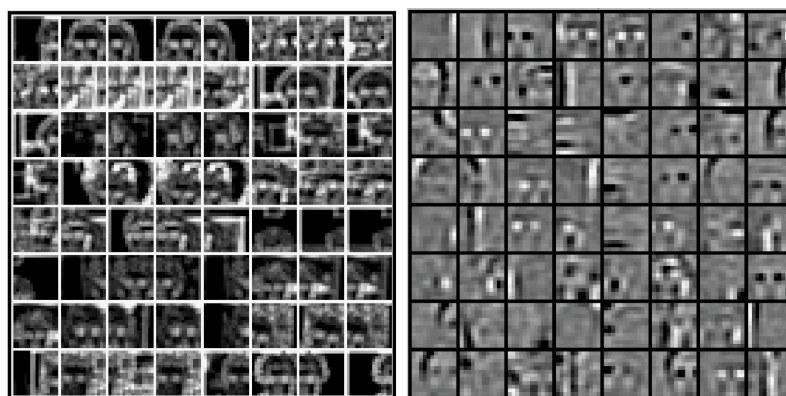
$$279 \quad M_{\theta,\sigma}^{S1} = |F_{\theta,\sigma}^{S1} * I|.$$

280 *Layer C1.*– C1 pools values of $M_{\theta,\sigma}^{S1}$ to produce $M_{\theta,\sigma}^{C1}$ maps with a decreased resolution but
281 an increased tolerance of features to shift and size (but not to orientation). Among the
282 different pooling functions, max pooling (keeping the maximum value only) works best
283 in generating invariant features (Scherer, Müller & Behnke 2010). For each of the t
284 values of the orientation θ , a max filter $F^{C1} \in \mathbb{R}^{r \times r}$ (with $r \propto q$, see Theriault, Thome &
285 Cord 2013) is applied simultaneously to neighbourhood values within a given
286 $M_{\theta,\sigma}^{S1}$ feature map (thus gaining position invariance), and to pairs of maps with two
287 consecutive scales (thus gaining scale invariance). The max filter is moved in steps of 1
288 between scales, and around a given feature map in steps of magnitude r . Note that due to
289 the pairwise pooling in scale, in C1 there is only $SC-1$ remaining values for σ .

290

291 *Layer S2.*– V4/IT/isthmic neurons are modelled in $S2$ by a set \mathbf{F}^{S2} of filters, which are
292 selective to more complex features (curved lines, shapes) compared to $F_{\theta,\sigma}^{S1}$ (segment
293 lines; Figure 2). Motivated by the biological evidence that shape-selectivity is largely
294 tuned by adaptation or experience to environmental stimulation in higher levels of the
295 visual pathway, \mathbf{F}^{S2} is generated in an initial feature learning stage using a set of training
296 images. This stage includes four steps. First, $M_{\theta,\sigma}^{S1}$ and then $M_{\theta,\sigma}^{C1}$ are generated for each
297 learning image. Second, the coordinates (x,y) of a filter centre are randomly drawn in
298 $M_{1,1}^{C1}$. Third, one filter size q' is randomly drawn, with the constraint that the different
299 possible values of q' are all equally represented in \mathbf{F}^{S2} . F_1^{S2} is then extracted from $M_{\theta,1}^{C1}$
300 around (x,y) at the first scale $\sigma = 1$ and for all orientations. Steps two to four are repeated
301 to generate N filters F_n^{S2} ; typically $N = 10^3$. Note that each F_n^{S2} is thus constructed as a 3D
302 filter: $\mathbb{R}^{q' \times q' \times t}$. In contrast to $S1$, the learning stage of $S2$ allows shape-selective filters to
303 adapt to dominant shapes within training images, which can be the same as the target
304 stimulus or other images depicting, e.g., environmental scenes.

305



306

307

308 **Figure 2.** Filters in $S2$ (\mathbf{F}^{S2}) learned on the human face category of Caltech101 image
309 dataset with classical HMAX (left) and *sparse*-HMAX (right). Filters in $S2$ model neurons
310 selective to complex features and with large receptive fields (here, regions of faces).

