

# Anticipating critical transitions in epithelial-hybrid-mesenchymal cell-fate determination <sup>1</sup>

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## Abstract

In the vicinity of a tipping point, critical transitions occur when small changes in an input condition causes sudden, large and often irreversible changes in the state of a system. Many natural systems ranging from ecosystems to molecular biosystems are known to exhibit critical transitions in their response to stochastic perturbations. In diseases, an early prediction of upcoming critical transitions from a healthy to a disease state by using early warning signals is of prime interest due to potential application in forecasting disease onset. Here, we analyze cell-fate transitions between different phenotypes (epithelial, hybrid epithelial/mesenchymal (E/M) and mesenchymal states) that are implicated in cancer metastasis and chemoresistance. These transitions are mediated by a mutually inhibitory feedback loop microRNA-200/ZEB driven by the levels of transcription factor SNAIL. We find that the proximity to tipping points enabling these transitions among different phenotypes can be captured by critical slowing down based early warning signals, calculated from the trajectory of ZEB mRNA level. Further, the basin stability analysis reveals the unexpectedly large basin of attraction for a hybrid E/M phenotype. Finally, we identified mechanisms that can potentially elude the transition to a hybrid E/M phenotype. Overall, our results unravel the early warning signals that can be used to anticipate upcoming epithelial-hybrid-mesenchymal transitions. With the emerging evidence about the hybrid E/M phenotype being a key driver of metastasis, drug resistance, and tumor relapse; our results suggest ways to potentially evade these transitions, reducing the fitness of cancer cells and restricting tumor aggressiveness.

*Keywords:* critical transition | indicators of critical slowing down | alternative stable states | epithelial-hybrid-mesenchymal transition | cancer biology

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### 33 **Significance Statement**

34 Epithelial-hybrid-mesenchymal transitions play critical roles in cancer metastasis, drug resistance, and tumor  
35 relapse. Recent studies have proposed that cells in a hybrid epithelial/mesenchymal phenotype may be more  
36 aggressive than those on either end of the spectrum. However, no biomarker to predict upcoming transitions  
37 has been identified. Here, we show that critical slowing down based early warning signals can detect sudden  
38 transitions among epithelial, hybrid E/M, and mesenchymal phenotypes. Importantly, our results highlight  
39 how stable a hybrid E/M phenotype can be, and how can a transition to this state be avoided. Thus, our  
40 study provides valuable insights into restricting cellular plasticity en route metastasis.

### 41 **Introduction**

42 Biological systems often display nonlinear dynamics and emergent complex behavior, and consequent multi-  
43 stability [1, 2]. This nonlinear behavior in many cases leads to ‘tipping points’ - threshold values at which  
44 the system abruptly shifts from one state to another, in response to small stochastic perturbations [3]. Such  
45 changes - referred to as critical transitions - have been observed in multiple instances of ecosystems, climate,  
46 financial markets [4, 5, 6], and more recently in many cases of health and disease [7, 1]. The consequences of  
47 critical transitions are often large and undesirable, for instance, the switch from a healthy state to a diseased  
48 state such as the onset of type-2 diabetes [8] or that of depression [9]. Moreover, these transitions are often  
49 difficult to reverse, potentially due to self-reinforcing positive feedback [10], thus, predicting the ‘tipping  
50 points’ can be crucial for preventing such catastrophic changes.

51 A critical transition is usually identified after a tipping point and is difficult to predict beforehand,  
52 because the equilibrium state of the system stays relatively unchanged until the tipping point is reached  
53 [1]. Thus, static observations may not be sufficient to predict these abrupt transitions. Many indicators of  
54 changing system dynamics have been suggested as early warning signals (EWS) for the impending critical  
55 transitions and have been experimentally shown to predict transitions in alternative states in yeast cultures  
56 [11] and plankton chemostats [12]. The most important clues for EWS arise from critical slowing down of the  
57 system as it approaches the tipping point. At the onset of a tipping point, the rate of return of the system to  
58 the current equilibrium state upon a random disturbance decreases as the dominant eigenvalue approaches  
59 zero, and eventually, this equilibrium state is replaced by the alternative state. Thus, under conditions of  
60 critical slowing down, the state of the system at a given time becomes increasingly like that at a previous  
61 moment, leading to higher temporal autocorrelation. Similarly, due to moving into a shallower well closer  
62 to the bifurcation point, the variance in data is increased [6]. Hence, two canonical statistical measures that  
63 are mostly used as EWS to indicate the proximity of a system to a tipping point are increasing variance and  
64 temporal lag-1 autocorrelation - AR(1) [3]. Few other measures used as EWS are recovery rate/return time  
65 [13, 12], skewness [14], conditional heteroskedasticity [15], spectral reddening [16], likelihood ratio [17] and  
66 interaction network based indicators [18].

67 While EWS and critical transitions have been well-studied in ecological and climate systems, their appli-  
68 cation in predicting disease onset is relatively recent and remains largely conceptual [1, 10]. Particularly, in  
69 cancer, critical transitions have been predicted in metabolic reprogramming [19] - a hallmark of cancer [20].  
70 Here, we investigate critical transitions and EWS in another hallmark of cancer - invasion and metastasis.  
71 Metastasis - the spread of cancer cells from one organ to another - accounts for nearly all cancer related  
72 deaths in solid tumors [21]. Despite extensive genomic efforts, no specific mutational signatures have been yet  
73 identified for metastasis [22], thus limiting the druggable targets to restrict metastasis. Therefore, identifying  
74 tipping points for predicting and preventing metastasis can be beneficial in curbing tumor aggressiveness.

75 Most solid tumors originate in epithelial organs where cells do not typically migrate or invade, rather  
76 maintain tight cell-cell adhesion and a specific tissue organization. Thus, to metastasize, they typically  
77 undergo a phenotypic switch known as epithelial-mesenchymal transition (EMT) where they lose cell-cell  
78 adhesion and gain the traits of migration and invasion [23]. Cells undergoing EMT get launched into the  
79 bloodstream, and also gain the ability to initiate new tumors at metastatic sites, gain resistance against  
80 multiple drugs [24], and evade attacks by the immune system [25]. Thus, EMT provides multiple survival  
81 advantages to disseminated cells that typically undergo a mesenchymal-epithelial transition (MET) to col-  
82 onize distant organs. Recent investigations, including ours, have identified that EMT and MET need not  
83 be binary processes, instead cells can undergo partial EMT/MET and stably maintain one or more hybrid-  
84 epithelial/mesenchymal (E/M) phenotype(s) [23]. Importantly, cells in hybrid-E/M phenotype(s), i.e. those  
85 that undergo partial EMT, may be even more aggressive than cells that have undergone full EMT [26, 27].  
86 However, no specific biomarker has been identified that can a priori predict the onset of transitions among  
87 epithelial, mesenchymal and hybrid-E/M states. Thus, identifying EWS for transitions among these cell  
88 states can be a valuable contribution towards restricting them.

