Ultrafast Structural Changes Decomposed from Serial Crystallographic Data

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Summary

Direct imaging of ultrafast and subtle structural events during a biochemical reaction, such as a single electronic transition from one atomic or molecular orbital to another, is highly desirable but has been beyond the reach of protein crystallography. It entails the capability of observing changes in electronic distributions at both an ultrafast time scale and an ultrahigh spatial resolution (Itatani et al., Nature 432, 867, 2004). The recent developments in femtosecond serial crystallography at X-ray free electron lasers (XFELs) have brought the achievable temporal resolution within a striking distance. This paper presents the electron density map decomposed from the XFEL data that shows the formation of several $3d$ atomic orbitals of the heme iron tens of femtoseconds after the laser-triggered photolysis. A key strategy that has enabled the findings here is a numerical deconvolution to resolve concurrent variations in a series of time-resolved electron density maps so that the image depicting an electron transfer event can be isolated as a partial change from the overwhelming presence of the bulk electrons that are not directly involved in bonding. The subtle changes in electron distribution due to a spin crossover can be decoupled from far greater changes due to atomic displacements. Direct observations of electronic orbitals could offer unprecedented mechanistic insights into a myriad of chemical and biochemical reactions such as electron transfer in redox reactions, and formation, rupture, and isomerization of chemical bonds.

Ligand photodissociation in carbonmonoxy myoglobin (MbCO) has been a

benchmark for studying ultrafast protein dynamics in a biological system (from

Šrajer et al., Science 274, 1726, 1996 to Barends et al., Science 350, 445, 2015). This

paper presents an in-depth analysis of the serial crystallographic datasets of

MbCO that Barends & Schlichting et al. (2015) contributed to the Protein Data

Bank. First, an atomic resolution image clearly shows the reappearance of the

high-spin $3d$ orbitals of the heme iron upon the photodissociation of the CO

ligand. Second, the dynamic behaviors of these newly regained $3d$ orbitals

within picoseconds after the photolysis provide long-awaited structural

validation for previous spectroscopic observations and computational

simulations. Specifically, the newly formed densities are oscillating with the

heme iron at a high frequency and developing a positional modulation during

the first a few picoseconds (Champion, Science 310, 980, 2005). The dominant

oscillation at a low frequency previously detected by coherence spectroscopy can

be clearly resolved from the time series of electron density maps. The associated

changes in electron density during the first cycle of the oscillation are largely

located in the solvent rather than on the protein or heme, which suggests that the

low frequency oscillations in a number of heme proteins, including MbCO, likely

originate from a photolysis triggered pressure wave propagating in the solvated

protein. Finally, these findings of chemical signals are isolated from coexisting

thermal artifacts also by the numerical deconvolution. It is modeled in this study

that the ultrashort XFEL pulses cause a transient spike of the local temperature at

the heme site for hundreds of K.

Keywords

ultrafast dynamics

heme protein

X-ray free electron laser

serial crystallography

singular value decomposition

Abstract

Direct visualization of electronic events during chemical and biochemical

reactions will offer unprecedented mechanistic insights. This has been long

believed to be beyond the reach of protein crystallography, as it requires the

ability to capture alterations in electronic distributions at an ultrafast time scale

and an ultrahigh spatial resolution. Ultrashort pulses produced by X-ray free

electron lasers (XFELs) offer an unprecedented opportunity for direct

observations of transient electronic and molecular events as short-lived as tens of

femtoseconds. This paper presents an in-depth analysis of the serial

crystallographic datasets collected by Barends & Schlichting et al. (2015) that

probe the ligand photodissociation in carbonmonoxy myoglobin (MbCO), a long-

serving hallmark for observing ultrafast dynamics in a biological system. This

analysis reveals orbital densities that depict the formation of high-spin $3d$ atomic

orbitals of the heme iron upon the CO departure and the dynamic behaviors of

these newly formed orbitals in a time series within the first a few picoseconds.

The regained $3d$ orbitals are found vibrating with the heme iron at a high

frequency and developing a positional modulation. These new findings here

provide long-awaited visual validations for previous works using ultrafast

spectroscopy and molecular dynamics simulations. This analysis also extracts

electron density changes largely in the solvent during the first period of a low

frequency oscillation previously detected by coherence spectroscopy. This work

demonstrates the power and importance of the analytical methods in detecting

and isolating transient, often very weak signals of electronic changes arising

from chemical reactions in proteins.

Introduction

One of the major applications of X-ray free electron lasers (XFELs) is to directly

observe ultrafast protein dynamics at atomic resolution during photochemical

reactions by serial femtosecond (fs) crystallography (SFX). Time-resolved

experiments (Barends et al., 2015; Coquelle et al., 2017; Nango et al., 2016; Nogly

et al., 2018; Pande et al., 2016; Tenboer et al., 2014) have been successfully

conducted by several groups at XFELs to follow the photodissociation of

carbonmonoxy myoglobin (MbCO; Fig. S1), photoisomerization of the p-

coumaric acid chromophore in photoactive yellow protein (PYP), and the

hydroxybenzylidene imidazolinone chromophore in a photoswitchable

fluorescent protein (Levantino et al., 2015a). While the time resolution, that is,

the ability to differentiate transient changes, is limited to tens of fs determined by

the duration of the XFEL pulses, the reported time resolution for MbCO and PYP

was around hundreds of fs due to some timing jitter between a short laser pump

and an XFEL probe.

It is well established that the CO ligand parks at a nearby docking site upon

photodissociation, while the heme iron moves out of the heme plane towards the

proximal side due to a spin crossover resulting in a high-spin, deoxy iron. As the

porphyrin structure becomes buckled, several side chains and helices of the

globin move accordingly (Fig. 1). These light-induced structural changes are

highly reproducible and have been observed by both static and time-resolved

crystallography, repeatedly (Bourgeois et al., 2003, 2006; Chu et al., 2000;

Kachalova et al., 1999; Schotte et al., 2003; Šrajer et al., 1996, 2001) . Recently,

they have been captured once again at a sub-picosecond (sub-ps) time scale in

MbCO crystals using XFEL pulses (Barends et al., 2015), a major achievement in

SFX. However, no significantly new structural finding is beyond those already

obtained by time-resolved Laue diffraction at synchrotrons (Bourgeois et al.,

2007; Clifton et al., 1997; Ren et al., 1999), which is one thousand times slower at

a time resolution of 100 ps limited by the bunch length of the synchrotrons

(Aranda et al., 2006; Bourgeois et al., 2006; Jung et al., 2013; Ren et al., 2012;

Schotte et al., 2012).

The signals of ultrafast structural events could be too weak to be observed

clearly by the standard crystallographic analyses. In this work, I reexamine the

time-resolved XFEL datasets of MbCO deposited in the Protein Data Bank (PDB;

5cmv, 5cn4-9, and 5cnb-g) by Barends & Schlichting et al. (2015) with a

previously developed analytical method based on singular value decomposition

(SVD; Ren, 2013a; Ren et al., 2012, 2013; Schmidt et al., 2003). This SVD analysis

unravels several ultrafast events that have been identified by spectroscopy and

computational studies but have not been directly observed as images of electron

density distribution. The new findings along with several previously known

structural events upon photolysis of MbCO are summarized in a Gantt chart

(Fig. 1).

Heterogeneous signals in dynamic crystallography

Compared to the well-established structural events of MbCO photolysis such as

ligand docking and heme doming, the ultrafast signals are small in amplitude

and low in population. As a result, they do not stand out at a given time point,

thus went unnoticed in the conventional analyses. As demonstrated below, these

weak yet consistent signals can be manifested as major components resulting

from SVD when multiple datasets collected at different time points are jointly

examined. This necessity for decomposition presents both a challenge and a

unique opportunity for dynamic crystallography, broadly defined as

experimental and analytical approaches that tightly join crystallographic datasets

and their metadata (Ren et al., 2013). We have developed a numerical process of

deconvolution, which is able to identify and extract the dynamic signals from a

large collection of datasets acquired at varying experimental conditions such as

time points and temperatures (Bandara et al., 2017; Kim et al., 2017; Ren, 2016;

Ren et al., 2012; Yang et al., 2011; Zeng et al., 2015). In essence, this strategy takes

advantage of the structural heterogeneity, rather than preemptively avoids them

as in the common practice of static protein crystallography. In a nutshell,

dynamic crystallography consists of two major steps: first, to experimentally

introduce a functionally-relevant structural heterogeneity; and second, to

numerically resolve the heterogeneous conformational species as long as their

compositions vary as a function of an experimental parameter (Ren et al., 2013).

Here, time-resolved serial crystallography, with the variable being the ultrashort

time delay, is simply a special case in the broader scope of dynamic

crystallography. Therefore, the previously established methodology is largely

applicable (Methods).

Ultrafast dynamics during photodissociation of MbCO

The departure of the CO ligand from the iron porphyrin system has been found

within the first 15 fs upon photolysis (Dunietz et al., 2003; Falahati et al., 2017).

