An active inference approach to modeling concept learning

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Abstract

Within computational neuroscience, the algorithmic and neural basis of concept learning remains poorly understood. Concept learning requires both a type of internal model expansion process (adding novel hidden states that explain new observations), and a model reduction process (merging different states into one underlying cause and thus reducing model complexity via meta-learning). Although various algorithmic models of concept learning have been proposed within machine learning and cognitive science, many are limited to various degrees by an inability to generalize, the need for very large amounts of training data, and/or insufficiently established biological plausibility. In this paper, we articulate a model of concept learning based on active inference and its accompanying neural process theory, with the idea that a generative model can be equipped with extra (hidden state or cause) 'slots' that can be engaged when an agent learns about novel concepts. This can be combined with a Bayesian model reduction process, in which any concept learning – associated with these slots – can be reset in favor of a simpler model with higher model evidence. We use simulations to illustrate this model's ability to add new concepts to its state space (with relatively few observations) and increase the granularity of the concepts it currently possesses. We further show that it accomplishes a simple form of 'one-shot' generalization to new stimuli. Although deliberately simple, these results suggest that active inference may offer useful resources in developing neurocomputational models of concept learning.

Keywords: Model Expansion; Structure Learning; Concepts; Computational Neuroscience; Active Inference

Introduction

The ability to conceptualize and understand regularly observed patterns in co-occurring feature observations is central to human cognition. For example, we do not simply observe particular sets of colors, textures, shapes, and sizes – we also observe *identifiable objects* such as, say, a 'screwdriver'. If we were tool experts, we might also recognize particular types of screwdrivers (e.g., flat vs. Phillip's head), designed for a particular use. This ability to recognize co-occurring features under conceptual categories (as opposed to just perceiving sensory qualities; e.g., red, round, etc.) is also highly adaptive. Only if we know an object is a screwdriver could we efficiently infer that it affords putting certain structures together and taking them apart; and only if we know the specific type of screwdriver could we efficiently infer, say, the artefacts to use it on. Many concepts of this sort require experience-dependent acquisition (i.e., learning).

From a computational perspective, the ability to acquire a new concept can be seen as a type of Bayesian model comparison or structure learning (Botvinick, Niv, & Barto, 2009; S. J. Gershman & Niv, 2010; MacKay & Peto, 1995; Salakhutdinov, Tenenbaum, & Torralba, 2013; Tervo, Tenenbaum, & Gershman, 2016). Specifically, concept acquisition can be cast as an agent learning (or inferring) that a new hypothesis (referred to here as a hidden cause or state) should be added to the internal or generative model with which she explains her environment, because existing causes cannot account for new observations (e.g., an agent might start out believing that the only tools are hammers and screwdrivers, but later learn that there are also wrenches). In other words, the structure of the space of hidden causes itself needs to expand to accommodate new patterns of observations. This model expansion process is complementary to a process called Bayesian model reduction (Karl Friston & Penny, 2011); in which the agent can infer that there is redundancy in her model, and a model with fewer states or parameters provides a more parsimonious (i.e. simpler) explanation of observations (KJ Friston, Lin, et al., 2017; Schmidhuber, 2006). For example, in some instances it may be more appropriate to differentiate between fish and birds as opposed to salmon, peacocks and pigeons. This reflects a reduction in model complexity based on a particular feature space underlying observations and thus resonates with other accounts of concept learning as dimensionality reduction (Behrens et al., 2018; Stachenfeld, Botvinick, & Gershman, 2016) – a topic we discuss further below.

A growing body of work in a number of domains has approached this problem from different angles. In developmental psychology and cognitive science, for example, probability theoretic (Bayesian) models have been proposed to account for word learning in children and the remarkable human ability to generalize from very few (or even one) examples in which both broader and narrower categorical referents could be inferred (Kemp, Perfors, & Tenenbaum, 2007; Lake, Salakhutdinov, & Tenenbaum, 2015; Perfors, Tenenbaum, Griffiths, & Xu, 2011; Xu & Tenenbaum, 2007a, 2007b). In statistics, a number of nonparametric Bayesian models, such as the “Chinese Room” process and the “Indian Buffet” process, have been used to infer the need for model expansion (S. Gershman & Blei, 2012). There are also related approaches in machine learning, as applied to things like Gaussian mixture models (McNicholas, 2016).

These are the generative models that underwrite various clustering algorithms that take sets of data points in a multidimensional space and divide them into spatially separable clusters. While many of these approaches assume the number of clusters is known in advance, various goodness-of-fit criteria may be used to determine the optimal number. However, a number of approaches require much larger amounts of data than humans do to learn new concepts (Geman, Bienenstock, & Doursat, 1992; Hinton et al., 2012; LeCun, Bengio, & Hinton, 2015; Lecun, Bottou, Bengio, & Haffner, 1998; Mnih et al., 2015). Many also require corrective feedback to learn and yet fail to acquire sufficiently rich conceptual structure to allow for generalization (Barsalou, 1983; Biederman, 1987; Feldman, 1997; Jern & Kemp, 2013; A. B. Markman & Makin, 1998; Osherson & Smith, 1981; Ward, 1994; Williams & Lombrozo, 2010).

One potentially fruitful research avenue that has not yet been examined is to explore concept learning from the perspective of Active Inference models based on the free-energy principle (KJ Friston, 2010; KJ Friston et al., 2016; KJ Friston, Lin, et al., 2017; KJ Friston, Parr, & de Vries, 2017). In this paper, we explore the potential of this approach. In brief, we conclude that structure learning is an emergent property of active inference (and learning) under generative models with ‘spare capacity’; where spare or uncommitted capacity is used to expand the repertoire of representations (Baker & Tenenbaum, 2014), while Bayesian model reduction (KJ Friston, Lin, et al., 2017; Hobson & Friston, 2012) promotes generalization by minimizing model complexity – and releasing representations to replenish ‘spare capacity’.

In what follows, we first provide a brief overview of active inference. We then introduce a model of concept learning (using basic and subordinate level animal categories) to produce synthetic cognitive (semantic) processes associated with adding new concepts to a state space and increasing the granularity of an existing state space. We then establish the validity of this model using numerical analyses of concept learning when repeatedly presenting a synthetic agent with different animals characterized by different combinations of observable features. We will demonstrate how particular approaches combining Bayesian model reduction and expansion can reproduce successful concept learning without the need for corrective feedback, and allow for generalization. We conclude with a brief discussion of the implications of this work.

An Active Inference model of concept learning

A primer on Active Inference

Active Inference suggests that the brain is an inference machine that approximates optimal probabilistic (Bayesian) belief updating across perceptual, cognitive, and motor domains. Active Inference more specifically postulates that the brain embodies an internal model of the world that is "generative" in the sense that it can simulate the sensory data that it should receive if its model of the world is correct. These simulated (predicted) sensory data can be compared to actual observations, and differences between predicted and observed sensations can be used to update the model. Over short timescales (e.g., a single observation) this updating corresponds to inference (perception), whereas on longer timescales it corresponds to learning (i.e., updating expectations about what will be observed later). Another way of putting this is that perception optimizes beliefs about the current state of the world, while learning optimizes beliefs about the relationships between the variables that constitute the world. These processes can be seen as ensuring the generative model (entailed by recognition processes in the brain) remains an accurate model of the world that it seeks to regulate (Conant & Ashbey, 1970).

Active Inference casts decision-making in similar terms. Actions can be chosen to resolve uncertainty about variables within a generative model (i.e., sampling from domains in which the model does not make precise predictions), which can prevent anticipated deviations from predicted outcomes. In addition, some expectations are treated as a fixed phenotype of an organism. For example, if an organism did not continue to "expect" to observe certain amounts of food, water, and shelter, then it would quickly cease to exist (McKay & Dennett, 2009) – as it would not pursue those behaviors that fulfill these expectations (c.f. the 'optimism bias' (Sharot, 2011)). Thus, a creature should continually seek out observations that support – or are internally consistent with – its own continued existence. Decision-making can therefore be cast as a process in which the brain infers the sets of actions (policies) that would lead to observations consistent with its own survival-related expectations (i.e., its "prior preferences"). Mathematically, this can be described as selecting sequences of actions (policies) that maximize "Bayesian model evidence" expected under a policy, where model evidence is the (marginal) likelihood that particular sensory inputs would be observed under a given model.

In real-world settings, directly computing Bayesian model evidence is generally intractable. Thus, some approximation is necessary. Active Inference proposes that the brain computes a quantity called "variational free energy" that provides a bound on model evidence, such that minimization of free energy indirectly maximizes model evidence (this is exactly the same functional used in machine learning where it is known as an evidence lower bound or ELBO). In this case, decision-making will be approximately (Bayes) optimal if it infers (and enacts) the policy that will minimize expected free energy (i.e., free energy with respect to a policy, where one takes expected future observations into account). Technically, expected free energy is the variational free energy averaged under the posterior predictive density over policy-specific outcomes.

Expected free energy can be decomposed in different ways that reflect uncertainty and prior preferences, respectively (e.g., epistemic and instrumental affordance or ambiguity and risk). This formulation means that any agent that minimizes expected free energy engages initially in exploratory behavior to minimise uncertainty in a new environment. Once uncertainty is resolved, the agent then exploits that environment to fulfil its prior preferences. The formal basis for Active Inference has been thoroughly detailed elsewhere (KJ Friston, FitzGerald, Rigoli, Schwartenbeck, & Pezzulo, 2017), and the reader is referred there for a full mathematical treatment.

When the generative model is formulated as a partially observable Markov decision process (a mathematical framework for modeling decision-making in cases where some outcomes are under the control of the agent and others are not, and where states of the world are not directly known but must be inferred from observations), active inference takes a particular form. Here, the generative model is specified by writing down plausible or allowable policies, hidden states of the world (that must be inferred from observations), and observable outcomes, as well as a number of matrices that define the probabilistic relationships between these quantities (see right panel of figure 1).