311

312 The output of $S2$, i.e., feature maps $M_{\sigma,n}^{S2}$, are eventually built by convoluting F_n^{S2}

313 with 3D stacks of $M_{\theta,\sigma}^{C1}$,

$$M_{\sigma,n}^{S2} = F_n^{S2} * M_{\theta,\sigma}^{C1},$$

314 using the radial basis function described in (Mutch & Lowe 2008) that calculates, at

315 every position (x,y) of the map, the response Re to the filter of a patch P from $M_{\theta,\sigma}^{C1}$ as

$$316 \quad Re_{(P,F_n^{S2})} = \exp\left(-\frac{\|P-F_n^{S2}\|^2}{2s^2\alpha}\right).$$

317 Mutch & Lowe (2008) proposed to set s to unit value and the normalization factor α to

318 $(q/4)^2$, with q set to the smallest possible value. The convolution operation eventually

319 generates $N \times (sc - 1)$ different maps.

320

321 *Layer C2.*– $C2$ pools values of $M_{\sigma,n}^{S2}$ feature maps to gain global invariance to position and

322 scale. For each n max pooling is applied across all positions and scales to produce a

323 vector $\vec{M}^{C2} \in \mathbb{R}^N$, the output of HMAX.

324

325 *sparse-HMAX*

326 Sparse coding can be added to HMAX at two different stages: during filter learning (e.g.,

327 Hu *et al.* 2014) or during stimulus encoding. In computer vision these stages are clearly

328 distinct: it is possible to sparsely encode a stimulus with Gabor filters, or to encode a

329 stimulus with a simple linear convolution with filters learned by applying a sparseness

330 constraint. In *sparse-HMAX*, the user can choose to impose sparseness during filter

331 learning, during stimulus encoding in layer $S2$ (Figure 1b), or at both steps. In filter

332 learning, m square patches are first extracted from $M_{\theta,\sigma}^{C1}$ of training images. Each patch is

333 then transformed into a vector $\vec{\xi} \in \mathbb{R}^k$, and all $\vec{\xi}$ are h-concatenated into a matrix

334 $\mathbf{X} \in \mathbb{R}^{k \times m}$. \mathbf{X} is then unity-based normalized and zero-centred. Learning sparse filters
335 requires solving the following optimisation problem:

$$\underset{\mathbf{F}^{S2}, \mathbf{S}}{\operatorname{argmin}} \left(\frac{1}{2\sigma^2} \|\mathbf{X} - \mathbf{F}^{S2} \mathbf{S}\|^2 + \beta \sum \|\mathbf{S}\|_1 \right)$$
$$\text{s. t. } \|\vec{f}_n^{S2}\|^2 \leq 1, \forall i = 1, \dots, N$$

336 where $\mathbf{F}^{S2} \in \mathbb{R}^{k \times N}$ is a matrix of N vectorised filters $\vec{f}_n^{S2} \in \mathbb{R}^k$ (the set of filters that are
337 learned) and $\mathbf{S} \in \mathbb{R}^{N \times m}$ a matrix of filter weights. The left part of the problem, called the
338 reconstruction error, is a least square minimisation of the difference between observed
339 patches and patches predicted by a linear combination of filters. The right part of the
340 problem, called penalty function, imposes sparseness to \mathbf{S} . Here the penalty function is a
341 ℓ_1 -norm regularisation, which is notoriously good at generating sparse weights while
342 being robust to irrelevant features (Ng 2004). The regularisation parameter β
343 determines the relative importance given to maximising either the reconstruction
344 accuracy or the sparseness. To prevent \mathbf{F}^{S2} from having arbitrarily large values that
345 would lead to arbitrarily small values in \mathbf{S} , its columns \vec{f}_n^{S2} are constrained to have their
346 ℓ_2 -norm less than or equal to one. Since the problem is convex in \mathbf{F}^{S2} (while holding \mathbf{S}
347 fixed) and convex in \mathbf{S} (while holding \mathbf{F}^{S2} fixed) but not convex in the two
348 simultaneously, it is necessary to iteratively and alternatively minimize with respect to
349 \mathbf{F}^{S2} or \mathbf{S} while holding the other element fixed. Theoretically, this iterative optimisation
350 could be achieved using a metaheuristic evolutionary algorithm using operators such as
351 reproduction, mutation and recombination, in order to simulate an evolutionary
352 adaptation of the neuronal selectivity to environmental features. In practice, however,
353 the time needed to solve the problem would be unrealistically long, and several more
354 efficient algorithms have been developed. In our implementation of *sparse-HMAX*, we
355 used the *Fast Sparse Coding* algorithm (Lee *et al.* 2007), which derives and solves the

