

1 Co-infections by non-interacting 2 pathogens are not independent & 3 require new tests of interaction

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20 **Abstract** If pathogen species, strains or clones do not interact, intuition suggests
21 the proportion of co-infected hosts should be the product of the individual
22 prevalences. Independence consequently underpins the wide range of methods for
23 detecting pathogen interactions from cross-sectional survey data. However, the
24 very simplest of epidemiological models challenge the underlying assumption of
25 statistical independence. Even if pathogens do not interact, death of co-infected
26 hosts causes net prevalences of individual pathogens to decrease simultaneously.
27 The induced positive correlation between prevalences means the proportion of
28 co-infected hosts is expected to be higher than multiplication would suggest. By
29 modeling the dynamics of multiple non-interacting pathogens, we develop a pair of
30 novel tests of interaction that properly account for non-independence. Our tests
31 allow us to reinterpret data from previous studies including pathogens of humans,
32 plants, and animals. Our work demonstrates how methods to identify interactions
33 between pathogens can be updated using simple epidemic models.

35 **Keywords:** coinfection, multiple pathogens, SIS epidemic, statistical independence, pathogen
36 association

37 **Author contributions:** Designed research: All authors. Performed mathematical analyses: F.M.Ha., L.J.A., N.J.C.
38 Explored and analyzed data: F.M.Ha., F.M.Hi., M.A.R., N.J.C. Wrote the paper: F.M.Ha., L.J.A., N.J.C.

39 **1 Introduction**

40 It is increasingly recognized that infections often involve multiple pathogen species
41 or strains/clones of the same species (Balmer and Tanner, 2011; Vaumourin et al.,
42 2015). Infection by one pathogen can affect susceptibility to subsequent infection by
43 others (Griffiths et al., 2011; Petney and Andrews, 1998). Co-infection can also affect
44 the severity and/or duration of infection, as well as the extent of symptoms and the
45 level of infectiousness (Graham et al., 2005). Antagonistic, neutral and facilitative
46 interactions are possible (Karvonen et al., 2018; Rigaud et al., 2010). Co-infection
47 therefore potentially has significant epidemiological, clinical and evolutionary impli-
48 cations (Susi et al., 2015; Hilker et al., 2017; Alizon et al., 2013).

49 However, detecting and quantifying biological interactions between pathogens is
50 notoriously challenging (Johnson and Buller, 2011; Hellard et al., 2015). In pathogens
51 of some host taxa, most notably plant pathogens, biological interactions can be
52 quantified by direct experimentation (Mascia and Gallitelli, 2016). However, often
53 ethical considerations mean this is impossible, and so any signal of interaction must
54 be extracted from population scale data. Analysis of longitudinal data remains the
55 gold standard (Fenton et al., 2014), although the associated methods are not infal-
56 lible (Telfer et al., 2010). However, collecting longitudinal data requires a dedicated
57 and intensive sampling campaign, meaning in practice cross-sectional data are often
58 all that are available. Methods for cross-sectional data typically concentrate on iden-
59 tifying deviation from statistical independence, using standard methods such as χ^2
60 tests or log-linear modelling to test whether the observed probability of co-infection
61 differs from the product of the prevalences of the individual pathogens (Booth and
62 Bundy, 1995; Howard et al., 2001; Bogaert et al., 2004; Raso et al., 2004; Regev-
63 Yochay et al., 2004; Nielsen et al., 2006; Chaturvedi et al., 2011; Rositch et al., 2012;
64 Degarege et al., 2012; Malagón et al., 2016; Teweldemedhin et al., 2018). Detecting
65 such a non-random statistical association between pathogens is then taken to signal
66 a biological interaction. The underlying mechanism can range, for example, from
67 individual-scale direct effects on within-host pathogen dynamics (Tollenaere et al.,
68 2016; Mascia and Gallitelli, 2016), to indirect within-host immune-mediated interac-
69 tions (de Roode et al., 2005), to indirect population-scale “ecological interference”
70 caused by competition for susceptible hosts (Rohani et al., 1998, 2003).

71 A well-known difficulty is that factors other than biological interactions between
72 pathogens can drive statistical associations. For instance, host heterogeneity – that
73 some hosts are simply more likely than others to become infected – can generate
74 positive statistical associations, since co-infection is more common in the most vul-
75 nerable hosts. Heterogeneity in host age can also generate statistical associations,
76 as infections accumulate in older individuals (Lord et al., 1999; Kucharski and Gog,
77 2012; Kucharski et al., 2016). Methods aimed at disentangling such confounding fac-
78 tors have been developed, but show mixed results in detecting biological interactions
79 (Pedersen and Fenton, 2007; Fenton et al., 2010; Hellard et al., 2012; Vaumourin
80 et al., 2014). Methods using dynamic epidemiological models to track co-infections
81 are also emerging, although more often than not require longitudinal data (Shrestha
82 et al., 2011, 2013; Reich et al., 2013; Man et al., 2018; Alizon et al., 2019).

83 More fundamentally, however, the underpinning and long standing assumption
84 that non-interaction implies statistical independence (Forbes, 1907; Cohen, 1973)
85 has not been challenged. Here we confront the intuition that biological interactions
86 can be detected via statistical associations, demonstrating how simple epidemiolog-
87 ical models can change the way we think about biological interactions. In particular,
88 we show that non-interacting pathogens should not be expected to have prevalences
89 that are statistically independent. Co-infection by non-interacting pathogens is more
90 probable than multiplication would suggest, invalidating any test invoking statistical
91 independence.

92 The paper is organized as follows. First, we use a simple epidemiological model
93 to show that the probability that a host is co-infected by both of a pair of non-
94 interacting pathogens is greater than the product of the net prevalences of the in-
95 dividual pathogens. Second, we extend this result to an arbitrary number of non-
96 interacting pathogens. This allows us to construct a novel test for biological inter-
97 action, based on testing the extent to which co-infection data can be explained by
98 our epidemiological models in which pathogens do not interact. Different versions of
99 this test, conditioned on the form of available data and whether or not co-infections
100 are caused by different pathogen species, allow us to reinterpret a number of pre-
101 vious reports (Chaturvedi et al., 2011; López-Villavicencio et al., 2007; Andersson
102 et al., 2013; Koepfli et al., 2011; Nickbakhsh et al., 2016; Seabloom et al., 2009;

103 Moutailler et al., 2016; Howard et al., 2001; Molineaux et al., 1980). Our examples
104 include plant, animal and human pathogens, and the methodology can potentially
105 be applied to any cross-sectional survey data tracking co-infection.

106 **2 Results**

107 **2.1 Two non-interacting pathogens**

108 **2.1.1 Dynamics of the individual pathogens**

109 We consider two distinct pathogen species, strains or clones (henceforth pathogens),
110 which we assume do not interact, i.e. the interaction between the host and one
111 of the pathogens is entirely unaffected by its infection status with respect to the
112 other. Epidemiological properties that are therefore unaffected by the presence or
113 absence of the other pathogen include initial susceptibility, within-host dynamics
114 including rates of accumulation and/or movement within tissues, host responses to
115 infection, as well as onward transmission. Assuming a fixed-size host population and
116 Susceptible-Infected-Susceptible (S-I-S) dynamics (Keeling and Rohani, 2007), the
117 proportion of the host population infected by pathogen $i \in \{1, 2\}$ follows

$$\dot{I}_i = \beta_i I_i (1 - I_i) - \mu I_i, \quad (1)$$

118 in which the dot denotes differentiation with respect to time, β_i is a pathogen-specific
119 infection rate, and μ is the host's natural death rate.

120 While natural mortality may be negligible for acute infections, it cannot be ne-
121 glected for chronic (i.e. long-lasting) infections, which are responsible for a large
122 fraction of co-infections in humans and animals (Griffiths et al., 2011; Gorsich et al.,
123 2018). Likewise, plants remain infected over their entire lifetime following infection
124 by most pathogens, including almost all plant viruses, as well as the anther smut
125 fungus, which drives one of our examples here (López-Villavicencio et al., 2007).

126 We assume that the disease-induced death rate (virulence) is zero, as otherwise
127 there would be ecological interactions between pathogens (Rohani et al., 2003).
128 However our model can be extended to handle pathogen-specific rates of clearance
129 (Supplementary Information: Sections S1.4, S1.5 and S2.3).