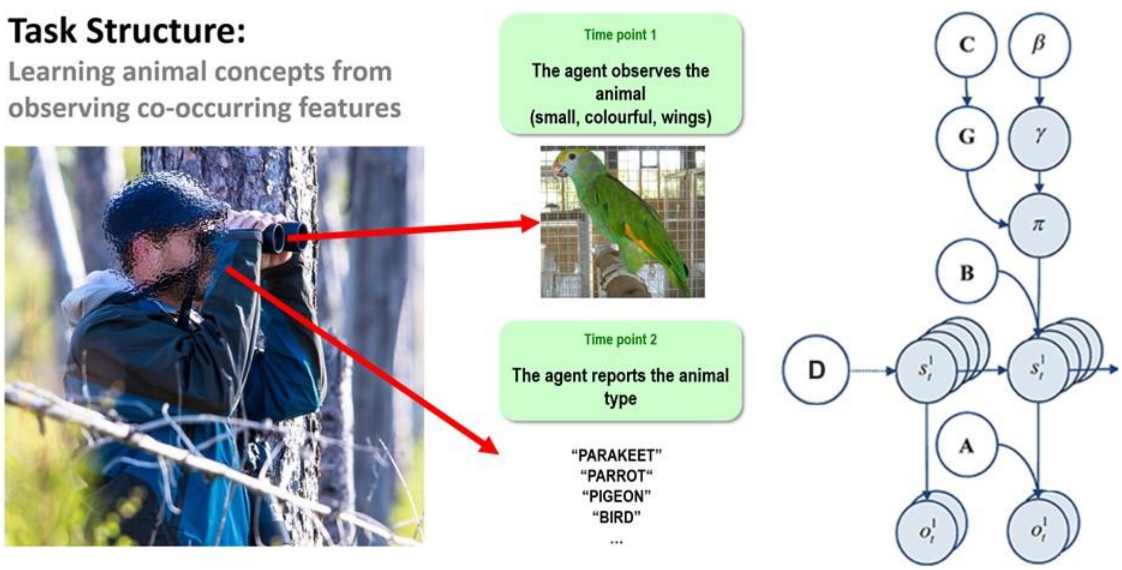


Figure 1. Left: Illustration of the trial structure performed by the agent. At the first time point, the agent is exposed to one of 8 possible animals that are each characterized by a unique combination of visual features. At the 2nd time point, the agent would then report which animal concept matched that feature combination. The agent could report a specific

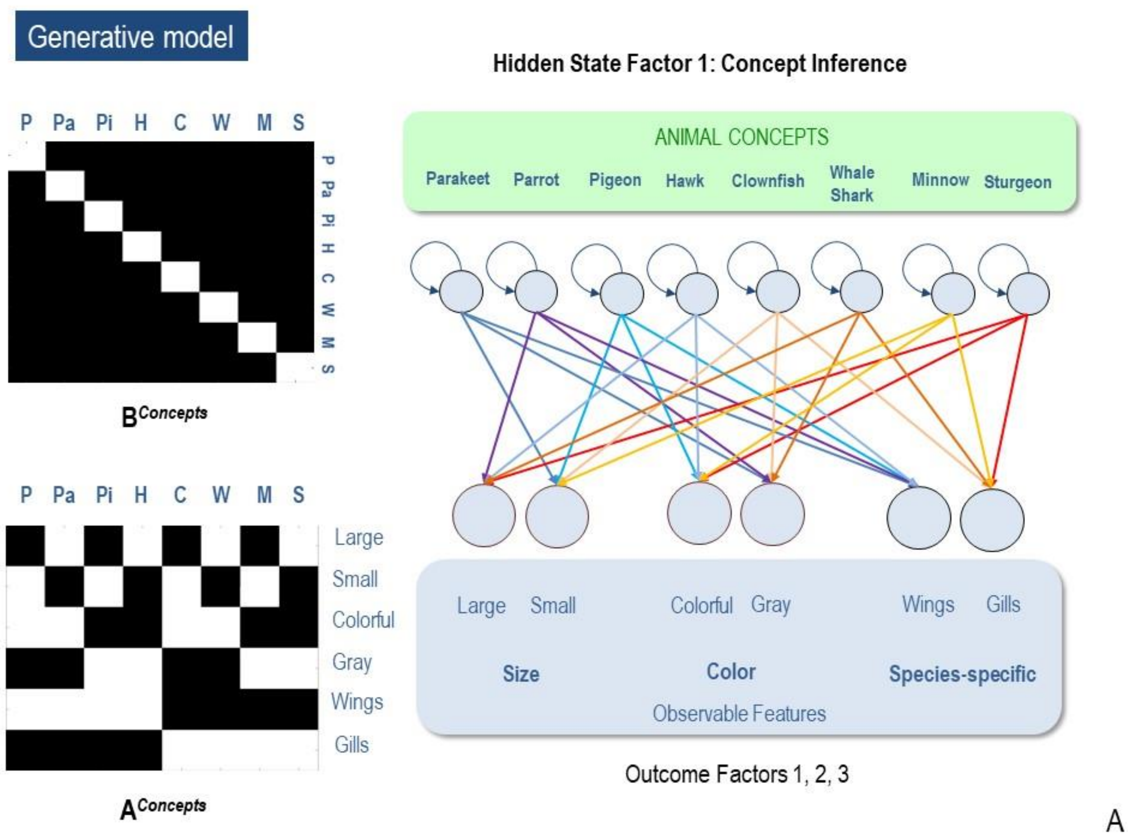
category (e.g., pigeon, hawk, minnow, etc.) or a general category (i.e., bird or fish) if insufficiently certain about the specific category. See the main text for more details. Right: Illustration of the Markov decision process formulation of active inference used in the simulations described in this paper. The generative model is heredepicted graphically, such that arrows indicate dependencies between variables. Here observations ($\mathbf{o}$) depend on hidden states ($\mathbf{s}$), as specified by the **A** matrix, and those states depend on both previous states (as specified by the **B** matrix, or the initial states specified by the **D** matrix) and the policies ($\boldsymbol{\pi}$) selected by the agent. The probability of selecting a particular policy in turn depends on the expected free energy ($\mathbf{G}$) of each policy with respect to the prior preferences ($\mathbf{C}$) of the agent. The degree to which expected free energy influences policy selection is also modulated by a prior policy precision parameter ($\boldsymbol{\gamma}$), which is in turn dependent on beta ($\boldsymbol{\beta}$) –where higher values of beta promote more randomness in policy selection (i.e., less influence of the differences in expected free energy across policies). For more details regarding the associated mathematics, see (KJ Friston, Lin, et al., 2017; KJ Friston, Parr, et al., 2017).

The ‘A’ matrix indicates which observations are generated by each combination of hidden states (i.e., the likelihood mapping specifying the probability that a particular set of observations would be observed given a particular set of hidden states). The ‘B’ matrix is a transition matrix, indicating the probability that one hidden state will evolve into another over time. The agent, based on the selected policy, controls some of these transitions (e.g., those that pertain to the positions of its limbs). The ‘D’ matrix encodes prior expectations about the initial hidden state the agent will occupy. Finally, the ‘C’ matrix specifies prior preferences over observations; it quantifies the degree to which different observed outcomes are rewarding or punishing to the agent. In these models, observations and hidden states can be factorized into multiple outcome *modalities* and hidden state *factors*. This means that the likelihood mapping (the ‘A’ matrix) can also model the interactions among different hidden states when generating outcomes (observations). In what follows, we describe how this type of generative model was specified to perform concept inference/learning.

A model of concept inference and learning

To model concept inference, we constructed a simple task for an agent to perform (see figure 1, left panel). In this task, the agent was presented with different animals on different trials and asked to answer a question about the type of animal that was seen. As described below, in some simulations the agent was asked to report the type of animal that was learned previously; in other simulations, the agent was instead asked a question that required conceptual generalization. Crucially, to answer these questions the agent was required to observe different animal features, where the identity of the animal depended on the combination of features. There were three feature categories (size, color, and species-specific; described further below) and two discrete time points in a trial (observe and report).

To simulate concept learning (based on the task described above) we need to specify an appropriate generative model. Once this model has been specified, one can use standard (variational) message passing to simulate belief updating and behavior in a biologically plausible way: for details, please see (KJ Friston, FitzGerald, et al., 2017; KJ Friston, Parr, et al., 2017). In our (minimal) model, the first hidden state factor included (up to) eight levels, specifying four possible types of birds and four possible types of fish (Figure 2A). The outcome modalities included: a feature space including two size features (big, small), two color features (gray, colorful), and two species-differentiating features (wings, gills). The 'A' matrix specified a likelihood mapping between features and animal concepts, such that each feature combination was predicted by an animal concept (Hawk, Pigeon, Parrot, Parakeet, Sturgeon, Minnow, Whale shark, Clownfish). This model was deliberately simple to allow for a clear illustration, but it is plausibly scalable to include more concepts and a much larger feature space. The 'B' matrix for the first hidden state factor was an identity matrix, reflecting the belief that the animal identity was conserved during each trial (i.e., the animals were not switched out mid-trial).



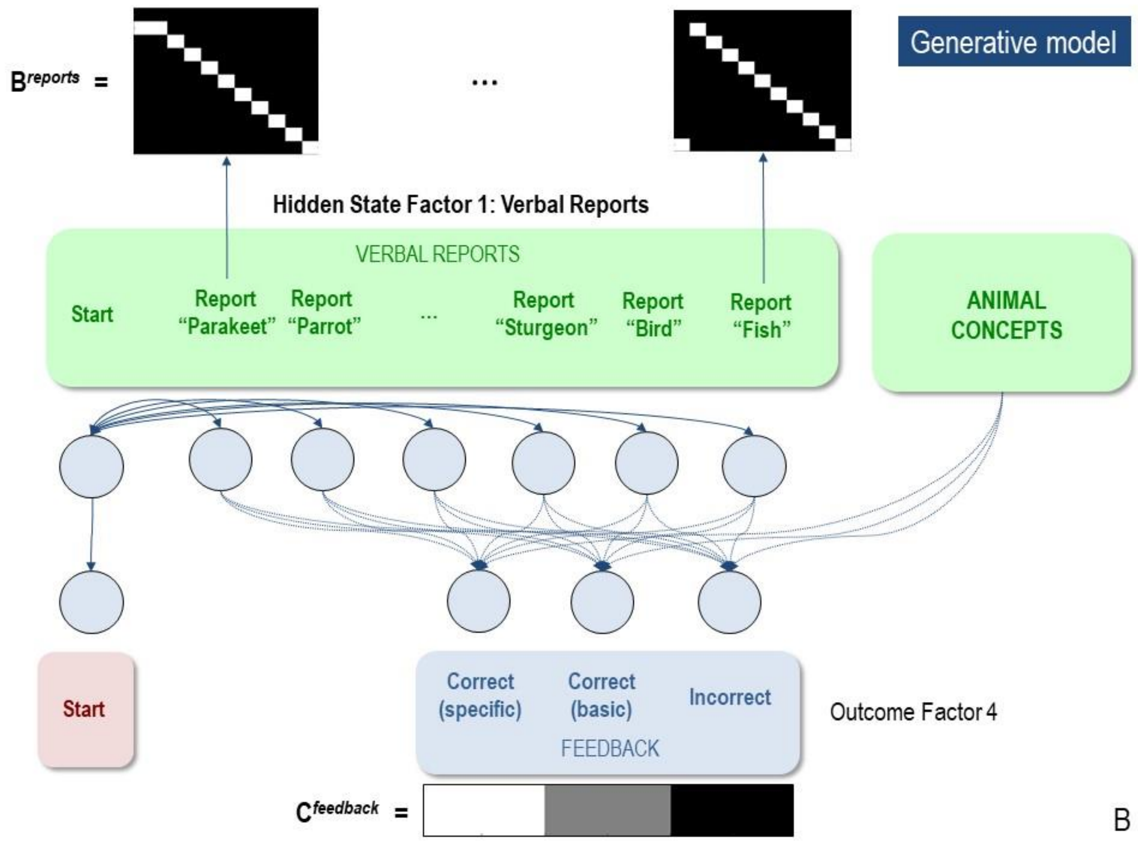


Figure 2. (A) Illustration of the first hidden state factor containing columns (levels) for 8 different animal concepts. Each of these 8 concepts generated a different pattern of visual feature observations associated with the outcome modalities of size, color, and species-specific features. The B matrix was an identity matrix, indicating that the animal being observed did not change within a trial (white = 1, black = 0). The A matrix illustrates the specific mapping from animal concepts to feature combinations. As depicted, each concept corresponded to a unique point in a 3-dimensional feature space. (B) illustration of the 2nd hidden state factor corresponding to the verbal reports the agent could choose in response to her observations. These generated feedback as to whether their verbal report was accurate with respect to a basic category report or a specific category report. As illustrated in the C matrix, the agent most preferred to be correct about specific categories, but least preferred being incorrect. Thus, reporting the basic categories was a safer choice if the agent was too uncertain about the specific identity of the animal.

The second hidden state factor was the agent's report. That this is assumed to factorise from the first hidden state factor means that there is no prior constraint that links the chosen report to the animal generating observations. The agent could report each of the eight possible specific animal categories, or opt for a less specific report of a bird or a fish. Only one category could be reported at any time. Thus, the agent had to choose to report only bird vs. fish or to report a more specific category. In other words, the agent could decide upon the appropriate level of coarse-graining of her responses (figure 2B).