356 Lagrange dual to learn the filters and applies a feature-sign search to learn the weights
357 (for details, see Lee *et al.* 2007).

358 To sparsely encode a target stimulus, as in training, a matrix \mathbf{X} of m square
359 patches is extracted in a similar way from $M_{\theta,s}^{C1}$ maps of the target image I , then
360 normalised and centred, and eventually the feature-sign search algorithm is used to
361 learn \mathbf{S} using \mathbf{F}^{S2} , the set of filters learned previously. In *sparse*-HMAX, the output of $S2$ is
362 thus \mathbf{S} , which is equivalent to a single feature map indicating the activation of each filter
363 for all patches (and thus for all scales and orientations in $C1$). The output \vec{M}^{C2} of *sparse*-
364 HMAX is eventually given by

$$\vec{M}^{C2} = \max_j |\mathbf{S}_{N,m}|, j = 1, \dots, m,$$

365 that is, by extracting the maximal activation of each filter over all patches.

366

367 *Modelling colour opponency*

368 Classical models of HMAX have been developed to process visual stimuli in the
369 luminance channel only. However, Zhang, Barhomi & Serre (2012) recently developed
370 coloured feature maps that model single- and double-opponent cells in primate V1, and
371 proposed an implementation of these maps into HMAX (see also Mély & Serre 2017). In
372 the following, we present the *SO-DO* feature maps of Zhang, Barhomi & Serre (2012)
373 generalized to any opponent function and any number of photoreceptor types used in
374 colour vision (e.g., to dichromatic mammals or to tetrachromatic birds; Figure 1c). Let
375 set $I \in \mathbb{R}^{h \times w \times npe}$ the input matrix of photoreceptor excitation maps $I_b \in \mathbb{R}^{h \times w}$,
376 with $b \in \mathbb{Z}^{npe}$. In Old World primates, $npe = 3$ with $b \in \{L, M, S\}$, which is often
377 approximated by $\{R, G, B\}$, the blue, green and red channels of a colour image. Moreover,
378 in some species like primates and fishes, the luminance channel is given by summing

379 two or more I_b (see *Layer SO*). In other species, e.g., in birds, a specific set of
 380 photoreceptors (the double cones) feeds the luminance channel. In this case, the
 381 photoreceptor excitation map of the luminance channel should be included as a specific
 382 I_b within I .

383

384 *Layer SO*.– The spatio-chromatic selectivity of *SO* cells is modelled using Gabor filters as
 385 in the non-colour HMAX. However, here one Gabor filter is decomposed into two filters:
 386 one keeping the positive part of the filter values only, to model an excitatory cell (other
 387 values are set to zero), the other one keeping the negative values to model an inhibitory
 388 cell. *SO* filters are thus defined by $F_{\theta^{SO},\sigma,*}^{SO} \in \mathbb{R}^{q \times q}$, with $\theta^{SO} \in \mathbb{R}^{t^{SO}}$, $\sigma \in \mathbb{R}^{sc}$ and
 389 $* \in \{\mathbb{R}^-, \mathbb{R}^+\}$ indicating the filter polarity (excitatory or inhibitory). Because *SO* cells are
 390 only weakly selective to orientation, Zhang, Barhomi & Serre (2012) set $t^{SO} = 2$ for
 391 humans, which likely also applies to other vertebrates. Having built the filter set \mathbf{F}^{SO} ,
 392 each filter is then convolved with each photoreceptor excitation map to generate a set
 393 \mathbf{M}^{SO} of $t^{SO} \times sc \times 2 \times npe$ feature maps $M_{\theta^{SO},\sigma,*,b}^{SO}$ such that

$$394 \quad M_{\theta^{SO},\sigma,*,b}^{SO} = \left| F_{\theta^{SO},\sigma,*}^{SO} * I_b \right|.$$

