

1 **K-FIT: An accelerated kinetic parameterization algorithm using steady-state** 2 **fluxomic data**

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9 **Abstract**

10 Kinetic models predict the metabolic flows by directly linking metabolite concentrations and
11 enzyme levels to reaction fluxes. Robust parameterization of organism-level kinetic models that
12 faithfully reproduce the effect of different genetic or environmental perturbations remains an
13 open challenge due to the intractability of existing algorithms. This paper introduces K-FIT, an
14 accelerated kinetic parameterization workflow that leverages a novel decomposition approach to
15 identify steady-state fluxes in response to genetic perturbations followed by a gradient-based
16 update of kinetic parameters until predictions simultaneously agree with the fluxomic data in all
17 perturbed metabolic networks. The applicability of K-FIT to large-scale models is demonstrated
18 by parameterizing an expanded kinetic model for *E. coli* (307 reactions and 258 metabolites)
19 using fluxomic data from six mutants. The achieved thousand-fold speed-up afforded by K-FIT
20 over meta-heuristic approaches is transformational enabling follow-up robustness of inference
21 analyses and optimal design of experiments to inform metabolic engineering strategies.

22 Keywords: Kinetic models of metabolism; parameterization; *E. coli*; Metabolic engineering

23 **Introduction and Background**

24 The pressing need for the rapid development of truly predictive models of metabolism to
25 accelerate build-design-test cycles for metabolic engineering has been widely reported (Cheng
26 and Alper, 2014; Dromms and Styczynski, 2012; Long et al., 2015). Advances in synthetic
27 biology (Chae et al., 2017; Cho et al., 2018; Stovicek et al., 2017) have alleviated the challenge
28 of genome editing, placing the onus on the identification of suitable genetic modifications in
29 metabolic engineering projects. Kinetic models of metabolism can alleviate the limitations of
30 stoichiometric strain design techniques by quantitatively describing the relationship between
31 fluxes, enzyme levels, and metabolite concentrations based on mechanistic and/or approximate
32 rate law formalisms. The promise of superior product yield and production rate prediction
33 offered by kinetic models comes at the expense of substantially increased experimental data
34 requirements and complexity in model assembly, parameterization and interpretation of results.
35 Long parameterization times stemming from poor scalability of existing parameterization
36 frameworks ultimately preclude the deployment of follow up statistical analyses for large-scale
37 kinetic models limiting insights into (i) the robustness of resolution of kinetic parameters given
38 mutant flux datasets, (ii) kinetic parameter confidence levels, and (iii) the need for follow up
39 measurements to improve prediction. These challenges motivate the development of K-FIT, a
40 decomposition-based approach for parameterization of kinetic models using steady-state
41 fluxomic and/or metabolomic data collected for multiple perturbation mutants. K-FIT builds
42 upon the concept of Ensemble Modeling (EM) by anchoring concentrations and kinetic
43 parameters to a reference strain. Unlike earlier efforts that employed a genetic algorithm
44 (Khodayari et al., 2014) to parameterize kinetic models, K-FIT achieves three orders of
45 magnitude improvement in efficiency by relying on a customized decomposition approach to

46 compute steady-state fluxes in mutant networks. K-FIT was first benchmarked against EM for
47 three test kinetic models of increasing size ranging from 100 to 953 kinetic parameters to
48 demonstrate the increase in computational savings with model size. K-FIT remained tractable
49 even for a near genome-scale kinetic model containing 307 reactions, 258 metabolites, and 2,367
50 kinetic parameters parameterized with 1,728 steady-state fluxes from six single gene-deletion
51 mutants determined using ¹³C-metabolic flux analysis (¹³C-MFA) (Long et al., 2018). The
52 parameterization was carried out 100 times with random initializations and was completed within
53 48 hours of cumulative computation time. The best solution was recovered 44 out of 100 times
54 providing confidence that convergence to the true optimum was indeed achieved. The kinetic
55 model k-ecoli307 accurately recapitulated fluxes to within 15 mmol/gdw-h of the fluxes reported
56 by ¹³C-MFA while also predicting fluctuations in glucose uptake in response to genetic
57 perturbation and flux rerouting through energy metabolism to meet biosynthetic NADPH
58 demands. The yield predictions of acetate, lactate, and malate for engineered strains were found
59 to be within 30% of the experimental yield for metabolites derived from central metabolism. K-
60 FIT offers, with no additional effort, local kinetic parameter sensitivity information through the
61 computation of gradients allowing for optimality testing of all obtained solutions. This gradient
62 output also enables the straightforward computation of parametric uncertainty, establishment of
63 confidence intervals for kinetic parameters and derivation of metabolic control coefficients. This
64 post-parameterization information can aid in the interpretation of kinetic model predictions,
65 suggest efficient experiment design to improve prediction fidelity and ultimately inform
66 metabolic engineering strategies.

67 **Results**

68 In this section, first a schematic representation of the workflow of K-FIT is described. The
69 performance of K-FIT is benchmarked against the Ensemble Modeling (EM) (Khodayari et al.,
70 2014) using three test kinetic models to assess the impact of model scale-up on the
71 computational savings afforded by K-FIT. The applicability of K-FIT to near genome-scale
72 models is then demonstrated by parameterizing an expanded kinetic model for *E. coli* (k-
73 ecoli307) containing 307 reactions, 258 metabolites, and 2,367 kinetic parameters parameterized
74 using ¹³C-labeling data for ten proteinogenic amino acids from six single gene-deletion mutants
75 (Long et al., 2018). Local regression based standard deviations for all estimated elementary
76 kinetic and Michaelis-Menten parameters are computed by leveraging the gradient calculations
77 embedded within K-FIT. The predictive capability of k-ecoli307 is then assessed by comparing
78 predicted product yields against experimentally measured yields for six over-producing strains
79 not used during model parameterization.

80 **The K-FIT Algorithm**

81 K-FIT is a gradient-based kinetic parameter estimation algorithm using steady-state flux
82 measurements from multiple genetic perturbation mutants. The schematic workflow for K-FIT is
83 shown in Figure 1 and Supplementary Figure S1. Reaction fluxes are related to metabolite
84 concentrations using mass-action kinetics after decomposition of the enzyme catalytic
85 mechanism into elementary steps (see supplementary methods for the detailed procedure for
86 elementary step decomposition) as described by Tran *et al* (Tran et al., 2008). Conservation of
87 mass across enzyme complexes and metabolites is therefore expressed as a system of bilinear
88 equations. This formalism was chosen because it is mechanistically sound, obeys mass
89 conservation laws and is inherently thermodynamically feasible (Saa and Nielsen, 2017). At the
90 same time, it allows for easy integration of allosteric regulation without the need to derive

91 cumbersome case-specific nonlinear rate laws. The K-FIT algorithm iteratively applies the
92 following three steps till the optimal set of kinetic parameters is found: (i) K-SOLVE, (ii)
93 Steady-State Flux Evaluator (SSF-Evaluator) and (iii) K-UPDATE.

94 The objective of K-SOLVE is to anchor kinetic parameters to a reference state which is typically
95 the wild-type (WT) network similar to the procedure described by Tran *et al.* (Tran et al., 2008).
96 This is needed because for arbitrary assignment of values to the elementary kinetic parameters
97 enzyme and metabolite conservation balances are not always satisfied. K-SOLVE uses as input
98 the WT reverse fluxes of all elementary steps and the WT enzyme fractions. WT net fluxes are
99 treated differently than mutant net fluxes. While net fluxes from the mutant networks form the
100 sum of squares objective function to be minimized, net fluxes in the WT network are fixed in the
101 model to their experimentally resolved values. This ensures that the inferred kinetic parameters
102 remain consistent with both experimental WT flux measurements and inherently satisfy mass
103 conservation laws for both enzymes and metabolites. Consequently, the specified (i) reverse WT
104 elementary fluxes and (ii) WT enzyme fractions along with (iii) the specified net WT fluxes fix
105 all degrees of freedom in the system of equations. Unique values for all kinetic parameters can
106 thus be assigned so as they inherently satisfy mass balances across metabolites and enzymes in
107 the WT network. This is an important consideration as mass balances are not always satisfied
108 under metabolic steady-state for an arbitrary assignment of values to the kinetic parameters.

109 Kinetic parameters anchored by K-SOLVE are then used by SSF-Evaluator to compute the
110 steady-state fluxes and concentrations across all mutants, one at a time. The system of bilinear
111 algebraic equations in metabolite and enzyme fractions resulting from the elementary step
112 decomposition of enzyme catalysis is partitioned into two sub-problems. The first bilinear sub-
113 problem, representing conservation of mass across enzyme complexes, is reduced to a system of

114 linear algebraic equations in enzyme fractions in the mutant network when the metabolite
115 concentrations in the mutant network are specified. Similarly, the second bilinear sub-problem
116 describing conservation of mass across all metabolites reduces to a system of linear algebraic
117 equations in metabolite concentrations in the mutant network when the enzyme fractions in the
118 mutant network are specified. By iterating between these two linear sub-problems, the steady-
119 state enzyme levels and metabolite concentrations in each mutant is identified. SSF-Evaluator
120 begins with fixed-point iterations and switches to Newton steps closer to the solution. A semi-
121 implicit Euler integration step is carried out whenever the Jacobian matrix in the Newton step
122 becomes singular. Convergence is achieved when the concentrations of enzyme complexes and
123 metabolites remain almost unchanged between successive iterations. This iterative scheme
124 therefore enables the direct evaluation of steady-state fluxes (almost) without the need to
125 integrate any ODEs and contributes to the speed-up of the kinetic parameterization process.

126 The calculated fluxes for all mutants are compared against the corresponding measured fluxes
127 and the sum of squared residuals (SSR) representing the variance-weighted squared deviation of
128 predicted fluxes from experimental data, as well as the first- and second-order gradients are
129 computed by K-UPDATE. The WT reverse elementary fluxes and WT enzyme fractions are then
130 updated using a Newton step and the core loop of K-FIT is repeated by returning to K-SOLVE
131 until the minimum deviation of predicted fluxes from experimental measurements is reached.
132 Note that the computation of first- and second-order gradients uses as input the local sensitivity
133 of fluxes with respect to kinetic parameters. These local sensitivities can readily be computed by
134 solving a system of linear equations (see supplementary methods) as opposed to having to
135 perform costly forward sensitivity analysis. This enables K-FIT to confirm that any reported
136 solution is indeed optimal while also allowing for the assembly of the covariance matrix from

137 which approximate confidence intervals for all estimated kinetic parameters can efficiently be
138 derived.

139 **Benchmarking K-FIT against Ensemble Modeling**

140 The computational performance of the K-FIT algorithm was first compared against solution with
141 a genetic algorithm (GA) operating on a population of models constructed using the Ensemble
142 Modeling (EM) approach. Three test models of increasing sizes were used to assess the impact of
143 model size on parameter estimation speed and solution reproducibility. The first small model
144 containing 14 reactions, 11 metabolites, and 100 kinetic parameters was adapted from the three
145 glycolytic pathways in *E. coli* and was parameterized using flux distributions from four single
146 gene-deletion mutants (Supplementary Fig. S2a). The second medium-sized kinetic model
147 containing 33 reactions, 28 metabolites, and 235 kinetic parameters was adapted from a previous
148 study (Greene et al., 2017) and was parameterized using flux distributions from seven single
149 gene-deletion mutants (Supplementary Fig. S2b). The third test model describing carbon flows
150 through central and amino acid metabolism was adapted from the model developed by Foster *et*.
151 *al.*, (2019 (Under Review)). This model (Supplementary Fig. S2c) contains 108 reactions, 65
152 metabolites, and 953 kinetic parameters and was parameterized using flux distributions from
153 seven single gene-deletion mutants. The test models along with the data required for kinetic
154 parameterization is provided in supplementary tables ST1 – ST12.