During learning trials, the policy space was restricted such that the agent could not provide verbal reports or observe corrective feedback (i.e., all it could do is “stay still” in its initial state and observe the feature patterns presented). This allowed the agent to learn concepts in an unsupervised manner (i.e. without being told what the true state was or whether it was correct or incorrect). After learning, active reporting was enabled, and the ‘C’ matrix was set so that the agent preferred to report correct beliefs. We defined the preferences of the agent such that she preferred correctly reporting specific category knowledge and was averse to incorrect reports. This ensured that she only reported the general category of bird vs. fish, unless sufficiently certain about the more specific category.

In the simulations reported below, there were two time points in each trial of categorisation or conceptual inference. At the first time point, the agent was presented with the animals features, and always began in a state of having made no report (the “start” state). The agent’s task was simply to observe the features, infer the animal identity, and then report it (i.e., in reporting trials). Over 32 simulations (i.e., 4 trials per animal), we confirmed that, if the agent already started out with full knowledge of the animal concepts (i.e., a fully precise ‘A’ matrix), it would report the specific category correctly 100% of the time. Over an additional 32 simulations, we also confirmed that, if the agent was only equipped with knowledge of the distinction between wings and gills (i.e., by replacing the rows in the ‘A’ matrix corresponding to the mappings from animals to size and color with flat distributions), it would report the generic category correctly 100% of the time but would not report the specific categories.[1] This was an expected and straightforward consequence of the generative model – but provides a useful example of how agents trade off preferences and different types of uncertainty.

Simulating concept learning and the acquisition of expertise

Having confirmed that our model could successfully recognize animals if equipped with the relevant concepts (i.e., likelihood mappings) – we turn now to concept learning.

Concept acquisition

We first examined our model's ability to acquire concept knowledge in two distinct ways. This included 1) the agent's ability to "expand" (i.e., fill in an unused column within) its state space and add new concepts, and 2) the agent's ability to increase the granularity of its conceptual state space and learn more specific concepts, when it already possessed broader concepts.

Adding Concepts

[1] However, "risky" reporting behavior could be elicited by manipulating the strengths of the agent's preferences such that she placed a very high value on reporting specific categories correctly (i.e., relative to how much it disliked reporting incorrectly).

To assess whether our agent could expand her state space by acquiring a new concept, we first set one column of the previously described model's 'A' matrix (mapping an animal concept to its associated features) to be a uniform distribution[2]; creating an imprecise likelihood mapping for one concept – essentially, that concept predicted all features with nearly equal probability. Here, we chose sturgeon (large, gray, gills) as the concept for which the agent had no initial knowledge (see Figure 3A, right-most column of left-most 'pre-learning' matrix). We then generated 2000 observations based on the outcome statistics of a model with full knowledge of all eight animals (the "generative process"), to test whether the model could learn the correct likelihood mapping for sturgeon (note: this excessive number of observations was used for consistency with later simulations, in which more concepts have to be learned, and also to evaluate how performance improved as a function of the number of observations the agent was exposed to; see figure 3B).

In these simulations, learning was implemented via updating (concentration) parameters for the model's 'A' matrix after each trial. For details of these free energy minimizing learning processes, please see (KJ Friston et al., 2016) as well as the left panel of Figure 8 and the associated legend further below. An intuitive way to think about this belief updating process is that the strength of association between a concept and an observation is quantified simply by counting how often they are inferred to co-occur. This is exactly the same principle that underwrites Hebbian plasticity and long-term potentiation (Brown, Zhao, & Leung, 2010). Crucially,

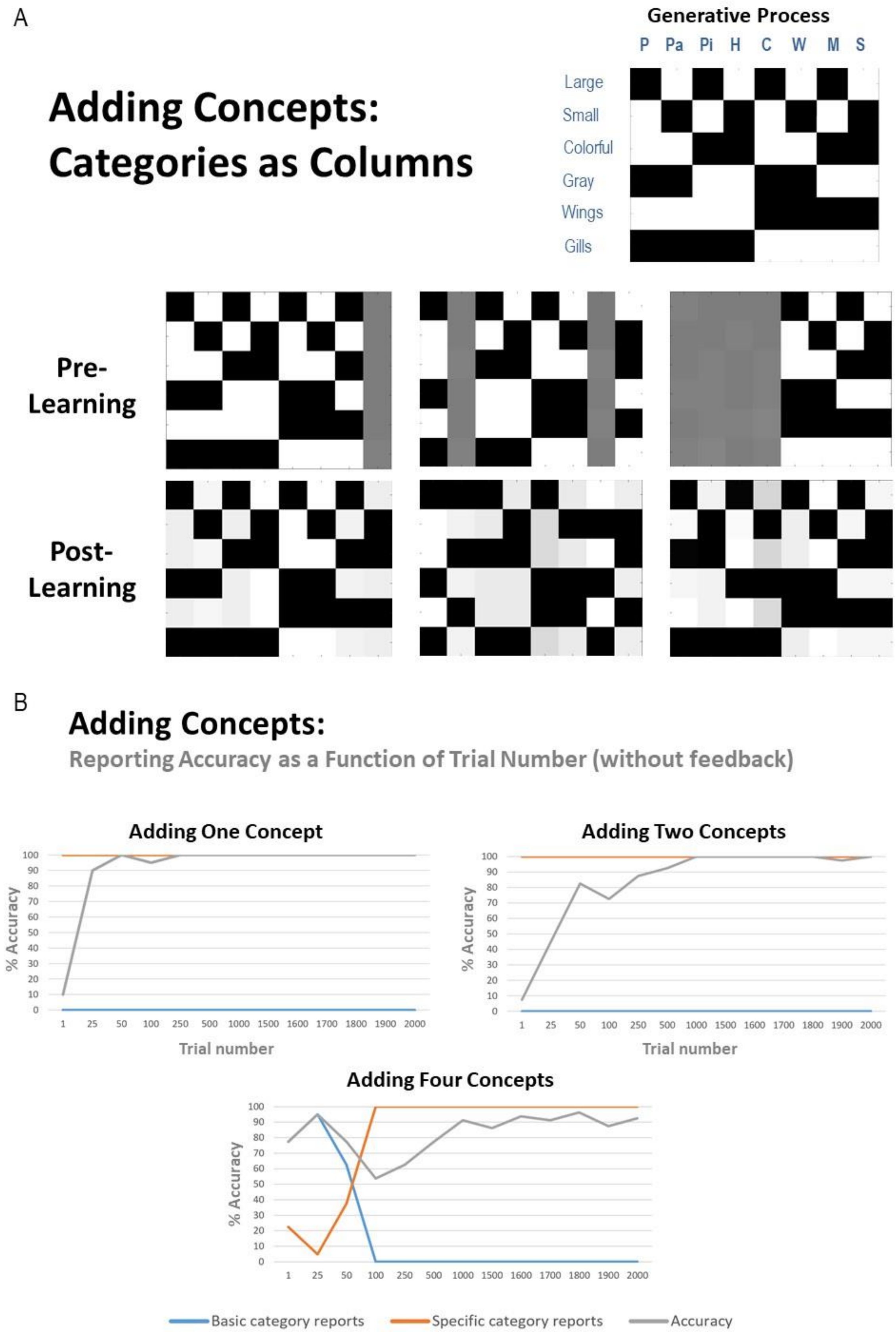
[2] To break the symmetry of the uniform distribution, we added small amounts of Gaussian noise (with a variance of .001) to avoid getting stuck in local free energy minima during learning.

policies were restricted during learning, such that the agent could not select reporting actions; thus, learning was driven entirely by repeated exposure to different feature combinations. We evaluated successful learning in two ways. First, we compared the 'A' matrix learned by the model to that of the generative process. Second, we disabled learning after various trial numbers (i.e., such that concentration parameters no longer accumulated) and enabled reporting. We then evaluated reporting accuracy with 20 trials for each of the 8 concepts.

As shown in Figure 3A, the 'A' matrix (likelihood) mapping – learned by the agent – and the column for sturgeon in particular, strongly resembled that of the generative process. When first evaluating reporting, the model was 100 % accurate across 20 reporting trials, when exposed to a sturgeon (reporting accuracy when exposed to each of the other animals also remain at 100%) and first reached this level of accuracy after around 50 exposures to all 8 animals (with equal probability) (figure 3B). The agent also always chose to report specific categories (i.e., it never chose to only report bird or fish). Model performance was stable over 8 repeated simulations.

Crucially, during learning, the agent was not told which state was generating its observations. This meant that it had to solve both an inference and a learning problem. First, it had to infer whether a given feature combination was better explained by an existing concept, or by a concept that predicts features uniformly. In other words, it had to decide that the features were sufficiently different – from things it had seen before – to assign it a new hypothetical concept. Given that a novel state is only inferred when another state is not a better explanation, this precludes learning 'duplicate' states that generate the same patterns of observations. The second problem is simpler. Having inferred that these outcomes are caused by something new, the problem becomes one of learning a simple state-outcome mapping through accumulation of Dirichlet parameters.

To examine whether this result generalized, we repeated these simulations under conditions in which the agent had to learn more than one concept. When the model needed to learn one bird (parakeet) and one fish (minnow), the model was also able to learn the appropriate likelihood mapping for these 2 concepts (although note that, because the agent did not receive feedback about the state it was in during learning, the new feature mappings were often not assigned to the same columns as in the generative process; see figure 3A). Reporting also reached 100% accuracy, but required a notably greater number of trials. Across 8 repeated simulations, the mean accuracy reached by the model after 2000 trials was 98.75% (SD = 2%).


A
Adding Concepts:
Categories as Columns
Generative Process
P Pa Pi H C W M S
Large
Small
Colorful
Gray
Wings
Gills
Pre-
Learning
Post-
Learning
B
Adding Concepts:
Reporting Accuracy as a Function of Trial Number (without feedback)
Adding One Concept
Adding Two Concepts
Adding Four Concepts
% Accuracy
Trial number
1 25 50 100 250 500 1000 1500 1600 1700 1800 1900 2000
Basic category reports
Specific category reports
Accuracy

375

376

Figure 3. (A) illustration of representative simulation results in which the agent successfully learned 1, 2, or 4 new animal concept categories with no prior knowledge beforehand. The generative process is shown in the upper right, illustrating the feature combinations to be learned. Pre-learning, either 1, 2 or 4 columns in the likelihood mapping began as a flat distribution with a slight amount of Gaussian noise. The agent was and then provided with 2000 observations of the 8 animals with equal probability. Crucially, the agent was prevented from providing verbal reports during these 2000 trials and thus did not receive feedback about the true identity of the animal. Thus learning was driven completely by repeated exposure in an unsupervised manner. Also note that, while the agent was successful at learning the new concepts, it did not always assign the new feature patterns to the same columns as illustrated in the generative process. This is to be expected given that the agent received no feedback about the true hidden state that generated her observations. (B) illustration of how reporting accuracy, and the proportion of basic category and specific category responses, changed as a function of repeated exposures. This was accomplished by taking the generative model at a number of representative trials and then testing it with 20 observations of each animal in which reporting was enabled. As can be seen, maximal accuracy was achieved much more quickly when the agent had to learn fewer concepts. When it had learned 4 concepts, it also began by reporting the general categories and then subsequently became sufficiently confident to report the more specific categories.