395 The set of single opponent and luminance maps $\mathbf{M}^{SO'}$ containing $t^{SO} \times sc \times 2 \times nch$ maps
 396 $M_{\theta^{SO},\sigma,ch}^{SO'}$ with $ch \in \mathbb{Z}^{nch}$ are then obtained by linearly combining $M_{\theta^{SO},\sigma,*,b}^{SO}$ such that

$$\mathbf{M}^{SO'} = \mathbf{\Omega} \mathbf{M}^{SO}$$

397 with $\mathbf{\Omega} \in \mathbb{R}^{npe \times nch}$ is a matrix of weights in which columns define the single opponent
 398 and luminance functions. For example, in Old World primates,

399

$$\mathbf{\Omega} = \begin{bmatrix} \pm 1 & \mp 1 & \pm 1 \\ \mp 1 & \mp 1 & \pm 1 \\ 0 & \pm 2 & \pm 1 \end{bmatrix}$$

400 thereby producing, for each orientation θ^{SO} and scale σ , four opponent colour channels:
 401 $L+/M-$ (red excitatory and green inhibitory), $M+/L-$, $S+/(M+L)-$, $(M+L)+/S-$; and two
 402 luminance channels: $L+M+S,-L-M-S$.

403 Each $M_{\theta^{SO},\sigma,*,ch}^{SO'}$ map is then rectified by half-squaring to maintain positive firing
 404 rate (Heeger 1992; Zhang, Barhomi & Serre 2012):

$$M_{\theta^{SO},\sigma,*,b}^{SO'} = \begin{cases} 0, \forall M_{\theta^{SO},\sigma,*,b}^{SO'} < 0 \\ \left(M_{\theta^{SO},\sigma,*,b}^{SO'}\right)^2, \forall M_{\theta^{SO},\sigma,*,b}^{SO'} > 0. \end{cases}$$

405 The last step of layer SO is a divisive normalisation that provides tolerance to small light
 406 intensity scaling:

$$M_{\theta^{SO},\sigma,*,ch}^{SO''} = \sqrt{\left(\frac{k' \times M_{\theta^{SO},\sigma,*,ch}^{SO'}}{\sigma'^2 + \sum_{*,ch} M_{\theta^{SO},\sigma,*,ch}^{SO'}}\right)}.$$

407 Note that in this layer normalisation is performed over maps of all (excitatory and
 408 inhibitory) channels at a given θ^{SO} and σ . Based on neurophysiological data from
 409 macaque, Zhang, Barhomi & Serre (2012) fixed the scale k' and semi-saturation σ'
 410 parameters to 1 and 0.225, respectively.

411

412 *Layer DO.*—Each map of SO is first convolved with the classical Gabor filters $F_{\theta,\sigma}^{S1}$ (thus not
 413 dissociating excitatory and inhibitory subunits), which produces $t^{SO} \times sc \times 2 \times nch \times t$
 414 maps $M_{\theta,\sigma,*,ch}^{DO}$ of \mathbf{M}^{DO} :

$$415 \quad M_{\theta,\sigma,*,ch}^{DO} = |F_{\theta,\sigma}^{S1} * M_{\theta,\sigma,*,ch}^{SO''}|.$$