89 Here, we identify critical slowing down based EWS in a core regulatory network of EMT/MET. Three  
90 well known indicators - lag-1 autocorrelation, variance and conditional heteroskedasticity - work well to  
91 forewarn upcoming transitions among epithelial, hybrid-epithelial/mesenchymal, and mesenchymal states,  
92 thus opening the possibility of considering EWS as biomarkers to forewarn cancer metastasis. We also  
93 calculate the basin stability measure to evaluate the probability of occurrence of a particular state in various  
94 multistable regions. A higher basin stability measure corresponding to a particular state determines larger  
95 possibility of attaining the state in a multistable region. Complementing our basin stability measures with  
96 potential landscapes and phase diagrams for EMT circuit, we identify how a monostable hybrid E/M state  
97 can be maintained and thus suggest mechanisms to avoid it. Overall, our results highlight the ability to  
98 predict cellular transitions in metastasis before they occur and may provide a dynamic biomarker to gauge  
99 metastatic potential.

## 100 **Model**

101 We consider an analytical model of microRNA (miR) based chimeric circuit developed by Lu et al. [28]. The  
102 model incorporates the features of miR mediated regulation in the translation-transcription processes and

103 captures the formation of various miR-mRNA complexes by the binding/unbinding dynamics of miR and  
 104 mRNA (see Fig. 1A). The deterministic equations of the circuit which govern the combined dynamics of miR  
 105 ( $\mu$ ), mRNA ( $m$ ) and TF protein ( $B$ ) are given by:

$$\frac{d\mu}{dt} = g_\mu - mY_\mu - k_\mu\mu, \quad (1a)$$

$$\frac{dm}{dt} = g_m - mY_m - k_m m, \quad (1b)$$

$$\frac{dB}{dt} = g_B mL - k_B B, \quad (1c)$$

106 where  $g_\mu$  and  $g_m$  are the synthesis rates of  $\mu$  and  $m$ , respectively, and  $g_B$  is the translation rate of protein  
 107  $B$  for each  $m$  in the absence of  $\mu$ .  $k_\mu$ ,  $k_m$  and  $k_B$  are the degradation rates of  $\mu$ ,  $m$  and  $B$ , respectively.  $Y_\mu$ ,  
 108  $Y_m$  and  $L$  are  $\mu$  dependent functions [28] denoting various effects of microRNA-mediated repression.

109 The corresponding chimeric tristable miR-200/ZEB circuit is modeled as:

$$\frac{d\mu_{200}}{dt} = g_{\mu_{200}} H^s(Z, \lambda_{Z, \mu_{200}}) H^s(S, \lambda_{S, \mu_{200}}) - Y_{\mu_{200}} - k_{\mu_{200}} \mu_{200}, \quad (2a)$$

$$\frac{dm_Z}{dt} = g_{m_Z} H^s(Z, \lambda_{Z, m_Z}) H^s(S, \lambda_{S, m_Z}) - Y_{m_Z} - k_{m_Z} m_Z, \quad (2b)$$

$$\frac{dZ}{dt} = L - k_Z Z, \quad (2c)$$

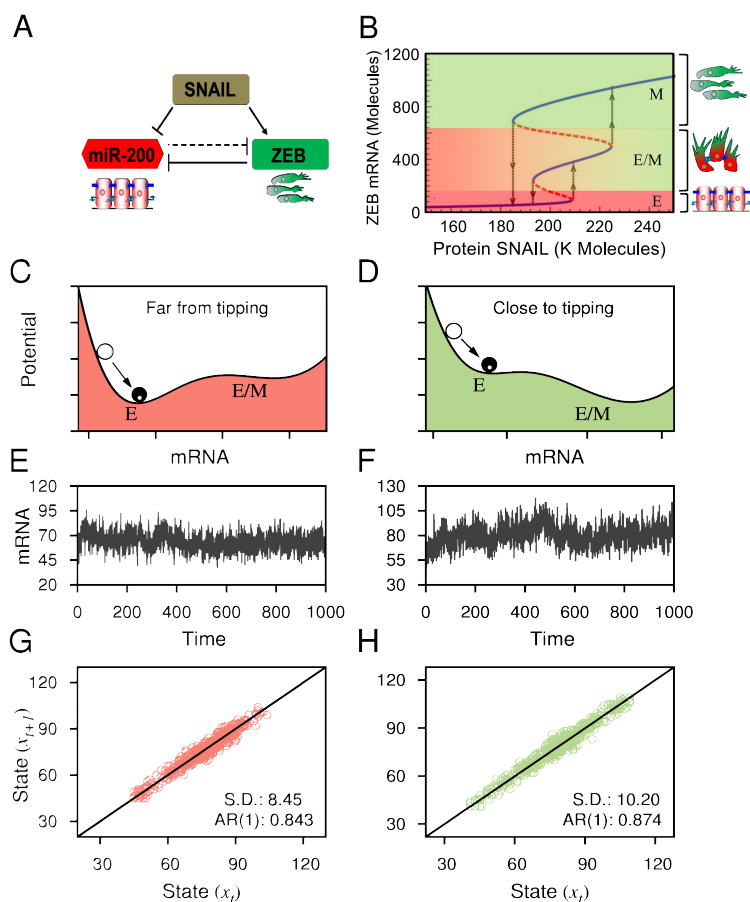
110 where  $H^s$  is the Hill function (details are in *SI Text, Sections 1 and 2*).

111 As a stochastic description of Eqs. (2) can accurately capture the dynamics of the system, we derive the  
 112 corresponding chemical Master equation which follows from birth-death processes [29]. The Master equation  
 113 is given by:

$$\begin{aligned} \frac{\partial p}{\partial t} = & g_{\mu_{200}}(Z) (p(\mu_{200}^0 - 1, m_Z, Z) - p(\mu_{200}^0, m_Z, Z)) + g_{m_Z} (p(\mu_{200}^0, m_Z - 1, Z) - p(\mu_{200}^0, m_Z, Z)) \\ & + k_{m_Z} ((m_Z + 1)p(\mu_{200}^0, m_Z + 1, Z) - m_Z p(\mu_{200}^0, m_Z, Z, \mu_0)) \\ & + k_Z ((Z + 1)p(\mu_{200}^0, m_Z, Z + 1) - Zp(\mu_{200}^0, m_Z, Z)) \\ & + k_{\mu_{200}} ((\mu_{200}^0 + 1)p(\mu_{200}^0 + 1, m_Z, Z) - \mu_0 p(\mu_{200}^0, m_Z, Z)) \\ & + L(\mu_{200}^0, m_Z) (p(\mu_{200}^0, m_Z, Z - 1) - p(\mu_{200}^0, m_Z, Z)) \\ & + \sum_{j=0}^n (\Lambda_{j m_Z}(\mu_{200}^0, m_Z + 1)p(\mu_{200}^0, m_Z + 1, Z) - \Lambda_{j m_Z}(\mu_{200}^0, m_Z)p(\mu_{200}^0, m_Z, Z)) \\ & + \sum_{j=0}^n (\Lambda_{j \mu_{200}}(\mu_{200}^0 + j, m_Z)p(\mu_{200}^0 + j, m_Z, Z) - \Lambda_{j \mu_{200}}(\mu_{200}^0, m_Z)p(\mu_{200}^0, m_Z, Z)). \quad (3) \end{aligned}$$

