Co-localization and confinement of diphosphohydrolases and ecto-nucleotidases modulate extracellular adenosine nucleotide pools

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$_{1}$ 1 Abstract

Nucleotides comprise small molecules that perform critical signaling and en-2 ergetic roles in biological systems. Of these, the concentrations of adenosine 3 and its derivatives, including adenosine tri-, di-, and mono-phosphate are dy-4 namically controlled in the extracellular-space by diphosphohydrolases and ecto-5 nucleotidases that rapidly degrade such nucleotides. In many instances, the close 6 coupling between cells such as those in synaptic junctions yields tiny extracellular 'nanodomains', within which the charged nucleotides interact with densely-8 packed membranes and biomolecules. While the contributions of electrostatic and steric interactions within such nanodomains are known to shape diffusion-10 limited reaction rates, less is understood about how these factors control the 11 kinetics of sequentially-coupled diphosphohydrolase/nucleotidase-catalyzed re-12 actions. To rank the relative importance of these factors, we utilize reaction-13 diffusion numerical simulations to systematically probe coupled enzyme activ-14 ity in narrow junctions. We perform these simulations in nanoscale geometries 15 representative of narrow extracellular compartments, within which we localize 16 sequentially- and spatially-coupled enzymes. These enzymes catalyze the con-17 version of a representative charged substrate such as adenosine triphosphate 18 (ATP) into substrates with different net charges, such as adenosine monophos-19 phate (AMP) and adenosine (Ado). Our modeling approach considers elec-20 trostatic interactions of diffusing, charged substrates with extracellular mem-21 branes, and coupled enzymes. With this model, we find that 1) Reaction rates 22 exhibited confinement effects, namely reduced reaction rates relative to bulk, 23 that were most pronounced when the enzyme was close to the pore size and 24 2) The presence of charge on the pore boundary further tunes reaction rates 25 by controlling the pooling of substrate near the reactive protein akin to ions 26 near trans-membrane proteins. These findings suggest how remarkable reaction 27 efficiencies of coupled enzymatic processes can be supported in charged and 28 spatially-confined volumes of extracellular spaces. 29

30 2 Introduction

Nucleotide signaling and regulation of cellular energy pools are reliant on the 31 diffusion of small molecules over micrometer-scale distances [1]. Examples of 32 processes reliant on nucleotides include signal transduction and regulation in 33 smooth muscle [2], network motifs in transcriptional regulation networks [3], 34 genomic regulatory networks [4], complexes of metabolic enzymes [5] and trans-35 membrane ligand-gated channels [6, 7]. Of the latter, many nucleotide-gated 36 channels and ATPases [8] reside within extracellular junctions formed between 37 cells in close apposition [9], such as synaptic junctions comprised of neurons 38 and glia [10]. Scanning electron microscopy has revealed that many of these 39 junctions are on the nanometer length-scale [11]. Within those spaces, we ex-40 pect that the free diffusion and apparent concentration of nucleotides will differ 41 substantially from bulk solutions, although the influence of electrostatics, and 42 confinement on diffusion properties and distributions of nucleotides have only 43 been examined in limited detail. 44

At the cell surface, nucleotide pools are controlled by phosphohydrolases and nucleotidases [12, 13]. These enzymes hydrolyze nucleotides including adenosine- and uracil-based molecules and thereby regulate the pool of nucleotides available for signaling and metabolism [14]. It has for instance been demonstrated that the nucleotide concentration can vary considerably at the cell surface on both cytoplasmic and extracellular sides, as measured by the

activity of ATPases and ATP-sensitive channels [15, 16]. A sub-class of phos-51 phohydrolases called ecto-nucleotidase (NDA)s are localized to the extracellular 52 surfaces of cell membranes. There, NDAs rapidly and dynamically control nu-53 cleotide concentrations adjacent to proteins that catalyze or are gated by these 54 molecules. These include proteins such as purinergic receptors triggered by 55 ATP and adenosine diphosphate (ADP) binding [14]. Although many NDAs are 56 relatively nonspecific in their affinities for adenosine phosphates, some classes 57 are selective for ATP and ADP, such as CD39a and CD39b, respectively [12, 58 17]. Subsequently, AMP hydrolysis into adenosine proceeds via the CD78 ecto-59 nucleotidase [12]. In this capacity, CD39 and CD78 catalyze the coupled, 60 sequential hydrolysis of ATP into adenosine, and thereby influence nucleotide 61 signaling at the membrane. However, no precise characterization is available 62 for ATP diffusion-limited reaction kinetics in nucleotide pools localized to the 63 membrane as a result of these enzymes' spatial configurations within junctions. 64 Thus, the pools that are actually encountered by proteins remain poorly re-65 solved. 66

A key foothold for understanding ATP pools is to examine factors that con-67 trol the coupling of sequential reactions, such as $ATP \rightarrow AMP$ and $AMP \rightarrow$ 68 Ado. The fundamental motif of a sequentially-controlled enzymatic process 69 consists of two enzymes, of which one enzyme generates a reaction intermediate 70 that is catalyzed by the second enzyme [5, 18]. For diffusion-limited reactions, 71 the efficiency of sequentially-coupled reactions is strongly determined by the 72 relative distance between enzymes, as well as the rates of substrate diffusion 73 toward their reactive centers [19]. Further, sequential enzyme reactivity de-74 pends on the transfer efficiency of intermediates, which can be facilitated by 75 molecular tunnels [20] or electrostatic channeling [19, 21]. An essential consid-76 eration is therefore how intrinsic rates of substrate diffusion in bulk solution are 77 modulated by steric and long-range electrostatic interactions between substrates 78 and target enzymes versus those with the surrounding cellular environment [22, 79 23]. For instance, nucleotides are generally negatively-charged and are thus at-80 tracted to positively-charged nucleotide binding sites of NDA[24]. Most notably, 81 cellular 'crowders' comprising enzymes, proteins, and macromolecules typically 82 reduce substrates' intrinsic diffusion rates, which in turn can manifest in al-83 tered enzyme kinetics [23, 25–29]. Additionally, diffusion limitations stemming 84 from densely packed media or impermeable membranes can confine substrates 85 to narrow 'microdomains', within which substrate concentrations are vastly dif-86 ferent from those in the bulk cytosol or extracellular medium [30]. Based on 87 these considerations, it is plausible that nucleotidases confined to narrow com-88 partments between cells will hydrolyze extant nucleotide pools with markedly 89 different kinetics than those observed in vitro. However, only recently has cou-90 pled NDA activity been examined [31] via numerical modeling; in that study, 91 Sandefur et al [31] developed a computational model of human airway surface 92 hydration that accurately described experimentally-observed trends in NDA ac-93 tivity. This study was an important first step toward describing coupled NDA 94 enzyme activities. Extension of this description to account for the extra-95 cellular environment between cells, including diffusion-limitation imposed by 96 97 macromolecules and long-range electrostatic interactions from charged proteins and membranes are expected to give local variations that strongly differ from 98 bulk measurements. 99

Reaction kinetics in biological media are inherently difficult to study, given the breadth of influential factors including weak interactions of substrates with lipid membranes or proteins, restricted accessible volumes owing to such crowding by adjacent enzymes, proximity between enzymes involved in catalysis, and long-range electrostatic interactions. Since the systematic control of these fac-

tors in experiments is challenging [32], numerical models of molecular diffusion 105 and reaction kinetics have been valuable in our understanding of catalysis in 106 107 biological systems. At the coarsest resolution of such numerical models are representations of processes as networks of reactions and network motifs [18, 108 33-36]; although these coarse representations often do not account for kinetics 109 or enzyme proximity, they have helped to establish bounds on the function of 110 strongly-coupled reaction networks [37]. More sophisticated models accounting 111 for enzyme size [38–41], charge [42, 43] and co-distribution [44] are based on or-112 dinary and partial differential equation formalisms that implicitly capture these 113 effects. Recent ordinary differential equations (ODE) approaches that implic-114 itly consider the distributions of finite-sized enzymes include a mean field the-115 ory from Rao et aland [45]. These models provided strong quantitative insights 116 into the efficiency of catalytic processes [46] and limits on efficiency gains for 117 sequentially-coupled enzymes [45], but only implicitly account for geometrical 118 and physiochemical factors. Explicit consideration of those factors for coupled 119 enzyme processes generally utilize partial differential equations or particle-based 120 solutions, which have afforded descriptions of how neighboring reactive enzymes 121 [29, 47, 48], feedback inhibition, [29, 46], protein geometry and electrostatic in-122 teractions [19, 21, 49, 50] contribute to enzyme activity. Still, a systematic 123 study of sequential reaction phenomena under physiological variations in sol-124 vent ionic strength and intracellular confinement is needed for probing how 125 nucleotide pools are regulated by sequential NDAs in geometrically-constrained 126 intercellular junctions. 127

Our key objective was therefore to quantify how cellular and organelle mem-128 branes tune NDA sequential enzyme activity and local nucleotide pools. This 129 study was investigated in a model, biomimetic material for which the mate-130 rial porosity and surface composition could be controlled. Our approach uses 131 a finite-element based partial-differential equation model developed in [51], for 132 which we introduced explicit enzymes [23] to quantify how conditions such as 133 lipid charge, ionic strength, porosity tune the efficiency of protein functions 134 that utilize diffusing molecular substrates. Our key findings are that NDA co-135 localization and their charge complementarity with substrates can offset reduced 136 reaction rates owing to their confinement in nanoscale volumes; moreover, tun-137 ing of pore surface properties further improve nucleotidase reaction efficiency. 138 Through precise control of co-localization, the ratios of signal can be controlled. 139 In this regard, NDA activity is strongly influenced by its environment, which 140 can lead to reaction kinetics that differ relative to *in vitro* measurements. 141

142 **3** Results

¹⁴³ 3.1 Overview

We have developed a computational model of sequential nucleotidase reactions confined to a narrow space between apposed cells. The reactions are activated by ATP diffusing from neighboring cells. This geometry consists of a pore with nanometer-scale radius spanning between two reservoirs to emulate a femtoliterscale volume confined between two coupled cells. We impose an ATP gradient oriented parallel to a micrometer-length pore to reflect nucleotide diffusion into the pore. We summarize the conditions run in Table S1.

