

# 1                    **A New Model for Single-Molecule Tracking Analysis**

## 2                    **of Transcription Factor Dynamics**

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26  
27 **Running title:** Power-law behavior in transcription factor dynamics

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29 chromatin, DNA binding, photobleaching, bi-exponential.

## 30 **Abstract**

31 Single-molecule tracking allows the study of transcription factor dynamics in the  
32 nucleus, giving important information regarding the search and binding behavior of  
33 these proteins with chromatin *in vivo*. However, these experiments suffer from  
34 limitations due to photobleaching of the tracked protein and assumptions about the  
35 exponential behavior required for data interpretation, potentially leading to serious  
36 artifacts. Here, we developed an improved method to account for photobleaching  
37 effects, theory-based models to accurately describe transcription factor dynamics, and  
38 an unbiased model selection approach to determine the best predicting model. A new  
39 biological interpretation of transcriptional regulation emerges from the proposed models  
40 wherein transcription factor searching and binding on the DNA results in a broad  
41 distribution of binding affinities and accounts for the power-law behavior of transcription  
42 factor residence times.

## 43 **Introduction**

44 Transcription factors (TFs) are key regulatory proteins responsible for turning  
45 genes “on” and “off” by binding to enhancer or promoter elements across the genome<sup>1</sup>.  
46 Fluorescence microscopy techniques have revolutionized our understanding of how TFs  
47 search and interact with chromatin<sup>2</sup>. Fluorescence recovery after photobleaching  
48 (FRAP) in live-cell systems unveiled the dynamic nature of these proteins, in contrast to  
49 the long-standing static model of TF-chromatin interactions<sup>3</sup>. The combined innovative  
50 technological improvements in fluorophore brightness and stability<sup>4</sup>, optical set-ups to  
51 increase the signal-to-noise ratio<sup>5</sup>, and camera speed and sensitivity now allow the

52 study of single-molecules at an unprecedented temporal and spatial resolution. Single-  
53 molecule tracking (SMT) is a powerful technique that allows the characterization of  
54 protein dynamics in single, live cells. It is based on detecting and following through time  
55 the traces produced by the light emitted from a single fluorophore. When applied to the  
56 study of TFs, important information regarding the search and binding dynamics of these  
57 proteins can be extracted<sup>2</sup>.

58 The SMT approach (**Fig. 1**) has now been applied for approximately two-dozen  
59 TFs in a variety of cellular systems<sup>6</sup>. The current consensus, based on empirical  
60 residence time distributions, describes TFs as able to transition between three different  
61 states: 1) unbound from DNA (diffusing in the nucleus), 2) non-specifically bound and 3)  
62 specifically bound to chromatin (i.e. interacting with specific response elements)<sup>7</sup>. This  
63 three-population model is based on the observation that TFs can intermittently stop, and  
64 then resume rapid diffusion<sup>8, 9</sup>. Thus, the empirical residence time distribution of the  
65 “stopped molecules” have been phenomenologically fitted to families of exponential  
66 distributions<sup>10-19</sup>, with no underlying normative model for the origins of these  
67 distributions<sup>8</sup>. Nevertheless, the bi-exponential fits to the distribution suggest that the  
68 DNA-bound population of molecules includes two distinct subpopulations: a short-lived  
69 fraction (‘fast stops’) and a longer-lived fraction (‘slow stops’). The slow fraction has  
70 been hypothesized to represent specific binding events associated with enhancers or  
71 promoters, while the fast fraction is hypothesized to represent non-specific binding to  
72 chromatin<sup>11, 12, 15, 17</sup>. Experiments wherein the DNA-binding domain of TFs has been  
73 mutated are consistent with this hypothesis as the longer events were reported to be  
74 dramatically reduced<sup>10, 12, 17, 18</sup>.

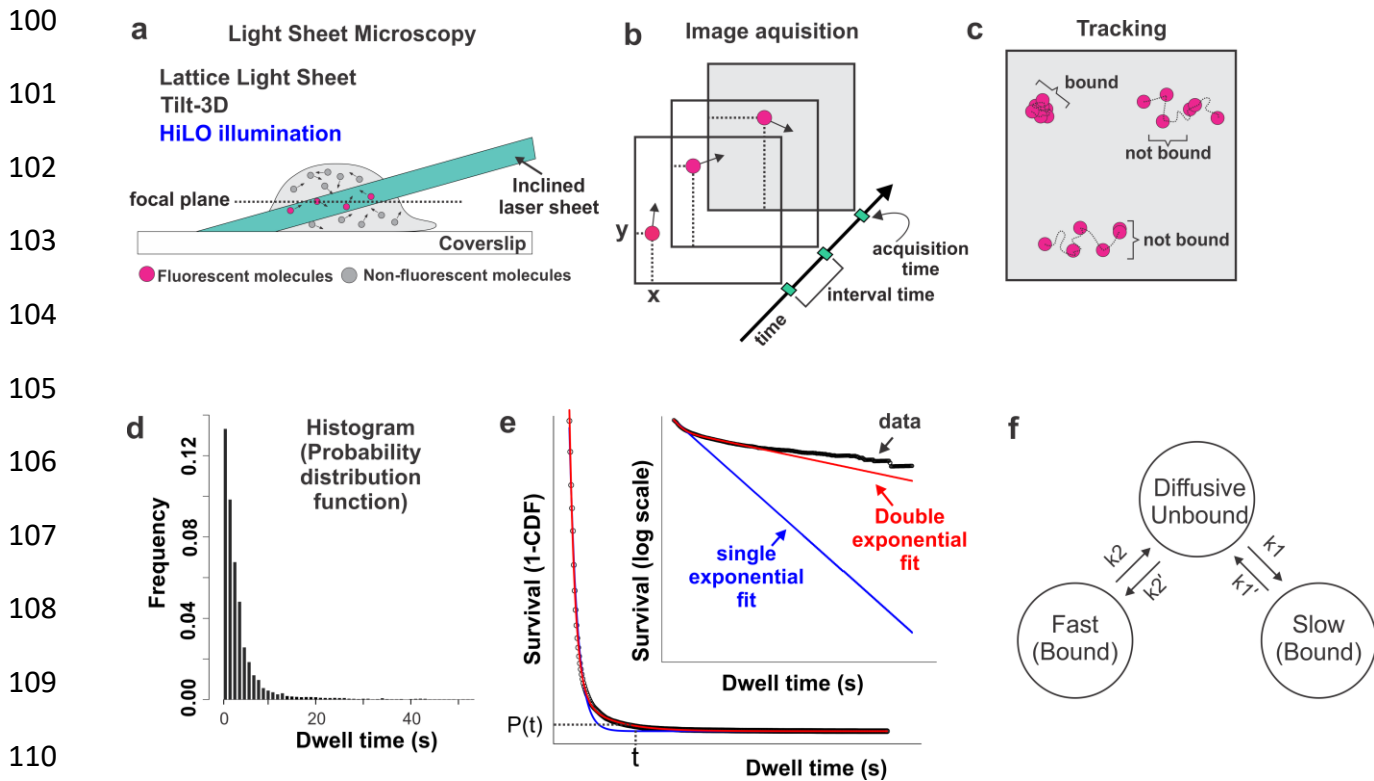
75           Despite the technological advances in studying TF dynamics, a major remaining  
76 limitation of the SMT approach is the relatively short fluorescence stability (or tendency  
77 to photobleach) of any fluorophore dye. This makes it difficult to distinguish whether a  
78 loss of signal is due to an unbinding event, drifting or photobleaching. Hence, careful  
79 corrections must be performed to accurately estimate TF dynamics. Unfortunately,  
80 photobleaching correction methods vary widely among research groups<sup>17, 20, 21</sup> which  
81 highlights the lack of a standard approach to overcome the photobleaching bias of SMT  
82 strategies<sup>9, 12, 18, 22</sup>.