When the model needed to learn all 4 birds, performance varied somewhat more when the simulations were repeated. The learned likelihood mappings after 2000 trials always resembled that of the generative process, but with variable levels of precision; notably, the model again assigned different concepts to different columns relative to the generative process, as would be expected when the agent is not given feedback about the state she is in. Over 8 repeated simulations, the model performed well in 6 (92.50 % – 98.8 % accuracy) and failed to learn one concept in the other 2 (72.50 % accuracy in each) due to overgeneralization (e.g., mistaking parrot for Hawk in a majority of trials; i.e., the model simply learned that there are large birds). Figure 3A and 3B illustrate representative results when the model was successful (note: the agent never chose to report basic categories in the simulations where only 1 or 2 concepts needed to be learned).

To further assess concept learning, we also tested the agent's ability to successfully avoid state duplication. That is, we wished to confirm that the model would only learn a new concept if actually presented with a new animal for which it did not already have a concept. To do so, we equipped the model with knowledge of seven out of the eight concept categories, and then repeatedly exposed it only to the animals it already knew over 80 trials. We subsequently exposed it to the eighth animal (Hawk) for which it did not already have knowledge over 20 additional trials. As can be seen in figure 4, the unused concept column was not engaged during the first 80 trials (bottom left and middle). However, in the final 20 trials, the agent correctly inferred that her current conceptual repertoire was unable to explain her new pattern of observations, leading the unused concept column to be adumbrated and the appropriate state-observation mapping to be learned (bottom right). We repeated these simulations under conditions in which the agent already had knowledge of six, five, or four concepts. In all cases, we observed that unused concept columns were never engaged inappropriately.

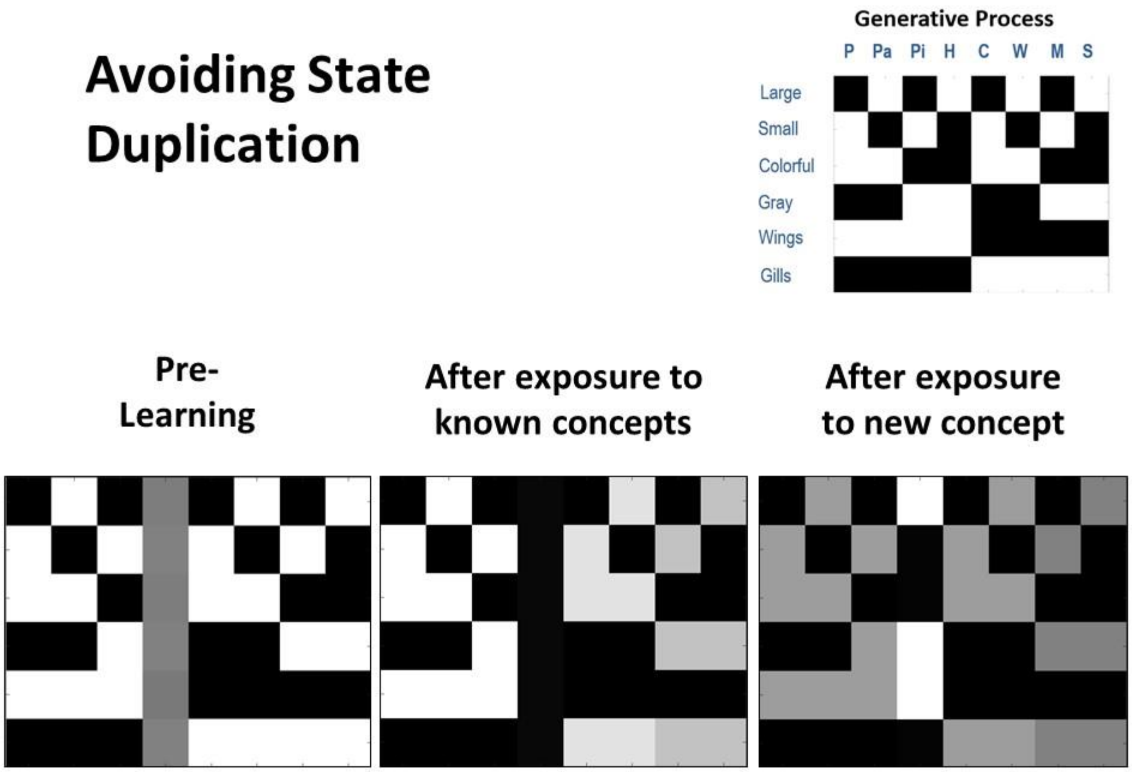


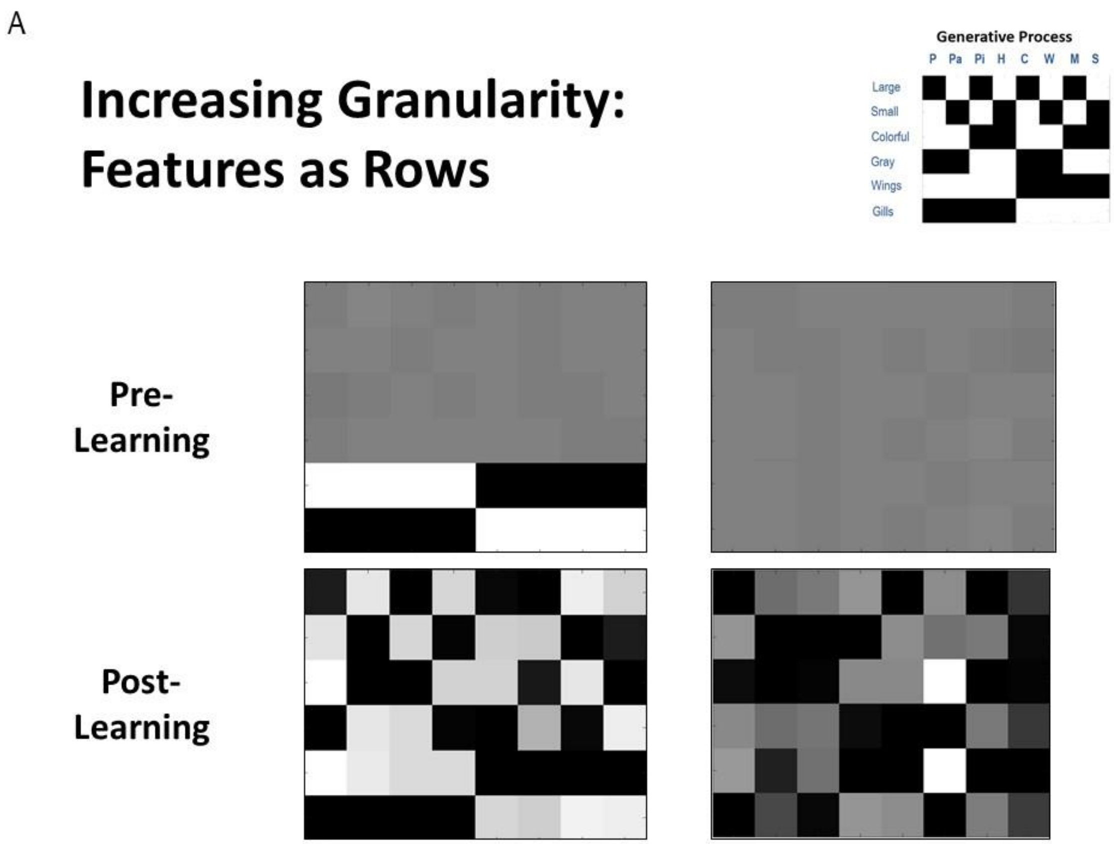
Figure 4. Illustration of representative simulation results when the agent had to avoid inappropriately learning a new concept (i.e., avoid state duplication)after only being exposed to animals for which it already had knowledge. Here the agent began with prior knowledge about seven concept categories, and was also equipped with an eighth column that could be engaged to learn a new concept category (bottom left). The agent was then presented with several instances of each of the seven animals that she already knew (80 trials in total). In this simulation, the agent was successful in assigning each stimulus to an animal concept she had already acquired, and did not engage the unused concept 'slot' (bottom middle). Finally, the agent was presented with a new animal (a hawk) that she did not already know over 20 trials. In this case, the agent successfully engaged the additional column (i.e., she inferred that none of the concepts she possessed could account for her new observations), and learn the correct state-observation mapping (bottom right).

Crucially, these simulations suggest that adaptive concept learning needs to be informed by existing knowledge about other concepts, such that a novel concept should only be learned if observations cannot be explained with existing conceptual knowledge. Here, this is achieved via the interplay of inference and learning, such that agents initially have to infer whether to assign an observation to an existing concept, and only if this is not possible an 'open slot' is employed to learn about a novel concept.

Increasing granularity

Next, to explore the model's ability to increase the granularity of its concept space, we first equipped the model with only the distinction between birds and fish (i.e., the rows of the likelihood mapping corresponding to color and size features were flattened in the same manner described above). We then performed the same procedure used in our previous simulations. As can be seen in Figure 5A (bottom left), the 'A' matrix learned by the model now more strongly resembled that of the generative process. Figure 5A (bottom) also illustrates reporting accuracy and the proportion of basic and specific category reports as a function of trial number. As can be seen, the agent initially only reported general categories, and became sufficiently confident to report specific categories after roughly 50 – 100 trials, but its accuracy increased gradually over the next 1000 trials (i.e., the agent reported specific categories considerably before its accuracy improved). Across 8 repeated simulations, the final accuracy level reached was between 93% – 98% in 7 simulations, but she failed to learn one concept in the 8th case, with 84.4% overall accuracy (i.e., a failure to distinguish between pigeon and parakeet, and therefore only learned a broader category of "small birds").

To assess whether learning basic categories first was helpful in subsequently learning specific categories, we also repeated this simulation without any initial knowledge of the basic categories. As exemplified in figure 5A and 5B, the model tended to perform reasonably well, but most often learned a less precise likelihood mapping and reached a lower reporting accuracy percentage after 2000 learning trials (across 8 repeated simulations: mean = 81.21%, SD = 6.39%, range from 68.80% – 91.30%). Thus, learning basic concept categories first appeared to facilitate learning more specific concepts later.



