Collapse and rescue of evolutionary food webs under global warming

Authors

Youssef Yacine [1], Korinna T. Allhoff [2], Avril Weinbach [1], Nicolas Loeuille [1]

[1] Institute of Ecology and Environmental Sciences Paris (iEES-Paris), 4 place Jussieu, 75252 Paris Cedex 5, France

[2] University of Tübingen, Institute of Evolution and Ecology, Auf der Morgenstelle 5, D-72076, Tuebingen, Germany

Corresponding author

Youssef yacine, youssef.yacine@upmc.fr

Institute of Ecology and Environmental Sciences Paris (iEES-Paris)

Tower 44-45, Floor 4, Room 413, case 237

4 place Jussieu, 75252 Paris Cedex 5, France

Running headline

Food web evolutionary rescue

Abstract

1. Global warming is severely impacting ecosystems and threatening ecosystem services and human well-being. While some species face extinction risk, several studies have demonstrated the possibility that fast evolution may allow species adaptation and survival despite environmental change.

2. We aim to assess how such evolutionary rescue dynamics extend to multitrophic communities and whether evolution systematically preserve biodiversity under global warming.

3. Using a simplified consumer-resource model, we analyze mathematically how evolution of body mass and of foraging strategies affect coexistence. We then extend our analysis to diverse, multi-trophic networks of co-evolving morphs.

4. Our simulations predict that the long-term diversity loss triggered by warming is considerably higher in scenarios where evolution is slowed down or switched off completely, indicating that eco-evolutionary feedbacks indeed help to preserve biodiversity. Evolving webs however experience vast disruptions in their structure and functioning and reversing warming may not be sufficient to restore previous structures.

5. Our findings thus highlight how the interaction between evolutionary rescue and changes in trophic structures affects our understanding of ecosystem responses under warming with important implications for conservation and management policies.

Keywords

Adaptive Dynamics, Biodiversity conservation, Body mass evolution, Climate Change, Coevolution, Consumer-Resource model, Eco-evolutionary tipping point, Evolutionary rescue, Preadaptation, Trophic networks

Introduction

Ecosystems support human societies through ecosystem services, such as resource provisioning, pollination, and geochemical recycling, amongst others (Costanza & Daly, 1992). Recent global changes, planetary-scale human-driven changes rivaling some of the great forces of Nature and deeply impacting the Earth system functioning, however severely impact ecosystems. Current extinction rates suggest a sixth extinction crisis is under way (Barnosky et al., 2011) and major losses of biodiversity are expected (Thomas et al., 2004). Such extinctions, but also more largely impacts on species demography, ranges and phenologies (Parmesan & Yohe, 2003), changes in ecological interactions (Tylianakis et al., 2008) imply significant ecological networks reorganizations potentially threatening ecosystems functioning.

As food webs describe the distribution of energy and matter flows in ecosystems, they provide useful tools to discuss such functional changes. Current observations highlight how global warming may affect trophic interactions (Both et al., 2009). Under such stress, species evolution (or coevolution) within the network may constrain its maintenance and stability.

Existing models investigating ecosystem responses to global warming often ignore evolutionary processes and species interactions. This is particularly true for "niche" or "envelop" models (Colwell & Rangel, 2009) that link data of species occurrence with climatic variables to build a multivariate statistical representation of species niches. Future species distributions are then predicted according to different climate change scenarios (e.g. Pearson et al., 2002), implicitly assuming that the niches are fixed. Evolution may however affect species fundamental niches, especially given current strong selective pressures, while community turnover, also expected given current changes (Tylianakis et al., 2008), may modify realized niches. While envelop models are important first steps, they are thus limited in their ability to understand the effects of global warming, and to provide relevant conservation policies (Lavergne et al., 2010).

Ignoring evolution is based on the controversial assumption that ecological and

evolutionary processes act on separate timescales, while recent studies show that evolution may

act within decades or even years (Koch et al., 2014; Olsen et al., 2004). Anthropogenic

disturbances create strong selective pressures that favor rapid evolution (Hendry et al., 2008)

emphasizing the importance of considering evolution when addressing global warming issues.

For instance, Bradshaw & Holzapfel (2001) showed that the genetically controlled

photoperiodic response of the mosquito, *Wyeomyia smithii,* has shifted towards shorter, more

southern day-lengths as growing seasons have become longer due to global warming. Their

experimental design allows them to attribute this shift, which is detectable over a time interval

as short as five years, to adaptive evolutionary change rather than phenotypic plasticity.

In some situations, evolution enables species to survive severe environmental change.

Gomulkiewicz & Holt (1995) theorized this concept as "evolutionary rescue". Adaptive

evolutionary change then restores positive growth to declining populations and prevents

extinction. Using experimental evolution, Bell & Gonzalez (2009) exposed hundreds of

populations of yeast to normally lethal conditions of salt and observed evolutionary rescue

within a few generations. For a review on the different factors involved and a discussion on the

empirical evidence for evolutionary rescue, see Carlson et al. (2014). Evolution may however

also threaten species maintenance. Environmental change can for example bring an

evolutionary attractor out of the viability area so that selection leads to a non-viable phenotype

given the new ecological conditions ("evolutionary trapping", Ferriere & Legendre, 2012) or

the evolution of one species can drive another interacting species to extinction ("evolutionary

murder", e.g. Dieckmann, Marrow, & Law, 1995). Accounting for evolution is therefore critical

to implement relevant conservation decisions (Stockwell, Hendry, & Kinnison, 2003).

Accounting for species interactions is equally crucial. Indeed, several examples

highlight that complex unexpected behaviors can arise from the interplay between evolution

and species interactions. For instance, Norberg et al. (2012) analyzed a spatially explicit multi-species model and demonstrated that evolution under climate warming can create extinction debts, long after climate stabilization, but only when competitive interactions are accounted for. Osmond, Otto, & Klausmeier (2017) showed that the maximal environmental change rate an evolving prey population can support before going extinct may increase in the presence of a predator, which seems rather surprising at first. In fact, in some cases (e.g. selective predation of maladapted preys), the presence of a predator accelerates prey evolution, enabling the prey to follow the environmental optimum more closely. Therefore, within a network context, the conditions for and the mechanisms underlying evolutionary rescue can be much more complex. A focal species could be rescued by the evolution of another interacting species, a concept called "indirect evolutionary rescue" by Yamamichi & Miner (2015). On the contrary, if evolutionary rescue happens for various species within the network, a focal species could still go extinct if its enemies (resp. positive interactors) recovery is too fast (resp. too slow) compared to its own (Loeuille, 2019). If in addition this species is key to the network, vast disruptions are to be expected due to secondary extinctions.

A key question is therefore whether evolutionary rescue, derived from a monospecific approach, can extend to ecological networks. Models that consider evolutionary processes within multi-species communities, such as Norberg et al. (2012) are currently rare. Single or few species models are useful because they focus on essential mechanisms happening in more complex networks. Nevertheless, they provide a limited view of the complex indirect interactions occurring in diverse networks (Ellner & Becks, 2011). This is particularly true for the theory of evolutionary rescue. The goal of the present study is therefore to expand existing knowledge from simplified mono-specific systems to complex multi-species networks. More precisely, we ask whether evolutionary rescue effects can promote food web persistence under warming.

We propose to address this question following a two-level approach. First, we

investigate the impact of temperature change on a simple consumer-resource model where the

consumer body mass and foraging strategy undergo evolution. Within this framework, we

analyze when evolution actually facilitates coexistence (trough evolutionary rescue) or

threatens it (through trapping). We first study body mass evolution, then let foraging strategies

evolve while fixing body mass. We finally give a brief glimpse on the two-trait coevolution

scenario.

We then discuss how complex feedbacks occurring in more complicated settings affect

our results, by allowing the food web module to evolve beyond the first diversification, resulting

in complex multi-trophic networks. Trophic structures emerging from such evolutionary

dynamics share many features with empirical food webs (Allhoff et al., 2015; Loeuille &

Loreau, 2005). Such evolutionary food web models therefore provide a powerful tool to study

community responses to changing environmental conditions. We investigate whether

evolutionary rescue is likely in such diverse, multi-trophic communities and compare the results

with the lower complexity approaches. Based on evolutionary rescue theory (Gomulkiewicz &

Holt, 1995), we hypothesize that faster evolution and lesser warming will help the rescue

process and facilitate community maintenance. We test this hypothesis using numerical

simulations, keeping in mind that indirect effects may modify this prediction in many ways. We

also expect upper trophic levels, more energy-limited and closer to the extinction threshold, to

be more impacted (Binzer et al., 2012; Weinbach et al., in prep).

Model and Methods

Model presentation

Ecological dynamics

We model the trophic network using ordinary differential equations:

$$\frac{dB_0}{dt} = r\left(1 - \frac{B_0}{K}\right)B_0 - \sum_i a_{i0}B_0 B_i \tag{1}$$

$$\frac{dB_i}{dt} = \sum_{j=ressource/prey} ea_{ij}B_j B_i - \sum_{j=predator} a_{ji}B_j B_i - \sum_{j=competitor} c_{ij}B_j B_i - d_i B_i \tag{2}$$

Where B_i is the biomass density of morph i; r and K are respectively the intrinsic growth rate and carrying capacity of the external resource; a_{ij} and c_{ij} are respectively the attack and competition rate of morph i on morph j; e is the conversion efficiency from prey to predator (set to 0.85 following Yodzis & Innes (1992)) and d_i is the death rate of morph i. Energy enters the system as an external resource of biomass density B_0 whose dynamics in the absence of interaction follows a logistic growth (1). Variations of consumer biomass density B_i (2) include four terms: a growth rate based on prey consumption, and loss terms due to predation, competition and natural death. See Table S1 (Supporting Information) for a complete list of variables and parameters.

Evolutionary processes and adaptive traits

Each consumer morph i has two adaptive traits: its body mass m_i and its feeding center f_i. Body mass largely constrain trophic interactions as observed empirically (Woodward et al., 2005) and within theoretical frameworks (Allhoff et al., 2015; Loeuille & Loreau, 2005). The feeding center of a consumer corresponds to the prey body mass allowing a maximal attack rate

173 and therefore describes its foraging strategy within our framework. The traits evolve following

174 a mutation-selection process. The external resource is characterized by its body mass m_0 that is

175 not allowed to evolve.