130 **2.1.2 Tracking co-infection**

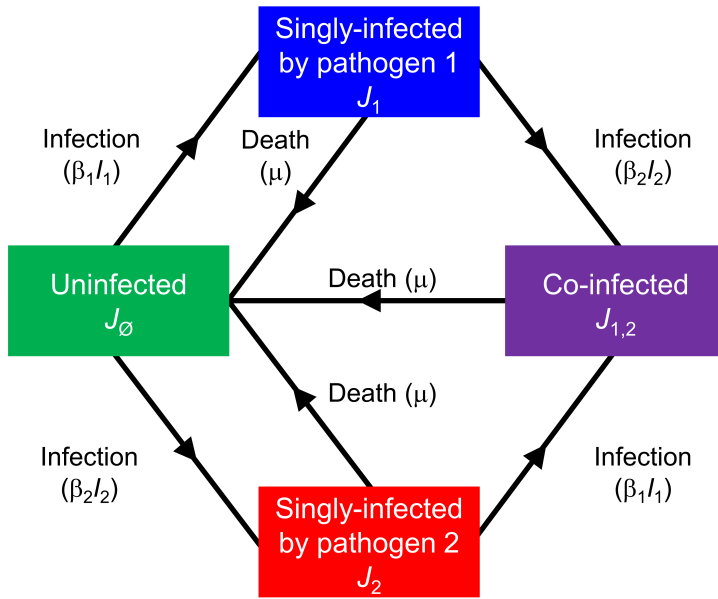


Figure 1: **Schematic of the model tracking a pair of non-interacting pathogens.** The model is defined in Eqs. (1-3): J_\emptyset denotes uninfected hosts, J_1 and J_2 are hosts singly infected by pathogens 1 and 2, respectively, $J_{1,2}$ are co-infected hosts, $I_1 = J_1 + J_{1,2}$ and $I_2 = J_2 + J_{1,2}$ are net densities of hosts infected by pathogens 1 and 2, respectively.

131 Making identical assumptions, but instead distinguishing hosts infected by differ-
 132 ent combinations of pathogens, leads to an alternate representation of the dynam-
 133 ics. We denote the proportion of hosts infected by only one of the two pathogens by
 134 J_i , with $J_{1,2}$ representing the proportion co-infected. Pathogen-specific net forces of
 135 infection are

$$F_i = \beta_i I_i = \beta_i (J_i + J_{1,2}), \quad (2)$$

136 and so

$$\begin{aligned} \dot{j}_1 &= F_1 J_\emptyset - (F_2 + \mu) J_1, \\ \dot{j}_2 &= F_2 J_\emptyset - (F_1 + \mu) J_2, \\ \dot{j}_{1,2} &= F_2 J_1 + F_1 J_2 - \mu J_{1,2}. \end{aligned} \quad (3)$$

137 in which $J_\emptyset = 1 - J_1 - J_2 - J_{1,2}$ is the proportion of hosts uninfected by either pathogen
 138 (Fig. 1).

139 **2.1.3 Prevalence of co-infected hosts**

140 We assume the basic reproduction number, $R_{0,i} = \beta_i / \mu > 1$ for both pathogens.
 141 Solving Eq. (3) numerically for arbitrary but representative parameters (Fig. 2A)
 142 shows the proportion of co-infected hosts ($J_{1,2}$) to be larger than the product of the

143 individual prevalences ($P = I_1 I_2$ from Eq. (1)). That $J_{1,2}(t) \geq P(t)$ for large t (for all
 144 parameters) can be proved analytically (Supplementary Information: Section S1.1).

145 Simulations of a stochastic analogue of the model (Fig. 2B) reveal the key driver
 146 of this behavior. The net prevalences of the pathogens considered in isolation, I_1 and
 147 I_2 , are positively correlated (Fig. 2C; Eq. (27) in Methods: Section 4.1.4, “Stochastic
 148 models”), due to simultaneous reductions whenever co-infected hosts die.

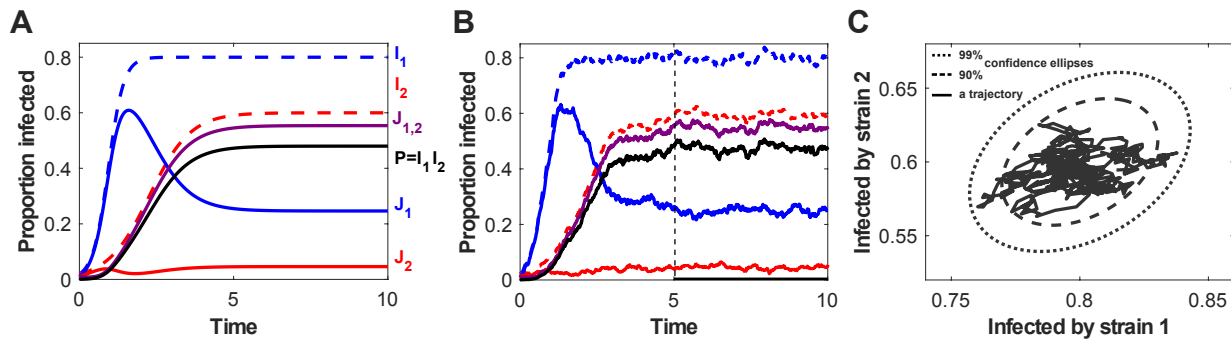


Figure 2: **Simulations of the two-pathogen model show that net densities of the two pathogens are positively correlated.** J_1 and J_2 are hosts singly infected by pathogens 1 and 2, respectively, $J_{1,2}$ are co-infected hosts, $I_1 = J_1 + J_{1,2}$ and $I_2 = J_2 + J_{1,2}$ are net densities of hosts infected by pathogens 1 and 2, respectively. (A) Dynamics of the deterministic model (1-3), with $\beta_1 = 5$, $\beta_2 = 2.5$, and $\mu = 1$ (parameters have units of inverse time). (B) Dynamics of a stochastic version of the model, in a population of size $N = 1000$ (see also Methods: Section 4.1.4, “Stochastic models”). (C) A single trajectory from the stochastic simulation (black line) in panel B (restricted to the time interval starting from the dashed line at $t = 5$) in the phase plane (I_1, I_2), and the 90% and 99% confidence ellipses (dashed and dotted curves, respectively) generated from an analytical approximation to the stochastic model.

149 2.1.4 Deviation from statistical independence

150 For $R_{0,i} > 1$ the relative deviation of the equilibrium prevalence of co-infection ($\bar{J}_{1,2}$)
 151 from that required by statistical independence ($\bar{P} = \bar{I}_1 \bar{I}_2$) is

$$\frac{\bar{J}_{1,2} - \bar{P}}{\bar{P}} = \frac{\mu}{\beta_1 + \beta_2 - \mu} = \frac{1}{R_{0,1} + R_{0,2} - 1} \geq 0 \quad (4)$$

152 (Eq. (9) in Methods: Section 4.1.1, “Equilibria of the two-pathogen model”). The
 153 deviation is therefore zero if and only if the host natural death rate $\mu = 0$.

154 The observed outcome would conform with statistical independence only for non-
 155 interacting pathogens where there is no host natural death (at the timescale of an

156 infection). This reiterates the role of host natural death in causing deviation from a
157 statistical association pattern.

158 This result (Eq. (4)) was first published by Kucharski and Gog (2012) in a differ-
159 ent context (model reduction in multi-strain influenza models). Moreover, using a
160 continuous age-structured model, Kucharski and Gog (2012) showed that one may
161 recover statistical independence within infinitesimal age-classes. The result in Eq.
162 (4) is related to aging, as individuals acquire more infections as they age. As age
163 increases, so does the probability of being infected with pathogens 1 and/or 2. There-
164 fore, the prevalences of pathogens 1 and 2 are positively correlated (Kucharski et al.,
165 2016). The reason for a greater deviation from independence as the mortality rate
166 μ increases is likely due to the fact that prevalence is increasing and concave with
167 respect to age, and saturates in older age-classes (Lord et al., 1999).

168 **2.2 Testing for interactions between pathogens**

169 Eq. (3) can be straightforwardly extended to track n pathogens which do not interact
170 in any way (including pairwise and three-way interactions). Equilibria of this model
171 are prevalences of different classes of infected or co-infected hosts carrying different
172 combinations of non-interacting pathogens. These can be used to derive a test for
173 interaction between pathogens which properly accounts for the lack of statistical
174 independence revealed by our analysis of the simple two-pathogen model.

175 **2.2.1 Modelling co-infection by n non-interacting pathogens**

176 We denote the proportion of hosts simultaneously co-infected by the (non-empty)
177 set of pathogens Γ to be J_Γ , and use $\Omega_i = \Gamma \setminus \{i\}$ (for $i \in \Gamma$) to represent combinations
178 with one fewer pathogen.

179 The dynamics of the $2^n - 1$ distinct values of J_Γ follow

$$180 \quad \dot{J}_\Gamma = \sum_{i \in \Gamma} F_i J_{\Omega_i} - \left(\sum_{i \notin \Gamma} F_i + \mu \right) J_\Gamma, \quad (5)$$

180 in which the net force of infection of pathogen i is

$$F_i = \beta_i I_i = \beta_i \sum_{\Gamma \in \mathcal{V}_i} J_\Gamma, \quad (6)$$

181 and ∇_i is the set of all subsets of $\{1, \dots, n\}$ containing i as an element. Eq. (5) can
182 be interpreted by noting

- 183 • the first term tracks inflow due to hosts carrying one fewer pathogen becoming
184 infected;
- 185 • the second term tracks the outflows due to hosts becoming infected by an
186 additional pathogen, or death.