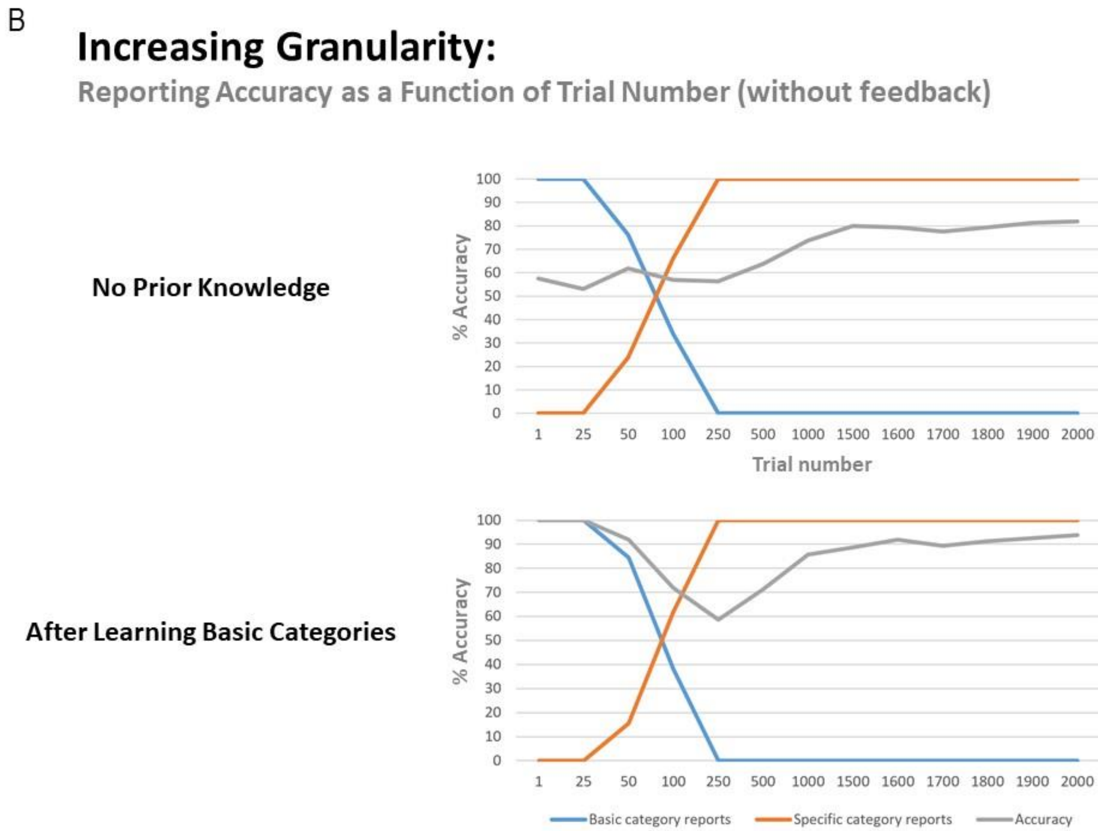


Figure 5. (A, left) Illustration of representative simulation results when the agent had to learn to increase the granularity of its concept space. Here the agent began with prior knowledge about the basic concept categories (i.e., it had learned the broad categories of "bird" and "fish") but had not learned the feature patterns (i.e., rows) that differentiate different types of birds and fish. Post learning (i.e., after 2000 exposures), the agent did successfully learn all of the more granular concept categories, although again note that specific concepts were assigned to different columns then depicted in the generative process due to the unsupervised nature of the learning. (A, right) illustration of the analogous learning result when the agent had to learn all 8 specific categories without prior knowledge of the general categories. Although moderately successful, learning tended to be more difficult in this case. (B) Representative plots of reporting accuracy in each of the 2 learning conditions as a function of the number of exposures. As can be seen, when the model starts out with prior knowledge about basic categories, it slowly become sufficiently confident to start reporting the more specific categories, and its final accuracy is high. In contrast, while the agent that did not start out with any prior knowledge of the general categories also grew confident in reporting specific categories over time, its final accuracy levels tended to be lower. In both cases, the agent began reporting specific categories before it achieved significant accuracy levels, therefore showing some initial overconfidence.

Overall, these findings provide a proof of principle that this sort of active inference scheme can add concepts to its state space in an unsupervised manner (i.e., without feedback) based purely on (expected) free energy minimization. In this case, it was able to accomplish this starting from a completely uninformative likelihood distribution. It could also learn more granular concepts after already acquiring more basic concepts, and our results suggest that learning granular concepts may be facilitated by first learning basic concepts (e.g., as in currently common educational practices).

The novel feature of this generative model involved 'building in' a number of "reserve" hidden state levels. These initially had uninformative likelihood mappings; yet, if a new pattern of features was repeatedly observed, and the model could not account for this pattern with existing (informative) state-observation mappings, these additional hidden state levels could be engaged to improve the model's explanatory power. This approach therefore accommodates a simple form of model expansion.

Integrating model expansion and reduction

We next investigated ways in which model expansion could be combined with Bayesian model reduction (KJ Friston, Lin, et al., 2017) – allowing the agent to adjust her model to accommodate new patterns of observations, while also precluding unnecessary conceptual complexity (i.e., over-fitting). To do so, we again allowed the agent to learn from 2000 exposures to different animals as described in the previous section – but also allowed the model to learn its 'D' matrix (i.e., accumulate concentration parameters reflecting prior expectations over initial states). This allowed an assessment of the agent's probabilistic beliefs about which hidden state factor levels (animals) it had been exposed to. In different simulations, the agent was only exposed to some animals and not others. We then examined whether a subsequent model reduction step could recover the animal concepts presented during the simulation; eliminating those concepts that were unnecessary to explain the data at hand. The success of this 2-step procedure could then license the agent to "reset" the unnecessary hidden state columns after concept acquisition, which would have accrued unnecessary likelihood updates during learning. Doing so would allow the optimal ability for those "reserve" states to be appropriately engaged, if and when the agent was exposed to truly novel stimuli.

The 2nd step of this procedure was accomplished by applying Bayesian model reduction to the 'D' matrix concentration parameters after learning. This is a form of post-hoc model optimization (K. J. Friston et al., 2016; Karl Friston, Parr, & Zeidman, 2018) that rests upon estimation of a 'full' model, followed by analytic computation of the evidence that would have been afforded to alternative models (with alternative, 'reduced', priors) had they been used instead. Mathematically, this procedure is a generalization of things like automatic relevance determination (Karl Friston, Mattout, Trujillo-Barreto, Ashburner, & Penny, 2007; Wipf & Rao, 2007) or the use of the Savage Dickie ratio in model comparison (Cornish & Littenberg, 2007). It is based upon straightforward probability theory and, importantly, has a simple physiological interpretation; namely, synaptic decay and the elimination of unused synaptic connections. In this (biological) setting, the concentration parameters of the implicit Dirichlet distributions can be thought of as synaptic tags. For a technical description of Bayesian model reduction techniques and their proposed neural implementation, see (KJ Friston, Lin, et al., 2017; Hobson & Friston, 2012; Hobson, Hong, & Friston, 2014a); also see the left panel of Figure 8 and the associated legend further below for some additional details).

The posterior concentration parameters were compared to the prior distribution for a full model (i.e., a flat distribution over 8 concepts) and prior distributions for possible reduced models (i.e., which retained different possible combinations of some but not all concepts; technically, reduced models were defined such that the to-be-eliminated concepts were less likely than the to-be-retained concepts). If Bayesian model reduction provided more evidence for one or more reduced models, the reduced model with the most evidence was selected. Note: an alternative would be to perform model reduction on the 'A' matrix, but this is more complex due to a larger space of possible reduced models; it also does not address the question of the number of hidden state levels to retain in a straightforward manner.

In our first simulation, we presented our agent with all animals except for parakeets with equal probability over 2000 trials. When compared to the full model, the winning model corresponded to the correct 7-animal model matching the generative process in 6/8 cases (log evidence differences ranged from -3.12 to -8.3), and in 2/8 cases it instead selected a 6-animal model due to a failure to distinguish between 2 specific concepts during learning (log evidence differences = -5.30, -

7.70). Figure 6 illustrates the results of a representative successful case). In the successful cases, this would correctly license the removal of changes in the model's 'A' and 'D' matrix parameters for the 8th animal concept during learning in the previous trials. Similar results were obtained whenever any single animal type was absent from the generative process.

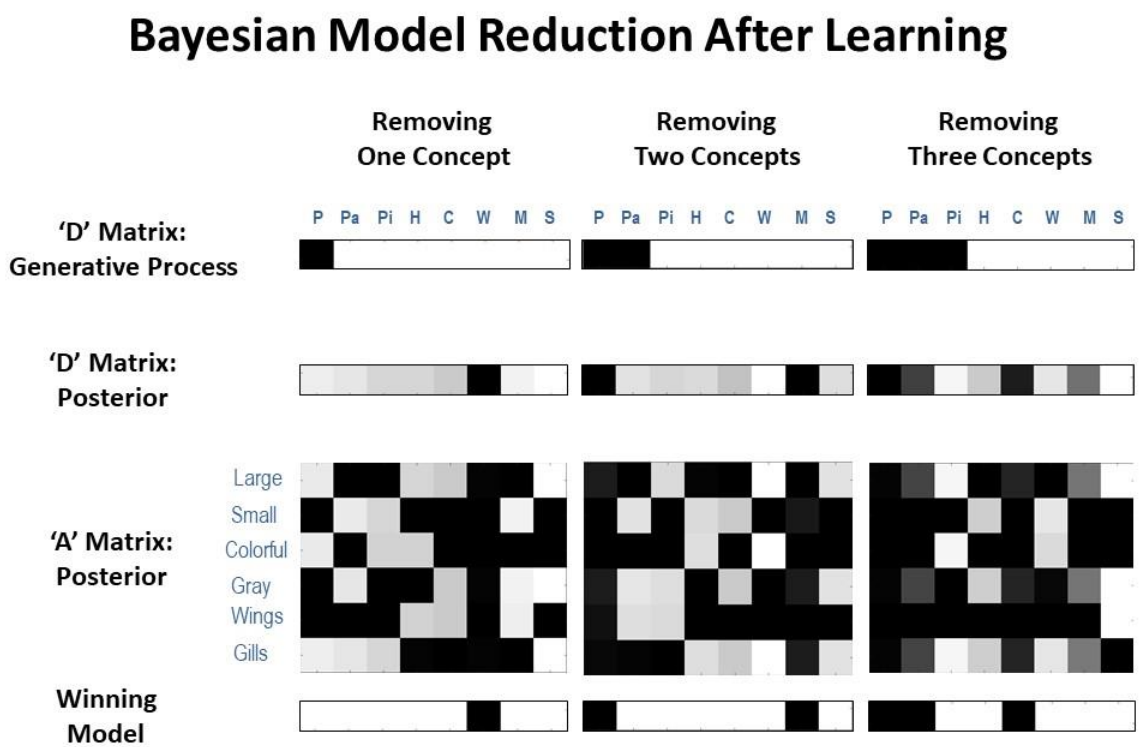


Figure 6. Representative illustrations of simulations in which the agent performed Bayesian model reduction after learning. In these simulations, the agent was first exposed to 2000 trials in which either 7, 6, or 5 animals were actually presented (i.e., illustrated in the top row, where only the white columns had nonzero probabilities in the generative process). In each case, model reduction was often successful at identifying the reduced model with the correct number of animal types presented (bottom row, where black columns should be removed) based on how much evidence it provided for the posterior distribution over hidden states learned by the agent (2nd row). This would license the agent to reset the unneeded columns in its likelihood mapping (3rd row) to their initial state (i.e., a flat distribution over features) such that it could be engaged if/when a new type of animal began to be observed (i.e., as in the simulations illustrated in the previous sections).

In a second simulation, the generative process contained 2 birds and all 4 fish. Here, the correct reduced model was correctly selected in 6/8 simulations (log evidence differences range from -.96 to -8.24, with magnitudes greater than -3 in 5/6 cases), whereas it incorrectly selected the 5-animal model in 2 cases (log evidence differences = -3.54, -4.50). In a third simulation, the generative process contained 1 bird and all 4 fish. Here, the correct reduced model had the most evidence in only 3/8 simulations (log evidence differences = -4.10, -4.11, -5.48), whereas a 6-animal model was selected in 3/8 cases and a 3-animal and 7-animal model were each selected once (log evidence differences > -3.0). Figure 6 also illustrates representative examples of correct model recovery in these 2nd and 3rd simulations.

While we have used the terms ‘correct’ and ‘incorrect’ above to describe the model used to generate the data, we acknowledge that ‘all models are wrong’ (Box, Hunter, & Hunter, 2005), and that the important question is not whether we can recover the ‘true’ process used to generate the data, but whether we can arrive at the simplest but accurate explanation for these data. The failures to recover the ‘true’ model highlighted above may reflect that a process other than that used to generate the data could have been used to do so in a simpler way. Simpler here means we would have to diverge to a lesser degree, from our prior beliefs, in order to explain the data under a given model, relative to a more complex model. It is worth highlighting the importance of the word *prior* in the previous sentence. This means that the simplicity of the model is sensitive to our prior beliefs about it. To illustrate this, we repeated the same model comparisons as above, but with precise beliefs in an 'A' matrix that complies with that used to generate the data. Specifically, we repeated the three simulations above but only enabled 'D' matrix learning (i.e., the model was already equipped with the 'A' matrix of the generative process). In each case, Bayesian model reduction now uniquely identified the correct reduced model in 100% of cases.