Eco-evolutionary framework

178 Many ecological parameters, such as mortality rates or attack rates, are known to scale

179 with body mass (Brown et al., 2004; Peters, 1983). Also, we assume strong competition

180 between two consumers with similar foraging strategies because of the resulting strong overlap

181 in their ecological niches (Macarthur & Levins, 1967). We therefore assume:

$$d_i = m_i^{-0.25}d \qquad (3)$$

$$a_{ij} = \frac{m_i^{-0.25}}{\sqrt{2\pi}s}e^{\frac{-(log(f_i)-log(m_j))^2}{2s^2}} \qquad (4)$$

$$c_{ij} = \frac{c_0}{\sqrt{2\pi}s}e^{\frac{-(log(f_i)-log(f_j))^2}{4s^2}} \qquad (5)$$

183 Where s is the feeding niche width (standard deviation of the gaussian feeding kernel (**Fig. 1**)),

184 assumed constant and identical for all morphs, and c_0 is a scaling constant for the interference

185 competition.

Temperature dependence

188 Biochemical reactions and thus metabolic rates are known to grow exponentially with

189 temperature. Following Brown et al. (2004),we incorporate temperature dependency as an

190 Arrhenius factor, into the resource growth rate r, and the mortality rate d_i of consumer i as

191 follows:

$$r(T) = r_0 e^{\frac{-E(T_0-T)}{kTT_0}} \qquad (6)$$

$$d_i(T) = d_0 m_i^{-0.25} e^{\frac{-E(T_0-T)}{kTT_0}} \qquad (7)$$

193 Following Fussmann et al. (2014), we assume the carrying capacity decreases exponentially

194 with temperature.

$$K(T) = K_0 e^{\frac{E(T_0-T)}{kTT_0}} \qquad (8)$$

196 Here, k is the Boltzmann constant and E the activation energy (Table S1). These relationships

197 hold over the temperature range of normal activity (0-40 °C for most organisms). Note that

198 while (6) and (7) are widely accepted, (8) is likely to be context-dependent (see Uszko et al.,

199 2017).

Framework to address evolutionary rescue

202 We first study the simple consumer-resource model (hereafter "Consumer-Resource"

203 model). Then, we analyze the complex networks that may emerge from the evolutionary

204 dynamics (hereafter Community Evolution Model, Loeuille & Loreau, 2010). **Fig. 1** shows a

205 schematic view of the two models. Using this two-step approach, we aim at providing an

206 encompassing view of the eco-evolutionary responses of food web to warming.

Figure 1: (Adapted from Allhoff et al., 2015). **a.** Schematic view of the Consumer-Resource model that allows for analytical investigations. The consumer (black triangle) is characterized by its body mass m_1 and its feeding center f_1 whose logarithm is the mean of the Gaussian function representing its attack rate kernel on the potential prey (black curve). Here the consumer feeds on an external resource (white triangle) with maximum attack rate. In the top right corner, the resulting two species trophic network is represented. **b.** Schematic view of the Community Evolution Model where large mutational steps are allowed facilitating diversification, emergence of higher trophic levels and leading to complex network structures. The snapshot here shows 3 morphs: morph 3 (black triangle) feeds on morph 1 and 2 (grey triangles) with respectively low and high attack rates, morph 2 and 1 feed on the external resource (white triangle) with respectively low and high attack rates. In the top right corner, the resulting trophic network is illustrated. Note that this is only a snapshot, the real network is dynamical and with typically contains many more morphs.

Consumer-Resource Model

The ecological dynamics of the Resource-Consumer model are derived from equation (1) and (2) in the case of only one consumer:

$$\frac{dB_0}{dt} = r\left(1 - \frac{B_0}{K}\right)B_0 - a_{10}B_1B_0 \tag{9}$$

$$\frac{dB_1}{dt} = ea_{10}B_0B_1 - (a_{11}(1-e) + c_{11})B_1^2 - d_1B_1 \tag{10}$$

We use adaptive dynamics (Dieckmann & Law, 1996; Metz, Nisbet, & Geritz, 1992) to investigate how warming affects the eco-evolutionary dynamics of the Consumer-Resource module. Two major assumptions are being made: first, evolution occurs at a much longer timescale than ecology and second, mutations are of small amplitude. The first assumption is in contradiction with the idea of evolutionary rescue that implies evolutionary and ecological

dynamics happen on overlapping timescales. We therefore relax this hypothesis when adaptive

dynamics indicate the system is supposed to go extinct (Ferriere & Legendre, 2012). Thanks

to the timescale separation hypothesis, the environment felt by the mutant can be clearly defined

as the trait of the resident population associated with ecological equilibrium (B_0^*, B_1^*) whose

expression can be found in the supporting information (Appendix S1 A.1).

In the case of body mass (m_1) evolution, the relative fitness $w(m_1^{mut}, m_1^{res})$ of a rare

mutant corresponds to its growth rate in the resident population:

$$w(m_1^{mut}, m_1^{res}) = \left[\frac{1}{B_1^{mut}} \frac{dB_1^{mut}}{dt} \right]_{B_0^*, B_1^*, B_1^{mut} \ll B_1^*} \tag{11}$$

243 Positive values of *w* indicate that the mutant frequency increases within the resident population,

244 eventually replacing it. The sequence of trait substitutions describes the evolutionary dynamics

245 of the system. Such a sequence can be approximated by the canonical equation (Dieckmann &

246 Law, 1996):

$$\frac{dm_1}{dt} = CN_1^* \mu \sigma^2 \left[\frac{\partial w(m_1^{mut}, m_1^{res})}{\partial m_1^{mut}} \right]_{m_1^{mut} = m_1^{res} = m_1} \tag{12}$$

247 where C, N_1^*, μ, σ are respectively a homogenizing constant, the resident density at ecological

248 equilibrium, the mutation rate and the amplitude of mutations. The zeros of equation (12) are

249 called evolutionary singularities, m_1^*, and verify:

$$\left[\frac{\partial w(m_1^{mut}, m_1^{res})}{\partial m_1^{mut}} \right]_{m_1^{mut} = m_1^{res} = m_1} = 0 \tag{13}$$

A singularity is convergent when evolutionary dynamics starting close to it select traits that are

even closer and it is invasible when, being at the singularity, nearby mutants can invade (Geritz

et al., 1998). The conditions for invasibility and convergence can be determined based on the

sign of the second derivatives of the invasion fitness function (11) (see Appendix S1 B.2.d). A

non-invasible and convergent evolutionary singularity is a Continuously Stable Strategy (CSS,

Eshel, 1983), which corresponds to an evolutionary endpoint (i.e., evolution stops at the

selected singularity). An invasible and convergent singularity is called a Branching Point and

may lead to a stable dimorphism in the population (i.e., emergence and coexistence of two

consumer populations that differ in their body mass, due to disruptive selection, (Geritz et al.,

1997)). An invasible and divergent singularity is called a Repellor.

We first fix the consumer feeding center and let only its body mass evolve, then, we fix the

consumer body mass and let the feeding center evolve. Finally, we give a glimpse on what can

happen when both traits coevolve. By analyzing how the pattern of evolutionary singularities

changes with temperature, we discuss the maintenance of the trophic module confronted with

warming.

266   **Community Evolution Model**

267   We let the Consumer-Resource module evolve (both body mass and feeding center)

beyond the first diversification, which leads to the emergence of a multi-trophic network

(Allhoff et al., 2015; Loeuille & Loreau, 2005). Simulations start with a single ancestor morph

with body mass $m_1 = 100$ and feeding center $f_1 = 1$ feeding on the external resource with

body mass $m_0 = 1$. We define a mutational frequency μ so that a mutation event takes place

every $1/\mu$ time steps. At each mutation event, a parent morph i is chosen randomly and

proportionally to its population density. The mutant j's traits are set as follows: $log10(m_j)$

(resp. $log10(f_j)$) is randomly selected from a normal distribution of mean $log10(m_i)$ (resp.

$log10(f_i)$) and variance σ^2. The value of σ is chosen (0.25) so that occasional big mutational

steps are allowed: 5% of the mutations result in $m_j < \frac{m_i}{3}$ or $m_j > 3m_i$. The mutant's initial

biomass density B_j is set to ε taken from the parent's biomass density B_i. ε is also the extinction

threshold below which a morph population is removed from to system (Table S1).

279   We define evolutionary rescue at the network scale starting from the most general

280 definition proposed by Ferriere & Legendre (2012): "evolutionary rescue occurs when a

281 population subject to environmental change 'performs better' under the operation of

282 evolutionary processes than without these processes". We adapt the definition by assessing not

283 the performance of a given population but rather the performance of the whole network, given

284 different rules of coevolution and different warming intensities.

285   Performance is measured based on the maintenance of diversity. The most

286 straightforward diversity measure in our model is the trait diversity, i.e. the cardinality of the

287 set of all (m_i, f_i) at a given time. While this is certainly a valuable measure from a functional

288 and structural point of view, a large focus exists in conservation biology on the preservation of

289 species diversity. Because our model ignores genetic details and focuses on phenotypes, the

290 definition of species is notoriously tricky. For lack of better criteria, we define species as

291 clusters in the phenotypic space (using the silhouette method (Rousseeuw, 1987) on the k-

292 means algorithm, Appendix S2).

293   Diversity is measured before and after warming. Due to the constant mutation/extinction

294 events, diversity measures are subject to stochastic variations. Therefore, we need a criterion to

295 assess whether the system has reached a 'stable state' before and after disturbance. We consider

296 the system to be stable when the ratio between the trait diversity standard deviation and mean

297 (coefficient of variation) over a time window of 2.10^6 mutation events is below 0.045. Such a

298 criterion allows for at least 95% of the observed trait diversity over this time window to be

299 within the range of $\pm 10\%$ of the observed trait diversity mean over the same period. We are

300 thus not focusing on the transient state following perturbation but rather build our community-

301 scale evolutionary rescue definition on the long-term response of the network to warming.

302   We consider different scenarios with evolution ("scenario E") and without evolution

303 ("scenario NE"). In a "scenario NE μ", the network is built up with a mutation frequency μ.