416 Each $M_{\theta,\sigma,*,ch}^{DO}$ map is then rectified by half-squaring as in SO , to produce $M_{\theta,\sigma,*,ch}^{DO'}$. A
 417 normalisation is further applied, but over orientations (instead of channels in SO):

$$418 \quad M_{\theta,\sigma,*,ch}^{DO''} = \sqrt{\left(\frac{k' \times M_{\theta,\sigma,*,ch}^{DO'}}{\sigma'^2 + \sum_{\theta} M_{\theta,\sigma,*,ch}^{DO'}}\right)}.$$

419 Last, feature maps $M_{\theta,\sigma,*,ch}^{DO''}$ with complementary channels (excitatory and inhibitory) are
420 summed in order to make DO cells insensitive to figure-ground reversal:

$$M_{\theta,\sigma,ch}^{DO'''} = \sum_* M_{\theta,\sigma,*,ch}^{DO''} .$$

421 From there, all of the $t \times sc \times nch$ maps of $\mathbf{M}^{DO''}$ are processed individually by $C1$, $S2$ and
422 $C2$, and all \vec{M}_{ch}^{C2} are concatenated to produce $\vec{M}^{C2} \in \mathbb{R}^{N \times nch}$, the final output of the HMAX
423 and *sparse*-HMAX.

424

425 **Example: estimating facial (dis)similarity in mandrills**

426 HMAX can be used to estimate the perceived resemblance between entire phenotypes.

427 To illustrate this, we computed the distance between vectors $C2$ encoding faces of

428 mandrills (*Mandrillus sphinx*) and compared the results with a method based on

429 manually-positioned landmarks that is widely used in primatology (Dal Martello &

430 Maloney 2006; Bower, Suomi & Paukner 2012). In addition, we compared HMAX to

431 another computer vision approach based on the number of shared, automatically

432 detected features, which has been previously used in evolutionary ecology (Stoddard,

433 Kilner & Town 2014; SURF method hereafter). We analysed 100 pictures of mandrill

434 faces depicting 75 different individuals. Twenty-six individuals were represented by

435 more than one portrait, taken between 6 and 36 months intervals. For each of the four

436 methods (landmark, SURF, classic HMAX and *sparse*-HMAX), we estimated the similarity

437 between all pairs of portraits (for details, see Supporting Information), we ordered pairs

438 by ascending value of similarity and summed the rank of pairs corresponding to

439 different pictures of the same individuals. The higher the rank sum, the best the method

440 to assign high similarity to same-individual portrait pictures. For all three methods, the

441 rank sum was higher than expected by chance (95% limit of a null distribution),

442 indicating that all methods recognized that same-individual pictures were more similar
443 than different-individual pictures (Table 2). The landmark and SURF methods reached
444 similar performance, but both methods were surpassed by HMAX. Performance of
445 *sparse*-HMAX was lower than that of HMAX but higher than that of landmark and SURF
446 methods.

447
448 **Table 2.** Results of the facial similarity analysis. The standardized (Sd) rank sum
449 corresponds to the rank sum divided by that of an ideal observer who would give the
450 highest rank to all same-individual pairs.

Method	Rank Sum ($\times 1e^3$)	Sd rank sum	<i>p</i> -value
95% limit	84.4	0.59	0.05
Landmark	99.5	0.70	1e-4
SURF	100.2	0.70	4e-5
HMAX	111.3	0.78	<1e-6
<i>sparse</i> -HMAX	104.9	0.73	<1e-6
Ideal observer	143.1	1	<1e-6

451
452
453 **Discussion**
454 With the increasing need to study colour patterns as animals perceive them, HMAX
455 offers a useful framework applicable to a wide array of vertebrate species. We provide a
456 flexible and fast (See performance tests in Supporting Information) implementation of
457 HMAX that can apply to RGB images, but also to the photoreceptor excitation spaces of
458 other vertebrates. Furthermore, photoreceptor excitations can be combined into any
459 opponent coding scheme. Knowing the visual acuity of a studied species (for a review,

460 see Caves, Brandley & Johnsen 2018), it should be possible to further set the scale
461 parameter of Gabor filters to realistically map the visual resolution of details in colour
462 patterns. Even without specific knowledge about the visual system of a species, HMAX
463 would still be a valuable tool to analyse the robustness of results (e.g., measures of
464 phenotypic similarity) to deviations from hypotheses about the visual processes; e.g.,
465 the uniform distribution of orientation-selective neurons, or the adaptation of neurons
466 in early cortical/tecto-isthmic areas to features of the environment.