This signal is clear in the earliest time point of ~30 fs (nominally -0.1 ps). A

transient charge transfer from the iron to porphyrin subsequently taking place

during 50-300 fs has been identified as a key event by resonance Raman spectra

(Franzen et al., 2001). A high-spin iron is resulting in the charge transfer from
the porphyrin back to the iron. The transient charge transfer state is not visible
in the maps calculated from the datasets of Barends & Schlichting et al. perhaps
due to a broad distribution of the transferred electron over the porphyrin.
However, the larger, high-spin iron is clearly manifested in the earliest maps.
Vibrational signals have been observed by ultrafast spectroscopy, which is
attributed to a discontinuity or "seam" at the intersection of two energy
landscapes of the photoexcited MbCO and the ligand dissociated product
(Champion, 2005). Dissipation of such vibrational energy in the protein matrix
and its solvent environment consists of at least two distinct components:
localized, high frequency vibration modes of the heme and global, low frequency
oscillations of the protein (Fujisaki and Straub, 2005). This analysis provides
direct visual evidences to validate both components and to delineate the low
frequency oscillations in greater detail.
182
The most observed high frequency modes range from hundreds to more than
a thousand wavenumbers (Armstrong et al., 2003; Kitagawa et al., 2002;
Mizutani, 1997). The corresponding vibrational periods of these high frequency
modes of 24-50 fs are comparable to the XFEL pulse duration. Hence an
individual cycle of such vibrations is beyond what can be traced out by the
current XFEL datasets. Nevertheless, the averaged effect of these high frequency
vibrations is detectable in the XFEL datasets of Barends & Schlichting et al.
Amplitude modulation and damping of the high frequency mode v_7 was
simulated with a period of 350 fs and a time constant of 1.3 ps (Barends et al.,
2015). There are no sufficient time points in the existing datasets to determine
these amplitude modulation and damping. However, a slower modulation of
the high frequency modes is clearly present in these datasets. These in-plane
modes are thought to be coupled to out-of-plane motions of the heme and its iron
(Nagy et al., 2005), which take place within a fraction of ps and continue to
develop non-exponentially for another hundreds of ps according to molecular
dynamics simulations (Henry et al., 1985; Kuczera et al., 1993). Recent
simulations (Barends et al., 2015; Falahati et al., 2017) suggested that many
transient species, including those with iron in or out of the plane, are mixed
together before 1 ps. This analysis of the XFEL datasets of Barends & Schlichting

et al. can confirm that the out-of-plane displacement of the iron is a highly
asynchronous event among a molecular population, but this event takes place
simultaneously with an in-plane shift of the porphyrin. The signal associated
with the iron displacement lingers on for many μs before the ligand rebinding as
captured by the time-resolved Laue diffraction (Šrajer et al., 2001).
207
At a low frequency region, the dominant oscillations range between 25 and 45
cm^{-1}, corresponding to a period around 1 ps (Gruia et al., 2008; Rosca et al., 2002).
Such a low frequency oscillation is within the reach of the sub-ps time resolution
achieved with XFEL pulses. Therefore, each individual cycle of the low
frequency oscillation should be, and indeed is, visible in the datasets of Barends
& Schlichting et al. Here I demonstrate that such an oscillation occurs more in
the solvent channels than in the protein or the heme at a low frequency
comparable to those detected by spectroscopy. This analysis supplies the long
missing visual evidence to support such a low-frequency oscillation in a solvated
protein.
218
Imaging the geometric shapes of atomic orbitals
The shape of a *d* orbital of copper was first experimentally imaged by Zuo et al.
in their difference electron density map (Humphreys, 1999; Zuo et al., 1999),
which is not, however, without controversy (Mulder, 2011; Scerri, 2000).
Nevertheless, experimenters continued to draw parallels between captured
images, sometimes with reconstructions, and atomic or molecular orbitals.
Molecular orbital tomography constructs images of highest occupied molecular
orbital from many projections of aligned molecules (Itatani et al., 2004; Ogilvie,
2011; Schwarz, 2006; Stapelfeldt, 2004; Vozzi et al., 2011). Both the highest
occupied and lowest unoccupied molecular orbitals of flat molecules, such as
pentacene, were scanned using scanning tunneling microscopy (Chaika, 2014;
Patera et al., 2019; Repp et al., 2005). Field-emission electron microscopy and
angle-resolved photoemission spectroscopy can directly capture or indirectly
reconstruct orbital images or transformation of orbital configurations
(Mikhailovskij et al., 2009; Puschnig et al., 2009). More recently, a
photoionization experiment showed the nodal structures of the atomic orbital of
excited hydrogen atoms (Stodolna et al., 2013). A charge-density analysis at an

ultrahigh resolution showed iron $3d$ and sulfur $3p$ densities (Hirano et al., 2016).

In all these achievements, the experimentally captured electron densities with

their unique shapes are directly influenced by electrons occupying specific

orbitals. Therefore, in this paper, the images and motion pictures of difference

electron densities caused by a superposition of several wave functions are

attributed to atomic orbital densities and their dynamics.

Results

Decomposition of difference maps and simulated annealing omit maps

To analyze the time-resolved serial crystallographic data of horse heart (hh)

MbCO, one dark and 12 light datasets are obtained from PDB (Fig. 1; Barends et

al., 2015), among which three light datasets collected at the shortest time delays

of 0 and ±0.1 ps span both sides of time 0 that is centered on the excitation laser

flash of 150 fs. These datasets are expected to capture ultrafast signals at delay

times as short as a few fs to tens of fs. However, these ultrafast signals are also

inevitably mixed with signals at longer delays as far as the X-ray pulses stretch

(Haldrup et al., 2011). These sub-ps time points were taken at the same

controlled time delay of 0.5 ps. But the diffraction images were binned according

to the timestamp records from an experimental timing tool (Bionta et al., 2011,

2014), which produced a number of short delays before 1 ps. Several longer time

points ranging from 3 to 150 ps are also included in this analysis (Fig. 1) along

with a deoxy hhMb dataset collected from a synchrotron beamline at 7°C, which

represents the end product of the MbCO photolysis, and two hhMb structures

(1dws/t) in the photolyzed state captured by cryo-trapping (Chu et al., 2000).

Two types of electron density maps, namely difference Fourier map and

simulated annealing omit map (SAOM), are produced and subjected to the SVD

analyses separately. Each type carries its own advantage (Methods). While

difference Fourier maps are sensitive for detecting weak signals, they are also

highly susceptible to systematic errors that often overwhelm the signal. It is

critically important to choose a proper reference dataset, which is preferably

obtained under the identical experimental condition, if not from the very same

crystal as the light dataset. An alternative is to use simulated annealing omit

maps (SAOMs), where a small portion of a protein structure is set aside from

structure refinement using the simulated annealing algorithm. In such omit

maps, the electron densities within the omitted region shall not be biased by the

protein model, thus are considered as the most authentic representation of the

experimentally observed electron densities (Hodel et al., 1992). A collection of

SAOMs within a common omitted region, although less sensitive to subtle

changes compared to difference maps, permits a joint analysis of maps derived

under very different experimental conditions, which is usually difficult with

difference maps.

Difference Fourier maps are synthesized using a Fourier coefficient set of

F_{light}-$F_{reference}$ combined with the best possible phase set. While $F_{reference}$ is typically

the structure factor amplitudes from a dark dataset (5cmv), a light dataset at a

given time point can also be used as a reference (Table S1) to reveal changes with

respect to that specific time point. Therefore, a collection of difference maps

produced with the same reference dataset constitutes a time series that show

continuous changes from the reference time point and on. For example, -F_0 series

refers to the difference maps with the reference dataset at the nominal 0 ps. For

omit maps, the heme and several side chains within its immediate vicinity are

omitted to produce SAOMs of the ligand binding site (Methods). A few static

structures determined at room temperature of sperm whale (sw) Mb in the

aquomet (1bz6), deoxy (1bzp), and CO-bound (1bzr) forms are also included

(Kachalova et al., 1999), along with three hhMb structures (1dwr/s/t) (Chu et al.,

2000) as control data points in this joint analysis. The resolutions of these static

datasets are truncated to 1.75 Å to match the XFEL datasets.

Altogether, a total of 93 difference maps (Table S1 and Fig. S2) and 20 omit

maps are obtained for two independent SVD analyses. The electron densities are

masked around the heme site unless stated otherwise (Methods). In this work,

the significant signals are identified by the top nine components from SVD (Fig.

2e). A numerical procedure called rotation in the SVD space plays an important

role to extract chemically sensible components from these (difference) electron

density maps. Proper SVD rotations reveal the correlation between the core data,

i.e., electron densities, and their metadata, i.e., delay time, sample temperature,

and other experimental parameters (Methods). Given their highly dynamic

nature, the ultrafast signals due to multiple causes are often entangled before

suitable rotations are found in the multi-dimensional SVD space (Methods).

Using this decomposition and rotation strategy, I identify signals that reveal the

electronic redistributions between high and low spin of the iron orbitals.

Molecular image of low-spin to high-spin crossover

The ultrafast signals within several fs to a few tens of fs are manifested in the

fourth component, denoted as U_4, of 93 difference maps. The component U_4

exists only in the map series with the dark dataset as reference (red, $-F_{dark}$ series

in Fig. 2). That is to say, all signals in this component have occurred by the time

of the very first time point at ~30 fs (nominally -0.1 ps), therefore the map

series $-F_{-0.1}$ shows the cancellation of this component (orange series in Fig. 2).