187 **Predicted prevalences.** If $R_{0,i} = \beta_i/\mu > 1$ for all $i = 1, \dots, n$, the equilibrium preva-
188 lence of hosts infected by any given combination of pathogens, \bar{J}_F , can be obtained
189 by (recursively) solving a system of 2^n linear equations (Eq. (16) in Methods: Section
190 4.1.2, “Equilibria of the n -pathogen model”).

191 These equilibrium prevalences are the prediction of our “Non-interacting Distinct
192 Pathogens” (NiDP) model, which in dimensionless form has n parameters (the $R_{0,i}$ ’s,
193 $i = 1, \dots, n$; Methods: Section 4.2.2, “Fitting the models”).

194 **Epidemiologically interchangeable pathogens.** If we simplify the model by as-
195 suming that all pathogen infection rates are equal (i.e. $\beta_i = \beta$ for all i), then if
196 $R_0 = \beta/\mu > 1$, the proportion of hosts infected by k distinct pathogens can be ob-
197 tained by (recursively) solving $n + 1$ linear equations (Eq. (22) in Methods: Section
198 4.1.3, “Deriving the NiSP model from the NiDP model”). This constitutes the predic-
199 tion of our “Non-interacting Similar Pathogens” (NiSP) model, a simplified form of the
200 NiDP model requiring only a single parameter (R_0).

201 **2.2.2 Using the models to test for interactions**

202 If either the NiSP or NiDP model adequately explain co-infection data, those data are
203 consistent with the underpinning assumption that pathogens do not interact. Which
204 model is fitted depends on the form of the available data.

205 **Numbers of distinct pathogens.** Studies often quantify only the number of dis-
206 tinct pathogens carried by individual hosts, without necessarily specifying the combi-
207 nations involved (López-Villavicencio et al., 2007; Seabloom et al., 2009; Chaturvedi

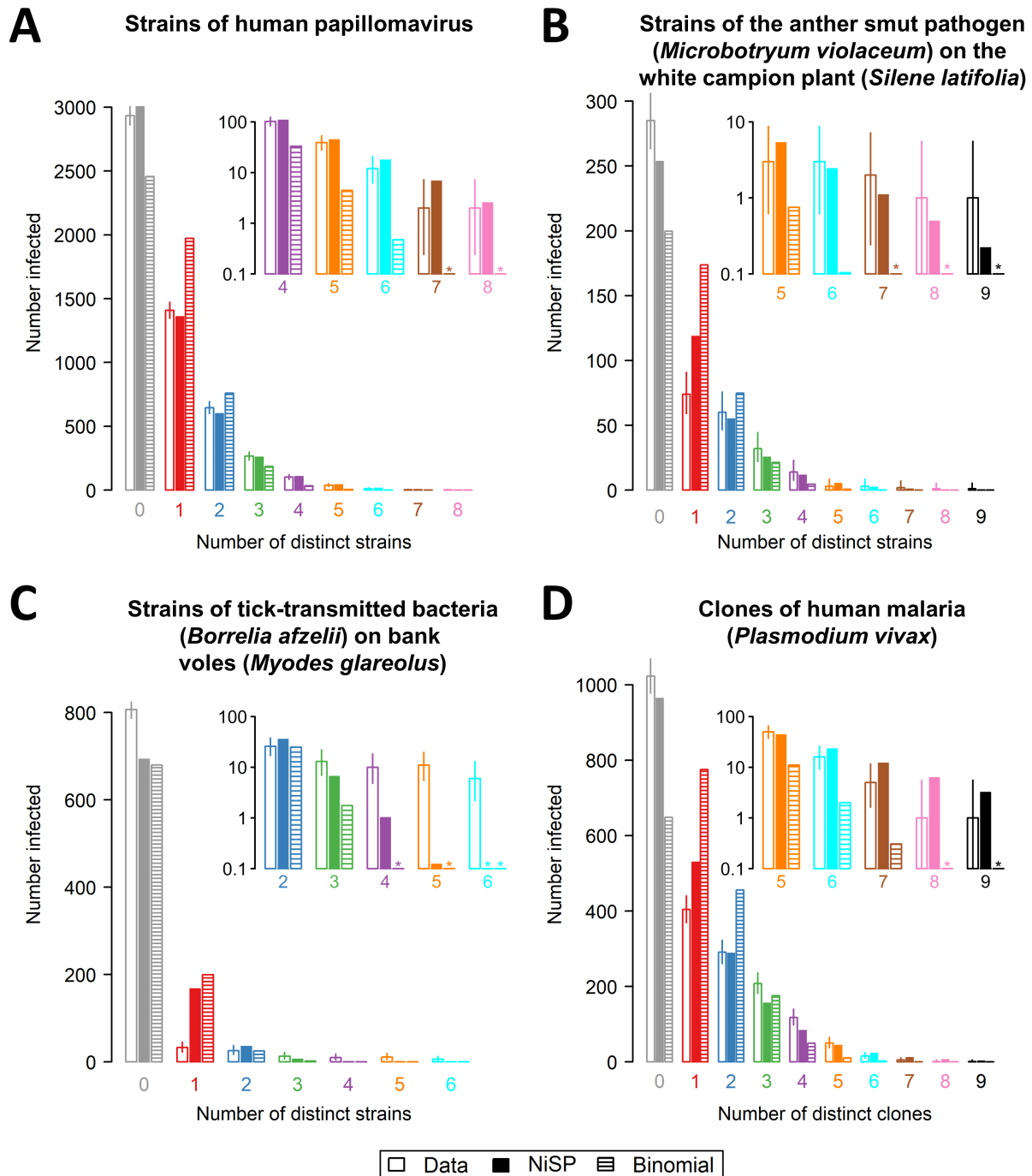


Figure 3: **Comparing predictions of the NiSP model with binomial models assuming statistical independence.** In using the NiSP model, pathogens are assumed to be epidemiologically interchangeable: we have therefore restricted attention to data sets concerning strains/clones of a single pathogen species. (A) strains of human papillomavirus (Chaturvedi et al., 2011); (B) strains of the anther smut pathogen (*M. violaceum*) on the white campion (*S. latifolia*) (López-Villavicencio et al., 2007); (C) strains of tick-transmitted bacteria (*B. afzelii*) on bank voles (*M. glareolus*) (Andersson et al., 2013); and (D) clones of malaria (*P. vivax*) (Koepfli et al., 2011). Insets to each panel show a “zoomed-in” section of the graph corresponding to high multiplicities of clone/strain co-infection. Asterisks indicate predicted counts smaller than 0.1. In all four cases, the NiSP model is a better fit to the data than the binomial model ($\Delta\text{AIC} = 572.8, 158.6, 293.8$ and 596.3 , respectively). For the data shown in panel (A), there is no evidence that the NiSP model does not fit the data (lack of goodness-of-fit $p = 0.08$), and so our test indicates the human papillomavirus strains do not interact. For the data shown in panels (B)-(D), there is evidence of lack of goodness-of-fit (all have lack of goodness-of-fit $p < 0.01$). Our test therefore indicates these strains/clones interact (or are epidemiologically different).

208 et al., 2011; Koepfli et al., 2011; Andersson et al., 2013; Moutailler et al., 2016; Nick-
209 bakhsh et al., 2016). There are insufficient degrees of freedom in such data to fit the
210 NiDP model, and so we fall back upon the NiSP model. In using the NiSP model, we
211 additionally assume all pathogens within a given study are epidemiologically inter-
212 changeable.

213 We identified four suitable studies reporting data concerning strains/clones of a
214 single pathogen, and tested whether these data are consistent with no interaction.
215 For all four studies (Fig. 3), the best-fitting NiSP model is a better fit to the data
216 than the corresponding binomial model assuming statistical independence (Eq. (28)
217 in Methods: Section 4.2.1, “Models corresponding to assuming statistical indepen-
218 dence”). Three additional examples for data-sets considering distinct pathogens,
219 which deviate more markedly from the epidemiological equivalence assumption, are
220 in the Supplementary Information (Section S2.1).

221 In one case – co-infection by different strains of human papillomavirus (Chaturvedi
222 et al., 2011) (Fig. 3A) – we find no evidence that the reported data cannot be ex-
223 plained by the NiSP model. These data therefore support the hypothesis of no inter-
224 action – and indeed no epidemiological differences – between the pathogen strains
225 in question.