These results demonstrate that – after a naïve model has expanded its hidden state space to include likelihood mappings and initial state priors for a number of concept categories – Bayesian model reduction can subsequently be used to eliminate any parameter updates accrued for *one or two* redundant concept categories. In practice, the 'A' and 'D' concentration parameters for these redundant categories could be reset to their default pre-learning values – and could then be re-engaged if new patterns of observations were repeatedly observed in the future. However, when three concepts should be removed, Bayesian model reduction was much less reliable. This appeared to be due to imperfect 'A' matrix learning, when occurring simultaneously with the (resultingly noisy) accumulation of prior expectations over hidden states – as a fully precise 'A' matrix led to correct model reduction in every case tested (i.e., suggesting that this type of model reduction procedure could be improved by first allowing state-observation learning to proceed alone, then subsequently allowing the model to learn prior expectations over hidden states, which could then be used in model reduction).

Can concept acquisition allow for generalization?

One important ability afforded by concept learning is generalization. In a final set of simulations, we asked if our model of concept knowledge could account for generalization. To do so, we altered the model such that it no longer reported what it saw, but instead had to answer a question that depended on generalization from particular cross-category feature combinations. Specifically, the model was shown particular animals and asked: “could this be seen from a distance?” The answer to this question depended on both size and color, such that the answer was yes only for colorful, large animals (i.e., assuming small or gray animals would blend in with the sky or water and be missed).

Crucially, this question was asked of animals that the model had not been exposed to, such that it had to generalize from knowledge it already possessed (see Figure 7). To simulate and test for this ability, we equipped the model’s ‘A’ matrix with expert knowledge of 7 out of the 8 animals (i.e., as if these concepts had been learned previously, as in our simulations above). The 8^{th} animal was unknown to the agent, in that it’s likelihood mapping was set such that the 8^{th} animal state “slot” predicted all observations roughly equally. In one variant, the model possessed all concepts except for “parrot,” and it knew that the answer to the question was yes for “whale shark” but not for any other concept it knew. To simulate one-shot generalization, learning was disabled and a parrot (which it had never seen before) was presented 20 times to see if it would correctly generalize and answer “yes” in a reliable manner. In another variant, the model had learned all concepts except “minnow” and was tested the same way to see if it would reliably provide the correct “no” response.

Here, we observed that in both of these cases (as well as all others we tested) the model generalized remarkably well. It answered “yes” and “no” correctly in 100% of trials. Thus, the agent did not simply possess concepts to explain things it saw. It instead demonstrated generalizable knowledge and could correctly answer questions when seeing a novel stimulus.

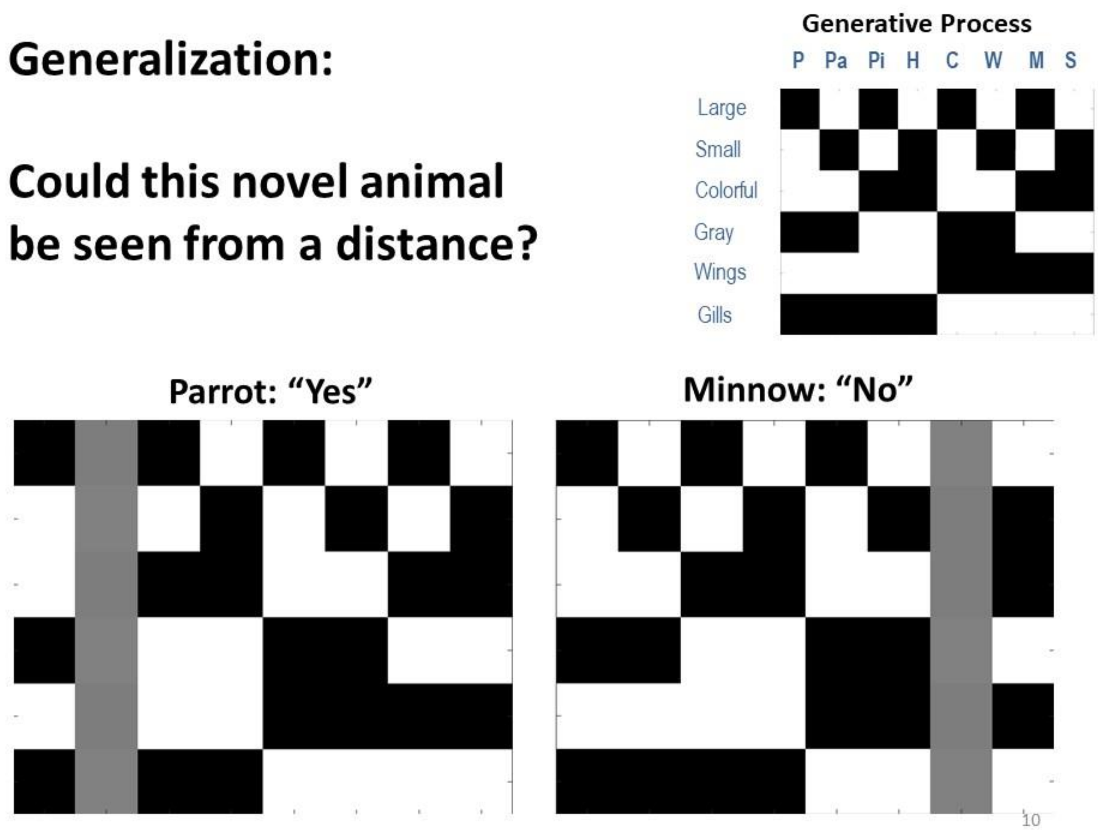


Figure 7. Depiction of simulations in which we tested the agents ability to generalize from prior knowledge and correctly answered questions about new animals to which it had not previously been exposed. In the simulations, the generative model was modified so that the agent instead chose to report either “yes” or “no” to the question: “could this animal be seen from a distance?” Here, the answer was only yes if the animal was both large and colorful. We observed that when the agent started out with no knowledge of parrots it still correctly answered this question 100% of the time, based only on its knowledge of other animals. Similarly, when it started with no knowledge of minnows, it also correctly reported “no” 100% of the time. Thus, the agent was able to generalize from prior knowledge with no additional learning.

Open questions and relation to other theoretical accounts of concept learning

As our simulations show, this model allows for learning novel concepts (i.e., novel hidden states) based on assigning one or more 'open slots' that can be utilised to learn novel feature combinations. In a simple example, we have shown that this setup offers a potential computational mechanism for 'model expansion'; i.e., the process of expanding a state space to account for novel instances in perceptual categorisation. We also illustrated how this framework can be combined with model reduction, which may be a mechanism for 're-setting' these open slots based on recent experience.

This provides a first step towards understanding how agents flexibly expand or reduce their model to adapt to ongoing experience. Yet, several open questions remain, which have partly been addressed in previous work. For example, the proposed framework resonates with previous similarity-based accounts of concept learning. Previous work has proposed a computational framework for arbitrating between assigning an observation to a previously formed memory or forming a novel (hidden) state representation (S. J. Gershman, Monfils, Norman, & Niv, 2017), based on evidence that this observation was sampled from an existing or novel latent state. This process is conceptually similar to our application of Bayesian model reduction over states. In the present framework, concept learning relies on a process based on inference and learning. First, agents have to *infer* whether ongoing observations can be sufficiently explained by existing conceptual knowledge – or speak to the presence of a novel concept that motivates the use of an 'open slot'. This process is cast as inference on (hidden) states. Second, if the agent infers that there is a novel concept that explains current observations, it has to *learn* about the specific feature configuration of that concept (i.e., novel state). This process highlights the interplay between inference, which allows for the acquisition of knowledge on a relatively short timescale, and learning, which allows for knowledge acquisition on a longer and more stable timescale.

Similar considerations apply to the degree of 'similarity' of observations. In the framework proposed here, we have assumed that the feature space of observations is already learned and fixed. However, these feature spaces have to be learned in the first place, which implies learning the underlying components or feature dimensions that define observations. This relates closely to notion of structure learning as dimensionality reduction based on covariance between observations, as prominently discussed in the context of spatial navigation (Behrens et al., 2018; Dordek, Soudry, Meir, & Derdikman, 2016; Stachenfeld et al., 2016; Whittington, Muller, Mark, Barry, & Behrens, 2018).

Another important issue is how such abstract conceptual knowledge is formed across different contexts or tasks. For example, the abstract concept of a 'bird' will be useful for learning about the fauna in a novel environment, but specific types of birds – tied to a previous context – might be less useful in this regard. This speaks to the formation of abstract, task-general knowledge that results from training across different tasks, as recently discussed in the context of meta-reinforcement learning (Ritter, Wang, Kurth-Nelson, & Botvinick, 2018; J X Wang et al., 2016) with a putative link to the prefrontal cortex (Jane X. Wang et al., 2018). In the present framework, such task-general knowledge would speak to the formation of a hierarchical organisation that allows for the formation of conceptual knowledge both within and across contexts. Also note that our proposed framework depends on a pre-defined state space, including a pre-defined set of 'open slots' that allow for novel context learning. The contribution of the present framework is to show how these 'open slots' can be used for novel concept learning and be re-set based on model reduction. It will be important to extend this approach towards learning the structure of these models in the first place, including the appropriate number of 'open slots' (i.e., columns of the A-matrix) for learning in a particular content domain and the relevant feature dimensions of observations (i.e., rows of A-matrix).

This corresponds to a potentially powerful and simple application of Bayesian model reduction, in which candidate models (i.e., reduced forms of a full model) are readily identifiable based upon the similarity between the likelihoods conditioned upon different hidden states. If two or more likelihoods are sufficiently similar, the hidden states can be merged (by assigning the concentration parameters accumulated during experience-dependent learning to one or other of the hidden states). The ensuing change in model evidence scores the reduction in complexity. If this reduction is greater than the loss of accuracy – in relation to observations previously encountered – Bayesian model reduction will, effectively, merge one state into another; thereby freeing up a state for the learning of new concepts. We will demonstrate this form of structure learning via Bayesian model reduction in future work.

Potential advantages of the approach

The present approach may offer some potential theoretical and empirical advantages in comparison to previous work. One theoretical advantage corresponds to the parsimony of casting this type of structure learning as an instance of Bayesian model selection. When integrated with other aspects of the active inference framework, this entails that perceptual inference, active learning, and structure learning are all expressions of the same principle; namely, the minimization of variational free energy, over three distinct timescales. A second, related theoretical advantage is that, when this type of structure learning is cast as Bayesian model selection/reduction, there is no need to invoke additional procedures or schemes (e.g., nonparametric Bayes or 'stick breaking' processes; (S. Gershman & Blei, 2012)). Instead, a generative model with the capacity to represent a sufficiently complex world will automatically learn causal structure in a way that contextualizes active inference within active learning, and active learning within structure learning.

One potential empirical advantage of the present approach stems from the fact that active inference models have a plausible biological basis that affords testable neurobiological predictions. Specifically, these models have well-articulated companion micro-anatomical neural process theories, based on commonly used message-passing algorithms (KJ Friston, FitzGerald, et al., 2017; Parr & Friston, 2018; Parr, Markovic, Kiebel, & Friston, 2019). In these process theories, for example, the activation level of different neural populations (typically portrayed as consisting of different cortical columns) can encode posterior probability estimates over different hidden states. These activation levels can then be updated by synaptic inputs with particular weights that convey the conditional probabilities encoded in the 'A' and 'B' (among other) matrices described above, where active learning then corresponds to associative synaptic plasticity. Phasic dopamine responses also play a particular role in these models, by reporting changes in policy precision (i.e., the degree of confidence in one policy over others) upon new observations (see Figure 8 and the associated legend for more details).