Some of these signals start to decay slightly before 1 ps and diminish

significantly after 3 ps (Fig. 2cd). The corresponding map of U_4 reveals the

previously well-established events arising from the CO docking, out-of-plane

motion of the iron, and an uneven heme doming upon photolysis of MbCO (Fig.

3). The negative densities associated with the pyrrole rings and their propionic

acid side chains increase in intensity from ring A to B to C to D (Fig. 3c). Positive

densities are found at a cross section 1.1 Å above from the heme plane towards

the distal side (Fig. 3b).

More remarkably, U_4 captures some hitherto unseen signals characterized by

a cubic-shaped network of positive densities that cage the iron atom. To

facilitate a more concise description, a cross section of $Z = 0$ is defined as the

heme plane and the cross section of $X = 0$ is the dividing plane between two

propionic acid side chains, from which the third orthogonal cross section of $Y = 0$

can be derived (Fig. S1). The eight corners of the positive cube are oriented along

the X and Y cross sections. Strong positive densities appear in the middle of four

six-membered rings encircled by adjacent pyrroles, the connecting methane

bridge, and the heme iron (marked 1-4 in Fig. 3c). These positive features reach

their four minima when they cross the heme plane. These positive densities

become more prominent above and below the heme plane (Fig. 3). The eight

corners of this cube are located at 1.7±0.2 Å away from the iron. This cubic-

shaped network is asymmetric with a notable skew towards CD rings (Fig. 3fh).

Meanwhile, the iron skews in the opposite direction towards the AB rings with

an out-of-plane displacement towards the proximal side (Fig. 3e). But the major

features of this cube seem more symmetric in the Y cross section.

Similar features are also evident in the seventh component U_7 from an

independent SVD analysis of 20 SAOMs. It is clear that the seventh dimension

after a proper rotation (Methods) represents the reaction coordinate of the ligand

dissociation. Mb structures along this reaction coordinate form a lineup on c_7

dimension regardless of sample temperatures: from the ligated structures of

MbCO, to several photoexcited species including the static structures Mb*CO

and **CO, to the time-resolved structures, eventually to deoxy Mb (Fig. 4e). U_7 of

the omit maps contains the usual signals of ligand dissociation, such as strong

negative densities on the ligand indicating dissociation and a positive layer on

the distal side of the heme indicating its doming (Fig. 5), except that the quality

of these signals are not as good as those from difference maps because the

SAOMs are derived from datasets in very different experiments. Despite the

systematic errors, the seventh component U_7 of the omit maps again clearly

shows the cubic network of positive densities around the iron. This cubic

network also skews towards the CD rings of the heme. This is a reoccurrence of

the same signals shown in U_4 of the difference maps (compare Figs. 3 and 5). The

eight corners of the positive cube are 1.9±0.2 Å away from the iron.

It is clear that the positive densities arranged in the cubic network around the

iron, either in the difference maps or in the omit maps, originate from the

intrinsic signals induced by light. While multiple interpretations of these distinct

cube-shaped signals may apply, this positive network clearly indicates a larger,

high-spin iron as an immediate overall effect upon photolysis. Here I attribute

the distinct shape of the positive feature to instantaneous regaining of the high-

spin $3d$ orbitals of the heme iron. The theoretical radii of the $3d$ orbitals for an

isolated Fe(II) cation are about 0.36 Å at the maximum probability of these

orbitals (Waber and Cromer, 1965). Iron $3d$ orbital densities in a Fe_4S_4 cluster

span no further than 1 Å away from the iron as measured in a protein (Hirano et

al., 2016). The observed distances between the iron and the corners of the cube

are significantly larger. Therefore, the observed cubic densities here are not an

image of a probability distribution of a $3d$ electron. They are resulted from a net
gain of electron densities as a fully ligated iron transitions to the high-spin deoxy
state when several $3d$ orbitals are reoccupied by unpaired electrons. Second, the
deoxy iron remains ligated to the five nitrogen atoms after the ligand
dissociation, which is very different from an isolated iron. A calculated image of
a combined probability distribution of the textbook $3d_{xz}$ and $3d_{yz}$ with $3d_{xy}$ or
$3d_{x^2-y^2}$ orbitals show features highly similar to the observed cubic-shaped map
(Fig. 3lm). The strongest electron densities are located at eight corners of the
cube where the lobes of $3d$ orbitals intercept. It is entirely possible that the
captured positive cube is due to a greater influence of the regained high-spin
electrons in several $3d$ orbitals at greater distances from the iron. The observed
cubic densities result from not only a superposition of several $3d$ orbitals but also
an average of multiple electronic configurations. Such orbital changes have not
been captured in protein crystals by X-ray crystallography. Since the non-
bonding inner electrons remain largely constant in their distributions, their
presence as a majority can easily overwhelm small changes in the minority of
electrons involved in bonding. Therefore, it usually requires ultrahigh resolution
to observe orbital densities (Hirano et al., 2016). It is very difficult, if not
impossible, to detect small orbital changes upon bond rupture using regular
electron density maps such as a 2Fo-Fc map. This difference map derived from
an SVD component represents a partial, variable signal in addition to the total
signal that is largely constant regardless of the bonding change. After the
decomposition and a proper rotation, it is highly sensitive to small but consistent
changes such as the low-spin to high-spin transition of the heme iron.
396
Interestingly, the signals in U_4 of difference maps decay after a few ps leading
up to the static structures Mb*CO, **CO, and the deoxy Mb (Fig. 2cd). Given in-
plane vibration modes (Abe et al., 1978; Nagy et al., 2005) with time periods
comparable to the XFEL pulse duration, the averaged electron density between
vibrating atoms is expected to increase, as shown in the six-membered rings.
The observed decay of U_4 can be interpreted as the damping of these vibrations
at high frequencies (see below). These in-plane vibrations trigger the iron to pop
out of the heme plane (Franzen et al., 1995). However, it remains unclear from
this observation which, or a combination of which vibration modes, plays the

dominant role. Importantly, this decay does not reach zero even for the static
deoxy Mb (Fig. 2de), which suggests that the above interpretation of the cubic
network of positive densities as the consequence of the spin crossover largely
holds.
410

Development, modulation, and damping after 1 ps
If the cubic network of positive densities is indeed influenced by the newly
formed $3d$ orbitals, the dynamic behaviors of these high-spin electrons should be
visible in the time points shortly after the initial formation. The observed
evolution of the cubic network provides the necessary validation. Along the
dimension c_1 in the SVD space of 93 difference maps, all time points after 1 ps,
including the deoxy Mb (5d5r, representing a time point of ∞), are well separated
from the sub-ps time points around $c_1 = 0$ (Fig. 2ac). Therefore, the first
component U_1 of the difference maps represents more permanent changes. The
cubic positive densities in U_4 have no sign in U_1. Among the major features
captured by U_1 (Fig. S3), the most noticeable is a rather skewed displacement of
the iron towards the AB rings of the heme. In the meanwhile, the motion of
His93 corroborates the skewed displacement of the iron (Fig. S3ae). It is also
shown below that the entire heme moves in its plane towards the AB rings as
well. Such long-lived signals in U_1 are in good agreement with a wide
distribution of individual trajectories and interquartile range of iron
displacement predicted by molecular dynamics simulations (Barends et al., 2015;
Henry et al., 1985; Kuczera et al., 1993). However, the cryo-trapped structures,
Mb*CO and **CO, are two exceptions. They exhibit very little characteristics in
U_1 (Fig. 2ac), even though they presumably represent long time delays. It can be
reasoned that the skewed displacement of iron requires concerted motions in the
proximal His93 and helix F, which are evidently forbidden at cryogenic
temperatures.
434

Another major signal revealed after 1 ps is a modulation of the new formation
of $3d$ orbitals imaged by the cubic-shaped positive densities around the iron.
This is a positional modulation manifested in the sixth and eighth components of
the difference maps. While the corresponding coefficients c_6 and c_8 are nearly
constant in the sub-ps range, they start to ramp up after 1 ps and oscillate at a

period of 220±20 ps (Fig. 2f). The U_6 features a distorted network of positive

densities around the heme iron (Fig. S4) similar to U_4 but with an inclination

towards ring B. In U_6, the strong negative density on iron is skewed towards the

proximal side in contrast to U_4 (compare Figs. 3e and S4c). In a linear

combination sense, U_6 and U_8 can be interpreted as variations that are combined

into U_4 resulting in an oscillation of a period at a few hundreds of ps (Fig. 2f).

Specifically, the cubic-shaped densities lean towards ring D from 0.5 to 3 ps,

return to the mean position after 50 ps, then sway towards ring B in the opposite

direction (Fig. 2f and Movie S1). Such oscillating behavior suggests modulation

and damping of the vibrational modes. In other words, the cubic network of

positive densities is not only observed in the earliest time points. The motions of

this network are captured in later time points as well. This 220-ps oscillation is

largely along BD cross section (Fig. S1), while damping of the vibrational

amplitude was only recorded up to 150 ps thus insufficient to depict a complete

picture. Two other more significant components U_2 and U_3 also show an

oscillation of a distinct type as discussed below (Fig. 2b).