226 In the three other cases we considered – strains of anther smut (*Microbotryum*
227 *violaceum*) on the white campion (*Silene latifolia*) (López-Villavicencio et al., 2007)
228 (Fig. 3B); strains of the tick-transmitted bacterium *Borrelia afzelii* on bank voles (*My-
229 odes glareolus*) (Andersson et al., 2013) (Fig. 3C); and clones of a single malaria pa-
230 rasite (*Plasmodium vivax*) infecting children (Koepfli et al., 2011) (Fig. 3D) – despite
231 outperforming the model corresponding to statistical independence, the best-fitting
232 NiSP model does not adequately explain the data. We therefore reject the hypothe-
233 ses of no interaction in all three cases, noting that our use of the NiSP model means
234 it might be epidemiological differences between pathogen strains/clones that have
235 in fact been revealed.

236 **Combinations of pathogens.** Other studies report the proportion of hosts in-
237 fected by particular combinations (rather than counts) of pathogens, although many
238 of those concentrate on helminth macroparasites for which our underlying S-I-S

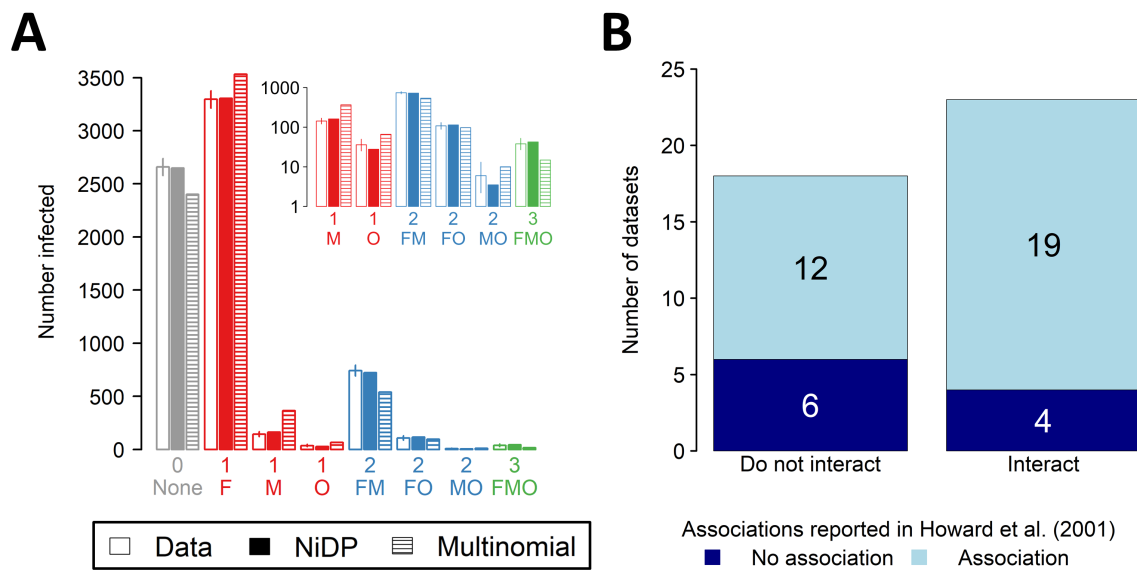


Figure 4: **Using the NiDP model to re-analyse malaria data sets considered by Howard et al. (2001).** In using the NiDP model there is no need to assume malaria-causing *Plasmodium* spp. are epidemiologically interchangeable. (A) Comparing the predictions of the NiDP model with a multinomial model of infection (i.e. statistical independence) for the data set on *P. falciparum* (F), *P. malariae* (M) and *P. ovale* (O) co-infection in Nigeria reported by Molineaux et al. (1980). The NiDP model is a better fit to the data than the multinomial model ($\Delta AIC = 326.2$); additionally, there is no evidence of lack of goodness-of-fit ($p = 0.40$). This data set is therefore consistent with no interaction between the three *Plasmodium* species. (B) Comparing the results of fitting the NiDP model and the methodology of Howard et al. (2001) based on log-linear regression and so statistical-independence. For 16 (i.e. $12 + 4$) out of the 41 data sets we considered, the conclusions of the two methods differ.

239 model is well-known to be inappropriate (Anderson and May, 1991).

240 However, a methodological article by Howard et al. (2001) introduces the use
241 of log-linear modeling to test for statistical associations. Conveniently, that article
242 reports the results of that methodology as applied to a large number of studies
243 focusing on *Plasmodium* spp. causing malaria.

244 By interrogating the original data sources (Methods: Section 4.3.2, “Combinations
245 of pathogens (NiDP model)”), we found a total of 41 studies of malaria reporting the
246 disease status of at least $N = 100$ individuals, and in which three of *P. falciparum*,
247 *P. malariae*, *P. ovale* and *P. vivax* were considered. Data therefore consist of counts
248 of the number of individuals infected with different combinations of three of these
249 four pathogens, a total of eight classes. There were sufficient degrees of freedom
250 to fit the NiDP model, which here has three parameters, each corresponding to the
251 infection rate of a single *Plasmodium* spp. Fig. 4A shows the example of fitting the
252 NiDP model to data from a study of malaria in Nigeria (Molineaux et al., 1980).

253 Fitting the NiDP model allows us to test for interactions between *Plasmodium*
254 spp., without assuming they are epidemiologically interchangeable. In 18 of the 41
255 cases we considered, our methods suggest the data are consistent with no inter-
256 action (Fig. 4B). We note that in 12 of these 18 cases the methodology based on
257 statistical independence of Howard et al. (2001) instead suggests the *Plasmodium*
258 spp. interact.

259 **3 Discussion**

260 We have shown that pathogens which do not interact and so have uncoupled preva-
261 lence dynamics (Eq. (1)) are not statistically independent. For two pathogens,
262 the prevalence of co-infection is always greater than the product of the preva-
263 lences (Eq. (4)), unless host natural death does not occur. This result was first pub-
264 lished in an age-structured, multi-strain influenza model (Kucharski and Gog, 2012).
265 Pathogens share a single host in co-infections, and so when a co-infected host dies,
266 net prevalences of both pathogens decrease simultaneously. The prevalences of in-
267 dividual pathogens, regarded as random variables, therefore co-vary positively. A
268 related interpretation is due to Kucharski and Gog (2012): the prevalences of the

269 pathogens are positively correlated through a single independent variable, namely
270 the age of the hosts. As a side result, we note our analysis indicates a high-profile
271 model of May and Nowak (1995) is based on a faulty assumption of probabilistic in-
272 dependence (Supplementary Information: Section S1.3). More importantly, our anal-
273 ysis shows that statistically independent pathogens may well be interacting (Supple-
274 mentary Information: Section S1.5) which confirms that statistical independence is
275 far from equivalent to the absence of biological interaction between pathogens.

276 We extended our model to an arbitrary number of pathogens to develop a novel
277 test for interaction that properly accounts for statistical non-independence. Many
278 data sets summarize co-infections in terms of multiplicity of infection, regardless
279 of which pathogens are involved. Since there would be as many epidemiological
280 parameters as pathogens in our default NiDP model, and so as many parameters as
281 data-points, the full model would be over-parameterised. We therefore introduced
282 the additional assumption that all pathogens are epidemiologically interchangeable.
283 This formed the basis of the parsimonious NiSP model, which is most appropriate for
284 testing for interactions between strains or clones of a single pathogen species.

285 Despite the strong and perhaps even unrealistic assumption that strains/clones
286 are interchangeable, the NiSP model outperformed the binomial model assuming
287 statistical independence for all four data sets we considered. In particular, the NiSP
288 model successfully captured the fat tails characteristic of observed multiplicity of
289 infection distributions. All four data sets therefore support the idea that co-infection
290 is far more frequent than statistical independence would imply.

291 For the data set concerning co-infection by different strains of human papillo-
292 mavirus (Chaturvedi et al., 2011), the NiSP model also passed the goodness-of-fit
293 test, allowing us to conclude strains of this pathogen do not interact. Goodness-
294 of-fit for such a simple model is a particularly conservative test, especially for the
295 NiSP model, when we assume pathogens clones/strains are epidemiologically inter-
296 changeable.

297 We illustrated our methods via case studies for which suitable data are readily
298 available, and our purpose was not to come to definitive conclusions concerning any
299 particular system. That would require dedicated studies. However, by fitting even
300 a highly-simplified version of our model to data, we have demonstrated how results

301 of simple epidemiological models challenge previous methods based on statistical
302 independence.

303 To explore further the implications of our findings, we analyzed available data sets
304 tracking combinations of pathogens involved in each occurrence of co-infection. For
305 methodological comparison purposes, we restricted ourselves to data referenced in
306 Howard et al. (2001), concerning interactions between *Plasmodium* spp. causing
307 malaria. Relaxing the assumption of epidemiological interchangeability (i.e. using
308 the NiDP model), we found that 43.9% (i.e. 18/41) of data sets considered in Howard
309 et al. (2001) are consistent with no interaction.