155 Kinetic parameters were estimated in 9 minutes, 30 minutes, and 4 hours for the three models,
156 respectively, using K-FIT. In contrast, GA required 60 hours, 726 hours, and 4,278 hours,
157 respectively, to parameterize the same three models. Computational speed-up increased from
158 100-fold for the small model to 1000-fold for the core kinetic model upon switching from GA to
159 K-FIT. This dramatic reduction in parameterization time arises from the (largely) integration-free

160 steady-state flux evaluation using the SSF-Evaluator step and the fact that K-FIT traverses the
161 variable space in a highly economical manner (i.e., Newton steps) requiring fewer than 500
162 steady-state flux evaluations to identify the optimal solution. In contrast, the GA approach relies
163 on iterative recombination of kinetic parameter vectors and required as many as 20,000 steady-
164 state flux evaluations before finding the same solution (Khodayari et al., 2014) but without
165 confirming optimality. SSF-Evaluator evaluated steady-state fluxes, on average in 0.42 seconds,
166 1.13 seconds and 6.12 seconds, respectively, for the three models, whereas, numerical integration
167 required 3.5 seconds, 120 seconds and 440 seconds, respectively. Bypassing integration also
168 enables SSF-Evaluator to handle stiff systems of ODEs arising from the large dynamic range of
169 kinetic parameters and ensuring that steady-state fluxes are always within a mass imbalance of
170 just 0.001 mol%. It is important to note that Newton's method can only guarantee convergence
171 to a local and not necessarily the global minimum of SSR. As a safeguard against failure to reach
172 the true minimum, models were parametrized using K-FIT from 100 random initial starting
173 points to determine the reproducibility of the obtained best solution. For the three test models K-
174 FIT exhibited a best solution recovery of 98%, 93%, and 60%, respectively. This high solution
175 reproducibility provides confidence that K-FIT is able to consistently converge to the lowest
176 SSR of 0 for the small model, 8.9 for the medium-sized model, and 1.3 for the core model,
177 respectively. Notably, no alternate optima in the vicinity of the best solution (within an SSR of
178 100) was detected for any of the three models implying that the best SSR minimization solution
179 is the only good kinetic parameterization candidate. The inherent ability of K-FIT to quickly
180 calculate local sensitivities of predicted fluxes to metabolite concentrations was leveraged to
181 confirm whether SSF-Evaluator reported steady-state concentrations that are stable. To this end,
182 the eigenvalues of the Jacobian matrix (Greene et al., 2017) at metabolic steady-state were

183 calculated to confirm that the real part of all eigenvalues were strictly negative for all iterations
184 and problems solved.

185 **Parameterization of a kinetic model (k-ecoli307) for *E. coli* with near-genome-wide** 186 **coverage**

187 Following the application to three test models of increasing models, K-FIT was then deployed
188 for the parameterization of k-ecoli307, an *E. coli* kinetic model with near-genome-wide coverage
189 similar to k-ecoli457 (Khodayari and Maranas, 2016). The expanded model containing 307
190 reactions, 259 metabolites and 2,367 kinetic parameters, and encompasses central metabolism,
191 expanded amino acid, fatty acid, and nucleotide pathways and lumped pathways for
192 peptidoglycan biosynthesis. Compared to k-ecoli457 (Khodayari and Maranas, 2016), k-ecoli307
193 lacks the pathways for anaerobic metabolism and secretion of organic acids as it was
194 parameterized using data under aerobic growth only. Flux data for six single gene-deletion
195 mutants were computed using ¹³C-Metabolic Flux Analysis (¹³C-MFA) to recapitulate the
196 measured labeling distribution of 10 proteinogenic amino acids grown with 1,2-¹³C-glucose as
197 the carbon tracer (Long et al., 2018). This provided a total of 1,728 MFA-determined fluxes
198 across six single gene-deletion mutants from upper glycolysis for kinetic parameterization. All
199 69 substrate level regulatory interactions for 26 reactions in the expanded model were transferred
200 from k-ecoli457. Complete cofactor balances were not included in k-ecoli307. Instead, k-
201 ecoli307 only accounts for the net moles of ATP, NADH and NADPH produced or consumed in
202 any reaction and disregards the recycling of ADP, AMP, NAD and NADP back to the
203 phosphorylated/reduced form of the cofactor. This simplification is necessary to avoid metabolite
204 pool dependencies in the stoichiometry matrix. These dependencies force the total concentration
205 of (ATP + ADP + AMP), (NAD + NADH), and (NADP + NADPH) to be held constant across

206 all mutants (Vallabhajosyula et al., 2006) while allowing only the ratios of metabolite
207 concentrations ATP/ADP, ADP/AMP, NAD/NADH and NADP/NADPH to vary across mutants
208 (Greene et al., 2017).

209 The expanded model was parameterized using flux distributions from the six mutants Δpgi ,
210 Δgnd , Δzwf , Δeda , Δedd , and Δfbp with 288 fitted fluxes per mutant. 100 fluxes were inferred
211 using ^{13}C labeling data whereas 207 reactions were growth-coupled. Parameterization using the
212 K-FIT algorithm was completed in 48 hours on an Intel-i7 (4-core processor, 2.6GHz, 12GB
213 RAM) computer with a minimum SSR of 131.05 and a solution reproducibility of 44%. The
214 recapitulation of the experimentally measured fluxes by K-FIT for the six mutants is shown in
215 Supplementary Figure S3. All predicted fluxes were within 15 mmol/gdw-h of their
216 corresponding flux reported by ^{13}C -MFA. This corresponds to a maximum deviation of only
217 10% from the experimentally determined fluxes. Flux distributions for Δeda , Δedd , and Δfbp
218 mutants were largely unchanged from WT (Supplementary Figures S3a, S3b, and S3c) alluding
219 to the dispensability of the corresponding genes. In contrast, carbon flux was significantly
220 rerouted in response to the knockout of *pgi*, *zwf*, and *gnd* genes. Glucose uptake remained similar
221 to WT for the Δzwf mutant but routed completely via the EMP pathway (Supplementary Figure
222 S3d). The non-oxidative pentose phosphate pathway (TKT and TAL reactions) operated in
223 reverse to generate ribose-5-phosphate for nucleotide biosynthesis. Glucose catabolism solely via
224 the EMP pathway increased acetate and biomass yields by 10%. The expanded model also
225 revealed that the loss of NADPH production via the oxidative pentose phosphate pathway was
226 compensated by a 90% increase in the flux through the transhydrogenase reaction in the Δzwf
227 strain (Supplementary table ST20). Glucose uptake for the Δgnd mutant was decreased by less
228 than 10% compared to WT (Supplementary Figure S3e). Δgnd was the only strain with a

229 measurable flux through the ED pathway by rerouting 24.9 mmol/gdw-h of flux through EDD
230 and EDA reactions is a ten-fold increased flux through the ED pathway relative to the WT
231 network. Similar to Δzwf , the reversal of flux through the non-oxidative pentose phosphate
232 pathway generated the required ribose-5-phosphate for nucleotide biosynthesis.

233 Of all the mutants, Δpgi involved the most significant flux rerouting relative to WT. Glucose
234 uptake was reduced by 75% compared to WT resulting in a 70% reduction in growth rate
235 (Supplementary Figure S3f). Flux redirection through the glyoxylate shunt and reduction of
236 acetate secretion improved carbon routing towards biomass precursors, thereby increasing the
237 biomass yield by 22% compared to WT. Interestingly, the ED pathway was found to carry only 2
238 mmol/gdw-h of flux with almost all of the carbon being metabolized via the pentose phosphate
239 pathway (Supplementary Figure S3f). In addition, an 80% reduction in flux through glycolysis
240 along with the absence of acetate secretion lowers overall glycolytic ATP production. This loss
241 is compensated by the reversal of flux through the transhydrogenase reaction relative to WT
242 which allows the oxidation of excess NADPH generated by the oxidative pentose phosphate
243 pathway (Supplementary Table ST20). *k-ecoli307* captured that non-competitive inhibition of
244 the EDA reaction by glyceraldehyde-3-phosphate limits flux through the ED pathway in all
245 strains but Δgnd . In the Δgnd mutant, a 37-fold increase in the concentration of 6-
246 phosphogluconate provided the necessary driving force to overcome this product inhibition,
247 thereby allowing a flux of 24.9 mmol/gdw-h through the ED pathway. In contrast, in mutant
248 Δpgi a two-fold increase in the concentration of glyceraldehyde-3-phosphate maintains the
249 inhibition on the ED pathway which could not be overcome by a 40% increase in the
250 concentration of 6-phosphogluconate (Hoque et al., 2011).

251 A total of 2,461 K_m and V_{max} values were subsequently assembled using the estimated
252 elementary kinetic parameters (Supplementary Table ST13). Contrary to conventional thought,
253 the total number of Michaelis-Menten parameters (i.e., K_m and V_{max}) exceeded the number of
254 elementary kinetic parameters (i.e., k_p). This is because the number of elementary kinetic
255 parameters per reaction increases linearly with the number of participating species (reactants,
256 products or regulators) (see section 1 in the Supplementary methods). However, the Michaelis-
257 Menten formalism requires parameters for every combination of species in the rate expression
258 yielding a quadratic increase in parameters (Cleland, 1963). For example, for an ordered bi-
259 substrate reaction $A + B \rightleftharpoons C + D$, ten elementary kinetic parameters are needed as described in
260 the Supplementary methods. Using the Michaelis-Menten formalism requires two V_{max} terms
261 (one for the forward reaction and one for the reverse reaction) and ten K_m terms (four K_m s for
262 individual metabolite concentrations $[A]$, $[B]$, $[C]$ and $[D]$ and six K_m s one for each pair $[A][B]$,
263 $[C][D]$, $[A][C]$, $[B][D]$ and triplet $[A][B][C]$ and $[B][C][D]$). Note that the Michaelis-Menten
264 formalism requires two fewer parameters than using elementary kinetics for a reversible, uni-uni
265 reaction (Cleland, 1963) and only one fewer parameter for a bi-uni or uni-bi reaction. However,
266 for bi-bi and higher-order reaction mechanisms, the Michaelis-Menten formalism always
267 requires more parameters than the elementary kinetics description (Cleland, 1963). These two
268 competing effects yield the relatively close number of elementary and MM parameters for k-
269 *ecoli307*. Nevertheless, the true number of Michaelis-Menten parameters would have been much
270 larger than the ones required for the elementary kinetics description if complete cofactor
271 metabolism, protons, water and phosphate groups were included in k-*ecoli307*. The larger
272 number of Michaelis-Menten parameters may explain the reported highly correlated Michaelis-
273 Menten parameters leading to multicollinearity (Heijnen and Verheijen, 2013).