Vibration induced by XFEL pulses and instantaneous local temperature jump

An independent SVD analysis of 20 simulated annealing omit maps (SAOMs)

masked around the heme site (Methods) offers a direct comparison of the XFEL

data with previous synchrotron data. Among the top eight components of the

SAOMs, the first component reveals a well-defined molecular image of the heme

and its surrounding (Fig. S5); and the second and fourth components apparently

account for differences between hhMb and swMb. It is the third component U_3

that clearly distinguishes the XFEL datasets from the synchrotron datasets in a

temperature-dependent manner (Fig. 4ace). In other words, the cryo structures

(1dwr-t), ambient temperature structures (1bz6/p/r) including the deoxy hhMb at

7°C (5d5r), and the room temperature XFEL structures line up in an ascending

order along the dimension c_3. Not incidentally, the photolyzed structures

Mb*CO and **CO have the most negative coefficient c_3 because they were

obtained at temperatures even lower than 100 K (Chu et al., 2000). The

corresponding decomposed map U_3 reveals the thermal effects in electron

densities of the heme as a nearly symmetric shell of spherical positive densities

around the iron with very little positive or negative density on the iron itself (Fig.

6), which transitions into a square shape with four corners centered at the six-

membered rings in the heme plane (marked 1-4 in Fig. 6e). This symmetric U_3 of

the omit maps shows no usual signals for ligand dissociation and heme doming

therefore contrasts with the asymmetric U_4 of the difference maps characterized

by the distinct cube-shaped positive density maxima at the eight corners. Two

sheets of positive densities are found at the cross sections parallel to, and 1.3 Å

away from, the heme plane on both sides. They are connected via pillar-shaped

positive densities that run through the six-membered rings (marked 1-4 in Fig.

6e) and the pyrrole rings (marked 5-8 in Fig. 6e) along with some other positive

densities outside the heme (marked 9-20 in Fig. 6e). It is noteworthy that such

thermal effects do not stand out as a major component in the SVD analysis of

difference maps because they are largely self-canceled during the subtraction of

the reference.

The temperature dependency and the symmetry of the signals in U_3 strongly

suggest that these features are related to the local temperature at which the

MbCO molecules were probed, rather than artifacts arising from structural

refinement or difference in spatial resolutions among datasets. All datasets used

for the analysis are unified to a similar spatial resolution (Methods). Both the

positional and thermal parameters of the model within the omitted region are

intentionally "forgotten" in the calculation of the omit maps. The large c_3

coefficients of the XFEL datasets are due to instantaneous temperature elevation

at the heme site upon exposure to each ultrashort XFEL pulse, which introduces

isotropic vibrations at high frequencies to the entire tetrapyrrole cofactor as

captured by U_3. This molecular image of U_3 shows an increasing intensity from

cryo to ambient temperatures and to a very high local temperature induced by

XFEL pulses (Movie S2). Based on an estimate by linear extrapolation, the

instantaneous temperature of the heme may exceed 500 K during an 80-fs XFEL

pulse (Fig. 6i).

Identification of such symmetric and temperature-dependent signals is

important for interpretation of the highly dynamic ultrafast signals. The fact that

the symmetric U_3 and the asymmetric U_7 of the omit maps are orthogonal to each

other mathematically guarantees that these coexisting components are not

interchangeable and not cross contaminated between them (Methods). The cubic

shaped U_7 of the omit maps and its counterpart U_4 of the difference maps are the

chemical consequence of ligand dissociation; the symmetrical and spherical

shaped U_3 originates from thermal consequence. One cannot compensate for the

other and both coexist in all maps. The equivalents can be stated: The cubic

positive densities influenced by the $3d$ orbitals are not due to thermal vibrations;

the symmetrically enlarged iron and the thickened heme due to an instantaneous

temperature elevation have been isolated from the chemically relevant signals. It

is worth noting that the ultrafast onset of U_4 of the difference maps (Fig. 2cd) is

not in contradiction with U_7 of the omit maps contained not only in time-

resolved XFEL datasets but also in the static structures of deoxy Mb and

photolyzed Mb*CO and **CO at different temperatures (Fig. 4e). Instead,

regaining of the high-spin $3d$ orbitals of the iron occurs before the first time point

at tens of fs; this permanent change lasts throughout the dissociation reaction

into the deoxy product.

The asymmetric U_7 of the omit maps also describes an important but

unfamiliar structural event upon the ligand dissociation – the heme in-plane

movement. This type of motion is harder to capture in difference maps due to

little net gain or loss of electron density. With respect to the protein framework,

the heme moves towards its AB rings in addition to the other changes (Movie

S3). Heme in-plane sliding is consistent with the strong signals of the skewed

iron displacement and the proximal His93 (Fig. S3) that are closely related to

structural responses in the globin. Similar in-plane sliding is even more

significant in tetrameric hemoglobins (Baldwin and Chothia, 1979), which is the

origin of the cooperative oxygen affinity (Ren, 2013b). However, the functional

role of the heme in-plane sliding in Mb is not yet clear and this discussion

branches away from the topic here on ultrafast structural changes.

Global oscillation at low frequency

A recent solution scattering experiment using XFEL pulses showed that the

radius of gyration R_g first increases within 1 ps upon photodissociation of MbCO

and then oscillates with a period of 3.6 ps, equivalent to 9 cm^{-1}, a frequency lower

than those observed by coherence spectroscopy (Levantino et al., 2015b). The

molecular volume increases afterward with a phase shift. These observations

indicate a wavelike motion propagating outward in the protein. Recent

molecular dynamics simulations have confirmed that a pressure wave generated

by photolysis reaches the molecular surface at 300 fs along the direction

perpendicular to the heme plane (Brinkmann and Hub, 2016), which corresponds

to a frequency of 28 cm^{-1} if the wave is reflected by the surface and oscillates. It

was found by the simulations that the oscillation in solvent would be

overdamped thus dissipate quickly.

Some SVD components derived here display strict correlation suggesting

their dependency on each other. The U_2-U_3 pair of the difference maps (Fig. 2b)

is evidently equivalent to the U_5-U_8 pair of the omit maps (Fig. 4b). When the

difference maps of the entire globin are analyzed, the same signals emerge as the

top-ranked U_1-U_3 pair suggesting that the local signals at the heme site concur

with global structural changes (Fig. 7). Interestingly, this pair of components

displays a near-perfect circular correlation at sub-ps delays but is almost absent

at longer delays > 1 ps (Fig. 7afgh). Such an oscillation at a low frequency is

depicted by at least two orthogonal components. The oscillating signals have

spread over the entire globin at sub-ps delays while their amplitudes decay to

nearly zero after a few ps. Based on least-squares fittings of the corresponding

coefficients (Methods), the frequency of the oscillation is determined to be 36±1

cm^{-1}, corresponding to a period of 0.93 ps. The equivalent oscillation frequencies

determined from the difference maps and omit maps at the heme site are 42±2

cm^{-1} (a period of 0.8 ps) and 37±1 cm^{-1} (a period of 0.89 ps), respectively.

The low frequency oscillation determined from the XFEL datasets of Barends

& Schlichting et al. is highly comparable to those measured by fs vibrational

coherence spectroscopy (Gruia et al., 2008; Rosca et al., 2002). I postulate that this

low frequency reflects the molecular size of the solvated Mb rather than the

property of the heme. First, the corresponding period of 0.93 ps is about the time

that a mechanical wave takes to traverse 14 Å, that is, an average radius of Mb,

given a speed of sound at 1.5 km/s or nm/ps (Fig. S1). Second, the molecular

sizes of several heme proteins, such as cytochrome c, Mb, and cystathionine β-

synthase, show good correlation with their dominant low frequency of 25, 40,

and 44 cm^{-1}, respectively (Gruia et al., 2008; Karunakaran et al., 2010, 2014).
Third, no association between the oscillating signals and specific structural
features cab be found, such as the heme, ligand, surrounding side chains, or
helices (Fig. 7ik). These oscillating signals sharply contrast with strong, non-
oscillating signals associated with the heme and various helices evidenced by the
second and fourth components of the difference maps (Fig. 7jl), where helices E,
G, and H move away from the heme in response to ligand dissociation (Barends
et al., 2015). These motion signals of various structural elements are much
enhanced by SVD.
585
The oscillating signals at low frequency seem to propagate via solvent within
the protein matrix rather than the globin. Strong positive and negative peaks in
U_5, U_6, and U_8 of the omit maps are consistently found in the solvent region away
from the heme and protein. However, a specific description on how the solvent
responses to ligand dissociation is difficult, since the solvent structure is far less
well determined than the globin structure. The total contribution of U_5, U_6, and
U_8 can be judged by a sum of the absolute values $|U_5|+|U_6|+|U_8|$ that enhances
the distribution of signals. The heme and globin are located in low intensities of
these oscillating components (Fig. 7b-e). These global oscillations occur only a
few times before dissipation (Gruia et al., 2008; Rosca et al., 2002). However,
there are not enough time points to determine the damping time constant, which
is roughly estimated to be 1-4 ps. Contrary to the common belief that low
frequency oscillations are due to the properties of the heme, here I attribute the
low frequency oscillation to the global response of protein and solvent as a whole
to a sudden molecular event – photodissociation of the ligand, where the
vibrational energy released from the photodissociation is quickly dissipated into
the environment.
603
Discussion
Spatial and temporal resolutions vs. resolution of heterogeneity
Ultrashort XFEL pulses improve the time resolution to sub-ps or even tens of fs
compared to 100 ps at synchrotrons. This ultrafast time resolution, when
combined with the spatial resolution of crystallography, offers an unprecedented
opportunity for direct observation of transient strucutral changes in a

photochemical reaction as detail as individual electrons. However, due to their

highly dynamic and heterogeneous nature, it is difficult to experimentally isolate

short-lived structural species along the temporal dimension regardless of the

ultrafast time resolution. As demonstrated by this work, analytical methods

such as SVD are needed to deconvolute them from mixtures of intermediate

species. The strategy we proposed for resolution of structural heterogeneity (Ren

et al., 2013) is highly effective for separating independent electronic and atomic

events that often coexist during a chemical reaction or a biological process.