310 One may wonder whether focusing on age classes may be sufficient to correct
311 for the positive correlation between non-interacting pathogens (Lord et al., 1999).
312 Of the 41 data sets identified by Howard et al. (2001) that we analysed, 14 focused
313 only on data collected from children, and therefore, associations are less likely less
314 likely to emerge solely by the confounding effect of age (Fenton et al., 2010). Of
315 these 14 studies, we came to the same conclusion as Howard et al. (2001) in only 6
316 cases. We identified 2 cases in which our methods suggest there is an interaction in
317 which Howard et al. (2001) concluded no interaction (studies 71 and 77), as well as
318 6 cases in which we conclude no interaction whereas Howard et al. (2001) conclude
319 there is an interaction (studies 76, 68, 69, 70, 79 and 80). Thus, focusing on discrete
320 and arbitrary age classes may not be sufficient to correct for the positive correlation
321 between non-interacting pathogens.

322 Again we do not intend to conclusively demonstrate interactions – or lack of in-
323 teractions – for malaria. Instead what is important is that our results very often
324 diverge from those originally reported in Howard et al. (2001) using a method based
325 on statistical associations, namely log-linear regression. Log-linear regression suf-
326 fers from well-acknowledged difficulties in cases in which there are zero counts (i.e.
327 certain combinations of pathogens are not observed) (Fienberg and Rinaldo, 2012).
328 Such cases often arise in epidemiology. Methods based on epidemiological models
329 therefore offer a twofold advantage: biological interactions are not confounded with
330 statistical associations, and parameter estimation is well-posed, irrespective of zero
331 counts.

332 Moreover, simple epidemiological models (with no explicit age structure) intrin-

333 sically correct the bias due to the positive correlation between age and prevalence,
334 which makes it unnecessary to control for age. Therefore, and this may be our main
335 conclusion: although age is an evident confounding factor, epidemiological models
336 make it unnecessary to keep track of the age of infected hosts. This is made possible
337 by replacing the paradigm of “statistical Independence and random distributions”
338 with “model-based distributions in absence of biological interactions.”

339 We focused here on the simple S-I-S model, since it is sufficiently generic to be
340 applicable to a number of systems. However, an important assumption of our model
341 is that natural mortality occurs at a time scale comparable to that of an infection.
342 Our model is therefore tailored for chronic (i.e. long-lasting) infections, which rep-
343 resent a large fraction of of co-infections in humans, animals, and plants. Also,
344 our study is restricted to nonlethal infections, as otherwise there may be ecological
345 interactions between pathogens. In future work focusing on particular pathogens,
346 models including additional system-specific detail would of course be appropriate.
347 We leave further analysis of more complex underpinning epidemiological models to
348 such future research.

349 Lastly, we speculate our results may have implications beyond epidemiology.
350 After all, pathogens are species which form meta-populations occupying discrete
351 patches (hosts) (Seabloom et al., 2015). Meta-community ecology has long been
352 concerned with whether interactions between species can be detected from co-
353 occurrence data (Forbes, 1907; Caswell, 1976; Connor and Simberloff, 1979) and
354 most existing methods are based on detecting statistical associations (Gotelli, 2000;
355 Gotelli and Ulrich, 2012), but see Hastings (1987). Our dynamical modeling approach
356 may also provide a new perspective in this area.

357 **4 Methods**

358 **4.1 Mathematical analyses**

359 **4.1.1 Equilibria of the two-pathogen model**

360 The 2-pathogen model is given by Eq. (1-2-3). Since the population size is constant,
 361 $J_{\emptyset} = 1 - J_1 - J_2 - J_{1,2}$, and so it follows that

$$\dot{J}_{\emptyset} = \mu(J_1 + J_2 + J_{1,2}) - (F_1 + F_2)J_{\emptyset} = \mu(1 - J_{\emptyset}) - (F_1 + F_2)J_{\emptyset}. \quad (7)$$

362 It is well-known (Keeling and Rohani, 2007) that if $R_{0,i} = \beta_i/\mu > 1$ and $I_i(0) > 0$,
 363 the prevalence of pathogen i will tend to an equilibrium $\bar{I}_i = 1 - 1/R_{0,i}$.

364 Since $F_i = \beta_i I_i$ and $J_i = I_i - J_{1,2}$, the rate of change of co-infected hosts in Eq. (3)
 365 can be recast as

$$\dot{J}_{1,2} = \beta_2 I_2 (I_1 - J_{1,2}) + \beta_1 I_1 (I_2 - J_{1,2}) - \mu J_{1,2}. \quad (8)$$

366 The equilibrium prevalence of co-infected hosts ($\bar{J}_{1,2}$) can therefore be written in
 367 terms of the individual net prevalences at equilibrium (\bar{I}_1 and \bar{I}_2),

$$\bar{J}_{1,2} = \left(\frac{\beta_1 + \beta_2}{\beta_1 + \beta_2 - \mu} \right) \bar{I}_1 \bar{I}_2. \quad (9)$$

368 This immediately leads to the result concerning the deviation of $\bar{J}_{1,2}$ from $\bar{P} = \bar{I}_1 \bar{I}_2$
 369 (i.e., the expected prevalence of co-infected hosts given statistical independence)
 370 quoted in Eq. (4).

371 **4.1.2 Equilibria of the n -pathogen model**

372 The n -pathogen model is given by Eq. (1-5-6). Since the host population size is
 373 constant, $J_{\emptyset} = 1 - \sum_{\Gamma \in \nabla} J_{\Gamma}$, where ∇ is the set of all $2^n - 1$ sets with infected or co-
 374 infected hosts. It is also true that

$$\dot{J}_{\emptyset} = \mu(1 - J_{\emptyset}) - \left(\sum_{i=1}^n F_i \right) J_{\emptyset}. \quad (10)$$

375 At equilibrium, Eq. (5) becomes

$$0 = \sum_{i \in \Gamma} \bar{F}_i \bar{J}_{\Omega_i} - \left(\sum_{i \notin \Gamma} \bar{F}_i + \mu \right) \bar{J}_{\Gamma}, \quad (11)$$

376 in which \bar{J}_{Ω_i} and \bar{J}_{Γ} are equilibrium prevalences, and \bar{F}_i is the force of infection of
377 pathogen i at equilibrium, i.e.

$$\bar{F}_i = \beta_i \bar{I}_i = \beta_i \left(1 - \frac{\mu}{\beta_i} \right) = \beta_i - \mu. \quad (12)$$

378 Since these forces of infection are constant and do not depend on the equilibrium
379 prevalences, the set of $2^n - 1$ equations partially characterizing the equilibrium is
380 linear, with

$$0 = \sum_{i \in \Gamma} (\beta_i - \mu) \bar{J}_{\Omega_i} - \left(\sum_{i \notin \Gamma} (\beta_i - \mu) + \mu \right) \bar{J}_{\Gamma}. \quad (13)$$

381 Similarly, Eq. (10) is linear

$$0 = \mu(1 - \bar{J}_{\emptyset}) - \left(\sum_{i=1}^n (\beta_i - \mu) \right) \bar{J}_{\emptyset}. \quad (14)$$

382 The equilibrium prevalences can be written very conveniently in a recursive form
383 (i.e. using the first equation to fix \bar{J}_{\emptyset} , using \bar{J}_{\emptyset} to independently calculate all values
384 of \bar{J}_{Γ} for $|\Gamma| = 1$, then using the set of values of \bar{J}_{Γ} when $|\Gamma| = 1$ to independently
385 calculate all values of \bar{J}_{Γ} for $|\Gamma| = 2$, and so on). A recurrence relation to find all
386 equilibrium prevalences can therefore be initiated with the following expression for
387 the density of uninfected hosts:

$$\bar{J}_{\emptyset} = \frac{\mu}{\mu + \sum_{i=1}^n (\beta_i - \mu)} = \frac{1}{1 + \sum_{i=1}^n (R_{0,i} - 1)}. \quad (15)$$

388 Then, one may recursively use the following equation, equivalent to Eq. (13):

$$\bar{J}_{\Gamma} = \frac{\sum_{i \in \Gamma} (\beta_i - \mu) \bar{J}_{\Omega_i}}{\mu + \sum_{i \notin \Gamma} (\beta_i - \mu)} = \frac{\sum_{i \in \Gamma} (R_{0,i} - 1) \bar{J}_{\Omega_i}}{1 + \sum_{i \notin \Gamma} (R_{0,i} - 1)}. \quad (16)$$

389 Since the densities in Eq. (16) are entirely in terms of the equilibrium densities
390 of hosts carrying one fewer pathogen (\bar{J}_{Ω_i}), this allows us to recursively find the
391 densities of all pathogens given pathogen-by-pathogen values of $R_{0,i}$.