83           In this work, we propose a new and improved photobleaching correction method  
84 that uses fluorescence dynamics of histones measured at the focal plane under precise  
85 SMT acquisition conditions. We then use the glucocorticoid receptor (GR), a ligand  
86 regulated transcription factor<sup>23</sup>, as a model TF to validate and test the new  
87 methodology. This approach unexpectedly reveals a novel, power-law behavior of GR  
88 residence time distributions. We then derive theory-based models for TF dynamics and  
89 a principled method to obtain optimal model parameters from empirical residence time  
90 distributions, using Bayesian statistics. We show that a model of TF searching and  
91 binding or a model of a nucleus with broad distribution of binding affinities accounts for  
92 the power law behavior of GR residence times obtained after implementing the modified  
93 photobleaching correction method. These models exhibit a broad effective distribution of  
94 binding affinities, thus challenging the established model with two discrete, bound  
95 populations.

96 **RESULTS**

97 **Photobleaching: a source of error in single-molecule tracking of TFs**

98 When tracking TFs at the single-molecule level, binding events can be observed  
 99 as stationary spots (**Fig. 1a-c, Supplementary Video 1**).



111 **Figure 1. The current SMT pipeline and interpretation of TF dynamics.** (a) A HiLO set-up is most  
 112 commonly implemented to increase signal-to-noise ratio (shown in panel). A laser beam is tilted and hits  
 113 the sample creating a thin illumination layer in the focal plane. Any light sheet microscope may be used.  
 114 (b) Several images are taken at specific yet variable acquisition and interval time conditions. (c) A  
 115 tracking algorithm is used to follow each individual molecule and classify them as either bound or  
 116 unbound. (d) From the bound population, a histogram is usually plotted which shows the frequency of TF  
 117 molecules that are bound for a specific time (dwell time). (e) Fitting of the data is performed on the  
 118 survival plot, which corresponds to 1-CDF, where CDF is the cumulative distribution function. This plot  
 119 represents the probability  $P$  that a molecule will last  $t$  number of time points, or longer. This distribution  
 120 has been phenomenologically fitted to either a single or a bi-exponential distribution. The better fit to a bi-  
 121 exponential function gave rise to the notion that two-distinct bound populations (fast and slow) exist. (f)  
 122 The current model states that TFs can be found in three different states: unbound from the DNA (diffusing  
 123 in the nucleus), specifically bound (slow stops), and non-specifically bound (fast stops). The latter can be  
 124 composed of TFs sliding and hopping on the DNA to facilitate searching of specific sites. Due to the  
 125 resolution limit, any transition between specific and non-specific bound states cannot be distinguished.

126  
 127 The experimental information recovered is the time the molecule “remains” visible

128 before it bleaches or moves out of the focal plane. From these observations, one can

129 obtain a local dwell time for TFs which is defined as the time interval between a single  
130 molecule transitioning from a diffusive state to a bound state and its subsequent  
131 unbinding from DNA. The definition of dwell time presented here differs of the one used  
132 in the field (residence time) in that we are considering any process occurring in a point  
133 spread function as a single observable binding event in the calculation of the dwell time  
134 distribution, not just discrete binding states. The dwell time distribution is generated by  
135 integrating the ensemble-averaged distribution of bound times (**Fig. 1d**,  
136 **Supplementary Note 1.1**). Most often, a “survival” distribution, defined as 1-CDF,  
137 where CDF is the empirical cumulative distribution function of dwell times, is used for  
138 further analysis (**Fig. 1e**). This survival distribution is usually fit to a bi-exponential  
139 distribution, interpreted as the “three population model” (i.e. diffusive, fast bound, slow  
140 bound) illustrated in **Fig. 1f**.