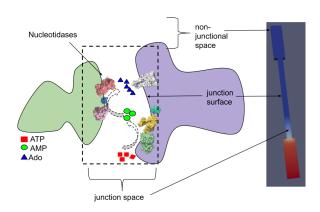


Figure 1: Left) Schematic of the synapse-like junctional space formed between the membranes of adjacent cells. Nucleotidases confined within the junctional space hydrolyze ATP into AMP and Ado. Right) A model geometry based on the schematic, for which the reservoirs correspond to the non-junctional space. The spatial and electrostatic configuration of the mock synapse influence the reactivity of confined nucleotidases CD39 and CD78.

Within the pore we consider two sequential, CD39- and CD73-catalyzed ATP 151 and AMP hydrolysis reactions that are in steady-state. With this model, we 152 examine how enzyme co-localization, 'tethering' the enzymes to the pore wall, 153 and charges on the enzyme and pore surfaces shape enzyme kinetics within the 154 idealized pore volume. Although we assume that the enzymes are spherical 155 with uniform reactivity and charge, we have found that such representations 156 are reasonable approximations of structurally-detailed, non-uniformly charged 157 proteins we have examined in other studies [52, 53], but are considerably less 158 computationally expensive to evaluate. 159

¹⁶⁰ 3.2 Effects of molecular pore confinement on enzymatic activity

Coupled enzyme reactions have been widely studied in a variety of contexts, in-162 cluding isolated globular enzymes and along surfaces. Here we extend these 163 approaches to examine nucleotide hydrolysis reaction kinetics for NDA en-164 zymes embedded within nanoscale gaps between cells, which we emulated with 165 nanopores of varying radii (see Fig. 1). These pores are representative of small, 166 well-contained extracellular volumes, such as junctions formed between adjacent 167 cells or synapsing neurons. We vary the relative distance between enzymes to 168 investigate how co-localization impacts reaction efficiency, as well as their dis-169 tance to the pore as to simulate surface-tethered versus freely-floating enzymes. 170 In such geometries, substrate access to the enzyme is restricted to a narrow 171 volume, which is expected to decrease the diffusion-limited reaction rate. To 172 quantify the dependence of enzyme reactivity on nanopore volume, we numer-173 ically solved Equation 12 for substrate species ATP, AMP and Ado subject to 174 the boundary conditions defined in Table 1. We defined the reaction efficiency 175 as the ratio of the substrate Ado production rate coefficient, $k_{prod,C}$ over the 176 substrate ATP association rate coefficient, $k_{on,A}$. Since the association rate of 177 AMP is equal to the Ado production rate, the reaction efficiency highlights how 178 the reactivity of AMP is shaped by the system configuration independent of the 179 ATP reaction rate. This ratio is identical to the ratio of the association rate coefficient of AMP to the production rate coefficient of AMP . We further note 180 181 that this ratio of rate coefficients is identical to the ratio of particle flux values. 182 We will use this identity to simplify the calculations, as the flux values are a 183 more direct result of the numerical simulations. 184

We first validate our model against an analytical solution for the diffusion limited reaction rate coefficient on a uniformly reactive sphere. [54] Here, the association rate, k_{on} , for the reactive sphere embedded in an infinite domain is given by

$$k_{on} = 4\pi R D \tag{1}$$

where R is the radius of the enzyme and D is the substrate diffusion coeffi-185 cient. For the purpose of validation, we evaluate this rate at the sphere (0.5)186 nm radius) by assuming a uniform concentration for ATP (1.0 mM) at both the 187 reservoir and pore (11.5 nm diameter). We will later assume no-flux (reflective) 188 boundary conditions for the concentration along the pore to emulate a typical 189 non-reactive pore boundary. Under the aforementioned conditions, we numerically estimated a rate of $k_{on,ATP}=3.611 \times 10^{-3} \text{ nm}^3 \text{ ns}^{-1}$, which is within 5% 190 191 of the analytical estimate of $k_{smol,bulk} = 3.768 \times 10^{-3} \,\mathrm{nm^3 \, ns^{-1}}$. The minor dis-192 crepancy can be attributed to the nonspherical domain used for the numerical 193 simulation, whereas a radially-symmetric domain is assumed in Eq. 1. We here-194 after refer to this as the 'bulk' configuration, for which the enzyme concentration 195 corresponds to roughly 0.19 mM. To establish a frame of reference for our re-196

> ¹⁹⁷ sults using reflective pore boundaries (see Table 1), in Fig. 2 we demonstrate a ¹⁹⁸ numerical prediction of $k_{on,ATP}=1.151 \times 10^{-3} \text{ nm}^3 \text{ ns}^{-1}$, which is within 1.8% ¹⁹⁹ of the analytical result, $k_{smol,local}=1.13 \times 10^{-3} \text{ nm}^3 \text{ ns}^{-1}$, for which we used the ²⁰⁰ concentration . For this estimate, we average the bulk ATP concentration of ²⁰¹ the sphere centered within the pore. Both approaches confirm the reliability ²⁰² of the model for reproducing analytic results for diffusion-limited association ²⁰³ reactions.

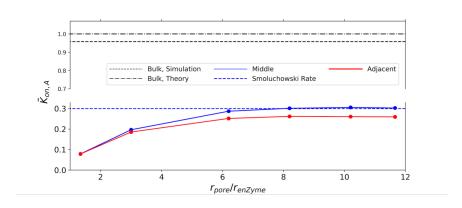


Figure 2: Effects of confinement and proximity Normalized reaction rate coefficients of ATP to CD39, including comparison of simulation results to the analytical value for bulk conditions based on the Kimball-Collins relation. The vertical axis is broken to emphasize the dependence on pore radius.

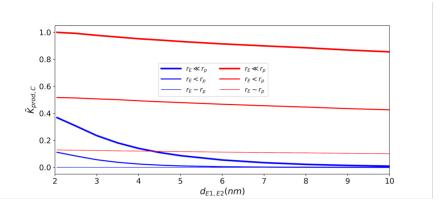


Figure 3: Effects of confinement and proximity Normalized reaction rate coefficient for the second enzyme at different pore sizes and enzyme proximities. Blue lines are for absorbing boundary conditions to emulate an open, bulk-like configuration, and red lines are for reflecting boundary conditions. \tilde{k} is normalized with respect to the maximal Kprod value at min d max rp.

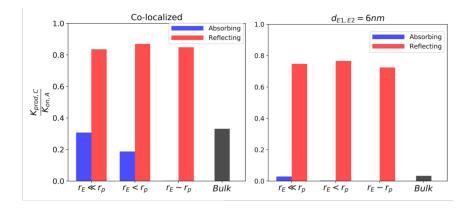


Figure 4: Effects of confinement and proximity Efficiency of reactivity of second enzyme respect to first enzyme. Left panel: enzymes co-localized. Right panel: enzymes separated by 6 nm. Efficiencies for the absorbing case approach 0 as $r_p \rightarrow r_E$.

> Using our validated model we investigated how the reaction of substrate ATP 204 on CD39 is influenced by confinement within a nanoscale channel. These sim-205 ulations were conducted assuming a constant concentration gradient along the 206 dominant axis of the channel. In Fig. 2) we present normalized association rates 207 for ATP with enzyme CD39, $k_{on} \equiv k_{on,A}/k_{on,Bulk}$, subject to a constant en-208 zyme radius ($r_E = 1.0 \text{ nm}$) and varied pore diameters ($r_E \approx r_p$ with $r_p = 1.3 \text{ nm}$ 209 , $r_E < r_p$ with $r_p = 3.0$ nm, and $r_E \ll r_p$ with $r_p = 5.5$ nm). Confinement of 210 the enzyme to the pore reduced the reaction rate coefficient by roughly 70%211 relative to the corresponding rate in bulk ($\tilde{k}_{on} = 1$). This can be qualitatively 212 rationalized by the concentration profiles manifest in the channel (see Fig. S1). 213 The concentration profile decreases from $A=6.0 \times 10^{-4} \text{ nm}^{-3}$ at the right-hand 214 side reservoir (Γ_R) and approaches zero at the left-hand side reservoir (Γ_L) . As 215 the pore radius decreases from $r_p \gg r_E$ to $r_p > r_E$, the concentration of ATP 216 within the pore decreased relative to the reservoir. Hence, pore confinement 217 in essence reduces the substrate concentration at the enzyme surface, which 218 219 culminates in a reduced $k_{on,ATP}$.

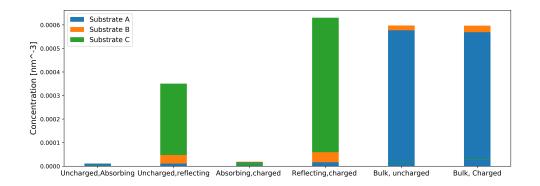


Figure 5: **ATP**, **AMP** and **Ado** concentrations at midpoint between enzymes for different pore wall boundary conditions. CD39 and CD73 are separated by a distance of 6.0 nm.

We additionally varied the proximity of the enzyme to the pore surface. This variation serves as a proxy for probing the reactivity of enzymes that are essentially free floating within the pore interior versus immobilized to the pore surface. The reactivity of CD39 was additionally reduced, albeit negligibly, as CD39 was localized to the pore surface. This can be rationalized by noting the similarity between the time-independent diffusion equation and the Laplace equation commonly used in electrostatics (see Eq. 15 with $\kappa = 0$.).