The first time stability is reached, evolution stops ($\mu = 0$). When a new equilibrium is reached

(pre-disturbance state), warming occurs and we reach a third equilibrium situation (post-

disturbance state). This separation in two steps allows to distinguish the effect of warming from

the effect of competitive exclusion between mutant and resident morphs that would happen

regardless of changes in temperature. Effects of warming are here based on the comparison of

diversity between post and pre-disturbance states. In a "scenario E μ", the network is built up

with a mutation frequency μ. The first time stability is reached, warming occurs. The second

time, the simulation ends.

All in all, we launched 420 simulations, with initial temperature 275 K, with 10 replicas

per condition. Conditions are with versus without evolution (scenario NE or E), mutation

frequency ($10^{-1}, 10^{-2}, 10^{-3}$) and warming intensity (0,2,4,6,8,10 or 20 K). We assess trait and

species diversity maintenance comparing their means before (d_{bef}) and after the warming

(d_{aft}) through their quotient d_{aft}/d_{bef} (hereafter "persistence"). We use statistical models

implemented in R-software to explain this persistence by the different conditions.

Results

Resource-Consumer model

331   For convenience, the following notations are introduced:

$$M_1 \overset{\text{def}}{=} log_{10}(m_1) \qquad\qquad F_1 \overset{\text{def}}{=} log_{10}(f_1) \tag{14}$$

332   Considering the evolution of m_1 (resp. f_1) is thus equivalent to considering the evolution of

333   M_1 (resp. F_1).

How does the selected body mass depend on temperature?

The evolving trait is the consumer body mass M_1 while the feeding center F_1 is fixed

and matches the resource body mass $log_{10}(m0) = 0$.

Two important results arise. First, conditions for $B_1^* > 0$ show that above a critical

temperature $T_{\lim}$, the consumer population goes extinct no matter its body mass so that no

evolutionary rescue is possible (Appendix S1 A.2). Second, using adaptive dynamics, we derive

equation (15) (Appendix S1 B.1.b) and conclude that the body mass at the evolutionary

singularity does not depend on temperature since temperature does not influence any parameter

of this equation.

$$c_0 + a_{11}(M_1^*) * \left(1 - \frac{4(M_1^* - F_1)}{s^2 \, ln(10)} \right) = 0 \tag{15}$$

Fig. 2 is an E^3-diagram showing the dependence of evolutionary singularities on

temperature. The temperature T is represented by the x-axis while the evolving trait is

represented by the y-axes. The curves correspond to the evolutionary singularities and the trait

types and colors indicate their nature. Numerical calculations show there are two singularities,

a repellor and a CSS, whose values do not depend on the temperature in agreement with

equation (15). Above temperature T_{lim} (see Appendix S1 A.2 for an analytical expression), the

net energy flow from resource to consumer is not sufficient to compensate for consumer

mortality losses.

At T_{lim}, the CSS collide with the non-viability area. This is a case of "evolutionary

trapping" (Ferriere & Legendre, 2012). No evolutionary rescue is possible since the CSS is

within the non-viability area no matter the temperature above T_{lim}. Hence, consumer survival

depends on the intensity of warming: a warming from T_1 to T_2 will not threaten the consumer

population while a warming from T_1 to T_3 will make the consumer population go extinct.

359
360

361 **Figure 2:** E³-diagram Body Mass-Temperature. Nature of singularities is given by line type and color. Grey area
362 is the non-viability area (i.e. representing consumer extinction). The arrows indicate the direction of evolutionary
363 trajectories. The temperatures T_1, T_2, T_3 are used to discuss the impact of warming. Parameters values used are
364 the ones in table S1.

How does the selected foraging strategy depend on temperature?

The feeding center F_1 now evolves while the body mass M_1 is fixed (0 for small masses and 2 for big masses). As detailed in Appendix S1 (B.2.b), evolutionary singularities F_1^* verify:

$$(M_1 - F_1^*)a_{11}B_1^* - (F_1^* - M_0)a_{10}B_0^* = 0 \tag{16}$$

Given equation (16), we prove analytically that at least one evolutionary singularity exists and that all singularities verify $M_0 < F_1^* < M_1$ (Appendix S1 B.2.b).

For convenience, we define:

$$\alpha \epsilon \mathbb{R} \text{ such as: } F_1 \stackrel{\text{def}}{=} \alpha M_0 + (1-\alpha)M_1 \qquad L \stackrel{\text{def}}{=} \frac{M_1 - M_0}{s} \tag{17} \tag{18}$$

α characterizes the consumer foraging strategy. Such strategies vary between two extreme scenarios: $\alpha = 1$ (the consumer specializes on the resource) and $\alpha = 0$ (the consumer specializes on cannibalism). Equation (17) being a bijection, considering the evolution of F_1 is equivalent to considering the evolution of α.

Since $B_0^*(T)$ and $B_1^*(T)$ depend on the temperature, equation (16) indicates the latter has an impact on the value of the evolutionary singularities α^* and hence the evolutionary dynamics. Equation (16) is rewritten separating the temperature T and the singularity α^* which allows for further insights regarding the effects of warming. The mathematical analysis (Appendix S1 B.2.c) shows there are four possible scenarios corresponding to four different qualitative outcomes of warming. Scenarios are presented in table 1, with their biological meaning. The scenarios differ in the consumer to resource body mass ratio (parameter L) and/or consumer body mass M_1. For a given value of M_1, we note $L^c(M_1)$ the critical value of L at which the change of outcome occurs.

391  When the ratio between consumer and resource body masses is small (scenario A),

392 warming leads to a CSS that increases with temperature (**Fig. 3a**). Selected foraging strategies

393 therefore increasingly rely on resource consumption under warming.

394  For intermediate values of consumer to resource body mass ratio (scenario B), in

395 addition to increasing the selected α, warming also changes the nature of the singularity from

396 branching point to CSS (**Fig. 3c**). To further investigate the consequences of branching events

397 happening at low and intermediate temperatures, we simulated the corresponding eco-

398 evolutionary dynamics. Diversification leads to the emergence of a new upper trophic level as

399 presented in **Fig. 4**. The new module is a three-level food chain consisting of a consumer

400 specialized in its resource ($\alpha \simeq 1$ corresponding to the first trophic level and another consumer

401 with the same body mass specialized in the former consumer and himself ($\alpha \simeq 0$). Warming

402 from T_1 to T_2 on **Fig. 3a** therefore reduces diversity by impeding the branching (3-level food

403 chain) and replacing it by a CSS (2-level food chain).

404  Finally, consider scenarios with big consumer to resource body mass ratios. At small

405 consumer masses (scenario C), three singularities exist for low temperatures: a branching point,

406 a repellor and a CSS. Branching point and CSS values increase with temperature while the

407 value of the repellor decreases with temperature. The branching point and the repellor

408 eventually collide, letting the system with only a CSS close to one: a consumer specialized on

409 its resource (**Fig. 3f**). Again, we investigated the branching outcome: the emerging food chain

410 is qualitatively similar to the one obtained after branching in the case of intermediate CR body

411 masses ratio (**Fig. 4b**). At large masses (Scenario D, **Fig. 3h**), the system is characterized by an

412 increasing CSS with warming.

In all scenarios, a limit temperature T_{lim} exists (see Appendix S1 A.2 for an analytical

expression) above which the consumer population cannot survive whatever the value of the

foraging trait α. No evolutionary rescue can happen for temperatures above T_{lim} .

If the final temperature is below T_{lim}, evolutionary rescue is always possible. In all four

scenarios, consumers might adapt fast enough to the new conditions and avoid extinction.

For instance, at small CR body mass ratios (**scenario A, Fig. 3a**), a warming from T_1 to

T_3 leads the system to enter the non-viability area (black circle). The food chain can however

persist if evolutionary rescue occurs (white arrow) as larger values of α are selected and

eventually allow consumer survival. Such an outcome is simulated on **Fig. 3b**: warming (sun

symbol) provokes the collapse of the consumer population (black curve) but if α increases fast

enough (grey curve), population recovery occurs. Note that given the very small population size

after disturbance, in nature, the consumer population may still go extinct due to demographic

stochasticity or genetic drift (not accounted for here).

It should nevertheless be noted that if warming is progressive from T_1 to T_2 first and

then from T_2 to T_3 (**Fig. 3a**), the consumer population is never endangered as the trait remains

close to the CSS curve, with a small lag: progressive warming decreases extinction risk, in

accordance with both theoretical (Gomulkiewicz & Holt, 1995) and empirical works (Bell &

Gonzalez, 2009). This is true for all 4 scenarios.

Rescue is not the only evolutionary process constraining diversity and persistence.

Consider intermediate body mass ratios (scenario B, **Fig. 3c**), and a warming from T_1 to T_2

occurring just before (**Fig. 3d**) or long after branching (**Fig. 3e**). Because branching changes

the food chain structure (**Fig. 4b**), the time of warming matters. Warming before branching

leads to potential evolutionary rescue as described above (**Fig. 3d**). Warming occurring after

branching is however less threatening as the consumer population now occupies two ecological

niches unlikely to be simultaneously destroyed by warming (**Fig. 4b**). Note that the higher

branch (**Fig. 4a**) has a trait value close to the strategy selected under warmed conditions. **In Fig. 3e,** such a scenario is simulated: population of the top trophic level (low branch) decreases while the population of the intermediate consumer (high branch) increases under warming; total consumer biomass increases. Branching therefore allows a preadaptation in scenario C.

Under scenario C, warming can be responsible for crossing eco-evolutionary tipping points (here around 300 K). A warming from T_1 to T_2 (**Fig. 3f**) utterly modifies the number of convergent singularities. At T_1, the system is characterized by the existence of three singularities (CSS, repellor and branching point) while at T_2 only the CSS still exists. Consequently, if initially the system is in the branching attraction area, such a warming would be responsible for a reduction of diversity, as the system moves from a polymorphic situation (branching) to a monomorphic situation (CSS). While similar to scenario B, a major difference exists. Given the convergent properties of the CSS, decreasing temperature will not recover the initial diversity, as phenotypes remain at the CSS. This is a case of evolutionary hysteresis. We simulated such a scenario in **Fig. 3g**: starting near branching, the system is warmed from T_1 to T_2 (sun symbol), note that the branching is lost. When the system is cooled back to T_1 (snowflake symbol), it remains monomorphic.