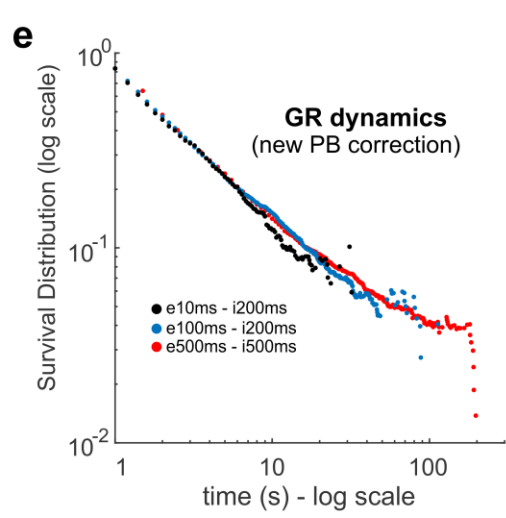
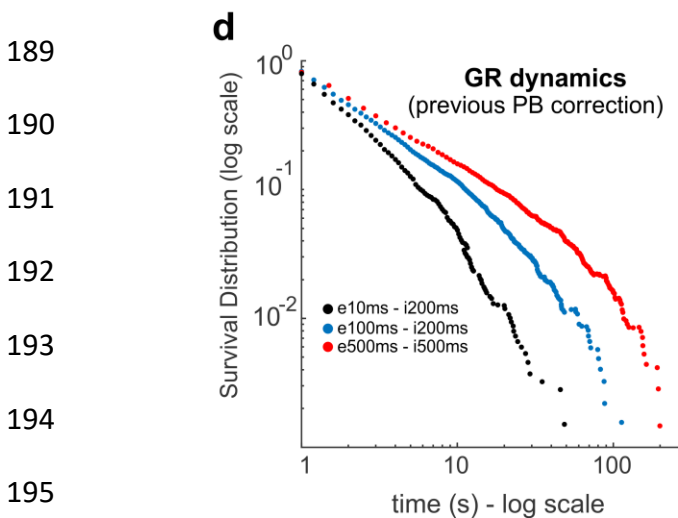
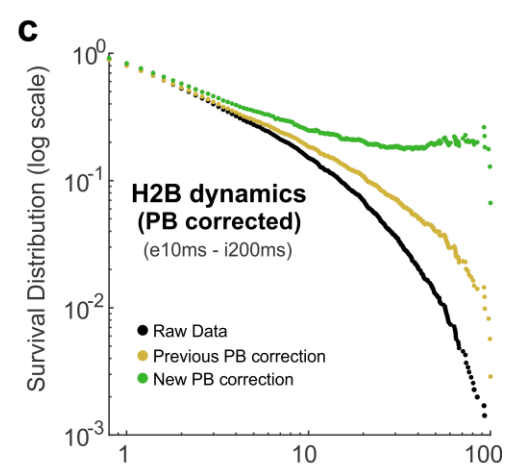
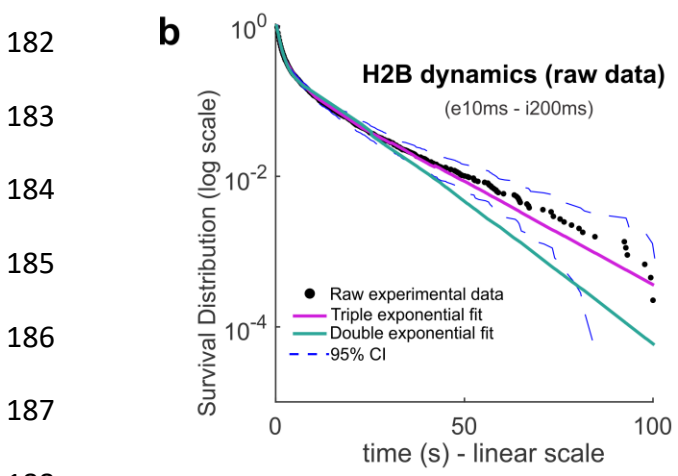
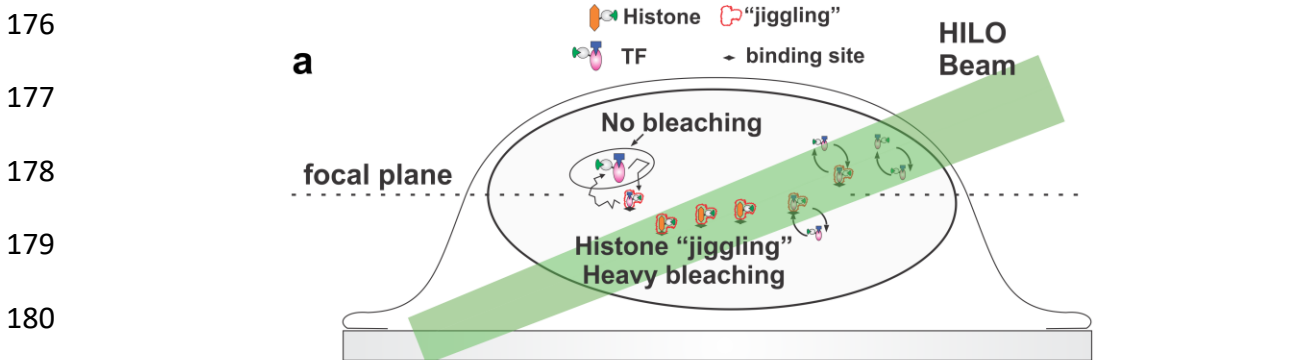
141 The upper temporal limit in SMT experiments is ultimately determined by the  
142 intrinsic photostability of the chosen fluorophore. When the affinity of bound TFs leads  
143 to dwell times longer than the averaged photostability of their fluorescent dyes,  
144 residence times cannot be resolved. Importantly, even when bound molecules have  
145 relatively lower affinities, they will appear to have shorter experimental dwell times due  
146 to photobleaching (PB) bias. To demonstrate this, we conducted single-molecule  
147 imaging of the glucocorticoid receptor (GR), a ligand-dependent transcription factor<sup>23</sup>,  
148 tagged with HaloTag-Janelia Fluor 549 (JF<sub>549</sub>)<sup>7</sup>. If we artificially modulate the PB  
149 conditions by changing acquisition parameters (exposure time, interval time, laser  
150 power), the resulting kymographs (**Supplementary Fig. 1, Supplementary Videos 1-4**)

151 appear to have originated from different TFs. Therefore, PB must be properly corrected  
152 to prevent artifacts in the analysis of SMT data.

153         Currently, there is no consensus on the proper method to estimate PB rates. One  
154 common approach is to simply count, frame-by-frame, the number of particles in the  
155 focal plane and then fit the time-dependent number to a bi-exponential model<sup>8, 10, 11, 16,</sup>  
156 <sup>17</sup>. However, this method underestimates the real PB that stably-bound proteins  
157 undergo at the focal plane. The dwell time of diffusive (unbound) molecules is much  
158 shorter than that of bound molecules (**Fig. 2a**) in the focal plane. Therefore, stably-  
159 bound molecules will photobleach faster than diffusive particles. Moreover, the number  
160 of particles in the focal plane will be heavily dominated by the diffusive component,  
161 which will also distort PB rates derived from the fits.

162         Another strategy uses histones as a proxy for obtaining PB rates. Histones are a  
163 good representation of stably-bound proteins because, after integration into chromatin,  
164 they have a residence time much longer than the photostability of any currently  
165 available organic fluorophore<sup>24</sup>. Therefore, by measuring the residence time of histones,  
166 we can obtain a direct representation of PB for particles in the focal plane, as the  
167 disappearance of a long-lived particle will most likely represent a PB event. This method  
168 is widely used in the SMT field<sup>12, 19, 21 13, 14, 25</sup> but, as we will demonstrate below, it has  
169 not been properly implemented. In current models, photobleaching kinetics are  
170 characterized by an exponential parameter  $k_{his}$  and the slow component of a TF is  
171 characterized by another exponential parameter  $k_{TF}$  (see **Supplementary Note 2** for  
172 details). This rate  $k_{TF}$  is corrected for PB to give the “real residence time” ( $k_{TFreal}$ ) of the  
173 transcription factor by:  $k_{TFreal} = k_{TF} - k_{his}$ . However, the correction assumes that TF

174 kinetics come from an exponential family since PB kinetics is exponential. Therefore,  
 175 the parameter  $k_{TFreal}$  may emerge as an artifact of this assumption.