392 4.1.3 Deriving the NiSP model from the NiDP model

393 If all pathogens are interchangeable, and so have identical values of $R_{0,i} = R_0 \forall i$,
 394 then for any pair of combinations of infecting pathogens, Γ_1 and Γ_2 , it must be the
 395 case that $\bar{J}_{\Gamma_1} = \bar{J}_{\Gamma_2}$ whenever $|\Gamma_1| = |\Gamma_2|$. This means the equilibrium prevalences of
 396 hosts infected by the same number of distinct pathogens must all be equal, irrespec-
 397 tive of the particular combination of pathogens that is carried. In this case solving
 398 the system is much simpler. First, Eq. (11) can be rewritten as

$$0 = |\Gamma| \bar{F} \bar{J}_{\Omega_i} - ((n - |\Gamma|) \bar{F} + \mu) \bar{J}_{\Gamma}, \quad (17)$$

399 in which $\bar{F} = \beta - \mu$. The net prevalence of hosts infected by k distinct pathogens is

$$\bar{M}_k = \sum_{\Gamma \in \nabla(k)} \bar{J}_{\Gamma}, \quad (18)$$

400 in which $\nabla(k)$ is the set of combinations of $\{1, \dots, n\}$ with k elements. Since the
 401 form of Eq. (17) depends only on $|\Gamma|$, all individual prevalences involved in \bar{M}_k are
 402 identical, and so

$$\bar{M}_k = C_k^n \bar{J}_{\Gamma,k}, \quad (19)$$

403 in which C_k^n is a combinatorial coefficient, and $\bar{J}_{\Gamma,k}$ is any of the individual prevalences
 404 for which $|\Gamma| = k$. The ratio between successive values of \bar{M}_k is given by

$$\frac{\bar{M}_k}{\bar{M}_{k-1}} = \frac{C_k^n \bar{J}_{\Gamma,k}}{C_{k-1}^n \bar{J}_{\Gamma,k-1}} = \frac{n-k+1}{k} \frac{\bar{J}_{\Gamma,k}}{\bar{J}_{\Gamma,k-1}}. \quad (20)$$

405 From Eq. (15), it follows that

$$\bar{M}_0 = \frac{\mu}{\mu + n\bar{F}} = \frac{1}{1 + n(R_0 - 1)}, \quad (21)$$

406 in which $R_0 = \beta/\mu$. For $1 \leq k \leq n$, Eq. (17) and (20) together imply

$$\bar{M}_k = \frac{(n-k+1)\bar{F}}{(n-k)\bar{F} + \mu} \bar{M}_{k-1} = \frac{(n-k+1)(R_0 - 1)}{(n-k)(R_0 - 1) + 1} \bar{M}_{k-1}, \quad (22)$$

407 a form which admits a simple recursive solution.

408 4.1.4 Stochastic models

409 Figs. 2B and 2C were generated by simulating the stochastic differential equation
 410 corresponding to Eq. (3); simulating a continuous time Markov chain model using
 411 Gillespie's algorithm gave consistent results. Confidence ellipses were obtained from
 412 an approximate expression for the covariance matrix at equilibrium (see below).

413 **Continuous-time Markov chain.** The continuous-time Markov chain model cor-
 414 responding to the unscaled version of Eq. (3-7) tracks a vector of integer-valued
 415 random variables $X(t) = (J_\emptyset(t), J_1(t), J_2(t), J_{1,2}(t))$. Defining $\Delta X = X(t + \Delta t) - X(t) =$
 416 $(\Delta J_\emptyset, \Delta J_1, \Delta J_2, \Delta J_{1,2})$, changes of ± 1 to each element of $X(t)$ occur in small periods
 417 of time Δt at the rates given in Table 1. Stochastic trajectories from this model can
 418 conveniently be simulated via the Gillespie algorithm (Gillespie, 1977). Note that
 419 the numeric values of the infection rates and the host birth rate must be altered to
 420 account for the scaling by population size.

Table 1: Transitions in the two-pathogen stochastic models. The prevalence of uninfected host is J_\emptyset , the prevalence of each class of singly-infected hosts is J_i (for $i \in [1, 2]$), and the prevalence of co-infected host is $J_{1,2}$. The net force of infection of pathogen i is $F_i = \beta_i I_i / N = \beta_i (J_i + J_{1,2}) / N$ (note the scaling by the population size N relative to the forces of infection as used in the deterministic version of the model). To ensure a constant host population size, we have made the simplifying assumption that removal and replacement occur simultaneously; this has no effect on our qualitative results.

Event number	Event	Rate	Change(s) to state variable(s) (ΔX)
1	Infection of uninfected host by pathogen 1	$F_1 J_\emptyset \Delta t + o(\Delta t)$	$J_\emptyset \rightarrow J_\emptyset - 1$ $J_1 \rightarrow J_1 + 1$
2	Infection of uninfected host by pathogen 2	$F_2 J_\emptyset \Delta t + o(\Delta t)$	$J_\emptyset \rightarrow J_\emptyset - 1$ $J_2 \rightarrow J_2 + 1$
3	Infection by pathogen 1 of host singly-infected by pathogen 2	$F_1 J_2 \Delta t + o(\Delta t)$	$J_2 \rightarrow J_2 - 1$ $J_{1,2} \rightarrow J_{1,2} + 1$
4	Infection by pathogen 2 of host singly-infected by pathogen 1	$F_2 J_1 \Delta t + o(\Delta t)$	$J_1 \rightarrow J_1 - 1$ $J_{1,2} \rightarrow J_{1,2} + 1$
5	Death of host singly-infected by pathogen 1 and replacement with an uninfected host	$\mu J_1 \Delta t + o(\Delta t)$	$J_1 \rightarrow J_1 - 1$ $J_\emptyset \rightarrow J_\emptyset + 1$
6	Death of host singly-infected by pathogen 2 and replacement with an uninfected host	$\mu J_2 \Delta t + o(\Delta t)$	$J_2 \rightarrow J_2 - 1$ $J_\emptyset \rightarrow J_\emptyset + 1$
7	Death of co-infected host and replacement with an uninfected host	$\mu J_{1,2} \Delta t + o(\Delta t)$	$J_{1,2} \rightarrow J_{1,2} - 1$ $J_\emptyset \rightarrow J_\emptyset + 1$

421 **Stochastic differential equations.** The model can also be written as a system
 422 of stochastic differential equations (SDEs), an approximation to the continuous-time
 423 Markov chain that is valid for sufficiently large N (Kurtz, 1970) and which is partic-
 424 ularly well-suited for simulation of the stochastic model when the population size is

425 large. This form of the model again tracks the seven events in Table 1, although in
 426 the SDE formulation the random variables in $X(t)$ are continuous-valued. A heuristic
 427 derivation is based on a normal approximation described below. Alternately, the for-
 428 ward Kolmogorov differential equations in the continuous-time Markov chain model
 429 are closely related to the Fokker Planck equation for the probability density function
 430 of the SDE model (Allen et al., 2008).

431 The expected change $\mathbb{E}(\Delta X)$ and covariance of the changes $\mathbb{V}(\Delta X)$ can be com-
 432 puted from Table 1 to order Δt via

$$\mathbb{E}(\Delta X) \approx \tilde{f}\Delta t \text{ and } \mathbb{V}(\Delta X) \approx \mathbb{E}(\Delta X[\Delta X]^T) = \Sigma\Delta t, \quad (23)$$

433 where $dJ = \tilde{f}dt$ is the unscaled version of the deterministic model as specified in Eq.
 434 (3-7) with $N = J_\emptyset + J_1 + J_2 + J_{1,2}$ (a constant) and $F_i = \beta_i(J_i + J_{1,2})/N$. In addition, the
 435 matrix Σ is given by

$$\begin{bmatrix} \mu(N - J_\emptyset) + (F_1 + F_2)J_\emptyset & -F_1J_\emptyset - \mu J_1 & -F_2J_\emptyset - \mu J_2 & -\mu J_{1,2} \\ -F_1J_\emptyset - \mu J_1 & F_1J_\emptyset + (F_2 + \mu)J_1 & 0 & -F_2J_1 \\ -F_2J_\emptyset - \mu J_2 & 0 & F_2J_\emptyset + (F_1 + \mu)J_2 & -F_1J_2 \\ -\mu J_{1,2} & -F_2J_1 & -F_1J_2 & F_2J_1 + F_1J_2 + \mu J_{1,2} \end{bmatrix}. \quad (24)$$