Protein crystallography is proven to be a powerful tool for capturing atomic

motions involving displacements of tens of electrons such as side chain rotation

or heme iron movement by a few tenths of an Å. However, it has been long

believed that ultrahigh spatial resolution is required for direct observation of

changes involving fewer electrons such as a single electron transfer. Such a

notion contradicts the fact that there is no discontinuity in the probability

distribution of any electronic orbital. Therefore, a reasonably high spatial

resolution shall be sufficient to analytically describe the smooth function of a

gain or loss of an electron. In reality, any gain or loss of a few electrons would be

overwhelmed by the strong presence of static distributions of the bulk electrons.

Difference Fourier maps must be used to reveal such small changes.

Furthermore, difference signals are often dominated by displacements of the

nuclei that carry the displacements of many electrons, which in turn conseals

subtle changes of electronic configuration. Therefore, the small amplitude of the

desired changes over a much greater known and variable quantity hinders a

reliable observation more than the lack of high spatial frequencies does. This

work presents the effectiveness of the strategy that enables numerical

deconvolution to resolve concurrent events so that the image of a partial change,

such as a single electron transfer, can be isolated from the total change.

This work shows that it is possible to observe structural changes as small as a

gain or loss of individual electrons by protein crystallography. Small structural

signals can be correlated with their corresponding metadata, thus allowing the

origin of these signals be revealed. I posit that all observations so far are the

consequence, rather than the cause, of the ligand dissociation. The proposed

transient event of metal-to-ligand charge transfer is not imaged here, presumably

because the transferred electron is delocalized all over the porphyrin (Franzen et

al., 2001). Therefore, the fundamental questions remain for photolysis of MbCO:

Upon the absorption of a quantum of energy from a green photon, what initial

electronic events lead to a partitioning of the bonding electrons in two separate

atoms, that is, the bond rupture? What electronic events cause spin crossover?

To image these events directly, even higher time resolution is required to capture

electron density changes prior to and during bond rupture.

Future outlook of ultrafast dynamic crystallography

Despite the recent developments in SFX and its applications to time-resolved

studies of a few model systems, it is yet premature to declare that the current

SFX platform applies to other systems of more significant biological importance

such as visual and microbial rhodopsins, plant and bacterial phytochromes,

photosynthetic reaction centers, and light-dependent DNA repair enzymes.

There are at least three major challenges. First, it is not practical to produce and

inject an astronomical number of nano or microcrystals of these proteins into an

XFEL beam to support data collections of a time series. The current serial

protocol used to obtain the MbCO and other data has a diminishingly small yield

of useful diffraction data, based on the ratio between the number of diffracting

crystals and the total number of crystals produced (Ren et al., 2018). That is to

say, the vast majority of the crystals have no chance to diffract X-ray even when

they are injected into the X-ray beam. Such a yield, defined differently from the

hit rate, is not sustainable for other photosensitive protein crystals. Several

"fixed target" implementations of crystal delivery systems have been reported

recently, including one that we have recently proposed (Ren et al., 2018).

However, it remains to be seen how effective they are in achieving a higher yield

while maintaining the diffraction quality.

Second, since the XFEL pulses usually carry insufficient bandwidths to fully

integrate the Bragg reflections and in the meanwhile angular rotation of a crystal

during an ultrashort XFEL pulse is not attainable, tens to hundreds of thousands

of diffraction images are required to obtain integrated intensities for all Bragg

reflections in a complete sampling of the reciprocal space. A possibility to solve

this partiality problem is to use an X-ray beam with a significant convergence of
about a degree (Ho et al., 2002; Spence et al., 2014), which will produce
completely integrated intensities with a far fewer number of diffraction images,
thereby alleviating the demand for a large quantity of purified proteins.
682
Third, data merging from a large crystal pool is a major source of error in
producing difference signals for photosensitive crystals, particularly those with
much lower diffraction power than Mb crystals. To address this challenge, we
also proposed a protocol to obtain difference signals that bypasses the integrated
intensities of Bragg reflections. We reason that integrated intensities are not
necessary if the dark and light diffraction images can be obtained within a single
exposure of an XFEL pulse because the percentage change in integrated
intensities of a given reflection, that is, the time-resolved signal, is already
captured even if this reflection is partially measured in both dark and light
conditions. A plan has been laid to implement this protocol, based on a
dichromator instrument that achieves angular split and temporal delay at the
same time (Ren and Yang, 2016). When each of these technical hurdles is
overcome, direct observations of atomic and electronic movements during
ultrafast processes will be closer to broad applications for many light sensitive
crystals of great biological and medical significance.
698
Methods
Difference Fourier maps
Difference Fourier maps are commonly used in protein crystallography. A
difference Fourier map is synthesized from a Fourier coefficient set of F_{light}-$F_{reference}$
with the best available phase set. Before Fourier synthesis, F_{light} and $F_{reference}$ must
be properly scaled to the same level so that the distribution of difference values
is centered at 0 and not skewed either way. All available datasets from Barends
& Schlichting et al. (2015) are carefully scaled and difference Fourier maps are
calculated using a software package dynamiX™ according to previously
published protocols including a weighting scheme under the assumption that a
greater amplitude of a difference Fourier coefficient F_{light}-$F_{reference}$ is more likely
caused by noise than by signal (Ren et al., 2001, 2013; Šrajer et al., 2001; Ursby
and Bourgeois, 1997).

Both the dark and light datasets can serve as a reference in difference maps.

If a light dataset at a certain delay is chosen as a reference, the difference maps

show the changes since that delay time but not the changes prior to that delay.

These 91 maps in a full matrix provide independent observations for all changes

(Table S1). This strategy is important to the subsequent singular value

decomposition (SVD) analysis. In addition, two more difference maps of the

photolyzed Mb*CO and **CO (1dws/t) are also included in the $-F_{dark}$ series for

SVD analysis.

Each difference map is masked around the heme site. All densities at those

grid points are set to zero, if they are greater than 5 Å away from the heme, the

bound or docked CO, the side chain of the proximal His93, and that of the distal

His64. Difference maps around the heme site include the most significant signals

and exclude noises in the protein and solvent. However, the unmasked

difference maps in the entire globin are also used a separate SVD analysis to

include the protein and solvent regions for observation of global changes (Fig. 7i-

l).

Simulated annealing omit maps

The heme, the ligand, the side chains of proximal His93 and distal His64 are

removed from all structures listed in Table S2. The remaining structures are

refined using simulated annealing method with phenix.refine starting from a

simulated temperature of 5,000 K. Fourier coefficients are obtained from the

differences between the structure factor amplitudes $F_{observed}$ observed in the time-

resolved experiments and the structure factor amplitudes $F_{calculated}$ calculated from

the omitted structures. The resulting $F_{observed}$-$F_{calculated}$ maps, known as simulated

annealing omit maps (SAOMs), reveal the unbiased electron densities in the

omitted region. Since the sperm whale (sw) Mb structures are in a different unit

cell compared to the horse heart (hh) Mb structures, the SAOMs are aligned

together according to a least-squares fitting of the main chain atoms in residues

2-151. The heme and ligand are not used in the alignment procedure. Therefore,

the motion of heme found in these omit maps is with respect to the globin

framework (Fig. 5 and Movie S3). 20 aligned omit maps are subjected to the

subsequent SVD analysis.

Singular value decomposition of (difference) electron density maps

The principle of SVD can be found in texts on linear algebra. SVD analysis of

(difference) electron density maps was previously described (Ren et al., 2013;

Schmidt et al., 2003). Here I briefly recap this numerical procedure with an

emphasis on the physical meaning of the outcome.

An electron density map, sometimes a difference map, consists of density

values on a collection of grid points within a mask (see above), also known as

voxels that are equivalent to pixels in a digitized image. All M voxels in a three-

dimensional map can be serialized into a one-dimensional sequence of density

values according to a specific protocol. It is not important what the protocol is as

long as a consistent protocol is used to serialize all maps of the same grid setting

and size, and a reverse protocol is available to erect a three-dimensional map

from a sequence of M densities. Therefore, a set of N serialized maps, also

known as vectors in linear algebra, can fill the columns of a data matrix $\mathbf{A}$ with

no specific order so that the width of $\mathbf{A}$ is N columns and the length is M rows.