Warming impacts the CR evolutionary dynamics: the 4 possible scenarios

Scenario	CR body mass ratio	Consumer body mass	Mathematical significance	Qualitative outcomes
A	Small	Any	$L \ll L^c(M_1)$	• One singularity: a CSS that increases with temperature
B	Intermediate	Any	$L < L^c(M_1)$ $L \approx L^c(M_1)$	• One singularity whose value increases with warming • Warming switches the selection regime from disruptive (BP) to stabilizing (CSS)
C	Big	Small	$L > L^c(M_1)$	• Three singularities (CSS, Repellor and BP) for low temperatures and only one (CSS) for high temperatures • Evolutionary hysteresis
D		Big		• One singularity: a CSS that increases with warming

467 **Table 1:** The four scenarios with their biological and mathematical significances. $L^c(M_1)$ is the value of L,

468 determined numerically and M_1-dependant, around which the behavioral change occurs. CR stands for

469 Consumer-resource and BP for branching point.

Figure 3: a-c-f-h E³ diagrams corresponding respectively to scenarios A, B, C and D (**Table 1**). Grey areas are non-viability regions; the curves indicate the evolutionary singularities with the type of lines indicating their nature. Thin black arrows indicate the direction of evolution and the big white arrows indicates potential evolutionary rescues. **b-d-e** Model simulations with evolutionary rescue occurring (sun symbols indicate warming times). Evolutionary rules: $\mu = 10^{-2}$ and small mutational steps: $5\% f_1$. **g-** Model simulation with evolutionary hysteresis occurring (sun: warming; snowflake: cooling). Evolutionary rules: $\mu = 10^{-2}$ and small mutational steps: $1\% f_1$. All parameters values are the ones found in **table S1.**

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Figure 4: Branching point issued from numerical simulation of the Consumer-Resource model with
consumer body mass $m_1 = 100$, intermediate CR body masses ratio $L = 2.2$ and temperature $T = 275K$.
Mutation rate of 10^{-2} and mutational steps of $\pm 1\%$. **a. Trait alpha through time**: branching point occurring
for $\alpha \simeq 0.5$ and time $t_1 \simeq 10^6$. **b. Schematic view of the trophic network before and long after**
branching. The arrows thickness indicates the intensity of trophic interactions. The size of each circle is
proportional to the biomass density of the morph it represents. The branching leads to the emergence of a
new upper trophic network and intensifies trophic interactions within the network. Values for other
parameters are in **table S1.**

Co-evolution of body mass and foraging strategy under warming

While the results presented so far provide a first understanding of evolutionary responses

to warming, they have to be considered with caution in the context of complex coevolving

networks. Especially, the separation between body mass and feeding center evolution is utterly

artificial: in nature, both coevolve and exert selective pressures one on another. We simulated

two coevolutionary scenarios that differ in the mutational frequencies (10^{-2} for both traits in

scenario 1 and 10^{-2} (resp. 10^{-3}) for body mass (resp. feeding center) in scenario 2 and a warming

from 280 to 316.4 K (slightly below T_{lim})). A graphical visualization of the evolutionary

trajectories is available in the supporting information (Fig. S1). Evolutionary rescue occurs and

enables food chain persistence. Note two important results. First, the two scenarios yield

different evolutionary trajectories. The relative speed of evolution between the two evolving

traits therefore affect evolutionary dynamics, possibly constraining evolutionary rescue.

Second, while we would have expected no evolution of body mass under warming from the

previous analysis, such an evolution occurs when we allow coevolution. Both body mass and

feeding center are affected by warming (squares and triangles trajectories in **Fig. S1a**).

Evolutionary rescue trajectories (**Fig. S1b**) then involve the effective evolution of both traits.

498   **Evolutionary rescue at the community scale**

We now compare the diversity response within multi-trophic networks with and without

evolution (**Fig 5**). We assume a 20K warming. While this value is beyond current predictions

of temperature change, we choose it for illustrative purposes. Results remain qualitatively

similar, though less impressive, for smaller warmings. In both cases, warming is responsible

for a loss of trait and species diversity (**Figs 5a&d).** The drop is of comparable sizes but in

evolutionary scenarios, we observe a recovery so that the final state is as diverse as before the

perturbation. Upper trophic levels suffer most (**Figs 5b&e**). The whole fourth trophic level goes

extinct, but recovers if evolution is allowed. Large body masses and/or large feeding centers,

mainly corresponding to upper trophic levels, are primarily affected (**Figs 5c&f**). Some of them

go extinct; others evolve. Small body masses and/or small feeding centers are less affected

(**figures 5c&f**). Evolution seems to eventually rescue the network.

We now analyze how warming intensity and evolutionary potential affect the rescue

process. Boxplots are shown in **figures 5g&h**: as expected, the bigger the temperature

difference, the higher the diversity loss. This is particularly obvious in the absence of evolution.

Persistence is higher when evolution is allowed, especially at high levels of warming.

For each mutational frequency, we fitted two ANCOVAs to link trait or species diversity

persistence with evolutionary scenarios (evolution vs no evolution) and warming intensity. All

details are presented in Appendix S2 and **table S2** summarizes the whole statistical analysis

results. For brevity, we present here the main conclusions on trait diversity persistence when

mutational frequency is $\mathbf{10^{-2}}$. Results for other evolution rates and diversity measures are

qualitatively similar.

The model explains more than half of the observed variance. Evolution has a positive

effect on diversity persistence. This effect is both direct ($Evolution_{Effect} = 0.06$, $p_{value} =$

0.0204, $Var_{explained} = 28.4\%$) and indirect, increasing in importance with warming

($Interaction_{Effect} = 0.017, p_{value} < 10^{-8}, Var_{explained} = 9.5\%$). The intensity of warming

has a negative effect on diversity persistence when no evolution occurs ($Warming_{Effect} =$

$-0.024, p_{value} < 2.10^{-16}, Var_{explained} = 30.3\%$). With evolution, warming effects are still

significantly negative but much less pronounced ($Warming_{Effect} + Interaction_{Effect} =$

$-0.07, p_{value} < 10^{-3}$). Note that for other evolution rates, this effect is no longer significantly

different from 0 indicating evolution totally mitigates diversity losses in these cases.

Evolutionary rescue therefore clearly acts at the community scale.

An additional ANCOVA was fitted to assess whether the evolution rate (i.e. the

mutational frequency) has an effect on diversity persistence (Appendix S2). Here, the

explanatory variables are the evolution rate (3 levels: $10^{-1}, 10^{-2}, 10^{-3}$) and the warming

intensity. No significant effect of the evolution rate has been detected for both diversity

measures ($Evolution\ Rate\ _{Effect}: p_{value} > 0.12$; $Interaction_{Effect}: p_{value} > 0.08$). Using

the Mann–Whitney U test for each warming scenario, we confirmed this result by a pairwise

comparison between the different evolution rates.

Figure 5: Diversity response to warming. a-b-c: Diversity and evolutionary dynamics of a simulation where evolution has been stopped (prohibition symbol indicates stopping time) before warming from 275 K to 295 K (sun symbol); **d-e-f:** Same outputs with ongoing evolution ($\mu = 10^{-2}$) at warming. **g-h:** Boxplots of trait and species diversity persistence according to different warming intensities for scenarios with or without evolution. Each box corresponds to 10 independent replicas. See Table S1 for parameters values.

Discussion

In this study, we investigated the effect of global warming on food webs, incorporating evolution explicitly into our framework. Our results suggest that evolution can mitigate warming effects through two main mechanisms: evolutionary rescue and preadaptation. Nevertheless, warming is still expected to negatively impact trophic networks, as evolutionary trapping is always observed above a critical temperature. Consistent with previous works, we also find that higher trophic levels are more prone to extinction (Binzer et al., 2012; Weinbach et al., in prep). Warming can even reduce the potential for diversification with long-lasting consequences for the recovery of the trophic community. Our results have important implications for the conservation and the management of networks in the context of global change.

Evolution can help consumers to persist as shown within our consumer-resource framework. For instance, the evolution of foraging strategies leads to higher resource consumption and lower cannibalism under warming conditions. Provided such an adaptation occurs fast enough, it may allow the consumer maintenance (see white arrows in **Fig. 3**). In such instances, evolution allows higher energy acquisition, balancing the increase in metabolic requirements due to warming. Changes in foraging strategies in response to climate change have also been documented in nature. For instance, in the United Kingdom, Pateman et al. (2012) showed that the butterfly *Aricia agestis*, originally a specialist of *Helianthemum nummularium* as a larval host plant, now also largely use *Geranium molle,* the latter being more abundant in warmer climates. This evolutionary adaptation (Buckley, Butlin, & Bridle, (2012) showed a genetic basis) from specialism to generalism not only permitted *A.agestis* to survive but to extend its geographical range.

Next to evolutionary rescue, our work reveals another evolutionary process favoring persistence: preadaptation. Preadaptation happens when evolutionary dynamics allow the

system to reach a pre-warming state that, once warming happens, will be resistant to the

temperature change. Branching and dimorphism (or polymorphism) here favor preadaptation,

by allowing the emergence of contrasted strategies. Pre-warming evolutionary dynamics then

lead to a system in which a variety of niches are occupied, with a higher likelihood that some

of these selected strategies are resistant to temperature changes, a mechanism akin to the

insurance hypothesis (Yachi & Loreau, 1999).

Despite these positive effects (rescue and preadaptation), evolution does not

systematically save the consumer population. Above a critical warming, evolutionary trapping

is inevitable. Evolution, no matter how fast, then cannot prevent the consumer extinction when

disturbances are large enough. In some cases, evolutionary dynamics even constrain persistence

or recovery. Indeed, warming can change the selection regime from disruptive to stabilizing,

reducing polymorphism and diversification potentials (as shown in **fig. 3c**). If we are to

conserve biodiversity, fostering conditions under which eco-evolutionary processes maintain

genetic diversity and facilitate speciation should also be preserved. Losing such processes

potentially has major consequences as biodiversity loss may deteriorate ecosystem functioning

and stability (see Hooper et al. (2005) for a review). In particular, less diverse ecosystems are

expected to be more vulnerable to other disturbances such as pests (Wasilewska, 1995) or

invasive species (Tilman, 1997). Amplifying interactions between different drivers of global

change have already been observed and research needs to further investigate such scenarios

(Brook, Sodhi, & Bradshaw, 2008). As conservation ecology moves to focus less on diversity

patterns and more on preserving processes (Smith, Bruford, & Wayne, 1993), our results point

out how interactions between warming and evolution (or coevolution (Northfield & Ives,

2013)) should be carefully assessed.