The total electric flux is dependent on the capacitance; as the sphere ap-227 proaches an insulator wall $(J \cdot n = 0)$, the capacitance decreases [55], which 228 decreases the total electric flux. The total electric flux is the electrostatic equiv-229 alent of the concentration flux in substrate diffusion, hence, the numerically-230 estimated $k_{on,ATP}$ values were smaller for immobilized enzymes relative to those 231 far from the surface. Altogether, these results demonstrate that restricting the 232 diffusion of ATP within the pore and to a slightly greater extent, near the pore 233 wall, suppress $k_{on,ATP}$ relative the bulk. 234

Reduction of $k_{on,ATP}$ through enzyme confinement of enzymes is expected 235 to subsequently suppress production rates for AMP and Ado. However, co-236 localization of enzymes within 'nano reactors' is a common approach to tune 237 production rates of desired chemical products [44, 46, 56]. We therefore intro-238 duced a second enzyme, CD73, into the pore and simulated the steady state 239 reactions $ATP \rightarrow AMP$ at CD39 and $AMP \rightarrow Ado$ at CD73. In Fig. 3) we 240 first report Ado production rate coefficients, $k_{prod,Ado}$, as a function of enzyme 241 separation and for non-reactive (reflective) and reactive (absorbing) pore sur-242 faces. These values are normalized with respect to the $k_{prod,Ado}$ value obtained 243 for $r_p \gg r_E$ and maximal enzyme separation $(d_{CD39,CD73}=1)$. The normalized 244 $k_{prod,Ado}$ rate coefficients are negligibly impacted as enzyme separation is re-245 duced within a boundary that is nonreactive with AMP (e.g. reflective); this 246 indicates that enzyme colocalization has negligible impact on $k_{prod,Ado}$ if AMP 247 does not interact with the pore boundary (e.g. reflective). If absorbing condi-248 tions are assumed, that is AMP is depleted at the pore, then there is a kinetic 249 advantage to enzyme colocalization; this is demonstrated by the increased rates 250 with reduced $d_{CD39,CD73}$ in Fig. 3) that are accentuated for increasing pore 251 sizes. Hence, as the domain approaches the bulk-like system where AMP can 252 escape from the reaction complex, the advantage of enzyme proximity becomes 253 apparent and is consistent with recent studies of enzyme co-localization [19, 254 45, 46]. In summary, the nature of the intermediate, AMP, interactions with 255 the surface appear to determine the relative advantage of enzyme colocaliza-256 tion in closed, nanoscale domains. For example, in a scenario where enzymes 257 auxiliary to NDAs are depleting nucleotides, NDA enzymes would benefit from 258 co-localization. 259

To delineate the effects of pore confinement and enzyme colocalization on 260 $k_{on,AMP}$ and $k_{prod,Ado}$ independent of $k_{on,ATP}$, we report in Fig. 4) the Ado 261 production efficiency, k_{eff} , which we define as $k_{eff} \equiv k_{prod,Ado}/k_{on,ATP}$. The 262 efficiencies reported for co-localized enzymes (left panel) are consistently higher 263 than those for separated enzymes (right panel), with the largest increases demon-264 strated for reactive pore boundaries (blue) or open (red, bulk) configurations. 265 Hence, under circumstances that permit intermediates to diffuse away from or 266 compete with the reactive centers, there is a clear advantage to colocalization, 267 akin to findings in [45]. However, in the absence of substrate interactions with 268 the pore surface, confinement leads to higher overall efficiencies that monotoni-269 cally decrease with increasing pore radius, but have little dependence on enzyme 270 proximity. 271

3.3 Effects of surface charge on reaction rate coefficient

In the previous section we demonstrated that confinement of enzymes like the 273 ectonucleotidases to nano-scale extracellular domains suppresses the overall re-274 action rate coefficient of uncharged substrates. Co-localization of the reactive 275 centers mitigated this reduction to a modest extent. Naturally, the adenosine 276 substrates are charged, with ATP having the most negative charge and AMP 277 the least. Hence, their concentrations and diffusion rates are expected to be sen-278 sitive to the charge configuration of their binding partners and the surrounding 279 lipid bilayer environment. It is well-known, for instance, that many enzymes 280 have evolved to exploit electrostatic interactions to accelerate substrate bind-281 ing [42]. Further, there is strong evidence that local ionic concentration near 282 charged membranes yield concentrations of Ca²⁺ and Na⁺ that deviate sig-283 nificantly from the bulk [57, 58]. We therefore expanded the approach in the 284 previous section to consider competing or complementary effects of electrostatic 285 interactions in coupled enzyme kinetics. 286

In this section, we model the contribution of electrostatic interactions be-287 tween substrates and their environment using the Smoluchowski electro-diffusion 288 equation (see Eq. 8), for which the electrostatic potential was modeled using the 289 linearized Poisson-Boltzmann equation (see Eq. 15). We first validate our imple-290 mentation under dilute solvent conditions ($\kappa \to 0$) and assume that the pore and 291 CD73 are uncharged. The association rate for a substrate with charge q_A with 292 a spherically-symmetric enzyme of radius, r_E and charge q_E can be analytically 293 determined [19]: 294

$$k = 4\pi D \left(\frac{Q_s \exp(Q_s/r_D)}{\exp(Q_s/r_E) - \exp(Q_s/r_D)} \right)$$
(2)

with $Q_S \equiv q_A q_E \lambda_B$, where r_D is the radius of the domain within which the reaction is confined and λ_B is the Bjerrum length. Accordingly, we demonstrate in Fig. S2 that as $\kappa \to 0$ (log(κ) $\to -\infty$) the numerically-predicted k_{on,ATP} rates for z_{ATP} =-1 and $\Phi_{CD39} = 25$ mV approach the analytical estimate within 6% percent, and thus reasonably validate the electrostatic model.

Using the validated electro-diffusion model, we first examined changes in 300 CD39 reactivity by confinement within an electrically-neutral pore, subject to 301 electrostatic interactions between a negatively charged substrate A, positively 302 charged CD39 and variably charged CD73. We assumed surface potentials of 303 ± 19.2 mV for the enzymes based on ζ potentials measured for proteinaceous solutions by Salgn *et al et al* [59]. Further, although adenosine metabolite 304 305 charges vary from -4 to 0, and because Ado is commonly chelated by Mg^{2+} 306 [60], we used charges of -2, -1 and 0 for ATP, AMP and Ado, respectively, 307 to exemplify effects on reactivity. Under these conditions, $k_{on,ATP}$ decreases 308 as the pore radius to enzyme radius decreases as was observed for the neutral 309 system (see Fig. 6). Importantly, $k_{on,ATP}$ for the charged system does assume 310 a higher rate coefficient than the neutral system. Hence, the electrostatic in-311 teractions in essence counterbalance the reduction in reaction rate coefficients due to confinement. Moreover, NDAs complementary charge with nucleotides 312 313 exploits fast association. Additionally, consistent with [19], when CD73 and 314 CD39 assume the same charge complementary to the intermediate, increased 315 $k_{on,ATP}$ rate coefficients result, owing to the attraction between enzymes and 316 the substrate. We next imposed a negative electric potential on the pore surface and 317

We next imposed a negative electric potential on the pore surface and present the resulting reaction rate coefficients (red in Fig. 7). We chose surface charge densities consistent with biological membranes reported by surface

conductivity microscopy such as DPTAP=15.1 $\rm mC\,m^{-2}$, DPPE=5.3 $\rm mC\,m^{-2}$ 321 and $DPPG=-44.0\,\mathrm{m}\tilde{\mathrm{C}\,\mathrm{m}^{-2}}$ for positively charged, zwitterionic and negative 322 charged lipid bilayers, respectively [61]. We first examine effects of the pore 323 electric potential, Φ_{pore} , on reaction kinetics, assuming CD73 is uncharged. 324 In Fig. 7 we demonstrate that in general $k_{on,ATP}$ monotonically decreases re-325 gardless of the membrane charge. In the event that the pore interactions with 326 substrate ATP are repulsive ($\Phi_p < 0$), the reaction rate coefficient decreases at 327 a faster rate. However, in certain regimes the charge complementarity of the 328 pore surface was found to greatly accelerate k_{on.ATP} relative to the neutral pore, 329 whereas a repulsive pore ($\Phi_{pore} < 0$, blue) attenuated $k_{on,ATP}$ by roughly 17% 330 or $3.77\times 10^{-1}\,{\rm nm^3\,ns^{-1}}$ (see Fig. 6) for $r_p\sim 6r_E$). We attribute the enhanced 331 reaction rate coefficient for the positively charged membrane to the elevated 332 concentration adjacent to the membrane relative to the uncharged membrane, 333 which effectively raised its average concentration within the pore (). That is, 334 the complementary charged pore surface drew ATP into the pore interior and 335 thereby facilitated the reaction on CD39. Hence, the charge of the pore sur-336 face stemming from different phospholipid compositions can strongly influence $k_{on,ATP}$, and in turn control AMP productions ATP degradation via CD39. 337 338