Often, $M \gg N$, thus $\mathbf{A}$ is an elongated matrix. In this work, $N = 93$ for the

difference maps and $N = 20$ for the omit maps. If a consistent protocol of

serialization is used, the corresponding voxel in all N maps occupies a single row

of matrix $\mathbf{A}$. For example, the voxels at the heme iron in all maps could occupy

the jth row of matrix $\mathbf{A}$. This strict correspondence in a row of matrix $\mathbf{A}$ is

important. Changes of the density values in a row are due to either signals,

systematic errors, or noises.

SVD of the data matrix $\mathbf{A}$ results in $\mathbf{A} = \mathbf{UWV}^{\mathrm{T}}$, also known as matrix

factorization. Matrix $\mathbf{U}$ has the same shape as $\mathbf{A}$, that is, N columns and M rows.

The N columns contain decomposed components $\boldsymbol{U}_k$, known as left singular

vectors of length M, where $k = 1, 2, …, N$. Therefore, each component $\boldsymbol{U}_k$ can be

erected using the reverse protocol to form a three-dimensional map. This

decomposed elemental map can be presented in the same way as the original

maps, for example, rendered in molecular graphics software such as Coot and

PyMol. The second matrix $\mathbf{W}$ is a square matrix that contains all zeros except for
N positive values on its major diagonal, known as singular values w_k. The
magnitude of w_k is considered as a weight or significance of its corresponding
component $\boldsymbol{U}_k$. The third matrix $\mathbf{V}$ is also a square matrix of $N \times N$. Each column
of $\mathbf{V}$ or row of its transpose $\mathbf{V}^{\mathrm{T}}$, known as a right singular vector, contains the
relative compositions of $\boldsymbol{U}_k$. That is to say, the N values in a column $\boldsymbol{V}_k$ describe
the relative compositions of the component $\boldsymbol{U}_k$ in the N original maps. A singular
triplet denotes 1) a decomposed component $\boldsymbol{U}_k$, 2) its singular value w_k, and 3) the
composition function $\boldsymbol{V}_k$. Singular triplets are often sorted in a descending order
of their singular values w_k. Only a small number of n significant singular triplets
identified by the greatest singular values w_1 through w_n can be used in a linear
combination to reconstitute a set of composite maps that closely resemble the
original ones in matrix $\mathbf{A}$, where $n < N$. For example, the original map in the ith
column of matrix $\mathbf{A}$ can be closely represented by the ith composite map $w_1 v_{1i} \boldsymbol{U}_1 +$
$w_2 v_{2i} \boldsymbol{U}_2 + \ldots + w_n v_{ni} \boldsymbol{U}_n$, where $(v_{1i}, v_{2i}, \ldots)$ is the ith row of matrix $\mathbf{V}$. $c_k = w_k v_{ki}$ in
front of $\boldsymbol{U}_k$ is the coefficients for the linear combination. Excluding the
components after $\boldsymbol{U}_n$ in this linear combination is based on an assumption that
the singular values after w_n are very small relative to those from w_1 through w_n.
As a result, the structural information evenly distributed in all N original maps is
effectively concentrated into a far fewer number of n significant components,
known as information concentration or dimension reduction. In this study, the
difference maps seem to contain 15 major components; the top nine are the most
significant (Fig. 2e). The components 10 through 15 may very likely harbor less
and less signals or systematic differences.
803
On the other hand, the trailing components in matrix $\mathbf{U}$ contain inconsistent
fluctuations and random noises. Excluding these components effectively rejects
noises. However, no clear boundary is guaranteed between signals, systematic
errors, and noises. Systematic errors could be more significant than the desired
signals, for example, the second and fourth components $\boldsymbol{U}_2$ and $\boldsymbol{U}_4$ of the omit
maps describe significant differences between hhMb and swMb, a valid scientific
point to investigate but outside the scope of this paper.
811

The orthonormal property of SVD

The solution set of SVD must guarantee that the columns in **U** and **V**, the left and

right singular vectors $\mathbf{U}_k$ and $\mathbf{V}_k$, are orthonormal, that is, $\mathbf{U}_h \bullet \mathbf{U}_k = \mathbf{V}_h \bullet \mathbf{V}_k = 0$

(ortho) and $\mathbf{U}_k \bullet \mathbf{U}_k = \mathbf{V}_k \bullet \mathbf{V}_k = 1$ (normal), where $h \neq k$ but both are from 1 to N. The

orthonormal property also holds for the row vectors. As a result, each

component $\mathbf{U}_k$ is independent of the other components. In other words, a

component cannot be represented by a linear combination of any other

components. For example, the symmetrical, spherical, and thermal component

$\mathbf{U}_3$ (Fig. 6) and the asymmetrical, cubic component of $\mathbf{U}_7$ (Fig. 5) of the omit maps

are orthonormal. Both components show positive densities around the iron. But

the cubic positive densities in $\mathbf{U}_7$ are associated with the strong negative densities

on the ligand depicting dissociation, which is a direct link from ligand

dissociation to regaining of the high-spin $3d$ orbitals of the iron. On the other

hand, no sign of ligand dissociation is found in the component $\mathbf{U}_3$ that features

the spherical densities, which strongly suggests that this component describes

the thermal effect. These two components are not interchangeable despite some

similarity in the arrangement of positive densities, that is, one of the physical

meanings of the orthonormal property of SVD.

Due to the orthonormal property of SVD, an N-dimensional Euclidean space

is established, and the first n dimensions define its most significant subspace.

Each coefficient set $c_k = w_k v_{ki}$ of the ith composite map is located in this subspace.

All coefficient sets for $i = 1, 2, …, N$ in different linear combinations to

approximate the N original maps can be represented by N points in the n-

dimensional Euclidean subspace. In this work, many two-dimensional

orthographical projections of this n-dimensional subspace are presented as

scatter plots. These scatter plots are highly informative to reveal the relationship

between the (difference) electron density maps and their metadata. For example,

the difference maps at short time delays < 1 ps are consistently different from

those at long time delays > 1 ps in a certain aspect (Fig. 2ac) and that two major

components are circularly correlated but only before 1 ps, that is, a damped

oscillation (Fig. 2b).

Rotation in SVD space

Dimension reduction is indeed effective in data analysis of dynamic

crystallography when many datasets are evaluated at the same time. However,

the interpretation of a given component U_k, that is, "what-does-it-mean",

requires a clear demonstration of the relationship between the core data, here

(difference) electron density maps, and their metadata, such as time,

temperature, and other experimental conditions. The outcome of SVD does not

provide a direct answer to "what-does-it-mean". The factorized set of matrices

U, **W**, and **V** from SVD is not a unique solution. That is to say, they are not the

only solution to factorize matrix **A**. The default solution set from SVD may not

carry an obvious physical meaning to depict "what-does-it-mean", thus its

usefulness is very limited to merely a mathematical construction. This is also the

origin of the debate on whether atomic orbitals as a basis set of mathematical

constructions are physical and observable (Scerri, 2000). It is very important to

find one or more alternative solution sets that can elucidate a structural

interpretation. This process, called rotation in SVD space, also known as Givens

rotation, is a connection between the analytical tool SVD and scientific findings.

This rotation shall not be confused with a rotation in the three-dimensional real

space, in which a molecular structure resides.

A rotation in the n-dimensional Euclidean subspace is necessary to change

the perspective before a clear relationship emerges to elucidate scientific

findings. It can be shown that two linear combinations are identical before and

after a rotation of an angle θ in a two-dimensional subspace of h and k. That is,

$$c_h U_h + c_k U_k = f_h R_h + f_k R_k, \tag{1}$$

871   where c_h and c_k are the coefficients of the components U_h and U_k before the

872   rotation as defined above for the ith map; and f_h and f_k are the coefficients of the

873   rotated components R_h and R_k, respectively. The same Givens rotation is applied

874   to both the components and their coefficients:

$$\begin{cases} R_h = U_h \cos\theta - U_k \sin\theta; \\ R_k = U_h \sin\theta + U_k \cos\theta. \end{cases} \tag{2}$$

877   Obviously, the rotated components R_h and R_k remain mutually orthonormal and

878   orthonormal to other components. And

$$\begin{cases} f_h = s_h t_h = c_h \cos\theta - c_k \sin\theta = w_h v_h \cos\theta - w_k v_k \sin\theta; \\ f_k = s_k t_k = c_h \sin\theta + c_k \cos\theta = w_h v_h \sin\theta + w_k v_k \cos\theta. \end{cases} \tag{3}$$

Here $s_{h|k} = \sqrt{\sum f_{h|k}^2}$ are the singular values that replace w_h and w_k, respectively,

after the rotation. They may increase or decrease compared to the original

singular values so that the descending order of the singular values no longer

holds. $T_{h|k} = (t_{h|k1}, t_{h|k2}, …, t_{h|kN}) = (f_{h|k1}, f_{h|k2}, …, f_{h|kN})/s_{h|k}$ are the right singular

vectors that replace V_h and V_k, respectively. T_h and T_k remain mutually

orthonormal after the rotation and orthonormal to other right singular vectors

that are not involved in the rotation.