Our results have important implications for conservation and restoration policies.

Consequences of warming can be long lasting, difficult to predict and to reverse. Our results

for instance suggest the possibility of evolutionary hysteresis **(Figs 3f&g)**. In our case, warming

led to a switch from a dimorphic (branching) to a monomorphic (CSS) situation. Convergent

properties then prevent branching recovery even after cooling the system. While such tipping

points are often described based on the ecological state of the system (Suding & Hobbs, 2009),

evolutionary dynamics may hasten or delay ecological tipping points (Dakos et al., 2019). In

our case, tipping points in fact emerge from the presence of alternative convergent strategies.

In a context of global change where selective pressures are likely strong, our work suggests that

related eco-evolutionary dynamics can lead to tipping points.

In our multi-trophic networks, warming impact is trophic-level dependent, upper trophic

levels being more likely to go extinct. Because the ratio of ingestion to metabolism decreases

with warming (Binzer et al., 2012), energy flows across food chains do not compensate

increased respiration rate under warming. Nevertheless, evolution in such networks can help to

restore biodiversity in the long-term. Statistical analysis shows this positive effect is both direct

and indirect, increasing with warming (**figure 5g&h, table S2**) with important consequences

for our prediction of future diversity. For instance, this suggests that predictions based on

models that ignore evolutionary dynamics may be increasingly inaccurate when warming is

expected to be intense (IPCC, 2014). This emphasizes the limits of the widely used climatic

niche models and the need for a better integration of evolution and species interactions into the

predictive frameworks on which decision makers rely.

It is rather surprising that evolutionary rescue applies, within our framework, both at the

consumer-resource and at the community scale. We would have expected, given the many

indirect effects within complex networks, evolutionary rescue may not apply so well (Loeuille

2019). Our hypothesis is that, by allowing big mutational steps, we may have facilitated rescue

processes. Future work is needed to investigate how the mutational regime impacts the role of

evolution in maintaining diversity at the community scale.

That being said, we emphasize that such network-level rescues need to be considered

with caution, as does the optimistic view that evolution eventually restores trophic networks.

First, before the network recovery, the transient state, characterized by a very fragile

community, is likely unable to support proper ecosystem functioning and associated services.

While we assume populations go extinct when very low, we do not explicitly model ecological

effects happening at such low levels (Allee effect, demographic stochasticity…). Therefore, we

might be overestimating the transient state diversity and ignoring cases of total collapse, in

which evolution would have no effect on diversity persistence since there is no diversity left

for evolutionary processes to act on. Large density variations and large extinction rates such as

the ones we observe in the transient state may also impact the evolution of other interactions

we do not model here. For instance, it has been argued that evolutionary deterioration of

mutualistic interactions may happen under global change (Toby Kiers et al., 2010), further

impacting ecological networks. Finally, our approach of species diversity is clearly

oversimplified as we do not model speciation explicitly, but rather phenotypic differentiation.

As speciation also requires time for genetic isolation among incipient species to happen, we are

likely underestimating the time needed for species diversity to recover in real systems. Apart

from that, we only analyzed diversity but we did not investigate whether warming leaves a

fingerprint on community structure and composition (see Tylianakis et al., 2008).

Our results may be difficult to test empirically, as separating warming from other effects

of global changes in empirical datasets is not straightforward. Microcosm or mesocosm

experiments are more appropriate. Trophic network consisting of rotifer-alga (Yoshida et al.,

2003) or phytoplankton-zooplankton-fish (Hulot et al., 2000) are already being used for

experimental evolution. The main challenge is to be able to modulate evolutionary speed. This

necessitate cooperation between geneticists and evolutionary biologists (Kokko et al., 2017).

Manipulating the number of clones can allow experimental manipulation of evolutionary

potential (e.g. Yoshida et al., 2003). Several of our results could then be tested: the positive

effect of evolution on the long-term diversity persistence, the differential impact of warming

on trophic levels or the selection toward less cannibalism and more resource consumption due

to warming as many zooplankton species are cannibalistic.

All in all, our work reveals a potential for evolutionary rescue and preadaptation in food

webs confronted with global warming. This optimistic message is however to be modulated by

the fact that above a certain temperature, evolution does not act efficiently and because transient

collapse is still expected. The variety of possible responses to warming that we observed in our

study furthermore suggests that the consequences of warming for a specific ecosystem might

be more than challenging to forecast, since too many factors play a role in shaping these eco-

evolutionary dynamics. In general, we can only predict that these consequences can be dramatic

and long lasting, potentially threatening ecosystem stability and functioning with large

consequences for human populations. Conservation and biodiversity management policies

should integrate such evolutionary components to better address the issues raised by global

changes.

Acknowledgement:

This work was funded by the National Research Agency of France (ANR) as part of the ARSENIC project (ANR grant no. 14-CE02-0012). We also would like to thank the HPCave computational facilities of Sorbonne Université on which the simulations were run.

Author's contributions statement:

NL, KTA and YY conceived the ideas and designed methodology. The analysis was conducted by AW (body mass evolution) and YY (feeding center evolution and coevolution of body mass and feeding center). Concerning the community evolution model, KTA developed the code used and adapted by YY to do the simulations. Results analysis was done by NL, KTA and YY. YY wrote the first draft of the manuscript that has been subsequently reviewed and edited by NL and KTA. All authors contributed critically to the drafts and gave final approval for publication.

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Supporting Information

Table S1: List of all the parameters and variables in the model with biological

significance, values and units.

Appendix S1: Eco-evolutionary dynamics of the Consumer-Resource model

Appendix S2: Community Evolution Model: species diversity and statistical

analysis

Fig. S1: Consumer Resource model: the two-trait coevolution scenario

Table S2: Diversity persistence in the community evolution model according to

evolutionary scenarios: statistical analysis summary

Table S1: List of all the parameters and variables in the model with biological
significance, values and units.

Variables and parameters		Biological meaning	Value	Unit
Variables	B_i	Biomass density of morph i		$Kg.m^{-2}$
	m_i	Body mass of morph i		Kg
	f_i	Feeding center of morph i		Kg
Temperature parameters	T	Temperature	$[275,320]$	K
	T_0	Reference temperature of empirical parameters	293	K
	E	Energy of activation	0.65	eV
	k	Boltzmann Constant	$8.617\ 10^{-5}$	$eV.K^{-1}$
Ecological parameters	r_0	Growth rate at temperature T_0	1	s^{-1}
	$r(T)$	Growth rate of external resource		s^{-1}
	K_0	Carrying capacity at temperature T_0	10	$Kg.m^{-2}$
	$K(T)$	Carrying capacity of external resource		$Kg.m^{-2}$
	d_0	Mortality rate scaling term at temperature T_0	0.3	$Kg^{0.25}.s^{-1}$
	$d_i(T)$	Mortality rate of morph i		s^{-1}
	e	Conversion efficiency	0.85	1
	c_0	Intra-morph competition	0.15	$m^2.s^{-1}$
	c_{ij}	Competition between morph i and morph j		$Kg^{-1}.m^2.s^{-1}$
	a_{ij}	Attack rate of morph i on morph j		$Kg^{-1}.m^2.s^{-1}$
	s	Feeding range or width of consumption niche	0.25	Kg
	ε	Extinction threshold	$2\ 10^{-6}$	$Kg.m^{-2}$
Evolutionary parameters	μ	Mutational frequency	$(10^{-1}, 10^{-2}, 10^{-3})$	s^{-1}
	σ	Mutational amplitude	0.25	Kg

Table S1: Parameter dependence on temperature is explicitly indicated. The values are the ones used in the numerical simulations except explicit mention.

Appendix S1: Eco-evolutionary dynamics of the Consumer-Resource model

This document is dedicated to the analysis of the consumer-resource eco-evolutionary dynamics. Throughout this analysis, the equations presented in the main document are proved. In order to avoid confusion, these equations are indexed using the same number used in the main document while new equations are indexed using a letter. The same applies for figures or tables. Important equations are framed.

A. Ecological dynamics:

Population dynamics are given by the following equations:

$$\frac{dB_0}{dt} = r\left(1 - \frac{B_0}{K}\right)B_0 - a_{10}B_1B_0 \tag{9}$$

$$\frac{dB_1}{dt} = ea_{10}B_0B_1 - (a_{11}(1-e) + c_{11})B_1^2 - d_1B_1 \tag{10}$$

1. Ecological equilibriums:

Ecological equilibria are given by resolving $\frac{dB_0}{dt} = 0$ and $\frac{dB_1}{dt} = 0$ which leads to 4 possible solutions:

$$\{B_0^* = 0 \; ; B_1^* = 0\}$$

$$\{B_0^* = K \; ; B_1^* = 0\}$$

$$\left\{B_0^* = 0 \; ; B_1^* = \frac{-d_1}{(a_{11}(1-e) + c_{11})}\right\}$$

$$\left\{B_0^* = K\left(1 - \frac{a_{10}\left(ea_{10} - \frac{d_1}{K}\right)}{\frac{R}{K}(a_{11}(1-e) + c_0) + ea_{10}^2}\right) ; B_1^* = \frac{R(ea_{10} - \frac{d_1}{K})}{\frac{R}{K}(a_{11}(1-e) + c_0) + ea_{10}^2}\right\}$$

The third equilibrium is not reachable because $B_1^* < 0$. The fourth equilibrium is called the coexistence equilibrium.

2. Condition for coexistence and expression of T_{lim}:

The coexistence equilibrium is possible when $B_0^* \geq 0$ and $B_1^* \geq 0$.