196 **Figure 2. The new proposed photobleaching correction method.** (a) TFs will not photobleach  
 197 uniformly in the nucleus. A group of TFs stay bound in the focal plane until they bleach, another group  
 198 stay bound in the focal plane for a short time and diffuse away from it, another group never enters the



199 focal plane and a final group diffuses through the focal plane. **(b)** Fit of H2B survival distribution to a  
200 double exponential and triple exponential. A triple exponential better represents the experimental data  
201 where the slower component corresponds to the photobleaching rate in the focal plane. **(c)** Raw data for  
202 H2B survival distribution (black), previous correction using number of molecules decayed (yellow) which  
203 shows that H2B after correction still have a finite dwell time. Upgraded photobleaching correction for  
204 histone H2B representing two different regimes: stably incorporated histones that have a longer residence  
205 time that photobleaching and a dynamic regime representing unincorporated histones nonspecifically  
206 interacting in the nucleus. **(d)** Single-molecule tracking data of the glucocorticoid receptor (GR) activated  
207 with corticosterone (Cort). GR acquired with different photobleaching kinetics and corrected using the  
208 number of particles decayed in the focal plane (previous PB correction). **(e)** Same experimental data as  
209 (d) but corrected using the upgraded photobleaching correction, showing similar dynamics independent of  
210 photobleaching.  
211

## 212 **Improving photobleaching correction**

213 We propose a new PB correction method also based on histone data as a proxy  
214 of the fluorophore stability. A detailed derivation is described in **Supplementary Note 2**.  
215 The main step involves SMT of histones under the same conditions that the TF of  
216 interest will be imaged. We tracked individual H2B, H3 or H4 molecules using highly  
217 inclined and laminated optical sheet (HiLO) illumination<sup>5</sup> by sub-optimal transient  
218 transfection of HaloTag-fused histones, labeled with JF<sub>549</sub> HaloTag ligand<sup>26</sup> (**see online**  
219 **Methods**). The three histone variants we tested presented statistically similar dynamics  
220 (**Supplementary Fig. 2a**). We continued with H2B for all further experiments.

221 Histone genes are primarily transcribed upon entry into S-phase of the cell  
222 cycle<sup>27</sup>. Due to our transient transfection approach, HaloTag-H2B proteins will be  
223 translated during interphase and therefore some histones will not be incorporated into  
224 chromatin at the time of acquisition (**Supplementary Video 5**). Hence, survival  
225 distribution of H2B will be composed of PB kinetics and a diffusive/transient binding  
226 component. To account for this behavior and assuming PB kinetics at the single-  
227 molecule level is exponentially distributed, the survival distribution of H2B is fit to an  
228 exponential family with three components (**Fig. 2b, Supplemental Note 2**). The faster

229 components characterize the dynamics of histones that have not been stably  
230 incorporated into chromatin, while the third (slower) component describes the PB  
231 kinetics of the fluorophore. To confirm that our method quantifies PB kinetics and not  
232 intrinsic dynamics of the histone H2B, we calculated PB lifetimes using histones H3 and  
233 H4 with the same statistical results (**Supplementary Fig. 2b**). Consequently, artificially  
234 modifying PB kinetics (by changing acquisition conditions) modulates the H2B survival  
235 distribution (**Supplementary Fig. 2c**) and, accordingly, the mean PB lifetimes  
236 (**Supplementary Fig. 2d, Supplementary Videos 5-8**). This method for estimating PB  
237 rates has some drawbacks when a non-uniform illumination in the focal plane is used  
238 (as in HiLO microscopy), since the calculated rate corresponds to the average  
239 fluorophore decay in the focal plane, which may differ slightly in each location<sup>7</sup>.

240 To test this new PB correction method, we first analyzed how histone data itself  
241 changes after correction. Comparisons between H2B survival distribution with either our  
242 previous correction methods (**Supplementary Fig. 2e**) or with no correction at all,  
243 reveals a predictable upward shift of the distribution (**Fig. 2c**, compare yellow and black  
244 datapoints). However, H2B data still artifactually resembles the dynamics of a TF with a  
245 relatively short residence time. Conversely, if the newly proposed PB correction is used  
246 (**Fig. 2c**, green datapoints), a plateau in survival probability appears, indicating that H2B  
247 dwell times are now longer than those resulting from the photostability of the  
248 fluorophore. The high fluctuations at the tail of the distribution are likely due to noise in  
249 the data and the appearance of multiple particles within the point spread function  
250 (**Supplementary Fig. 2f**). For reproducibility and reliability purposes, multiple biological  
251 replicates are always taken, hence the ensemble average is used as the survival

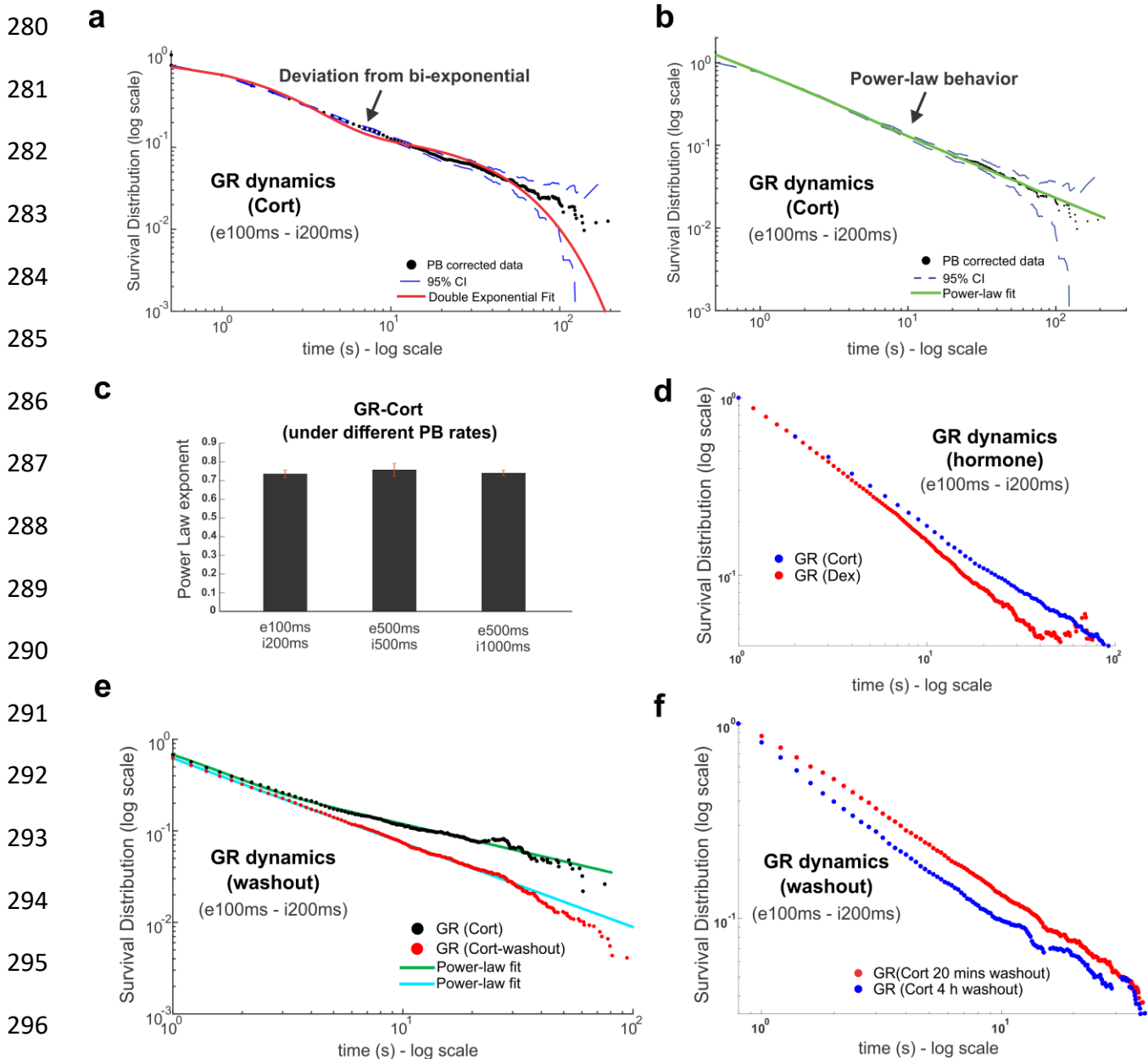
252 distribution for analysis (**Supplementary Fig. 2g**, all data). Even though the decay in  
253 number of particles at the focal plane (previous method) could potentially be used to  
254 correct for differences in laser illumination between replicates, we found that it makes  
255 the data more heterogenous (*c.f.* **Supplementary Fig. 2h** and **2i**), suggesting this is not  
256 a suitable measure for PB estimates.