114 where  $p(\mu_{200}^0, m_Z, Z)$  is the grand probability function. The Eq. (3) is a birth-death process for the proba-  
 115 bilities of the separate states specified by the values of  $(\mu_{200}^0, m_Z, Z)$ . All the terms appear in the equation  
 116 as pairs: (i) birth of a state  $(\mu_{200}^0, m_Z, Z)$  due to transition from other states  $(\mu_{200}^0, m'_Z, Z')$ , and (ii) death

117 due transition from  $(\mu_{200}^0, m_Z, Z)$  into other states. There are ten such processes associated with birth and  
 118 death of miR, mRNA and ZEB in our model (see also *SI Text, Section 3* for details). We have simulated  
 119 this Master equation with Gillespie algorithm [30] to obtain the stochastic trajectory of the system (*SI Text,*  
 120 *Section 3A*). The stochastic trajectory of the system identifies the occurrence of critical transition between  
 121 different phenotypes and using critical slowing down based EWS we are able to forecast such transitions  
 122 beforehand.



**Figure 1.** (A) Schematic diagram of the microRNA-based chimeric circuit. (B) Bifurcation diagram depicting the changes in ZEB mRNA levels with variations in the levels of SNAIL. E, hybrid E/M and M denote epithelial state, hybrid epithelial/mesenchymal state and mesenchymal state, respectively: lowest levels of ZEB mRNA correspond to epithelial state, intermediate levels to a hybrid E/M state, and highest ones to mesenchymal state, as shown by corresponding cartoons. (C-F) An overview of critical transition in the circuit which has multistability. Schematic potential landscapes representing two stable states (i.e. E and hybrid E/M) of deterministic system: (C) high resilience of the E state when it is far from the tipping point, and (D) low resilience of the state close to a tipping point, when the system approaches a sudden shift from E to hybrid E/M state. Stochastic time series of the system (2): (E) with  $S=197K$  (far from the tipping point) and (F) with  $S=207K$  (close to the tipping point), respectively. (G, H) In the vicinity of a tipping point, due to decreasing resilience the system has stronger memory for perturbation in comparison to that of far from a tipping point and that are characterized by larger standard deviation (S.D.) and lag-1 autocorrelation (AR(1)). All other parameters for the circuit are given in SI.

## 123 Results and Discussion

### 124 *Bifurcation-induced tipping signs in epithelial-hybrid-mesenchymal transition*

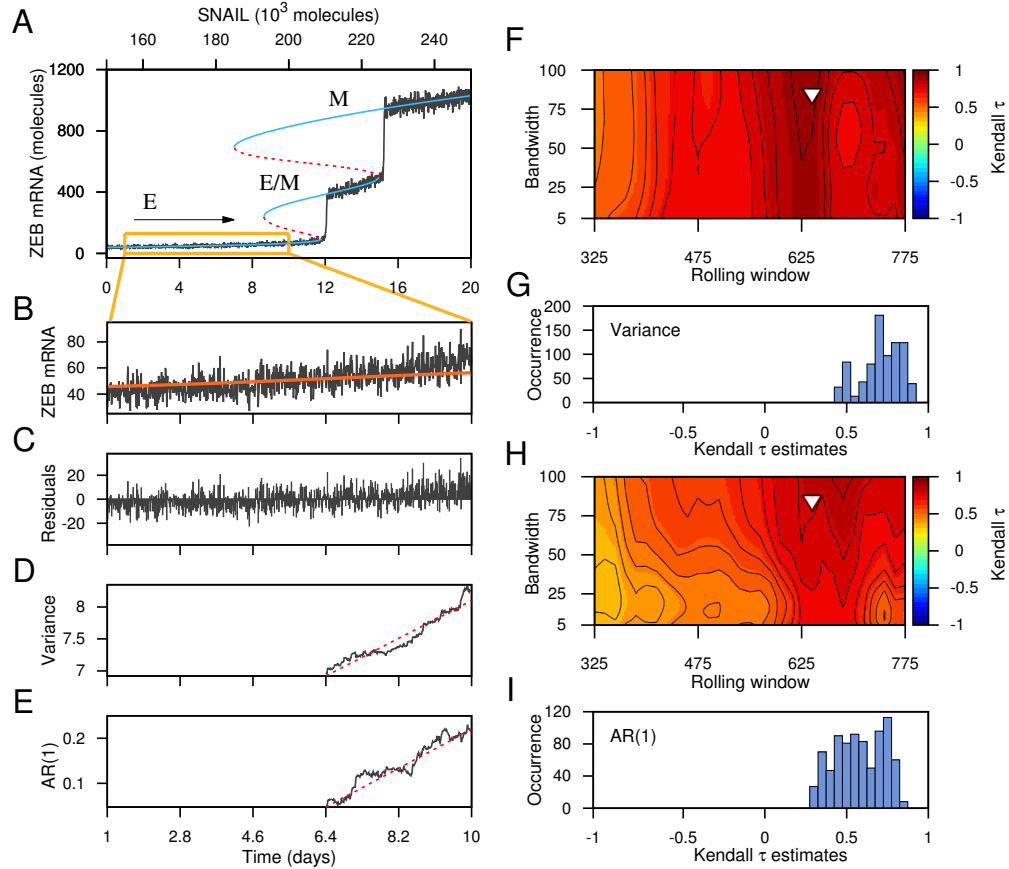
125 A mutually inhibitory feedback loop between members of ZEB transcription factor and those of microRNA  
 126 (miR)200 has been postulated to govern EMT/MET; ZEB can drive EMT by inhibiting cell-cell adhesion

127 and cell polarity, while miR-200 tend to maintain an epithelial phenotype [23]. Unlike mutually inhibiting  
128 feedback loops where both players are transcription factors, this loop is chimeric, i.e. it contains both tran-  
129 scriptional and translational regulation [28, 31]. First, we perform the bifurcation analysis of this determinis-  
130 tic tristable chimeric circuit Eqs. (2) with variations in the SNAIL concentration (S) (see Fig. 1B). The values  
131 of all the other model parameters of this circuit are presented in the *SI Text, Table S1-S3*. We denote three  
132 coexisting stable states: (high miR-200/low ZEB), (low miR-200/high ZEB), and (medium miR-200/medium  
133 ZEB). These states correspond to epithelial (E) and mesenchymal (M), and hybrid-epithelial/mesenchymal  
134 (E/M) phenotypes respectively [32, 23]. For increasing levels of S, the circuit first exhibits monostable E  
135 state; an increase in S leads to bistability between E and M states; a further increase enables tristability  
136 between E, hybrid-E/M and M states; then bistability between hybrid-E/M and M states, and finally a  
137 monostable M state. The existence of multistable regions includes the appearance of saddle-node bifurca-  
138 tions and hysteresis loops that triggers the possibility of occurrence of catastrophic critical transitions in the  
139 presence of intrinsic stochastic perturbations [33].