Fist, we demonstrate: $\forall T \geq 0, B_0^* \geq 0$ at the coexistence equilibrium.

$$B_0^* > 0 \Leftrightarrow \frac{R}{K}(a_{11}(1-e) + c_{11}) + ea_{10}^2 - a_{10}\left(ea_{10} - \frac{d_1}{K}\right) > 0$$

$$\Leftrightarrow \frac{R}{K}(a_{11}(1-e) + c_{11}) + a_{10}\frac{d_1}{K} > 0$$

The last relation is always true which ends the proof. For B_1^* we have:

$$B_1^* \geq 0 \Leftrightarrow ea_{10} \geq \frac{d_1}{K} \Leftrightarrow \frac{e}{\sqrt{2\pi}s}\exp\left(-\frac{(F_1 - M_0)^2}{2s^2}\right) \geq \frac{d_0}{K_0}\exp\left(\frac{-2E(T_0 - T)}{kTT_0}\right)$$

$$\Leftrightarrow \frac{(F_1 - M_0)^2}{2s^2} \leq \ln\left(\frac{eK_0}{\sqrt{2\pi}sd_0}\right) + \frac{2E(T_0 - T)}{kTT_0}$$

$$\Leftrightarrow T \leq \frac{2E}{\frac{2E}{T_0} + k\left[\frac{(F_1 - M_0)^2}{2s^2} + \ln\left(\frac{\sqrt{2\pi}sd_0}{eK_0}\right)\right]}$$

In the case of body mass evolution, $F_1 = M_0$ so that the condition for coexistence can be rewritten:

$$T \leq \frac{2E}{\frac{2E}{T_0} + k\left(\ln\left(\frac{\sqrt{2\pi}sd_0}{eK_0}\right)\right)} \stackrel{\text{def}}{=} T_{lim} \tag{B}$$

With the standard parameter values presented in table S1, $T_{lim} \approx 316.4$ K.

In the case of feeding center evolution, F_1 can evolve. The condition for coexistence can be rewritten:

$$\frac{(F_1 - M_0)^2}{2s^2} \leq \ln\left(\frac{eK_0}{\sqrt{2\pi}sd_0}\right) + \frac{2E(T_0 - T)}{kTT_0}$$

$$\Leftrightarrow \left(\begin{array}{c} \ln\left(\frac{eK_0}{\sqrt{2\pi}sd_0}\right) + \frac{2E(T_0 - T)}{kTT_0} \geq 0 \\ -\sqrt{2s^2\left[\ln\left(\frac{eK_0}{\sqrt{2\pi}sd_0}\right) + \frac{2E(T_0 - T)}{kTT_0}\right]} \leq F_1 - M_0 \leq \sqrt{2s^2\left[\ln\left(\frac{eK_0}{\sqrt{2\pi}sd_0}\right) + \frac{2E(T_0 - T)}{kTT_0}\right]} \end{array} \right)$$

$$\boxed{\left(\begin{array}{c} T \leq \dfrac{2E}{\dfrac{2E}{T_0} + k\left(\ln\left(\dfrac{\sqrt{2\pi}sd_0}{eK_0}\right)\right)} \stackrel{def}{=} T_{lim} \\ -\sqrt{2s^2\left[\ln\left(\dfrac{eK_0}{\sqrt{2\pi}sd_0}\right) + \dfrac{2E(T_0 - T)}{kTT_0}\right]} \leq F_1 - M_0 \leq \sqrt{2s^2\left[\ln\left(\dfrac{eK_0}{\sqrt{2\pi}sd_0}\right) + \dfrac{2E(T_0 - T)}{kTT_0}\right]} \end{array} \right) \quad (C)}$$

B. Evolutionary dynamics:

966 Reminder: we define M_0 , M_1 , F_1 by $M_1 \stackrel{def}{=} log_{10}(m_1)$ $F_1 \stackrel{def}{=} log_{10}(f_1)$

1. How does the selected body mass depend on temperature?

a. Fitness of invasion

Here, we expose the whole approach to determine the invasion fitness function in the adaptive

dynamics' framework:

We consider a rare mutant 1' appearing in the resident population 1, given the population

dynamics equations, we have:

$$\frac{1}{B_{1'}}\frac{dB_{1'}}{dt} = \underbrace{ea_{1'0}B_0}_{ressource} - \overbrace{(c_{1'1'}B_{1'} + c_{1'1}B_1)}^{competition} + \underbrace{(ea_{1'1}B_1 - a_{11'}B_1 - (1-e)a_{1'1'}B_{1'})}_{comsumption\ interaction\ between\ 1\ and\ 1'} - d_{1'}$$

Given 1 and 1' have the same feeding center, we have $c_{1'1'} = c_{1'1} = c_0$ and $a_{11'} = a_{1'1'}$.

Also, $B_{1'} \ll B_1$ (rare mutant). Hence: $c_{1'1'}B_{1'} \ll c_{1'1}B_1$ and $(1 - e)a_{1'1'}B_{1'} \ll a_{11'}B_1$

All in all, we deduce:

980
$$\frac{1}{B_{1'}}\frac{dB_{1'}}{dt} = ea_{1'0}B_0 - c_0B_1 - a_{11'}B_1 + ea_{1'1}B_1 - d_{1'}$$

981 Which leads to:

983
$$w(M_{1'}, M_1) \stackrel{\text{def}}{=} \left[\frac{1}{B_{1'}}\frac{dB_{1'}}{dt}\right]_{\substack{B_{1'} \ll B_1 \\ B_0^*, B_1^*}} = ea_{1'0}B_0^* - c_0B_1^* - a_{11'}B_1^* + ea_{1'1}B_1^* - d_{1'}$$

985 *b. Evolutionary singularities (proof of equation (15))*

987 We first need to determine the derivative of the fitness invasion function with respect to the

988 mutant body mass (first variable):

990
$$\frac{\partial w(M_{1'}, M_1)}{\partial M_{1'}} = eB_0^*\frac{\partial a_{1'0}}{\partial M_{1'}} - \underbrace{\frac{\partial c_0 B_1^*}{\partial M_{1'}}}_{0} - B_1^*\frac{\partial a_{11'}}{\partial M_{1'}} + eB_1^*\frac{\partial a_{1'1}}{\partial M_{1'}} - \frac{\partial d_{1'}}{\partial M_{1'}}$$

991
$$= -\frac{\ln(10)}{4}ea_{1'0}B_0^* - \frac{(F_1 - M_{1'})}{s^2}a_{11'} - \frac{\ln(10)}{4}ea_{1'1}B_1^* - \frac{\ln(10)}{4}d_{1'}$$

993 Thus:

994
$$\left[\frac{\partial w(M_{1'}, M_1)}{\partial M_{1'}}\right]_{M_{1'}=M_1} = -\frac{\ln(10)}{4}(ea_{10}B_0^* + ea_{11}B_1^* - d_1) - \frac{(F_1 - M_1)}{s^2}a_{11}B_1^*$$

996 At ecological equilibrium $\frac{dB_1}{dt} = 0$ gives $ea_{10}B_0^* + ea_{11}B_1^* - d_1 = c_0B_1^* + a_{11}B_1^*$ and hence:

998
$$\left[\frac{\partial w(M_{1'}, M_1)}{\partial M_{1'}}\right]_{M_{1'}=M_1} = B_1^*\left(-\frac{\ln(10)}{4}(c_0 + a_{11}) - \frac{(F_1 - M_1)}{s^2}a_{11}\right)$$

1000 Evolutionary singularities occur when the selection gradient vanishes:

1001

1002 $$\left[\frac{\partial w(M_{1'}, M_1)}{\partial M_{1'}}\right]_{M_{1'}=M_1} = 0 \Longleftrightarrow -\frac{\ln(10)}{4}(c_0 + a_{11}) - \frac{(F_1 - M_1^*)}{s^2}a_{11} = 0$$

1003 $$\Longleftrightarrow c_0 + a_{11}\left(1 - \frac{4(M_1 - F_1)}{s^2\ln(10)}\right) = 0$$

1004 Noting explicitly the dependence of a_{11} on M_1, we deduce evolutionary singularities M_1^* verify:

$$c_0 + a_{11}(M_1^*) * \left(1 - \frac{4(M_1^* - F_1)}{s^2\ln(10)}\right) = 0 \qquad (15)$$

1005

1006 This ends the proof of equation (15).

1007

2. How does the selected foraging strategy depend on temperature?

1008

1009 *a. Fitness of invasion*

1010

1011 Here, the evolving trait is f_1 or equivalently F_1. As we did before, we consider the appearance

1012 of a new morph 1' issued from 1 by a random small mutation.

1013 Given our model, its biomass follows the equation (note that $d_1 = d_{1'}$):

1014

1015 $$\frac{1}{B_{1'}}\frac{dB_{1'}}{dt} = ea_{1'0}B_0 + ea_{1'1}B_1 - (1-e)a_{1'1'}B_{1'} - a_{11'}B_1 - c_{1'1'}B_{1'} - c_{11'}B_1 - d_1$$

1016

1017 Using $B_{1'} \ll B_1$ and $F_1 \approx F_{1'}$ (small mutation hypothesis), we find:

1018

1019 $$\frac{1}{B_{1'}}\frac{dB_{1'}}{dt} = ea_{1'0}B_0 + (ea_{1'1} - a_{11'})B_1 - c_{11'}B_1 - d_1$$

1020

1021 When considered at ecological equilibrium, the previous expression gives the invasion fitness

1022 function:

1023

1024 $$w(F_{1'}, F_1) \overset{\text{def}}{=} \left[\frac{1}{B_{1'}}\frac{dB_{1'}}{dt}\right]_{\substack{B_{1'}\ll B_1 \\ B_0^*, B_1^*}} = ea_{1'0}B_0^* + (ea_{1'1} - a_{11'})B_1^* - c_{11'}B_1^* - d_1$$

1025

1026 *b. Evolutionary singularities (proof of equation (16)):*

1027 When the selection gradient vanishes:

1028

1029
$$\left[\frac{\partial w(F_{1'},F_1)}{\partial F_{1'}}\right]_{F_{1'}=F_1} = 0 \Leftrightarrow \left[eB_0^*\frac{\partial a_{1'0}}{\partial F_{1'}} + eB_1^*\frac{\partial a_{1'1}}{\partial F_{1'}} - B_1^*\underbrace{\frac{\partial a_{11'}}{\partial F_{1'}}}_{0} - B_1^*\underbrace{\frac{\partial c_{11'}}{\partial F_{1'}} - \frac{\partial d_1}{\partial F_{1'}}}_{0}\right]_{F_{1'}=F_1} = 0$$