257       Next, we apply our new PB correction to GR as a model TF. As with histones, we  
258 transiently transfected 3617 cells with GR fused to HaloTag (HaloTag-GR), incubated  
259 the cells with JF<sub>549</sub>, and activated the receptor with its natural ligand, corticosterone  
260 (Cort, 600 nM). We artificially varied the PB kinetics of the experiment by changing the  
261 exposure time and laser power (**see Methods**). When the previous PB correction was  
262 applied, GR survival curves show clear dependence on acquisition conditions (**Fig. 2d**),  
263 illustrating the artifact produced by PB. Conversely, with the new PB correction, we can  
264 now retrieve the same underlying distribution (**Fig. 2e**), independently of the acquisition  
265 conditions.

### 266 **New photobleaching correction reveals power-law behavior of TF dynamics**

267       Surprisingly, after correcting for PB with the new method, GR's SMT data now  
268 deviates clearly from a bi-exponential distribution (**Fig. 3a**). However, the data looks  
269 strikingly linear in a log-log plot (**Fig. 3b**), which suggests a power-law behavior. A  
270 similar distribution is observed upon GR activation with dexamethasone (Dex, 100 nM),  
271 a more potent, synthetic hormone (**Supplementary Fig. 3a**). To rule out any artifacts  
272 from acquisition conditions, SMT experiments for GR-Cort complexes were acquired at  
273 different time intervals and exposure times (**Supplementary Fig. 3b-d**). Independent of  
274 the sampling times, GR's survival distribution appears power-law distributed, with a

275 plateau in the tail due to a few molecules staying immobilized longer than the  
 276 characteristic photobleaching time of the fluorophore (illustrated in **Supplementary Fig.**  
 277 **2f**). An acquisition rate of 1000 ms allows the observation of the long-lived events in the  
 278 tail of the power law distribution (**Supplementary Fig. 3d**, independent replicates are  
 279 plotted as well to demonstrate reproducibility).



297 **Figure 3. Impact of photobleaching correction on GR dynamics.** (a-b) Single-molecule tracking data  
 298 of GR activated with corticosterone (Cort). Data was acquired at 100 ms exposure time (e100ms) in 5Hz  
 299 interval (i200ms). The survival distribution is shown, fitted to either a bi-exponential (a) or a power-law (b).

300 (c) Power law exponent of GR-Cort SMT data under different photobleaching rates, artificially generated  
301 by modulation of acquisition conditions. (d) GR dynamics under Cort or dexamethasone (Dex) activation.  
302 Power-law exponents of 0.71 +/- 0.05 and 0.81 +/- 0.01 respectively. The natural hormone exhibits longer  
303 dwell times. (e) Comparison of GR dynamics under Cort or washout of the hormone, which inactivates the  
304 TF. (f) GR dynamics under a standard 20 min or a more stringent 4h washout protocol.  
305

306 A random variable  $t$  follows a power law<sup>28</sup> for  $t > t_{min}$  if  $f(t) = At^{-\beta}$ , where  $A$  is  
307 a constant and  $\beta \in \mathbb{R}^+$  corresponds to the critical exponent, also known as the scaling  
308 parameter. Power laws are heavy tailed (right-skewed) and  $\beta$  is a measure of this  
309 skewness. Quantification of the critical exponent of the power-law shows that GR under  
310 different acquisition conditions (i.e. under different PB rates) exhibits the same  
311 dynamics (**Fig. 3c**). Surprisingly, GR activated with Cort shows an upward-shifted  
312 (lower  $\beta$ ) distribution compared to the more potent hormone dexamethasone (**Fig. 3d**),  
313 suggesting longer residence times for the less potent ligand. These differences were not  
314 apparent in previous studies<sup>10, 17</sup> likely due to inaccurate PB correction and/or a partial  
315 exploration of the dynamic range of the protein of interest in the SMT experiments.

316 Previous research has assumed that the dynamics of non-specific binding is well  
317 described by a single exponential component with a much shorter dwell time than  
318 specific binding<sup>8, 12, 17</sup>. To understand the dynamics of non-specific binding, we  
319 inactivated GR by washing out the hormone<sup>29</sup> for 20 minutes, which greatly reduces  
320 specific binding measured by chromatin immunoprecipitation<sup>30</sup>. Interestingly, GR still  
321 exhibits power law behavior, although with shorter dwell times as indicated by an  
322 increase in the power law exponent (**Fig. 3e**). While longer washouts (four hours) shows  
323 further reduction in dwell times, GR still exhibits power-law dynamics (**Fig. 3f**,  
324 **Supplementary Fig. 3e, Supplementary Video 9**). In conclusion, the new PB  
325 correction method has a major impact on the *distribution* of TF residence times (power-

326 law instead of bi-exponential). This, in turn, questions the interpretation of specific and  
327 non-specific binding as two distinct populations with discrete (and measurable)  
328 residence times.