140 Since this feedback loop exhibits tristability, it may pass through two critical points (or tipping points)  
141 and, therefore can reach two alternative states, one after another. A systematic analysis of such critical  
142 transition is commonly done by analysing stochastic trajectory. In Fig. 1, we have presented a brief overview  
143 of critical transition in the EMT circuit from pure E to hybrid-E/M phenotype transition with variations in  
144 the levels of protein SNAIL, when the system is far from or close to a tipping point (see Figs. 1C,D). More  
145 specifically, larger variance and increased lag-1 autocorrelation determine the proximity to a tipping point  
146 (see Figs. 1G,H). With increasing SNAIL value the system may experience two subsequent transitions, one  
147 from E to hybrid-E/M state and another from hybrid-E/M to M state. However, while decreasing SNAIL  
148 value results in a direct transition from M to E state which bypasses the hybrid-E/M state.

#### 149 *Early warning signals for transitions among epithelial, hybrid-E/M and mesenchymal states*

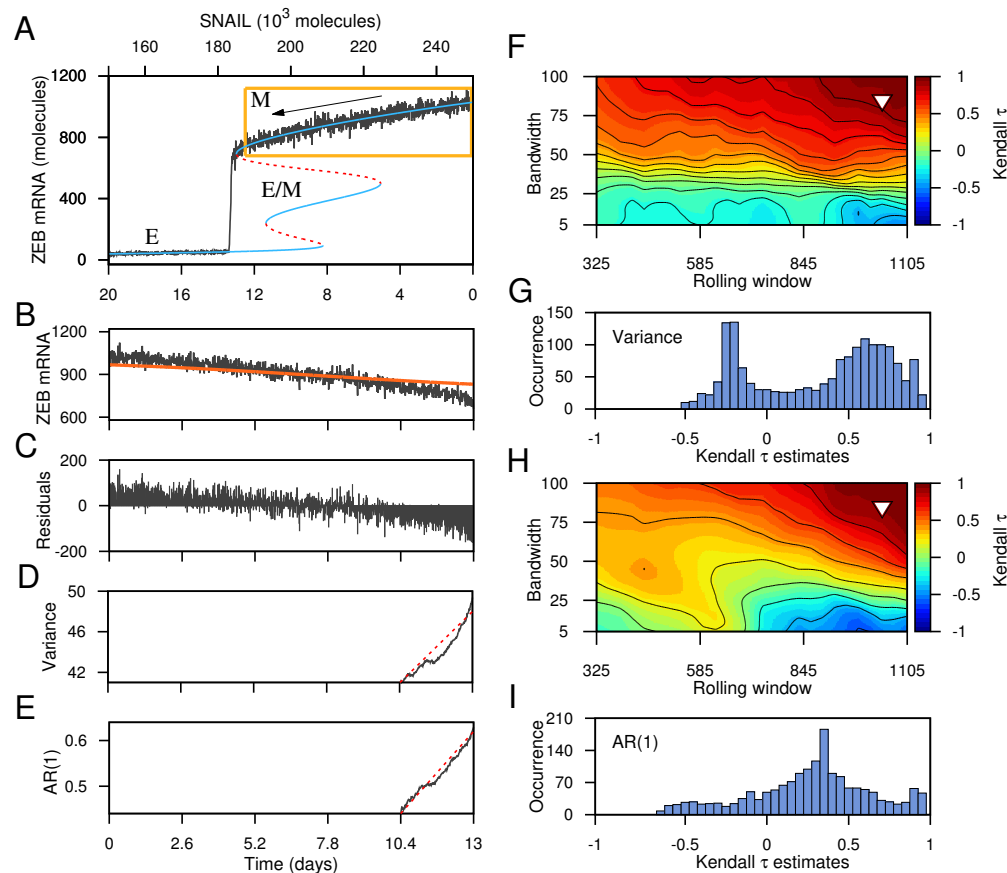
150 We began our search for signals of critical slowing down by calculating EWS of critical transitions in data  
151 sets obtained from stochastic simulations (see *Materials and Methods section*) of the chimeric circuit. The  
152 stochastic trajectory (time series) representing ZEB mRNA levels, with continuously increasing SNAIL value,  
153 exhibits sudden transitions from E state to hybrid-E/M state and further hybrid-E/M state to M state (See  
154 Fig. 2A). The trajectory is generated with time varying signal SNAIL. The SNAIL level starts at 150K  
155 molecules at day 0 and then increases upto 250K molecules at day 20. This increase in SNAIL levels can  
156 drive EMT in a cell, i.e. moving from monostable epithelial region to a monostable mesenchymal region  
157 (Fig. 1B), and the timescale over which SNAIL levels are varied are commensurate with those over which  
158 EMT is observed [34, 35].



**Figure 2.** Critical transitions between different cell states of the regulatory circuit that are driven by *forward* change in the control parameter SNAIL, and indicators of critical slowing down. (A) Transitions from E state to hybrid E/M state and hybrid E/M state to M state. (B) Stochastic time series segment of the system before the transition to hybrid E/M state (a segment as indicated by the boxed region in (A)). (C) Residual time series after applying Gaussian filter (red curve in (B)) is the trend used for filtering). EWS calculated from the filtered time series after using a rolling window of 60% of the data length: (D) variance and (E) AR(1). (F-I) Sensitivity analysis of the filtering bandwidth and the rolling window size used to calculate the EWS. Contour plots reveal the effect of variable rolling window size and filtering bandwidth on the observed trend in the EWS, (F) variance and (H) AR(1), for the filtered data as measured by the Kendall- $\tau$  value. The triangles indicate the rolling window size and bandwidth used to calculate the EWS. Frequency distributions of Kendall- $\tau$  values for (G) variance and (I) AR(1).

159 First we evaluate the effectiveness of different EWS to positively alarm an impending sudden catastrophic  
 160 transition from E state to hybrid-E/M state, by tracking the values of ZEB mRNA. For EWS analysis, we  
 161 consider a time series segment before the transition to hybrid-E/M state (see Fig. 2B). To filter possible  
 162 non-stationarities in the data we subtracted a Gaussian kernel smoothing function across the time series  
 163 segment and used the remaining residuals (Fig. 2C) for EWS analysis [36]. We calculate the variance and  
 164 lag-1 autocorrelation (AR(1)) (see *SI Text, Section 4*) values with a rolling window having a length of 60%  
 165 the length of the residual time series segment and found both the variance and AR(1) value to be increasing  
 166 (see Figs. 2D-E). A concurrent increase in the EWS is an well known indicator of an upcoming critical  
 167 transition [3, 5]. The performance of EWS is in general known to be sensitive to the choice of the filtering  
 168 bandwidth used in Gaussian kernel smoothing and also on the rolling window size [37, 38]. The bandwidth of  
 169 kernel smoothing determines the degree of data smoothing without filtering the low frequencies from the data  
 170 and the choice of rolling window size depends on a trade-off between data resolution and reliability of the

171 estimation of EWS. Therefore, rather than choosing arbitrary values, here we perform sensitivity analysis, of  
 172 the filtering bandwidth and rolling window size (see Figs. 2F-I). For sensitivity analysis the rolling window  
 173 size was varied from 25% to 75% of the data length in increments of 15 points, together with variations in  
 174 the filtering bandwidth ranging from 5 to 100 in increments of 10. For all possible combinations of these two  
 175 parameters, the observed trends in variance and AR(1) were quantified using the non-parametric Kendall's  
 176  $\tau$  rank correlation coefficient. A positive Kendall's  $\tau$  determines increasing trend in the EWS prior to a  
 177 critical transition. To maximise the estimated trends for the EWS, we have used the sensitivity plot to select  
 178 a particular filtering bandwidth and window size (see Fig. 2F for variance and Fig. 2H for AR(1)) (for details  
 179 see *SI Text, Section 4B*). The frequency distributions of the Kendall's trend statistic for the variance and  
 180 the AR(1) are presented in Fig. 2G, I, respectively.