1030
$$\Leftrightarrow \frac{-eB_0^*}{s^2}a_{10}(F_1 - M_0) - \frac{eB_1^*}{s^2}a_{11}(F_1 - M_1) + \frac{B_1^*}{2s^2}\underbrace{(F_1 - F_1)}_{0}c_0 = 0$$

1031

1032

1033   So that evolutionary singularities F_1^* verify:

1034

$$(M_1 - F_1^*)a_{11}B_1^* - (F_1^* - M_0)a_{10}B_0^* = 0 \qquad\qquad (16)$$

1035

1036   This ends the proof of equation (16).

1037

1038   *Proof of the existence of at least one singularity:*

1039

1040   Here, we proof that equation (16) has at least one solution and that all solutions F_1^*verify: $M_0 <$

1041 $F_1^* < M_1$. For convenience we define:

1042
$$\alpha\ such\ as: F_1 = \alpha M_0 + (1 - \alpha)M_1 \qquad ie: \qquad \alpha \stackrel{\text{def}}{=} \frac{M_1 - F_1}{M_1 - M_0}$$

1043

1044
$$L \stackrel{\text{def}}{=} \frac{M_1 - M_0}{s}$$

1045   Equation (16) can be rewritten with these new notations, by noting that $M_1 - M_0 > 0$

1046   (hypothesis of the model):

$$\alpha \exp\left(\frac{-\alpha^2 L^2}{2}\right)B_1^*(\alpha, T, M_1, L) - (1 - \alpha)\exp\left(\frac{-(1 - \alpha)^2 L^2}{2}\right)B_0^*(\alpha, T, M_1, L) = 0 \qquad (D)$$

1047   We define:

1048 $z(\alpha, T, M_1, L) \stackrel{\text{def}}{=} \alpha \exp\left(\frac{-\alpha^2 L^2}{2}\right)B_1^*(\alpha, T, M_1, L) - (1 - \alpha)\exp\left(\frac{-(1 - \alpha)^2 L^2}{2}\right)B_0^*(\alpha, T, M_1, L)$

1049

We have, $z(0, T, M_1, L) < 0, z(1, T, M_1, L) > 0$ and, given $T, M_1, L \quad z_\alpha: \alpha \to z(\alpha, T, M_1, L)$

is a continuous function. So, by the Intermediate Value Theorem, there exists α^* such that

$z_\alpha(\alpha^*) = z(\alpha, T, M_1, L) = 0$. This is equivalent to say that equation (D) or (16) has at least one

solution. Moreover, under coexistence (i.e. $B_0^* > 0$ and $B_1^* > 0$), if $\alpha \leq$

0 *then* $z(\alpha, T, M_1, L) < 0$ and if $\alpha \geq 1$ *then* $z(\alpha, T, M_1, L) > 0$ so that we have necessarily

$0 < \alpha^* < 1$. This is equivalent to $M_0 < F_1^* < M_1$. This ends the proof.

1056

1057 *c. Effect of warming on feeding center evolution and emergence of 4 scenarios:*

1058

Since do depend on the temperature, equation (16) indicates temperature has an impact

on the singularities and consequently on evolutionary dynamics. In order to go further, we

rewrite (16) replacing B_0^* and B_1^* by their explicit expressions and separating temperature and

evolutionary singularity:

$$equation\ (16) \Leftrightarrow equation\ (D) \Leftrightarrow \frac{B_1^*(\alpha, T, M_1, L)}{B_0^*(\alpha, T, M_1, L)} = \frac{(1-\alpha)\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right)}{\alpha \exp\left(\frac{-\alpha^2 L^2}{2}\right)}$$

$$\Leftrightarrow \frac{ea_{10} - \dfrac{d_\infty 10^{\frac{-M_1}{4}}}{K_\infty}\exp\left(\dfrac{-2E}{kT}\right)}{a_{11}(1-e) + c_0 + \dfrac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}a_{10}} = \frac{(1-\alpha)\exp\left(\dfrac{-(1-\alpha)^2 L^2}{2}\right)}{\alpha \exp\left(\dfrac{-\alpha^2 L^2}{2}\right)}$$

$$\Leftrightarrow e \exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right)$$

$$- \frac{\sqrt{2\pi}s d_\infty}{K_\infty}\exp\left(\frac{-2E}{kT}\right)$$

$$= \left[(1-e)\exp\left(\frac{-\alpha^2 L^2}{2}\right) + \sqrt{2\pi}s c_0 10^{\frac{M_1}{4}}\right.$$

$$\left. + \frac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right)\right]\left[\frac{(1-\alpha)\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right)}{\alpha \exp\left(\frac{-\alpha^2 L^2}{2}\right)}\right]$$

1069

1070 $$\Leftrightarrow e \exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right) - \frac{\sqrt{2\pi}s d_\infty}{K_\infty}\exp\left(\frac{-2E}{kT}\right)$$

1071 $$= (1-e)\frac{(1-\alpha)}{\alpha}\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right) + \sqrt{2\pi}s c_0 10^{\frac{M_1}{4}}\frac{(1-\alpha)}{\alpha}\exp\left(\frac{-(1-2\alpha)L^2}{2}\right)$$

1072 $$+ \frac{(1-\alpha)}{\alpha}\frac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}\exp\left(\frac{-(2(1-\alpha)^2 - \alpha^2)L^2}{2}\right)$$

1073

1074 We define $C(M_1) = \sqrt{2\pi}s c_0 10^{\frac{M_1}{4}}$ and have the equivalence with:

$$\frac{\sqrt{2\pi}s d_\infty}{K_\infty} \exp\left(\frac{-2E}{kT}\right)$$

$$= \frac{-1}{\alpha}\left[(1-\alpha)C(M_1)\exp\left(\frac{-(1-2\alpha)L^2}{2}\right)\right.$$

$$+ (1-\alpha)\frac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}\exp\left(\frac{-(2(1-\alpha)^2 - \alpha^2)L^2}{2}\right)$$

$$\left. + \left[(-\alpha e + (1-e)(1-\alpha))\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right)\right]\right]$$

$$\Longleftrightarrow \frac{\sqrt{2\pi}s d_\infty}{K_\infty}\exp\left(\frac{-2E}{kT}\right)$$

$$= \frac{-1}{\alpha}\left[(1-\alpha)C(M_1)\exp\left(\frac{-(1-2\alpha)L^2}{2}\right) + (1-e-\alpha)\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right) + (1\right.$$

$$\left. - \alpha)\frac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}\exp\left(\frac{-(2(1-\alpha)^2 - \alpha^2)L^2}{2}\right)\right]$$

$$\boxed{\Longleftrightarrow Q(T) = G(\alpha, M_1, L) \qquad\qquad (E)}$$

Where the functions $Q(T)$ and $G(\alpha, M_1, L)$ are defined by:

$$Q(T) \overset{\text{def}}{=} \frac{\sqrt{2\pi}s d_\infty}{K_\infty}\exp\left(\frac{-2E}{kT}\right)$$

$$G(\alpha, M_1, L) \overset{\text{def}}{=} \frac{-1}{\alpha}\left[(1-\alpha)C(M_1)\exp\left(\frac{-(1-2\alpha)L^2}{2}\right) + (1-e-\alpha)\exp\left(\frac{-(1-\alpha)^2 L^2}{2}\right) + (1\right.$$

$$\left. - \alpha)\frac{d_\infty 10^{\frac{-M_1}{4}}}{R_\infty}\exp\left(\frac{-(2(1-\alpha)^2 - \alpha^2)L^2}{2}\right)\right]$$

To investigate the effect of temperature on the evolutionary dynamics, we analyze the functions:

$$T \rightarrow Q(T)$$

$$\alpha \rightarrow G(\alpha, M_1, L)$$

Study of Q as a function of T:

Q is increasing with the temperature T and has the following variation table:

Table A: Table of variation of $\mathbf{Q(T)}$. Note that $\mathbf{Q(T)}$ is always positive.

Study of G as a function of α, M_1 and L seen as parameters:

Analytically we cannot determine all the possible behaviors of G. Thus, we used the software Python to investigate its variation. **Figure A** shows the allure of G for different values of CR body masses ratio.

For small CR body masses ratios, G is increasing with α (fig A a-d). For big CR body masses ratios, G is not monotonous anymore (fig A-c, A-f). There is a value of the parameter L, M_1-dependant, we refer to as $L^c(M_1)$, around which the switch of behavior from monotonous to not monotonous occurs. We can show by studying the derivative $\frac{\partial G}{\partial \alpha}$ that $\forall M_1 \in \mathbb{R}, L^c(M_1) \geq \sqrt{2}$. Its value is around 2.2 for the values of M_1 we investigated ($M_1 = 0$ and $M_1 = 2$). In addition, just before the change of behavior occurs, the curvature of G changes (fig A-b, A-e) meaning its second derivative switches sign. This can have consequences on the nature of the singularities as this nature depends on the second derivatives of the fitness gradient. This led us to consider scenarios with an intermediate CR body masses ratio.

Figure A: Plots of $G(\alpha)$ for various values of L and M_1. CR stands for Consumer-Resource.

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1127 From the previous analysis, we deduce the following table of variation for G:

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Case	α	0	$\alpha_1(M_1, L)$	$\alpha_2(M_1, L)$	1
$L \le L^*(M_1)$	$G(\alpha)$				
$L > L^*(M_1)$	$G(\alpha)$				

Table B: Variation table of function G according to the value of L

If the consumer to resource body mass ratio is small, G is strictly increasing with α so that there
is necessarily only one singularity.

If the ratio between consumer and resource body mass is big, the behavior of G indicates
the existence of up to three singularities. The coexistence condition on temperature (T_{lim})
implies $Q(T) < e$ so that it is impossible to have two singularities (except one degenerate case)
Hence, in this case, there is one or three singularities, depending on
$G(\alpha_1(M_1, L)), G(\alpha_2(M_1, L))$ and T.