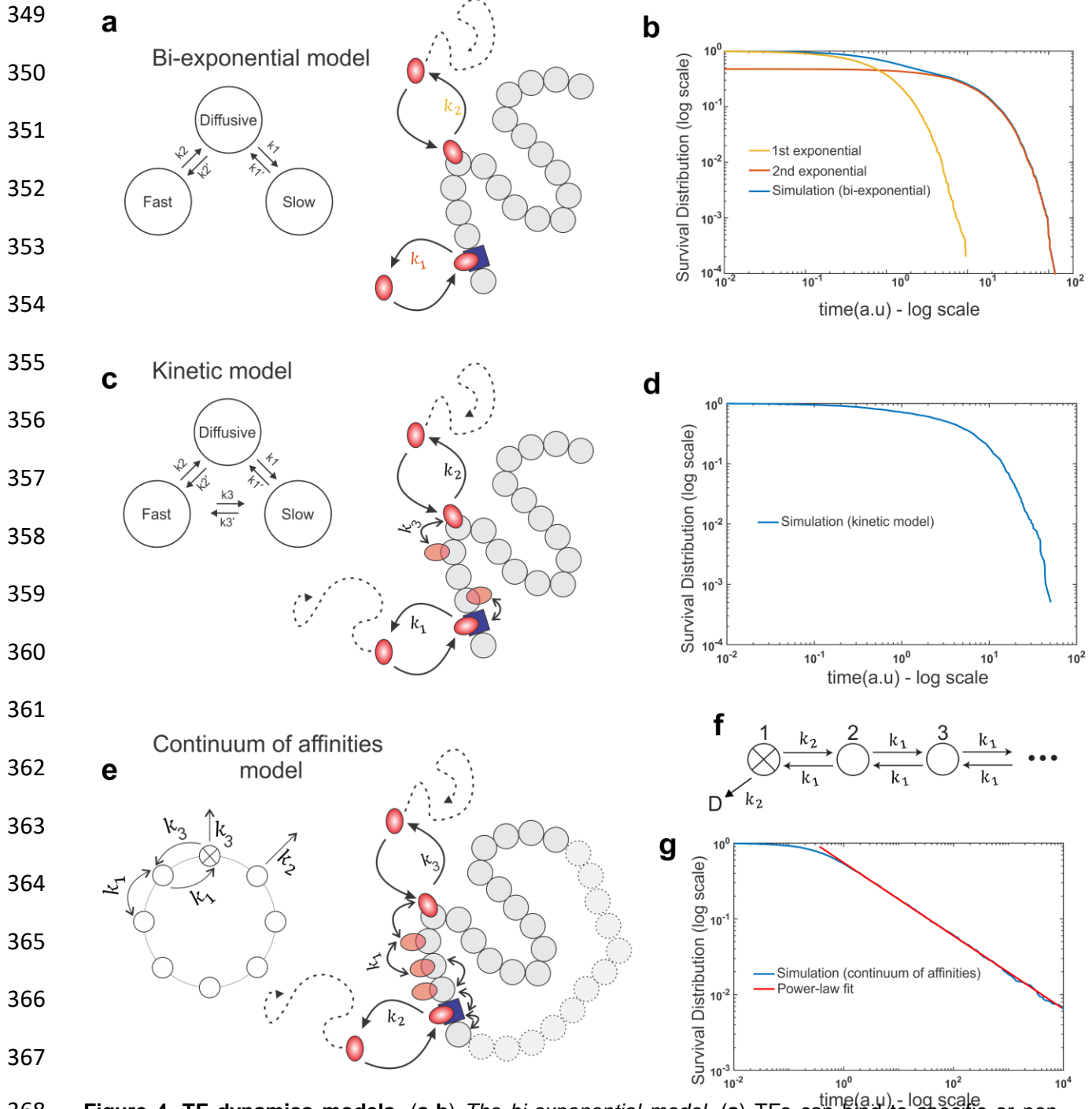
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### 330 **Re-interpretation of SMT data: new models for TFs kinetics**

331 To better understand the link between TF binding and the observed residence  
332 time distributions, we explored different theoretical models that may explain the  
333 emergence of different behaviors in the survival distribution. Calculation of dwell time  
334 distributions is a first-passage time problem in stochastic analysis and has been widely  
335 used to characterize the kinetic properties of molecular motors and ion channels<sup>31</sup>.

336 When simple kinetic schemes are involved, dwell time distributions can be calculated  
337 analytically. However, for more complex systems, other methods must be used. One  
338 particularly powerful approach is to assign one or more states to “act” as an absorbing  
339 boundary, and then solve the associated first-order kinetic equations to obtain dwell  
340 time distributions<sup>32</sup> (**Supplementary Note 1.1**). We assume that the diffusive state  
341 (unbound) corresponds to an absorbing boundary state since tracked particles end with  
342 such transitions. The single-molecule either photobleaches, disappears from the focal  
343 plane or begins diffusing. Any rebinding of the TF is considered an independent event.

344 We first examined the widely used bi-exponential model under this framework  
345 (**Fig. 4a**). According to this model, TFs can occupy three different states: diffusive, slow  
346 and fast. The diffusive state plays the role of an absorbing boundary state (i.e. exit from  
347 a bound state). In the current literature, it has been interpreted that the slow and fast  
348 states correspond to specific and nonspecific binding, respectively<sup>33</sup>.



368 **Figure 4. TF dynamics models.** (a-b) *The bi-exponential model.* (a) TFs can bind to specific or non-  
 369 specific sites with different affinities. After binding to DNA, they will unbind from DNA; transitions between  
 370 specific and non-specific sites are forbidden. (b) Numerical simulation showing the emergence of bi-  
 371 exponential behavior. (c-d) *The Kinetic model.* (c) TFs can transition from non-specific sites to specific  
 372 sites and vice versa. Transitions in the DNA are considered indistinguishable. (d) Simulation results of the  
 373 Kinetic model. (e-g) *The continuum of affinities model.* (e) TFs can diffuse on the DNA, and transition  
 374 between any state (Diffusive, specifically bound, nonspecifically bound). Dwell time is defined as the time  
 375 spent on the DNA, either bound or sliding. (f) Cartoon-simplification of (e), a TF arrives to a random site,  
 376 scans the DNA until it finds a specific site and it unbinds. This model can be solved analytically. (g)

377 Numerical simulation of (e) showing the emergence of power-law behavior (red line). See  
378 **Supplementary Note 1** for details.  
379

380 With this assumption of a well separated and narrow distribution of affinities, the  
381 expected behavior of the survival distribution corresponds to a double exponential with  
382 exponential parameters determining the average residence time of each state, as  
383 confirmed with stochastic simulations (**Fig. 4b**) using the Gillespie algorithm<sup>34</sup>.  
384 Importantly, this model does not allow for transitions between fast and slow states. For  
385 a complete derivation, see **Supplementary Note 1.2**.