Interestingly, attractive pore/ATP interactions initially accelerate $k_{on,ATP}$ 330 as the pore diameter is reduced, whereafter the rate declines. We find the 340 maximal acceleration is achieved when the pore size is roughly six-fold higher 341 than the enzyme radius (see Fig. S3). This maximum is dependent on the wall 342 potential amplitude, namely as the attractive wall potential amplitude increases, 343 the maxima shift to smaller pore/enzyme size ratios. In previous studies [51, 344 52, 62], it has been demonstrated that weakly attractive interactions with pore 345 boundaries can enhance diffusion and ion conductivities, which is consistent 346 with the initial increase in $k_{on,ATP}$ in our model. However, this acceleration in 347 diffusion due to attractive interactions is eventually outweighed by the hindrance 348 of diffusion as the pore is narrowed. Additionally, although diffusion is likely 349 accelerated, the amount of substrate able to interact with the target is reduced, 350 as we saw for uncharged cases. Hence, attractive pore potentials served to co-351 localize substrates near the pore wall and therefore offset the reduced reaction 352 volume that would otherwise decrease the overall reaction rate coefficient (see 353 Fig. S7), as might be expected for ATP with positively charged phospholipids. 354

We initially anticipated that positioning the protein directly adjacent to the 355 membrane would improve the reactivity relative to the pore center. Further, we 356 observe that $k_{on,ATP}$ can be amplified when the enzymes are tethered to the 357 pore surface under specific conditions, namely wide pores and strong attraction, 358 but this advantage is generally minor and thus of limited consequence to NDAs 359 (Fig. <u>S6</u>). We found little difference for $k_{on,ATP}$ at modest (< |25| mV) pore 360 potentials for far in Fig. S7). This appears to be consequence of reduced access 361 to the enzyme as it approaches the wall, which counterbalances the increased the 362 concentration of ATP near the surface due to attractive electrostatic infractions. 363 Electrostatic enhancement of $k_{on,ATP}$ is generally expected to promote $k_{on,AMP}$ and 364 $k_{prod,Ado}$, thus we examined the extents to which the intermediate species 365 charge and enzyme proximity control k_{eff} . We present results assuming a 366 positively-charged intermediate $(z_{AMP}=1)$ to emphasize the electrostatic con-367 trol of the sequential reactions. Consistent with findings from the neutral sys-368 tem and our previous studies of sequential enzyme channeling [19], we find that 369 $k_{prod,Ado}$ increases as the enzymes are brought into close proximity $(d_{E1E2} \rightarrow 0)$. 370 As observed in the preceding section, the absorbing pore boundaries show the 371 greatest sensitivity to enzyme distance, with favorable AMP /CD73 electrostatic 372

interactions for $\Phi_{CD73} < 0$ yielding faster $k_{prod,Ado}$ reaction rate coefficients rel-373 ative to neutral CD73, and conversely slower rates for positively-charged CD73. 374 The enhancement in the former case reflects both the electrostatic attraction of substrate AMP toward CD73, while ATP is electrostatically repelled toward 375 376 CD39, which culminate in increased $k_{on,AMP}$ and $k_{prod,Ado}$, respectively. In the 377 latter case, the positively charged CD73 repels AMP and attracts the negatively-378 charged A, which in effect competes with the reaction of ATP on CD39. Hence, 379 the enzymes' charge complementarity with their respective substrates enhances 380 the overall reaction rate coefficient relative to uncharged systems and thereby 381 offsets the suppressed rate due to confinement within the pore. In this regard, 382 co-localization of the nucleotidases can ensure reasonable reaction efficiency de-383 spite confinement in pore. Further, these results confirm trends identified in 384 [19, 45, 46] that enzyme colocalization can support higher overall reaction rate 385 coefficients under specific conditions, albeit here we consider such effects in the 386 context of confinement to the pore. 387

Attractive interactions between substrate ATP and a positively-charged sur-388 face lead to the greatest overall enhancements of $k_{prod,Ado}$. There is, however, 389 a limit to this acceleration, if the attractive interactions between the pore and 390 substrate are stronger than the target enzyme, as we demonstrate with a re-391 duction in $k_{on,ATP}$ demonstrated in Fig. S6. It is interesting that attractive 392 A/pore interactions dominate the sequential enzyme kinetics, given the likely 393 repulsion of cationic B species from the reactive centers, which could suppress 394 the overall reaction efficiency. To assess the extent to which AMP /pore interac-395 tions contribution to $k_{prod,Ado}$, we examined the k_{eff} ratios for the nonreactive pore boundaries. Based on the efficiencies reported in Fig. 8, most of the in-396 397 creased production rate can be attributed to $k_{on,ATP}$, as the efficiencies were 398 largely constant across the various charge configurations. However, efficiencies 399 were generally greater for co-localized enzymes versus separated configurations, 400 regardless of the pore charge For negatively-charged pores ($\Phi_{pore} < 0$), the re-401 action efficiency was enhanced for all configurations, except for colocalized en-402 zymes directly adjacent to the pore. In analogy to the effects of the pore charge 403 on ATP, the increased efficiency for positively-charged pores can be rational-404 ized by the pooling of anionic AMP within the pore, which increases $k_{on,AMP}$. 405 This concurs with our findings of enhanced reaction efficiency in dihydrofolate 406 reductase-thymidylate synthase (DHFR-TS) [21] owing to its complementary 407 surface charge to the dihydrofolate intermediate intermediate, in contrast to a 408 neutral or electrically-repulsive surface. For the co-localized cases at the mem-409 brane surface, it is likely that the membrane significantly competed with the 410 binding of B at CD73, which led to a reduced reaction rate coefficient. Over-411 all, these results suggest that favorable A/membrane interactions largely con-412 trol the absolute k_{prod,Ado} rate, with co-localization typically further enhancing 413 $k_{prod,Ado}$ and k_{eff} . The contributions of B/pore interactions play a compara-414 tively smaller role, perhaps given the limited extent to which B permeated the 415 pore relative to A. 416

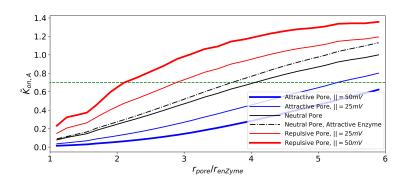


Figure 6: Effects of charge Effect of charge sign composition on reactivity, given $z_{ATP}=-1$, $z_{AMP}=+1$, $z_{Ado}=0$ and $\Phi_{CD39} > 0$ Effect of pore electrical potential on reactivity of enzyme for different sizes of the pore. We define an effective pore radius for charged pores, to be obtained from the green dashed line. For relatively small pores, the effective pore radius is larger for attractive pores, and smaller for repulsive pores. For relatively larger pores, competition between the pore wall and the enzyme becomes more significant, leading to a decline in the reactivity of attractive pores (see Fig. S3 in which with decreasing the size of enzyme we allow larger relative sizes of pore to enzyme radius).

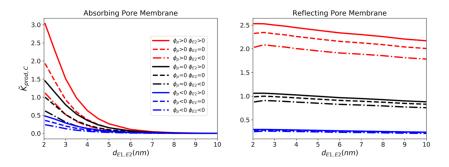


Figure 7: Effects of charge Effect of charge sign composition on reactivity, given z_{ATP} =-2, z_{AMP} =-1, z_{Ado} =0 and $\Phi_{CD39} > 0$ Normalized reaction rate coefficient for production of Ado as a function of distance between enzymes.

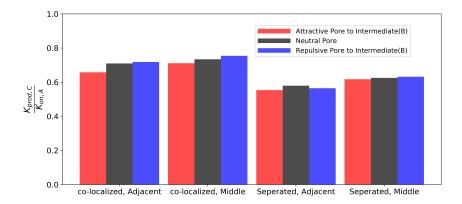


Figure 8: **Effects of charge** Efficiency of sequential enzymes for different pore wall electric potentials, given z_{ATP} =-2, z_{AMP} =-1, z_{Ado} =0 and $\Phi_{CD39} > 0$. Corresponding values for $k_{on,ATP}$ are provided in Fig. S7.

In the previous section, we highlighted reaction rate coefficients and efficiencies without accounting for electrostatic screening by common electrolytes. To model physiological conditions characterized by roughly 100 millimolar monovalent ion concentrations, we solved the linearized Poisson-Boltzmann equation, assuming Debye lengths on the order of 1 nm. This Debye length signifies that electrostatic interactions are significantly screened, which will in turn modulate reaction kinetics. To assess effects on $k_{on,ATP}$, it is helpful to compare rates as a function of $(1 + a\kappa)$, where a is the enzyme radius and κ is the inverse Debye length (see Fig. 9). This functional form is motivated by the relationship

$$\ln k_a = \ln k_{I=0} - \frac{A|z_1 z_2|\sqrt{I}}{1 + Ba\sqrt{I}}$$
(3)

(4)

where I is the ionic strength, while A and B are generally fitting parameters
introduced by Schreiber *et al*[63]. This latter term stems from the Debye-Huckel
treatment of electrolyte solutions.