274 At the optimal solution, the Hessian matrix computed by K-UPDATE represents the
275 inverse of the covariance matrix computed in linear regression (Wiechert et al., 1997). The
276 positive square root of the diagonal terms of this covariance matrix correspond to the standard
277 deviation, in the kinetic parameters. The relative uncertainty in parameter estimation is defined
278 as the ratio of standard deviation computed from the covariance matrix to the estimated
279 parameter value. The standard deviation conceptually quantifies the sensitivity of the SSR
280 objective function to local changes in the value of the inferred parameter. Large values for the
281 standard deviation imply insensitivity of the SSR objective function to changes in the value of
282 the inferred parameter. Note that this inference of standard deviation using linear regression is a
283 local approximation and generally should be viewed as an overestimate of the true uncertainty as
284 it captures neither covariances nor nonlinear effects for larger changes in parameter values.
285 Using this method, the standard deviation representing the uncertainty in estimation for WT
286 reverse elementary fluxes, enzyme fractions, and elementary kinetic parameters was computed.
287 All 1,129 reverse elementary fluxes in the WT network were resolved with a relative uncertainty
288 (σ_v/v_r) of less than 1%. Furthermore, the average standard deviation for the inferred enzyme
289 fractions (σ_e) was only 0.04 mol/mol-total enzyme. A total of 793 out of 1,238 enzyme fractions
290 had a standard deviation of less than 0.1 mol/mol-total enzyme and were resolved with high
291 precision (Supplementary Figure S4a). However, 208 enzyme fractions (mostly with near zero
292 values) were inferred with a standard deviation exceeding the corresponding parameter value
293 leading to a relative uncertainty (σ_e/e) exceeding 100% (Figure 2a) due to their small absolute
294 value. These enzyme fractions include fractional abundances corresponding to 126 free enzymes
295 and 19 inhibitor-bound enzyme complexes from the TCA cycle and aspartate metabolism, and 63
296 metabolite-bound enzyme complexes in the ED pathway, TCA cycle and aspartate metabolism.

297 The large relative uncertainty in inference for these enzyme fractions alludes to insufficient
298 fluxomic data for precise kinetic parameterization of these pathways. Uncertainty of inference
299 propagates from the enzyme fractions to the elementary kinetic parameters, leading to 393 out of
300 2,367 elementary kinetic parameters having a relative uncertainty (σ_{k_p}/k_p) exceeding 100%
301 (Figure 2b). Notably, 50 out of 69 kinetic parameters describing inhibition of enzyme catalysis
302 were resolved with a relative uncertainty of less than 10%. Well resolved inhibition kinetic
303 parameters include the inhibition of the oxidative pentose phosphate pathway by NADPH,
304 product inhibition of the ED pathway by glyceraldehyde-3-phosphate, and product inhibition of
305 cis-Aconitase by isocitrate. The narrow standard deviation also places a non-zero lower bound on
306 the 1σ -confidence interval for these inhibition constants implying the essentiality of regulatory
307 interactions in k-ecoli307 to explain the available experimental flux datasets.

308 Upon computing the uncertainty in estimation of Michaelis-Menten parameters, 231 of
309 the 570 V_{max} parameters from branched-chain amino acid and fatty acid biosynthetic pathways
310 were resolved with a relative uncertainty ($\sigma_{V_{max}}/V_{max}$) of less than 10% (Figure 2c). In contrast,
311 217 V_{max} parameters from nucleotide biosynthesis, aspartate metabolism, serine metabolism,
312 and the TCA cycle were resolved with a relative uncertainty exceeding 100%. Similarly, 893 of
313 1,891 K_m parameters from the EMP pathway, pentose phosphate pathway, TCA cycle and fatty
314 acid biosynthesis were resolved with a relative uncertainty (σ_{K_m}/K_m) of less than 10% (Figure
315 2d), whereas 633 K_m parameters from nucleotide biosynthesis, amino acid metabolism and the
316 glyoxylate shunt were resolved with a relative uncertainty exceeding 100%. As expected, the
317 estimation uncertainty for the elementary kinetic parameters propagated to K_m and V_{max} .
318 However, the fraction of unresolved Michaelis-Menten parameters was substantially higher
319 (33.4% of all K_m and 38.1% of all V_{max}) than the fraction of poorly resolved elementary kinetic

320 parameters (16.6%). The higher fraction of unresolved Michaelis-Menten parameters stems from
321 the fact that Michaelis-Menten parameters are assembled as a nonlinear combination of
322 elementary kinetic parameters due to which the high relative uncertainty for any one elementary
323 kinetic parameter can propagate to multiple Michaelis-Menten parameters. It is important to note
324 that the approach used for computing standard deviations is based on statistical analysis of linear
325 regression which only holds true in the neighborhood of the optimal solution. The analysis can
326 be confounded by the nonlinear behavior of the kinetic model upon moving further away from
327 the optimal solution. In addition, by looking at a single parameter at a time information of
328 covariances between parameters is lost and the inferred uncertainty significantly overestimates
329 the true parametric uncertainty. A more rigorous profile-likelihood method (Antoniewicz et al.,
330 2006) can be employed to account for the nonlinear structure of the kinetic model that generally
331 results in narrower confidence intervals as commonly seen in 13C-MFA.

332 The predictive capability of the model was evaluated by comparing the model prediction of
333 product yields in engineered strains with the corresponding experimental yield. The genetic
334 perturbation mutants considered for evaluation of predictive capability were not included in the
335 training dataset for kinetic parameterization. Of the six over-producing strains evaluated, the
336 kinetic model successfully predicted the yields of acetate, malate, and lactate to within 30% of
337 the reported experimental yield (Table 1). This indicates that the genetic perturbations in the
338 training dataset for parameterization and that the regulatory structure of the expanded kinetic
339 model is sufficient to explain the phenotypic response of *E. coli* to perturbations in the EMP
340 pathway. The yield predictions for acetate and malate were superior to those by k-ecoli457 due
341 to the fact that both the training dataset for parameterization of the expanded model and
342 cultivation of the engineered strains were at the same mid-exponential growth phase, whereas,

343 the training dataset for k-ecoli457 was generated during late exponential growth phase. The
344 transcriptomic and fluxomic differences between these two growth conditions limits the carbon
345 flux through acetate metabolism in the late exponential growth phase (Ishii et al., 2007). Unlike
346 predictions for core metabolism, product yields originating from peripheral metabolism were
347 poorly predicted by both models. This is because, growth-coupling of peripheral metabolic
348 pathways limits the flux through these pathways to less than 5 mmol/gdw-h in all mutants. The
349 limited redirection of fluxes within peripheral metabolism when compared to central metabolism
350 leads to insufficient fluxomic data for kinetic parameterization of peripheral metabolic pathways
351 and therefore, adversely impacts the prediction fidelity of both kinetic models.

352

353 **Discussion**

354 This manuscript details the development of K-FIT, an accelerated kinetic parameterization
355 algorithm based on steady-state fluxomic data. The K-FIT algorithm estimates kinetic parameters
356 by solving a nonlinear least-squares minimization problem to recapitulate experimentally
357 measured steady-state metabolite concentrations and fluxes using an iterative loop comprised of
358 three steps: K-SOLVE, SSF-Evaluator, and K-UPDATE. The computational savings afforded by
359 bypassing ODE integration improves parameterization speed of K-FIT by over three orders of
360 magnitude compared to the GA-based EM procedure for a core model of metabolism containing
361 953 kinetic parameters. These savings are likely to become even more pronounced for larger
362 models. These computational savings enable the evaluation of reproducibility of kinetic
363 parameterization within reasonable time even for large-scale models, which was previously not
364 possible using meta-heuristic methods. The parallelizable architecture of SSF-Evaluator

365 improves the scalability of the procedure while allowing compatibility with GPU-based
366 computing architectures which affords significant improvements in computation speed. The
367 iterative scheme presented in SSF-Evaluator is inherently numerically stable which allows it to
368 handle stiff systems of equations with ease while permitting reliable calculation of first- and
369 second-order gradients. Furthermore, the ability to calculate gradients enables local statistical
370 analysis of the inferred kinetic parameters.

371 The applicability of K-FIT to large-scale models was demonstrated using an expanded kinetic
372 model of *E. coli* containing 307 reactions, 258 metabolites, and 2,367 kinetic parameters,
373 parameterized using fluxes elucidated using 13C-MFA. In order to avoid any error propagation
374 arising from flux projection from simpler models, a recently developed two-step computational
375 pipeline (Foster et al., 2019 (Under Review)) was used for kinetic parameterization using 13C-
376 labeling data. First, fluxes were elucidated for the expanded model in the WT and six single
377 gene-deletion strains using 13C-MFA. The elucidated fluxes were then used to parameterize the
378 kinetic model corresponding to the same stoichiometric model. Although the expanded kinetic
379 model recapitulated the fluxes better than a core model for *E. coli*, product yield predictions in
380 engineered strains did not differ significantly compared to those predicted by the core model.
381 Compared to k-ecoli457, yield predictions for acetate and malate were better recapitulated by k-
382 ecoli307 because k-ecoli307 was parameterized using fluxomic data generated in the mid-
383 exponential growth phase. This similarity in growth conditions between the training dataset and
384 the engineered strains limits any proteome re-allocations that could lower the predictive
385 capabilities of parameterized kinetic models. Yield predictions for metabolites from peripheral
386 metabolism by both k-ecoli307 and k-ecoli457 differed from experimental yields by almost
387 100% (Table 1). This was traced back to insufficient fluxomic data from peripheral metabolism

388 across the considered mutants due to growth coupling. Since most amino acids are not
389 catabolized by *E. coli*, reliable parameterization of these pathways requires model expansion to
390 amino acid pool turnover by protein synthesis and degradation. Additional fluxomic and
391 metabolomic data from overproducing strains and auxotrophs will also help capture the link
392 between genetic perturbations and increased flux through peripheral metabolism as the WT strain
393 of *E. coli* does not secrete any amino acids during the mid-exponential growth phase.

394 A novel feature introduced by K-FIT is the rapid computation of local sensitivity of predicted
395 fluxes and metabolite concentrations to kinetic parameters, which enables easy computation of
396 the gradient and Hessian of the least-squares objective function for efficient traversal of the
397 feasible kinetic parameter space. Since the Hessian represents the inverse of the covariance
398 matrix in the neighborhood of the optimal solution, K-FIT is able to compute uncertainty in
399 parameterization estimation with no added cost. Interestingly, reverse elementary fluxes in the
400 WT strain were precisely estimated with a mean relative uncertainty of less than 1%. WT
401 enzyme fractions were also resolved with a mean standard deviation of 0.04 mol/mol-total
402 enzyme. However, the low abundance of 208 enzyme complexes relative to their corresponding
403 standard deviation leads to a relative uncertainty exceeding 100%. This large relative uncertainty
404 propagates to elementary kinetic parameters and Michaelis-Menten parameters due to nonlinear
405 mapping of WT enzyme fractions to kinetic parameters. It is important to note that uncertainty
406 analysis based on linear regression is accurate only near the optimal solution and a rigorous
407 analysis accounting for the nonlinear behavior of the kinetic model will be required to compute
408 the accurate confidence intervals further away from the optimal solution. Accurate confidence
409 intervals will provide insights into resolvability of kinetic parameters for the set of experimental
410 data and enable the identification of informative mutants (Zomorodi et al., 2013) and design of

411 experiments (Banga and Balsa-Canto, 2008) to pin down the poorly resolved kinetic parameters.
412 Furthermore, using accurate confidence intervals, additional insights into reaction reversibility
413 and importance of regulatory interactions can be gleaned. Currently, the statistical significance of
414 regulatory interactions can only be evaluated using frameworks such as SIMMER (Hackett et al.,
415 2016)

416 Overall, K-FIT highlights the data-demanding nature of the kinetic parameterization problem.
417 Although kinetic parameterization was performed using only steady-state flux data, steady-state
418 metabolite concentration data can also be included in the SSR objective function. In all studies
419 enzyme levels were assumed to remain the same in mutants as in WT with the exception of
420 enzymes that are associated with knock-out genes which were set to zero. Nevertheless, enzyme
421 levels in mutant strains can be pre-specified in K-FIT if the information is known *a priori*.
422 Ideally, one would want to integrate allosteric with transcriptional regulation so that the enzyme
423 concentrations in the mutant networks can be related to the altered metabolite concentrations
424 (Fuhrer et al., 2017). This would ultimately enable the integration of mutant network data
425 generated under both genetic and environmental perturbations and improve its predictive
426 capabilities. Furthermore, the local sensitivity of fluxes and metabolite concentrations with
427 respect to kinetic parameters directly map to elasticity coefficients used in metabolic control
428 analysis. They can thus be used to calculate flux and concentration control coefficients at
429 minimal additional cost to inform metabolic engineering strategies.