A rotation in two-dimensional subspace of h and k has no effect in other

dimensions, as the orthonormal property of SVD guarantees. Multiple Givens

rotations can be carried out in many two-dimensional subspaces consecutively to

achieve a multi-dimensional rotation. Rotation in the SVD space converts one

solution set $\mathbf{A} = \mathbf{UWV}^T$ to other alternative solutions $\mathbf{A} = \mathbf{RST}^T$ so that an

appropriate perspective can be found to elucidate the relationship between the

core data and metadata clearly and concisely. A solution derived from a rotation

retains the orthonormal property of SVD. SVD analyses presented in this paper

employ rotations extensively except that no distinction is made in the symbols of

components and coefficients before and after a rotation.

Least-squares fittings of oscillating components

Oscillating components are repeatedly found in the difference maps and omit

maps. Usually, a pair of components are oscillating with the same frequency,

which does not exclude the possibility that a third weaker component is also

oscillating at the same frequency. A pair of coefficients in a two-dimensional

damping oscillation of a period p is least-squares fitted as x and y:

$$\begin{cases} x = x_0 + ae^{-t/\tau} \sin(2\pi t/p + \varphi); \\ y = y_0 + be^{-t/\tau} \sin(2\pi t/p + \psi). \end{cases} \tag{4}$$

x_0 and y_0 are the center of the oscillation. a and b are the amplitudes of the

elliptical oscillation in two dimensions. They are damped according to an

exponential decay with a time constant τ. φ and ψ are the phase angles of the

oscillation in two dimensions. They are not strictly $\pi/2$ apart.

The sixth and eighth components of the difference maps appear to gain a

two-dimensional oscillation after 1 ps (Fig. 2f). Their coefficients are least-

squares fitted as x and y with an exponential onset:

$$\begin{cases} x = x_0 + a\left(1 - e^{-t/\tau}\right)\cos(2\pi t/p + \varphi)\,; \\ y = y_0 + a\left(1 - e^{-t/\tau}\right)\sin(2\pi t/p + \varphi)\,. \end{cases} \tag{5}$$

Due to the lack of sufficient time points, the oscillation is forced to take a circular

form.

Code availability

Computer software dynamiX™ is available from Renz Research, Inc.

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Competing interests

ZR is the founder of Renz Research, Inc. that currently holds the copyright of the

computer software dynamiX™.

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1134 **Figures and Legends**

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1138 Figure 1. A Gantt chart summarizing structural events during photodissociation

1139 of MbCO. Previously known structural events are in pink. The excitation laser

1140 pulse and XFEL pulses are in yellow. The newly observed structural events from

1141 decomposition of the serial crystallographic datasets are in dark blue.

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Figure 2. SVD of 93 difference maps around the heme site. These difference

maps are divided into several series. Each series consists of the difference maps

produced from subtracting the same reference (Methods and Table S1). Each of

the 93 difference maps can be expressed as a linear combination of many

components $c_1 U_1 + c_2 U_2 + \ldots$ after the decomposition. Among a common set of

components U_k ($k = 1, 2, \ldots$) shared by all difference maps, also known as left

singular vectors in linear algebra, the first a few components are the most

significant. Each of the difference maps requires a coefficient set c_k for the linear

combination. This coefficient set varies from one map to the other, that is, a

function of the delay time or other parameters associate with the datasets. Each

coefficient c_k indicates how much of the corresponding component U_k exists in

the linear combination. Each coefficient set is derived from a right singular

vector as traditionally known in linear algebra (Methods). These coefficients

produced by SVD should not be confused with the Fourier coefficients used to

synthesize each map. (a-d) Orthographic projections of coefficients in some of

the significant dimensions. Each difference map is represented by a dot at the

location that marks the coefficients associated with the major components U_1

through U_4. Difference maps in a series are connected by lines in a

corresponding color. The multi-dimensional space is projected to various planes

and tiled together so that any two adjacent panels from (a) to (d), horizontal or

vertical, can be imagined erecting a three-dimensional subspace. (e) Singular

values indicating the significance of the components are sorted in a descending

order of their values before a rotation. A multi-dimensional rotation alters their

values. (f) Coefficients c_6 and c_8 of the $-F_{-0.1}$ series in yellow are least-squares

fitted with a two-dimensional oscillation (Methods).

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1173 Figure 3. Ultrafast formation of high-spin 3*d* orbitals of the heme iron. The

1174 fourth component U_4 of the difference maps captures a cubic shaped positive

1175 density network. Each component in the linear combination resulting from SVD,

1176 that is, a left singular vector, can also be presented as a difference map. The

1177 fourth component U_4 is rendered as a difference map in two styles, two-

1178 dimensional cross sections and three-dimensional contours. The corresponding

1179 coefficient c_4 is plotted in Fig. 2cd. (a-h) Images of two-dimensional cross

1180 sections through the component difference map U_4. Cross sections defined in

1181 Fig. S1 are labeled in the top right corner of each panel. Non-centric cross

1182 sections are marked to indicate small offsets from the centric cross sections. The

1183 structure of the heme, CO ligand in both bound and docked positions, and some

1184 side chains are also projected onto the images. C, N, O, and Fe atoms are in

1185 black, blue, red, and rusty dots, respectively. The imidazole rings of the distal

1186 His64 and proximal His93 are approximately coplanar with X and AC cross

1187 sections, respectively. Two propionic acid side chains of AD rings extend to the

1188 left in (bcd). But they are not coplanar with these cross sections. These two-

1189 dimensional cross sections are able to reveal more details in difference Fourier

1190 maps. (i-k) Three-dimensional mesh contours superimposed over a structure are

1191 traditionally rendered to display electron density maps. Green and red meshes

1192 are positive and negative contours at $\pm 7\sigma$, respectively. (l and m) Theoretical

1193 contours of probability distributions of electrons in $3d$ orbitals of iron. The

1194 combined probability distributions of $3d_{xz}$ and $3d_{yz}$ with $3d_{xy}$ (l) or with $3d_{x^2-y^2}$

1195 (m) are both in the shape of a cube with the largest probability at the eight

1196 corners. A 1/16 cutaway in each contour shows the cross sections. This

1197 characteristic shape of electron distribution is observed in U_4 of the difference

1198 maps.

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Figure 4. SVD of 20 simulated annealing omit maps (SAOMs) around the heme
site. The first component is the average of all omit maps (Fig. S5). (a-e) Each

omit map is represented by a dot at the location that marks the coefficients

associated with the major components. SAOMs derived from the structures at

cryo and ambient temperatures are in red and blue, respectively. SAOMs from

the XFEL datasets are in green. The multi-dimensional space is projected to

various planes and tiled together so that any two adjacent panels from (a) to (e),

horizontal or vertical, can be imagined erecting a three-dimensional subspace.

(b) Coefficients c_5 and c_8 show a damped two-dimensional oscillation as marked

by the curved arrow. (f) Singular values indicating the significance of the

components are sorted in a descending order of their values before a multi-

dimensional rotation.

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U_7 of SAOMs at heme site

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Figure 5. The seventh component of SAOMs correlated with photolysis at all

temperatures. The coefficient corresponding to U_7 of SAOMs increases as the

reaction of ligand dissociation proceeds from MbCO to deoxy Mb in static, cryo-

trapping, and time-resolved XFEL experiments (Fig. 4e). The hypothetical

transition from MbCO to deoxy Mb at 0 K is shown in Movie S3. (a-h) Cross

sections through the seventh component U_7. A cubic network of positive

densities is marked 1-4 in (c). (i-k) Three-dimensional meshes of this component

in green and red are contoured at $\pm 15\sigma$, respectively. See Fig. 3 legend for more

detail.

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U_3 of SAOMs at heme site

Figure 6. Thermal vibrations isolated from the chemical signals of photolysis.

The third component of the SAOMs U_3 is highly correlated with the sample

temperature (Fig. 4ace). A linear extrapolation based on the datasets available at

cryo and ambient temperatures predicts that the instantaneous local temperature

jump of the heme may have exceeded 500 K in the XFEL datasets (i). (a) and (b)

Linear combinations of $900U_1$ - $586U_3$ and $900U_1$ + $476U_3$ reconstitute the omit

maps of the heme at 0 and 600 K, respectively, excluding all chemical signals

related to photolysis, where the coefficient c_1 = 900 of U_1 is nearly constant for all

omit maps. The transition from 0 to 600 K free of the chemical signals from

photolysis is shown in Movie S2. (c-h) Cross sections through the third

component U_3. Positive densities in 20 pillars connecting two sheets of positive

densities above and below the heme in the cross sections of Z±1.3 Å are marked

in (e). (j-l) Three-dimensional meshes of this component in green and red are

contoured at ±15σ, respectively. See Fig. 3 legend for more detail.

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$|\boldsymbol{U}_5|+|\boldsymbol{U}_6|+|\boldsymbol{U}_8|$ of SAOMs at heme site

Fitted wavenumber = 36.0 cm^{-1}; period = 0.93 ps; damping time constant = 1.5 ps

$\boldsymbol{U}_1$ through $\boldsymbol{U}_4$ of difference maps in the entire globin

Figure 7. Low frequency oscillations in the solvent. (a) SVD analysis of the

difference maps in the entire globin produces the first and third components

circularly oscillating before 1 ps and quickly diminishing during the first a few

ps. Similar oscillations are also observed only around the heme site in both the

difference maps (Fig. 2b) and the omit maps (Fig. 4b). (b-e) Cross sections

through an averaged map of both the positive and negative halves of U_5, U_6, and

U_8 of the SAOMs. Low densities in white and light green are located consistently

on the heme and protein, which suggests that the oscillating signals are more

associated with the solvent rather than the globin. (f) The circularly oscillating

coefficients c_1 and c_3 are plotted as function of delay time in many series. The

offset sinusoidal curves indicate a circular oscillation. Each map series is derived

from subtracting a different reference dataset as indicated by the color bar below.