In Fig. 9a), we demonstrate the scaling of $\ln[k_{on,A}]$ with respect to (1 + 1)420 $(\kappa a)^{-1} \rightarrow 1$, assuming either attractive or repulsive substrate interactions with 421 CD39 and pore. For attractive interactions, the maximum rate enhancement 422 relative to an electrically-neutral reference system is found under dilute condi-423 tions signified by $\kappa \to 0$ or equivalently, $(1 + \kappa a)^{-1} \to 1$. At high ionic strength 424 $(\kappa \to \infty)$, the rates approach those of the neutral system. By accounting for 425 electrolyte screening, reaction rate coefficients are somewhat attenuated, which 426 will in turn depress $k_{prod,Ado}$ and potentially k_{eff} . Accordingly, we reevaluated the effects of enzyme co-localization and pore proximity on $k_{prod,Ado}$ and k_{eff} , 427 428 subject to the 150 mM KCl background electrolyte. As anticipated, $\mathbf{k}_{prod,Ado}$ 429 generally scales proportionally to $k_{on,ATP}$ as a result of A/pore interactions 430 largely setting the overall reaction rate coefficient relative to the intermediate. 431 This is further evident in the negligible impact of ionic strength on k_{eff} . We 432 additionally found that the trends in k_{eff} and $k_{prod,Ado}$ relative to enzyme co-localization and pore proximity did not significantly deviate from those reported 433 434 in electrolyte-free conditions and are therefore not reported here. Overall, while 435 physiological ionic strength conditions modestly impact reaction kinetics rela-436 tive to dilute conditions, the changes are fairly insignificant and also insensitive 437 to moderate changes in ionic strength typical in physiological systems. In other 438 words, fluctuations in signaling ion concentrations are not likely to significantly 439 modulate NDA behavior. 440

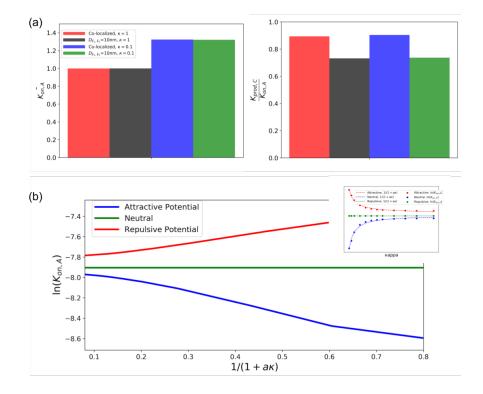


Figure 9: Effect of Debye Length on reactivity of sequential enzymes: a) Efficiency and first enzyme reactivity for large and small Debye lengths for co-localized and separated enzymes for $\Phi_{CD39} > 0$, $\Phi_p > 0$ and $\Phi_{CD73} < 0$ z_{ATP} =-1 and z_{AMP} =1 b)Relationship between $\ln(kon)$ and $(1 + a\kappa)^{-1}$ for three different pore electrostatic potentials. Subpanel shows fitting of curve to simulation data points.

441 4 Discussion

442 4.1 Summary

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In this study, we probed how nucleotide pools are controlled by ecto-nucleotidase 443 (NDA) and nucleotidase enzymes within narrow junctions formed between ad-444 jacent cells, similar to synaptic junctions comprising neurons. Here we used 445 numerical solutions of steady-state reaction-diffusion models of coupled nucleoti-446 dases that were recently studied in cystic fibrosis but assuming spatially uniform 447 behavior [31]. These simulations were performed under physiological conditions 448 that included confinement to narrow junctions bordered by cell plasma mem-449 brane and long-range, ionic strength-mediated electrostatic interactions. This 450 three-dimensional approach resolved important factors governing the fast re-451 action rates of NDAs in biological systems, that were not readily apparent in 452 prior numerical approaches based on ordinary (spatially-independent) differen-453 tial equations. Our key findings were 454

- that independent of NDA activity, nucleotide distributions within confined
 extracellular junctions can significantly differ relative to open, bulk-like
 configurations
 - that NDAsteady-state reaction rates are generally smaller when localized to small junctions, but reaction efficiency can improve by co-localizing coupled enzymes, and
 - that these reaction rates can be substantially accelerated when NDA and plasma membrane adopt charges complementary to reacting substrates, especially when the membrane attracts the relevant substrate.

Adenosine nucleotides encompass a set of small, polar molecules that are 464 critical for cellular signaling and metabolism [14]. These nucleotides are gener-465 ated or regulated by diverse processes, including secretion from neighboring cells 466 in tissue [64], as products of membrane-bound F1-ATPases [65], transport via 467 ectopic adenine nucleotide translocases [66] or hydrolyzed by ecto-nucleotidase 468 (NDA). For many cellular systems, these processes occur within femtoliter-scale 469 [11, 67] regions between neighboring cells, such as those characteristic of neu-470 ronal synapses [68]. Here, co-localization of NDAs including CD39 with puriner-471 gic receptors within caveolae [69] or with extracellular ATP release sites on 472 astrocytes [70] can give rise to 'compartmented' nucleotide pools [71] that can 473 strongly influence nucleotide-dependent signaling. The thermodynamics and ki-474 netics of molecular signaling in such compartments can differ considerably from 475 analogous processes in bulk solutions or in vitro. Myriad factors contribute to 476 these differences, including smaller compartment volumes that strongly amplify 477 substrate concentration gradients [30, 72], the presence of 'crowders' comprising 478 other small molecules, protein or nucleic acid that generally impede diffusion [28, 479 47, 73, 74], as well as rate enhancements typically exhibited for closely-apposed 480 enzymes [29, 42, 45, 50, 75] or those adopting electrostatic fields complementary 481 to a reacting species [38]. The relative contribution of these factors to coupled 482 NDA activity in a given multi-cellular domain has not been examined previously 483 and could ultimately determine the relative distribution of nucleotides. This 484 balance of nucleotide concentrations determine the extent to which membrane 485 bound, nucleotide receptors, ATPases and translocases are activated, that in 486 turn can shape diverse cellular processes, including migration [76] and cytokine 487 release [77]. 488

To systematically probe the potential for these factors to impact NDA activity and nucleotide distributions, we examined a sequential adenosine nucleotide

hydrolysis process as a model system for characterizing ecto-nucleotidase (NDA) 491 activity within synapse-like domains. We performed steady-state reaction-diffusion 492 simulations of two NDA enzymes that sequentially catalyze the hydrolysis pro-493 cesses $ATP/ADP \rightarrow AMP \rightarrow Ado$. Our system is modeled after the CD39 and 494 CD73 NDAs, though we utilize two arbitrary, spherical representations that are 495 selective for tri- and mono-phosphates, respectively, to generalize our results 496 to other coupled nucleotidases. Additionally, although the fully deprotonated 497 ATP anion assumes a charge of -4 [78], we assume in accordance with physio-498 logical systems that it is coordinated with magnesium [79] and thereby assumes 499 a net negative charge of -2. To orient our results, we note that increasing 500 $k_{on,ATP}$ rates (reaction of ATP at CD39) result in ADP accumulation, while 501 large $k_{on,AMP}$ and $k_{prod,Ado}$ correspond to high rates of ADP consumption and 502 AMP production at CD73. Since the latter rates are generally proportional to 503 $k_{on,ATP}$, we report the reaction efficiency, $k_{eff} \equiv k_{prod,Ado}/k_{on,ATP}$, to highlight 504 contributions specific to ADP consumption. In this regard, high k_{eff} rates gen-505 erally reflect significant consumption of extant ADP pools to form AMP. These 506 rates were measured for CD39 and CD73 in bulk solution and confined within 507 a nanometer-diameter pore, for which we varied the enzyme distributions, sub-508 strate/enzyme interactions, as well as the nanopore surface charge to emulate 509 typical phospholipid bilayers. Based on these variations, we discuss how the 510 relative nucleotide composition within confined domains is determined by phys-511 ical attributes of the nanoscale compartment, how co-distribution of coupled 512 hydrolysis reactions helps facilitate high reaction efficiency despite confinement 513 514 to nanoscale, extracellular junctions, how membrane surface charge localizes substrates so as to accelerate reaction rates, indirectly modulate enzyme kinet-515 ics. 516

⁵¹⁷ 4.2 Nucleotide transport and distribution (pools) within ⁵¹⁸ crowded extracellular junctions

We first discuss how nucleotide diffusion rates and distribution are influenced by physical attributes of the confined junctional geometry, including restricted 519 520 diffusional volumes and electrostatic interactions between substrates, reactive 521 enzymes and charged membrane surfaces. Independent of NDA activity, the 522 restricted volume of the junction relative to the surrounding substrate reservoir, 523 as well as the surface charge distribution within the junction, played key roles 524 in shaping the nucleotide distribution. In our model, nucleotides entered the 525 restrictive junctional domain from one of two reservoirs to emulate entrapment 526 of species generated from an external source, such ATP released from nearby 527 damaged cells or part of physiological processes including paracrinic release. In 528 the absence of nucleotide/surface interactions, the diffusion rate of nucleotides 529 through the junction decreases as its radius is reduced. This is easily rationalized 530 by noting that the substrate flux through a cylinder normal to the nucleotide 531 concentration gradient scaled proportionally to the cylinder's cross-sectional 532 area relative to the reservoir surface area [80]. The constriction of the substrate-533 accessible volume at the junction opening leads to a substantial reduction in 534 the amount of substrate available to the enzyme within the pore compared 535 to bulk conditions (see also [51]). For this reason, narrow junctions between 536 cells are anticipated to limit nucleotide pools available to ATPases and ATP-537 gated receptors localized to extracellular junctions. Hence, estimates of ATP 538 based on bulk (extracellular) measurements [79] are generally unrepresentative 539 of the local ATP pools formed within the compact interstitia between cells. This 540 deviation strongly justifies the use of localized measurements of nucleotides when 541 probing receptor activity in neural synapses for instance via microelectrodes [10]. 542