430 **Methods**

431 **Kinetic parameterization using K-FIT**

432 K-FIT is a gradient-based kinetic parameterization algorithm that minimizes the least-squares
433 objective function representing the weighted squared deviation between predicted and measured
434 steady-state metabolic fluxes (and possibly metabolite concentrations) across multiple genetic
435 perturbation mutants. The full mathematical description for the K-FIT algorithm is provided in
436 the supplementary methods. The least-squares NLP is solved using the Levenberg-Marquardt
437 algorithm (Madsen et al., 2004) in conjunction with the active-set method for enforcing linear
438 inequality constraints (Gill et al., 1984). K-FIT is encoded and implemented in MATLABTM and
439 run on an Intel-i7 (4-core processor, 2.6GHz, 12GB RAM) computer. K-FIT is tested using
440 kinetic models at different size scales. The full source-code is made available on GitHub.

441 Computation of standard deviations for estimated kinetic parameters was performed using linear
442 regression tools applied to a local linearization of mutant fluxes using Taylor series expansion
443 (Wiechert et al., 1997). Briefly, the Covariance matrix \mathbf{C} is computed by inverting the Hessian \mathbf{H}
444 computed by K-UPDATE. When the linear approximation holds, the diagonal of the covariance
445 matrix represents the estimation variance of kinetic parameters. The approximate standard
446 deviation of kinetic parameter k_p (σ_p) is evaluated as $\sigma_{k_p} = \sqrt{C_{pp}}$. The approximate 1σ
447 confidence interval is computed as $k_p \pm \sigma_{k_p}$. Note that this method of analysis does not capture
448 covariance between parameters which is expected to significantly lower uncertainty.

449 **Construction of the expanded kinetic model of *E. coli*, k-ecoli307**

450 The expanded metabolic model is constructed by de-lumping the central and peripheral
451 metabolic pathways in the core model (Foster et al., 2019 (Under Review)) based on the reported
452 biomass composition (Neidhardt and Curtiss, 1996). The expanded model contains 307 reactions
453 and 258 metabolites. Atom mapping for the additional reactions were obtained from the

454 previously published genome-scale carbon mapping model for *E. coli* (Gopalakrishnan and
455 Maranas, 2015). The amino acid labeling data for flux elucidation was obtained from the
456 published work by Long *et al* (2018). Metabolic fluxes and 95% confidence intervals were
457 elucidated using ¹³C-metabolic flux analysis as described earlier (Antoniewicz *et al.*, 2006;
458 Gopalakrishnan and Maranas, 2015). The mechanism and allosteric regulation of enzyme-
459 catalyzed reactions in the model were obtained from k-ecoli457, the near-genome-scale kinetic
460 model for *E. coli* (Khodayari and Maranas, 2016). The standard deviation σ_j corresponding to
461 the estimated flux V_j to be used as a weighting factor in the K-FIT algorithm is computed from
462 the lower and upper bounds of the confidence interval V_j^{LB} and V_j^{UB} reported by ¹³C-MFA as
463 $\sigma_j = \frac{V_j^{UB} - V_j^{LB}}{3.92}$. The input files containing the model, reaction mechanism descriptions, and
464 experimental data used for kinetic parameterization are provided in Supplementary Tables ST13
465 – ST16. Computed kinetic parameters were also packaged into Michaelis-Menten parameters as
466 described earlier (Khodayari and Maranas, 2016). The estimated kinetic parameters and
467 Michaelis-Menten parameters are reported in Supplementary Tables ST21 and ST22.

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540 ensemble modeling of metabolic networks. *Biotechnol J* 8, 1090-1104.

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Figure 1: Overview of the core loop of the K-FIT algorithm

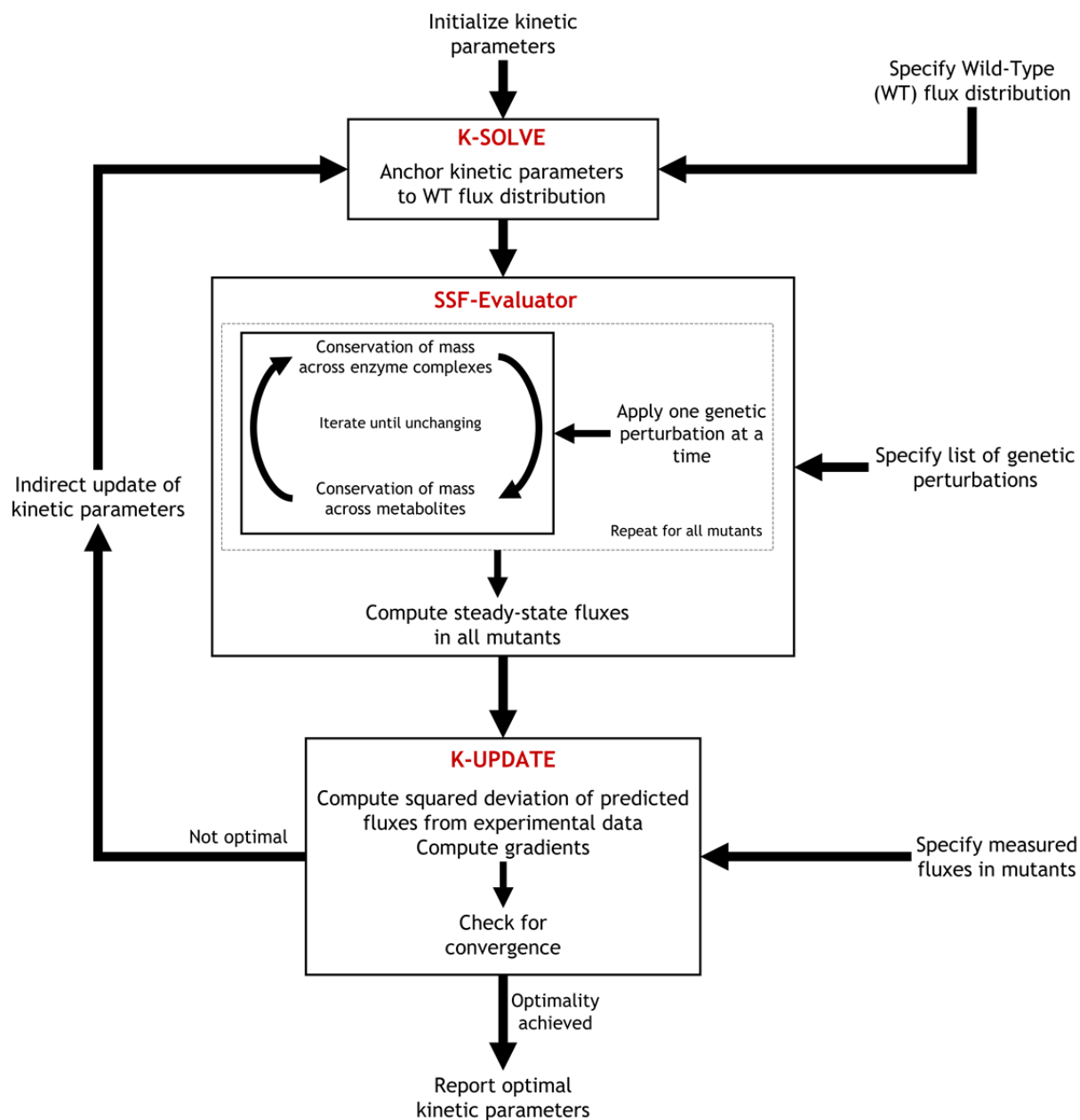
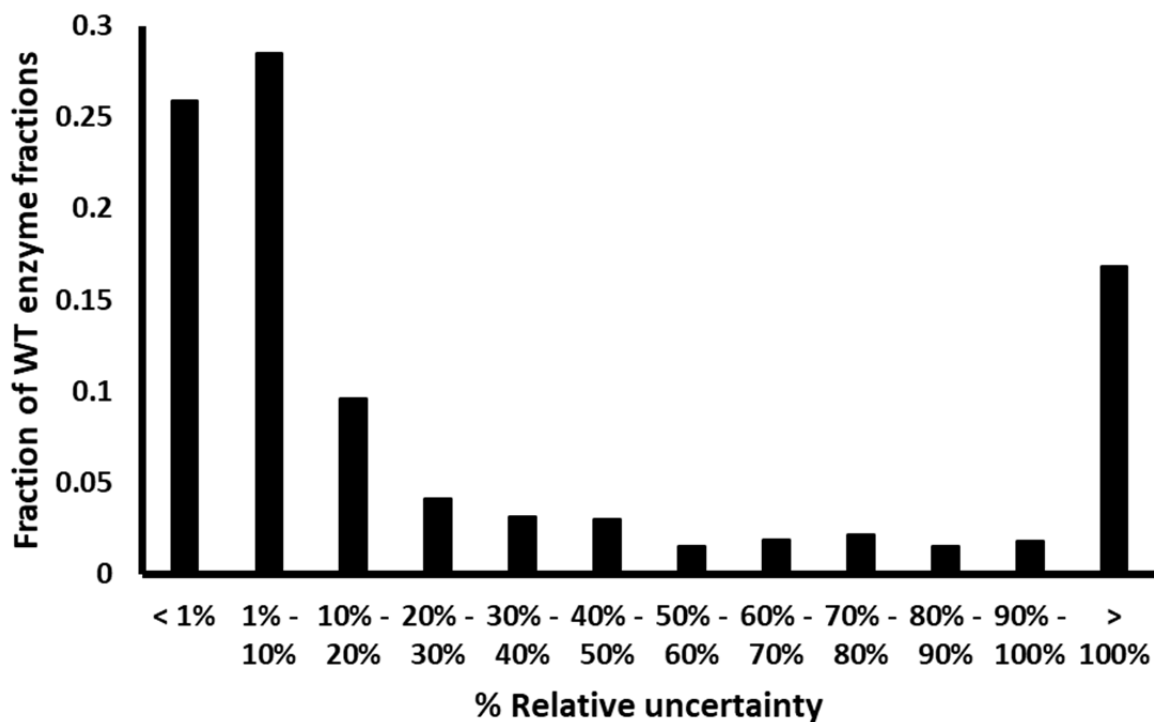
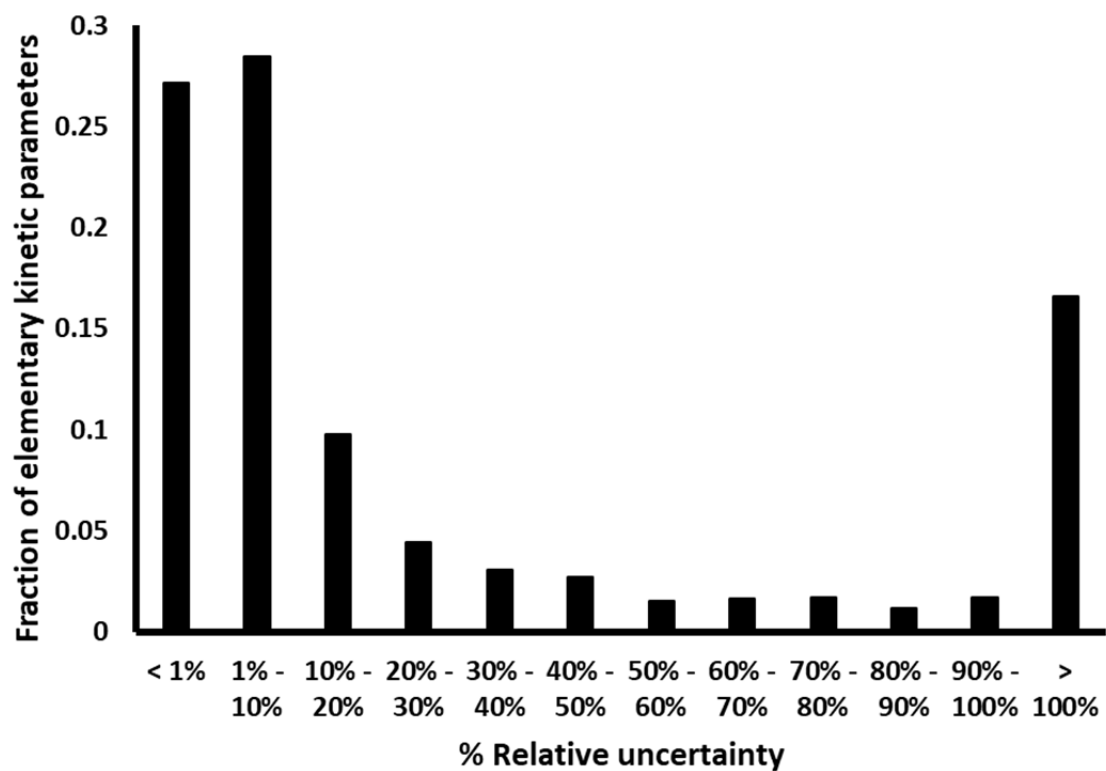


Figure 2: Distribution of relative uncertainty in estimation for (a) WT enzyme fractions, (b) elementary kinetic parameters, (c) V_{max} , and (d) K_m in k-ecoli307. (a) Distribution of values assumed by WT enzyme fractions (red bars) and standard deviations (green bars) for estimated enzyme fractions.