386 We next extended the bi-exponential model to allow for transitions between the  
387 slow and fast components (**Fig. 4c**). Due to the resolution limit (~30nm), any transitions  
388 between specific and non-specific bound states cannot be distinguished. The resulting  
389 survival distribution corresponds to a family of exponentials; we call this a *kinetic model*  
390 (**Supplementary Note 1.3**). Simulations were performed as before, and the expected  
391 distribution is displayed in **Fig. 4d**.

392 Finally, several theoretical studies have posited that TF search and “final” binding  
393 to its cognate site on the DNA involves a combination of bulk diffusion in the nucleus,  
394 1D sliding along the DNA, hopping and translocation, and theoretical search times for  
395 the TF to find specific sites in this framework have been estimated<sup>35-37</sup>. In this model,  
396 TFs will have a multiplicity of fast bound states that must be accounted for in the  
397 analysis of dwell time data. To do so, we modeled TF movement on the DNA as  
398 hopping on a circular chain composed of specific sites and non-specific sites (**Fig. 4e**).  
399 The main assumption in our “*continuum of affinities*” model derivation (**Supplementary**  
400 **Note 1.4**) is that the number of non-specific sites on the DNA is much larger than the  
401 number of specific sites. This is a biologically reasonable assumption as only a few to



402 tens of thousands of specific sites are bound by any TF according to genome wide  
403 studies<sup>38</sup>, while the entire genome contains millions of “other” potential chromatin sites.  
404 An analytical solution can be found for the simplest case where there is a single specific  
405 binding site and the TF unbinds from the specific site (**Fig. 4f, Supplementary Note**  
406 **1.4.3**). A simulation based on the model gives rise to asymptotic power law behavior at  
407 time scales compatible with specific binding (**Fig. 4g**).

408 To test which model better represents the observed behavior of GR dynamics,  
409 we used the Bayesian information criterion (BIC)<sup>39</sup> and Kolmogorov–Smirnov (KS) test  
410 to choose the best-predicted model (**see Methods**). Clearly, power-law behavior  
411 emerges as the best fit, as illustrated for both GR-Cort (**Supplementary Fig. 4a-c**) and  
412 GR-Dex (**Supplementary Fig. 4d-f**) data. Interestingly, power law distributions have  
413 infinite variance, which implies a finite probability of long-lived binding events. This  
414 could imply that productive binding events may be rare with dwell times much longer  
415 than previously appreciated, as indicated by the right-skewness of the distribution.  
416 Another possibility corresponds to a wide distribution of binding affinities in the nucleus  
417 due to heterogeneity in 1) binding affinities of individual response elements, and/or 2)  
418 local nuclear microenvironment.

419 In summary, by incorporating an improved PB correction method, we discovered  
420 that the survival distribution of GR dwell times does not follow a bi-exponential model. In  
421 fact, the data follows a power-law distribution, which we can derive using two theoretical  
422 models (**Supplementary Note 1.4-1.5**). Ultimately, if there is a way to define or  
423 distinguish non-specific from specific binding, our results indicate that it cannot be  
424 based on their global residence times. However, the slope of the residence time

425 distribution does provide an estimate of the overall affinity and can be used for  
426 comparing TFs and their function under different conditions.

427

## 428 **Discussion**

429 In the present study, we describe a new photobleaching correction method to  
430 prevent photobleaching-related bias of the dwell time distribution of TFs. This method is  
431 based on histone SMT dynamics, used here as a proxy for photobleaching kinetics  
432 within the focal plane (**Fig. 2**). When correcting H2B SMT data by itself with this new  
433 approach, we showed that the survival distribution of H2B exhibits a “plateau” (**Fig. 2c**).  
434 This indicates very slow dynamics for the core histone, much longer than our  
435 experimental time scale, and is fully compatible with previously published FRAP data<sup>40</sup>.  
436 Additionally, we are now able to reconcile data acquired under different experimental  
437 conditions (**Fig. 2d-e**) whereas previous attempts were not successful<sup>10, 20, 41, 42</sup>.

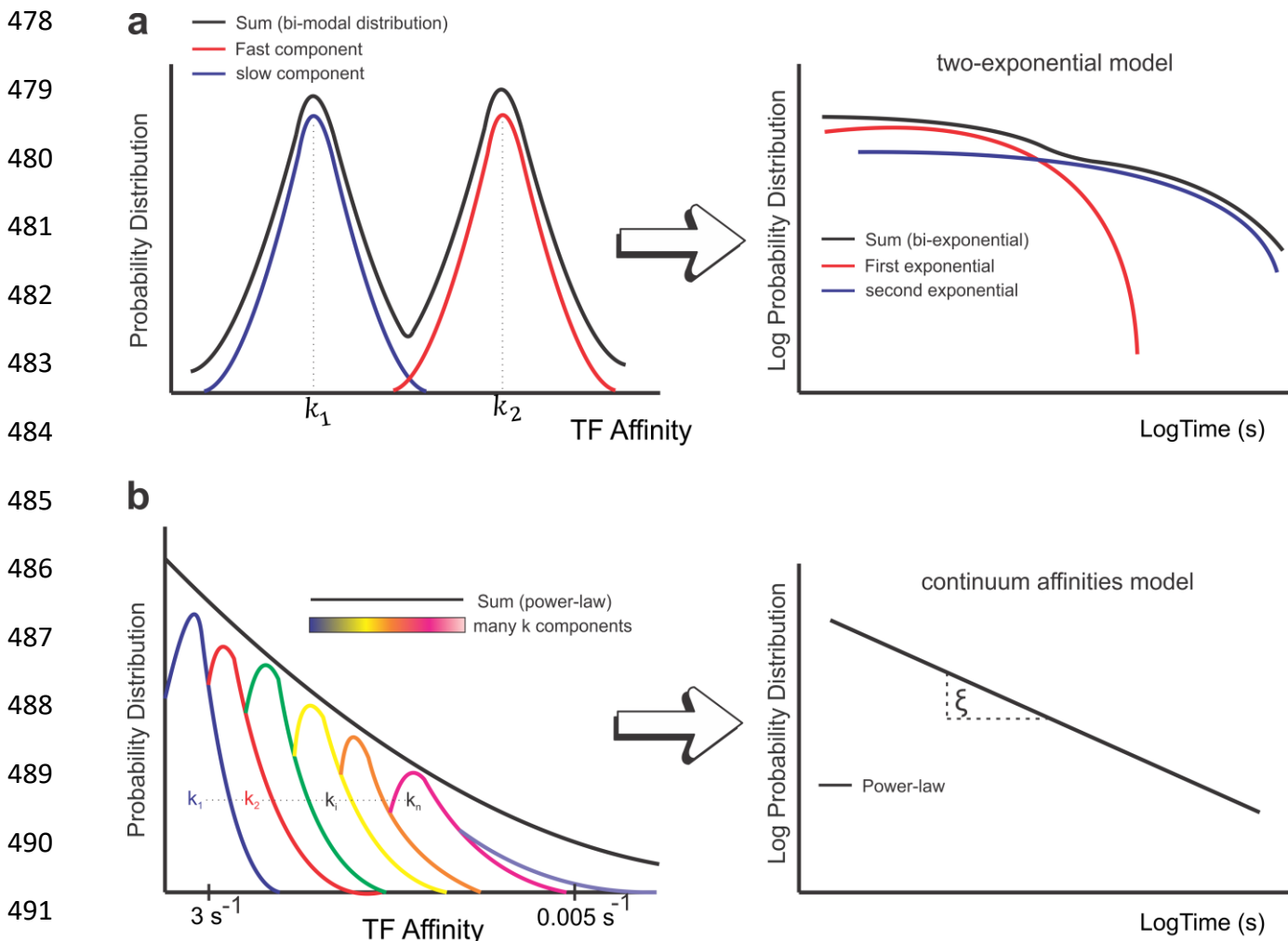
438 Given the current phenomenological interpretation of SMT data and the general  
439 lack of a model-based approach in the field, we derived theory-based models in an  
440 attempt to explain TF dynamics more accurately. We explored three different models:  
441 the classic bi-exponential model, a kinetic three-state model and a power-law model  
442 (**Fig. 4**). After correcting for photobleaching and implementation of Bayesian Information  
443 Criterion, we could identify the best predictive model that explained the residence time  
444 distribution of a paradigmatic TF, the GR. Surprisingly, the dwell time distribution of  
445 activated-GR is best described by a power-law. Although a recent study corrected PB of  
446 NF- $\kappa$ B dynamics by normalization with the H2B dwell time distribution, they still reported  
447 exponential behavior of the TF<sup>43</sup>. However, they did not consider alternative models and