⁵⁴³ 4.3 Contribution of NDAs to controlling nucleotide pools

A secondary focus of this computational study was to probe NDA dependent 544 modulation of steady-state nucleotide concentrations relative to those deter-545 mined by junction size and electrostatic charge alone. It is understood that 546 547 NDAs rapidly degrade nucleotides released in synaptic junctions [81]; hence, pulsatile release of ATP from post-synaptic neurons is followed by transient, 548 millisecond-scale upswings in the synaptic ATP concentration, owing to NDA 549 degradation [10]. However, the femtoliter volume of such spaces [11] and inter-550 cell separations within several fold of the Debye length suggest that NDA ac-551 tivity and resulting nucleotide pools will be sensitive to NDA colocalization, 552 strengths of substrate/enzyme electrostatic interactions and the junction vol-553 ume. Firstly, in analogy to the reduced nucleotide concentration reported at 554 555 the junction/reservoir boundary, we observed substantially lower $k_{on,ATP}$ rates for junction-confined CD39 relative to the bulk configuration. This behavior is 556 easily rationalized by the smaller cross-section of the pore relative to an open 557 system, which both reduces the concentration of substrate at the enzyme surface, 558 as well as the accessibility of reactive enzyme surface. Further, our predictions 559 are consistent with classic theoretical studies relating the dynamic accessibility 560 of gated protein active sites or substrate tunnels to observed enzyme activ-561 ity [82], as demonstrated in acetylcholinesterase [83] and the PutA peripheral 562 membrane flavoenzyme [84]. Since $k_{prod,Ado}$ scales proportionally to $k_{on,ATP}$, 563 reduced NDA rates owing to confinement suggests that in vitro characterization 564 of NDA activity in bulk media likely yield faster kinetics than would be expected 565 for strongly confined systems; As a consequence, ATP pools within the junction 566 were largely suppressed relative to those of the NDA-free system, while ADP 567 and AMP were considerably larger. Based on these predictions, we anticipate 568 that the degradation of adenosine phosphates to lower order molecules by ec-569 tonucleotidases proceeds more slowly in confined extracellular spaces relative to 570 bulk conditions. Further, this reduction in reactivity is largely determined by 571 the reaction rate of the first species, ATP. 572

In contrast to the consistent rate-limiting effect of enzyme confinement on 573 $k_{on,ATP}$, the efficiency of Ado production relative to bulk varied depending 574 on the nature of substrate/membrane interactions. We investigated this de-575 pendency assuming reflective (non-interacting) and absorbing boundary condi-576 tions on the membrane; the latter is representative of nucleotide depletion by 577 membrane-bound ATPases or translocases. We found that efficiency was maxi-578 mized when the nucleotides did not significantly interact with the membrane (re-579 flective). In this case, although CD39's confinement to the pore limited its access 580 to ATP, the membrane prevented intermediate diffusion away from CD73. This 581 established a relatively high intermediate concentration within the junction that 582 in turn increased $k_{prod,Ado}$. In contrast, efficiency was strongly reduced when 583 nucleotides were depleted at the surface (absorbing), as might be expected for 584 significant nucleotide uptake by plasma membrane adenine nucleotide translo-585 cases [66]. As discussed in the next section, this reduced efficiency for absorbing 586 membranes could be countered by co-localizing the two-enzymes to favor AMP 587 's reaction on CD73 relative to diffusing toward the membrane Ultimately, these 588 findings suggest that nucleotide pools capable of activating targets such as ADP 589 sensitive P2Y channels will be strongly regulated by the relative activity of pro-590 teins or transporters that reduce the di-phosphate concentration in the junction 591 and thereby compete with CD39. 592

Numerous biochemical processes that involve diffusing reactants rely on close
 spatial coupling of enzymes to promote efficient signaling. Examples of enzyme
 co-localization include formation of macro-molecular complexes [85, 86], confine-

ment in molecular 'tunnels' [87–89], the proximal reactive sites in the sulfate-596 activating complex [90], in addition to metabolic substrate channeling [91–93]. 597 We had thus expected that co-localizing NDAs within junctions would improve 598 reaction efficiency. However, we found that close spatial coupling was advanta-599 geous only when the junction membrane significantly interacted with the inter-600 mediate. Specifically, when the membrane either absorbed the intermediate or 601 attracted the intermediate through attractive electrostatic interactions, smaller 602 concentration gradients were evident at CD73 and thereby reduced $k_{on,AMP}$. 603 Co-localizing CD39 and CD73 minimized the intermediate's access to the mem-604 605 brane and thus facilitated faster $k_{on,AMP}$ rates than were evident at larger enzyme separations. This behavior is consistent with simulation studies by us and 606 others [19, 45, 82] for open (bulk) systems whereby co-localization of sequential 607 enzymes can enhance reaction rates. Based on our rationalization in the pre-608 ceding paragraph, co-localization of CD39 and CD73 for a reflective membrane 609 had minimal impact on the reaction efficiency, given that the intermediate had limited capacity to escape the reactive sites. Constructs including micelle- or 610 611 viral capsid-based nanoreactors that house enzymes, or enzymes immobilized 612 to linear or planar molecular assembles [56, 94] exhibit analogous increases in 613 efficiency through mitigating loss of intermediates to open boundaries. These 614 results therefore suggest that variations in NDA co-localization could provide 615 a means to tune the relative composition of nucleotide pools within junctions, 616 particularly for charged membranes or those with an abundance of proteins that 617 compete for nucleotides. 618

A central contribution from our study is to confirm the significant role of 619 electrostatics and intermediate channeling in facilitating coupled nucleotide hy-620 drolysis reactions catalyzed by NDAs in nanoscale volumes. Secondarily, we 621 demonstrate that tuning of the surface/enzyme and surface/substrate interac-622 tions can further optimize reaction rates. A third contribution of our study was 623 to systematically characterize how electrostatic interactions influence enzyme 624 kinetics under physiological conditions. It is clear from our simulation data 625 that a significant membrane charge can redistribute the populations of charged 626 substrates along the pore boundaries. Because diffusion-limited reaction rates 627 scale proportional to the substrate concentration gradient at the enzyme ac-628 tive site, it was expected that membrane charge configurations that localized 629 substrates to the pore and its surface would enhance the reaction rate. 630

From this standpoint, we can treat our predicted $k_{on,ATP}$ values as readouts 631 for the significance of local substrate concentrations to NDA activity, particu-632 larly in the context of electrostatic interactions. Such electrostatic interactions 633 have been speculated to contribute to the formation of 'micro-domains' local-634 ized to the membrane surface, such as for Ca^{2+} , Na^+ and to a lesser extent 635 [58, 72, 95, 96] following transient fluctuations in their concentrations. These 636 microdomains are strongly implicated in modulating the ion-dependent activa-637 tion of small proteins [97]. As an example, ATP has been suggested to assume 638 concentrations several-fold higher than the bulk cytosol, based on modeling and 639 ATPase enzyme assays [30, 93]. To the extent that the microdomains arise ex-640 clusively from electrostatic interactions, microdomain effects would be expected 641 to be maximal within the membrane's electric double layer that is approximately 642 1 nm at physiological ionic strength. 643

Therefore we sought to examine extent to which electrostatic interactions contribute to microdomains under steady state conditions. We found that the reaction rate coefficient had weak dependence on the enzymes' distance from the pore surface, regardless of ionic strength, which strongly suggests microdomains arise from a different basis. Consistent with this argument is our recent finding that ionic-strength-dependent changes in the Ca^{2+} at the membrane surface

has negligible impact on SERCA Ca²⁺ affinity [98]. This confirms that localized 650 substrate pools near the surface stem from non-equilibrium conditions, namely 651 a net flux of substrate from the extracellular or cytosolic domains toward the 652 membrane. For ATP, the steady-state flux toward the cytosolic side of the mem-653 brane could arise from the creatine and adenylate kinase shuttles [30, 93], while 654 localized ATP gradients on the extracellular side could result from F10ATPase 655 or translocase activity [8]. For ions such as Ca^{2+} , membrane-localized gradients 656 could arise from small inward fluxes of plasma membrane currents or leak from 657 compartments such as the endoplasmic reticulum [97]. 658

Our results suggest that the predominant effect of charging the membrane 659 is to increase the concentration of ATP within the entire pore interior. This 660 was evident in the predicted ATP concentration profile within the pore (see 661 Fig. S1), which varied significantly from the bulk reservoirs. This raises an 662 interesting possibility that NDA activity could be modulated through controlling 663 the surface charge by varying membrane lipid composition. Such variations in 664 lipid composition and surface charge are known to occur during phagocytosis 665 [99] and within neural synapses [67]. 666

In addition to substrate/surface interactions, we demonstrate that electro-667 static interactions between nucleotide substrates and their enzymes targets ac-668 celerate NDA activity. Favorable long-range electrostatic interactions between 669 enzymes and substrates are well known to optimize diffusion-limited reactions in 670 biological systems [63]. Chiefly, enzymes that bear charges complementary to 671 their substrates typically exhibit reaction rates that are several orders of mag-672 nitude higher than rates observed with neutral species or at high ionic strengths 673 that shield electrostatic interactions [51]. We specifically address this for CD39, 674 CD73 and charged membranes. CD39, for example, appears to have a slightly 675 greater density of positively-charged amino acids near the nucleotide binding 676 domain. We would expect this positive charge center to enhance the associ-677 ation rate via complementary electrostatic interactions through Arg56, Lys79, 678 Lys80, and Lys82. This was determined by visual inspection of the electrostatic 679 potential of a representative CD39 structure, the NTPDase2 from Legionella 680 pneumophila (PDB code 4BR7 [24]) using the Adaptive Poisson Boltzmann 681 Solver APBS[100]. Hence we expect this to facilitate the rapid reaction, though 682 to our knowledge rates with respect to ionic strength for this enzyme have not 683 been reported. 684

Beyond the role of electrostatics in shaping $k_{on,ATP}$, our results demonstrate 685 the kinetic advantage of co-localizing charged enzymes. When the enzymes 686 were co-localized, the influence of electrostatic interactions on the reaction rates 687 were most strongly evident, with the fastest rates reported for closely-opposed 688 enzymes that adopt surface charges complementary to their substrates. This 689 finding mirrors trends observed in other coupled enzymatic processes; namely 690 in the event that enzymes or reactive sites are sequentially aligned for cou-691 pled enzymatic reactions, electrostatic channeling of substrates is commonly 692 exploited in nature to optimize the rate or efficiency of substrate conversion [75, 693 101, 102]. As an example, a computational study of the DHFR-TS enzyme in 694 prokarvotes has revealed that tetrahydrofolate production rates are accelerated 695 by a patch of positively-charged amino acids between the thymidylate synthase 696 and dihydrofolate reductase reactive sites, which facilitate transfer of the an-697 ionic dihydrofolate intermediate [21]. Significantly, when the enzymes' charges 698 were complementary to those of the reactants, the overall reaction efficiency ex-699 ceeded predictions for the uncharged, confined enzymes and the enzymes in bulk 700 solution. Hence, these variations should profoundly influence the dynamics and 701 702 relative distribution of nucleotides within extracellular junctions.