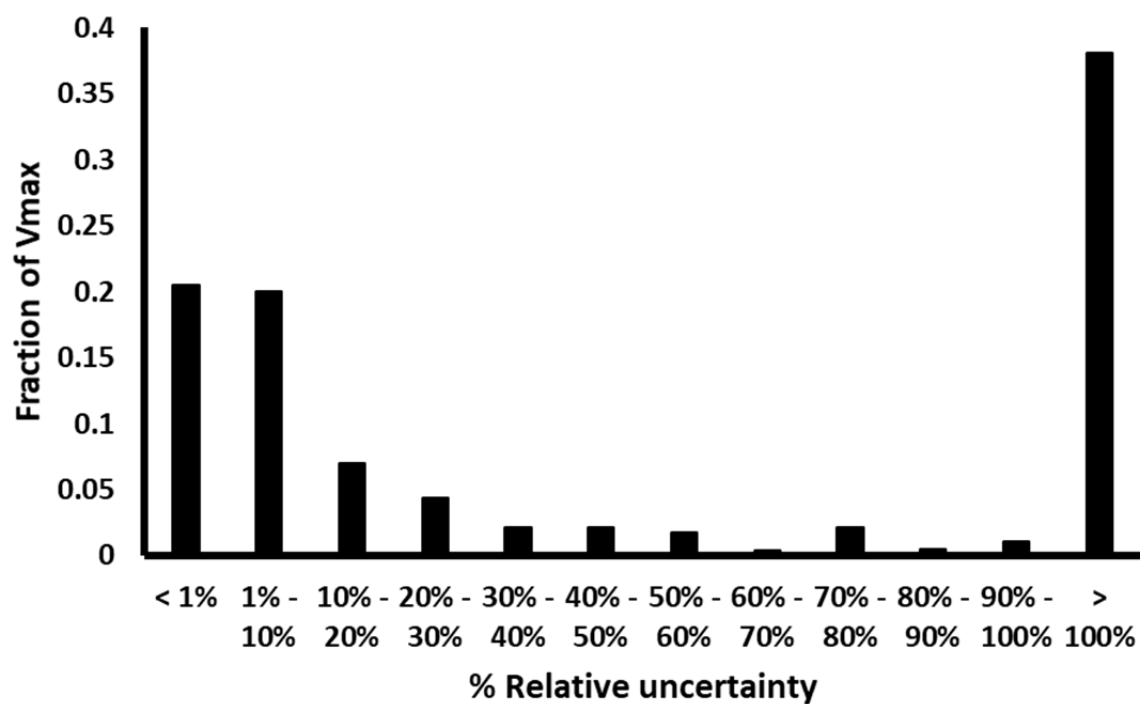
(a)



(b)



(c)



(d)

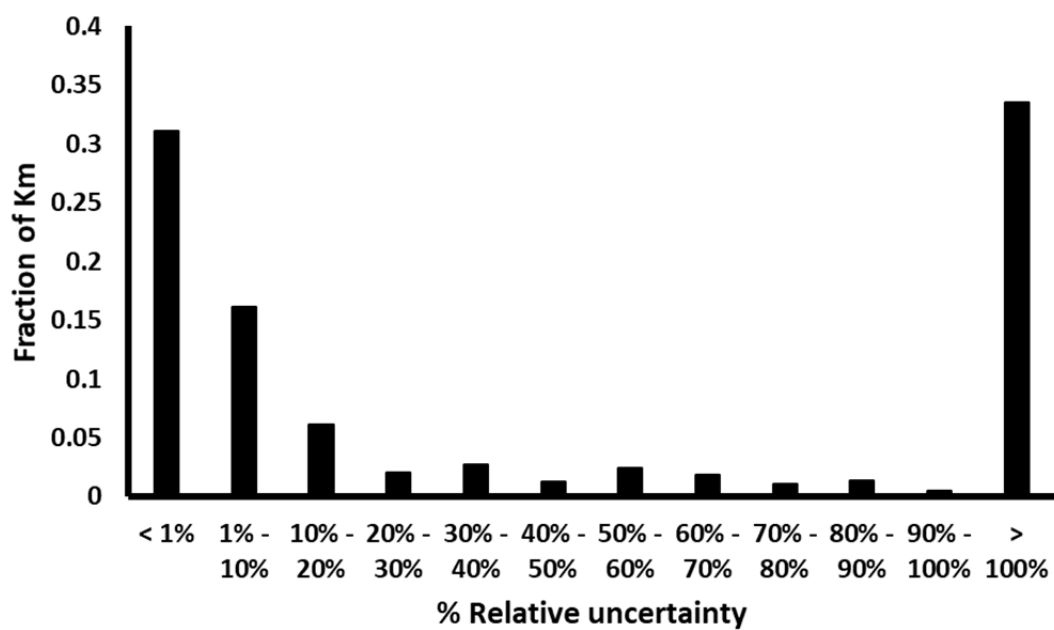


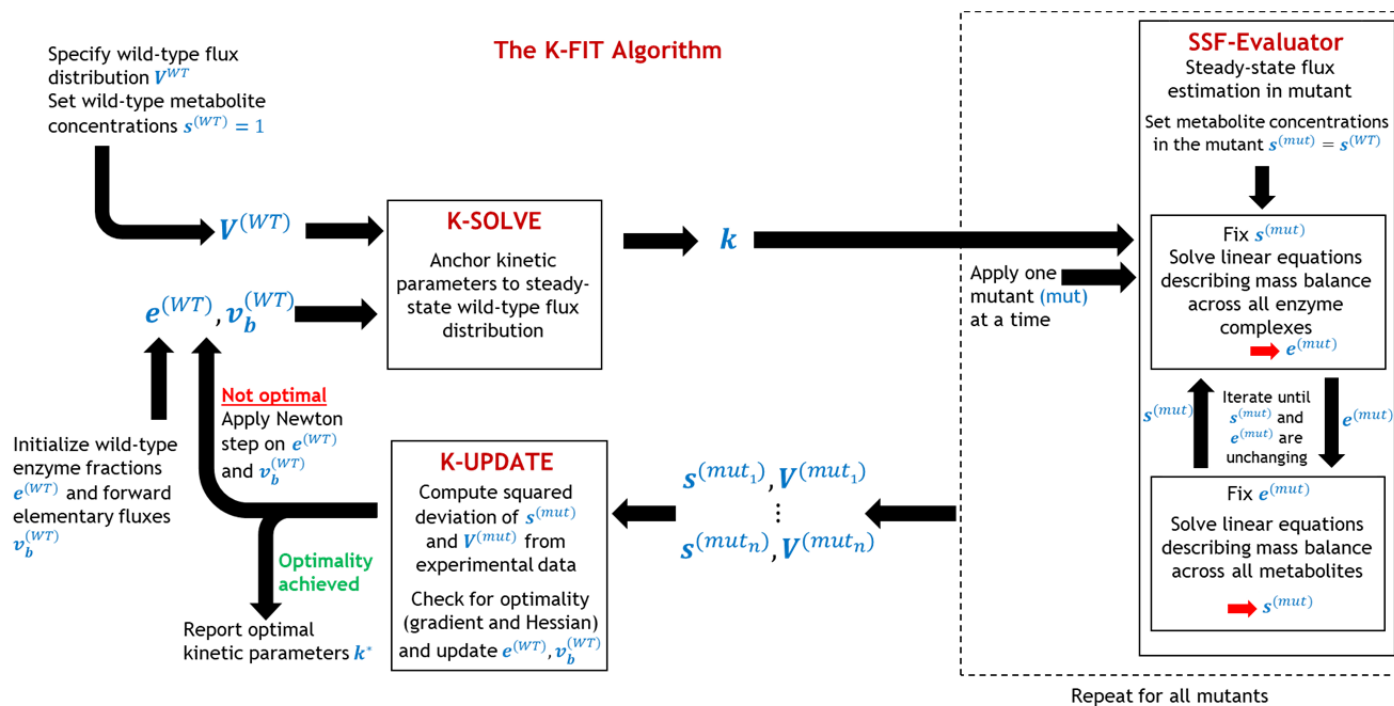
Table 1: Comparison of predicted product yields (mol/mol glucose) with experimental yields in engineered over-producing strains of *E. coli*. The experimental yields and predictions by k-ecoli457 were obtained from previously published data by Khodayari and Maranas (Khodayari and Maranas, 2016)

Product	Perturbed Enzyme	Predicted Yield	Predicted Yield (k-ecoli457)	Experimental Yield
Acetate	0.1x RPI	0.93	0.2	0.75
L-Valine	0.1x THRD	0.03	0.02	0.34
Lactate	0x ACKr	1.4	1.11	1.13
Malate	0.3x PTA; 10x PPCK	0.16	0.84	0.15
Artemisinin	2x PDH	0.17	0.03	0.38
Naringenin	2xACCOAC	0.026	0.012	0.008

References:

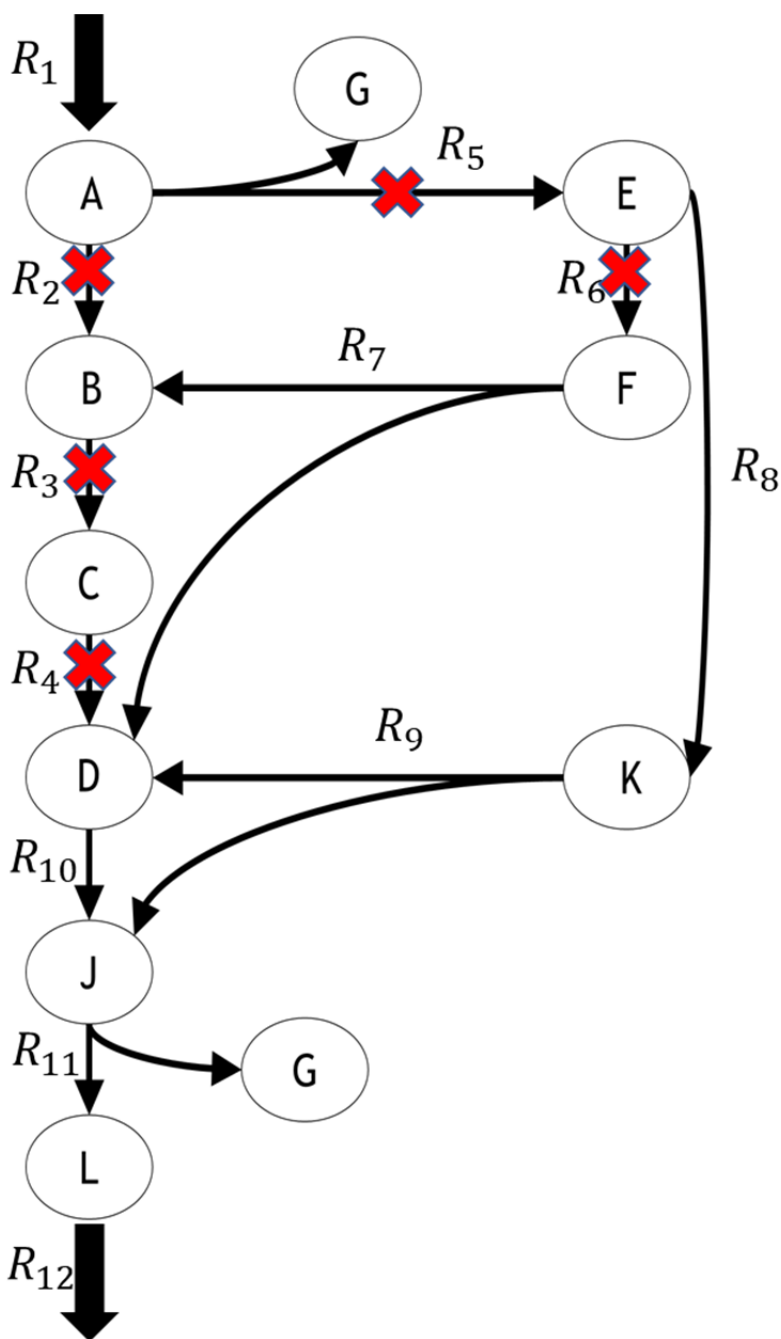
Khodayari, A., and Maranas, C.D. (2016). A genome-scale Escherichia coli kinetic metabolic model k-ecoli457 satisfying flux data for multiple mutant strains. *Nat Commun* 7, 13806.

Supplementary Figure S1: Overview of the K-FIT algorithm showing the flow of information between various components.

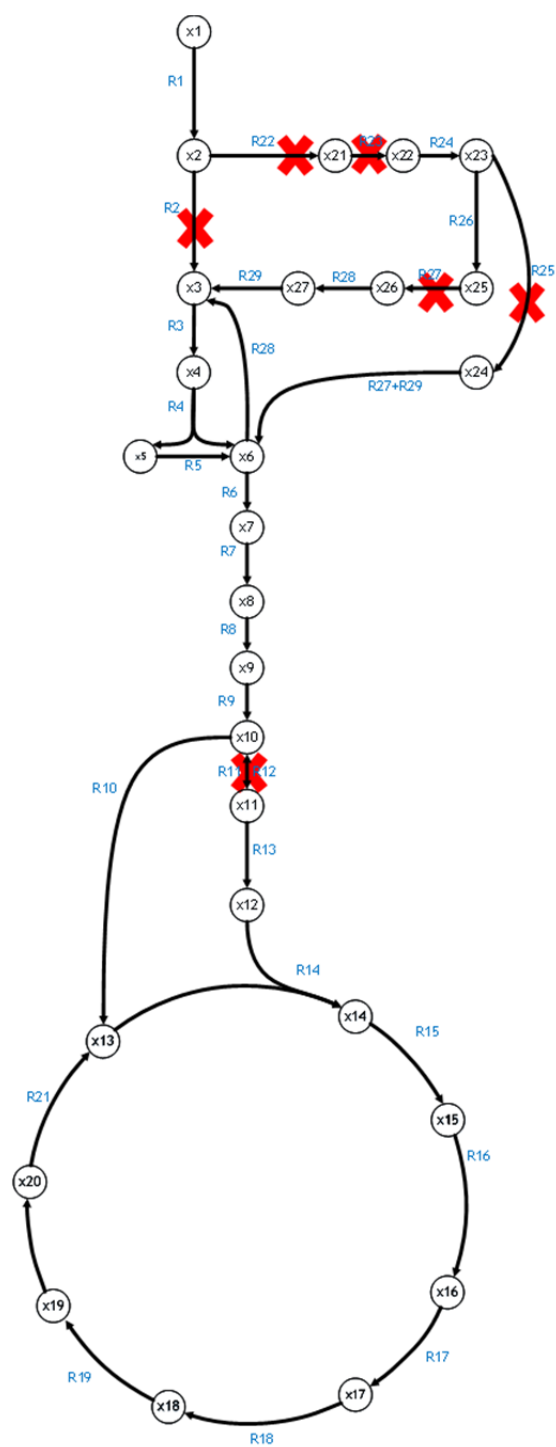


Supplementary Figure S2: Test models used to benchmark the performance of K-FIT against GA-based EM procedure. (a) Small model containing 14 reactions and 11 metabolites. (b) Medium-sized model containing 33 reactions and 28 metabolites. (c) Core model containing 108 reactions and 65 metabolites. Reactions knocked out in the single gene-deletion mutants are indicated using a red X.

(a)

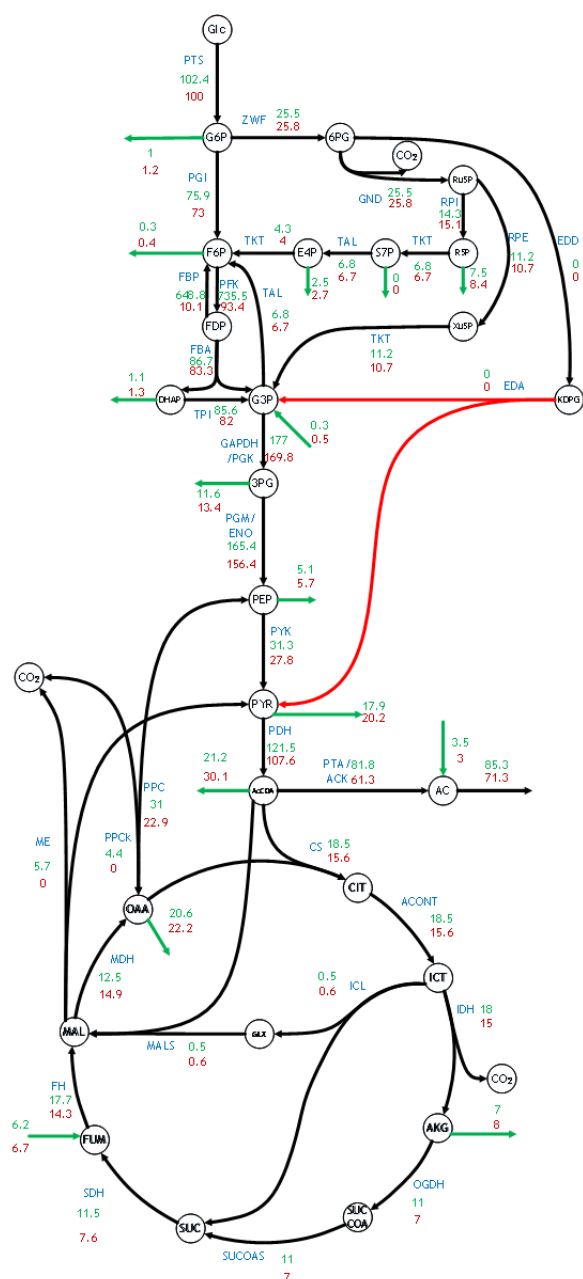


(b)

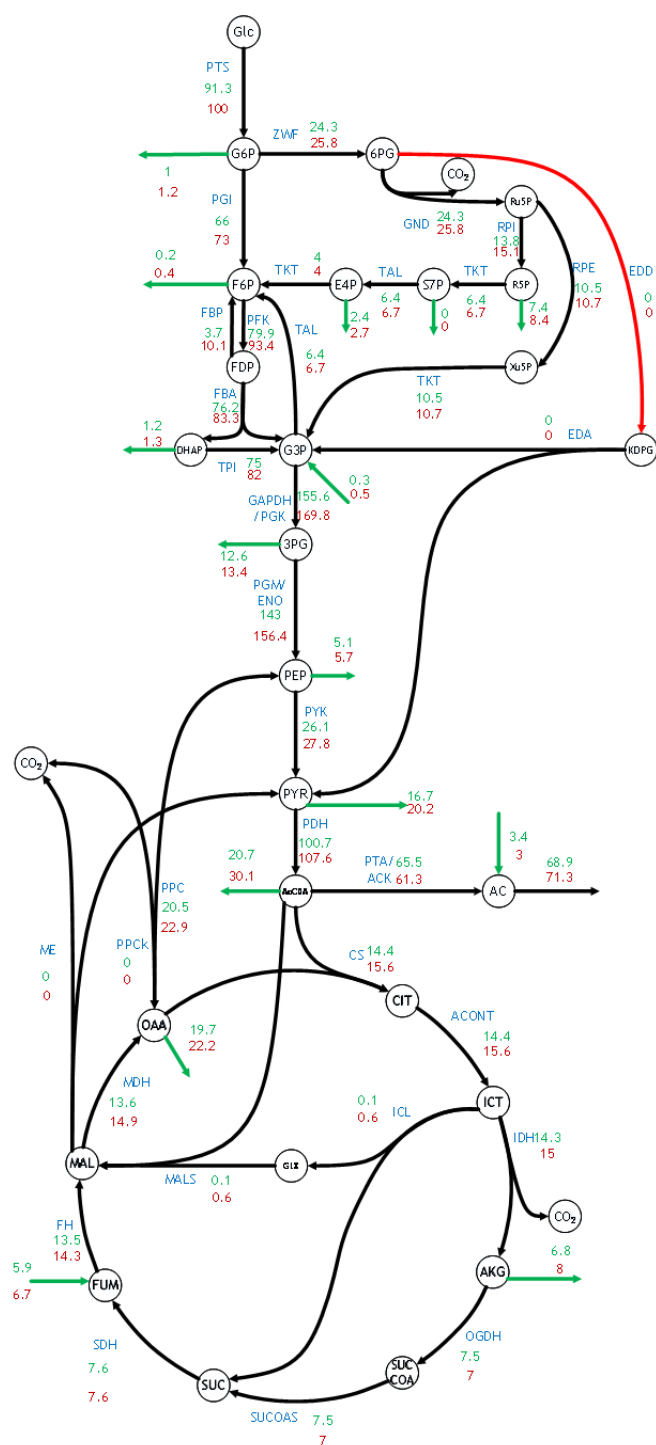


Supplementary Figure S3: Flux distribution through central metabolism of the expanded model for *E. coli* in (a) Δeda , (b) Δedd , (c) Δfbp , (d) Δzwf , (e) Δgnd , and (f) Δpgi mutant strains. Reactions representing metabolite flows between central and peripheral metabolism are indicated using green arrows. Fluxes elucidated using ^{13}C -MFA are shown in green and the corresponding flux prediction by the expanded kinetic model is shown in brown. Reactions corresponding to the knocked-out genes in each mutant strain are indicated using red arrows. Flux measurements for PFK and FBP were not fitted due to poor resolution ^{13}C -MFA