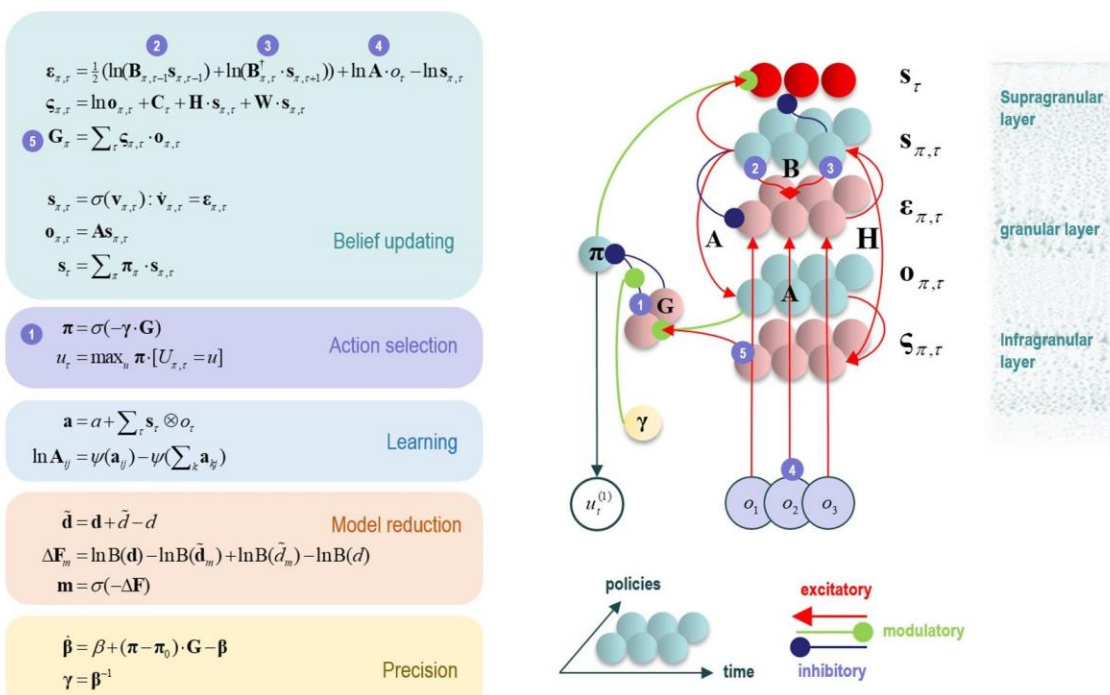


Figure 8. This figure illustrates the mathematical framework of active inference and associated neural process theory used in the simulations described in this paper. The differential equations in the left panel approximate Bayesian belief updating within the graphical model depicted in the right panel of Figure 1 via a gradient descent on free energy

($\mathbf{F}$). The right panel also illustrates the proposed neural basis by which neurons making up cortical columns could implement these equations. The equations have been expressed in terms of two types of prediction errors. State prediction errors ($\boldsymbol{\varepsilon}$) signal the difference between the (logarithms of) expected states ($\mathbf{s}$) under each policy and time point—and the corresponding predictions based upon outcomes/observations ($\mathbf{A}$ matrix) and the (preceding and subsequent) hidden states ($\mathbf{B}$ matrix, and, although not written, the $\mathbf{D}$ matrix for the initial hidden states at the first time point). These represent prior and likelihood terms respectively – also marked as messages 2, 3, and 4, which are depicted as being passed between neural populations (colored balls) via particular synaptic connections in the right panel. These (prediction error) signals drive depolarization ($\mathbf{v}$) in those neurons encoding hidden states ($\mathbf{s}$), where the probability distribution over hidden states is then obtained via a softmax (normalized exponential) function ($\boldsymbol{\sigma}$). Outcome prediction errors ($\boldsymbol{\varsigma}$) instead signal the difference between the (logarithms of) expected observations ($\mathbf{o}$) and those predicted under prior preferences ($\mathbf{C}$). This term additionally considers the expected ambiguity or conditional entropy ($\mathbf{H}$) between states and outcomes as well as a novelty term ($\mathbf{W}$) reflecting the degree to which beliefs about how states generate outcomes would change upon observing different possible state-outcome mappings (computed from the $\mathbf{A}$ matrix). This prediction error is weighted by the expected observations to evaluate the expected free energy ($\mathbf{G}$) for each policy ($\boldsymbol{\pi}$), conveyed via message 5. These policy-specific free energies are then integrated to give the policy expectations via a softmax function, conveyed through message 1. Actions at each time point ($\mathbf{u}$) are then chosen out of the possible actions under each policy ($\mathbf{U}$) weighted by the value (negative expected free energy) of each policy. In our simulations, the model learned associations between hidden states and observations ($\mathbf{A}$) via a process in which counts were accumulated ($\mathbf{a}$) reflecting the number of times the agent observed a particular outcome when she believed that she occupied each possible hidden state. Although not displayed explicitly, learning prior expectations over initial hidden states ($\mathbf{D}$) is similarly accomplished via accumulation of concentration parameters ($\mathbf{d}$). These prior expectations reflect counts of how many times the agent believes it previously occupied each possible initial state. Concentration parameters are converted into expected log probabilities using digamma functions ($\boldsymbol{\psi}$). The way in which Bayesian model reduction was performed in this paper is also written in the lower left (where B indicates a beta function, and $\mathbf{m}$ is the posterior probability of each model). Here, the posterior distribution over initial states ($\mathbf{d}$) is used to assess the difference in the evidence ($\boldsymbol{\Delta}\mathbf{F}$) it provides for the number of hidden states in the current model and other possible models characterized by fewer hidden states. Prior concentration parameters are shown in italics, posterior in bold, and those priors and posteriors associated with the reduced model are equipped with a tilde (**~**). As already stated, the right panel illustrates a possible neural implementation of the update equations in the middle panel. In this implementation, probability estimates have been associated with neuronal populations that are arranged to reproduce known intrinsic (within cortical area) connections. Red connections are excitatory, blue connections are inhibitory, and green connections are modulatory (i.e., involve a multiplication or weighting). These connections mediate the message passing associated with the equations in the left panel. Cyan units correspond to expectations about hidden states and (future) outcomes under each policy, while red states indicate their Bayesian model averages (i.e., a "best guess" based on the average of the probability estimates for the states and outcomes across policies, weighted by the probability estimates for their associated policies. Pink units correspond to (state and outcome) prediction errors that are averaged to evaluate expected free energy and subsequent policy expectations (in the lower part of the network). This (neural) network formulation of belief updating means that connection strengths correspond to the parameters of the generative model described in the text. Learning then corresponds to changes in the synaptic connection strengths. Only exemplar connections are shown to avoid visual clutter. Furthermore, we have just shown neuronal populations encoding hidden states under two policies over three time points (i.e., two transitions), whereas in the task described in this paper there are greater number of allowable policies. For more information regarding the mathematics and processes illustrated in this figure, see (KJ Friston, Lin, et al., 2017; KJ Friston, Parr, et al., 2017).

Based on these theories, the present model would predict that the brain contains “reserve” cortical columns and synapses (most likely within secondary sensory and association cortices) available to capture new patterns in observed features. To our knowledge, no direct evidence supporting the presence of unused cortical columns in the brain has been observed, although the generation of new neurons (with new synaptic connections) is known to occur in the hippocampus (Chancey et al., 2013). "Silent synapses” have also been observed in the brain, which does appear consistent with this prediction; such synapses can persist into adulthood and only become activated when new learning becomes necessary (e.g., see (Chancey et al., 2013; Funahashi, Maruyama, Yoshimura, & Komatsu, 2013; Kerchner & Nicoll, 2008)). One way in which this idea of “spare capacity” or “reserve” cortical columns might be tested in the context of neuroimaging would be to examine whether greater levels of neural activation – within conceptual processing regions – are observed after learning additional concepts, which would imply that additional populations of neurons become capable of being activated. In principle, single-cell recording methods might also test for the presence of neurons that remain at baseline firing rates during task conditions, but then become sensitive to new stimuli within the relevant conceptual domain after learning.

Figure 9 provides a concrete example of two specific empirical predictions that follow from simulating the neural responses that should be observed within our concept learning task under these process theories. In the left panel, we plot the firing rates (darker = higher firing rate) and local field potentials (rate of change in firing rates) associated with neural populations encoding the probability of the presence of different animals that would be expected across a number of learning trials. In this particular example, the agent began with knowledge of the basic categories of 'bird' and 'fish,' but needed to learn the eight more specific animal categories over 50 interleaved exposures to each animal (only 10 equally spaced learning trials involving the presentation of a parakeet are shown for simplicity). As can be seen, early in learning the firing rates and local field potentials remain at baseline levels; in contrast, as learning progresses, these neural responses take a characteristic shape with more and more positive changes in firing rate in the populations representing the most probable animal, while other populations drop further and further below baseline firing rates.

The right panel depicts a similar simulation, but where the agent was allowed to self-report what it saw on each trial (for clarity of illustration, we here show 12 equally spaced learning trials for parakeet over 120 total trials). Enabling policy selection allowed us to simulate expected phasic dopamine responses during the task, corresponding to changes in the precision of the probability distribution over policies after observing a stimulus on each trial. As can be seen, during early trials the model predicts small firing rate increases when the agent is confident in its ability to correctly report the more general animal category after observing a new stimulus, and firing rate decreases when the agent becomes less confident in one policy over others (i.e., as confidence in reporting the specific versus general categories becomes more similar). Larger and larger phasic dopaminergic responses are then expected as the agent becomes more and more confident in its ability to correctly report the specific animal category upon observing a new stimulus. It will be important for future neuroimaging studies to test these predictions in this type of concept learning/stimulus categorization task.