**Figure 3.** Critical transition between different cell states of the regulatory circuit that is driven by *backward* change in the control parameter SNAIL, and indicators of critical slowing down. (A) Transition from M state to E state that bypasses the hybrid E/M state. (B) Stochastic time series segment of the system before the transition to E state (a segment as indicated by the boxed region in (A)). (C) Residual time series after applying Gaussian filter (red curve in (B) is the trend used for filtering). EWS calculated from the filtered time series after using a rolling window of 80% of the data length: (D) variance and (E) AR(1). Contour plots reveal the effect of variable rolling window size and filtering bandwidth on the observed trend in the EWS, (F) variance and (H) AR(1), for the filtered data as measured by the Kendall- $\tau$  value. The triangles indicate the rolling window size and bandwidth used to calculate the EWS. Frequency distributions of Kendall- $\tau$  values for (G) variance and (I) AR(1).

181 The EWS work well for capturing the transition from hybrid-E/M state to M state (see *SI Text, Section 5*),



182 suggesting that transitions in the forward direction (i.e. increase in SNAIL) can be captured by stochastic  
183 time series of ZEB mRNA. We generate the stochastic time series of ZEB mRNA from the probabilistic  
184 model through the Monte Carlo simulations [30] which incorporates intrinsic cellular noise. We vary both  
185 the time and the parameter (the number of SNAIL molecules) together, which carries the signature of critical  
186 slowing down while shifting to an alternative stable state. We carried out our simulations for a period of 0  
187 to 20 days along with the simultaneous variations in the number of SNAIL molecules, that varies from 150K  
188 to 250K molecules.

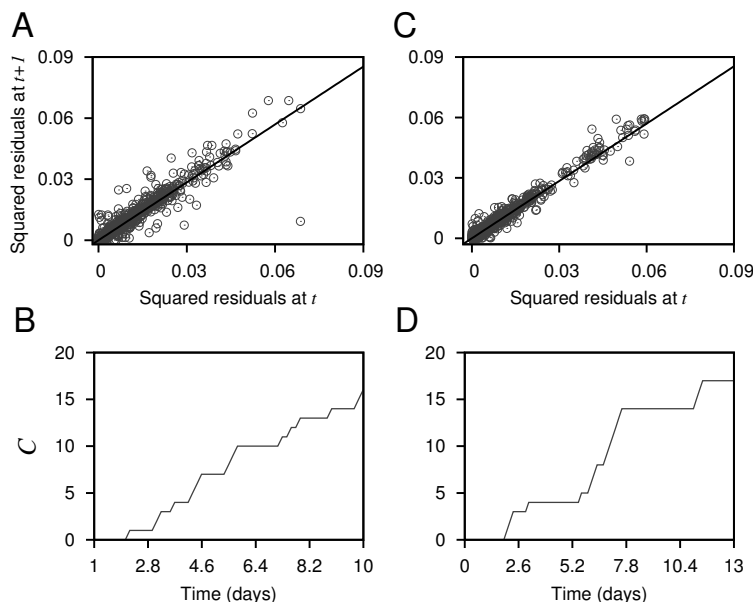
189 Next, we investigated whether these EWS can also be observed in backward transitions, i.e. with decreas-  
190 ing value of SNAIL (Fig. 3). Due to the hysteresis and asymmetry in transitions in both directions (E to M  
191 vs. M to E), we observe sudden direct transition from M to E state (see Fig. 3A) bypassing the hybrid-E/M  
192 state. We consider a time series segment prior the transition to E state (Fig. 3B) and further used the  
193 residual time series for EWS analysis (Fig. 3C). Importantly, both the EWS markers - variance and AR(1)  
194 - shown an increasing trend closer to the tipping point for this transition from M to E (see Figs. 3D-E).  
195 Reinforcing our previous analysis, these EWS were evaluated with specific choices of detrending bandwidth  
196 and rolling window size to maximise their trends. Put together, these results highlight that the transitions  
197 among E, hybrid-E/M and M states can be predicted before they occur, using EWS variance and AR(1).

198 Further, for the aforementioned three transitions, E to hybrid E/M, hybrid E/M to M and M to E state,  
199 we evaluate the robustness of EWS trends to all the rolling window sizes depicted as the distribution of  
200 the Kendall- $\tau$  statistic around their median, for both the 'original' and 'surrogate' time series (see *SI Text*,  
201 *Section 6* and *SI Fig. S2*). In the case of 'original' data sets, most of the trends for AR(1) and variance are  
202 robust to rolling window sizes as majority of the associated box-plots stays above the  $y$ -zero axes [39].

### 203 *Conditional heteroskedasticity applied as early warning signals*

204 To evaluate robustness of the predictions made by the EWS variance and AR(1), we calculate conditional  
205 heteroskedasticity - one of the other measures known to forewarn critical transitions [15]. Conditional het-  
206 eroskedasticity is indicated by the persistence in the conditional variance of the error term in time series  
207 models [40]. The advantage of this indicator over others is that it minimizes the chance of the occurrence  
208 of false positive signals in time series that does not have any critical transition. To calculate conditional  
209 heteroskedasticity, time series is modelled as an auto-regressive process and the residuals are obtained. The  
210 persistence of the conditional variance of the residuals then determine the conditional heteroskedasticity  
211 (see *SI Text, Section 4C* for details of the procedure). Prior to a critical transition, significant conditional  
212 heteroskedasticity is expected to be visible in the time series [15].

213 We consider the time series segments before the critical transitions for both the cases; E to hybrid-E/M  
214 transition and M to E transition (see Fig. 2B and Fig. 3B). Figure 4A presents the squared residuals from an  
215 auto-regressive lag-1 model applied to the time series segment of E to hybrid-E/M transition (Fig. 2B) plotted  
216 with the residuals at the next time step. The slanted line is the regression line. The positive correlation  
217 between the squared residuals at time step  $t$  and time step  $t + 1$  indicates conditional heteroskedasticity. We



**Figure 4.** (A, C) Squared residuals from an autoregressive lag-1 model plotted with the next squared residuals and (B, D) cumulative number of significant Lagrange multiplier test ( $C$ ), both applied to the data presented in Fig. 2B (for A, B) and Fig. 3B (for C, D), respectively. In (A, B), the black slanted lines are fitted regression lines at lag-1.