448 the range of detected experimental dwell times is relatively short (maximum ~10 s),  
449 which might have prevented observation (and verification) of asymptotic power law  
450 behavior.

451 Our observation of power law behavior of GR residence times suggests a model  
452 with a continuum of DNA-bound states rather than discrete non-specific/specific binding  
453 of TFs. Consistent with this model, wash-out of the hormone (**Fig. 3e**) revealed that the  
454 dwell time distribution also follows a power law, indicating no apparent dynamical  
455 differences between the so-called specific and non-specific binding. Nevertheless, the  
456 overall residence time decreases when the receptor is less active, suggesting that a  
457 majority of the longer events observed with the fully activated receptor are associated  
458 with productive transcription as previously reported<sup>8, 10, 12, 17, 43</sup>. However, non-specific  
459 binding can also result in TF binding events with long residence times, the implications  
460 of which are still not known. Critical efforts are required to investigate whether the  
461 slow(er) stops seen in SMT are matched exclusively to specific interactions with  
462 chromatin. Alternatively, a sub-population of these “stops” could correspond to  
463 microscopic regions in the nucleus where diffusion is severely impaired, or transient  
464 interaction with “clustered” structures such as foci observed for GR<sup>44</sup>, or another hitherto  
465 unknown mechanism.

466 The broad distribution of affinities is puzzling but may be explained by  
467 heterogeneity in the nuclear structure and chromatin environment. Targets for a  
468 searching TF certainly exist in a wide variety of chromatin states (compacted fibers,  
469 different nucleosome modification conditions, etc.). Also, affinities for the thousands of  
470 alternative binding sites in response elements must vary significantly. Furthermore,

471 recent work points to the presence of transcriptional hubs and liquid-liquid phase  
 472 separation domains<sup>45</sup> that contribute to the complexity of nuclear organization. If TFs  
 473 exhibit different dynamical properties in these structures, it is not surprising to find a  
 474 broad variation in binding affinities. The resulting broad distribution of binding affinities  
 475 in these scenarios goes against the widely-held assumption that TF dynamics on  
 476 chromatin results from well-separated and narrow distributions of specific and non-  
 477 specific binding (**Fig. 5**).



497 affinities may explain the emergence of power law behavior (characterized by the exponent,  $\xi$ ) in the  
498 residence time of TFs (right graph).

499

500

501 Given the heterogeneities in local organization and nuclear structure, TF binding sites

502 on chromatin can be viewed as a collection of traps with a distribution of trap depths

503 (analogous to binding affinities). In such a finite disordered system, the distribution of

504 trapping times asymptotically approaches a power law<sup>46, 47</sup> (**Fig. 5b, Supplementary**

505 **Note 1.5**)

506 Alternatively, but not mutually exclusive, heterogeneity in the searching

507 mechanism of TFs may affect the effective affinity constant observed in SMT

508 experiments. In support of the latter, the tetracycline repressor (TetR), a chimera

509 between a bacterial and a viral protein with no known endogenous targets in

510 mammalian cells, when used as a proxy to emulate TF dynamics, also showed power-

511 law behavior for “non-specific binding”<sup>48, 49</sup>. However, it could still be described as an

512 exponential on an artificially (and single) specific DNA binding array<sup>49</sup>. Thus, the intrinsic

513 nature of the searching mechanism of any DNA-binding protein may be governed by

514 power-law dynamics. In addition, the heterogeneity of dwell times in the thousands of

515 response elements for an endogenous TF could explain why GR can present power-law

516 tails as opposed to TetR, which can only bind to one artificial array site. Interestingly, a

517 recent study in yeast<sup>50</sup> reports that both the TF Ace1p and the chromatin remodeler

518 RSC binding follow a bi-exponential binding distribution in cells containing a natural

519 tandem of ten *CUP1* (Ace1p responsive) *genes*. This dynamic and discrete behavior, in

520 contrast with our GR data, can be explained by the particular and homogeneous

521 chromatin environment of single array of specific sites. Consequently, we speculate that

522 a broad distribution of binding affinities coming from a whole population of different  
523 binding sites (thousands in the case of GR) may result in a power-law behavior (**Fig. 5**).  
524 In summary, by the implementation of proper photobleaching kinetics, we reveal a new  
525 model of TF dynamics. Our findings suggest that, contrary to the established paradigm,  
526 TF dwell times follow a broad distribution with no evidence of binary, discrete  
527 populations.

528

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## 539 **Author Contributions**

540 D.A.G, G.F., D.M.P., and G.L.H. conceived the experiments and wrote the  
541 original draft with subsequent input and editing from all authors. D.A.G, G.F., and  
542 D.M.P. performed imaging experiments, analyzed data and prepared figures. V.P.  
543 participated in the early stages of the project. D.A.G conceived and wrote in MATLAB®  
544 the photobleaching correction method (with constant feedback from G.F and D.M.P.).

545 D.A.G and C.J. derived the theoretical models, performed the simulations, and  
546 implemented the Bayesian statistical analysis. A.U. supervised data analysis, theoretical  
547 models and participated in the manuscript preparation; G.L.H. supervised the project.

#### 548 **Competing Interests statement**

549 The authors declare no competing interests

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