(g) and (h) c_1 and c_3 of the $-F_{dark}$ series are least-squares fitted with a two-

dimensional damped oscillation (Methods). (i-l) Comparison of several

components of difference maps in the entire globin. U_1 and U_3 contain

widespread signals, as they are ranked the most significant components. But no

signal is concentrated on the heme or the protein. Instead, U_2 and U_4 clearly

contain strong signals associated with the heme and secondary structures of the

protein.

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1266 # Ultrafast Structural Changes Decomposed

1267 # from Serial Crystallographic Data

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1277 **Supplementary Table**

Table S1. The full matrix of difference Fourier maps

Reference (ps)	Dark	Time points (ps)												
[1]Dark (5cmv)	[2].	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	3	10	50	150	deoxy
-0.1 (5cn4)	[3]-	.	0	0.1	0.2	0.3	0.4	0.5	0.6	3	10	50	150	deoxy
0 (5cn5)	-	-	.	0.1	0.2	0.3	0.4	0.5	0.6	3	10	50	150	deoxy
0.1 (5cn6)	-	-	-	.	0.2	0.3	0.4	0.5	0.6	3	10	50	150	deoxy
0.2 (5cn7)	-	-	-	-	.	0.3	0.4	0.5	0.6	3	10	50	150	deoxy
0.3 (5cn8)	-	-	-	-	-	.	0.4	0.5	0.6	3	10	50	150	deoxy
0.4 (5cn9)	-	-	-	-	-	-	.	0.5	0.6	3	10	50	150	deoxy
0.5 (5cnb)	-	-	-	-	-	-	-	.	0.6	3	10	50	150	deoxy
0.6 (5cnc)	-	-	-	-	-	-	-	-	.	3	10	50	150	deoxy
3 (5cnd)	-	-	-	-	-	-	-	-	-	.	10	50	150	deoxy
10 (5cne)	-	-	-	-	-	-	-	-	-	-	.	50	150	deoxy
50 (5cnf)	-	-	-	-	-	-	-	-	-	-	-	.	150	deoxy
150 (5cng)	-	-	-	-	-	-	-	-	-	-	-	-	.	deoxy
Deoxy (5d5r)	-	-	-	-	-	-	-	-	-	-	-	-	-	.
PDB entry	5cmv	5cn4	5cn5	5cn6	5cn7	5cn8	5cn9	5cnb	5cnc	5cnd	5cne	5cnf	5cng	5d5r

[1]Traditionally, the dark dataset is chosen as the reference in calculation of difference Fourier maps.

[2]Dot (.) indicates that these are empty maps.

[3]Dash (-) indicates that these are negated maps.

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Table S2. Dataset labeling		
PDB entry	Label	Description
1bz6	aquomet	Aquomet Mb from sperm whale at room temperature
1bzp	swMb	Deoxy Mb from sperm whale at room temperature
1bzr	swMbCO	MbCO from sperm whale at room temperature
1dwr	MbCO	MbCO from horse heart at 100 K
1dws	Mb*CO	Photolyzed MbCO from horse heart at 22 K
1dwt	Mb**CO	Photorelaxed Mb from horse heart at 88 K
5cmv	dark	XFEL dark dataset of MbCO from horse heart
5cn4	-0.1	
5cn5	0	
5cn6	0.1	
5cn7	0.2	
5cn8	0.3	XFEL light datasets binned by delay time < 1 ps
5cn9	0.4	
5cnb	0.5	
5cnc	0.6	
5cnd	3	
5cne	10	
5cnf	50	XFEL light datasets with delay time > 1 ps
5cng	150	
5d5r	Mb	Deoxy Mb from horse heart at 7°C

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1282 **Supplementary Figures and Legends**

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Figure S1. Orientations in myoglobin. Myoglobin (Mb) is a 17 kDa protein in

muscle that carries molecular oxygen O_2. The protein structure is rendered in

ribbon and surface models in (a) and (b). The non-protein prosthetic group heme

in stick model is anchored to helix F of the protein through His93 also in sticks.

Two propionic acid side chains of the heme are facing the molecular surface. The

sixth coordination site of the heme iron in sphere could bind O_2, CO, and NO

ligands also in spheres. Both a bound CO and a dissociated CO docked nearby

are shown. Traditionally, the anchor side of the heme is called proximal with

respect to the opposite distal side, where the ligands bind. The most remote part

of the protein is 30 Å away from the heme iron. The proximal and distal

directions extend 19 and 20 Å in the protein from the heme iron, respectively.

The tetrapyrrole rings are traditionally named ABCD as marked in (c). In this

paper, the heme plane is called Z cross section. Other cross sections X, Y, AC,

and BD are defined in (c).

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Figure S2. The difference Fourier maps for SVD analysis. Difference maps are

calculated according to the protocol described in Methods. Only the series

from -0.1 to 150 ps with the dark dataset (5cmv) as the reference is displayed

here. Cross sections are marked on top. See the legends of Figs. S1 and 3 for

definition of cross sections. See Table S1 for the full matrix of difference maps. A

visual inspection of these difference maps at different delay times may conclude

that they are largely identical. Very subtle variations could be seen by a careful

comparison of these maps in a time series. A numerical deconvolution is

necessary to extract the inherent components.

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Figure S3. The most significant component U_1 of the difference maps. No doubt
the most significant event shown in the difference maps is the out-of-plane
displacement of the iron. This component shows the severely skewed iron
displacement. The corresponding coefficient c_1 is plotted in Fig. 2ac. This
component clearly separates the delays > 1 ps from those < 1 ps. (a-d) Images of
two-dimensional cross sections through the first component. (e) and (f) Three-
dimensional mesh contours of the component difference map. Green and red
meshes are positive and negative contours at $\pm 7\sigma$, respectively.

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Figure S4. Modification to the cubic positive densities around the iron in the
sixth component U_6 of the difference maps. The corresponding coefficient c_6 is
plotted in Fig. 2f. This component and the eighth component U_8 form a two-
dimensional oscillation that ramps up in the delays > 1 ps (Fig. 2f). The effect of
modulation and oscillation is shown in Movie S1. (a-d) Images of two-
dimensional cross sections through the sixth component. (e) and (f) Three-
dimensional mesh contours of the component difference map. Green and red
meshes are positive and negative contours at $\pm 7\sigma$, respectively.

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1335 Figure S5. The most significant component of SAOMs. Since an omit map is the

1336 least biased experimental electron density map, the most significant component

1337 of the omit maps, by far (Fig. 4f), is an averaged experimental electron density

1338 map of 20 omit maps. Only the ligand is not clear due to its variation. All other

1339 components will be added onto or subtracted from this average.

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Movie S1. Reconstituted difference maps showing modulation and oscillation.

Linear combination of components U_1, U_4, U_6, and U_8 of the difference maps span

from 0.02 to 500 ps with some extrapolation on both ends. Other major

components carrying systematic difference from the deoxy Mb or featuring

oscillations in the solvent are not included in the linear combination to produce

this mivie. The four panels are cross sections of Z+1.1 Å, Z, Z-1.1 Å, and Y,

respectively. See Figs. S1 and 3 for the definition of cross sections. The cubic

positive densities first lean towards ring D from 0.5 to 3 ps. Then the position

returns back to the mean position after 50 ps, sways in the opposite direction

towards ring B, and ramps up after 1 ps into an oscillation of a period of 220 ps

(Fig. 2f).

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Movie S2. Temperature effect on electron density map of the heme. The first and

third components of the SAOMs are recombined to show the temperature effect

only. The components related to the chemical signals of the photolysis are not

included. Therefore, this movie shows the pure thermal effect. Slight

extrapolation at both ends covers from 0 to 600 K. The two panels are cross

sections Z and Y, respectively. See also Fig. 6ab for electron density maps at 0

and 600 K.

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Movie S3. Comparison of electron density of the heme in ligated and unligated

deoxy states. The first, third, and seventh components of the SAOMs are

recombined to directly contrast the electron density maps of the heme in ligated

and deoxy states. The coefficients for the first and third components are constant

in the reconstituted maps at 0 K, that is to say, the thermal effect is excluded.

The coefficient c_7 for the seventh components is set to -400 for the ligated state

and 300 for the deoxy state (Fig. 4e). The panels are cross sections of Z+1.1 Å, Z,

Z-1.1 Å, and Y, respectively. The ligation state and the iron out-of-plane motion

are shown in cross section Y. Cross section Z+1.1 Å on the distal side clearly

shows greater densities of the heme in deoxy state, signaling the heme doming.

Most importantly, the heme is sliding towards the AB rings in deoxy state. This

is consistent with the skewed displacement of the iron and the motion of the

proximal His93.

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