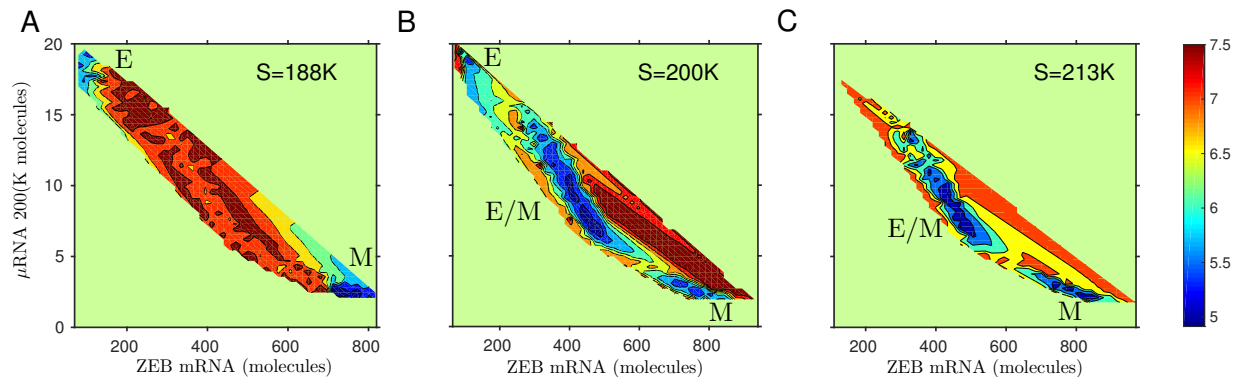
218 also apply the cumulative number of significant Lagrange multiplier test ( $C$ ) to the time series (Fig. 4B).  
 219 The cumulative increases prior to the critical transition indicating that significant number of tests shows  
 220 conditional heteroskedasticity in the time series. For the transition to M to E state, we get similar result  
 221 (Fig. 4C, D).

#### 222 *Stochastic potential and basin stability analyses reveal relative stability of the three cell states*

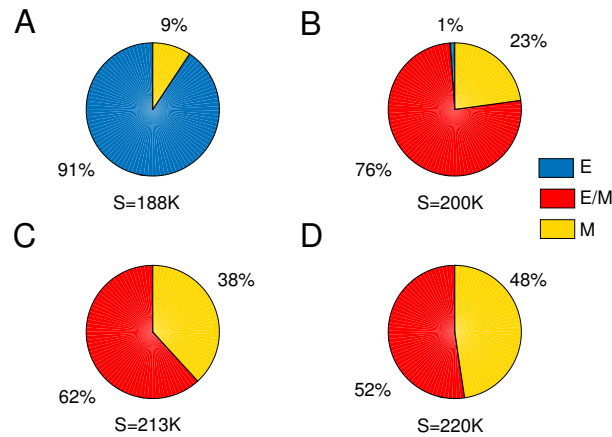
223 For a dynamical system, a potential well represents the existence of a steady state. Here, we projected the  
 224 stochastic potential of the system in ZEB mRNA –  $\mu$ RNA200 plane for different values of the parameter  
 225 SNAIL (Fig. 5). The lowest value of the potential corresponds to the existence of a deep well and hence  
 226 subsequently the existence of a steady state. Here, for different SNAIL values, the stochastic potentials clearly  
 227 exhibit bistable/multistable states. Consistent with the deterministic dynamics of the system (Fig. 1B), we  
 228 note the co-existence of E and M states (Fig. 5A), the co-existence of all the three E, hybrid-E/M and M  
 229 states (Fig. 5B), and the co-existence of hybrid-E/M and M states (Fig. 5C). The details of the method used  
 230 to calculate the stochastic potentials are given in the *SI Text, Section 7*.

231 Given that the hybrid-E/M state has been proposed to be the 'fittest' for metastasis [41] and that we  
 232 observed a relatively larger region denoting the stability of hybrid-E/M in the tristable region (Fig. 5), we  
 233 investigated the probability of attaining the hybrid-E/M state in a tristable region in the presence of random  
 234 perturbations. This probability can be calculated by performing basin stability measure [42].

235 For a complex system, basin stability is a measurement of the stability/resilience of a steady state in a  
 236 probability sense which pivots on the volume of the basin of attraction. In other words, it measures the  
 237 likelihood of return to a steady state after random, non-small perturbations. Thus, for a high-dimensional

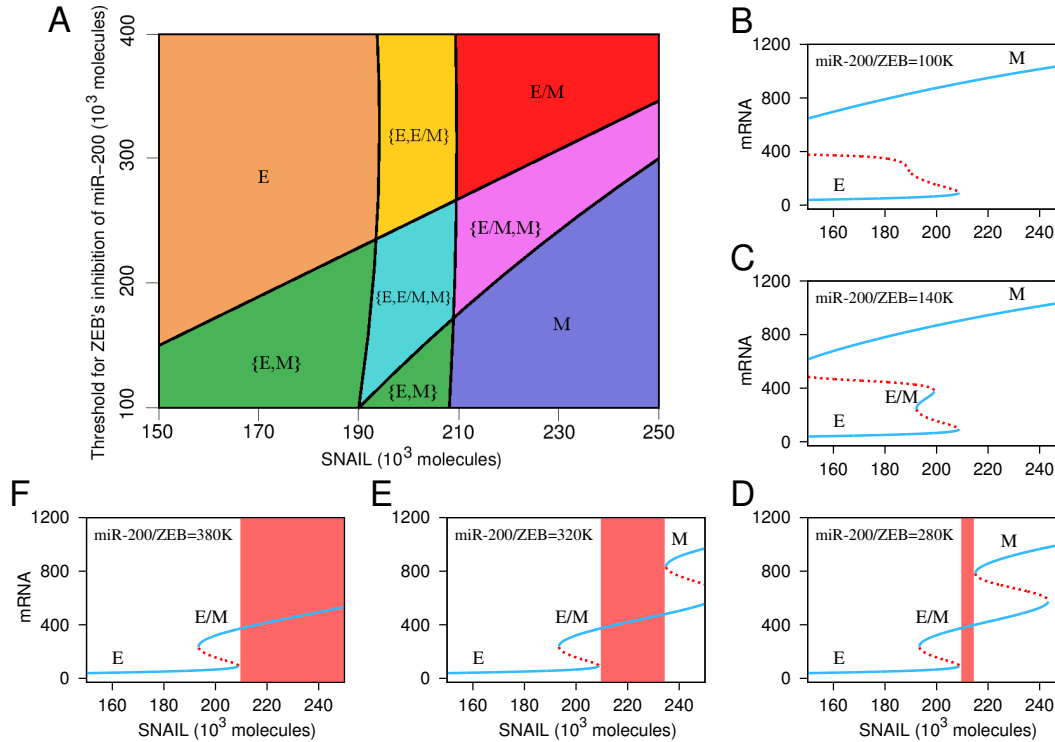


**Figure 5.** The potential landscapes of the genetic circuit in two-dimensional mRNA- $\mu$ RNA plane for different values of SNAIL (S). The blue regions represent lower potential and correspondingly higher probability of occurrence of a steady state. Identifying the existence of: (A) epithelial and mesenchymal states at S=188K, (B) epithelial, hybrid epithelial-mesenchymal and mesenchymal states at S=200K, and (C) hybrid epithelial-mesenchymal and mesenchymal states at S=213K.