436 The changes in a small time interval Δt are approximated by a normal distribution
 437 via the Central Limit Theorem: $\Delta X(t) - \mathbb{E}(\Delta X(t)) \approx \text{Normal}(0, \Sigma\Delta t)$, where $0 =$ zero
 438 vector. The covariance matrix Σ can be written as $\Sigma = GG^T$. Letting $\Delta t \rightarrow 0$, the SDE
 439 model can therefore be expressed as

$$dX = \tilde{f}dt + GdW. \quad (25)$$

440 The matrix G is not unique but a simple form with dimension 4×7 accounts for
 441 each event in Table 1 (Allen et al., 2008). Each entry in matrix G involves a square
 442 root and W is a vector of seven independent standard Wiener processes, where
 443 $dW_i \approx \Delta W_i(t) = W_i(t + \Delta t) - W_i(t) \sim \text{Normal}(0, \Delta t)$. An explicit form for the SDE

444 model in Eq. (25) is

$$\begin{aligned}
 dj_{\emptyset} &= \tilde{f}_0 dt - \sqrt{F_1 J_{\emptyset}} dW_1 - \sqrt{F_2 J_{\emptyset}} dW_2 + \sqrt{\mu J_1} dW_5 + \sqrt{\mu J_2} dW_6 + \sqrt{\mu J_{1,2}} dW_7, \\
 dj_1 &= \tilde{f}_1 dt + \sqrt{F_1 J_{\emptyset}} dW_1 - \sqrt{F_2 J_1} dW_4 - \sqrt{\mu J_1} dW_5, \\
 dj_2 &= \tilde{f}_2 dt + \sqrt{F_2 J_{\emptyset}} dW_2 - \sqrt{F_1 J_2} dW_3 - \sqrt{\mu J_2} dW_6, \\
 dj_{1,2} &= \tilde{f}_{1,2} dt + \sqrt{F_1 J_2} dW_3 + \sqrt{F_2 J_1} dW_4 - \sqrt{\mu J_{1,2}} dW_7.
 \end{aligned}
 \tag{26}$$

445 **Covariance matrix at the endemic equilibrium.** In Supplementary Information
 446 (Section S1.2) we show that the covariance between the prevalences of pathogen 1
 447 and pathogen 2 as they fluctuate in the vicinity of their equilibrium values is approx-
 448 imately

$$\text{cov}\left(\frac{I_1}{N}, \frac{I_2}{N}\right) = \frac{\mu \bar{J}_{1,2}}{N^2 [(\beta_1 - \mu) + (\beta_2 - \mu)]} = \frac{(\beta_1 + \beta_2)(\beta_1 - \mu)(\beta_2 - \mu)\mu}{N\beta_1\beta_2(\beta_1 + \beta_2 - \mu)(\beta_1 - \mu + \beta_2 - \mu)} \geq 0,
 \tag{27}$$

449 with equality if and only if $\mu = 0$ (assuming $\beta_i > \mu$, $i = 1, 2$). Only in the specific case
 450 $\mu = 0$, is the deviation from statistical independence equal to zero (Eq. (4)).

451 4.2 Statistical methods

452 4.2.1 Models corresponding to assuming statistical independence

453 If data are observations of numbers of individuals infected with k distinct pathogens,
 454 O_k , for $k \in [0, n]$, statistical independence corresponds to assuming the infection
 455 load of a single individual follows the one-parameter, binomial model $Bin(n, p)$, in
 456 which p is the pathogen prevalence (assumed identical for each pathogen, and fit-
 457 ted appropriately to the data), and n is the maximum number of infections that is
 458 possible (i.e. the total number of distinct pathogens under consideration). Model pre-
 459 dictions are then simply N samples from this binomial distribution, where $N = \sum_k O_k$
 460 is the total number of individuals observed in the data. One interpretation is as a
 461 multinomial model in which

$$O_k \sim Nq_k \quad \text{where} \quad q_k = C_k^n p^k (1-p)^{n-k}.
 \tag{28}$$

462 For the data for malaria corresponding to numbers of individuals, O_Γ , infected by dif-
463 ferent sets of pathogens, Γ , statistical independence corresponds to an n -parameter
464 multinomial model, parameterised by the prevalences of the individual pathogens p_i
465 (again fitted to the data), i.e.

$$O_\Gamma \sim N \prod_{i \in \Gamma} p_i \prod_{i \notin \Gamma} (1 - p_i). \quad (29)$$

466 **4.2.2 Fitting the models**

467 The host natural death rate, μ , can be scaled out of the equilibrium prevalences
468 by rescaling time. Fitting the models therefore corresponds to finding value(s) for
469 scaled infection rate(s) β_i , i.e. $R_{0,i} = \beta_i/\mu$ (all are equal for the NiSP model).

470 The method used to fit the model does not depend on whether the data are
471 numbers of hosts infected by a particular combination of pathogens, or numbers of
472 hosts carrying particular numbers of distinct pathogens, since both can be viewed
473 as N samples drawn from a multinomial distribution, with q_j observations of the j^{th}
474 class. If the corresponding probabilities generated by the model being fitted are p_j ,
475 then the log-likelihood is

$$L = \sum_j q_j \log(p_j). \quad (30)$$

476 The models were fitted by maximizing L via `optim()` in R (R Core Team, 2016).
477 Convergence to a plausible global maximum was checked by repeatedly refitting
478 the model from randomly chosen starting sets of parameters. All models were fitted
479 in a transformed form to allow only biologically-meaningful values of parameters;
480 that is, the basic reproduction numbers were estimated after transformation with
481 $\log(R_{0,i} - 1)$ to ensure $R_{0,i} > 1$.

482 **4.2.3 Model comparison**

483 To compare the best-fitting NiSP or NiDP model and an appropriate model assuming
484 statistical independence (binomial or multinomial), we use the Akaike Information
485 Criterion $AIC = 2k - 2\hat{L}$, in which \hat{L} is the log-likelihood of the best-fitting version of
486 each model and k is the number of model parameters. This is necessary since these
487 comparisons involve pairs of models that are not nested.

488 **4.2.4 Goodness-of-fit**

489 We use a Monte-Carlo technique to estimate p -values for model goodness-of-fit, gen-
490 erating 1,000,000 independent sets of samples of total size N from the multino-
491 mial distribution corresponding to the best-fitting model, calculating the likelihood
492 (Eq. (30)) of each of these synthetic data sets, and recording the proportion with a
493 smaller value of L than the value calculated for the data (Sokal and Rohlf, 2012). This
494 was done using the function `xmonte()` in the R package `XNomial` (Engels, 2015).

495 **4.3 Sources of data and results of model fitting**

496 **4.3.1 Numbers of distinct pathogens (NiSP model)**

497 Results of fitting the NiSP model to data from four publications for strains of a single
498 pathogen are presented in Figure 3. Error bars are 95% confidence intervals using
499 exact methods for binomial proportions via `binconf()` in the R package `Hmisc` (Har-
500 rell Jr et al., 2016). Results for three further data sets concerning different pathogens
501 of a single host (Andersson et al., 2013; Moutailler et al., 2016; Nickbakhsh et al.,
502 2016) are provided as Supplementary Information (Section S2.1).

503 For convenience the raw data as extracted for use in model fitting are re-tabulated
504 in Table 2. Results of model fitting are summarized in Table 3. We used the value
505 $n = 102$ for the number of distinct strains in López-Villavicencio et al. (2007) follow-
506 ing personal communication with the authors; there might be undetected genetic
507 differences due to missing data – which would require a larger value of n in our
508 model fitting procedure – but we confirmed that our inferences are unaffected by
509 taking any value of $n \in [100, 200]$.

510 **4.3.2 Combinations of pathogens (NiDP model)**

511 Howard et al. (2001) report results of analyzing 73 data sets concerning multiple
512 *Plasmodium* spp. causing malaria (rows 68–140 of Table 1 in that paper). We re-
513 analyzed the subset of these studies satisfying certain additional constraints as de-
514 tailed in the main text (see Supplementary Information: Section S2.2 for a full de-
515 scription of how the studies were filtered). This left a final total of 41 data sets taken
516 from 35 distinct papers: 24 data sets considering the three-way interaction between

Table 2: Sources of data for fitting the NiSP model in which pathogen types, clones or strains are assumed to be epidemiologically-interchangeable. The data sets include human papillomavirus (Chaturvedi et al., 2011), anther smut (*M. violaceum*) (López-Villavicencio et al., 2007), *Borrelia afzelii* on bank voles (Andersson et al., 2013), and malaria (*Plasmodium vivax*) (Koeplfi et al., 2011).

Pathogens with n distinct types, strains or clones	n	Observed counts, O_k										Total N
		0	1	2	3	4	5	6	7	8	9	
Human papillomavirus	25	2933	1409	646	267	102	39	12	2	2	-	5412
Anther smut (<i>M. violaceum</i>)	102	285	74	60	32	14	3	3	2	1	1	475
<i>Borrelia afzelii</i> on bank voles	7	807	33	26	13	10	11	6	-	-	-	906
Malaria (<i>Plasmodium vivax</i>)	57	1023	404	291	208	118	50	16	5	1	1	2117

Table 3: Fitting the NiSP model. The NiSP model was highly supported over the binomial model ($\Delta AIC \gg 10$) in all cases tested. The final column of the table – GoF – corresponds to the goodness-of-fit test of the NiSP model; values $p > 0.05$ correspond to lack of evidence for failure to fit the data, and so the NiSP model is adequate for the data concerning human papillomavirus (Chaturvedi et al., 2011).