Finally, while there is no qualitative difference between the case of a small consumer
body mass and a big one when the CR body mass ratio is either small (fig A-a, A-d) or
intermediate (fig A-b, A-e), there is one in the case it is big. Indeed, in that case, when the
consumer body mass is small $G(\alpha_1) > 0$ (fig Ac) while when the consumer body mass is big
$G(\alpha_1) < 0$ (fig A-f). This is important since $Q(T)$ is always positive (see Table A). This means
there can potentially be 3 singularities in the case of a small consumer body mass while there
is only one in the case of a big consumer body mass.

1144

1145 All this analysis of equation (F) leads to distinguish 4 scenarios that differs in the impact
1146 of warming on feeding center evolution: small CR body mass ratio **(scenario A)**, intermediate
1147 CR body mass ratio **(scenario B)**, big CR body mass ratio and small consumer body mass
1148 **(scenario C)**, big CR body mass ratio and big consumer body mass **(scenario D)**. Those are
1149 the scenarios presented in **Table 1.**

 d. Nature of the evolutionary singularities

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1152   Let F_1^* be the evolutionary singularity.

1153   Non-invasibility corresponds mathematically to:

1154

1155
$$\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}}\right]_{F_{1\prime}=F_1=F_1^*} < 0$$

1156   Moreover, we have the following expression obtained by derivation:

1157

1158
$$\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}}\right]_{F_{1\prime}=F_1} = \frac{e}{s^2} a_{10} B_0^* ((1-\alpha)^2 L^2 - 1) + \frac{e}{s^2} a_{11} B_1^* (\alpha^2 L^2 - 1) + \frac{B_1^* c_{11}}{2s^2}$$

1159   We used this expression to determine numerically trough Python software the invasibility

1160   properties.

1161

1162   Convergence corresponds mathematically to:

1163
$$\left[\frac{d}{dF_1}\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}}\right]_{F_{1\prime}=F_1}\right]_{F_1=F_1^*} < 0$$

1164

1165   In the case of one singularityF_1^*, it can be shown without calculation that the singularity will be

1166   convergent stable.

1167

Proof:

Let $F < M_0$ and $0 < \varepsilon < \frac{M_0 - F}{2}$, we have $w(F + \epsilon, F) > 0$ because the mutant with trait $(F +$

$\epsilon)$ has better attack rates on both the resource and the resident F and experience less competition

with the resident F than experienced by the resident ($c_{11} = c_0$).

Hence, $\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}}_{F_{1\prime}=F_1}\right]_{F_1=F} > 0$.

In a same way, we have: $\forall\ F > M_1$, $\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}}_{F_{1\prime}=F_1}\right]_{F_1=F} < 0$

Moreover, the function:

1175
$$\mathbb{R} \rightarrow \mathbb{R}$$

1176
$$F_1 \mapsto \left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}} \right]_{F_{1\prime} = F_1}$$

1177

1178 is continuous and only vanishes at $F = F_1^* \in \,]M_0, M_1[$.

1179 Hence, $\forall \, F < F_1^*, \left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}} \bigg|_{F_{1\prime} = F_1} \right]_{F_1 = F} > 0,$ $\quad \forall \, F > F_1^*, \left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}} \bigg|_{F_{1\prime} = F_1} \right]_{F_1 = F} < 0$ and

1180 $\left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}} \bigg|_{F_{1\prime} = F_1} \right]_{F_1 = F_1^*} = 0$ thus: $\left[\frac{d}{dF_1} \left[\frac{\partial w(F_{1\prime}, F_1)}{\partial F_{1\prime}} \bigg|_{F_{1\prime} = F_1} \right] \right]_{F_1 = F_1^*} < 0$

1181

1182 In the case of three singularities $F_1^* < F_2^* < F_3^*$, with the same kind of arguments, we know at

1183 least one singularity is convergent stable. It appears numerically that $F_1^* \; and \; F_3^*$ are convergent

1184 stable while F_2^* is not.

1185

1186 This analysis gives the qualitative outcomes of warming for each scenario presented in **Table**

1187 **1**.

1188

Appendix S2: Community Evolution Model: species diversity and statistical analysis

Our community evolution model starts with a consumer feeding on an external resource. The evolution of the consumer body mass and foraging strategy (feeding center) follows a mutation/selection process leading to the emergence of a complex multi-trophic network of approximately 30 to 40 morphs and 4 to 5 trophic levels. A morph corresponds to an adaptive phenotype, that is to say, a couple ($body\ mass, feeding\ center$).

Definition of species diversity:

At first, we only have one straightforward diversity measure: the trait diversity (i.e. the number of morphs in the trophic network at a given time). Because our model ignores genetic details and focuses on phenotypes, the definition of species is notoriously tricky. For lack of better criteria, we define species as clusters in the phenotypic space. The silhouette method (Rousseeuw, 1987) is used to determine the best number of clusters on the k-means clustering algorithm applied on the set of body masses and feeding centers (m_i, f_i) corresponding to the trait diversity. The number of clusters gives what we define as species diversity.

We consider a set of observations and a clustering. For each observation i, we note C_i the cluster to which i is affected. For each observation i, the silhouette width $S(i)$ is a measure of how much the observation is close its cluster C_i in comparison with other clusters. It is defined as follow:

$$S(i) = \frac{b(i) - a(i)}{\max(a(i), b(i))}$$

Where:

- $a(i)$ is the average dissimilarity between i and the other observations in c_i
- $b(i)$ is the average dissimilarity between i and the observations in the closest cluster to i, C, with $C \neq c_i$ (ie $b(i) = \min_C d(i, C)$)

$S(i)$ is between -1 and 1 and the more it is close to 1, the better is the affectation of i in the

1243   clustering. We use $\underset{i}{\text{mean}}\, S(i)$ as a measure of the performance of a given clustering. We

1244   compare the different clustering obtained by the k-means algorithm for different values of k.

1245

1246   We implemented the algorithm in R-software using the packages "Cluster" by Martin

1247   Maechler, Peter Rousseeuw Anja Struyf, Mia Hubert, Kurt Hornik, Pierre Roudier, Juan

1248   Gonzalez (2017).

1249

1250   <u>**Statistical analysis:**</u>

1251   We want to compare the diversity response to warming within our multi-trophic network

1252   with and without evolution.

1253

1254   For trait and species diversity, three models were fitted ($\mu = 10^{-1}, 10^{-2}, 10^{-3}$) in order

1255   to contrast diversity persistence for different evolutionary scenarios (scenario E μ against

1256   scenario NE μ). 6 ANCOVAs were fitted with evolution as factor (two levels: "evolution"

1257   versus "no evolution") and intensity of warming as quantitative variable ($0,2,4,6,8,10,20K$).

1258   The model is written as follows:

1259

1260   <u>Model:</u> $DiversityPersistence_{ij} = \mu + \alpha_i + (\beta + \gamma_i)WarmingIntensity_{ij} + \varepsilon_{ij}$

1261   As indicated in **table S2**, index 1 (resp. index 2) corresponds to the level "no evolution" (resp.

1262   "evolution") so that α_2 corresponds to the direct effect of evolution ($\alpha_1 = 0$) and γ_2

1263   corresponds to the interaction term between evolution and warming or indirect effect of

1264   evolution ($\gamma_1 = 0$). β describes the direct effect of warming.

1265

An additional ANCOVA was fitted in order to compare the diversity persistence

according to the evolution rate μ (i.e. mutational frequency). Here, diversity persistence is

explained by the rate of evolution ($\mu = 10^{-1}, 10^{-2}, 10^{-3}$) as a three-level factor and the

intensity of warming (0,2,4,6,8,10,20 K) as a quantitative variable. The model is written as

follows:

<u>Model:</u> $DiversityPersistence_{ij} = \mu + \alpha_i + \beta * WarmingIntensity_{ij} + \varepsilon_{ij}$

A graphical visualization showed all hypothesis for ANCOVA were verified for each

fitted model. All statistical analyses were conducted using R-software (version 3.5.2). The

"Anova" function of the R-package "car" was also used.

Fig. S1: Consumer Resource model: the two-trait coevolution scenario

1276

Coevolution of body mass and feeding center   Zoom on the evolutionary rescue

Fig. S1: a. Coevolution of body mass and feeding center under two scenarios. **Scenario 1**: body
mass and feeding center evolve at rate $\mu = 10^{-2}$ (squares). **Scenario 2**: body mass evolves at
$\mu = 10^{-2}$ while feeding center evolves at $\mu = 10^{-3}$ (triangles). Unfilled grey triangle: initial
conditions. The simulations started at 280 K and a warming occurred (switch from grey symbols
to black symbols) to 316.4 K (almost maximal sustainable temperature). Light grey area is the
non-viability area at 316.4 K. The increase in symbol size indicates time direction. **b.** Zoom to
see the evolutionary rescue consecutive to warming. Parameter values are in **table S1**.

1284

1285

1286 **Table S2:** Diversity persistence in the community evolution model according to

1287 evolutionary scenarios: statistical analysis summary

1288

Mutational frequency	Direct effect of evolution		Effect of warming intensity		Interaction term		Adjusted R^2
	α_2	Effect size	β	Effect size	γ_2	Effect size	
$\mu = 10^{-1}$	0.051	32.3%	-0.029***	11.5%	0.031***	15.6%	58.6 %
	-0.026	27.1%	-0.037***	27.9%	0.036***	24.8%	79.4%
$\mu = 10^{-2}$	0.060*	28.4%	-0.024***	30.3%	0.017***	9.5%	67.6%
	0.126**	33.4%	-0.035***	22.6%	0.027***	8.9%	64.1%
$\mu = 10^{-3}$	0.062**	39.8%	-0.019***	18.5%	0.017***	13.1%	70.7%
	0.196***	37.3%	-0.029***	12.2%	0.023***	5.8%	54.4%

1289 **Table S2:** Outputs of the 6 ANCOVAs. Index 1 (α_1) corresponds to the level « No evolution »
1290 while index 2 (α_2) corresponds to the level « Evolution ». All models have a **p-value below**
1291 **2.10^{-16}.** For each value of mutational frequency, the first line corresponds to trait diversity
1292 persistence while the second line corresponds to species diversity persistence. Significance
1293 code: $<10^{-3}$ '***', $<10^{-2}$ '**', <0.05 '*