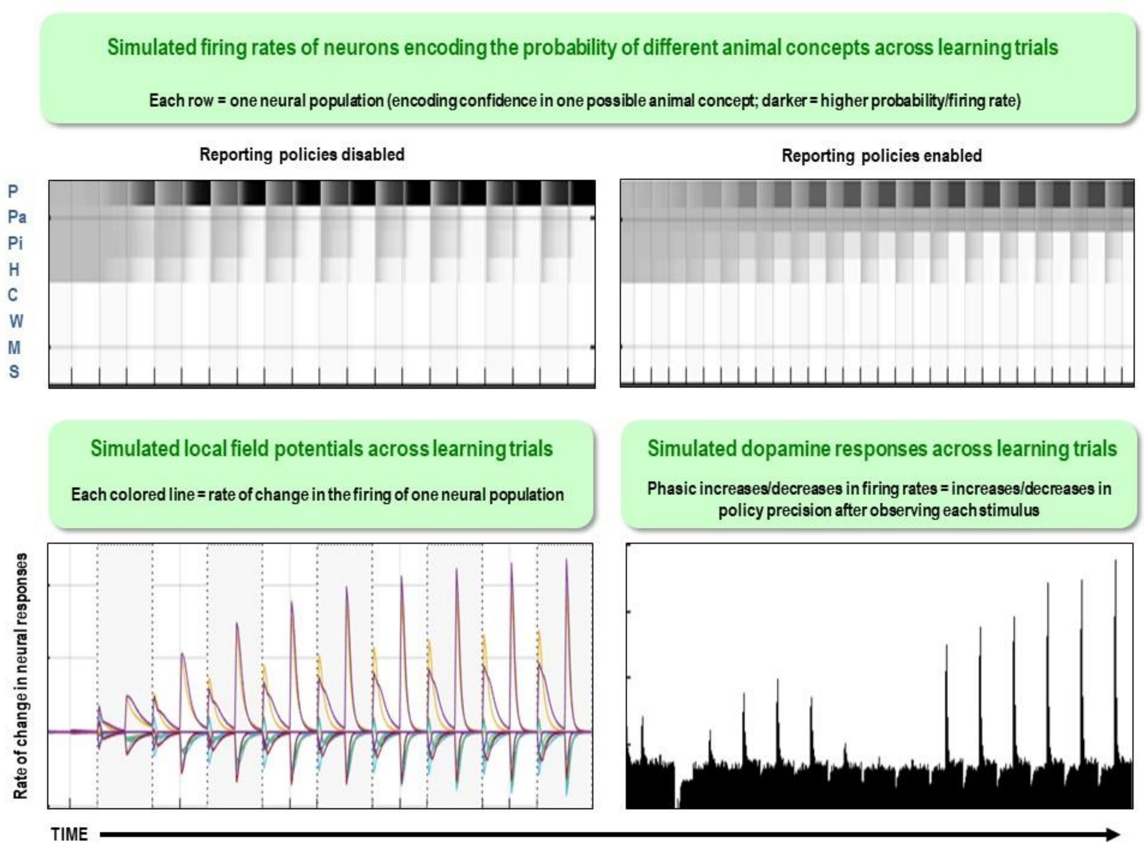


Figure 9. Simulated neuronal firing rates, local field potentials, and dopaminergic responses across learning trials based on the neural process theory associated with active inference summarized in Figure 8. The top left panel displays the predicted firing rates (darker = higher firing rate) of neural populations encoding the probability of each hidden state over 50 interleaved exposures to each animal (only 10 equally spaced learning trials involving the presentation of a parakeet are shown for simplicity) in the case where the agent starts out with knowledge of the basic animal categories but must learn the more specific categories. As can be seen, initially each of the four neural populations encoding possible bird categories (i.e., one row per possible category) have equally low firing rates (gray); as learning continues, firing rates increase for the 'parakeet' population and decrease for the others. The bottom left panel illustrates the predicted local field potentials (based on the rate of change in firing rates) that would be measured across the task. The top right panel displays the predicted firing rates of neural populations in an analogous simulation in which reporting policies were enabled (for clarity of illustration, we here show 12 equally spaced learning trials for parakeet over 120 total trials). Enabling policy selection allowed us to simulate the phasic dopaminergic responses (reporting changes in the precision of the probability distribution over policies) predicted to occur across learning trials; here the agent first becomes confident in its ability to correctly report the general animal category upon observing a stimulus, then becomes unsure about reporting specific versus general categories, and then becomes confident in its ability to report the specific categories.

Discussion

The Active Inference formulation of concept learning presented here demonstrates a simple way in which a generative model can acquire both basic and highly granular knowledge of the hidden states/causes in its environment. In comparison to previous theoretical work using active inference (e.g., (M. Mirza, Adams, Mathys, & Friston, 2016; Parr & Friston, 2017; Schwartenbeck, FitzGerald, Mathys, Dolan, & Friston, 2015)), the novel aspect of our model was that it was further equipped with "reserve" hidden states initially devoid of content (i.e., these states started out with uninformative likelihood mappings that predicted all outcomes with roughly equal probability). Over multiple exposures to different stimuli, these hidden states came to acquire conceptual content that captured distinct statistical patterns in the features of those stimuli. This was accomplished via the model's ability to infer when its currently learned hidden states were unable to account for a new observation, leading an unused hidden state column to be engaged that could acquire a new state-observation mapping.

Crucially, the model was able to start with some concepts and then expand its representational repertoire to learn others – but would only do so when a new stimulus was observed. This is conceptually similar to nonparametric Bayesian learning models, such as the "Chinese Room" process and the "Indian Buffet" process, that can also infer the need to invoke additional hidden causes with additional data (S. Gershman & Blei, 2012). These statistical learning models do not need to build in additional "category slots" for learning as in our model and can, in principle, entertain infinite state spaces. On the other hand, it is less clear at present how the brain could implement this type of learning. An advantage of our model is that learning depends solely on biologically plausible Hebbian mechanisms (for a possible neural implementation of model reduction, see (KJ Friston, Lin, et al., 2017; Hobson & Friston, 2012; Hobson, Hong, & Friston, 2014b)).

The distinction between nonparametric Bayesian learning and the current active learning scheme may be important from a neurodevelopmental perspective as well. In brief, structure learning in this paper starts with a generative model with 'spare capacity', where uncommitted or naive conceptual 'slots' are used to explain the sensorium, during optimization of free energy or model evidence. In contrast, nonparametric Bayesian approaches add new slots when appropriate. One might imagine that neonates are equipped with brains with 'spare capacity' (Baker & Tenenbaum, 2014) that is progressively leveraged during neurodevelopment, much in the spirit of curriculum learning (Al-Muhaideb & Menai, 2011). In this sense, the current approach to structure learning may be better considered as active learning with generative models that are equipped with a large number of hidden states, which are judiciously reduced – via a process of Bayesian model reduction. Furthermore, as in the acquisition of expertise, our model can also begin with broad category knowledge and then subsequently learn finer-grained within-category distinctions, which has received less attention from the perspective of the aforementioned models. Reporting broad versus specific category recognition is also a distinct aspect of our model – driven by differing levels of uncertainty and an expectation (preference) not to incorrectly report a more specific category.

Our simulation results also demonstrated that, when combined with Bayesian model reduction, the model can guard against learning too many categories during model expansion – often retaining only the number of hidden causes actually present in its environment – and to keep "reserve" hidden states for learning about new causes if or when they appear. With perfect "expert" knowledge of the possible animal types it could observe (i.e., fully precise likelihood mappings matching the generative process) this was true in general. Interestingly, however, with an imperfectly learned likelihood mapping, model reduction only succeeded when the agent had to remove either 1 or 2 concepts from her model; when 3 potential categories needed to be removed, the correct reduced model was identified less than half the time. It would be interesting to empirically test whether similar learning difficulties are present in humans.

Neurobiological theories associated with Active Inference also make predictions about the neural basis of this process (Hobson & Friston, 2012; Hobson et al., 2014b). Specifically, during periods of rest (e.g., daydreaming) or sleep, it is suggested that, because sensory information is down-weighted, learning is driven mainly by internal model simulations (e.g., as appears to happen in the phenomenon of hippocampal replay; (Feld & Born, 2017; Lewis, Knoblich, & Poe, 2018; Pfeiffer & Foster, 2013)); this type of learning can accomplish a model reduction process in which redundant model parameters are identified and removed to prevent model over-fitting and promote selection of the most parsimonious model that can successfully account for previous observations. This is consistent with work suggesting that, during sleep, many (but not all) synaptic strength increases acquired in the previous day are attenuated (Tononi & Cirelli, 2014). The role of sleep and daydreaming in keeping “reserve” representational resources available for model expansion could therefore be especially important to concept learning – consistent with the known role of sleep in learning and memory (Ackermann & Rasch, 2014; Feld & Born, 2017; Perogamvros & Schwartz, 2012; Stickgold, Hobson, Fosse, & Fosse, 2001; Walker & Stickgold, 2010).

In addition, an emergent feature of our model was its ability to generalize prior knowledge to new stimuli to which it had not previously been exposed. In fact, the model could correctly generalize upon a single exposure to a new stimulus – a type of “one-shot learning” capacity qualitatively similar to that observed in humans

(Landau, Smith, & Jones, 1988; E. Markman, 1989; Xu & Tenenbaum, 2007b). While it should be kept in mind that the example we have provided is very simple, it demonstrates the potential usefulness of this novel approach. Some other prominent approaches in machine-learning (e.g., deep learning) tend to require larger amounts of data (Geman et al., 1992; Hinton et al., 2012; LeCun et al., 2015; Lecun et al., 1998; Mnih et al., 2015), and do not learn the rich structure that allows humans to use concept knowledge in a wide variety of generalizable functions (Barsalou, 1983; Biederman, 1987; Feldman, 1997; Jern & Kemp, 2013; A. B. Markman & Makin, 1998; Osherson & Smith, 1981; Ward, 1994; Williams & Lombrozo, 2010). Other recent hierarchical Bayesian approaches in cognitive science have made progress in this domain, however, by modeling concepts as types of probabilistic programs (Ghahramani, 2015; Goodman, Tenenbaum, & Gerstenberg, 2015; Lake et al., 2015).

It is important to note that this model is deliberately simple and is meant only to represent a proof of principle that categorical inference and conceptual knowledge acquisition can be modeled within this particular neurocomputational framework. We chose a particular set of feature combinations to illustrate this, but it remains to be demonstrated that learning in this model would be equally successful with a larger feature space and set of learnable hidden causes.

Finally, another topic for future work would be the expansion of this type of model to context-specific learning (e.g., with an additional hidden state factor for encoding distinct contexts). In such cases, regularities in co-occurring features differ in different contexts and other cues to context may not be directly observable (e.g., the same species of bird could be a slightly different color or size in different parts of the world that otherwise appear similar) – creating difficulties in inferring when to update previously learned associations and when to instead acquire competing associations assigned to new contexts. At present, it is not clear whether the approach we have illustrated would be successful at performing this additional function, although the process of inferring the presence of a new hidden state in a second hidden state factor encoding context would be similar (for related work on context-dependent contingency learning, see (S. J. Gershman et al., 2017; S. Gershman, Jones, Norman, Monfils, & Niv, 2013)). Another point worth highlighting is that we have made particular choices with regard to various model parameters and the number of observations provided during learning. Further investigations of the space of these possible parameter settings will be important. With this in mind, however, our current modelling results could offer additional benefits. For example, the model's simplicity could be amenable to empirical studies of saccadic eye movements toward specific features during novel category learning (e.g. following the approach of (M. B. Mirza, Adams, Mathys, & Friston, 2018)). This approach could also be combined with measures of neural activity in humans or other animals, allowing more direct tests of the predictions highlighted above. In addition, the introduction of exploratory, novelty-seeking, actions could be used to reduce the number of samples required for learning, with agents selecting those data that are most relevant.

In conclusion, the Active Inference scheme we have described illustrates feature integration in the service of conceptual inference: it can successfully simulate simple forms of concept acquisition and concept differentiation (i.e. increasing granularity), and it spontaneously affords one-shot generalization. Finally, it speaks to empirical work in which behavioral tasks could be designed to fit such models, which would allow investigation of individual differences in concept learning and its neural basis. For example, such a model can simulate (neuronal) belief updating to predict neuroimaging responses as we illustrated above; i.e., to identify the neural networks engaged in evidence accumulation and learning (Schwartenbeck et al., 2015). In principle, the model parameters (e.g., 'A' matrix precision) can also be fit to behavioral choices and reaction times – and thereby phenotype subjects in terms of the priors under which they infer and learn (Schwartenbeck & Friston, 2016). This approach could therefore advance neurocomputational approaches to concept learning in several directions.

Software note

Although the generative model – specified by the various matrices described in this paper – changes from application to application, the belief updates are generic and can be implemented using standard routines (here **spm_MDP_VB_X.m**). These routines are available as Matlab code in the SPM academic software: http://www.fil.ion.ucl.ac.uk/spm/. The simulations in this paper can be reproduced (and customised) via running the Matlab code included here is supplementary material (**Concepts_model.m**).

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