**Figure 6.** Pie diagrams representing basin stability of the system for different values of SNAIL (S): (A) S=188K, (B) S=200K, (C) S=213K and (D) S=220K. The percentage of  $10^4$  simulations with random initial conditions reaching to a particular steady state in a bistable/multistable region. Blue, red and yellow regions correspond to the % of simulations reaching to any one of the E, hybrid-E/M and M states, respectively.

238 multistable system, it is a powerful tool to measure the basin volume (see *SI Text, Section 8*). For our  
 239 system, we observe multistability for different parameter values of SNAIL(S). For S=188K (see Fig. 1B), the  
 240 system has coexisting E and M states. Basin stability measures that for a sufficiently large set of random  
 241 initial conditions, E and M states have probabilities 0.91 and 0.09 of return to their original state, i.e. among  
 242 all random initial conditions 91% and 9% trajectories will reach E and M states (Fig. 6A), respectively. For  
 243 S=200K, system have probabilities 0.1, 0.76 and 0.23 of reaching to E, hybrid-E/M and M states (Fig. 6B),  
 244 respectively from a set of random initial states. Similarly for S=213K, the corresponding probabilities of  
 245 return to hybrid-E/M state and M state are 0.62 and 0.38 (Fig. 6C), respectively. Further, increase in the  
 246 levels of SNAIL at S=220K reduces the probability of attaining hybrid-E/M state which becomes 0.52 and  
 247 remaining 0.48 is the probability of attaining the M state (Fig. 6D), indicating that as we proceed from a



**Figure 7.** The phase diagram and corresponding bifurcation diagrams of the genetic circuit. (A) The phase diagram of the genetic circuit with variations in SNAIL and miR-200/ZEB levels. Each phase corresponds to either any of the monostable state or coexisting bistable/multistable states. For example, in E the epithelial state is stable, in {E, hybrid-E/M} both epithelial and hybrid-epithelial mesenchymal states coexist. (B-F) Bifurcation diagrams of mRNA with variations in the level of SNAIL for different values of miR-200/ZEB: (B) 100K, (C) 140K, (D) 280K, (E) 320K and (F) 380K. As we move from (D) to (F), the monostable region for hybrid-E/M state increases.

248 bistable M- E/M phase to a monostable M phase, the basin stability of E/M decreases, being conceptually  
 249 consistent with the mean residence time analysis for this circuit [43].

250 Hence, the basin stability results suggest that an E state is more stable in bistable region containing both  
 251 E and M states, but the hybrid E/M state is more stable for the two later cases. Thus, in the (miR-200/ZEB)  
 252 loop, chances of getting a hybrid E/M state seems relatively very high compared to the other two states  
 253 (see *SI Fig. S3*). This result is reminiscent of mean residence times calculations for E, hybrid-E/M and M  
 254 states [43], and suggests that hybrid-E/M state is not perhaps as ‘metastable’ as was initially postulated  
 255 experimentally [23].

#### 256 *Identifying mechanisms to evade the transition into aggressive hybrid E/M state*

257 Next, we sought after mechanisms to evade transition to a hybrid-E/M state, given its association with higher  
 258 aggressiveness and worse patient survival. We first identified what mechanisms can lead to stabilized hybrid-  
 259 E/M state. So far, our results have identified monostable E, monostable M, and other bistable and tristable  
 260 regions, but not a monostable hybrid-E/M state. Including other factors such as GRHL2, NUMB in the  
 261 network can enable the existence of a monostable hybrid-E/M region [27]. Here, we analyzed the parameter  
 262 space of the miR-200/ZEB feedback loop to identify regions enabling the existence of a hybrid-E/M state as

263 a monostable phase, without adding more components in the network. We varied the levels of SNAIL, and  
264 the threshold (half-maximal concentration) value of ZEB in the shifted Hill function corresponding to ZEB  
265 inhibiting miR-200, and calculated the phase diagram shown in Fig. 7. The different phases in the diagram  
266 are separated by four saddle-node bifurcation curves. We could identify a large parameter region in which the  
267 monostable hybrid-E/M phase appears - high levels of both SNAIL and the threshold of ZEB (see Fig. 7A).  
268 This result suggests that as the strength of inhibition of miR-200 by ZEB is weakened, the progression to a  
269 complete EMT may be halted and cells can stably occupy a hybrid-E/M state for higher values of SNAIL  
270 (Fig. 7A). Conversely, as this inhibition is made stronger, the stability of the hybrid-E/M state gradually  
271 decreases (Fig. 7C) and eventually the hybrid-E/M state disappears (Fig. 7B). Here, the hybrid-E/M state  
272 disappears when the systems response curve changes from 'folded' to 'smooth', in response to the variations  
273 in the input condition. In fact the folded response curve looks like a typical first-order (i.e. abrupt) or  
274 discontinuous transition (Fig. 7C), however contains two unstable states and one stable hybrid-E/M state  
275 which in general shows two stable and one unstable states in most of the studies on critical transitions  
276 [44, 45]. The smooth response curve corresponds to second-order (i.e. continuous) phase transition that has  
277 only one unstable state here (Fig. 7B), in contrast to a bistable system which has a stable state. Therefore  
278 the dynamical mechanism behind the disappearance of the hybrid-E/M state is the changeover from first-  
279 to second-order phase transition in the systems response curve.

280 Similarly, we also varied the levels of SNAIL and the threshold of self-activation of ZEB (see *SI Text*,  
281 *Section 9*). Reduced threshold, i.e. stronger self-activation, enable a monostable hybrid E/M phase at  
282 lower SNAIL values, while it disappears with increased threshold, i.e. weaker self-activation of ZEB (see *SI*  
283 *Fig. S4*). Increasing SNAIL values and weakening self-activation drive the system towards a bistable E, M  
284 phase, i.e. disappearance of E/M state. These results suggest that a balance between strengths of mutual  
285 inhibition and self-activation can enable the existence of a hybrid E/M phenotype [31].

## 286 Discussion

287 Anticipating critical transitions remains an extremely challenging task in multiple scenarios including eu-  
288 tropication of lakes, crash of financial markets, and more importantly, in onset of disease. The system  
289 typically displays almost no sign of the impending transition until it happens, thus using early warning sig-  
290 nals (EWS) such as variance, autocorrelation and conditional heteroskedasticity can be used to forecast the  
291 critical transitions which are often catastrophic. Here, we show that these EWS can capture the transitions  
292 among epithelial, mesenchymal and hybrid epithelial/mesenchymal phenotypes. This phenotypic plasticity  
293 drives cancer metastasis and drug resistance in cancer - the cause of almost all cancer-related deaths. Given  
294 that no unique mutational signature has been yet identified for metastasis, despite extensive genomic efforts,  
295 these EWS that can predict the onset of these cellular transitions that govern metastasis can serve as poten-  
296 tially important dynamic biomarkers. Recent efforts have focused on identifying such dynamic biomarkers  
297 in the context of pulmonary metastasis of hepatocellular carcinoma [46]. With more single-cell dynamic  
298 data emerging in the context of epithelial-hybrid-mesenchymal transitions [47], using EWS signals can help

299 predict the tipping point of metastasis initiation.