⁷⁰³ 4.4 Limitations and Future directions

In order to work with the system that was numerically solvable, we made several 704 assumptions. Firstly, we assumed all enzymatic reactions were fast compared to 705 the diffusion of nucleotides between reactive centers. NDAs are known to rapidly 706 manage nucleotide pools with reaction rates on the order of $1 \,\mu M \, s^{-1} [31]$. Since 707 the intrinsic reaction rates of these enzymes vary quite considerably depending 708 on the isoform and cell type, we assumed reaction-limited conditions for simplic-709 ity and generality. It may also be appropriate to consider feedback inhibition, 710 given evidence that productions can hinder NDA-catalyzed AMP hydrolysis. 711 We additionally assumed spherical shapes for the proteins; while this may seem 712 to miss important details, our previous studies have indicated that the native 713 structure of the proteins has a marginal influence on reaction dynamics [52]. 714 We additionally assumed constant membrane potentials, although this can be 715 expected to vary in real lipid systems such as during phagocytosis [99]. We addi-716 tionally considered all reactions to be in steady state for ease of simulation and 717 analysis for simplicity, though effects of micro domains expected to be most evi-718 dent under non-linear, non-steady state conditions that permit significant ATP 719 accumulation. We also limited our membrane potentials to modest ranges for 720 which the linearized Poisson-Boltzmann equation was appropriate. For stronger 721 potentials, the full PB equation would be more appropriate but also more com-722 putationally expensive [103]. Further, in highly charged and confined domains, 723 the diffusing substrate can be expected to contribute to the shielding of electric 724 charge, which advocates for the use of the Poisson-Nernst-Planck formalism (see 725 [51]726

Although the enzyme and pore representations were simplistic in this study, 727 our finite element modeling approach can be refined with detailed structural 728 models of enzymes and their cellular environment. As an example, we have pre-729 viously used finite element models to probe Ca²⁺ binding rates to myofilament 730 proteins bound to actin chains [52, 53, 104] at atomic resolution, using mesh 731 building software [105] applied to structures found in the Protein Databank. 732 A similar approach could be used to probe the dynamics of other enzyme-733 catalyzed reactions in detailed molecular environments, such as those based 734 on atomistic-resolution simulations of a crowded cytoplasm [106]. Along these 735 lines, simulations of NDA activity in crowded synapses are warranted. 736

737 5 Conclusions

Sequentially-coupled enzymatic processes have been extensively probed in the 738 literature. Our contribution in this paper complement these studies through 739 offering insights into nucleotide distributions and NDA activity within extracel-740 lular junctions. Our results are consistent with the well-established notions of 741 electrostatic channeling for accelerating reaction rates and for the benefits of co-742 localization. Our approach is unique in its basis in a finite-element framework 743 that allows for the direct incorporation of electron or confocal microscopy data, 744 such as for serial block images of neurons or neuronuscular junctions [107], or 745 for atomistic resolution molecular structures of NDA s. In order to generalize 746 our results, we utilized a simplified pore/spherical enzyme framework for which 747 we could easily vary system parameters such as distances and radii, which could 748 not be afforded with structurally detailed models. Based on our earlier studies, 749 while details of the surface charge density of an active site can influence reaction 750 rates, nuances of the protein structure generally do not substantially impact the 751 results. This can be easily seen in our plots, which show relatively small changes 752 in reactivity and efficiency as a function of significant changes in enzyme sizes 753

or separation distances. This observation is helpful in reducing the spatial com-754 plexity of reaction simulations in complex media. As we demonstrated in [23], 755 effective diffusion rates of small molecules in structurally-realistic crowded so-756 lutions did not significantly differ from those computed using perfect spheres of 757 fixed radii at a similar packing fraction. This is in agreement with [52, 104, 108]. 758 As shown in our results, this permits the contributions of crowders to diffusion-759 limited association rates to be accounted for in a rather simple implicit form, 760 namely by accounting for configurational entropy of the reacting substrate. 761

These simulations of steady-state NDA nucleotide hydrolysis activity in 762 nanoscale porous geometry mimic gap junctions between cells. We found that 763 confinement and high charge densities within confined domains alter nucleotide 764 concentrations relative to bulk, independent of NDA activity. Additionally, we 765 demonstrate that for NDAs localized to the pore, confinement of NDA reduces 766 activity relative to bulk, and the nature of surface interactions determines the 767 advantage of co-localization on reaction efficiency. Of these, charges substan-768 tially influence reaction kinetics, particularly contributions of complementary 769 membrane charge to ATP in the initial reaction. 770

We believe these findings provide new insights into the activity of purinergic 771 receptors and other proteins that respond to extracellular ATP concentrations. 772 Incorporating these features could expand our capacity to probe physiological 773 phenomena *in vivo*, monitor and tailor drug delivery kinetics (reviewed in [109]), 774 and engineer biosynthetic pathways, especially those utilizing immobilized en-775 zymes [44, 91, 110]. Additional future directions include probing reaction dy-776 namics relative to enzyme activity. We could further compare how the dynamic 777 nucleotide signals influence the activity of purinergic receptors and ATPase ac-778 tivity on the extracellular domain, like F_1 - F_O ATP synthase[8]. 779

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788 7 Method

789 7.1 Overview

The purpose of this study is to understand the steady-state properties of nu-790 cleotide hydrolysis by NDAs in nanoscale extracellular domains. This was 791 achieved through computer simulations of electrokinetic transport, similar to 792 those in [51], modified to handle reaction equilibria. The theoretical model 793 includes partial differential equations defined on a continuum problem domain. 794 These equations are solved numerically using the Finite Element Method. Steady-795 state conditions were assumed. The equations were solved for three-dimensional 796 geometries resembling the porous materials in [51, 80]. Rather than attempting 797 to simulate the intricate geometries of in vivo systems, the geometry of these 798 porous materials was chosen in order to reduce the computational cost of the 799 simulations, simplify interpretation of the results, and improve the reproducibil-800 ity of our findings. The porous material is modeled as a thin membrane sepa-801

rating two aqueous reservoirs. A concentration gradient can be created between 802 the two reservoirs to drive transport across the membrane. The simulations 803 study diffusion taking place inside circular pores in the membrane. The compu-804 tational model allows for the control of key geometric, electrostatic, and kinetic 805 parameters, so that the effects of various phenomena can be resolved. Predicted 806 substrate gradients that develop in the materials were used to estimate reac-807 tion rate coefficients, which directly relate to the kinetics of substrate transport. 808 Additional physics was enabled in the computational model that accounted for 809 electrostatic interactions and surface reactions. Chief outcomes of this study are 810 1) a computational model for molecular transport and 2) quantitative data for 811 describing how confined domains and electrostatic interactions control molec-812 ular transport. For comparisons against bulk conditions, we assumed enzyme 813 densities corresponding to millimolar concentrations. 814

815 7.2 Theory

In these simulations, the enzymes are held in fixed positions while the substrates
are allowed to diffuse. The problem domain is approximated as a continuum,
with the diffusing chemical species considered to be point particles. Under these
circumstances, the reaction encounter distance is just the radius of the enzyme.
The enzymes are therefore approximated as spheres.

Three species of substrate are included: A, AMP, and Ado. ATP is converted to the AMP product when it encounters the surface of CD39, followed by AMP's conversion to adenosine on enzyme CD78:

$$\begin{array}{c} \text{ATP} & \xrightarrow{CD39} & \text{AMP} \\ \text{AMP} & \xrightarrow{CD78} & \text{Ado} \end{array}$$
(5)

We define the concentration of a given species S as c_S , which is an unknown spatial function to be found by solving the governing diffusion equation. The ion flux of species S is a vector field, $\vec{j_S}$, related to the change in concentration with respect to time through a continuity equation,

$$\frac{\partial c_S}{\partial t} = -\nabla \cdot \vec{j_S} \tag{6}$$

The diffusion of ions in a fixed electrostatic field is described by the Smoluchowski equation [112], where the flux includes both a Fickian diffusion term and a term due to the electrostatic force:

$$\vec{j}_S = -D_S \left(\nabla c_S + \beta z_S c_S \nabla \Phi\right) \tag{7}$$

⁸²⁸ Where z_S is the electric charge of species S, Φ is the electric potential as a ⁸²⁹ scalar field, D_S is the Fickian diffusion coefficient for species S in the relevant ⁸³⁰ media, and β is $1/k_BT$ for temperature T and Boltzmann constant k_B . In this ⁸³¹ equation, the diffusion coefficient is assumed to be homogeneous and isotropic. ⁸³² Using this flux in the continuity equation, the Smoluchowski equation can ⁸³³ be written as:

$$\frac{\partial c_S}{\partial t} = \nabla \cdot \left(D_S \left(\nabla c_S + \beta z_S c_S \nabla \Phi \right) \right) \tag{8}$$

Under steady state conditions the concentration of species S does not vary in time, and so the governing differential equation is:

$$0 = \nabla \cdot \left(D_S \left(\nabla c_S + \beta z_S c_S \nabla \Phi \right) \right) \tag{9}$$

834

To reduce the computational burden, an alternate form of the Smoluchowski

> equation is used. The substrate flux is expressed as: 835

$$\vec{j_S} = -D_S e^{-\beta z_S \Phi} \nabla \left(e^{\beta z_S \Phi} c_S \right) \tag{10}$$