467 HMAX also allows analysing communication signals independently of their
468 rotation, distance and illumination. For visual ecology, this is a critical advantage over
469 the classical landmark-based methods because it allows working with non-standardized
470 images, e.g., pictures collected on the World Wide Web, thereby opening up the
471 possibility to analyse very large number of images or rare species represented by low-
472 quality photographs only. Furthermore, automatic analysis of features will save
473 considerable amount of time compared to positioning landmarks manually.

474 Over the last few years, computer scientists have gradually shifted away from
475 HMAX models to favour artificial neural networks with deeper architectures (i.e. deep
476 convolutional neural networks; ConvNets), reaching performances in object and
477 individual recognition that match and even surpass those of humans (LeCun, Bengio &
478 Hinton 2015). ConvNets have also recently gained the interest of visual scientists with
479 their ability to predict the selectivity of biological neurons in different brain areas
480 (Kriegeskorte 2015). However, HMAX models have two main advantages over ConvNets
481 that make them highly valuable for ecologists and evolutionary biologists. Contrary to
482 ConvNets, which require a huge amount of data for training (i.e. to learn filter
483 selectivities), *S2* filters in HMAX can be learned from a few images, and possibly a single
484 image. Moreover, contrary to ConvNets, with HMAX it is straightforward to visualize the

485 selectivity of neurons (i.e. filters) and to analyse which neurons are activated and which
486 are not. This is convenient, e.g., for revealing those features that most influence the
487 similarity between two phenotypes. HMAX models thus have high explanatory power,
488 which ConvNets still critically lack.

489 Compared to the classic HMAX, *sparse*-HMAX showed reduced performance both
490 for estimating facial similarity in mandrills and in performance tests (see SI). However,
491 our goal in proposing a sparse implementation of HMAX was not to maximize
492 performance but to make the model biologically more realistic. Furthermore, *sparse*-
493 HMAX provides a tool for estimating neuronal sparseness and thus efficiency in
494 information processing (Renoult & Mendelson 2019). A growing body of psychological
495 studies suggests that, when given a choice, humans tend to prefer stimuli that are
496 efficiently processed by the brain (Winkielman *et al.* 2003; Reber, Schwarz &
497 Winkielman 2004; Redies 2007). This finding is appealing for the field of evolutionary
498 biology as it could shed light on the mechanisms underlying the evolution of complex
499 and extravagant communication signals in animals (Renoult & Mendelson 2019).
500 Studying the efficiency of information processing outside laboratories of
501 neurophysiology and in non-model animal species, however, will require models such as
502 *sparse*-HMAX that quantify processing efficiency in animal brains. Visual ecology and
503 evolutionary biology are only beginning to embrace the benefits of methods developed
504 in computer vision and computational neuroscience. One aim of this article was to make
505 one step forward toward a better connection between these fields of research.

506 **Acknowledgements**

507 The authors thank Alexandre Faucher for preliminary studies on sparseness and HMAX.
508 This study is funded by National Science Foundation grant IOS-1708543, and by the
509 LABEX NUMEV, AGRO and CEMEB from University of Montpellier.

510

511 **Authors' contributions**

512 JPR and TM designed the study and wrote the first draft of the article; JPR performed the
513 facial similarity analyses; BG wrote the Matlab program including its documentation,
514 made performance tests and is managing the GitHub repository; JD, FG and FM
515 developed the *sparse*-HMAX algorithm; AP performed the landmark analysis; all authors
516 contributed to the final version of the manuscript.

517

518 **Data repository**

519 MATLAB codes and documentation available at
520 <https://github.com/EEVCOM-Montpellier/HMAX>

521

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