300 Cancer metastasis has been long thought to be driven solely by individual cell migration (i.e. a mes-  
301 enchymal state), however, recent studies have questioned this dogma, highlighting that not only clustered  
302 cell migration can be possible during metastasis, but also it can be the predominant driver of metastasis  
303 [48, 49]. These clusters, typically 5-8 cells large, can pass through capillaries by arranging themselves tran-  
304 siently into a single-file chain [50], and can contain non-cancerous cells that can facilitate metastasis [51].  
305 A hybrid-E/M phenotype has been associated with such collective/clustered cell migration [52, 53], thus,  
306 our analysis identifying the relatively high basin stability of the hybrid-E/M phenotype can help explain the  
307 ability of cancer cells to form clusters of circulating tumor cells.

308 Here, our analysis focused on temporal dynamics of a gene regulatory network for EMT; however, EWS  
309 have also been identified in spatiotemporal dynamics, particularly in ecology [54, 55]. Thus, EWS can also  
310 be potentially identified in a spatially extended regulatory networks for EMT, for instance, investigating  
311 the varying extents of EMT induction in different parts of a tissue [56, 57] or identifying critical transitions  
312 in cancer-immune interplay [58]. Further, besides EMT, there are other axes of phenotypic plasticity in  
313 cancer, such as metabolic plasticity, switching back and forth between a cancer stem cell (CSCs) and a  
314 non-cancer stem cell. With recent developments in identifying the multistable dynamics of the networks  
315 regulating these transitions [59]. EWS analysis can be applied to these networks to identifying promising  
316 novel dynamic biomarkers. However, an open question remains: can we identify the strongest and most  
317 robust signal of critical transition, among many which might show EWS? For instance, during metastasis,  
318 players involved in EMT, CSCs, and metabolic plasticity may all show EWS and vary dynamically, but  
319 which among these interconnected axes can be considered as the Achilles' heel of metastatic potential needs  
320 to be identified rigorously?

321 Majority of the critical slowing down based EWS used to predict critical transitions in natural systems  
322 involves saddle-node bifurcation under the presence of white noise (temporally uncorrelated noise) that per-  
323 turbs the abundance of the system [60]. For a large class of systems that exhibit other type of bifurcations  
324 apart from the saddle-node, the effectiveness of EWS remains largely unknown. For different type of bi-  
325 furcations (e.g. transcritical, pitchfork, supercritical Hopf bifurcation) with diverse noise (e.g. coloured  
326 (temporally correlated) noise) EWS do not always work reliably to forecast sudden critical transitions [61].  
327 They found to be very sensitive to the length of pre transition time series data, and also to other decisions  
328 like filtering bandwidth and rolling window size [38]. There also exist systems in which bifurcations occur  
329 without critical slowing down, such as in a structured consumer resource model where the upper point equi-  
330 librium coexists with a lower limit cycle [62], occurrence of basin boundary collisions [63] and as a result in  
331 these systems EWS do not work properly. In fact in general EWS work well for the situations when critical  
332 transition and critical slowing down co-occur [61]. Although robustness of EWS have been successfully shown  
333 in some cases [3, 60], a detail analysis of their effectiveness is still an open challenge [17, 64, 61].

334 In summary, our analysis strongly indicates the presence of EWS during epithelial-hybrid-mesenchymal

335 transitions - a central motor of cellular plasticity during cancer metastasis and emergence of therapy resis-  
336 tance [65]. We show that many robust measures of EWS - increased variance, autocorrelation and conditional  
337 heteroskedascity - vary dynamically as cells transition among these three phenotypes. Our results also iden-  
338 tify increased basin stability of a hybrid-E/M phenotype - considered to be the 'fittest' for metastasis - and  
339 suggests ways how to elude transitions into the hybrid-E/M state, potentially restricting cancer spread.

## 340 **Materials and Methods**

### 341 *Numerical simulations and bifurcation diagrams of the deterministic system*

342 We have used Matlab (R2015b) for numerical simulations of the deterministic system (Eq. 2). The codimension-  
343 one bifurcation diagrams involving two or more saddle-node bifurcation points were obtained using the  
344 continuation package MATCONT [66]. The two parameter bifurcation diagram (i.e. the phase diagram)  
345 with variations in the parameters SNAIL and miR-200/ZEB was obtained through the calculations of mul-  
346 tiple codimension-one bifurcations points. Later, the bifurcation curves separating monostable, bistable  
347 and tristable existence regions of the steady states were presented by connecting multiple codimension-one  
348 bifurcations points.

### 349 *Stochastic system and Monte Carlo simulations*

350 The time series of ZEB mRNA levels was generated from the probabilistic model through Monte Carlo  
351 simulations [30] which incorporates the intrinsic cellular noise. The algorithm considers each of the reaction  
352 events as individual realisations of Markov process. The time and species numbers are updated stochastically  
353 by choosing a random reaction event. The miR( $\mu$ ) based chimeric tristable miR-200/ZEB circuit is simulated  
354 by realising ten reaction events as a function of the number of SNAIL molecules. The reaction events are  
355 listed in the *SI Table S4*. All biochemical parameters are based on [32] and those are listed in the *SI*  
356 *Table S1, S2 and S3* for completeness. Both the time and the parameter (number of SNAIL molecules) are  
357 varied together to obtain the time series of ZEB mRNA levels that carries the signature of critical slowing  
358 down while shifting to an alternative stable state. In particular, we perform our simulations for a period of  
359 20 days along with the simultaneous variations in the number of SNAIL molecules, that ranges from 150K  
360 to 250K molecules. The chosen time period and the range of SNAIL molecules are in consistent in the  
361 context of epithelial to mesenchymal transition period [32]. More details of the simulation is presented in  
362 the *SI Text, Section 3*.

### 363 *Statistical analysis of CSD indicators*

364 In the stochastic time series analysed here, we first visually identified shifts between E to E/M state and M  
365 to E state. Then we took time series segments (the regions marked with boxes in Figs. 2 and 3) prior to a  
366 critical transition and examined them for the presence of EWS. For stationarity in residuals, we used Gaussian  
367 detrending before performing any statistical analysis of the data. The residuals were then used to calculate  
368 the EWSs variance, lag-1 autocorrelation and conditional heteroskedasticity. The time series analysis have  
369 been performed using the "Early Warning Signals Toolbox" (<http://www.early-warning-signals.org/>). A

370 concurrent rise in the variance and/or lag-1 autocorrelation forewarn an upcoming regime shift. The indicator  
371 conditional heteroskedasticity also works similarly (for details see *SI Text, Section 4*).

### 372 **Author contributions**

373 S.K.S., H.L., M.K.J., and P.S.D. designed research; S.S., S.K.S., M.K.J., and P.S.D. performed research; S.S.,  
374 S.K.S., H.L., M.K.J., and P.S.D. analyzed data; and S.K.S., M.K.J., and P.S.D. wrote the paper.

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