	NiSP		Binomial		$\Delta AIC=2\Delta L$	GoF p
	R_0	L	p	L		
Human papillomavirus	1.032	-6580.9	0.031	-6868.8	575.8	0.077
Anther smut (<i>M. violaceum</i>)	1.008	-614.0	0.008	-693.3	158.6	0.001
<i>Borrelia afzelii</i> on bank voles	1.044	-652.1	0.040	-799.0	293.8	0.000
Malaria (<i>Plasmodium vivax</i>)	1.021	-3169.2	0.021	-3467.3	596.3	0.000

517 *P. falciparum*, *P. malariae* and *P. vivax* and 17 data sets considering the three-way
518 interaction between *P. falciparum*, *P. malariae* and *P. ovale*.

519 We used our method based on the NiDP model to test whether any of these data
520 sets were consistent with no interaction between the *Plasmodium* spp. considered
521 (Table 4). We found 15 data sets for which the NiDP model was: i) a better fit than
522 the multinomial model as indicated by $\Delta AIC \geq 2$; ii) sufficient to explain the data
523 as revealed by our goodness-of-fit test. In these 15 cases our methods therefore
524 support the hypothesis of no interaction. For 11 of these 15 data sets (76, 109, 118,
525 130, 132, 68, 69, 70, 79, 95, 97, 98, 99, 100, 102) the results as reported in (Howard
526 et al., 2001) instead suggest the strains interact.

527 **5 Data availability**

528 The cross-sectional survey data extracted from previous publications which we have
529 used to test our methodology are tabulated in Table 2 in the main text, and Tables
530 S2 and S4 in the Supplementary Information. These data are available in electronic
531 format as .csv files from the corresponding author upon reasonable request.

Table 4: Fitting the NiDP model. Data sets which are consistent with no interaction between the *Plasmodium* spp. considered are highlighted in grey. Such data sets have both p -values for the goodness-of-fit test of the NiDP model $p(\text{GoF}) > 0.05$, and $\Delta\text{AIC} \geq 2$, meaning the NiDP model is adequate. The multinomial model corresponds to the statistical independence hypothesis. Parameters $R_{0,1}$ and $R_{0,2}$ are associated with *P. falciparum* and *P. malariae*, respectively. Parameter $R_{0,3}$ corresponds either to *P. vivax* (upper part of the table, data sets 74–137) or to *P. ovale* (lower part of the table, data sets 68–103). The final column contains a tick whenever at least one association between a pair of pathogens was assessed to be significant in Howard et al. (2001). Red ticks correspond to possible statistical associations that are consistent with our no-interaction model (NiDP), i.e. cases in which our methods lead to results diverging from those reported in Howard et al. (2001).

	NiDP					Multinomial					ΔAIC	Association(s) in Howard et al. (2001)
	$R_{0,1}$	$R_{0,2}$	$R_{0,3}$	L	$p(\text{GoF})$	p_1	p_2	p_3	L	$p(\text{GoF})$		
74	1.764	1.256	1.004	-340.7	0.000	0.468	0.220	0.004	-311.0	0.000	-59.3	✓
75	1.694	1.248	1.022	-194.8	0.000	0.445	0.215	0.022	-177.4	0.000	-34.8	✓
76	1.235	1.019	1.005	-492.5	0.251	0.190	0.019	0.005	-493.7	0.098	2.4	✓
82	1.776	1.165	1.108	-996.0	0.000	0.463	0.147	0.101	-936.3	0.000	-119.2	✓
84	1.212	1.017	1.207	-684.2	0.000	0.180	0.017	0.177	-660.4	0.000	-47.6	✓
88	1.296	1.120	1.260	-314.7	0.000	0.242	0.111	0.217	-295.0	0.000	-39.3	✓
106	1.818	1.146	1.055	-4105.2	0.000	0.442	0.125	0.052	-4296.6	0.000	382.9	✓
108	1.241	1.024	1.096	-1147.5	0.000	0.197	0.023	0.089	-1132.1	0.721	-30.9	✗
109	1.023	1.013	1.045	-359.3	0.866	0.023	0.013	0.043	-361.1	0.343	3.5	✓
111	1.198	1.005	1.786	-1929.2	0.000	0.175	0.005	0.467	-1798.8	0.000	-260.7	✓
112	1.307	1.086	1.056	-119.6	0.115	0.241	0.080	0.054	-116.6	0.552	-6.0	✗
113	1.213	1.007	1.119	-1324.1	0.000	0.179	0.007	0.108	-1290.8	0.000	-66.6	✓
114	1.615	1.084	1.038	-1224.4	0.000	0.392	0.080	0.037	-1182.6	0.000	-83.6	✓
116	1.780	1.124	1.100	-1035.1	0.000	0.471	0.116	0.094	-953.5	0.000	-163.2	✓
117	1.072	1.000	1.268	-31530.5	0.000	0.068	0.000	0.214	-30958.7	0.000	-1143.5	✓
118	1.085	1.039	1.171	-225.3	0.990	0.078	0.037	0.146	-227.5	0.515	4.5	✓
119	1.433	1.164	1.375	-265.7	0.000	0.325	0.146	0.291	-249.0	0.146	-33.6	✓
123	1.016	1.055	1.098	-6684.7	0.000	0.016	0.052	0.090	-6623.5	0.000	-122.4	✓
124	1.254	1.100	1.082	-3600.6	0.000	0.206	0.092	0.076	-3541.3	0.017	-118.7	✓
127	1.341	1.005	1.266	-1087.4	0.000	0.265	0.005	0.219	-1039.0	0.000	-96.8	✓
130	1.013	1.002	1.350	-352.7	0.978	0.013	0.002	0.259	-353.7	0.636	2.0	✗
132	1.397	1.027	1.074	-591.8	0.347	0.285	0.026	0.068	-594.3	0.067	4.9	✓
133	1.571	1.022	1.332	-687.9	0.000	0.375	0.022	0.257	-676.2	0.001	-23.4	✓
137	1.196	1.005	1.130	-2356.8	0.000	0.166	0.005	0.117	-2309.6	0.000	-94.3	✓
68	1.910	1.091	1.021	-152.0	0.200	0.469	0.082	0.020	-157.8	0.002	11.7	✓
69	4.827	1.443	1.036	-177.2	0.822	0.796	0.310	0.035	-181.4	0.121	8.5	✓
70	4.612	1.203	1.089	-239.2	0.953	0.781	0.168	0.082	-247.4	0.012	16.4	✓
71	6.070	1.370	1.181	-310.1	0.001	0.822	0.261	0.148	-336.2	0.000	52.1	✗
77	14.275	1.383	1.142	-155.3	0.032	0.944	0.286	0.127	-150.4	0.931	-9.9	✗
78	4.171	1.178	1.006	-166.2	0.264	0.773	0.153	0.006	-163.2	0.997	-5.8	✗
79	1.855	1.033	1.005	-1260.1	0.969	0.461	0.032	0.005	-1263.4	0.224	6.6	✓
80	1.546	1.062	1.021	-715.1	0.735	0.355	0.059	0.020	-715.2	0.675	0.2	✓
95	1.855	1.033	1.005	-1260.1	0.970	0.461	0.032	0.005	-1263.4	0.224	6.6	✗
96	1.910	1.071	1.017	-240.4	0.019	0.469	0.065	0.016	-248.9	0.000	17.1	✗
97	1.952	1.077	1.004	-242.6	0.568	0.486	0.071	0.004	-246.7	0.031	8.3	✓
98	1.662	1.014	1.018	-183.7	0.373	0.396	0.013	0.018	-187.0	0.030	6.6	✗
99	1.627	1.019	1.019	-133.7	0.823	0.384	0.019	0.019	-135.6	0.332	3.8	✗
100	1.037	1.003	1.000	-432.1	0.254	0.035	0.003	0.000	-433.2	0.083	2.3	✓
101	3.590	1.269	1.063	-11014.1	0.000	0.720	0.211	0.060	-11392.7	0.000	757.3	✓
102	2.473	1.153	1.027	-8188.9	0.403	0.595	0.132	0.027	-8352.0	0.000	326.2	✓
103	1.798	1.180	1.015	-7425.7	0.000	0.437	0.150	0.015	-7736.6	0.000	621.8	✓

532 **6 Code availability**

533 Code illustrating all statistical methods is freely available at:
534 <https://github.com/nikcunniffe/Coinfection>.

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