The equivalence of these two expressions for the flux can be readily verified 836 using the product rule for gradients. The advantage of this alternate form is 837 that it allows for the application of the Slotboom transformation [113] [114]: 838

$$\overline{D}_{S} = D_{S}e^{-\beta z_{S}\Phi}$$

$$\overline{c}_{S} = c_{S}e^{\beta z_{S}\Phi}$$
(11)

After applying this transformation, the Smoluchowski equation is expressed 839 in a form analogous to a simple Fickian diffusion equation: 840

$$\nabla \cdot \left(\overline{D}_S \nabla \left(\overline{c}_S \right) \right) = 0 \tag{12}$$

Equation 12 must be solved for each species. 841

We also define the integrated flux over any surface Γ as 842

$$J_S = \int_{\Gamma} \vec{j_S} \cdot \hat{n} d\Gamma \tag{13}$$

where \hat{n} is the unit normal to the surface Γ . The reaction kinetics at an enzyme are assumed to follow a simple rate law. 843 For the reactions in Equation 5, the rate laws are given by

$$-J_{ATP} = J_{AMP} = k_{CD39}c_{ATP}$$

$$-J_{AMP} = J_{Ado} = k_{CD73}c_{AMP}$$
(14)

which defines reaction rate coefficients $k_{CD39} = k_{on,ATP} = k_{prod,AMP}$ and 844 $k_{CD73} = \mathbf{k}_{on,AMP} = \mathbf{k}_{prod,Ado}.$ 845

The calculation procedure begins by solving Equation 12 for \overline{c} for each 846 species, then computing the flux vector fields from the Slotboom transformation 847 of Equation 7, then integrating the flux over the enzyme surface, and using the 848 integrated flux to calculate the rate coefficient. The rate laws of Equation 14 849 are enforced at each enzyme by requiring that $\vec{j}_{ATP} \cdot \hat{n} = -\vec{j}_{AMP} \cdot \hat{n}$ on the surface of CD39, and $\vec{j}_{AMP} \cdot \hat{n} = -\vec{j}_{Ado} \cdot \hat{n}$ on the surface of CD73. A summary of concentration boundary conditions applied to the model is 850 851

852 presented in Table 1. 853

Boundary Surface	Γ_{CD39}	Γ_{CD73}	Γ_R	Γ_L
Substrate ATP	$c_{ATP} = 0$	$j_{ATP} = 0$	$c_{ATP} = C_0$	$c_{ATP} = 0$
Substrate ADP	$j_{AMP} = -j_{ATP}$	$c_{AMP} = 0$	$c_{AMP} = 0$	$c_{AMP} = 0$
Substrate Ado	$j_{Ado} = 0$	$j_{Ado} = -j_{AMP}$	$c_{Ado} = 0$	$c_{Ado} = 0$

Table 1: Concentration boundary conditions for the nanoporous system. Boundary conditions on Γ_P vary. Here, $j_S = \vec{j}_S \cdot \hat{n}$.

The solution of Equation 12 requires knowledge of the electric potential Φ 854 throughout the model. The electric potential is found by solving the linearized 855 Poisson-Boltzmann equation: 856

$$\nabla^2 \Phi = \kappa^2 \tag{15}$$

where κ is ionic strength which is the inverse of the Debye length. For pure 857 aqueous solvent, $\kappa = 0$ and therefore Eq. 15 reduces to the Poisson equation 858 commonly used in electrostatics. 859

7.3 Numerical approach

Methodologies are generally as described in Sun et al., [51].

The system of partial differential equations and boundary conditions described above was solved numerically using the Finite Element Method. The open-source finite element package FEniCS[115], version 2017.2.0 was used to conduct the simulations. This software is publicly available at fenicsproject.org.

A second-order polynomial (Lagrange) basis set was used for all finite elements. The differential equations to be solved were all linear, so no nonlinear solution schemes were required. Various linear solvers and preconditioners were employed in order to obtain solutions.

Python-based analysis routines were used to set up, solve, and post-process the finite element models. All code written in support of this publication is publicly available at https://bitbucket.org/pkhlab/pkh-lab-analyses. Simulation input files and generated data are available upon request.

874 Supplement

S Supplementary Information (SI): Co-localization and confinement of ecto-nucleotidases modulate extracellular adenosine nucleotide pools

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$_{^{884}}$ S.1 Tables

Fig (Sect)	qA/EA	qB/Eb	Pore	Crowders	λ_D
Fig. ?? (1)	-1/0	+1/0	0	0	
2	-1/+1	+1/-1	var.	0	?
3	-1/0	+1/0	0	var.	?
Fig. ??c	-1/0	var./0	0	var.	

Table S1: Summary of cases run

885 S.2 Figures

Fig. S1

Ds ~ Dp	Ds < Dp	Ds << Dp
High	High	High
Low	Low	Low

Figure S1: Concentration profile for three different values of pore radius.

886

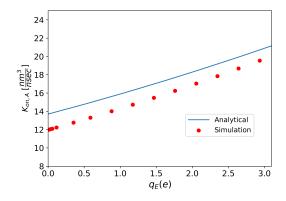


Fig. S2

Figure S2: A comparison between theoretical (see Eq. 2) and simulation result which validates our model of simulation. .

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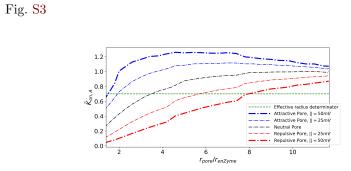


Figure S3: Effective pore radius for larger relative size of the pore to enzyme radius. .

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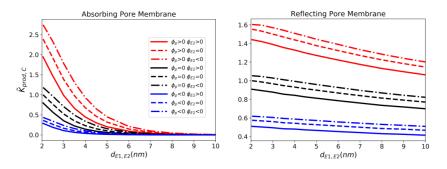


Figure S4: **Effects of charge** Effect of charge sign composition on reactivity, given z_{ATP} =-1, z_{AMP} =+1, z_{Ado} =0 and $\Phi_{CD39} > 0$ Normalized reaction rate coefficient for production of Ado as a function of distance between enzymes.

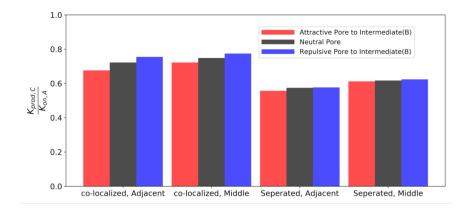


Figure S5: **Effects of charge** Efficiency of sequential enzymes for different pore wall electric potentials, given z_{ATP} =-1, z_{AMP} =+1, z_{Ado} =0 and $\Phi_{CD39} > 0$

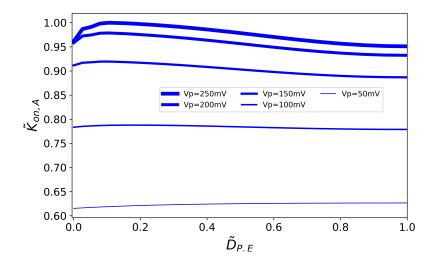


Figure S6: For large potential of the pore in comparison with enzyme potential, as the enzyme get close to the pore membrane, due to the attraction between pore membrane and substrate ATP the concentration of ATP is more and so it leads to an increase to the reactivity of enzyme. However, when the enzyme is getting too closer to the pore membrane, the competition between pore membrane and enzyme, especially for very large potential on membrane, is increasing, leading to a sudden decrease in reactivity. This obtained for kappa=1, and Dp=5DE.

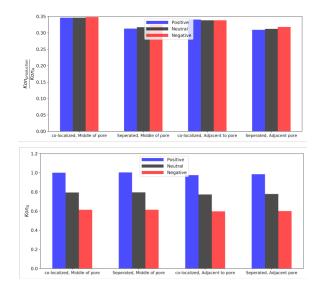


Figure S7: **Coloc/tethering effects on keff.** These data obtained for the case which maximize the efficiency with VE1 > 0, VE2 < 0, Dp=8 and Vp=25mV. The konA does not change when enzymes get close to the wall for all different charges of the pore wall. However, I have tried a case when Dp=11, kappa=1 and Vp=100 and saw that the konA increase as the enzymes get close tho the wall for attractive case between species ATP and pore wall (here positive), although the difference is not considerable(around 1.5 percent change). There is two factors affecting konA when it get close to the wall: The capacitance change when the enzyme get close to the wall. However, the concentration of ATP species is more near the wall of the pore.

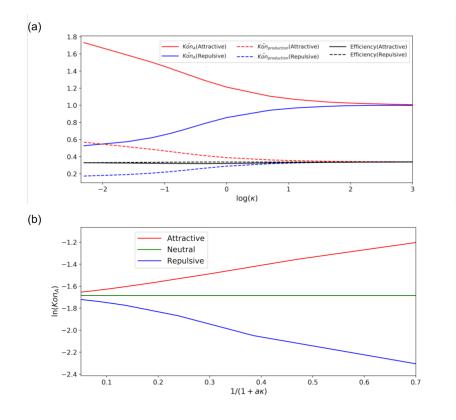


Figure S8: Effect of Debye Length on reactivity of sequential enzyme: a) Reactivity of the first enzyme and second enzyme as a function of pore to enzyme distance for electrostatical compositions with maximum (red) and minimum (blue) reactivity along with the efficiency (black) c) b)effective pore radius based on potential of the pore and Debye length. The green curve shows the data for uncharged reactivity of the first enzyme as a function of real physical size. based on the data we obtained from charged simulation, we can match an effective pore size which for attractive is bigger and for repulsive is less than its real size. .

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