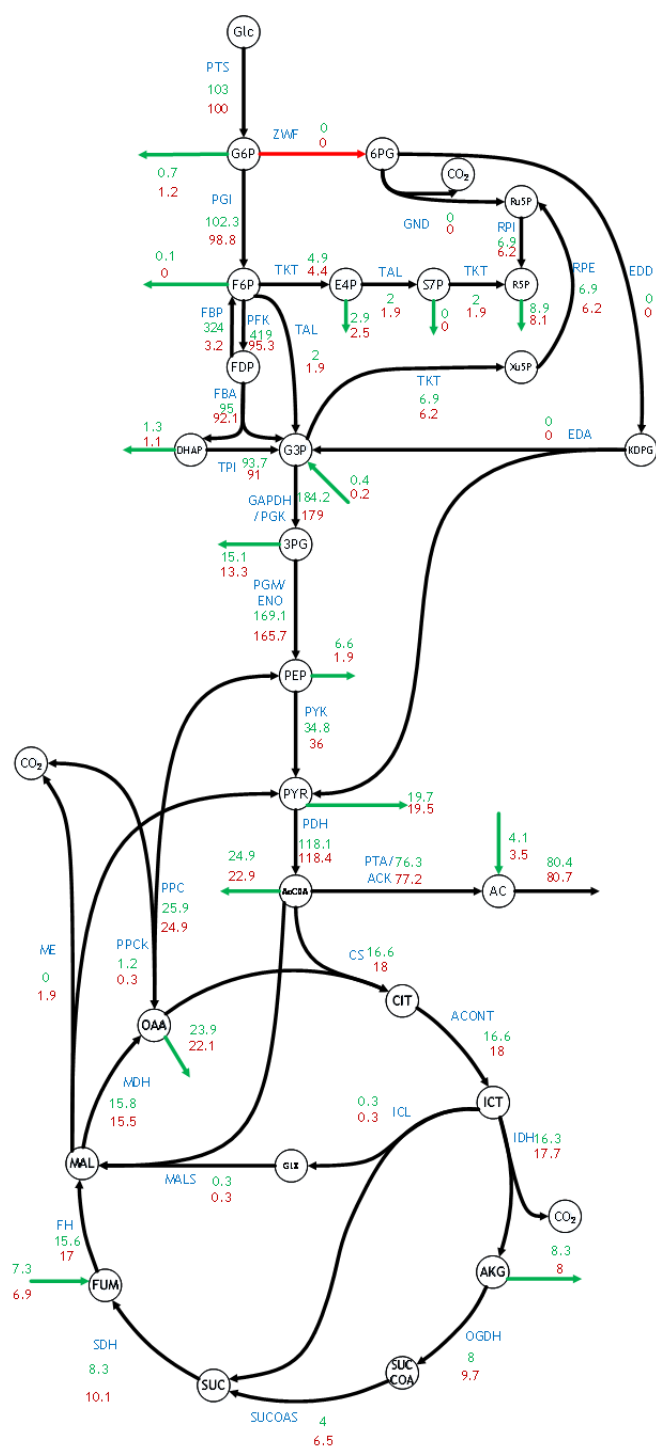
(a)



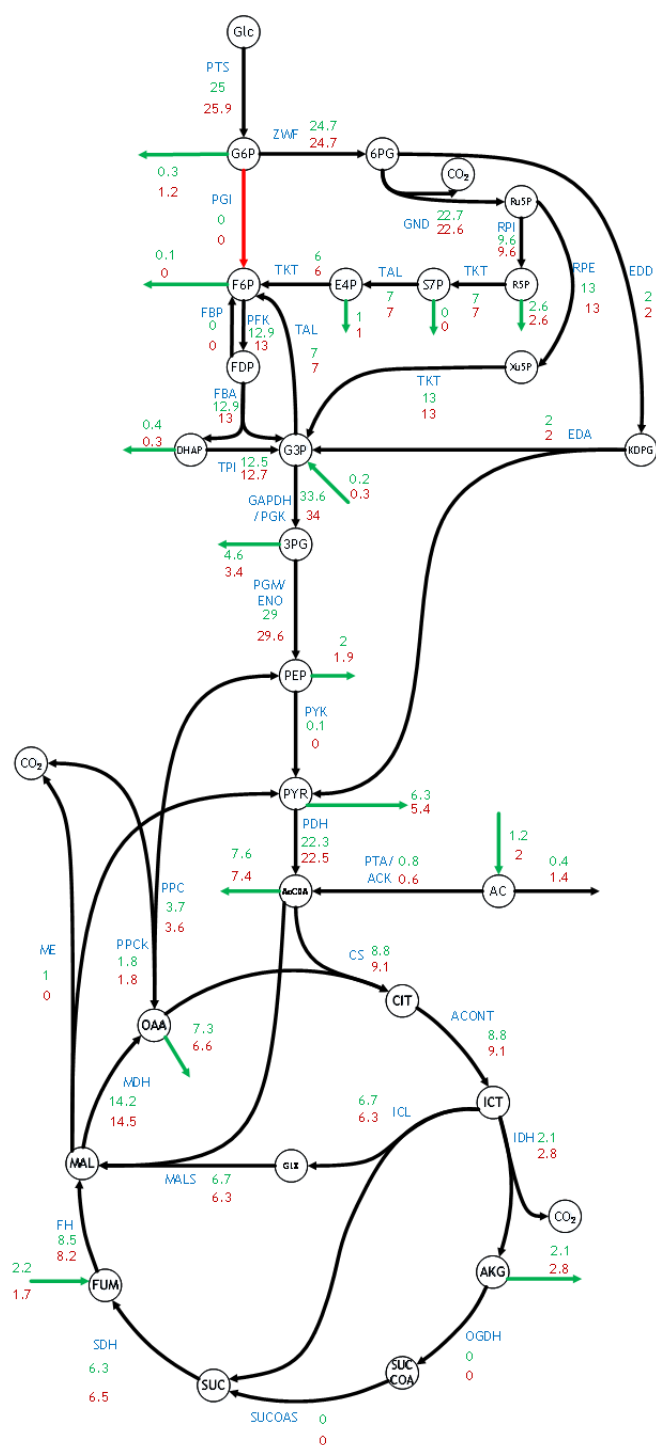
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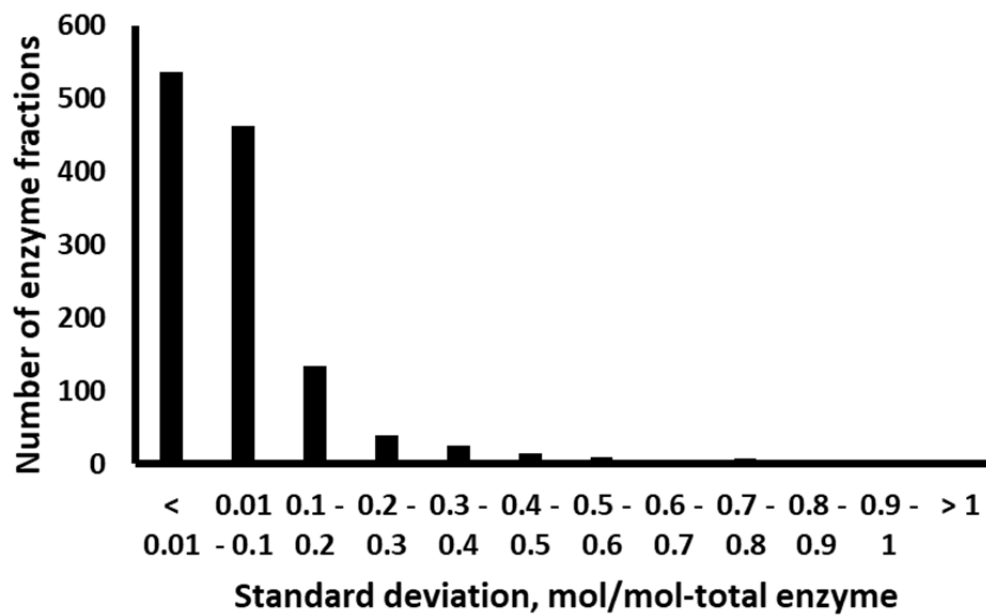


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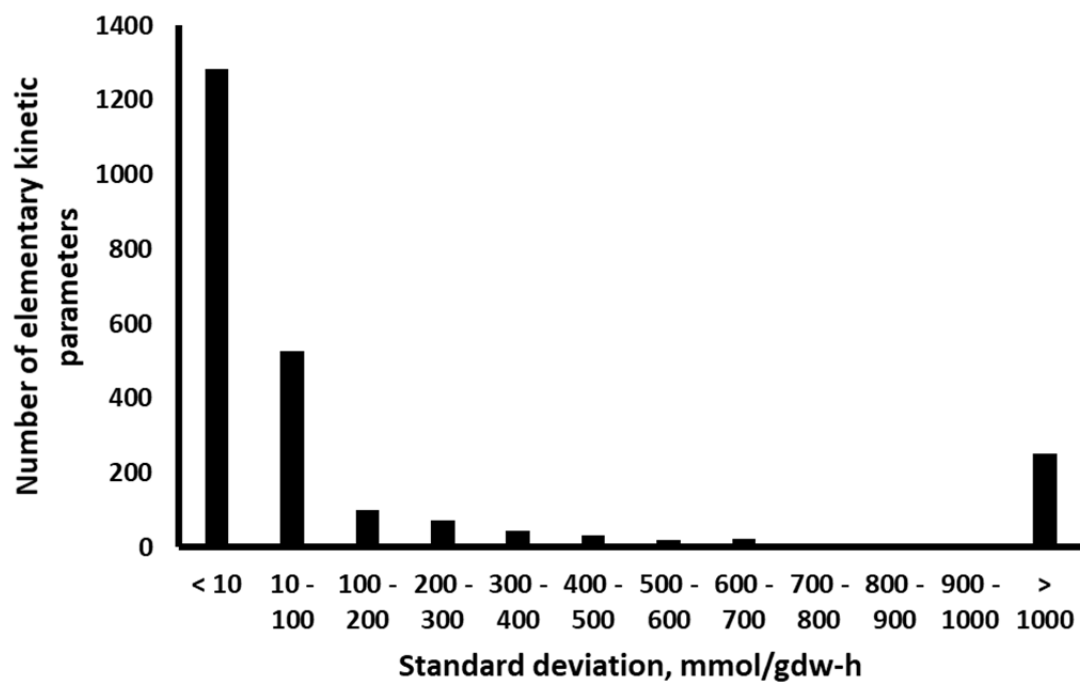


Supplementary Figure S4: Distribution of standard deviations for estimated (a) WT enzyme fractions, (b) elementary kinetic parameters, (c) K_m , and (d) V_{max} in k-ecoli307

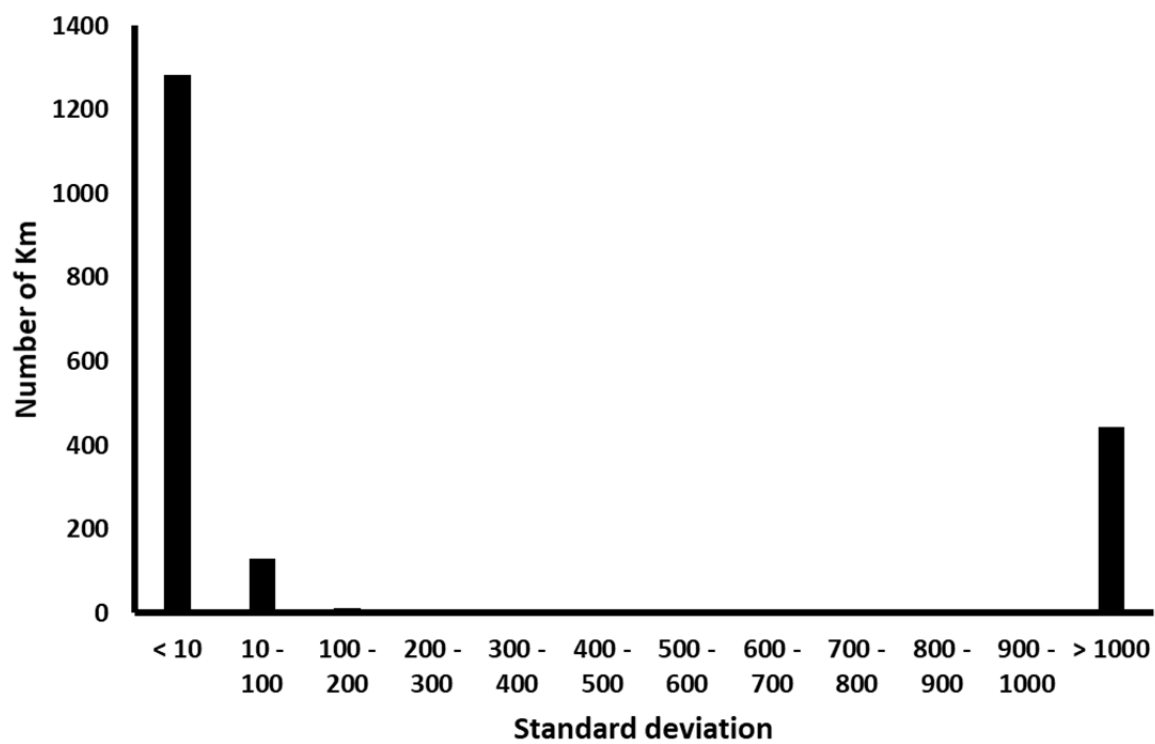
(a)



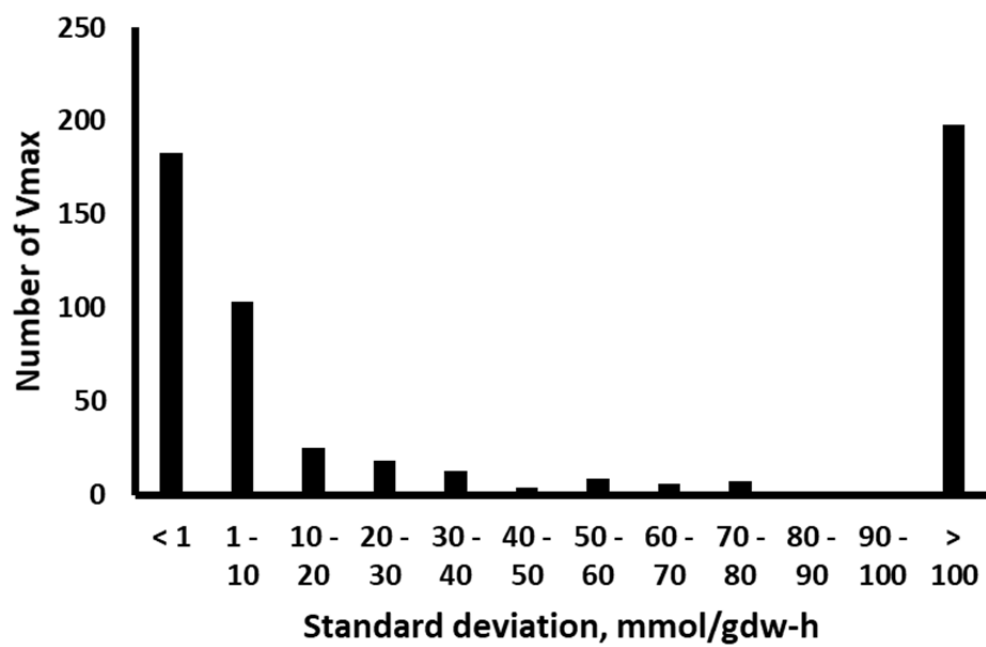
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(c)

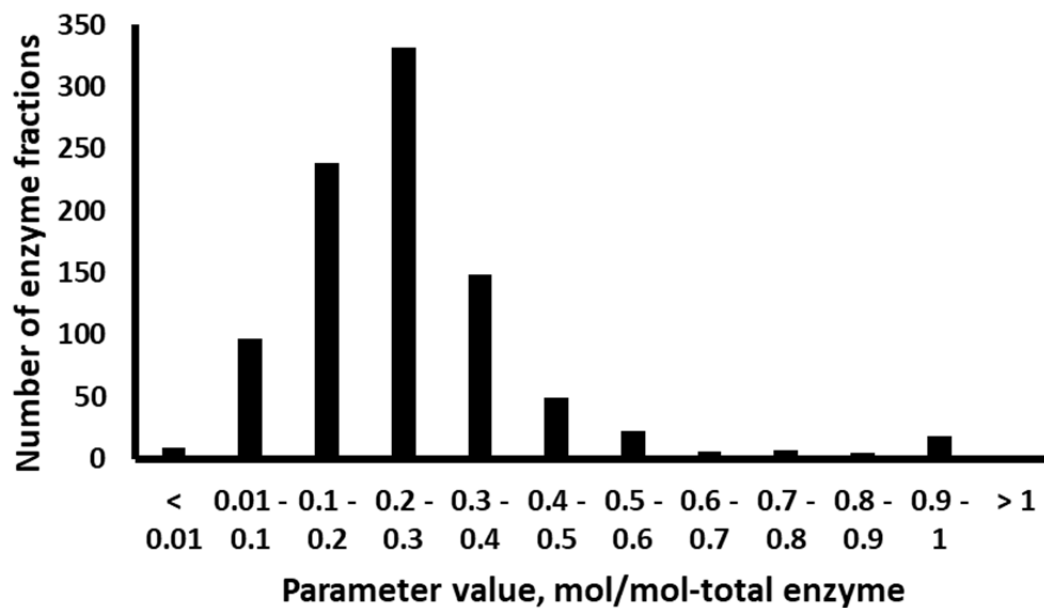


(d)

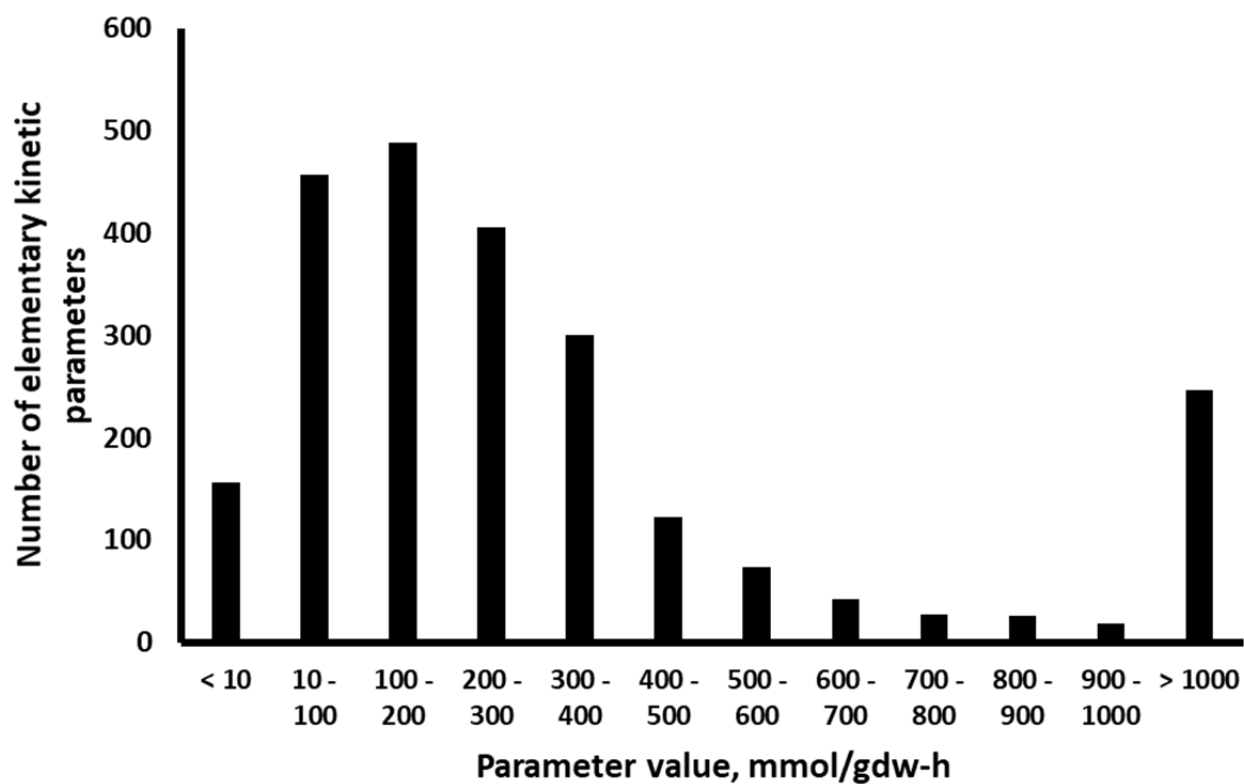


Supplementary Figure S5: Distribution of values assumed by (a) WT enzyme fractions, (b) elementary kinetic parameters, (c) K_m , and (d) V_{max} in k-ecoli307

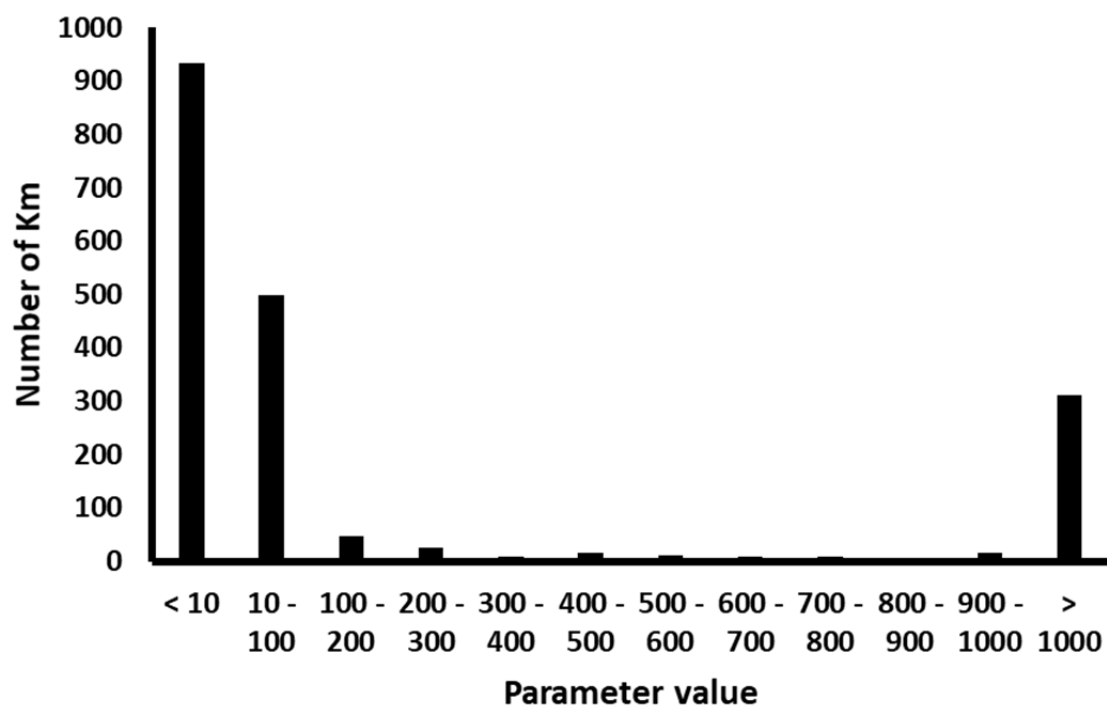
(a)



(b)



(c)



(d)

